THE UNIVERSITY OF MICHIGAN

SYSTEMS ENGINEERING LABORATORY

Department of Electrical Engineering

SEL Technical Report no. 41

METHODS OF IMPROVING TRANSIENT STABILITY OF A SYNCHRONOUS GENERATOR

Chaman N. Kashkari

July 1969

sponsored by

Consumers Power Company

Jackson, Michigan

ACKNOWLEDGEMENTS

The author wishes to express his appreciation to Professors J. J. Carey, C. L. Dolph, W. D. Getty, K. B. Irani, E. L. McMahon and R. F. Mosher for serving on his doctoral committee. In particular, the author wishes to record his debt of gratitude to:

Professor J. J. Carey, Chairman of the committee, without whose inspiring influence and invaluable guidance, this work would not have been possible.

Professor R. F. Mosher for his active help and close supervision at all stages of this work.

Professor K. B. Irani for his encouragement, inspiration and assistance.

Professor K. Chuang for many fruitful discussions during the course of this research.

Sincere thanks are due to Professor A. J. Pennington, formerly of the University of Michigan, for his assistance in the early stages of the author's graduate work.

Special thanks are due to Miss Joyce Doneth for her assistance in preparing the manuscript.

The support given by Consumers Power Company for this research is appreciated.

Finally, the author is indebted to his wife for her patience, understanding, and continued encouragements.

TO SHEILA AND MEERA

TABLE OF CONTENTS

List of Symbols

A_1 - A_6	Gain in Excitation System Auxiliary Signal Channels
E_d	Direct-Axis Component of Excitation Voltage
E'_d	Direct-Axis Component of Transient Internal Voltage
E_{ex}	Exciter Voltage Referred to Armature Circuit
E_i	Voltage Behind Transient Reactance
E_q	Voltage Behind Synchronous Impedance
E'_q	Voltage Proportional to Field Flux Linkage
E_{qd}	Voltage Behind Quadrature-Axis Synchronous Reactance
E_{qo}	Steady State Value of E_q
E_x	Exciter Field Voltage
E_1	Voltage Behind Transient Reactance
E_2	Infinite Bus Voltage
G	Machine Rating
H	Inertia Constant
i_d	Direct-Axis Component of Armature Current
i_f	Current in Machine Field Winding
i_q	Quadrature Axis Component of Armature Current
K_1, K_2	Design Parameters in Auxiliary Signals Channels
L_f	Inductance of Field Winding
M	Inertia Constant of the Machine
N	Slope of the Phase Trajectory

P_1, P_2, P_i, P	Input Power
P_{m1}, P_{m2}, P_{m3}	Amplitudes of Prefault, During Fault and Post Fault Power Angle Curves
P_u, P_o	Output Power
r_a	Armature Resistance
r_f	Resistance of Field Winding
S_d	Direct Axis Saturation Factor
S_q	Quadrature Axis Saturation Factor
T'_{do}	Open Circuit Transient Time Constant
T_e	Exciter Field Constant
T'_{qo}	Quadrature Axis Transient Open-Circuit Time Constant
v_b	Busbar Voltage
v_d	Direct Axis Component of Terminal Voltage
v_{dm}	Direct Axis Component of Machine Terminal Voltage
v_{dp}	Direct Axis Component of Potier Voltage
v_m	Machine Terminal Voltage
v_{qm}	Quadrature Axis Component of Machine Terminal
v_{qp}	Quadrature Axis Component of Potier Voltage
v_q	Quadrature Axis Component of Terminal Voltage
v_r	Reference Voltage
ω	Frequency in Radians per Second

List of Symbols (Continued)

x_d	Direct Axis Synchronous Reactance
x'_d	Direct Axis Transient Reactance
x_e	External Reactance
x_p	Potier Reactance
x_q	Quadrature Axis Synchronous Reactance
x'_q	Quadrature Axis Transient Reactance
δ	Torque Angle
δ_o	Steady-State Torque Angle
ϕ	Magnetic Flux
μ_v	Gain in the Voltage Channel

List of Figures

List of Figures (Continued)

List of Figures (Continued)

ABSTRACT

This work is an investigation of three techniques for enhancing transient stability of a synchronous machine connected to an infinite bus through parallel transmission lines. These techniques are as follows:

1. Application of Auxiliary Signals in Excitation Systems

   This research includes an analytical study of the effects of auxiliary signals used in Excitation Systems. These signals are proportional to derivatives of torque angle. A very interesting conclusion of the study is that if the signals are of correct magnitude, then the oscillations will be strongly damped. The mathematical techniques used in this study can be applied while designing an excitation system. These techniques are based on Phase Plane Analysis.

2. Use of Phase-Shifting Transformers

   Another interesting method for controlling transients utilizes phase-shift windings in step-up transformers. This research is concerned with an analytical study of the effect of phase-shifting transformers on the transient stability of a Synchronous Generator. Since the power is carried by the phase-shift winding during transients only, it can be designed for short time rating with resultant economy.

3. Switched Series Capacitors

   An experimental and an analytical investigation has been made on switched series capacitors. The analytical study includes some application problems associated with the capacitors. The results indicate that it is technically feasible to use switched series capacitors in long transmission lines.

Chapter I

SCOPE OF INVESTIGATION

Introduction

Since the early days of alternating current power generation and transmission, engineers have been faced with the problem of power system stability. During the past few years this problem has assumed new importance due to system interconnections, which have created complex networks having load areas far away from the generating plants. The Northeast blackout of November 9, 1965, created a new awareness of this problem in the minds of the public, the government and the electric utilities. The effects of the blackout were so profound that the utilities cannot afford to have another such occurence. Engineers must therefore search for new techniques to improve the reliability of electric power supply.

Definitions

According to the American Standards Association[1], stability with reference to power systems is defined as follows:

> "Stability when applied to a system of two or more synchronous machines through an electrical network, is the condition in which the difference of angular positions of the machines either remains constant while not subject to a disturbance or becomes constant following an aperiodic distrubance.
>
> "Transient Stability is a condition which exists in a power system if, after an aperiodic disturbance, the system regains its steady-state stability.
>
> "Steady State Stability is a condition which exists in a power system if it operates with stability when not subjected to an aperiodic disturbance."

It will be observed that depending upon the nature of the disturbance, stability can be classified in the following categories:

1. Steady-state-stability under slow or gradual load change.
2. Transient-stability-when the system is subjected to sudden and large magnitude disturbance.

By means of an automatic voltage regulator, it is possible to create stability of a generator under conditions for which the generator is inherently unstable. Regulator induced generator stability is referred to as "Dynamic Stability".

Methods of Improving Stability

During system planning, a stability study is required in order to ensure that it will be able to ride through the dynamic oscillations following any system disturbance and maintain stability. The power capability of long, interregional tie lines is usually limited by the transient stability limits. Such economic factors as the high cost of long lines and the additional revenue obtainable from the delivery of additional power provides a strong incentive to explore all economically and technically feasible means of raising the stability limits.

The methods of increasing the stability limits may be classified as follows:

1. Switching
2. Excitation systems
3. Compensation
4. Energy control.

1. Switching includes rapid clearing of faults, increasing the number of intermediate switching stations in the case of two or more parallel lines and the use of reclosing breakers. These methods have been traditionally used by power system engineers.

2. Excitation systems. Fast excitation systems have considerable effect on stability, particularly dynamic stability. There is however considerable interest in the development of faster excitation systems which would improve both dynamic and transient stability.

3. Compensation. By compensation is meant the introduction of series capacitors or shunt reactors.

4. Energy control. This includes braking resistors and rapid reduction of input power by fast valve action, on the prime mover.

Scope of Investigation

The aim of this research is to develop new techniques for improving transient stability of a synchronous generator, connected to an infinite bus by means of a double circuit transmission line. The following research was carried out:

(1) Application of Auxiliary Signals

The main function of an excitation system is to regulate the machine terminal voltage. A voltage signal proportional to the difference between the magnitudes of reference voltage and the terminal voltage is applied to the field circuit of a control exciter, thereby correcting the terminal voltage. The possibility of using signals other than the terminal voltage as input signals to the regulator has been mentioned recently.[2-5] These investigators have reported that the transient performance of the synchronous generator is improved if additional signals are applied to the excitation system.

This research is concerned with an analytical study of the fundamental effects of auxiliary signals, proportional to derivative of torque angle δ, on the transient performance of a synchronous generator. The study is based on phase plane techniques of nonlinear control systems. Previous investigations of excitation system problems involved either linearizing of the differential equations or use of empirical methods. By the approach followed in this report, expressions for appropriate signals are developed in a straightforward manner. The gains in various feedback channels can be determined from the knowledge of system parameters. This research lays foundations for future research on excitation system design.

(2) Use of Phase-Shifting Transformers

An interesting method is developed which may have considerable application in long distance transmission of power. This involves the use of auxiliary windings in the sending-end step-up transformer. This winding provides phase shift. During a transient, power flows through this winding, instead of the normal winding, with resultant damping of the oscillations and enhancement of the stability limit. Since the auxiliary winding carries power during the transient only, it can be short-time rated with resultant economy.

(3) Use of Switched-Series Capacitors

Kimbark[6] and Smith[12] have suggested inserting capacitors in the lines to compensate for the increase in line reactance due to switching out of a line section. The author has extended this idea by an experimental and theoretical study of a typical installation using switched series capacitors. Some application problems, inherent in use of switched series capacitors have been taken into consideration, thus establishing the feasibility of using this technique in modern installations.

Chapter II

MATHEMATICAL REPRESENTATION OF THE SYSTEM

Introduction

This study is based on the classical work of Park[7] who developed a mathematical model for an ideal synchronous machine. Details of the assumptions on which the model is based and the derivation of the equations are given in Appendix I. A simplifying approximation in the model is that the magnetic saturation has been neglected. In this chapter the machine equations have been suitably modified to account for saturation. This chapter also gives a description of the system under consideration and develops mathematical models for the components so that the system can be represented by a set of differential equations. The equations derived here have been used in Chapters III to V, for digital simulation of the system, to investigate the effects of feeding auxiliary signals to the excitation system.

Description of the System

The elementary power system which has been considered for the purpose of analysis is shown in figure 2.1. It consists of a salient-pole synchronous generator, connected to an infinite bus by a double circuit transmission line. This is a classical configuration for such a study.

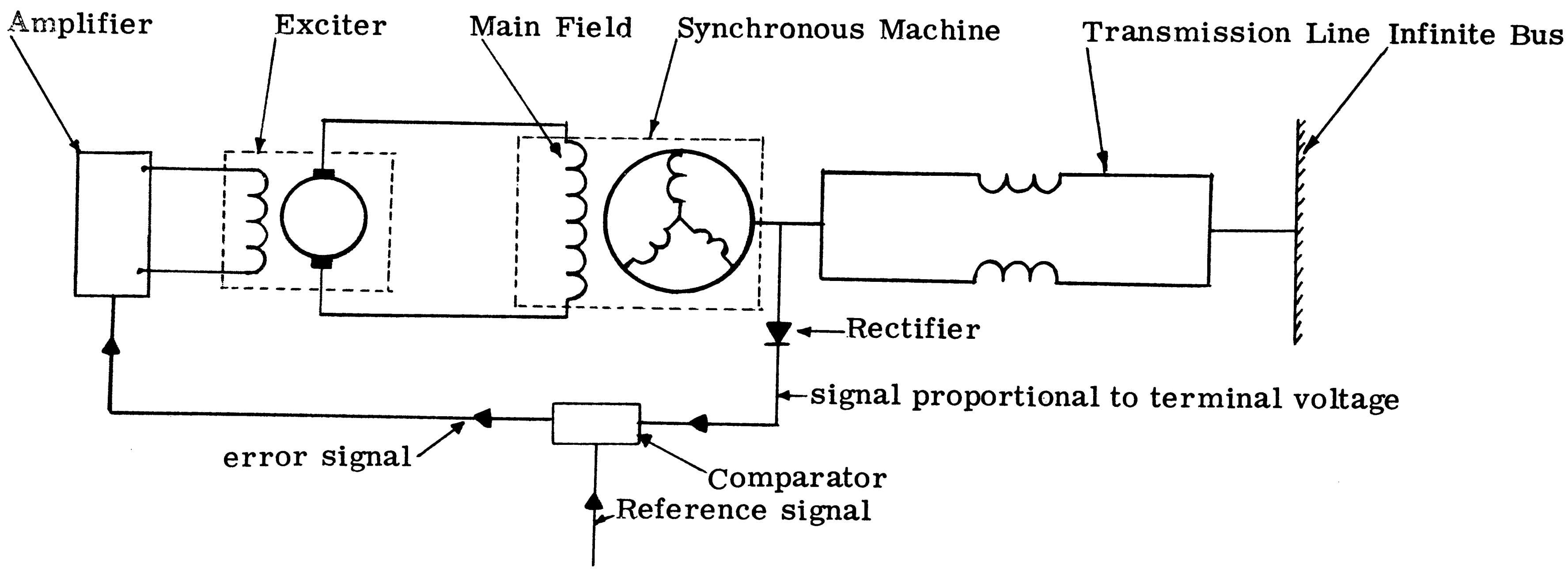


Fig 2. 1 Elementary Power System

Machine Equations and Saturation

The equations of an ideal synchronous machine have been developed in Appendix I. It will now be necessary to modify these equations so that saturation can be taken into account. Many methods, all approximate, have been given in literature to include saturation. The method used here is that outlined by Olive[8] and is based upon a correction of the flux linkages within the machine. It is approximate in that leakage flux has been assumed known for stator circuits only and has been assumed zero for rotor circuits. The saturation factor S_d or S_q at any specified voltage for the direct or quadrature axis, is defined as

$$\frac{\text{Field current to give that voltage on the magnetization curve}}{\text{Field current to give that voltage on the airgap line}} - 1$$

The direct axis saturation factor may be obtained directly from the machine open circuit curve corresponding to the value of the quadrature axis component of the Potier voltage or at

$$v_{qp} = v_q + r_a i_q + i_d x_p$$

where

$$S_d = f(v_{qp})$$

v_{qp} is the quadrature axis component of Potier voltage, v_q is the quadrature axis component of terminal voltage, r_a is the armature resistance, i_q is the quadrature axis component of current, i_d is the direct axis component of current, x_p is the Potier reactance and f is a symbol for a certain function.

The value of the quadrature axis saturation factor may also be obtained from the same open-circuit curve by multiplying the value found at the quadrature axis component of Potier voltage by the ratio x_q/x_d, where x_q and x_d are quadrature axis and direct axis synchronous reactances of the machine.

$$v_{dp} = v_d + r_a\, i_d - x_p\, i_q$$

where v_{dp} is the direct axis component of Potier voltage and v_d is the direct axis component of terminal voltage.

$$S_q = \frac{x_q}{x_d}\, f(v_{dp})$$

The equations of a synchronous machine as developed in Appendix I are

$$v_q = -r_a\, i_q - x_d\, i_d + E_q \qquad (2.1)$$

$$v_d = -r_a\, i_d + x_q\, i_q + E_d \qquad (2.2)$$

$$E'_q = -(x_d - x'_d)\, i_d + E_q \qquad (2.3)$$

$$E'_d = (x_q - x'_q)\, i_q + E_d \qquad (2.4)$$

$$\frac{d}{dt} E'_q = \frac{1}{T'_{do}} [E_{ex} - E_q] \qquad (2.5)$$

$$\frac{d}{dt} E'_d = -\frac{E_d}{T'_{qo}} \qquad (2.6)$$

In the above equations, E_q and E_d are the quadrature and direct axis components of excitation voltage. E'_q and E'_d are the quadrature and direct-axis components of transient internal voltage. x'_q and x'_d are the quadrature and direct axis transient reactances. T'_{qo} and T'_{do} are the quadrature and direct axis open circuit time constants. E_{ex} is the exciter voltage referred to the armature circuit.

Since the machine under consideration is a salient pole synchronous machine ($x_q = x'_q$), $E'_d = E_d = 0$ and if the resistance terms are neglected, the equations reduce to

$$v_q = E_q - i_d x_d \tag{2.7}$$

$$v_d = x_q i_q \tag{2.8}$$

$$E'_q = E_q - (x_d - x'_d)\, i_d \tag{2.9}$$

$$\frac{d E'_q}{dt} = \frac{[E_{ex} - E_q]}{T'_{do}} \tag{2.10}$$

If saturation is included, the equations modify to the following

$$v_q = E_q - [\frac{x_d - x_p}{1 + S_d} + x_p] i_d \tag{2.11}$$

$$v_d = [\frac{x_q - x_p}{1 + S_q} + x_p] i_q \tag{2.12}$$

$$E'_q = E_q - \frac{(x_d - x'_d)}{(1 + S_d)}\, i_d \tag{2.13}$$

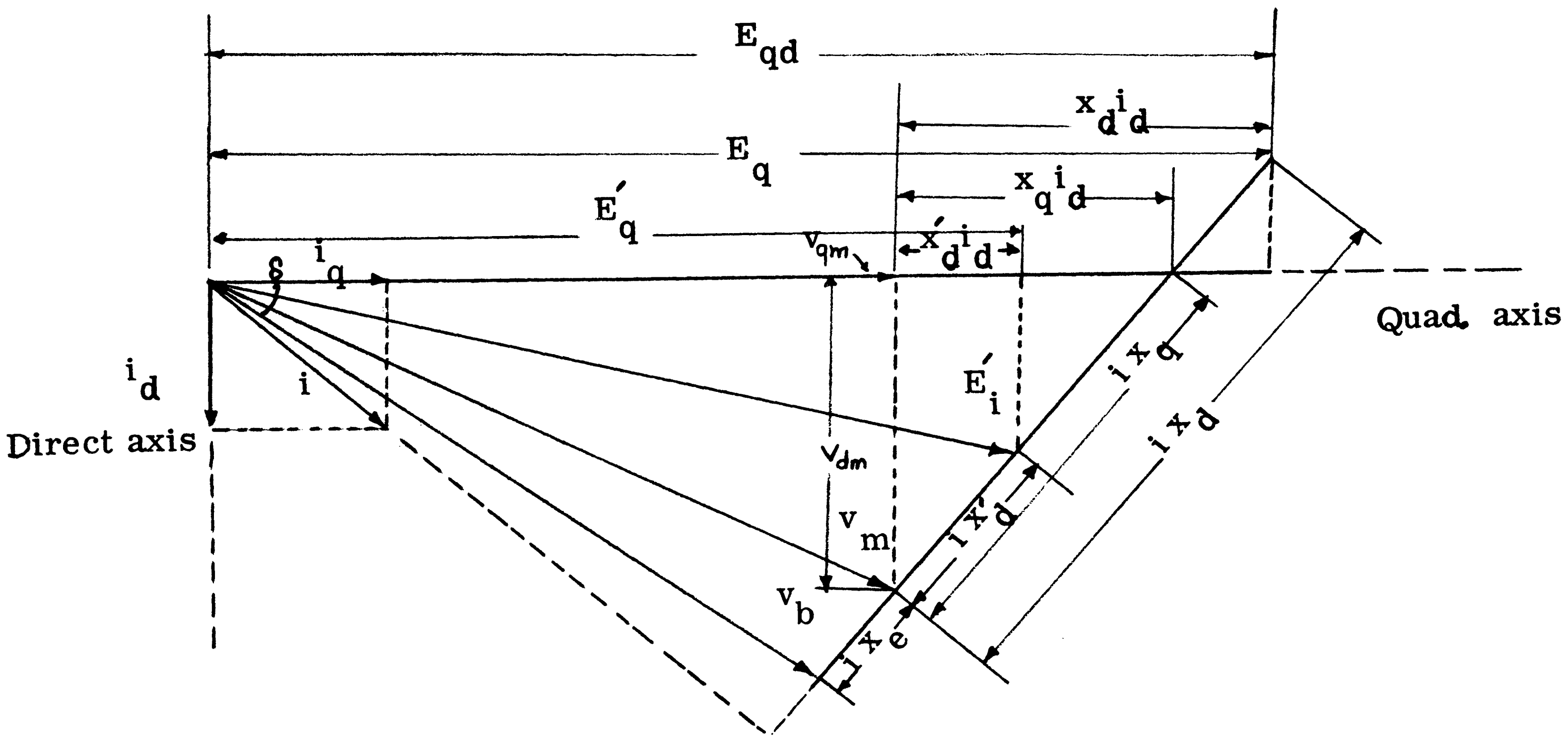


Fig. 2.2 Phasor Diagram of the System

$$\frac{d E'_q}{dt} = \frac{1}{T'_{do}} [E_{ex} - (1+S_d) E_q] \tag{2.14}$$

A phasor diagram showing the above situation is shown in figure 2.3.

Power Output Relationships (Neglecting Saturation)

The power output from a salient pole machine during a transient is given by

$$P_u = \frac{v_b E'_q}{x'_d + x_e} \sin \delta + v_b^2 \frac{(x'_d - x_q)}{2(x_q + x_e)(x'_d + x_e)} \sin 2\delta \tag{2.15}$$

where v_b is the busbar voltage, δ is the angle between the infinite bus and the quadrature axis of the rotor and x_e is the external reactance.

Since the rate of change of E'_q involves E_q, it would be convenient to relate E_q and E'_q by certain known quantities as follows. Refer to figure 2.2

$$E_{qd} - i_d(x_q + x_e) = v_b \cos \delta \tag{2.16}$$

where E_{qd} is the voltage behind the quadrature-axis synchronous reactance. Therefore

$$i_d = \frac{E_{qd} - v_b \cos \delta}{(x_q + x_e)} \tag{2.17}$$

But

$$E'_q = E_{qd} - i_d(x_q - x'_d) \tag{2.18}$$

Substituting equation (2.17) in equation (2.18),

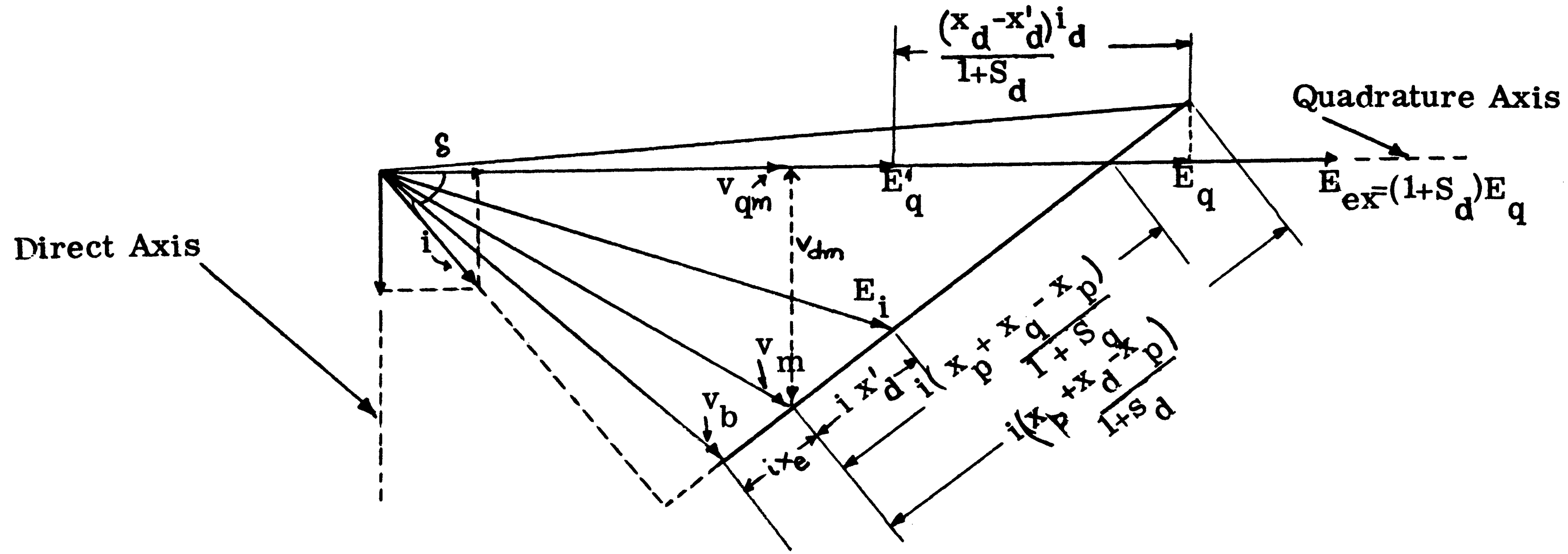


Fig. 2. 3 Phasor Diagram of the System with Saturation

$$E'_q = E_{qd} - \frac{E_{qd} - v_b \cos \delta}{x_q + x_e}(x_q - x'_d)$$

$$= E_{qd} \frac{(x'_d + x_e)}{(x_q + x_e)} + v_b \cos \delta \frac{(x_q - x'_d)}{(x_q + x_e)} \tag{2.19}$$

Again from the figure,

$$E_q = E_{qd} + i_d(x_d - x_q) \tag{2.20}$$

and

$$E_{qd} - E'_q = i_d (x_q - x'_d) \tag{2.18}$$

From the above equation,

$$i_d = \frac{E_{qd} - E'_q}{x_q - x'_d}$$

Substituting this expression for i_d in eq. (2.20), the result is

$$E_q = E_{qd} \frac{(x_d - x'_d)}{(x_q - x'_d)} - E'_q \frac{(x_d - x_q)}{(x_q - x'_d)} \tag{2.21}$$

From (2.19) and (2.21),

$$E_q = E'_q \frac{(x_d + x_e)}{(x'_d + x_e)} - v_b \cos \delta \frac{(x_q - x'_d)}{(x'_d + x_e)} \tag{2.22}$$

During a stability study, it will be necessary to use the equation involving rate of change of E'_q, namely

$$\frac{d E'_q}{dt} = \frac{1}{T'_{do}} [E_{ex} - E_q] \tag{2.5}$$

Since E_q is known from eq. (2.22), it will be necessary to know E_{ex}, the machine field voltage, which is also the exciter armature voltage. It is therefore, necessary to develop a mathematical model for the exciter as will be shown in the next section.

<u>Exciter Representation</u>

Figure 2.4 represents an exciter with a separately excited field winding. E_x is voltage applied to the field and E_{ex} is its armature voltage. Usually the machine voltage is regulated by controlling the exciter field voltage E_x.

If r_f, L_f are the resistance and the inductance of the exciter field winding and i_f is the field current, then the following relationships hold.

$$r_f\, i_f + L_f \frac{di_f}{dt} = E_x \tag{2.23}$$

$$E_{ex} = k\, \phi = k' i_f \tag{2.24}$$

where ϕ is the magnetization flux and k and k' are constants. The flux ϕ is proportional to i_f, as saturation in the exciter field is neglected. This is an acceptable approximation, since the ceiling voltage of modern exciters is many times the normal voltage.

From eqs. (2.23) and (2.24)

$$i_f = \frac{E_x}{r_f + p\, L_f}\ , \quad \text{p is operator } \frac{d}{dt}$$

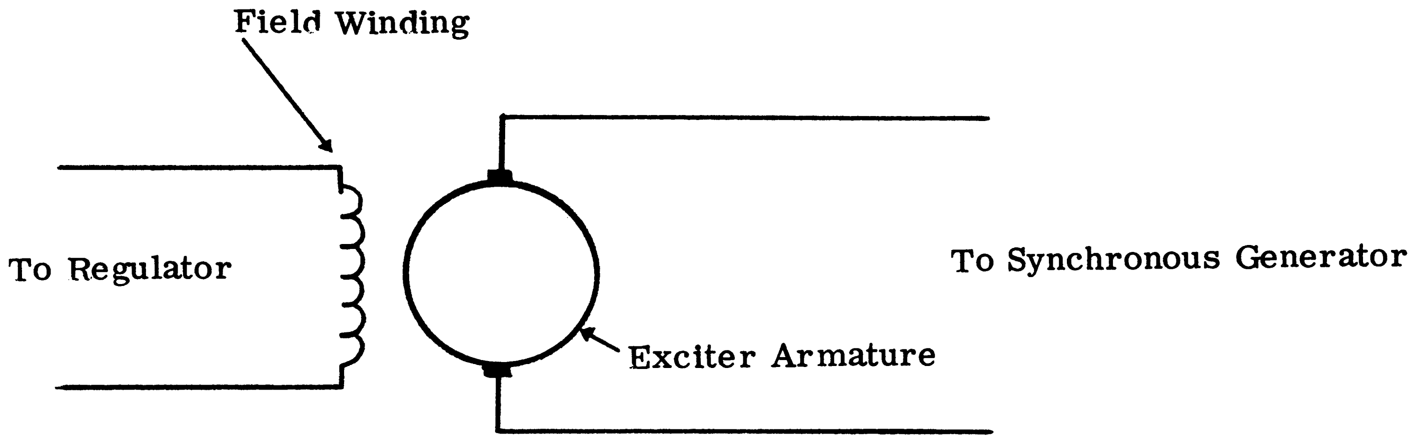


Fig. 2. 4 An Exciter for a Synchronous Generator

$$E_{ex} = \frac{k'E_x}{r_f + pL_f} = \frac{k'E_x}{r_f(1 + p\,T_e)}\,,\ T_e \text{ is time constant of the field winding}$$

If E_{ex} and E_x are in per unit system, then

$$E_{ex} = \frac{E_x}{1 + p\,T_e} \tag{2.25}$$

or

$$\frac{d\,E_{ex}}{dt} = \frac{1}{T_e}(E_x - E_{ex}) \tag{2.26}$$

E_x, the exciter field voltage will consist of a steady state component and a transient component proportional to the difference between terminal voltage and reference voltage, or

$$E_x = E_{xo} - \mu_v(v_m - v_r) \tag{2.27}$$

where μ_v is gain in voltage channel. From eq. (2.26) and eq. (2.5) the steady state component of E_x is equal to E_{qo}, the steady-state value of E_q. In any excitation control with auxiliary signals eq. (2.27) will be

$$E_x = E_{qo} - \mu_v\,(v_m - v_r) + f[\text{auxiliary signals}] \tag{2.28}$$

where f is a certain function.

Terminal Voltage

In eqs. (2.27) and (2.28), v_m is the machine terminal voltage and it is necessary to find an expression for v_m in terms of the known quantities. Referring to figure 2.2,

$$v_{dm} = v_b \sin\delta\,\frac{x_q}{x_q + x_e} \tag{2.29}$$

$$v_{qm} = v_b \cos\delta + i_d x_e \tag{2.30}$$

But

$$i_d = \frac{E_q - v_b \cos\delta}{x_e + x_d} \tag{2.31}$$

Therefore

$$v_{qm} = v_b \cos\delta \frac{x_d}{x_e + x_d} + \frac{E_q x_e}{x_e + x_d} \tag{2.32}$$

$$v_m^2 = v_{dm}^2 + v_{qm}^2 \tag{2.33}$$

<u>Machine Equations with Saturation</u>

Referring to figure 2.3, the following equations hold

$$E_q = E'_q + \frac{(x_d - x'_d)}{1 + S_d} i_d$$

$$v_b \cos\delta = E'_q - i_d (x'_d + x_e)$$

or

$$i_d = \frac{E'_q - v_b \cos\delta}{(x'_d + x_e)}$$

$$E_q = E'_q + \frac{(x_d - x'_d)}{(1 + S_d)} \left[\frac{E'_q - v_b \cos\delta}{x'_d + x_e} \right]$$

$$E_q = E'_q \left[1 + \frac{x_d - x'_d}{(x'_d + x_e)(1 + S_d)}\right]$$

$$- \frac{v_b \cos\delta \, (x_d - x'_d)}{(x'_d + x_e)\,(1 + S_d)} \tag{2.34}$$

$$\frac{v_{dm}}{v_b \sin\delta} = \frac{x_p + \dfrac{x_q - x_p}{1 + S_q}}{x_e + x_p + \dfrac{x_q - x_p}{1 + S_q}}$$

$$v_{dm} = \frac{x_p S_q + x_q}{(x_e+x_p) S_q+x_q+x_e} v_b \sin\delta \qquad (2.35)$$

Similarly

$$v_{qm} = v_b \cos\delta + i_d x_e$$

But

$$i_d = \frac{E_q - v_b \cos\delta}{x_e + x_p + \frac{x_d - x_p}{1 + S_d}}$$

Therefore

$$v_{qm} = v_b \cos\delta + \frac{E_q - v_b \cos\delta}{x_e + x_p + \frac{x_d - x_p}{1 + S_d}} x_e$$

$$= \frac{E_q x_e}{x_e + x_p + \frac{x_d - x_p}{1 + S_d}} + v_b \cos\delta\left[1 - \frac{x_e}{x_e + x_p + \frac{x_d - x_p}{1 + S_d}}\right] \qquad (2.36)$$

$$v_m^2 = v_{dm}^2 + v_{qm}^2 \qquad (2.33)$$

Similarly

$$\frac{d}{dt} E'_q = \frac{1}{T'_{do}} \left[E_{ex} - (1 + S_d) E_q\right] \qquad (2.37)$$

The equations derived in this chapter represent the mathematical model for digital simulation of the synchronous generator.

Chapter III

EXCITATION SYSTEMS

(PHASE PLANE ANALYSIS OF DERIVATIVE SIGNALS)

Introduction

The main function of an excitation system is to regulate the machine terminal voltage. A voltage signal proportional to the terminal voltage is compared with a reference voltage and the error is amplified and fed to the exciter field. Besides regulating the terminal voltage, an excitation system will have a beneficial effect on the stability of a synchronous machine. It will improve dynamic stability to an appreciable degree and transient stability to some extent by rapidly increasing the airgap flux following a fault which produces a depression of machine terminal voltage.

During the last few years many researchers have worked on the problem of improving transient stability by making improvements in the excitation systems[2-5]. They have attempted to feed auxiliary signals, besides the voltage deviation, to the excitation system. Notable among them are Langer et al[3] who have used linearized equations on an analog computer, feeding first and second derivatives of load angle to the regulator. Dineley et al[4] have used a digital computer to find the effect of varying gains in load angle channels.

This research is concerned with an analysis of nonlinear equations of a synchronous machine by the phase plane method and lays the foundations for a systematic study and design of amplification factors in derivative channels.

The Phase Plane [10]

The phase plane approach is a very convenient method of analyzing some non-linear systems. Consider the equation

$$A'_n \frac{d^n x}{dt^n} + A'_{n-1} \frac{d^{n-1} x}{dt^{n-1}} + \ldots + A'_1 x + A'_0 x = 0 \qquad (3.1)$$

If $x, \dot{x}, \ddot{x}, \frac{d^{n-1} x}{dt^{n-1}}$ are treated as co-ordinates of a certain phase space, then this equation can be represented by a single curve in the space. This curve is called a phase trajectory. In second order systems, it is fairly easy to obtain a phase trajectory.

In the above equation A'_n is the coefficient of the nth derivative.

If the differential equation is of second order, the equation may be

$$\ddot{x} + A\dot{x} + Bx = F.$$

A phase plane applied to this equation will have coordinates x and x.

The Swing Equation

The equation governing the oscillations of the salient-pole synchronous machine rotor is

$$M \frac{d^2 \delta}{dt^2} = P_i - \left[\frac{E'_q v_b}{X} \sin \delta - k \sin 2\delta \right]$$

where

M = inertia constant

δ = torque angle

t = time

P_i = input power

$E'q$ = voltage proportional to field flux linkages

v_b = infinite bus voltage

$X = x'_d + x_e$ where

x'_d = transient reactance

x_e = external reactance

$$k = v_b^2 \frac{x_q - x'_d}{2(x'_d + x_e)(x_q + x_e)} = \text{a constant, if } x'_d \text{ is assumed to be constant.}$$

Let $\omega = \frac{d\delta}{dt}$

$$\dot{\omega} = \frac{d\omega}{dt} = \frac{1}{M}\left[P_i - \frac{E'_q v_b}{X} \sin\delta + k \sin 2\delta\right]$$

or

$$\frac{d\delta}{d\omega} = \frac{\omega M}{P_i - \frac{E'_q v_b \sin\delta}{X} + k \sin 2\delta} \tag{3.3}$$

If E'_q, v_b X and M are constants, then the variables in eq. (3.3) can be separated to yield

$$M\omega d\omega = \left[P_i - \frac{E'_q v_b}{X} \sin\delta + k \sin 2\delta\right] d\delta$$

Integrating both sides

$$\frac{M\omega^2}{2} = P_i(\delta - \delta o) + \frac{E'_q v_b}{X} (\cos\delta - \cos\delta o)$$
$$- \frac{k}{2}\left[\cos 2\delta - \cos 2\delta o\right]$$

where initial values of δ and ω are δo and zero respectively.

Therefore

$$\omega^2 = \frac{2}{M} [P_i(\delta - \delta o) + \frac{E'_q v_b}{X} (\cos \delta - \cos \delta o) - \frac{k}{2} (\cos 2\delta - \cos 2\delta o)] \tag{3.4}$$

A phase plane plot was drawn for a synchronous machine system having the following constants and initial conditions.

$x_d = 1.25 \quad x'_d = .28 \quad v_b = 1.0 \quad x_q = .70 \quad x_e = .5$

H, the inertia constant = 3.0 G, the machine rating = 1.0

M, the inertia constant $= \frac{GH}{\pi f} = \frac{1.0 \times 3.0}{180 \times 60} = .000278$

Input power = 1.0 Power factor at infinite bus = 1.0

δ_o, the initial steady state torque angle = 50.2°

With reference to figure 2.2, the value of E'_q is calculated as follows:

$$E'_q = E_{qd} - (x_q - x'_d)\, i_d$$

$$\bar{E}_{qd} = \bar{v}_b + j(x_e + x_q)\, \bar{i}$$

$$= 1.00 + j(.5 + .7)\, 1.00 = 1.00 + j\ 1.2 = 1.57$$

$$i_d = i \sin \delta_o = .765$$

Therefore

$$E'_q = 1.57 - (.7 - .28)\, .765 = 1.245$$

The phase plane plot is shown in figure 3.1 when the input power was suddenly increased from 1.0 to 1.3. From the plot it

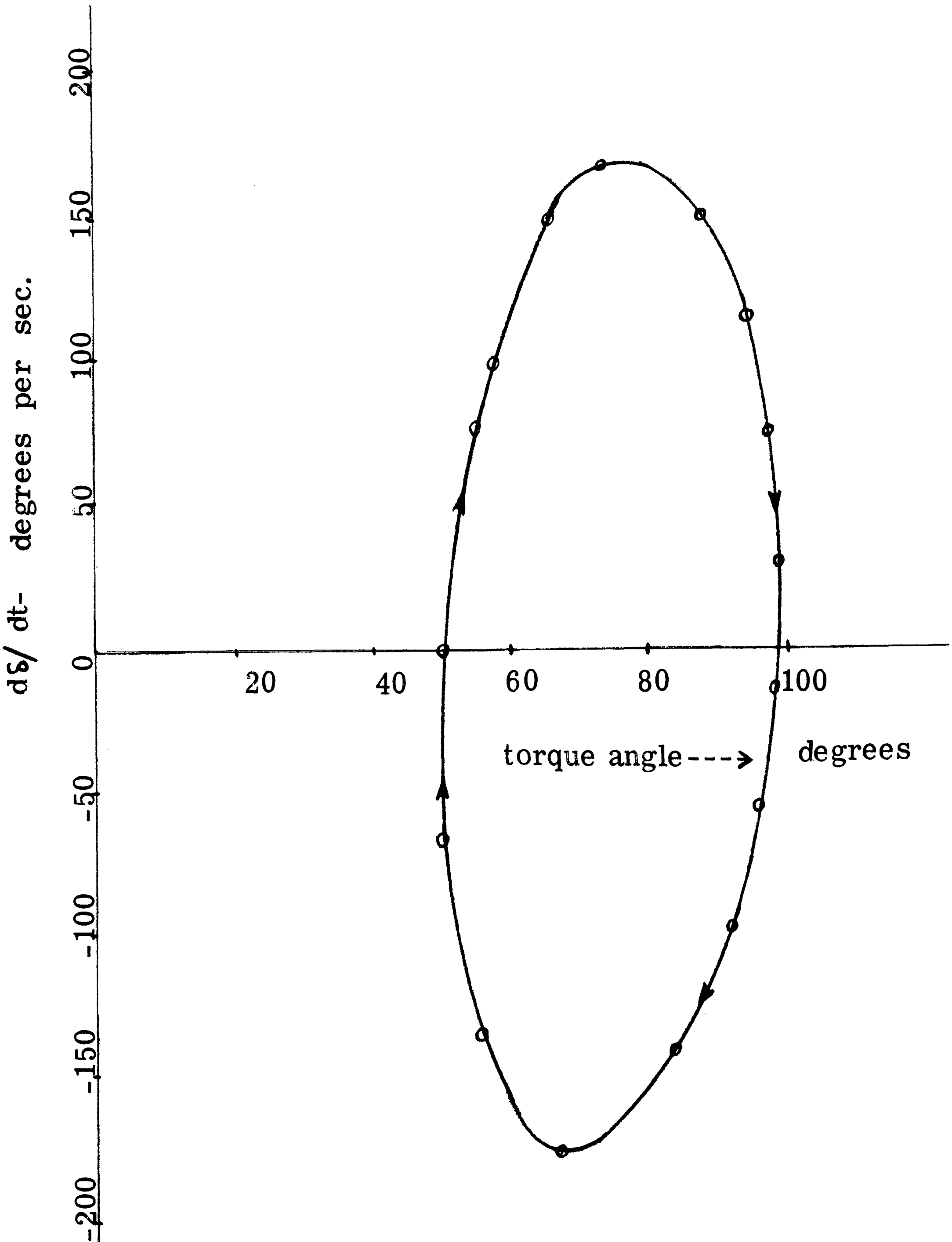


Fig. 3. 1 Torque angle/Rate of change of torque angle curve

is observed that the speed of the machine increases rapidly at the beginning of the transient and it reaches a maximum at an angle of about 70°, when it begins to decrease. The oscillations are from 50.2° to 87.0° and since damping has been neglected, these are continuous.

Auxiliary Signals

During the preceding analysis, it was assumed that the voltage behind transient reactance, E'_q was constant and figure 3.1 was drawn on the same basis. In this section, the influence of varying this voltage will be taken into account as follows:

Signal Proportional to Torque Angle

Assume that the voltage E'_q was regulated according to the following relationship

$$E'_q = E'_{qo} + K_1(\delta - \delta_o) \qquad (3.5)$$

where δ_o is the steady state value of δ before the transient occurred, E'_{qo} is the initial value of E'_q and K_1 is a constant, the value of which will be determined. δ and δ_o are in radians.

Substituting (3.5) in (3.3) gives

$$\frac{d\delta}{d\omega} = \frac{\omega M}{P_i - \frac{V_b}{X}\left[E'_{qo} + K_1(\delta-\delta_o)\right] \sin\delta + k \sin 2\delta} \qquad (3.6)$$

The effect of this signal is depicted in figure 3.2 which shows that the amplitude of the power angle curves increases from instant to instant due to action of the auxiliary signal. The operating point

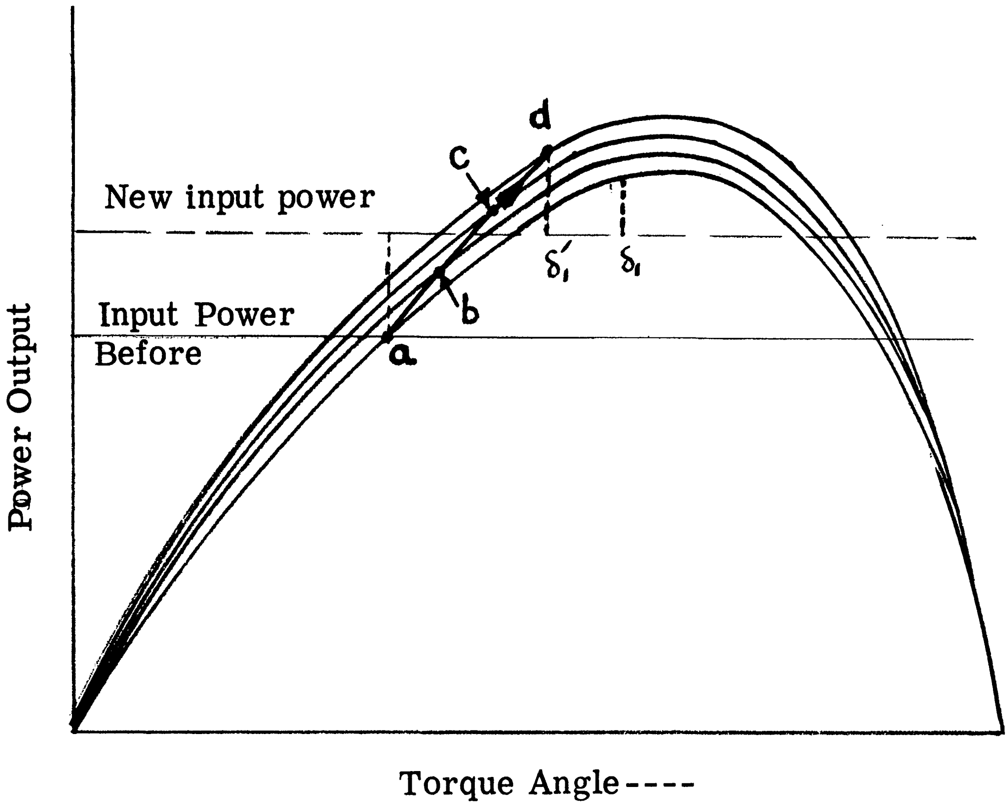


Fig. 3. 2 Effect of Excitation Control on operating point

moves from a to d through b and c . . It is observed that the amplitude of the oscillation is decreased from δ_1 to δ_1'.

Again separating the variables in eq. (3.6)

$$M\omega d\omega = [P_i - \frac{v_b}{X} [E'_{qo} + K_1(\delta-\delta_o)]\sin\delta + k\sin 2\delta]d\delta$$

Integrating this equation, the result is

$$\omega^2 \frac{M}{2} = P_i(\delta-\delta_o) + \frac{E'_{qo} v_b}{X}(\cos\delta-\cos\delta_o)$$

$$+ \frac{K_1 v_b}{X}[(\delta - \delta_o)\cos\delta + (\sin\delta_o - \sin\delta)]$$

$$- \frac{k}{2}(\cos 2\delta-\cos 2\delta_o) \qquad (3.7)$$

Comparing equations (3.4) and (3.7), the extra term in (3.7) is

$$\frac{K_1 v_b}{X}[(\delta - \delta_o)\cos\delta + (\sin\delta_o - \sin\delta)]$$

For positive value of K_1, this expression will have a negative value, hence this type of control will decrease the rotor speed and it will also decrease the angular swing. A phase plane plot for this condition is shown in figure 3.3 for a value of $K_1 = .01\times \frac{180}{\pi}$.

From the above analysis it is clear that if a signal proportional to the rotor angle is utilized, such that the voltage behind the transient reactance is changed, according to eq. (3.5), a substantial improvement in the reduction of transient oscillations will result. The improvement will depend on the value of K_1 which will have to be carefully chosen.

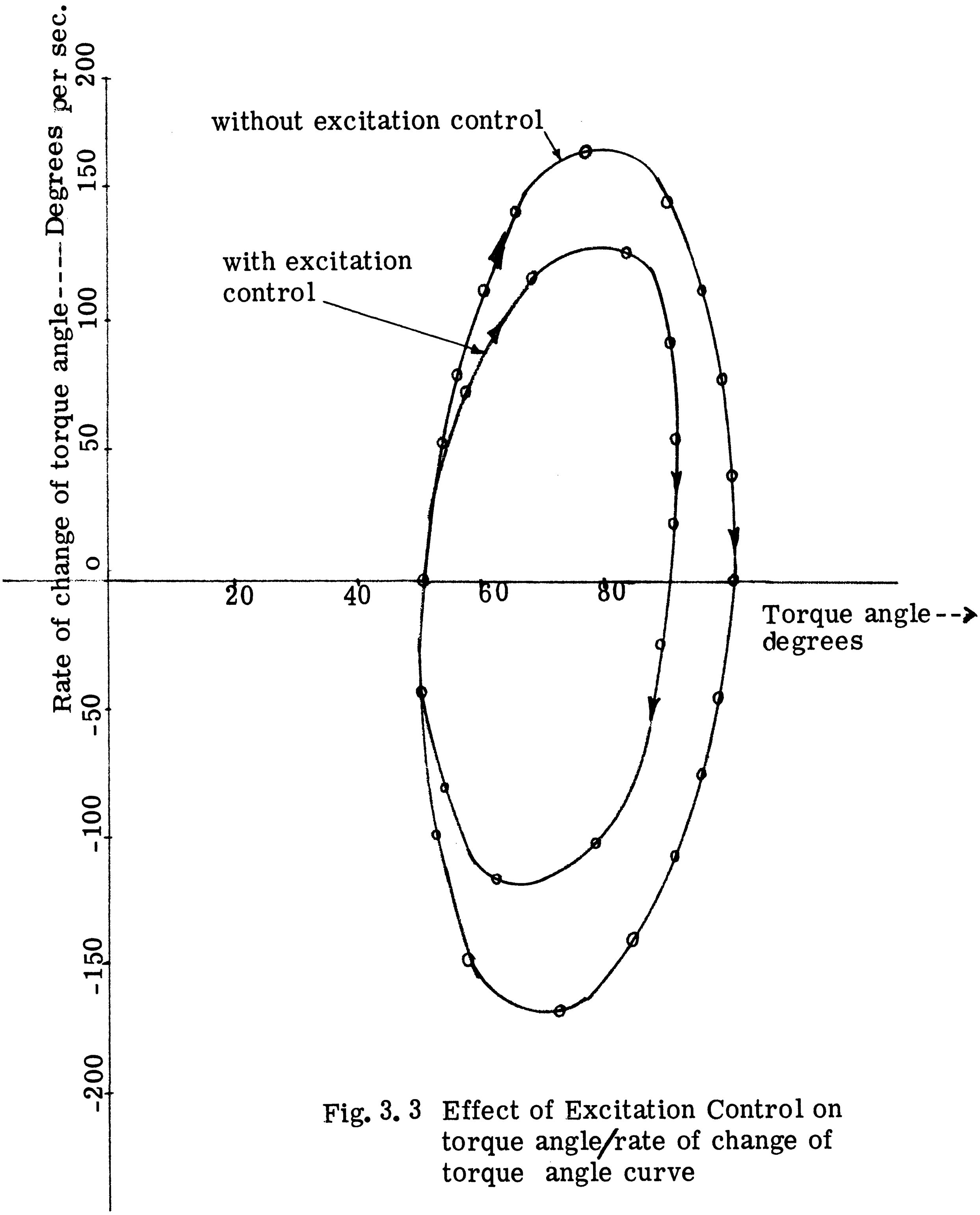


Fig. 3. 3 Effect of Excitation Control on torque angle/rate of change of torque angle curve

Signal Proportional to Rate of Change of Torque Angle

Assume now that a signal proportional to the rate of change of torque angle was used in such a way that $E'_q = E'_{qo} + K_2 \frac{d\delta}{dt}$, in which K_2 is a design parameter. $\frac{d\delta}{dt}$ is in radians/sec.

A great advantage of this type of control is that is is possible to design the parameter K_2 such that, for maximum value of transmitted power, the transient is controlled in a dead-beat non-oscillatory fashion. This can be readily observed from figure 3.4. At point a', the value of $\frac{d\delta}{dt} = 0$ but very soon $\frac{d\delta}{dt}$ becomes positive, and the amplitude of the power angle curve continues to increase rapidly and the operating point a' moves along through b', c', d' and reaches e' which is the new operating point. It should be kept in mind that the maximum value of dδ/dt is at d' and while above the P_i line, it steadily becomes less. Hence if the value of K_2 is chosen properly, the movement from a' to e' can be non-oscillatory.

Consider eq. (3.3) again and substitute the new value for E'_q

$$\frac{d\delta}{d\omega} = \frac{\omega M}{P_i - \frac{v_b}{X}\left[E'_{qo} + K_2 \frac{d\delta}{dt}\right] \sin \delta + k \sin 2\delta}$$

or

$$\left[k \sin 2\delta + P_i - \frac{v_b}{X}\left[E'_{qo} + K_2 \frac{d\delta}{dt}\right] \sin \delta\right] d\delta = M\omega d\omega$$

since

$$\frac{d\delta}{dt} = \omega$$

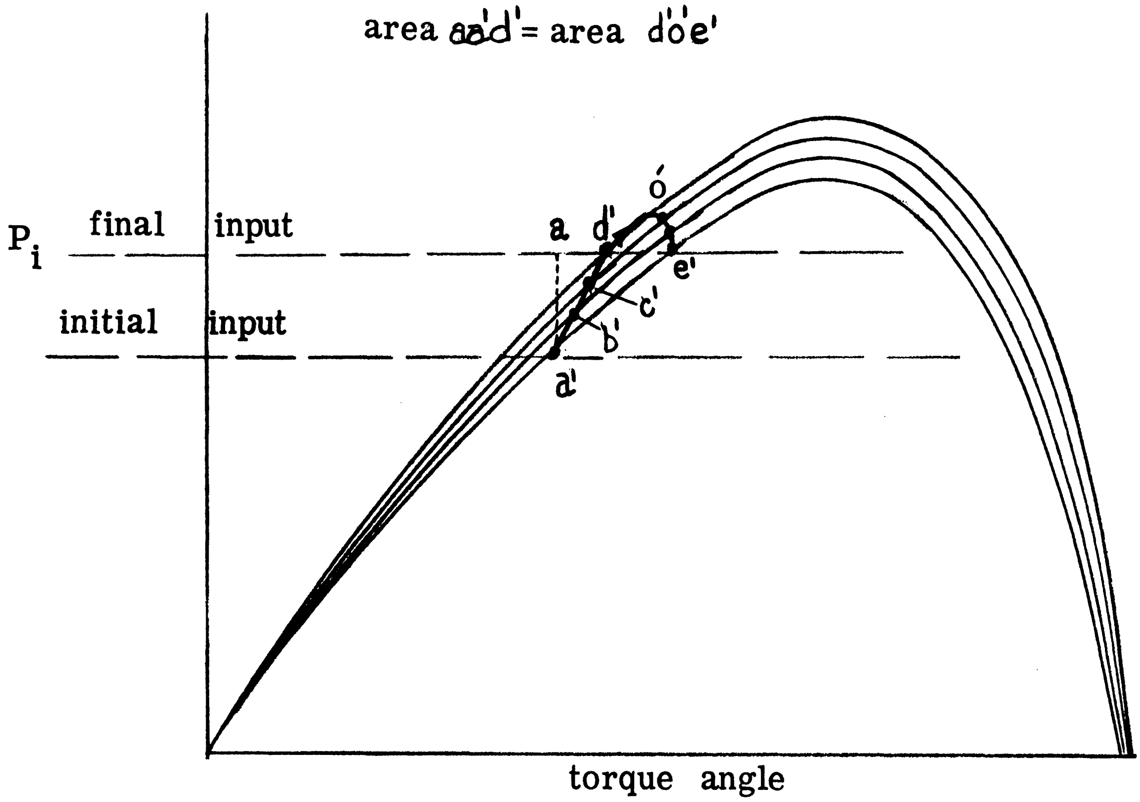


Fig. 3. 4 Effect of Excitation Control
on operating point

$$[k \sin 2\delta + P_i - \frac{V_b}{X}[E'_{qo} + K_2\omega] \sin \delta]\, d\delta = M\omega d\omega$$

or

$$\frac{d\omega}{d\delta} = \frac{P_i - \frac{V_b}{X}[E'_{qo} + K_2\omega] \sin \delta + k \sin 2\delta}{M\omega}$$

$$= [A - B[C + K_2\omega] \sin \delta + k' \sin 2\delta] \frac{1}{\omega} \qquad (3.8)$$

where A, B, C and k' are constants.

The variables of this equation cannot be separated and hence another method will be used to draw the phase trajectory.

The Method of Isoclines [10]

The method of isoclines can be illustrated by considering the following equation

$$\frac{dy}{dx} = \frac{Q(x,y)}{P(x,y)} \qquad (3.9)$$

There are three variables x, y and $\frac{dy}{dx}$ in this equation. $\frac{dy}{dx}$ is the slope of the curve (x,y) in the x,y plane. Asume that the slope is constant and equal to N_1, then

$$\frac{Q(x,y)}{P(x,y)} = N_1 \qquad (3.10)$$

which is a function of x and y only. This curve is called an "isocline". If a number of isoclines are drawn, the phase trajectory can be constructed from the given initial conditions by drawing straight line segments directed according to the slope of the isoclines.

The method of drawing isoclines is illustrated in figure 3.5. The initial operating point is A. The phase trajectory should cross isocline N_1 at a slope N_1. Therefore one dotted line is drawn at a slope N_1. But the trajectory should cross isocline N_2 at a slope N_2. Accordingly another dotted line is drawn at a slope N_2. Point B is therefore chosen as the mid point of the segment intercepted by the dotted lines. Similar procedure is followed in locating points C and D.

It may be pointed out that this is an approximate procedure to draw the trajectory but the error is very small. A great advantage of this method is that a number of phase trajectories may be drawn with little more effort than is required for a single trajectory.

Consider eq. (3.8) again,

$$\frac{d\omega}{d\delta} = \{A - B(C + K_2\omega)\sin\delta + k'\sin 2\delta\} / \omega$$

Let

$$\frac{d\omega}{d\delta} = N_1 = \text{a constant}$$

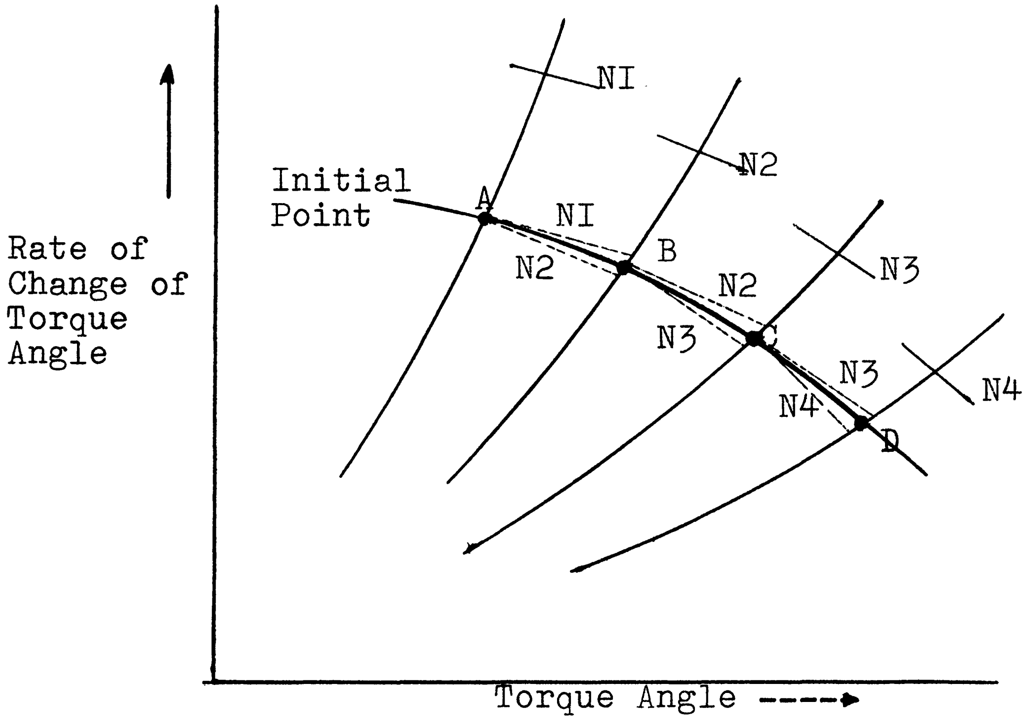


Fig.3.5 Construction of a Phase Trajectory using the method of Isoclines.

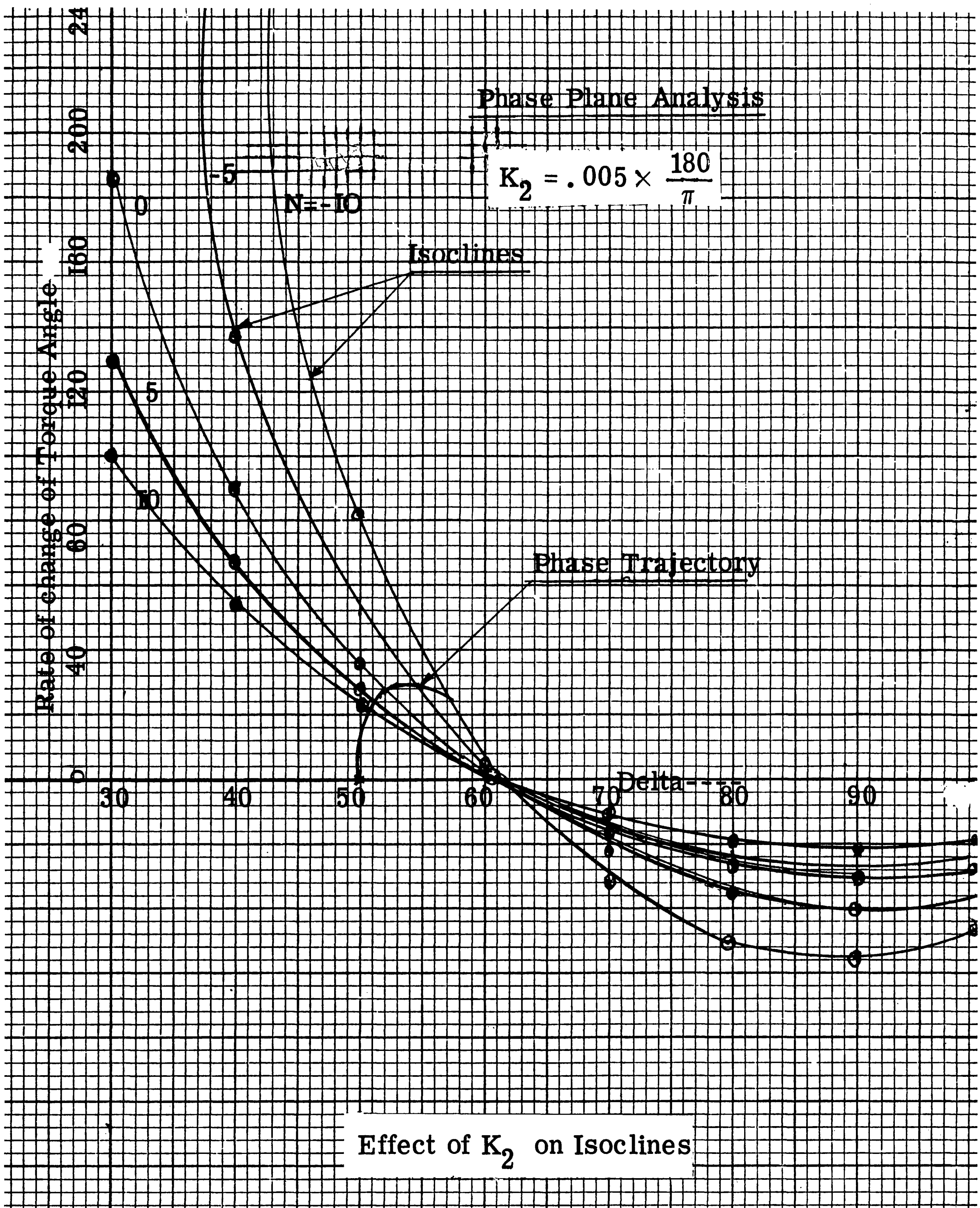

Phase Plane Analysis
$K_2 = .005 \times \frac{180}{\pi}$
Isoclines
Phase Trajectory
Rate of change of Torque Angle
Delta
N=-10
-5
0
5
10
0
40
80
120
160
200
24
30
40
50
60
70
80
90
Effect of K_2 on Isoclines

Fig. 3. 6-a

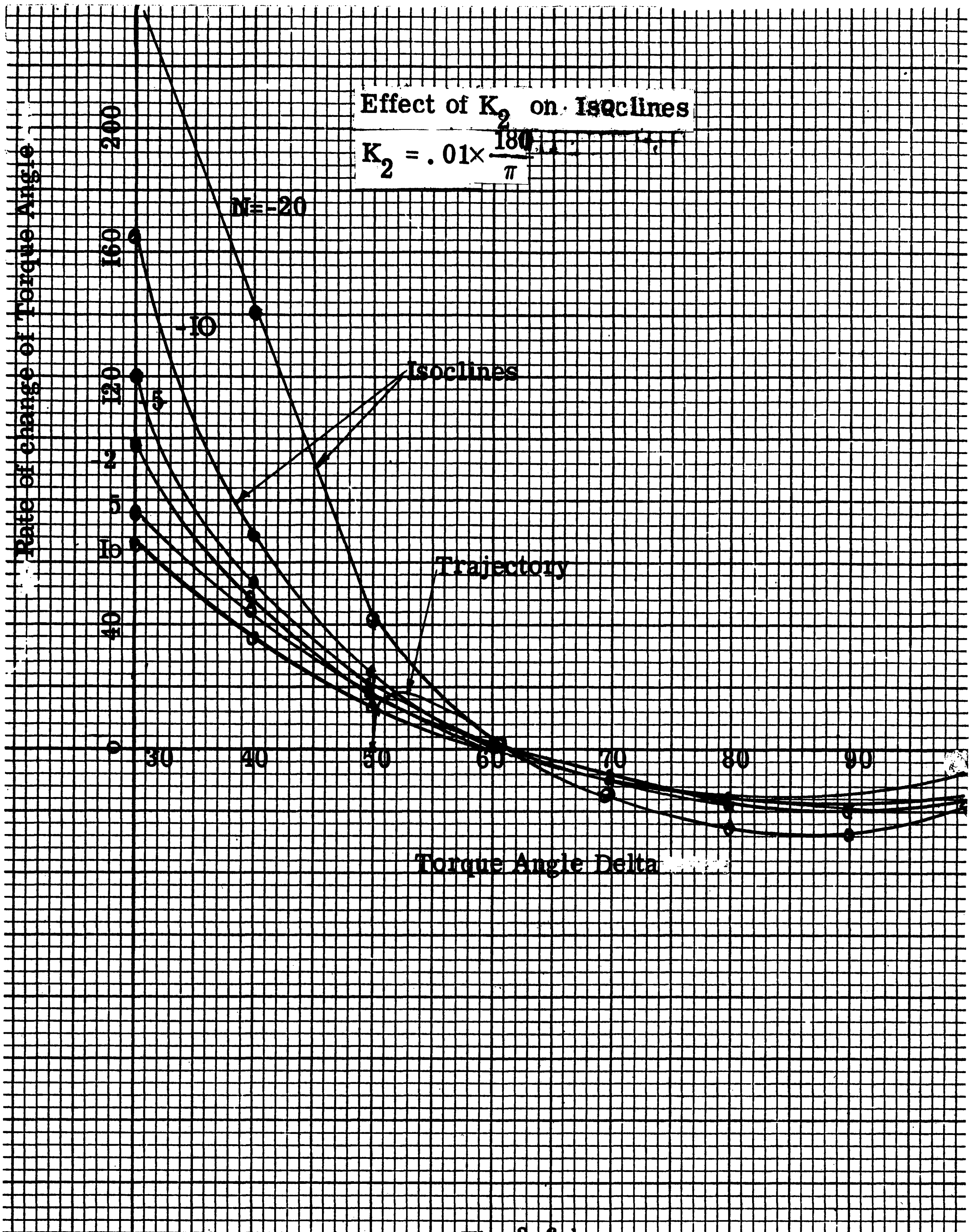

Effect of K_2 on Isoclines
$K_2 = .01 \times \frac{180}{\pi}$
N=-20
-10
-5
-2
5
10
Isoclines
Trajectory
Rate of change of Torque Angle
Torque Angle Delta
0
40
120
160
200
30
40
50
60
70
80
90

Fig. 3. 6-b

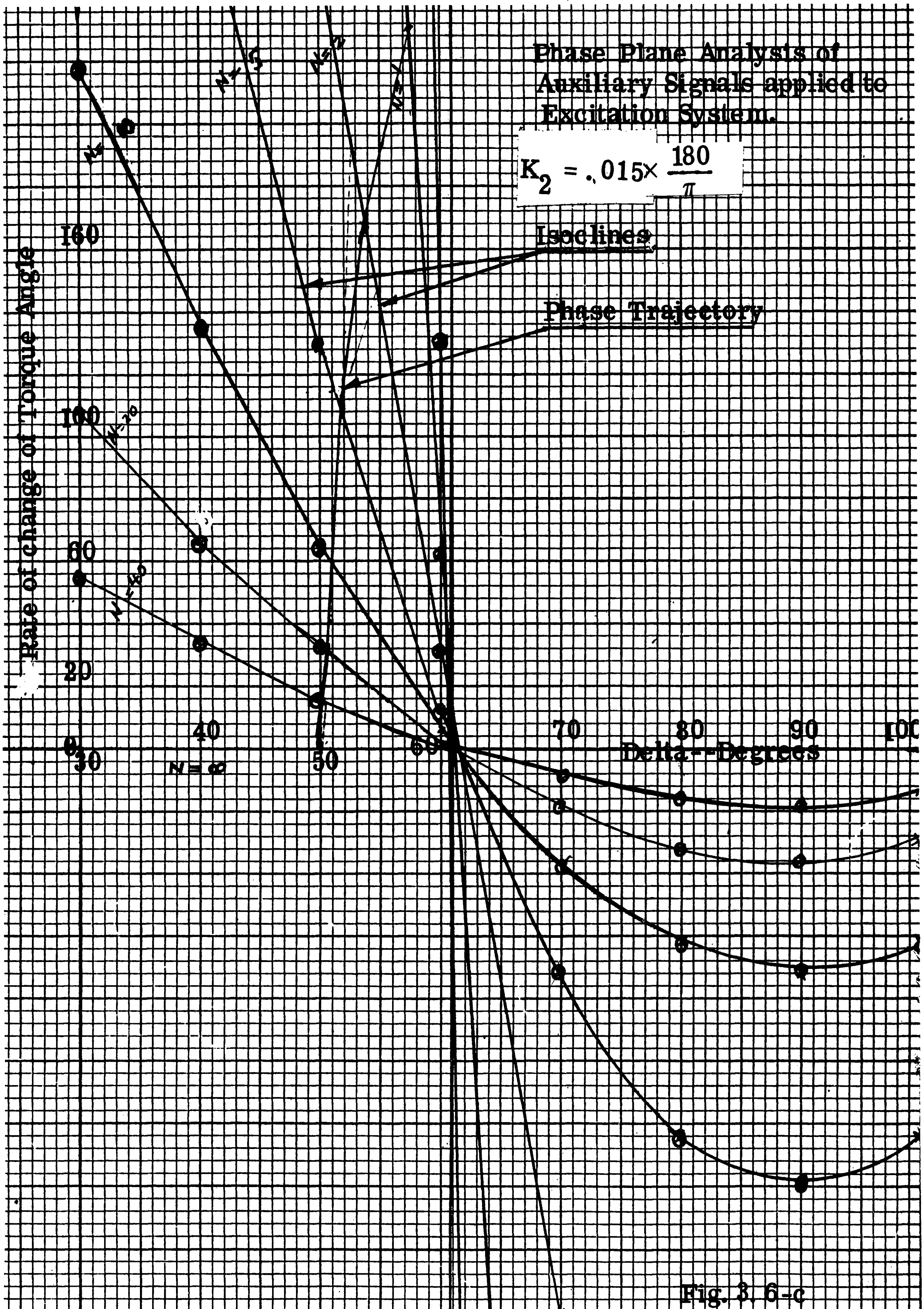

Phase Plane Analysis of Auxiliary Signals applied to Excitation System.
$K_2 = .015 \times \frac{180}{\pi}$
Isoclines
Phase Trajectory
Rate of change of Torque Angle
Delta--Degrees
160
100
60
20
0
30
40
50
60
70
80
90
100
N=5
N=2
N=1
N=10
N=20
N=40
N=∞

Fig. 3. 6-c

If N_1 is substituted for $\frac{d\omega}{d\delta}$ in eq. (3. 8) , then the following equation is obtained.

$$\omega N_1 = A - B(C + K_2\,\omega) \sin\delta + k' \sin 2\delta$$

$$\omega(N_1 + BK_2 \sin\delta) = A - BC \sin\delta + k' \sin 2\delta$$

or

$$\omega = \frac{A - BC \sin\delta + k' \sin 2\delta}{N_1 + B\,K_2 \sin\delta} \qquad (3.11)$$

Equation (3. 11) is the equation of the isocline. It would be possible to draw a set of isoclines for different values of N and to construct a phase trajectory from them. If K_2 is varied its effect can be conveniently analyzed from the shape of the phase trajectories.

Figure 3. 6a through 3. 6c show isoclines and trajectories drawn for the parameters under consideration. From these figures the best value of K_2 is found to be $.01 \times \frac{180}{\pi}$.

Exciter Input

In the previous sections, expressions were derived for the voltage behind the transient reactance to produce maximum damping. In a salient pole machine this voltage is equal to E'q which is proportional to field flux linkages. In a practical system, E'q is controlled by means of signals applied to the regulator. It is, therefore, necessary to derive expressions for voltage input to the regulator for the conditions of maximum damping.

As developed in Chapter II, the relationships between E'_q and the voltage applied to the exciter is as follows:

For Excitation Winding (Neglecting Saturation)

$$\frac{d\,E'_q}{dt} = \frac{1}{T'_{do}}\,(E_{ex} - E_q) \tag{3.12}$$

where E_{ex} = open-circuit excitation voltage in p.u.

E_q = voltage behind synchronous reactance in p.u.

T'_{do} = field time constant in seconds.

E'_q = voltage proportional to field flux linkages.

For Exciter

$$\frac{d\,E_{ex}}{dt} = \frac{1}{T_e}\,(E_x - E_{ex}) \tag{3.13}$$

where E_x = exciter field voltage in per unit.

T_e = exciter field time constant in seconds.

(a) Assume that a signal proportional to rotor angle deviation is applied to the excitation system such that

$$E'_q = E'_{qo} + K_1\,(\delta - \delta_o) \tag{3.4}$$

From eq. (3.12) $E_{ex} = E_q + T'_{do}\frac{d}{dt}E'_q$

$$E_q = E'_q + i_d\,(x_d - x'_d)$$

Therefore

$$E_{ex} = E'_q + i_d (x_d - x'_d) + T'_{do} \frac{d}{dt} E'_q \tag{3.14}$$

Substituting eq. (3.4) in eq. (3.14), the result is

$$E_{ex} = E'_{qo} + K_1 (\delta - \delta_o) + i_d (x_d - x'_d) + T'_{do} [K_1 \frac{d\delta}{dt}] \tag{3.15}$$

From eq. (3.13)

$$E_x = \text{exciter field voltage} = E_{ex} + T_e \frac{d}{dt} E_{ex}$$

$$= E'_{qo} + K_1 (\delta - \delta_o) + i_d (x_d - x'_d) + T'_{do} K_1 \frac{d\delta}{dt}$$

$$+ T_e \{K_1 \frac{d\delta}{dt} + (x_d - x'_d) \frac{di_d}{dt}$$

$$+ T'_{do} K_1 \frac{d^2\delta}{dt^2}\}$$

$$= \frac{d^2\delta}{dt^2} (T_e T'_{do} K_1) + \frac{d\delta}{dt} (T_e K_1 + T'_{do} K_1)$$

$$+ E'_{qo} + (\delta - \delta_o) K_1 + (x_d - x'_d) i_d$$

$$+ (x_d - x'_d) T_e \frac{di_d}{dt}$$

$$= A_1 \frac{d^2\delta}{dt^2} + A_2 \frac{d\delta}{dt} + A_3 \frac{di_d}{dt} + E_q \tag{3.16}$$

Therefore the voltage applied to the exciter field should have signals proportional to the second derivative of the torque angle, the first derivative of the torque angle and direct axis component of current.

(b) Assume that a signal is applied to the excitation system proportional to $\frac{d\delta}{dt}$ such that

$$E'_q = E'_{qo} + K_2 \frac{d\delta}{dt} \tag{3.17}$$

$$E_q = E'_q + i_d(x_d - x'_d) = E'_{qo} + K_2 \frac{d\delta}{dt} + i_d(x_d - x'_d)$$

$$E_{ex} = E'_q + i_d(x_d - x'_d) + T'_{do} \frac{d}{dt} E'_q \tag{3.14}$$

Substituting (3.17) in (3.14)

$$E_{ex} = E'_{qo} + K_2 \frac{d\delta}{dt} + (x_d - x'_d) i_d + T'_{do} [K_2 \frac{d^2\delta}{dt^2}]$$

$$E_x = E_{ex} + T_e \frac{d}{dt} E_{ex}$$

$$= E'_{qo} + K_2 \frac{d\delta}{dt} + (x_d - x'_d) i_d + T'_{do} [K_2 \frac{d^2\delta}{dt^2}]$$

$$+ T_e [K_2 \frac{d^2\delta}{dt^2} + (x_d - x'_d) \frac{di_d}{dt} + T'_{do} K_2 \frac{d^3\delta}{dt^3}].$$

$$= \frac{d^3\delta}{dt^3} (T_e T'_{do} K_2) + \frac{d^2\delta}{dt^2} (T_e K_2 + T'_{do} K_2)$$

$$+ (x_d - x'_d) T_e \frac{di_d}{dt} + E_q$$

$$= A_4 \frac{d^3\delta}{dt^3} + A_5 \frac{d^2\delta}{dt^2} + A_6 \frac{di_d}{dt} + E_q \tag{3.18}$$

where A's are constants.

Regulator Input (with saturation)

(a) Assume that a signal is applied to the excitation system such that

$$E'_q = E'_{qo} + K_1(\delta - \delta_o)$$

$$E_{ex} = (1 + S_d) E_q + T'_{do} \frac{d}{dt} E'_q$$

$$E_q = E'_q + i_d\left(\frac{x_d - x'_d}{1 + S_d}\right)$$

$$E_{ex} = \left\{E'_q + i_d \frac{(x_d - x'_d)}{1 + S_d}\right\}(1 + S_d) + T'_{do}\frac{d}{dt}E'_q$$

$$= E'_q(1 + S_d) + i_d(x_d - x'_d) + T'_{do}\frac{d}{dt}E'_q$$

$$= [E'_{qo} + K_1(\delta - \delta_o)](1 + S_d) + i_d[x_d - x'_d] + T'_{do}\left[K_1\frac{d\delta}{dt}\right]$$

$$E_x = E_{ex} + T_e\frac{d}{dt}E_{ex}$$

$$= [E'_{qo} + K_1(\delta - \delta_o)](1 + S_d) + i_d(x_d - x'_d)$$

$$+ T'_{do}K_1\left\{\frac{d\delta}{dt}\right\} + T_e\left\{K_1(1 + S_d)\frac{d\delta}{dt}\right\} + T_e\{(x_d - x'_d)\}\frac{d}{dt}i_d$$

$$+ T'_{do}T_eK_1\frac{d^2\delta}{dt^2}$$

$$= A_1\frac{d^2\delta}{dt^2} + A_2\frac{d\delta}{dt} + A_3\frac{di_d}{dt} + (1 + S_d)E_q \qquad (3.19)$$

(b) Assume that a signal proportional to $\frac{d\delta}{dt}$ is applied to the excitation system such that

$$E'_q = E'_{qo} + K_2\frac{d\delta}{dt}$$

$$E_q = E'_q + i_d\frac{(x_d - x'_d)}{1 + S_d}$$

$$E_{ex} = (1 + S_d)E_q + T'_{do}\frac{d}{dt}E'_q$$

$$E_{ex} = \left\{E'_q + i_d\frac{(x_d - x'_d)}{(1 + S_d)}\right\}(1 + S_d) + T'_{do}\frac{d}{dt}E'_q$$

$$= E'_q(1 + S_d) + (x_d - x'_d)i_d + T'_{do}\frac{d}{dt}E'_q$$

$$= (1 + S_d)\ \{E'_{qo} + K_2 \frac{d\delta}{dt}\} + (x_d - x'_d)\, i_d + T'_{do}\, K_2 \frac{d^2\delta}{dt^2}$$

$$E_x = E_{ex} + T_e \frac{d}{dt} E_{ex}$$

$$= (1 + S_d)(E'_{qo} + K_2 \frac{d\delta}{dt}) + (x_d - x'_d)\, i_d + T'd_o\, K_2 \frac{d^2\delta}{dt^2}$$

$$+ T_e\{(1 + S_d) K_2 \frac{d^2\delta}{dt^2} + (x_d - x'_d) \frac{di_d}{dt} + T'_{do}\, K_2 \frac{d^3\delta}{dt^3}\}$$

$$= \frac{d^3\delta}{dt^3} T_e\, T'_{do}\, K_2 + \frac{d^2\delta}{dt^2}\{T_e(1 + S_d)\ K_2 + T'_{do}\, K_2\}$$

$$+ \frac{d\delta}{dt}\{(1 + S_d) K_2\} + T_e(x_d - x'_d) \frac{di_d}{dt}$$

$$+ (x_d - x'_d)\, i_d$$

$$+ (1 + S_d)\, E'_{qo}$$

$$= \frac{d^3\delta}{dt^3} (T_e\, T'_{do}\, K_2) + \frac{d^2\delta}{dt^2} \{T_e(1 + S_d) K_2 + T'_{do}\, K_2\}$$

$$+ T_e(x_d - x'_d) \frac{di_d}{dt} + (1 + S_d) E_q$$

$$= A_4 \frac{d^3\delta}{dt^3} + A_5 \frac{d^2\delta}{dt^2} + A_6 \frac{di_d}{dt} + (1 + S_d) E_q \qquad (3.20)$$

Amplification Factors (without saturation)

Two modes of controlling transients have been developed. In one mode, signals proportional to $\frac{d\delta}{dt}$, $\frac{d^2\delta}{dt^2}$ and $\frac{di_d}{dt}$ are needed and amplification factors in various channels are

$\frac{d^2\delta}{dt^2}$ amplification factor $A_1 = T_e\, T'_{do}\, K_1$

$\frac{d\delta}{dt}$ amplification factor $A_2 = T_e\, K_1 + T'_{do}\, K_1$

$\frac{di_d}{dt}$ amplification factor $A_3 = (x_d - x'_d)\, T_e$.

In the second mode, which results in a dead-beat transient control, for maximum transmitted power the amplification factors are

$\frac{d^3\delta}{dt^3}$ amplification factor $A_4 = T_e\, T'_{do}\, K_2$

$\frac{d^2\delta}{dt^2}$ amplification factor $A_5 = T_e\, K_2 + T'_{do}\, K_2$

$\frac{di_d}{dt}$ amplification factor $A_6 = (x_d - x'_d)\, T_e$.

(with saturation)

First Mode

$\frac{d^2\delta}{dt^2}$ amplification factor $A_1 = T_e\, T'_{do}\, K_1$

$\frac{d\delta}{dt}$ amplification factor $A_2 = T'_{do}\, K_1 + T_e\, K_1(1 + S_d)$

$\frac{di_d}{dt}$ amplification factor $A_3 = T_e(x_d - x'_d)$

Second Mode

$\frac{d^3\delta}{dt^3}$ amplification factor $A_4 = T_e\, T'_{do}\, K_2$

$\frac{d^2\delta}{dt^2}$ amplification factor $A_5 = T_e(1 + S_d)\, K_2 + T'_{do}\, K_2$

$\frac{di_d}{dt}$ amplification factor $A_6 = (x_d - x'_d)\, T_e$.

Conclusion

An analytical method has been developed to determine the correct magnitudes of feedback signals, in excitation systems, to produce the desired control of swings. This information will now be applied, in Chapters IV and V, to a typical synchronous machine system.

Chapter IV

EXCITATION SYSTEMS (DIGITAL COMPUTER SIMULATION)

Introduction

The theory developed in Chapter III is applied to the system of figure 2.1. The equations describing the system were developed in Chapter II. The excitation system utilized auxiliary derivative signals as developed in Chapter III. The system was simulated on a digital computer.

For the investigation of the transient stability of the generator, several tests may be applied but all are concerned with the behavior of the system subsequent to a sudden disturbance. The latter may be a fault on a transmission line, the dropping of a line section, a sudden application of load or a change of input power.

The change in input power is the method used in this study. The generator is adjusted initially to operate at rated power output and terminal voltage and is then subjected to a sudden increase in the power input. Transient stability is said to exist, if the machine regains a state of equilibrium after such a disturbance.

Excitation Control

In Chapter III two modes for input voltage to the exciter field were developed which are as follows:

Mode I

$$E_x = A_1 \frac{d^2\delta}{dt^2} + A_2 \frac{d\delta}{dt} + A_3 \frac{di_d}{dt} + E_q$$

Mode II

$$E_x = A_4 \frac{d^3\delta}{dt^3} + A_5 \frac{d^2\delta}{dt^2} + A_6 \frac{di_d}{dt} + E_q$$

There are some practical considerations in applying the above theory to actual cases, as can be observed from the following:

a) There are upper and lower limits to the voltage that can be applied to the exciter field.

b) In actual cases it will be necessary to provide a signal proportional to the difference between the magnitudes of the terminal voltage and the desired terminal voltage.

c) Circuits have been designed to measure quickly and with reasonable accuracy the phase angle[21]. Researchers[5] have experimented with signals based on speed, from which it appears that it is possible to obtain with reasonable accuracy $\frac{d\delta}{dt}$ and $\frac{d^2\delta}{dt^2}$ signals. It is not, however, certain as yet whether a noise free $\frac{d^3\delta}{dt^3}$ signal can be obtained in actual practice.

At this stage it was therefore decided to restrict the computer investigation to application of $\frac{d\delta}{dt}$ and $\frac{d^2\delta}{dt^2}$ signals only, or Mode 1.

d) The influence of saturation depends largely on the shape of the saturation curve. In Chapter II and III, machine equations were modified to take saturation into account. So that the true effects of derivative signals are not masked, it was decided to neglect saturation in the computer study. Other investigators have followed the same procedure[3-5]. It can, however, be taken into account if desired, by using the modified equations of Chapters II and III.

System Equations

$$P_u = \frac{v_b E'_q}{x'_d + x_e} \sin \delta + \frac{v_b^2 (x'_d - x_q)}{2(x_q + x_e)(x'_d + x_e)} \sin 2\delta \tag{2.15}$$

$$E_q = \frac{E'_q (x_d + x_e)}{(x'_d + x_e)} - v_b \cos \delta \, \frac{(x_q - x'_d)}{(x'_d + x_e)} \tag{2.22}$$

$$\frac{dE_{ex}}{dt} = \frac{1}{T_e} (E_x - E_{ex}) \tag{2.26}$$

The exciter field voltage is E_x and by equation 3.16

$$E_x = A_1 \frac{d^2\delta}{dt^2} + A_2 \frac{d\delta}{dt} + A_3 \frac{di_d}{dt} + E_q \tag{3.16}$$

In the steady-state condition $E_{xo} = E_{qo}$ where subscript o denotes the steady-state value in the pre-fault state. On account of automatic voltage regulator requirement, a signal proportional to $(v_r - v_m)$ will be inserted into the exciter.

Therefore

$$E_x = E_q + A_1 \frac{d^2\delta}{dt^2} + A_2 \frac{d\delta}{dt} + A_3 \frac{di_d}{dt} + \mu_v(v_r - v_m)$$

where μ_v is the gain in the voltage channel.

The above equation can be written as follows:

$$E_x = E_{qo} + (E_q - E_{qo}) + A_1 \frac{d^2\delta}{dt^2} + A_2 \frac{d\delta}{dt} + A_3 \frac{di_d}{dt} + \mu_v(v_r - v_m)$$

In the above equation the terms $(E_q - E_{qo})$ and $A_3 \frac{di_d}{dt}$ are small as compared to $A_1 \frac{d^2\delta}{dt^2}$ and $A_2 \frac{d\delta}{dt}$. If these are neglected, for simplification, the equation reduces to

$$E_x = E_{qo} + A_1 \frac{d^2\delta}{dt^2} + A_2 \frac{d\delta}{dt} + \mu_v (v_r - v_m)$$

The input signal to the exciter is, therefore,

$$E_x = E_{xo} + A_1 \frac{d^2\delta}{dt^2} + A_2 \frac{d\delta}{dt} + \mu_v(v_r - v_m), \qquad (4.01)$$

since $E_{qo} = E_{xo}$ as shown in Chapter II.

In eq. 4.01 $A_2 = K_1 (T'_{do} + T_e)$ and $A_1 = T_e T'_{do} K_1$ as derived in Chapter III.

Chapter V

EXCITATION SYSTEMS (DIGITAL COMPUTER RESULTS)

Introduction

It is desired to study the transient stability of a synchronous generator, connected to an infinite bus, by means of a double circuit transmission line, when the input power of the generator is suddenly increased.

The results given in this chapter were obtained by simulating the system on a digital computer. Input power was increased suddenly until the system became unstable. The system was simulated with a conventional Automatic Voltage Regulator (A. V. R.) to maintain the machine terminal voltage constant. Curves were drawn from the data abtained with and without the use of auxiliary signals to make a comparison between the two cases. In this particular study, transient instability occurred when the input power was suddenly increased by 40 percent.

System Equations and Constraints

The investigation is based on the equations developed in previous chapters and for ready reference they are reproduced hereunder:

$$P_u = \frac{v_b E'_q}{x'_d + x_e} \sin\delta + \frac{v_b^2 (x'_d - x_q)}{2(x_q + x_e)(x'_d + x_e)} \sin 2\delta \tag{2.15}$$

$$E_q = \frac{E'_q (x_d + x_e)}{(x'_d + x_e)} - v_b \cos\delta \, \frac{(x_q - x'_d)}{(x'_d + x_e)} \tag{2.22}$$

$$\frac{dE'_q}{dt} = \frac{1}{T'_{d_o}} (E_{ex} - E_q) \qquad (2.5)$$

$$\frac{dE_{ex}}{dt} = \frac{1}{T_e} (E_x - E_{ex}) \qquad (2.26)$$

$$E_x = E_{xo} + \mu_v (v_r - v_m) + A_1 \frac{d^2\delta}{dt^2} + A_2 \frac{d\delta}{dt} \qquad (4.01)$$

$$A_1 = T_e \, T'_{do} \, K_1 \, , \; A_2 = (T'_{do} + T_e) \, K_1 \qquad (3.16)$$

A typical machine was selected for the investigation with the constants as given below. All values are in per unit system.

$x_d = 1.25$ $H = 3.0$ $x_q = .70$ $T'_{do} = 5.0$

$x'_d = .28$ $T_e = .15$ $x_e = .5$

Exciter Field Ceiling voltage Max = 3.5 Min = -1

The following operating conditions are assumed.

$P_u = 1.0$, $v_b = 1.0$ $v_m = 1.18$ $f = 60$ c/s. Power Factor=1.0.

The value of K_1 used in eq. (3.16) was $.01 \times \frac{180}{\pi}$. This value was selected by plotting eq. (3.7) for this system as shown in figure 3.3.

By calculations, the initial values of E'_q, δ, E_{qd}, and E_q were found as follows:

$E'_q = 1.245$ $\delta = 50.2°$ $E_{qd} = 1.562$ $E_q = 1.986$

$A_2 = .01(5 + .15) = .0515 \times \frac{180}{\pi}$ $A_1 = .01(5 \times .15) = .0075 \times \frac{180}{\pi}$

The computer program is enclosed with the appendix.

Computer Results:

Figure 5.1 shows the Swing Curve, when the input power is raised by 30 percent. It can be seen that the auxiliary signals have a significant damping effect on the oscillations.

Figure 5.2 shows the variation of terminal voltage with time during a transient. Without the derivative signals, the terminal voltage is higher but it has large fluctuations. With the derivative signals, the fluctuations have almost disappeared although the magnitude of the voltage is slightly less. The terminal voltage begins to rise as the oscillations of the rotor disappear. It is, therefore, observed that the quick damping of rotor oscillations is achieved at the expense of slight drop in terminal voltage during the transient.

Figure 5.3 shows the variation of field voltage with time. When auxiliary signals are used, the machine field voltage undergoes rapid changes, even reversing at a particular instant. This shows that to attain quick damping of rotor oscillations, the field voltage should be increased quickly to the maximum positive value reversed and again increased in the positive direction. This is an application of the classic "Bang-Bang" principle.

Figure 5.4 shows two swing curves drawn when the input power was 1.4 p.u. Both curves are for a system which includes the normal Automatic Regulator Action. It is observed that the auxiliary signals have significant effect on the damping of the oscillations.

Figure 5.5 is a graph of exciter field voltage and time. If there were no ceiling limit on the exciter field voltage, then the magnitude of this voltage would rise to as high a value as 9.5 per unit. Dut to the ceiling limit, it does not go above 3.5 per unit. The results of this limit is that the damping of the oscillations is not as rapid as it would be if there was no ceiling on the exciter field voltage.

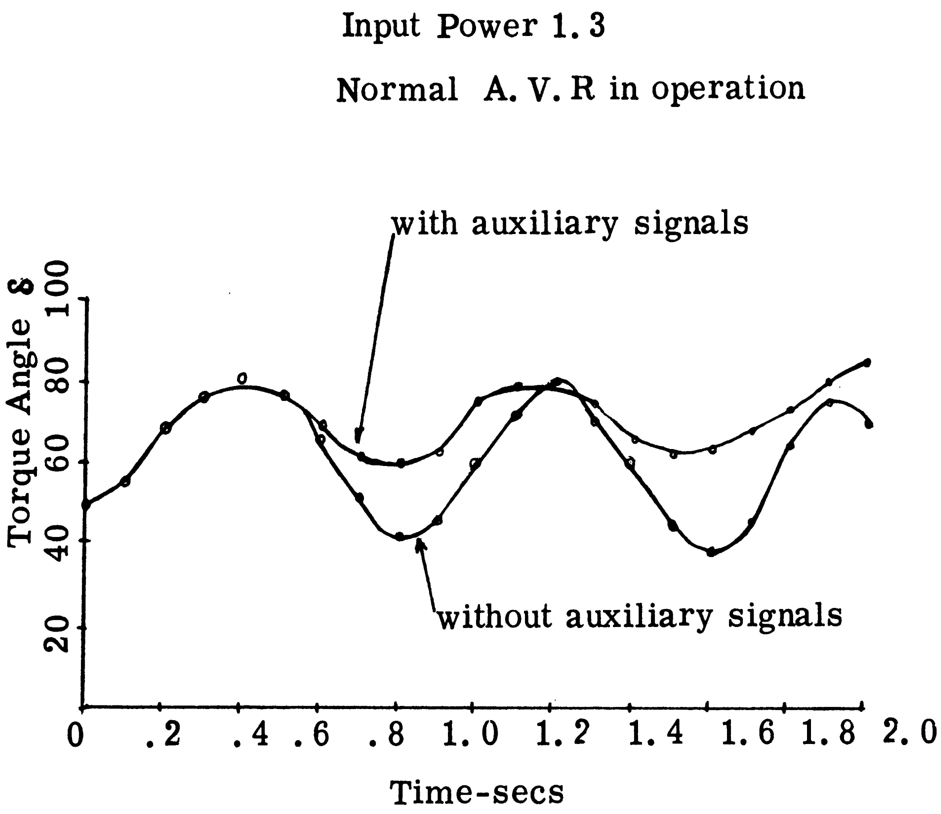


Fig. 5. 1 Swing Curves

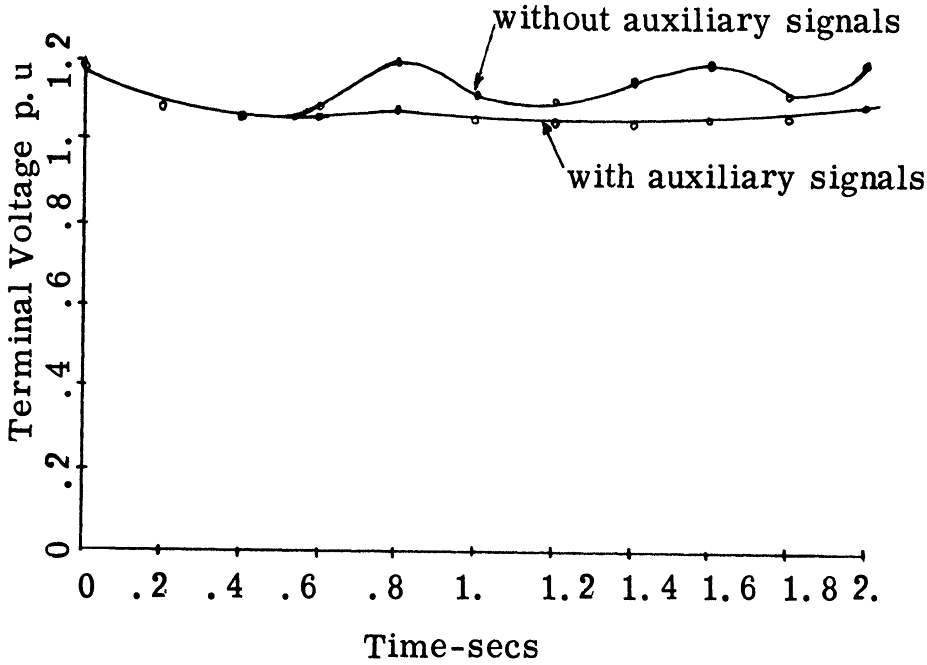


Fig. 5. 2 Sending End Terminal Voltage

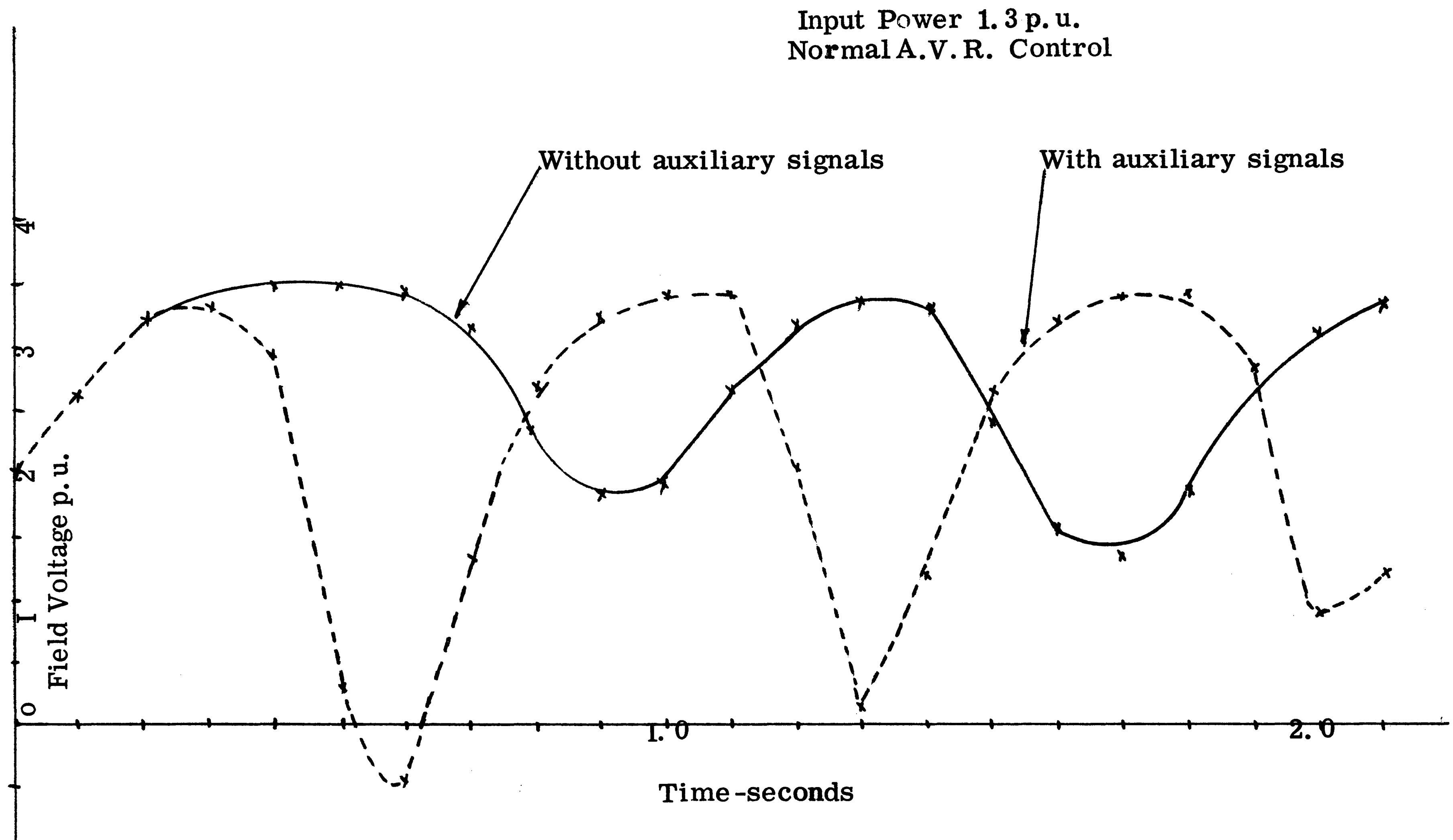


Fig. 5.3 Machine Field Voltage

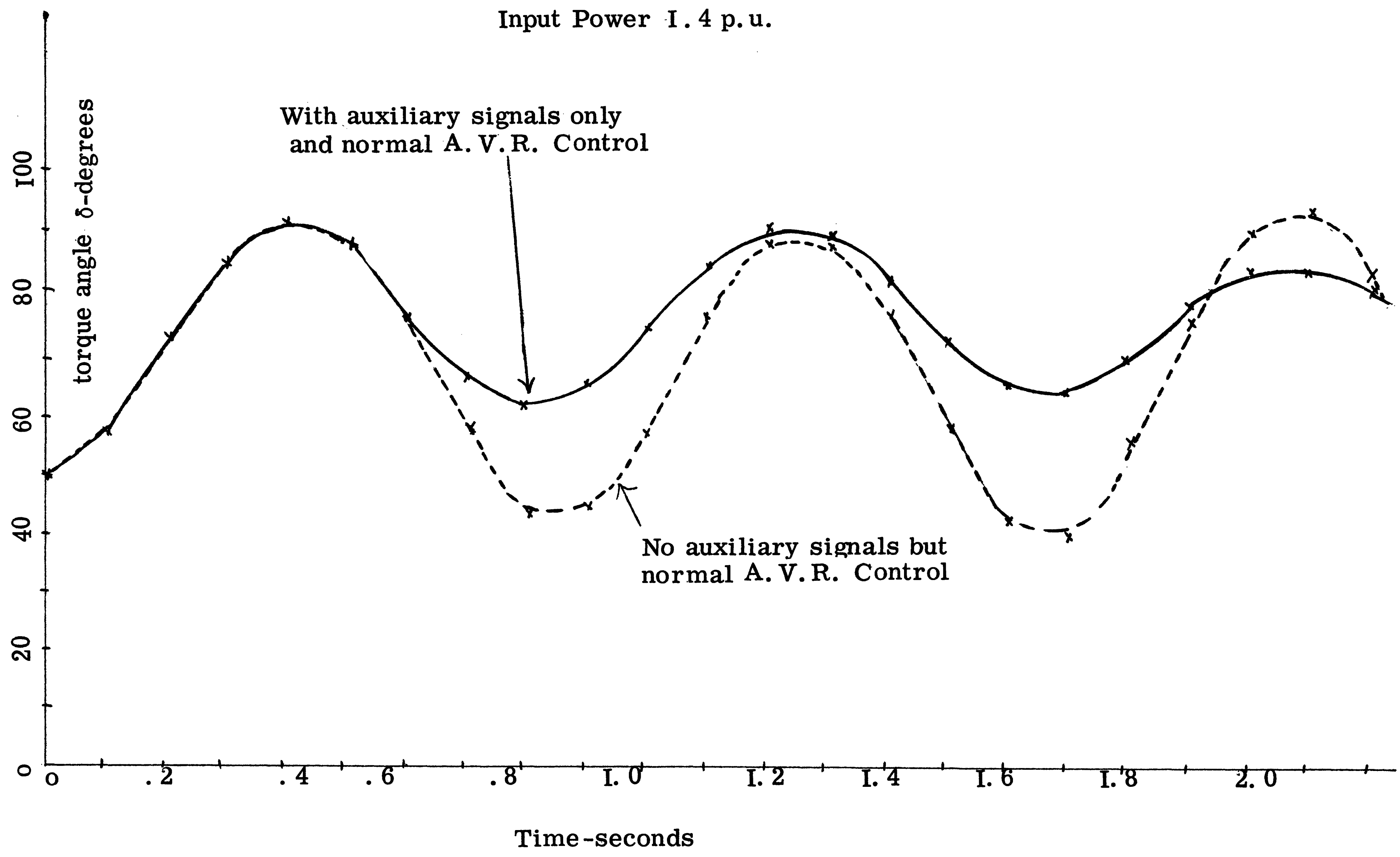


Fig. 5.4 Swing Curve for input 1.4

Input Power 1.3 p.u.   Normal A.V.R. Control

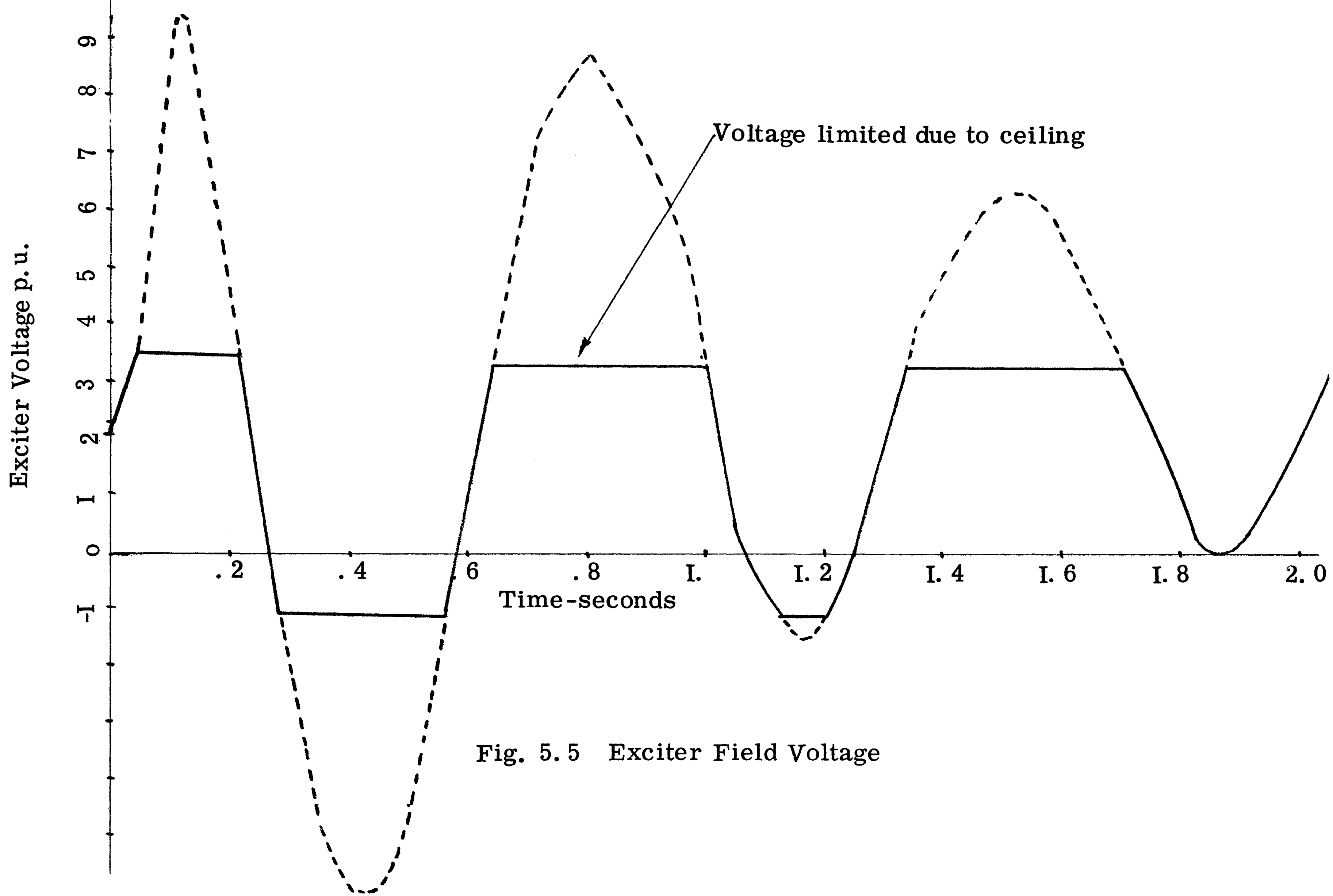


Fig. 5.5   Exciter Field Voltage

CHAPTER VI

NORMALIZED STABILITY MARGIN

Introduction

Since digital computers are becoming commonplace in utility system operation, there is a strong possibility that during the next decade they will be used to make decisions and initiate corrective action during transient periods. It will therefore be convenient to have a numerical technique by which relative stability of a system can be conveniently determined. Baba[11] et al suggested the use of phase-plane technique to develop an expression for stability margin of a synchronous generator. In this chapter the familiar equal-area criterion will be used to define stability margin and an expression will be developed for what the writer will call "Normalized Stability Margin", wherein the positive limit of stability margin will be unity. The concept of normalized stability margin will be used in Chapters VII and VIII to evaluate the effect of Phase-Shifting transformers.

Stability Margin

Figure 6-1 shows pre-fault and post-fault power angle curves. The amplitude of the post-fault curve is less due to the fact that post-fault reactance is higher, due to switching out of a line section. For simplicity, fault clearing time, being very small, is neglected but it will be taken into account later on.

According to equal area criterion,

if area $A_2 >$ area A_1, the system is stable

if area $A_2 <$ area A_1, the system is unstable.

$$\text{Area } A_1 = \int_{\delta_1}^{\delta_2} (P_1 - P_{m2} \sin\delta)\,d\delta = P_1(\delta_2 - \delta_1) + P_{m2}(\cos\delta_2 - \cos\delta_1)$$

$$\text{Area } A_2 = \int_{\delta_2}^{\pi-\delta_2} (P_{m2} \sin\delta - P_1)\,d\delta = P_{m2}(\cos\delta_2 + \cos\delta_2) - P_1(\pi - 2\delta_2)$$

$$\text{Therefore } A_2 - A_1 = P_{m2}(\cos\delta_2 + \cos\delta_1) + P_1(\delta_1 + \delta_2 - \pi) \quad (6.1)$$

It will be convenient to normalize expression 6.1 so that maximum stability margin will be unity which will occur at no load.

The lesser the load, the larger will be the area A_2 and smaller the area A_1; hence, maximum value of $(A_2 - A_1)$ will occur when $P_1 = 0$ It will be convenient to designate the maximum value as unity; therefore, if the expression 6.1 is divided by maximum value of A_2 (or total area under the post-fault curve), the expression is normalized. The area under post-fault curve is equal to $2P_{m2}$.

Therefore normalized stability margin

$$= \frac{1}{2}\left\{(\cos\delta_2 + \cos\delta_1) + \frac{P_1}{P_{m2}}(\delta_1 + \delta 2 - \pi)\right\} \quad (6.2)$$

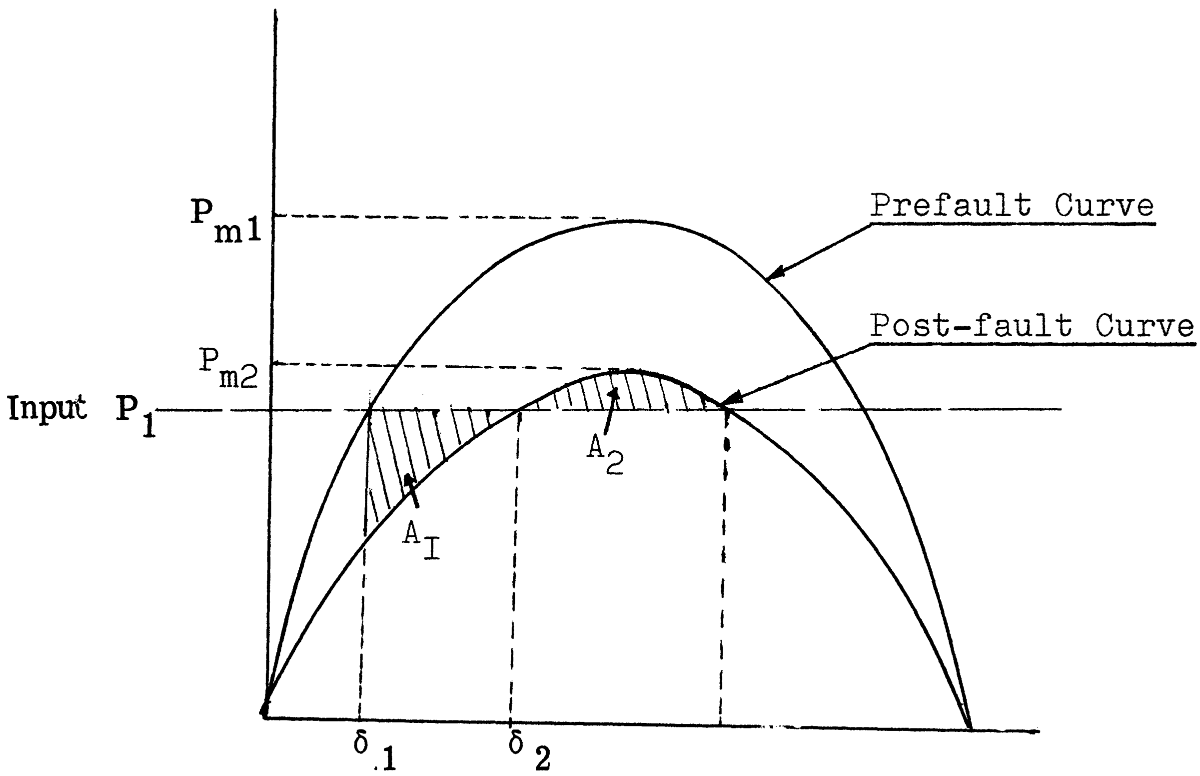


Fig.6. 1-Determination of Stability Margin

Derivation of Formula for Stability Margin when Fault Clearing Time is Taken into Account

Let P_{m1}, P_{m2} and P_{m3} be the maximum power transferrable under pre-fault, during-fault and post-fault conditions. Refer to figure 6.2. Let P be the input power in this case.

$$\text{Area } A_1 = \int_{\delta_1}^{\delta_2} (P - P_{m2} \sin \delta) d\delta + \int_{\delta_2}^{\delta_3} (P - P_{m3} \sin \delta) d\delta$$

$$= P(\delta_2 - \delta_1) + P_{m2}(\cos \delta_2 - \cos \delta_1) + P(\delta_3 - \delta_2)$$
$$+ P_{m3}(\cos \delta_3 - \cos \delta_2)$$
$$= P(\delta_3 - \delta_1) + P_{m2}(\cos \delta_2 - \cos \delta_1) + P_{m3}(\cos \delta_3 - \cos \delta_2)$$

$$\text{Area } A_2 = \int_{\delta_3}^{\pi - \delta_3} [P_{m3} \sin \delta - P] d\delta$$

$$= - P_{m3}[\cos(\pi - \delta_3) - \cos \delta_3] - P[\pi - \delta_3 - \delta_3]$$

$$= 2P_{m3} \cos \delta_3 - P[\pi - 2\delta_3]$$

$$\text{Therefore } A_2 - A_1 = P[\delta_3 + \delta_1 - \pi] + P_{m2}[\cos \delta_1 - \cos \delta_2]$$
$$+ P_{m3}[\cos \delta_3 + \cos \delta_2]$$

Dividing it by area under post-fault curve equal to 2 P_{m3}

$$\text{Stability Margin} = \frac{1}{2}[\cos \delta_2 + \cos \delta_3] + \frac{P_{m2}}{2P_{m3}}[\cos \delta_1 - \cos \delta_2]$$
$$+ \frac{P}{2P_{m3}} [\delta_1 + \delta_3 - \pi] \qquad (6.3)$$

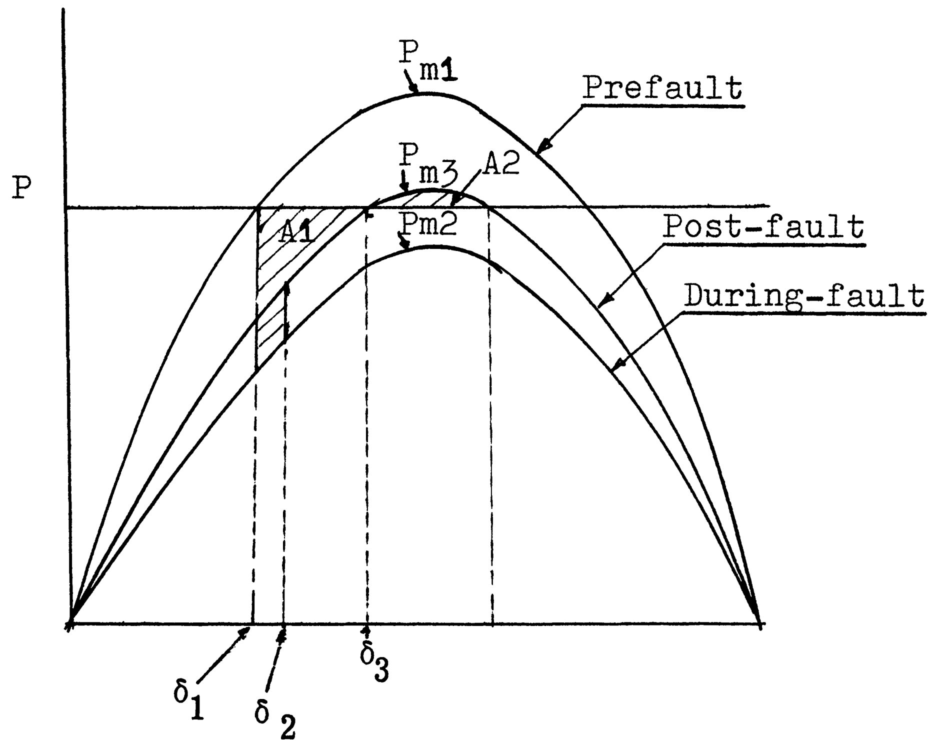


Fig. 6.2 Pre-fault, During-fault, and Post-fault Power Angle Curves

Example 6.1

The following example illustrates the application of the concent of normalized stability margin.

The equations describing the power flow from a generator are

Before the fault $P_o = 1.735 \sin \delta$

During the fault $P_o = .42 \sin \delta$

After the fault $P_o = 1.25 \sin \delta$

where P_o is the power output from the generator.

Assume fault removal when $\delta = 45^\circ$. Calculate the stability margin, if generator is delivering 1.0 per unit power.

$$P_{m1} = 1.735, \delta_1 = \sin^{-1} \frac{1.0}{1.735} = 35.2^\circ = .615 \text{ radian}$$

$$P_{m2} = .42, \delta_2 = 45^\circ = .785 \text{ radian}$$

$$P_{m3} = 1.25, \delta_3 = \sin^{-1} \frac{1.0}{1.25} = 53.1^\circ = .928 \text{ radian}$$

$$\text{Stability Margin} = \frac{1}{2}\{.60 + .707\} + \frac{.42}{2.50}\{.818 - .70\} + \frac{1.0}{2.50}\{.61 + .928 - 3.141\}$$

$$= .20 \text{ unit.}$$

If the fault is removed at an angle $\delta = 51.6^\circ$, then stability margin

$$= \frac{1}{2}\{.60 + .622\} + \frac{.42}{2.50}\{.818 - .622\} + \frac{1.0}{2.50}\{.615 + .928 - - 3.141\} = 0.0 \text{ per unit}$$

Hence this corresponds to critical switching angle.

Chapter VII

PHASE SHIFTING TRANSFORMERS (BASIC ANALYSIS)

Introduction

Another method of improving transient stability in a power system is investigated. This method involves using phase-shifting transformers to shift the power angle curve by a certain angle to produce the desired control of swings. The phase-shift windings of the transformers can be short time rated which will reduce their cost.

Principle of the Method

Figure 7.1 shows a generator G supply power to an infinite bus through a double circuit transmission line. On the sending end there is a transformer T which has a phase shifting arrangement as shown in the figure. A is the main breaker, while B is an additional breaker. The two breakers have an interlocking device so that at any time when one breaker is closed, the other is open and vice versa. The generator is supplying power P_1. The voltage behind transient reactance of the generator is E'_q and that of the infinite bus is v_b. X is the sum of the line reactance, the transformer reactance and the transient reactance. Therefore

$$P_1 = \frac{E'_q v_b}{X} \sin \delta$$

where δ is phase angle between the two voltages.

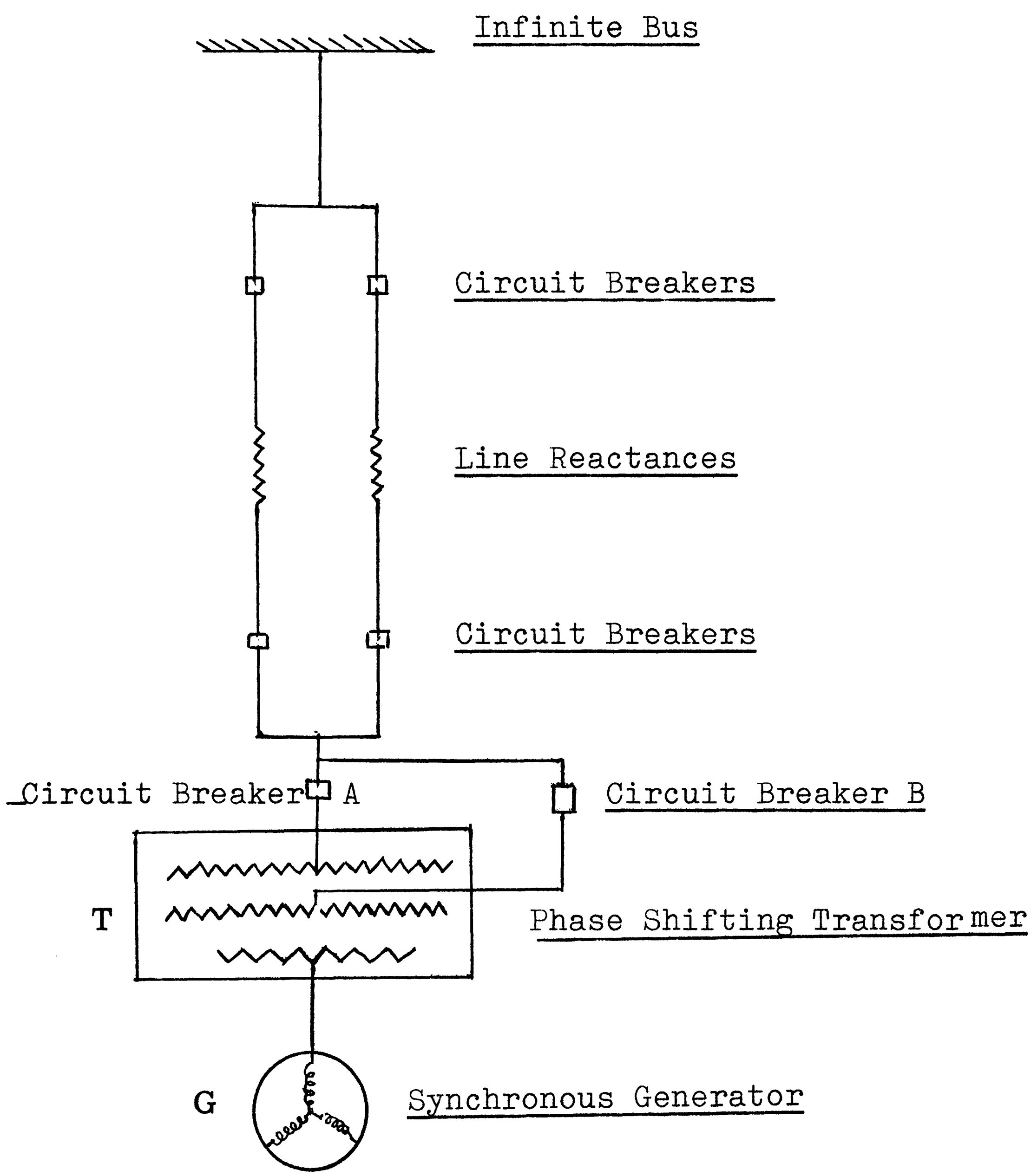


Fig. 7.1 Application of P.S. Transformers in a Typical System

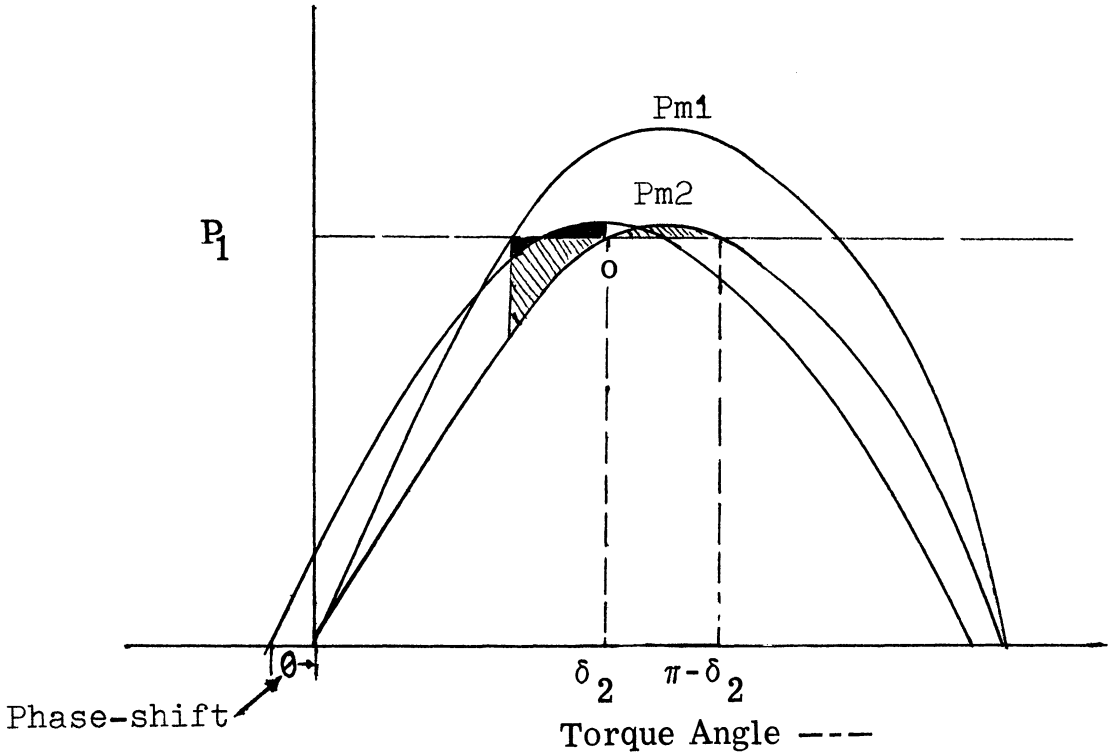


Fig. 7.2 Effect of Phase Shift on Power Angle Curve

The usual power angle curve is shown in figure 7.2. A fault occurs on one of the lines which is switched out due to the action of the fast-acting breakers. The loss of the line increases the reactance between the two voltages and operation moves from pre-fault to post-fault power angle curve. Since the duration of the fault is very short, no "during-fault" curve is shown for the sake of simplicity. As expected, the generator rotor begins to accelerate. In the case illustrated in the figure, the system is unstable since P_1 is greater than transient stability limit of the system.

If it is desired to control the swings, breaker A opens and B closes at the same time that the line is switched out. This is not difficult, since the relays which switch out the line can open A and close B also. Due to the phase-shifting property of the windings, the post-fault power angle is shifted towards the left by an angle θ as shown in figure 7.2. The result of this shift is the accelerating area is reduced while the decelerating area is increased, thereby increasing the stability margin. The magnitude of the swing is reduced too. If the angle θ is of correct magnitude, it is possible to bring the rotor to point o such that $\frac{d\delta}{dt} = 0$ at o. Under that condition, if the phase-shift is removed when the operating point reaches o, there will be no back swing. Thus the use of phase shifting transformers has resulted in bringing the operating point to o in a dead-beat non-oscillatory manner.

Phase-Plane Analysis of the Effect of Phase Shifting

To illustrate the basic effects of phase-shifting transformers, the phase-plane[10] technique is used to analyze the swing equation. The following assumptions are made.

1. Damping is ignored.
2. The synchronous machine is represented by a fixed voltage behind the transient reactance.
3. Saturation is neglected.
4. The prime mover input remains constant during a swing.
5. The machine has a smooth rotor.

Therefore

$$M \frac{d^2\delta}{dt^2} = P_1 - P_{m1} \sin \delta \tag{7. 1}$$

where $P_{m1} = \dfrac{E'_q v_b}{X}$

The steady-state condition is given by

$$P_1 = \frac{E'_q v_b}{X} \sin \delta_0 \tag{7. 2}$$

The representative point is shown in phase-plane figure 7.3, by A. A 3-phase fault occurs on the system which reduces the output to zero, hence

$$M \frac{d^2\delta}{dt^2} = P_1 \text{ during the fault}$$

$$M \frac{d\delta}{dt} = P_1 t + C \tag{7.3}$$

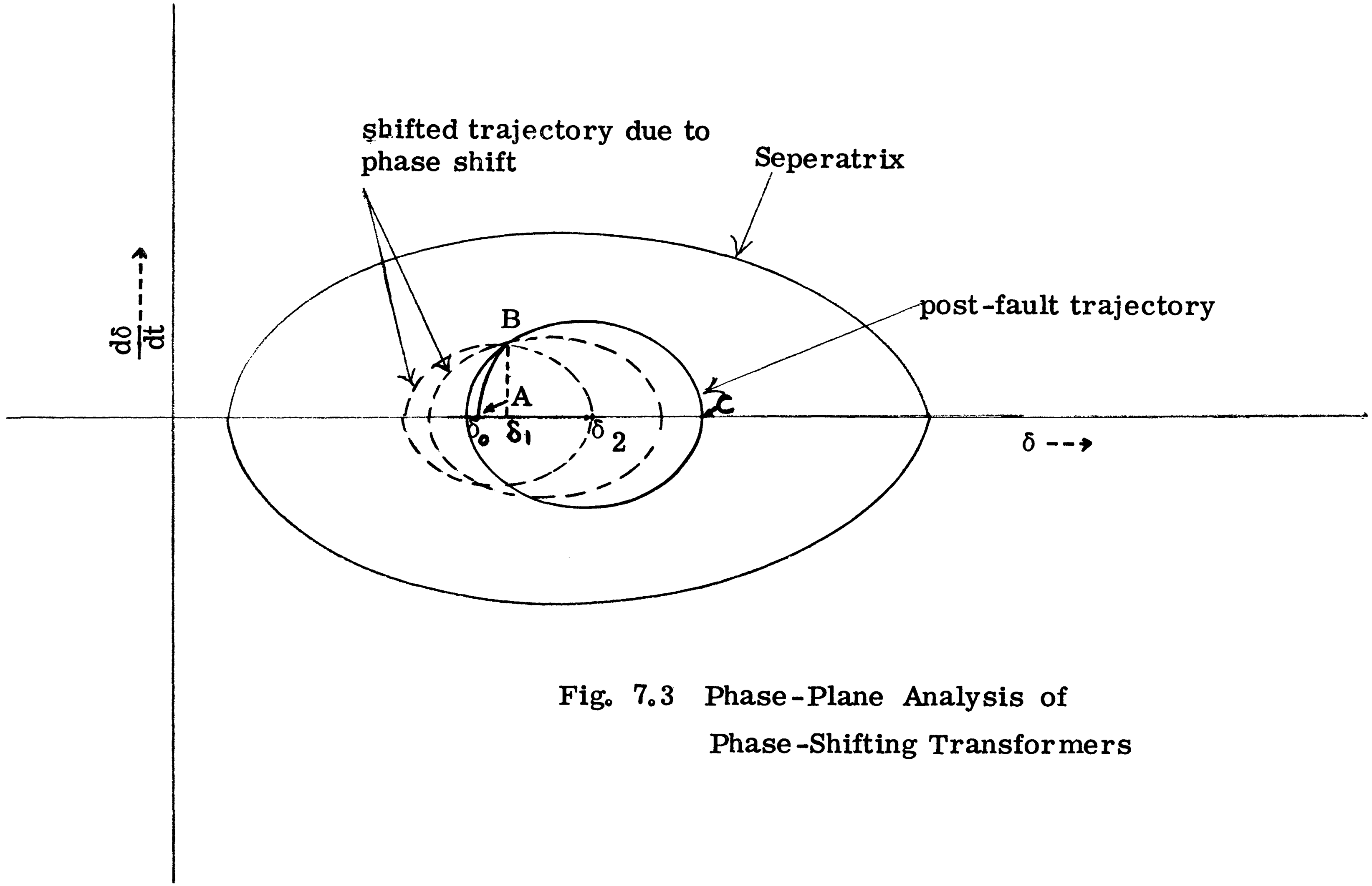


Fig. 7.3 Phase-Plane Analysis of Phase-Shifting Transformers

t = time after occurrence of the fault

or $M\omega = P_1 t + C$, where $\omega = \frac{d\delta}{dt}$, C is a constant. When $t = 0$, $\omega = 0$, therefore C=0. Therefore the phase-trajectory equation during the fault is

$$M\omega = P_1 t$$

$$\text{or } \omega = \frac{P_1 t}{M} = \frac{d\delta}{dt} \tag{7.4}$$

Integrating again the above equation

$$\delta = \frac{P_1}{2M} t^2 + C_1. \quad C_1 \text{ is a constant. When } t = 0, \ \delta = \delta_0,$$

therefore $C_1 = \delta_0$. Hence

$$\frac{P_1}{2M} t^2 = \delta - \delta_0 \tag{7.5}$$

Eliminating the variable t in equations (7. 4) and (7. 5), the result is

$$\delta - \delta_0 = \frac{P_1}{2M} \frac{\omega^2}{P_1^2} M^2 = \frac{M}{2P_1} \omega^2$$

$$\text{or} \quad \omega^2 = \frac{2P_1}{M} (\delta - \delta_0) \tag{7.6}$$

In the phase-plane, equation (7. 6) is represented by a trajectory AB, which represents the "during fault" portion of the trajectory.

After a time t_1, the fault is removed and the swing equation becomes

$$M \frac{d^2\delta}{dt^2} = P_1 - P_{m2} \sin \delta \qquad (7.7)$$

where $P_{m2} = \frac{E'_q v_b}{X'}$ and X' is the post-fault reactance.

The initial conditions for equation 7.7 are: when $t = 0$, $\delta = \delta_1$, $\omega = \omega_1$, where $\omega_1 = \frac{P_1}{M} t_1$, $\delta_1 = \frac{M\omega_1^2}{2P_1} + \delta_0$ from equations 7.4 and 7.6.

Assume the new steady-state point in the phase plane to be $(\delta_2, 0)$. Then

$$P_{m2} \sin \delta_2 = P_1.$$

Also,

$$\omega = \frac{d\delta}{dt}$$

$$\frac{d^2\delta}{dt^2} = \frac{d\omega}{dt} = \frac{1}{M}[P_1 - P_{m2} \sin \delta]$$

or $$\frac{d\omega}{d\delta} = [P_1 - P_{m2} \sin \delta] \frac{1}{\omega M}$$

or $$\omega\, d\omega = [\frac{P_1}{M} - \frac{P_{m2}}{M} \sin \delta]d\delta$$

Integrating, $$\frac{\omega^2}{2} = \frac{P_1}{M}\delta + \frac{P_{m2}}{M} \cos \delta + C_2 \qquad (7.8)$$

C_2 is a constant. For the transient stability limit $\omega = 0$ when $\delta = \pi - \delta_2$, which follows from the application of equal area criterion to figure 7.2.

$$C_2 = -\frac{P_1}{M}(\pi - \delta_2) - \frac{P_{m2}}{M}[\cos(\pi - \delta_2)]$$

$$= -\frac{P_1}{M}(\pi - \delta_2) + \frac{P_{m2}}{M}\cos\delta_2$$

or $$\frac{\omega^2}{2} = \frac{P_1}{M}\delta + \frac{P_{m2}}{M}\cos\delta + \frac{P_{m2}}{M}\cos\delta_2 - \frac{P_1}{M}(\pi - \delta_2)$$

$$= \frac{P_1}{M}(\delta - \pi + \delta_2) + \frac{P_{m2}}{M}(\cos\delta + \cos\delta_2) \qquad (7.9)$$

This is the equation of the Seperatrix. If the fault is removed while the representative point is within the area enclosed by the seperatrix, the system is stable, otherwise it is unstable.

Suppose that the fault is removed when the representative point is at B, which has coordinates ω_1, δ_1. Substituting this condition in equation 7. 8.

$$\frac{\omega_1^2}{2} = \frac{P_1}{M}\delta_1 + \frac{P_{m2}}{M}\cos\delta_1 + C_2$$

$$C_2 = \frac{\omega_1^2}{2} - \frac{P_1}{M}\delta_1 - \frac{P_{m2}}{M}\cos\delta_1$$

Therefore $$\frac{\omega^2}{2} = \frac{P_1}{M}(\delta - \delta_1) + \frac{P_{m2}}{M}(\cos\delta - \cos\delta_1) + \frac{\omega_1^2}{2} \qquad (7.10)$$

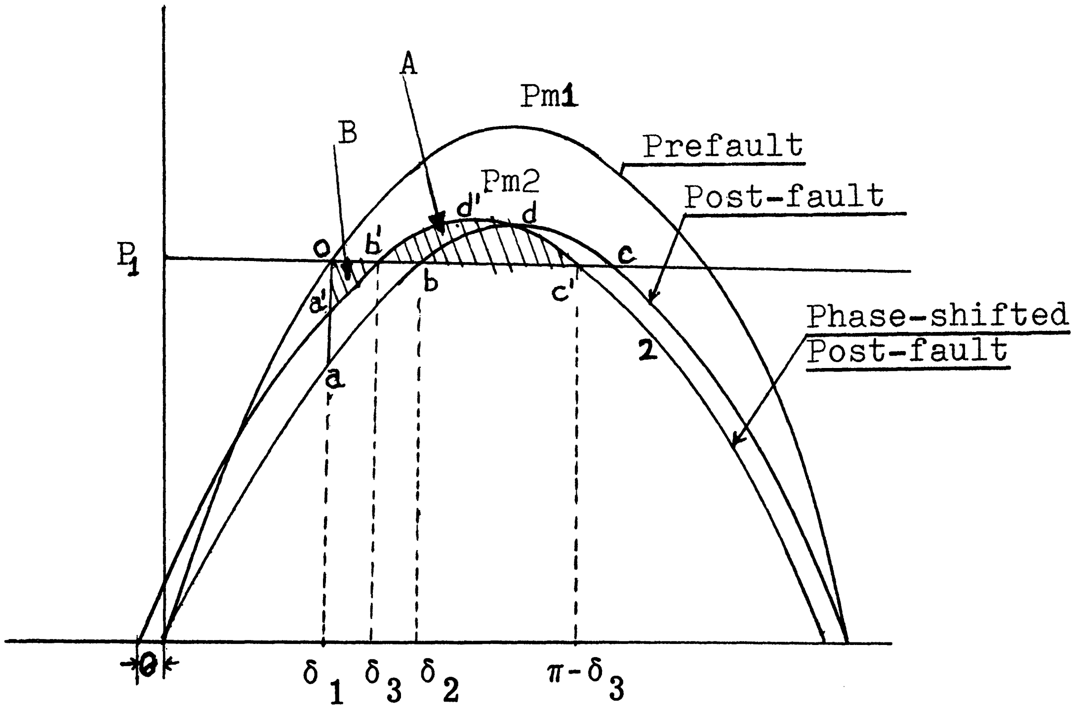


Fig. 7.4 Determination of the Effect of Phase-Shift on Stability Margin

This is shown in figure 7.3 by post-fault trajectory. It will be seen that the representative point oscillates continuously around the new steady state point δ_2.

Suppose that at the time of switching out of the line, the power angle is shifted towards the left by an angle θ. Then the swing equation becomes

$$M \frac{d^2\delta}{dt^2} = P_1 - P_{m2} \sin(\delta + \theta)$$

The apparent singular point is $\delta_3 = \delta_2 - \theta$. The trajectory equation (7.10) becomes

$$\frac{\omega^2}{2} = \frac{P_1}{M} (\delta - \delta_1) + \frac{P_{m2}}{M} (\cos(\delta + \theta) - \cos(\delta_1 + \theta)) + \frac{\omega_1^2}{2} \quad (7.11)$$

The new trajectory is again oscillatory and is shown in the dotted lines. Figure 7.3 shows qualitatively how the trajectories are shifted towards the left by the action of the phase shifters.

If the phase shift is of such a magnitude that the point C passes through δ_2, then it would be possible to remove the phase shift at that instant and the representative point will stay there without further movement, so that the transient is controlled in a dead-beat manner.

<u>Effect of Phase Shift on Stability Margin</u>

In chapter VI, the concept of stability margin was introduced as it gives a quantitative idea of stability. Referring to figure 7.4

$$\begin{array}{l}\text{Stability Margin} \\ \text{with phase shifting}\end{array} = \frac{\text{area b'd'c' - area o a'b'}}{\text{Total area under post-fault curve}} \quad (7.12)$$

Stability margin without phase shifting

$$= \frac{\text{area bdc - area oab}}{\text{Total area under post-fault curve}} \qquad (7.13)$$

Therefore increase in stability margin

$$= (7.12) - (7.13)$$

$$= \frac{\text{area oab - area oa'b'}}{\text{Total area under post-fault curve}}$$

(since area b'd'c' = area bdc)

$$\text{area oab} = \int_{\delta_1}^{\delta_2} (P_1 - P_{m2} \sin \delta)\, d\delta$$

$$= P_1(\delta_2 - \delta_1) + P_{m2}(\cos \delta_2 - \cos \delta_1)$$

Similarly

$$\text{area oa'b'} = P_1(\delta_3 - \delta_1) + P_{m2}(\cos \delta_3 - \cos \delta_1)$$

Total area under post-fault curve = $2\ P_{m2}$ approximately with and without phase shift.

$$\text{Therefore increase in stability margin} = \frac{P_1(\delta_2 - \delta_3) + P_{m2}(\cos \delta_2 - \cos \delta_3)}{2\ P_{m2}}$$

Since $\delta_3 = \delta_2 - \theta$

$$\text{Increase in stability margin} = \frac{P_1\ \theta + P_{m2}(\cos \delta_2 - \cos(\delta_2 - \theta))}{2\ P_{m2}}$$

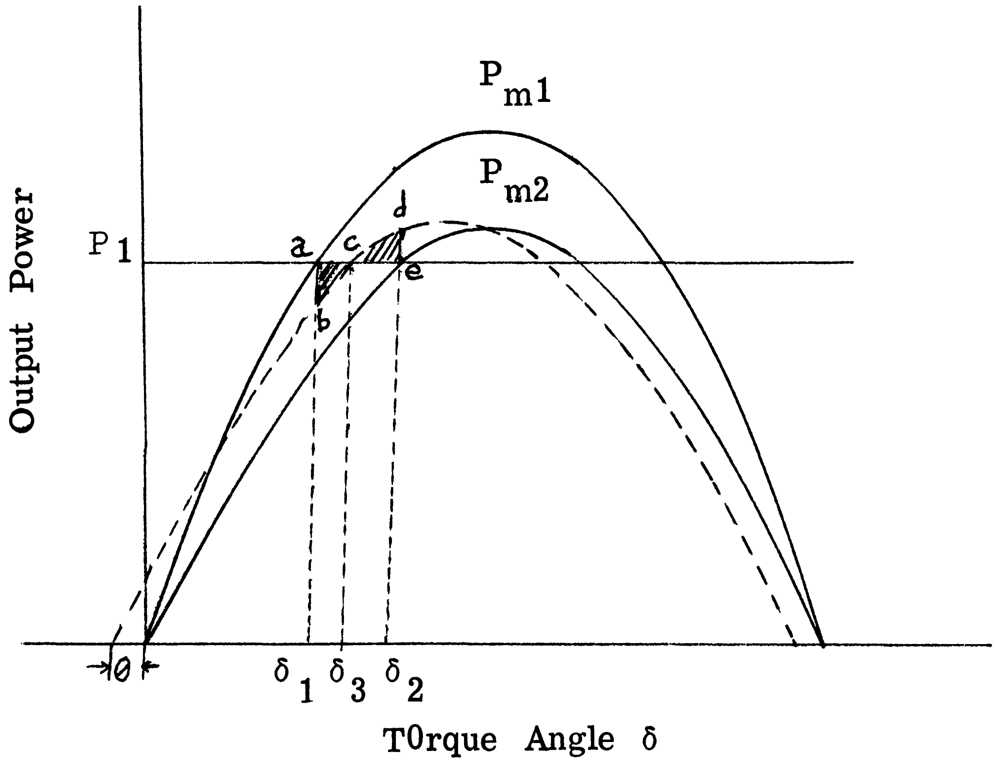


Fig. 7.5 θ for Dead-Beat Control

Determination of θ for Dead-beat Transient Control

Figure 7.5 shows the usual pre-fault and post-fault power angle curves. The apparent power angle curve, while the phase shifters are energized, is shown in the dotted curve. It is desired to find the value of θ which moves the representative point from a to e in a dead-beat non-oscillatory fashion, if the phase-shift is removed when it reaches d.

The criterion for the above is that the area abc = the area cde such that when the representative point reaches d, its velocity is zero. If the phase-shift is removed at d, it will occupy point e and stay there permanently, since power input equals output. Therefore,

$$\int_{\delta_1}^{\delta_3} [P_1 - P_{m2} \sin(\delta + \theta)]d\delta = \int_{\delta_3}^{\delta_2} [P_{m2} \sin(\delta + \theta) - P_1]d\delta$$

or $P_1(\delta_3 - \delta_1) + P_{m2} [\cos(\delta_3 + \theta) - \cos(\delta_1 + \theta)]$

$$= - P_{m2}[\cos(\delta_2 + \theta) - \cos(\delta_3 + \theta)] - P_1[\delta_2 - \delta_3]$$

or $P_1 [\delta_2 - \delta_1] = P_{m2} [\cos(\delta_1 + \theta) - \cos(\delta_2 + \theta)]$

or $\dfrac{P_1}{P_{m2}} [\delta_2 - \delta_1] = \cos\delta_1 \cos\theta - \sin\delta_1 \sin\theta - \cos\delta_2 \cos\theta + \sin\delta_2 \sin\theta$

$$= \cos\theta [\cos\delta_1 - \cos\delta_2] - \sin\theta [\sin\delta_1 - \sin\delta_2]$$

$$\text{Let}\quad \left.\begin{aligned} \cos\delta_1 - \cos\delta_2 &= A\sin\alpha \\ \sin\delta_1 - \sin\delta_2 &= A\cos\alpha \end{aligned}\right\} \text{ or } \tan\alpha = \frac{\cos\delta_1 - \cos\delta_2}{\sin\delta_1 - \sin\delta_2} \text{ and}$$

$$A^2 = (\cos\delta_1 - \cos\delta_2)^2 + (\sin\delta_1 - \sin\delta_2)^2$$

$$= 4\sin^2\left(\frac{\delta_1 - \delta_2}{2}\right)$$

$$\text{or } A = 2\sin\left(\frac{\delta_1 - \delta_2}{2}\right)$$

Then

$$\frac{P_1}{P_{m2}}[\delta_2 - \delta_1] = [\cos\theta\sin\alpha - \sin\theta\cos\alpha]A$$

$$= A\sin(\alpha - \theta)$$

$$\alpha - \theta = \sin^{-1}\frac{P_1}{A\,P_{m2}}[\delta_2 - \delta_1]$$

$$\text{or}\quad \theta = \alpha - \sin^{-1}\frac{P_1}{A\,P_{m2}}[\delta_2 - \delta_1]$$

$$\text{or}\quad \theta = \tan^{-1}\frac{\cos\delta_1 - \cos\delta_2}{\sin\delta_1 - \sin\delta_2} - \sin^{-1}\frac{P_1}{A\,P_{m2}}[\delta_2 - \delta_1] \qquad (7.16)$$

Increase in Power Transmitted

Figure 7.6 represents a pre-fault, a post-fault without phase shift and a post-fault with phase shift power angle curves. Without phase shift control, the maximum power transmitted will be P_1

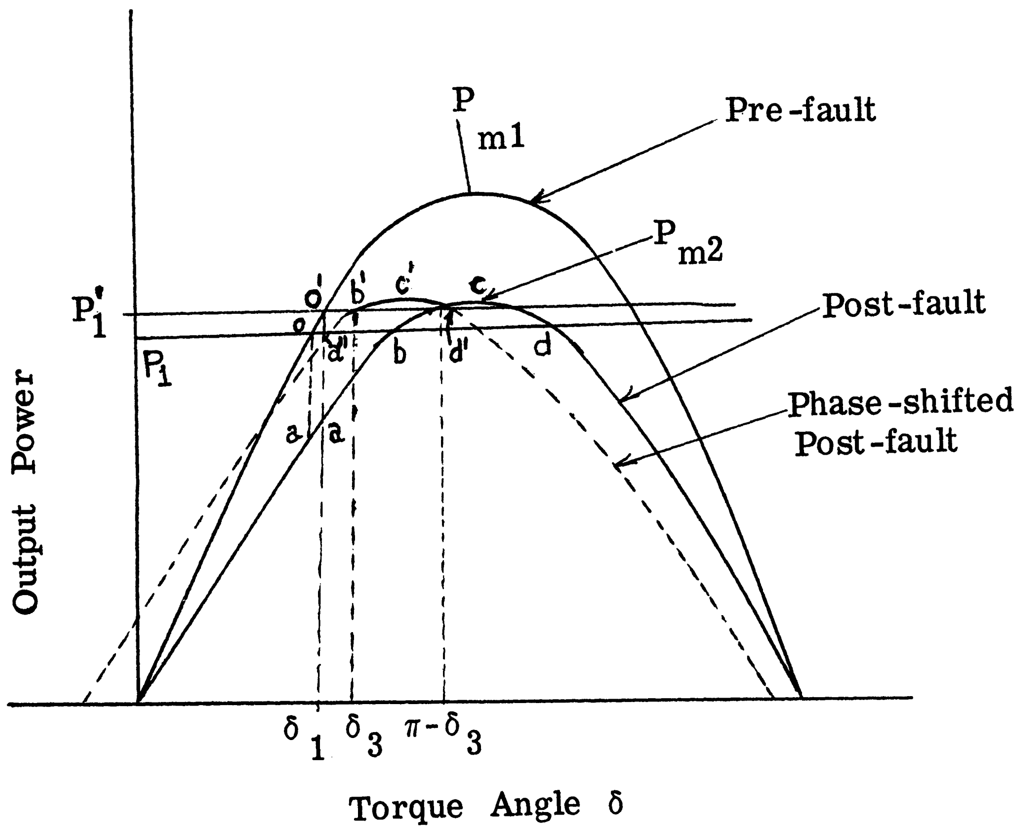


Figure 7.6 Determination of Minimum θ

which is determined graphically, satisfying the equal area criterion, such that area oab = area bcd.

It is desired to transmit power greater than P_1 over the system by employing phase shifting technique. Suppose it is desired to increase P_1 to P_1', then the power angle curve needs to be shifted by an amount θ such that area o'a''b' = area b'c'd' which is possible for some value of θ, as can be observed from the figure. Thus the phase shifting transformers have made it possible to raise the power transmitted to a level more than the transient stability limit P_1, in the case under consideration.

The maximum value of the power transmitted is P_{m2}, the steady-state limit in the post-fault condition. It will not be practicable to raise the power to this value. The maximum value of the transmitted power will depend upon the maximum angle δ_2 desired in the post-fault state, which should be less than 90^0. Hence if δ_2 be the maximum angle desired, then $P_1' = P_{m2} \sin \delta_2$. The phase-shifting transformer windings will be designed in such a way that power P_1' can be transmitted.

The minimum angle of phase shift will be necessary if area o'a''b' = area b'c'd' or that full area above the power line is used in decelerating the rotor.

Referring again to figure 7.6, area o'a''b' = area b'c'd' or

$$\int_{\delta_1}^{\delta_3} [P_1' - P_{m2} \sin(\delta + \theta)]d\delta = \int_{\delta_3}^{\pi - \delta_3 - \theta} [P_{m2} \sin(\delta + \theta) - P_1']d\delta$$

This gives

$$\frac{P_1'}{P_{m2}}[\pi - \delta_1 \underset{-\theta}{-} \delta_3] = [\cos(\delta_1 + \theta) + \cos \delta_3]$$

But

$$P_1' = P_{m2} \sin \delta_2 = P_{m1} \sin \delta_1$$

$$\text{and } \delta_2 = \delta_3 + \theta$$

Therefore

$$\frac{P_1'}{P_{m2}}[\pi - \delta_1 - \delta_2] = [\cos(\delta_1 + \theta) + \cos(\delta_2 - \theta)] \tag{7.17}$$

$$\frac{P_1'}{P_{m2}}[\pi - \delta_1 - \delta_2] = \cos \delta_1 \cos \theta - \sin \delta_1 \sin \theta + \cos \delta_2 \cos \theta + \sin \delta_2 \sin \theta$$

$$= [\cos \delta_1 + \cos \delta_2] \cos \theta + [\sin \delta_2 - \sin \delta_1] \sin \theta.$$

Let

$$\cos \delta_1 + \cos \delta_2 = A \sin \alpha \qquad \text{or} \qquad \tan \alpha = \frac{\cos \delta_1 + \cos \delta_2}{\sin \delta_2 - \sin \delta_1}$$

$$\sin \delta_2 - \sin \delta_1 = A \cos \alpha$$

$$\text{or } \alpha = \tan^{-1} \frac{\cos \delta_1 + \cos \delta_2}{\sin \delta_2 - \sin \delta_1}$$

and

$$A = 2 \cos \frac{(\delta_1 + \delta_2)}{2}$$

Therefore

$$\frac{P_1'}{P_{m2}}[\pi - \delta_1 - \delta_2] = A[\sin(\alpha + \theta)]$$

or

$$\theta = \sin^{-1} \frac{P'_1 \left[\pi - \delta_1 - \delta_2\right]}{P_{m2}\, 2 \cos \frac{(\delta_1 + \delta_2)}{2}} - \tan^{-1} \frac{\cos \delta_1 + \cos \delta_2}{\sin \delta_2 - \sin \delta_1}$$

Application to a Typical System

Chapter VIII gives details of an actual system and computer details when the technique of phase shift control was applied to a typical system.

Chapter VIII

APPLICATION OF PHASE SHIFTING TRANSFORMERS

Introduction

This chapter gives details of an investigation of a typical system using the method of phase-shifting transformers as developed in the preceding chapter. The study which was done by digital simulation indicates that there is considerable potential in this technique and it is hoped that the electric companies will be encouraged to explore its application to their systems.

System Studied

The system consists of a synchronous generator supplying power to an infinite bus by means of a double circuit transmission line, as shown in Figure 8.1. A fault occurs at point P which results in opening of the circuit breakers adjacent to P thereby raising transfer reactance between the generator and the infinite bus. The equations for the power output of the generator are:

Before the fault $P_{m1} \sin\delta = 1.735 \sin\delta$

During the fault $P_{m2} \sin\delta = 0.42 \sin\delta$

After the fault $P_{m3} \sin\delta = 1.25 \sin\delta$

Input Power = 1.15, hence $\delta_1 = \sin^{-1} \frac{1.15}{1.735} = 41.4°$,

$\pi - \delta_2 = \sin^{-1} \frac{1.15}{1.25} = 113°$.

The power angle curves are shown in figure 6.2.

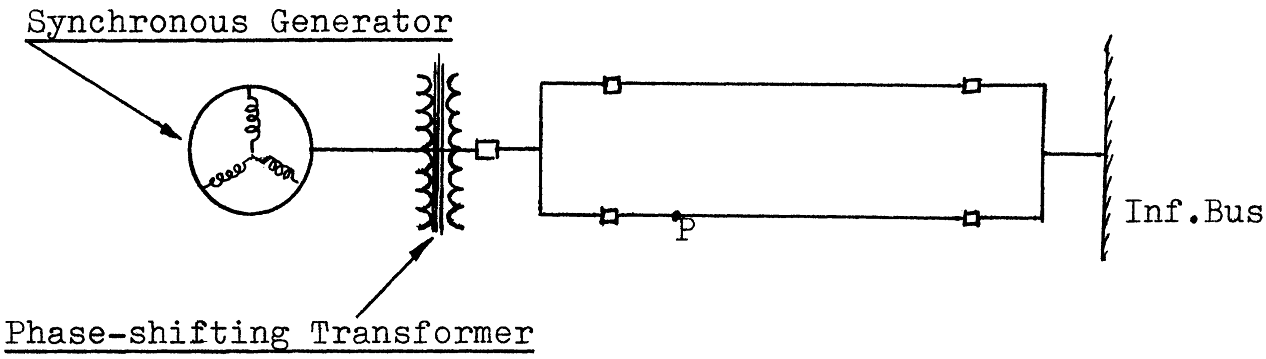


Fig. 8.1 A Typical System Using a P.S. Transformer

Modern beakers are capable of operating in 3 cycles (clearing time .05 sec.). From standard curves, the clearing angle for this case is 45.3°.

A computer program was written to calculate the data for the swing curve. The constants of the generator were as follows:

$$H = 3.0 \quad G = 1$$

Therefore

$$M = \frac{GH}{180 \times f} = 2.78 \times 10^{-4} \text{ per unit}$$

A time interval of 0.05 sec. was selected for the calculations.

Figure 8.2 shows the results of different values of phase shift on stability of the generator. The figure shows three curves obtained for phase shifts of 0°, 5°, and 10°. The effect of 10° phase shift is to decrease the maximum torque angle from 85° to 68°, a decrease of 17°. Similarly, the phase shift of 5° decreases the maximum torque angle by 10°. Therefore the amount of phase shift has considerable effect on the shape of the swing curve.

The computer program and the flow chart are enclosed in the appendix.

It may be mentioned here that there are many appli cation problems to be analyzed before the techniques of phase shifting transformers can be used in actual systems.

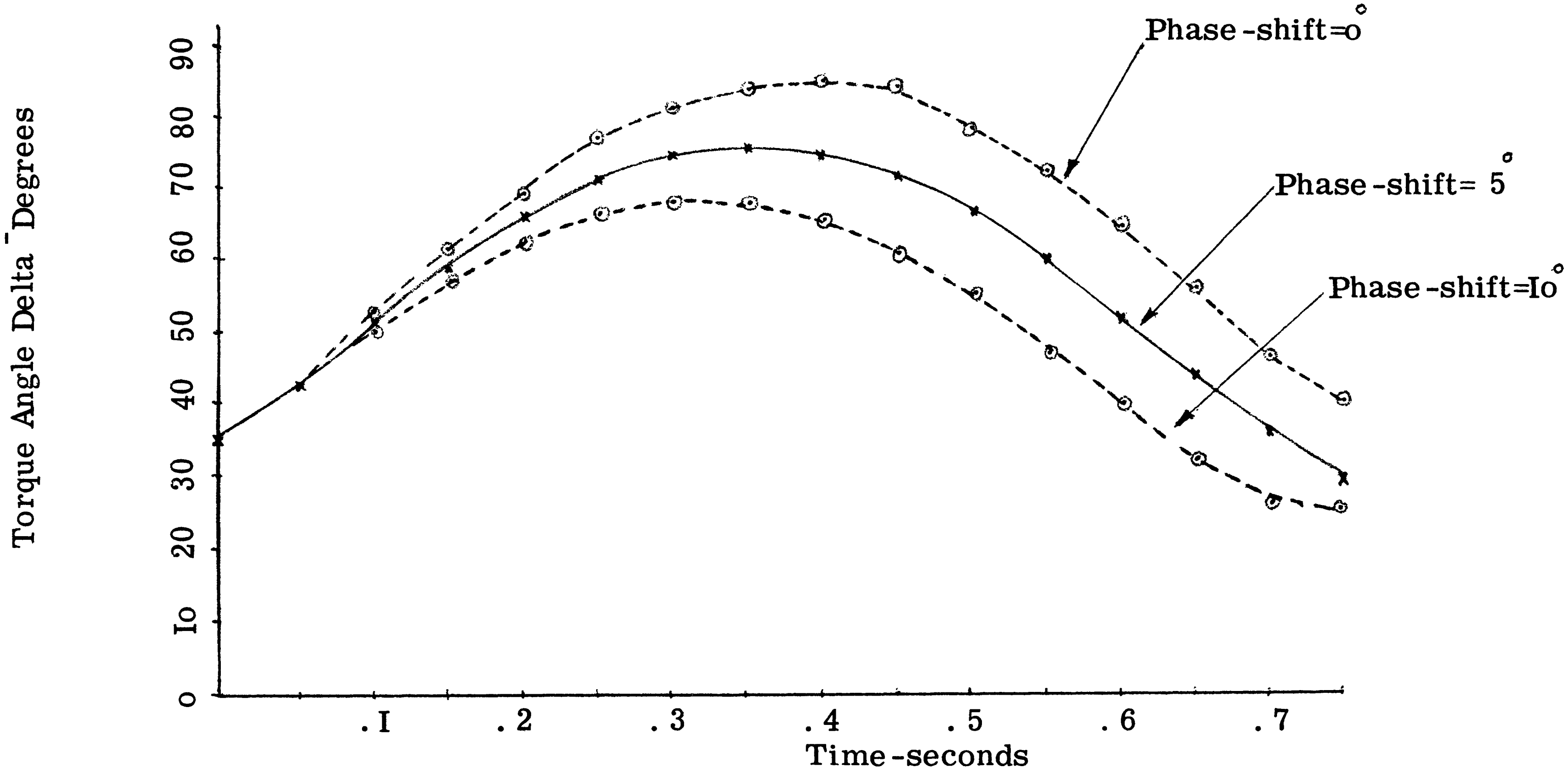


Fig. 8.2 Effect of Phase-shift Transformers on Swing Curve

Chapter IX

SWITCHED SERIES CAPACITORS

(APPROXIMATF ANALYSIS)

Introduction

A very effective method of raising stability limits of long transmission lines is by means of fixed series capacitors. There are some excellent papers on this subject in the literature.

In 1965 Kimbark[6] suggested the use of switched series capacitors which involves switching in a capacitor in series with the line as soon as a line section is switched out. The idea behind this scheme is that by switching in a capacitor, the increase in line reactance, caused by switching out of a section, is minimized. Kimbark's paper was followed by that of Smith[12]. He suggested that capacitors be switched in and out at appropriate instants to control the transients, the exact instants would be determined by a centralized control computer.

Scope of Investigation

Chapters IX to XI of this report relate to switched series capacitors as shown below:

Chapter IX. This chapter is concerned with an approximate analysis of the fundamental effects of switched series capacitors. As in Kimbark[6] and Smith's[12] papers, equal area criterion is the method used in the investigation. It is shown that the capacitive reactance needed to maintain stability can be minimized if dead-beat operation is not desired and formulas are derived for capacitor sizes under different modes of operation. A simple system is simulated on a digital computer to find the effect of switching in different sizes of capacitors.

Chapter X. This chapter is a detailed report on some experimental work performed in the Power Systems Laboratory of the University of Michigan. The experimental results are compared with the results obtained by digital simulation of the system.

Chapter XI. This chapter is concerned with the study of the following problems arising in application of switched series capacitors.

i. Effect, on system stability, of a secondfault which may occur while the capacitor is in circuit and the machines are swinging.

ii. Effect of Switching in a series capacitor on a circuit with distance relaying

Approximate Analysis

The aim of the approximate analysis is to investigate the fundamental effects of switched series capacitors. The investigation is restricted to a system which has two similar parallel lines such that the reactances of the two lines are equal.

Figure 9. 1 represents a double circuit transmission line connecting a generator to an infinite bus. X_1 is the reactance of each circuit. X_c is the reactance of the switched series capacitor. Circuit

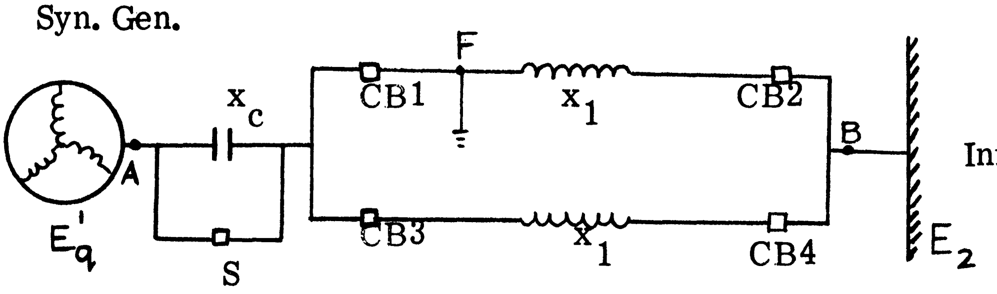


Fig. 9. 1 A Typical System Using Switched Series Capacitors

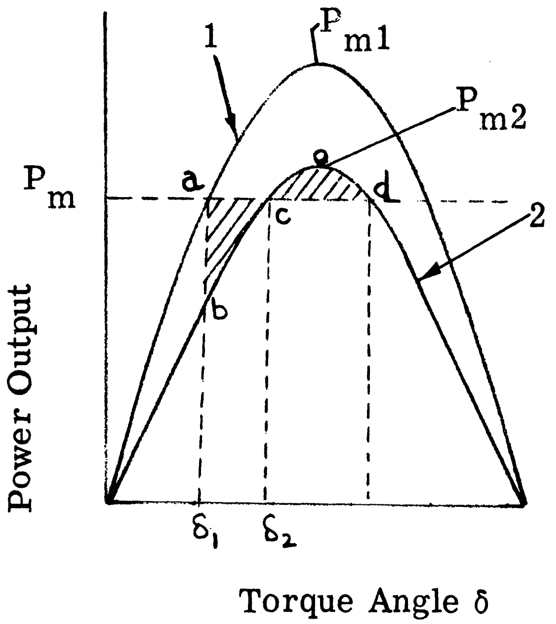


Fig. 9. 2 Power Angle Curves for the Above System

breaker S is normally closed and it opens when any of the circuit breakers CB-1 through CB-4 opens. It is assumed that the same relay initiates the tripping of the breakers S and any of the breakers CB-1 through CB-4.

When a fault occurs at point F, circuit breakers CB-1 and CB-2 trip, raising the line impedance from $\frac{x_1}{2}$ to x_1. Since circuit breaker S opens, the net reactance between A and B is raised from $\frac{x_1}{2}$ to $x_1 - x_c$. It is therefore clear that if the value of x_c is chosen to be $\frac{x_1}{2}$, the line reactance will remain unchanged although one circuit has been relayed out. It will however be shown that it is not necessary to have the capacitive reactance x_c equal to $\frac{x_1}{2}$, since a significant improvement in the transient stability limit will be achieved even when x_c is much lower than $\frac{x_1}{2}$.

Without Switched Capacitor

Figure 9.2 is the power angle diagram for the above system. Sine-curve no. 1 represents the prefault curve and no. 2 the post fault curve. The during-fault curve is neglected since fault duration is very small. P_{m1} and P_{m2} are the amplitudes of the power angle curves given by

$$P_{m1} = \frac{E'_q E_2}{x'_d + 0.5\,x_1}, \quad P_{m2} = \frac{E'_q E_2}{x'_d + x_1}$$

where

E'_q is the voltage behind the transient reactance of the generator

E_2 is the voltage of the infinite bus

x'_d is the transient reactance of the generator

x_1 is the reactance of each line.

Since it is an approximate method, the assumption will be made that x'_d is small in comparison with x_1 and hence may be neglected in the derivation to follow.

Let P_m be the transient stability limit derived on the basis of the equal area criterion such that area abc = area cod. P_m can then be determined from the following equation

$$P_m[\pi - \delta_2 - \delta_1] = P_{m2}[\cos \delta_2 + \cos \delta_1] \quad (3.1)$$

where δ_1 and δ_2 are given by the following equations

$$\delta_1 = \sin^{-1} \frac{P_m}{P_{m1}} \qquad \delta_2 = \sin^{-1} \frac{P_m}{P_{m2}}$$

Switched-Series Compensation

The effect of switched series compensation can be seen from figure 9. 3.

It is assumed that the circuit breaker S is operated by the same relay which operates either circuit breaker CB1 or CB2, thus the capacitor x_c is inserted as soon as the faulty circuit is taken out and reactance will increase from $\frac{x_1}{2}$ to $(x_1 - x_c)$. Consequently, the operation will be transferred from curve 2 to 2'.

The operating point moves from 1 to 2 and 2 to 3. The rotor starts to advance along the dashed curve from 3 to 4 at which point

$\frac{d\delta}{dt}$ is maximum. From 4, the operating point moves along the dashed curve to 5 at which point $\frac{d\delta}{dt} = 0$.

If the value of x_c is chosen such that 5 and 6 lie on the same vertical line, it is sufficient to remove the capacitor at 5 and the operating point will move from 5 to 6 and stay there, since 6 is the steady-state point in the post-fault condition. This method was originally suggested by Smith[12].

The operation has therefore moved from 1 to 6 in a dead-beat, non-oscillatory fashion with the aid of a switched-series capacitor. The value of the capacitor which will be required for the process will be unique for the loading P_m.

It can be argued from the above that, by means of a switched series capacitor, it will be possible to increase the transient stability limit up to the value of P_{m2}, with a maximum value of $\delta = 90°$. This is indicated in figure 9.4.

However, it would not be practicable to operate at $\delta = 90°$ and, therefore, the transient stability limit will be less than P_{m2}.

Selection of Capacitor Size

There are two criteria for selecting the capacitor size as follows:

(1) To produce dead-beat non-oscillatory transfer from pre-fault to post-fault state.

(2) To maintain stability but with operation which is not dead-beat. (This will result in a smaller capacitor size.)

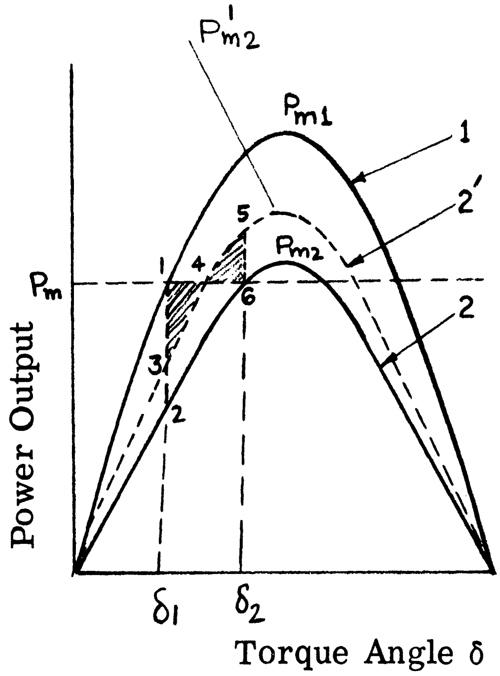


Fig. 9.3 Effects of Switched Series on Power Angle Curves

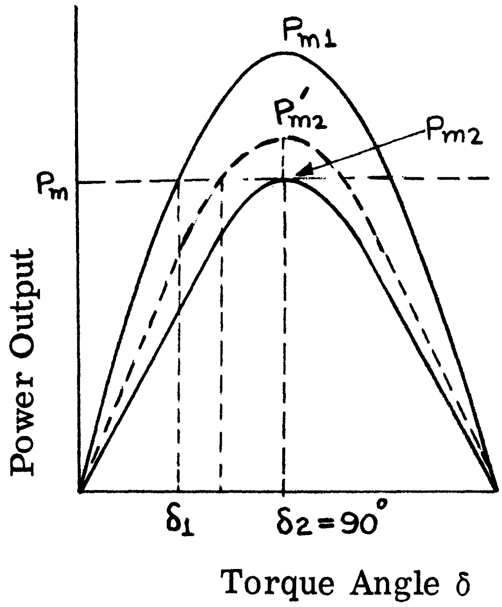


Fig. 9.4 Series Compensation for Maximum Transient Stability Limit

1. Dead-beat operation. Refer to figure 9. 3. Select δ_2 to be the maximum angle at which operation in the post-fault state may take place.

Then $\sin \delta_2 = P_m / P_{m2}$ or $P_m = P_{m2} \sin \delta$. Thus P_m is the maximum amount of power which is required to be transmitted along the lines.

$$\delta_1 = \sin^{-1} \frac{P_m}{P_{m1}}$$

$$P_m[\delta_2 - \delta_1] = P'_{m2} \int_{\delta_1}^{\delta_2} \sin \delta \, d\delta = -P'_{m2}[\cos \delta]_{\delta_1}^{\delta_2} = P'_{m2}[\cos \delta_1 - \cos \delta_2]$$

$$P_m[\delta_2 - \delta_1] = [\cos \delta_1 - \cos \delta_2] \, P'_{m2}$$

$$P'_{m2} = \frac{P_m [\delta_2 - \delta_1]}{[\cos \delta_1 - \cos \delta_2]}$$

$$\frac{E'_q \, E_2}{x'_2} = \frac{P_m [\delta_2 - \delta_1]}{[\cos \delta_1 - \cos \delta_2]}$$

where x'_2 is the total reactance when the capacitor is in the circuit.

$$\frac{E_q' E_2}{x_1} \quad \frac{x_1}{x_2'} = \frac{P_m[\delta_2 - \delta_1]}{[\cos \delta_1 - \cos \delta_2]}$$

$$\frac{P_{m1}}{2} \quad \frac{x_1}{x_2'} = \frac{P_m[\delta_2 - \delta_1]}{[\cos \delta_1 - \cos \delta_2]}$$

$$x_2' = \frac{P_{m1}}{2P_m} x_1 \quad \frac{[\cos \delta_1 - \cos \delta_2]}{[\delta_2 - \delta_1]}$$

But $x_2' = x_1 - x_c$

$$\text{or } x_1 - x_c = \frac{P_{m1}}{2P_m} x_1 \quad \frac{[\cos \delta_1 - \cos \delta_2]}{(\delta_2 - \delta_1)}$$

$$x_c = x_1\left[1 - \frac{P_{m1}}{2P_m} \frac{(\cos \delta_1 - \cos \delta_2}{\delta_2 - \delta_1}\right]$$

$$= x_1\left[1 - \frac{P_{m1}}{2P_m} \frac{\left[\sqrt{1 - \frac{P_m^2}{P_{m1}^2}} - \sqrt{1 - \frac{P_m^2}{P_{m2}^2}}\right]}{\delta_2 - \sin^{-1}\frac{P_m}{P_{m1}}}\right]$$

$$= x_1\left[1 - \frac{P_{m1}}{2P_m} \quad \frac{\frac{\sqrt{P_{m1}^2 - P_m^2}}{P_{m1}} - \frac{\sqrt{P_{m2}^2 - P_m^2}}{P_{m2}}}{\delta_2 - \sin^{-1}\frac{P_m}{P_{m1}}}\right]$$

$$= x_1 \left[1 - \frac{E'_q E_2}{x_1 \cdot P_m} \frac{\dfrac{\left[\sqrt{\dfrac{4 E'^2_q E_2^2}{x_1^2} - P_m^2}\right]}{\dfrac{2 E'_q E_2}{x_1}} - \dfrac{\left[\sqrt{\dfrac{E'^2_q E_2^2}{x_1^2} - P_m^2}\right]}{\dfrac{E'_q E_2}{x_1}}}{\delta_2 - \sin^{-1} \dfrac{P_m x_1}{2 E'_q E_2}} \right]$$

Since E_q, E_2, x_1, δ_2 and P_m are known, the value of capacitive reactance can be determined from the above mathematical relation.

2. Lowest Bound on the Capacitor Size.

As before δ_2 is selected as the maximum operating angle in the post-fault state. Instead of being dead-beat the rotor is allowed to swing up to angle $\alpha < \pi - \delta_2$, see figure 9. 5. The operating point moves from 1 through 2. 3. 4 to 5. At this stage the capacitor is taken out. The operation continues along curve 2 from 6 to 7 to 8 such that area 607 = area 789. The rotor oscillates around position 7 and finally settles down at 7 corresponding to angle δ_2.

The value of x_c which corresponds to the case of the rotor swinging upto an angle α is calculated on the next page.

As before, with reference to figure 9. 5, using the equal area criterion.

$$\int_{\delta_1}^{\delta'_2} (P_m - P'_{m2} \sin \delta)\, d\delta = \int_{\delta'_2}^{\alpha} (P'_{m2} \sin \delta - P_m)\, d\delta$$

or $\quad P_m(\delta'_2 - \delta_1) + P'_{m2} (\cos \delta'_2 - \cos \delta_1)$

$$= P'_{m2} (\cos \delta'_2 - \cos \alpha) - P_m(\alpha - \delta'_2)$$

$$P_m (\delta'_2 - \delta_1 + \alpha - \delta'_2) = P'_{m2} \{\cos \delta'_2 - \cos \alpha - \cos \delta'_2 + \cos \delta_1\}$$

$$P_m(\alpha - \delta_1) = P'_{m2}(\cos \delta_1 - \cos \alpha)$$

$$P'_{m2} = P_m \frac{(\alpha - \delta_1)}{\cos \delta_1 - \cos \alpha}$$

$$\frac{E'_q E_2}{x'_2} = P_m \frac{(\alpha - \delta_1)}{\cos \delta_1 - \cos \alpha}$$

$$x'_2 = \frac{E'_q E_2(\cos \delta_1 - \cos \alpha)}{P_m(\alpha - \delta_1)} = x_1 - x_c$$

Therefore

$$x_c = x_1 - \frac{E'_q E_2(\cos \delta_1 - \cos \alpha)}{P_m(\alpha - \delta_1)}$$

If in the limit α is allowed to approach a value of $\pi - \delta_2$, then the value of x_c in the above equation is the lowest bound on the size of the capacitor for system stability.

Application to an Actual System

Figure 9.6 represents a single machine connected to an infinite bus through parallel transmission lines. This system is studied with respect to the influence of switching in a capacitor in series with the transmission line. The numbers on the diagram indicate the values of the reactances in per unit. The breakers adjacent to a fault on both sides are arranged to clear simultaneously.

Resistance and shunt capacitance are neglected.

The following data are provided:

i. voltage behind transient reactance = 1.25 pu

ii. voltage of infinite bus = 1.0 pu

iii. power transferred = 1.0 pu.

The transient starts due to a 3 phase fault at P which is cleared by opening of the circuit breakers adjacent to P.

Positive sequence impedance diagram for the system is as shown in fig. 9.7.

Amplitudes of the power angle curves are: (See figure 6.2)

(A) Before fault = $\dfrac{1.0 \times 1.25}{.72} = 1.735$

(B) During fault = $\dfrac{1.0 \times 1.25}{2.98} = 0.420$

(C) After the fault = $\dfrac{1.0 \times 1.25}{1.00} = 1.25$

therefore
$$\frac{P_{m2}}{P_{m1}} = \frac{.42}{1.735} = .242$$
$$\frac{P_{m3}}{P_{m1}} = \frac{1.25}{1.735} = .720$$

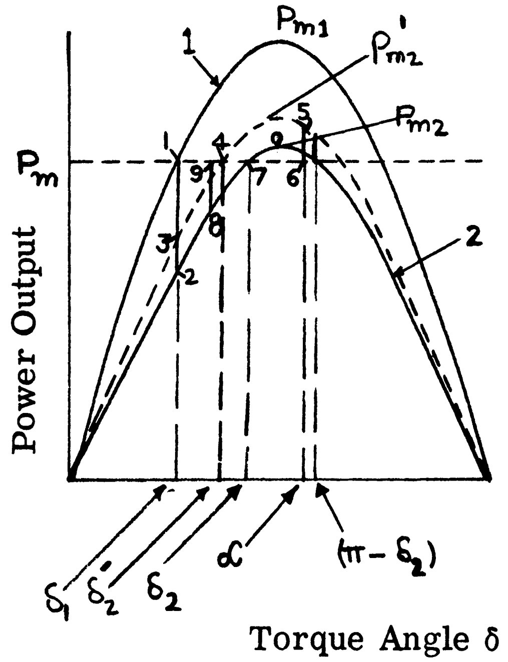


Fig. 9. 5 Determination of Minimum Capacitive Reactance

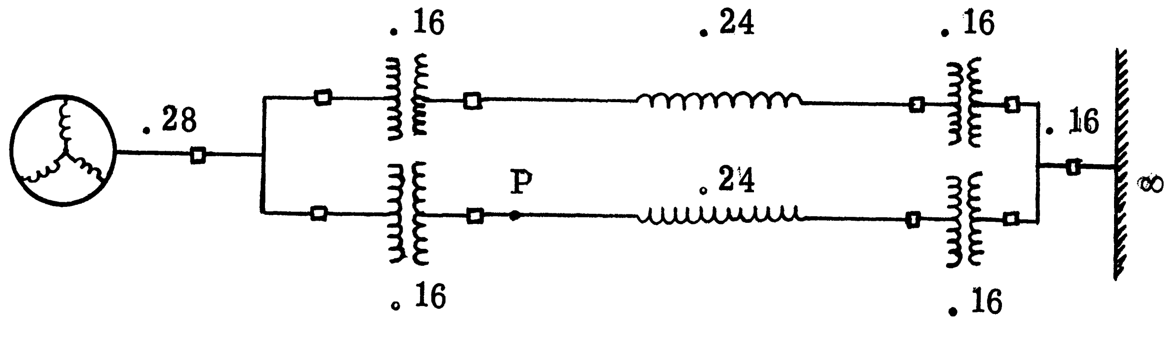


Fig. 9. 6 Single Machine System

$$\delta_1 = \sin^{-1} \frac{1.0}{1.735} = 35.2^{\circ} = .615 \text{ Radian}$$

$$\pi - \delta_3 = \sin^{-1} \frac{1.0}{1.25} = 126.9^{\circ} = 2.22 \text{ Radians}$$

$$\delta_2(\text{critical}) = \frac{\cos^{-1}\left(\frac{1.0}{1.735}\right)(2.22 - 0.615) + .72 \cos 126.9 - .242 \cos 35.2}{0.72 - .242}$$

$$= 51.6^{\circ}$$

Since 3 cycle breakers (0.05 sec. clearing time) are commonly used, it would be interesting to find out if the power level can be raised above 1.0 p.u. Initial angles for various power levels are as follows:

P	δ_1°	$(\pi-\delta_3)^{\circ}=\delta_m^{\circ}$	Clearing angle δ_2° (front standard curves)
1.0	35.2	127.0	38.6
1.05	37.2	122.8	40.7
1.10	39.3	118.0	43.9
1.15	41.4	113.0	45.3
1.20	43.7	106.5	47.7

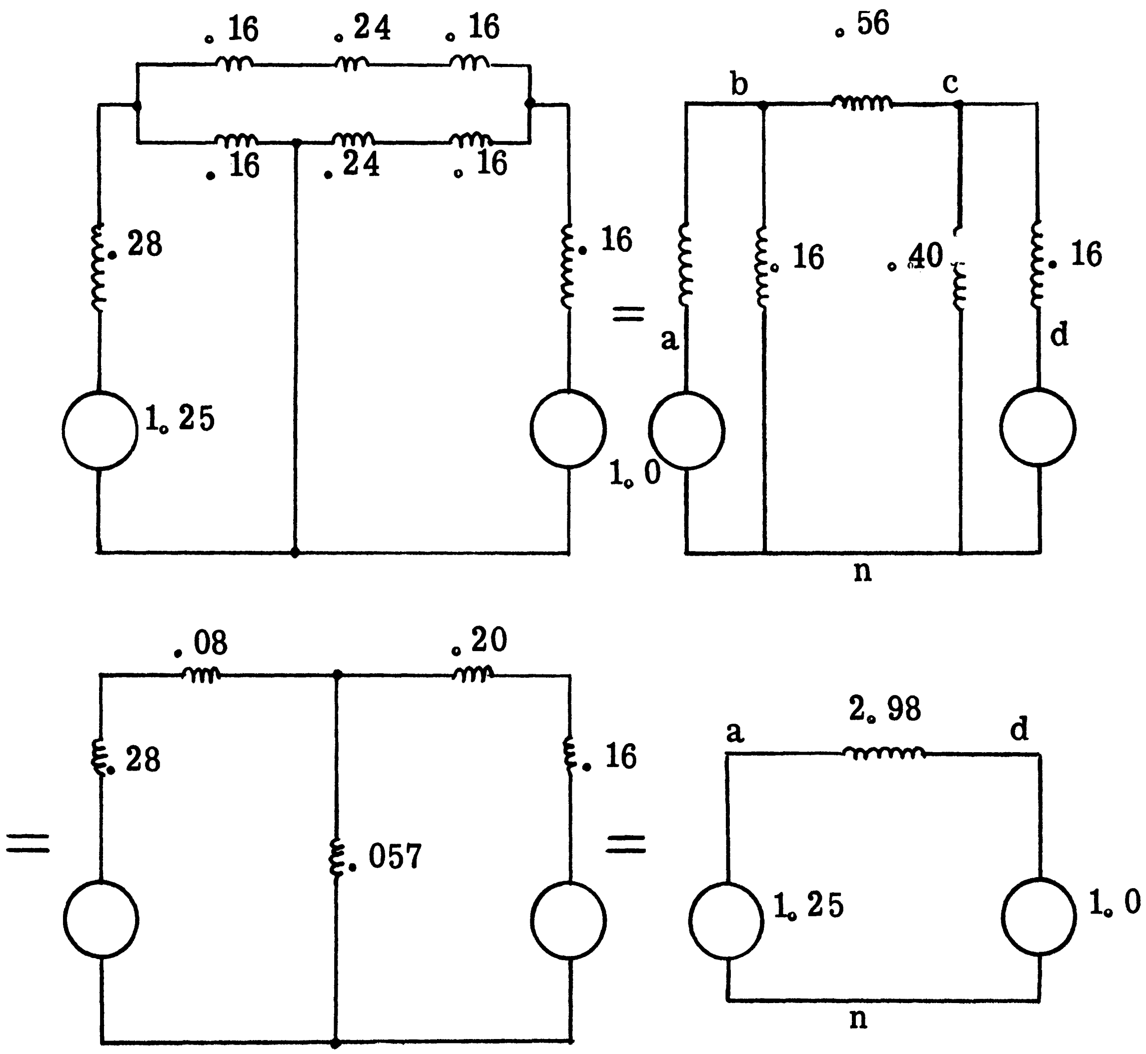


Figure 9.7 Positive Sequence Networks

Knowing the clearing angle corresponding to the minimum clearing time of 3 cycles, it is necessary to draw swing curves to check whether power level can be raised from 1.0 pu. to 1.20 pu. or an increase of 20%.

The data for the swing curves as obtained from digital calculations are given and swing curves plotted in figure 9.8.

<u>Fault cleared in 3 cycles</u>

	Power				
	a-1.0	b-1.05	c-1.10	d-1.15	e-1.20
Time-sec	δ^0	δ^0	δ^0	δ^0	δ^0
0.00	35.2	37.2	39.3	41.4	43.7
0.05	38.6	40.7	43.0	45.3	47.8
0.10	46.3	48.9	51.5	54.2	57.1
0.15	54.8	58.0	61.1	64.3	67.7
0.20	63.2	67.0	70.7	74.7	78.8
0.25	70.6	75.1	79.7	84.5	89.6
0.30	76.3	81.8	87.5	93.5	9.99
0.35	80.1	86.8	93.9	101.7	110.0
0.40	81.8	90.0	99.0	109.1	120.3
0.45	81.4	91.4	102.9	116.3	131.7
0.50	78.8	91.0	105.7	123.7	145.5
0.55	74.2	88.8	107.6	132.2	163.7
0.60	67.8	84.8	108.7	142.6	189.5
0.65	60.0	79.1	109.0	156.6	228.0
0.70	51.4	71.8	108.6	176.5	285.7
0.75	43.1	63.2	107.5	205.9	365.0

It is observed from the swing curves that cases d and e, corresponding to power transmitted of 1.15 and 1.20 respectively are unstable. So it has been possible to increase power by 10% by employing the fastest possible fault clearing.

Consider the effect of adding a suitable capacitor in series as soon as the line is tripped out. From the data provided, the amplitude of power angle curve during the post-fault condition is 1.25. Temporarily it is necessary to slightly increase this value to create stability for power levels of 1.15 and 1.20. Assume that the amplitude of the power angle curve during the post-fault condition is x, and if P_m is the input power then from equal area criterion

$$x = \frac{P_m(\delta_m - \delta_1) + .42\,(\text{Cos}\ \delta_2 - \text{Cos}\ \delta_1)}{\text{Cos}\ \delta_2 - \text{Cos}\ \delta_m}$$

The value of x as calculated from this equation is:

$P_m = 1.15$ pu $\quad x = 1.30$ pu

$P_m = 1.20$ pu $\quad x = 1.36$ pu

or

$$P_m = 1.15 \text{ pu}, \qquad \frac{E_q' \, E_2}{x_L - x_c} = 1.30 = \frac{1.25 \times 1.0}{1.0 - x_c}$$

$$x_c = 1.0 - \frac{1.25}{1.30} = 1.0 - .962 = .038 \text{ pu}$$

$P_m = 1.20$ pu

$$x_c = 1.0 - \frac{1.25}{1.36} = 1.0 - .883 = \underline{.117 \text{ pu}}$$

The swing curves are again calculated after including the series capacitors in the circuit and the following results are obtained.

	Power =	
	1.15 (x_c = .038)	1.20 (x_c = .117)
Time-sec	δ^0	δ^0
0.00	41.4	43.7
0.05	45.3	47.8
0.10	54.2	57.1
0.15	63.9	66.9
0.20	73.6	76.3
0.25	82.3	84.5
0.30	89.8	91.4
0.35	95.9	96.8
0.40	100.8	100.9
0.45	104.5	103.8
0.50	107.3	105.6
0.55	109.2	106.4
0.60	110.4	106.3
0.65	111.0	105.3
0.70	111.1	103.2
0.75	110.6	100.1

The swing curves are replotted on the same graph as broken lines from which the stability achieved by using capacitors can be observed.

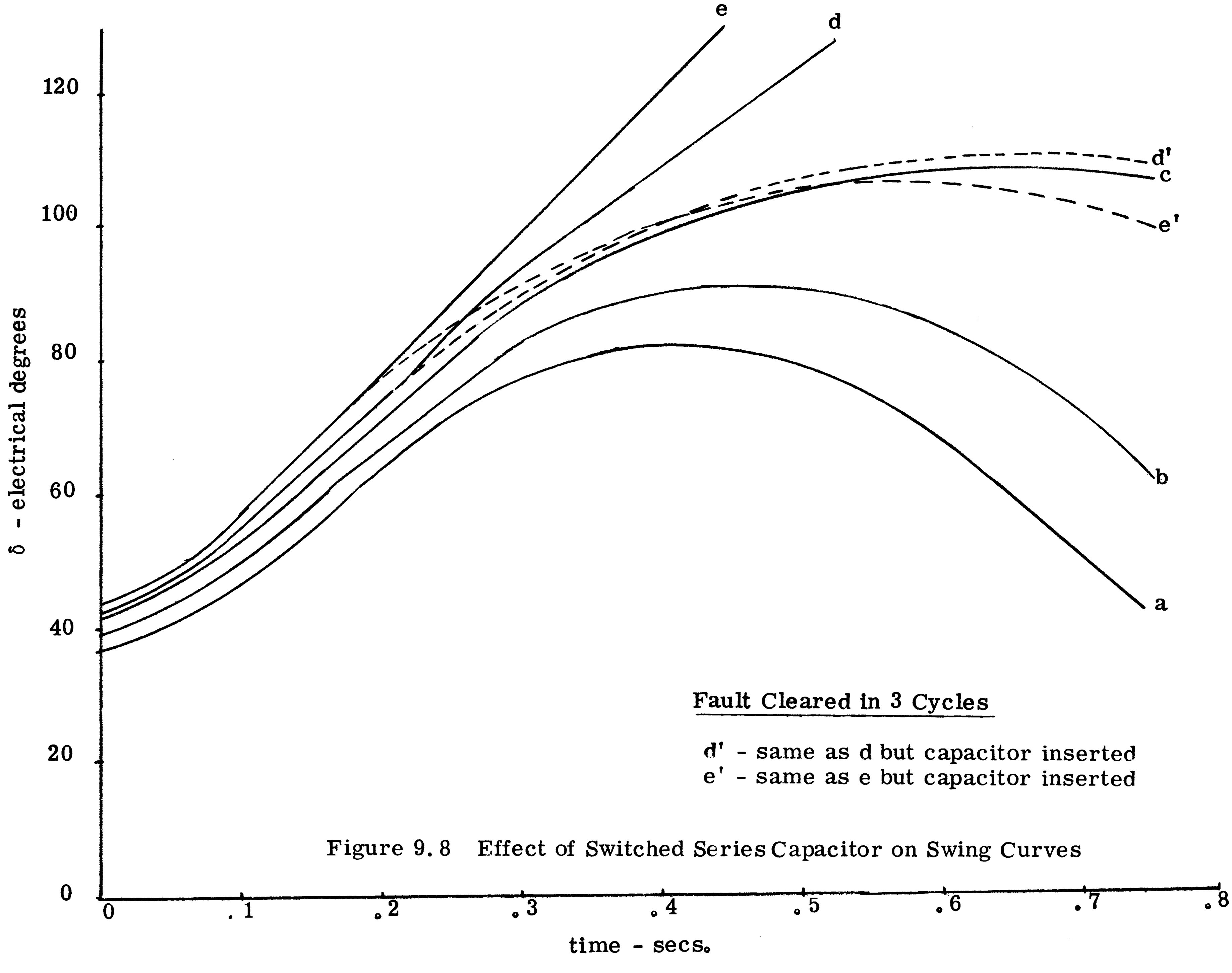


Figure 9.8 Effect of Switched Series Capacitor on Swing Curves

Chapter X

SWITCHED SERIES CAPACITORS (EXPERIMENTAL STUDY)

Introduction

This chapter describes some experimental work, on Switched Series Capacitors, which was performed in the Power Systems Laboratory of The University of Michigan. The purpose of the experiment was to make a comparison between the digital solution of the machine equations and the experimental results. It will be seen that there was fairly good agreement between the computer and the experimental results.

Brief Description of the Experiment

The experiment was performed on a synbhronous Generator/D.C. Motor set. The D.C. Motor which was a compound wound machine acted as the prime mover for the generator. The generator was synchronized to the A.C. Supply system through two parallel transmission lines, one consisting of reactors to simulate a long transmission system. The other a simple direct connection essentially zero reactance. Under steady state conditions, power was flowing through both lines. The direct connection was opened suddenly which created a transient and the generator began to oscillate. The oscillations were measured by a specially designed measuring apparatus. Different sizes of capacitors were introduced in the system and the oscillations were recorded. It was observed that for a certain value of the capacitive reatance, the oscillations were minimal. The experimental results were compared with the digital solution of the machine equations.

Machine Specifications

Synchronous Generator	Make Westinghouse
KVA 11	Excitation volts 125
Volts 240	Excitation amps 5.6
Amps 26.4	R. P. M. 1200
Phases 3	Cycles 60
Power factor .9	
D. C. Motor	Make Westinghouse
Compound Wound	
HP 15	
Volts 250	
Amps 52	
RPM 1200	
Auxiliary Generator	Make Dayton Electric
Volts 115	Phase 1
Amps 3.7	Cycles 60
Power Factor 1.0	RPM 3600
	VA 1000

Operation of the System

Figure 10. 1 is a schematic diagram of the system which was set up in the laboratory. The synchronous generator was connected to the infinite bus, which was the Detroit Edison supply in this case, through two parallel transmission lines. As shown in the schematic, line A was a direct connection but line B was made of lumped reactors to simulate a long transmission line. Line A had a contact 1 in series with it, which was controlled by coil CR1. Since CR1 was energized, this contact was closed; hence power was flowing from the synchronous generator to the infinite bus through line A. The coil CR1 had a push button 'a' in series with it which was normally closed. It was pushed by hand to de-energize the coil and open contact 1. Line A was thus switched out and power began to flow through line B. Since contact 3 was open, power was flowing through path CC' which was a series circuit comprising inductance and capacitance. The net reactance of path CC' was capacitive. It was necessary to keep a combination of inductance and capacitance in CC' since otherwise the value of capacitance required would have been very large.

Since coil CR1 was de-energized, it closed contact 1', which energized the timer. After a pre-set time delay, coil CR2 was energized which closed contact 2'. Closing of this contact energized coil CR3. As soon as this coil was energized, contact 3 was closed, shorting out the capacitor circuit.

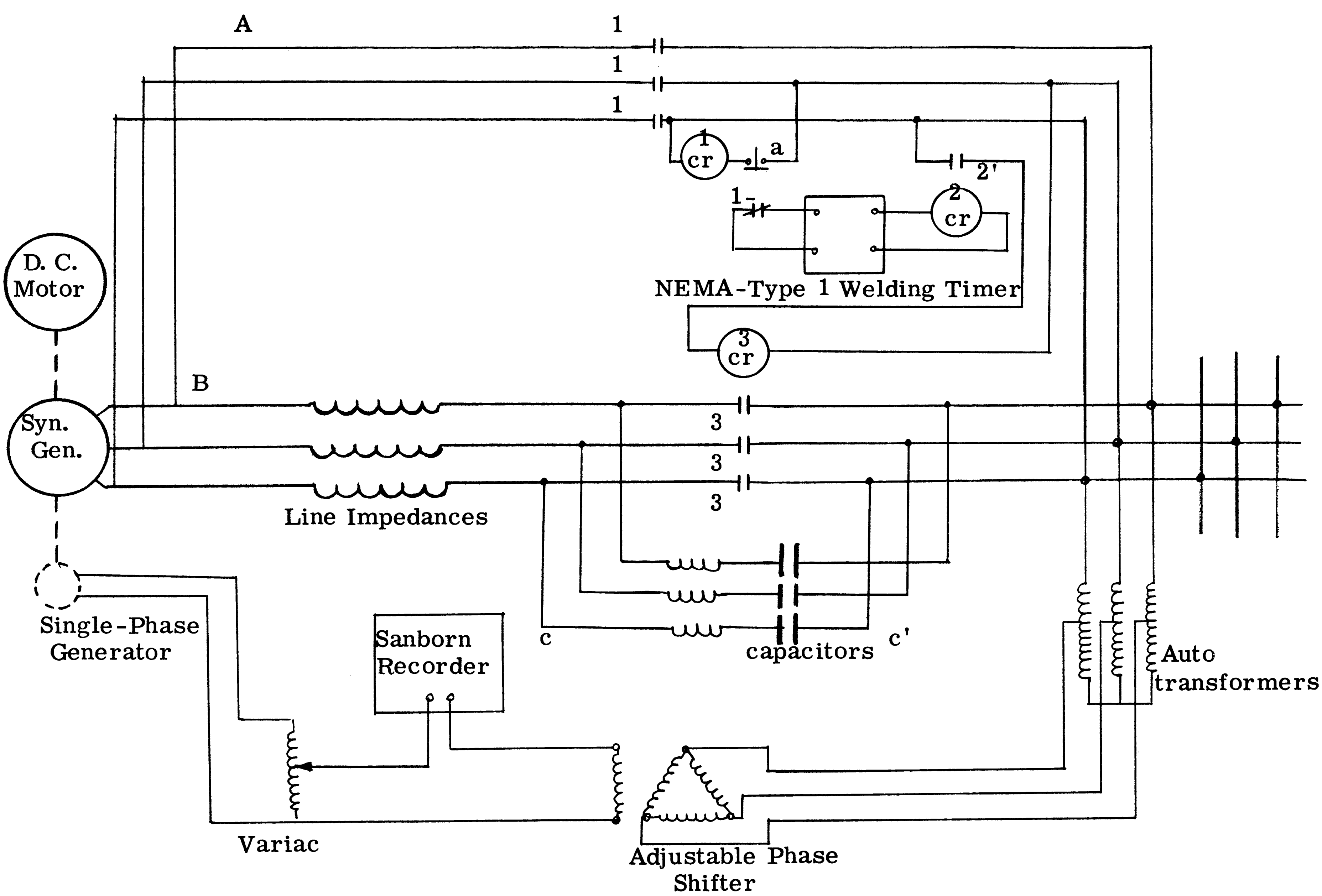


Fig. 10.1 Schematic Diagram of the Laboratory System

After a certain time when oscillations had disappeared and steady state conditions realized, push button 'a' was released which energized coil CR1, closing contact 1, thus reinserting line A and returning to the initial conditions.

The capacitor insertion time was varied by varying the time setting on the timer. The above procedure was repeated for different loads, capacitors and the capacitor insertion times.

Figures 10.2 through 10.5 are the photographs of the system as set up in the laboratory.

Torque Angle Measurement

The principle of the measurement of torque angle can be illustrated by reference to figure 10.6. OB and OA are two phasors representing two voltage sources which are assumed to remain constant in magnitude. OB is the voltage of the remote generating station and OA is the voltage of the receiving station. If δ is the torque angle, then AB is the phase difference between the two voltages.

If OB is equal in length to OA, then

$$AB = 2 \ OB \ \sin \frac{\delta}{2} = K \sin \frac{\delta}{2}$$

where K is a constant or

$$\delta = 2 \sin^{-1} \frac{AB}{K} \qquad (10.1)$$

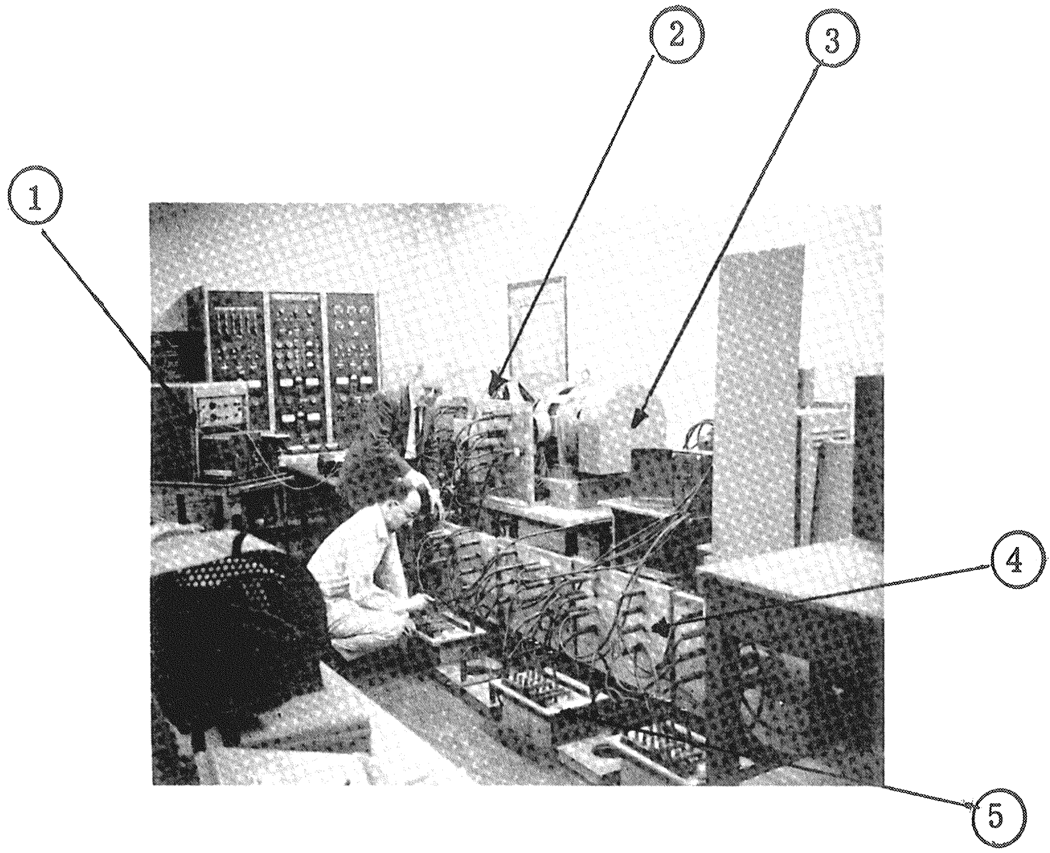


Fig. 10. 2

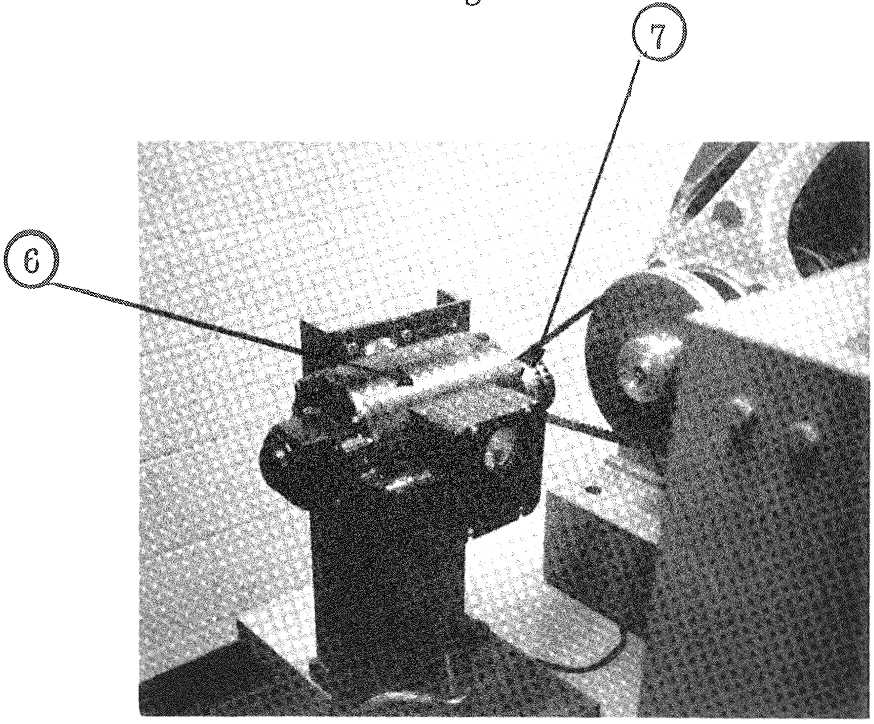


Fig. 10. 3

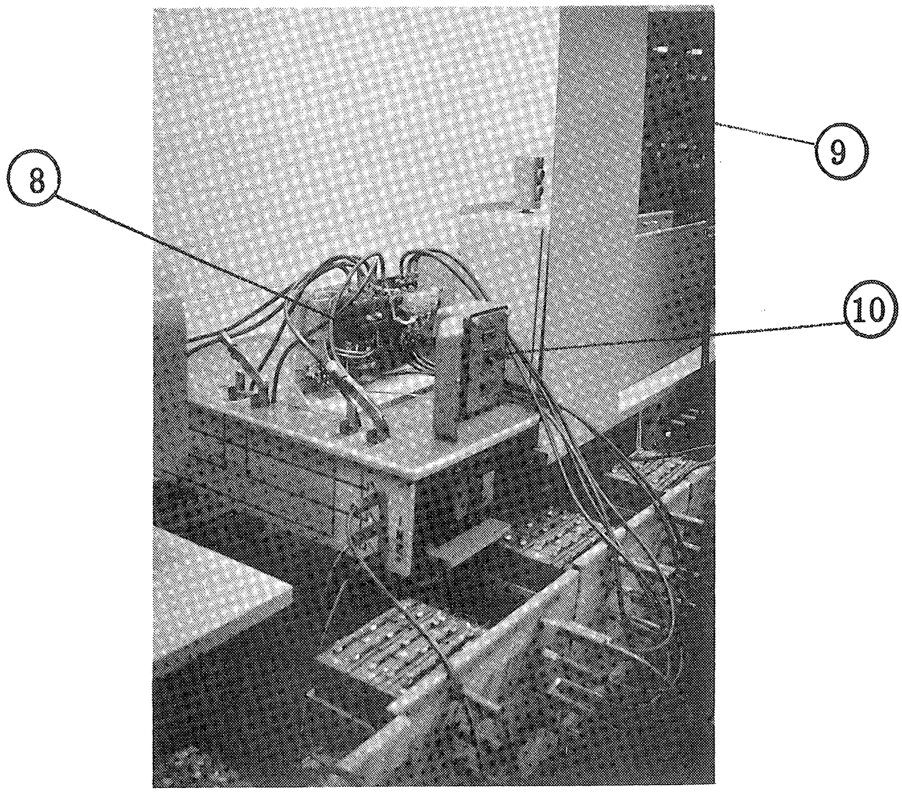


Fig. 10. 4

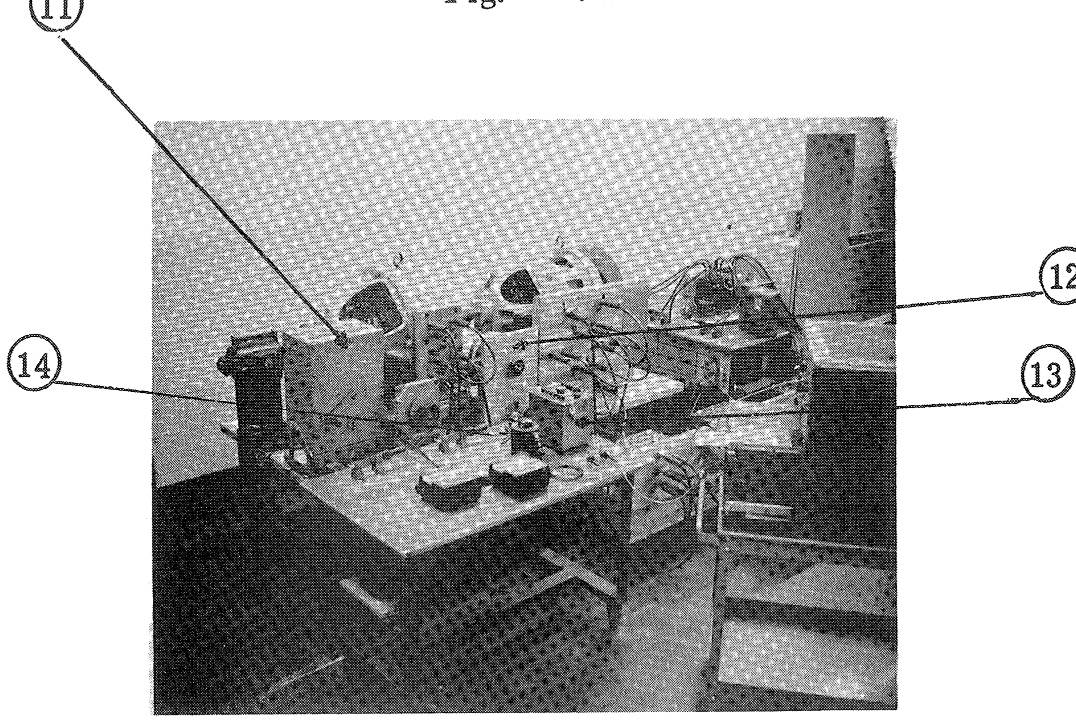


Fig. 10. 5

KEY TO FIGURES 10.2 , 10.3, 10.4, 10.5

1. Sanborn Recorder
2. D. C. Driving Motor
3. Synchronous Generator
4. Artificial Transmission Line
5. Switched Series Capacitors
6. Single Phase Reference Voltage Generator
7. Gear and Belt coupling
8. Control Contactor
9. Infinite Bus
10. Welding Timer
11. D. C. Motor Starter
12. Synchronous Generator Field Regulator
13. Phase Shifter
14. Variac

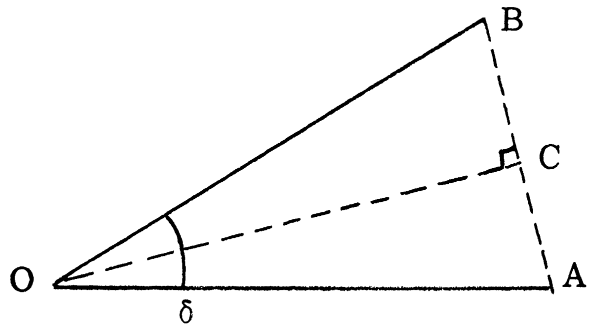


Fig. 10.6 Principle of Torque Angle Measurement

If the phase difference AB vs. time is recorded during a transient, by means of a recorder, it is possible to draw a torque angle vs. time curve by the equation 10.1.

The above principle was used to measure the torque angle during the transient. The voltage of an auxiliary generator was used as a reference signal. The auxiliary generator was coupled to the main shaft by gear and toothed belt drive as shown in figure 10.3, which was necessary since the auxiliary generator had a rated speed of 3600 rpm while the main shift was running at 1200 rpm.

Since there was no load on this generator, its voltage remained constant during a transient, but its phase changed with the position of the rotor. This was the voltage OB in figure 10.6. Voltage signal OA was obtained from the Edison supply, through an auto transformer as shown in figure 10.1. The voltage signal OB was adjusted by means of a variac such that OA = OB. A phase shifter was used to bring the two voltages in phase with each other when no power was transmitted across the line, so that δ was zero under no load condition.

Signal AB was connected across a Sanborn Recorder. As soon as the transient occurred, the fluctuations in the magnitude of this signal were recorded by this recorder. The equation 10.1 was then used to draw a torque angle/time curve from the recorder tracing.

Experimental Results

During the experiment, a number of machine loadings, capacitor sizes and switching times were used. A typical case was selected for analysis and comparison with the digital computer results. The data obtained from the Sanborn Recorder were converted to torque angles by equation 10.1. Experimental torque angle curves were then superimposed on the theoretical curves as obtained from digital computer analysis, the details of which are given in the following sections.

Theoretical Analysis

A theoretical study of the system was made by digital computer simulation and results were obtained as follows:

The digital computer investigation was to find the time solution of the swing equation

$$M \frac{d^2\delta}{dt^2} = P_i - P_o$$

where P_i and P_o are power input and power output of the machine. To find these quantities we proceed as follows:

Machine and System Parameters

Direct Axis Synchronous Reactance = 3.8 ohms per phase

Quadrature Axis Synchronous Reactance = 2.46 ohms per phase

Open Circuit Time Constant = 0.4 second

Direct Axis Transient Reactance = 0.89 ohm per phase

Line Reactance = 4 × 0.39 = 1.56 ohms

Additional Reactance = 0.48 ohm

Per Unit System

The following calculations are done on 11 KVA and 240 volt base.

$$\text{Per unit voltage} = \frac{240}{\sqrt{3}} = 138 \text{ V}$$

$$\text{Per unit current} = \frac{11000}{\sqrt{3} \times 240} = 26.5 \text{ amp.}$$

$$\text{Per unit impedance} = \frac{138}{26.5} = 5.236\ \Omega.$$

On the above per unit system, the values of the parameters of the system are as follows:

$x_d = 0.725$

$x_q = 0.47$

$x'_d = 0.17$

x_E(Line Reactance) = 0.300

Additional Reactance = 0.092

Figure 10.7 shows the reactances in per unit.

Determination of Inertia Constant

The inertia constant of the machine can be found from the following formula.

$$H = .231 \times WR^2 \times (\text{rpm})^2 \frac{\times 10^{-6}}{\text{KVA}}$$

where WR^2 is moment of inertia in lb-ft^2, H is the inertia constant in kw-sec/KVA and rpm is the speed of the machine in revolutions per minute.

The moment of inertia of the machine rotor and the coupled DC rotor was calculated experimentally and found equal to 32.9 lb-ft^2.

Therefore

$$H = \frac{0.231 \times 32.9 \times (1200)^2 \times 10^{-6}}{11}$$

$$= 0.995 \text{ kw/sec/KVA.}$$

Transient Stability Analysis

The machine was operated at full load and at unity power factor and suddenly line 1 was opened by pushing the button a. Figure 10.8 shows the phasor diagram of the system prior to the transient. OA is the voltage of the infinite bus as well as the terminal voltage of the machine, since line 1 is closed. OA is also the direction of the current since the power factor is unity.

$$v_m = v_b = 1.00 \angle 0$$

where v_m is the machine terminal voltage and v_b is the bus voltage.

$$i = 1.00 \angle 0^0 \qquad \phi = \text{power factor angle} = 0$$

$$\bar{E}_{qd} = \bar{v} + j\, x_q \bar{i}$$

$$= 1.00 \angle 0 + \angle 90^0 \text{ x } 0.47 \text{ x } 1.00 \angle 0$$

$$= 1.00 \angle 0 + .047 \angle 90^0$$

$$= 1.00 + j\, 0.47 = 1.105 \angle 25^0 \qquad \delta = 25^0$$

$$i_d = 1.00 \sin \delta = 1.00 \sin 25 = 0.423$$

$$E'_q = 1.105 - (x_q - x'_d)\, i_d$$

$$= 1.105 - (.47 - .17).423$$

$$= 0.9781$$

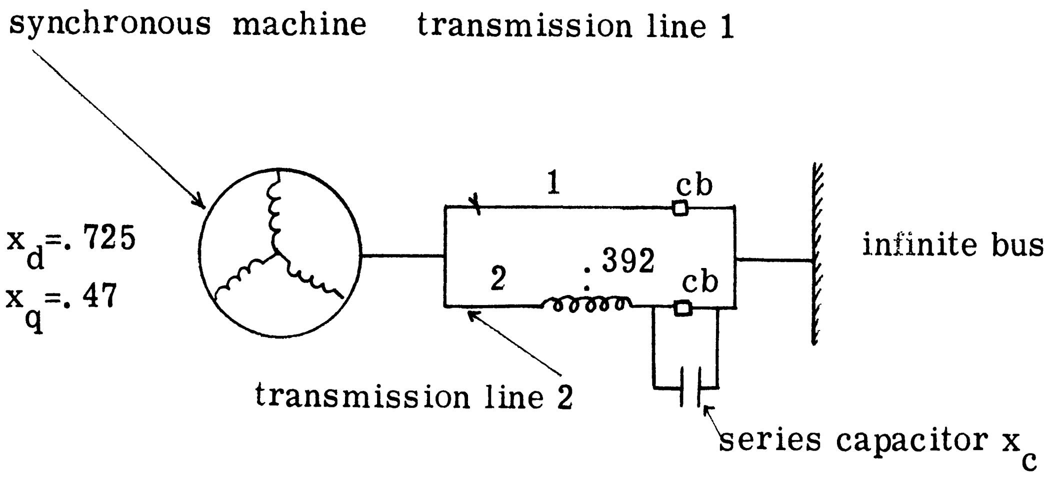


Fig. 10.7 Schematic Diagram of the System

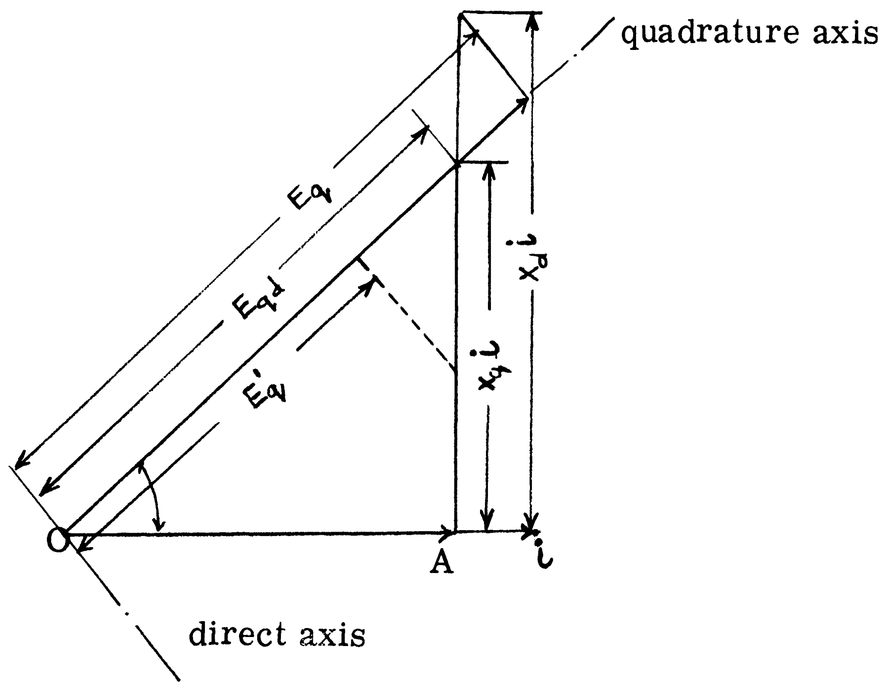


Fig. 10.8 Phasor Diagram

$$E_q = E_{qd} + i_d(x_d - x_q)$$

$$= 1.105 + 0.423(.725 - .47) = 1.213$$

A transient stability program was written and the swing curve was calculated by the help of this program. Figure 10.9 shows both the theoretical and the experimental results as explained below.

1. Experimental swing curve without capacitor insertion.
2. Theoretical swing curve without capacitor insertion.
3. Experimental swing curve with .1 capacitor inserted for 10 cycles.
4. Theoretical swing curve with .1 capacitor inserted for 10 cycles.

It will be observed from the figure that there is a reasonable amount of agreement between experimental and theoretical data.

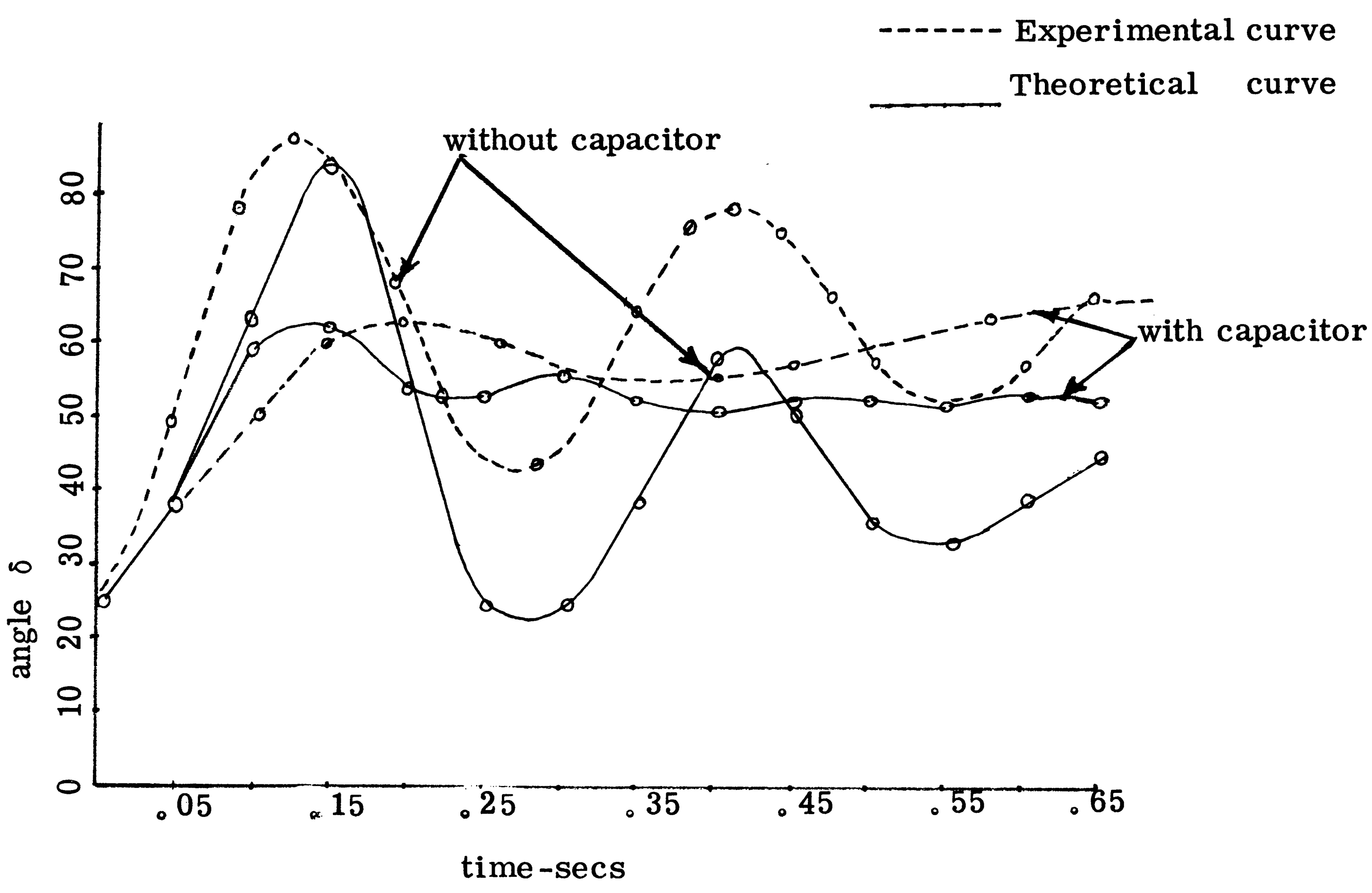
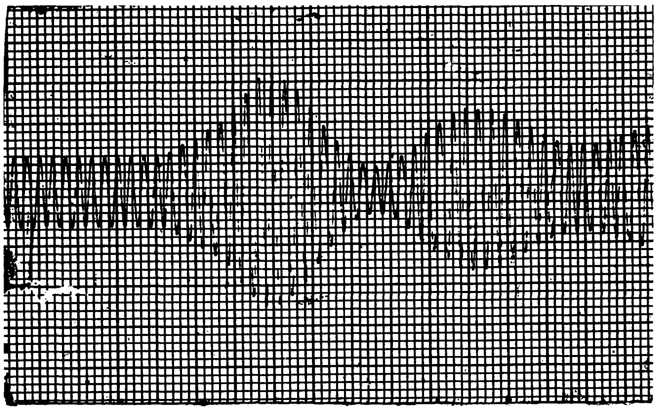
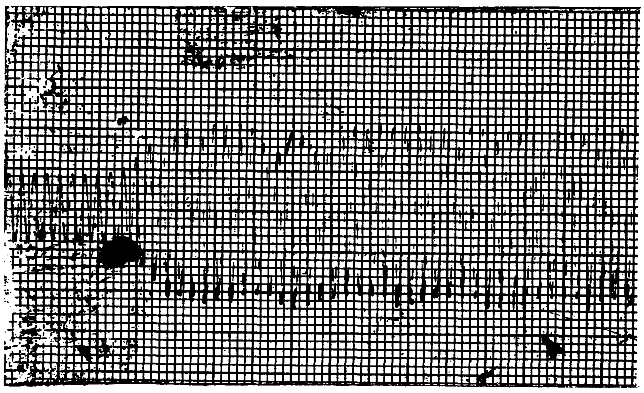


Fig. 10.9 Comparison of Theoretical and Experimental Curves



without capacitor



with capacitor insertion

Fig. 10. 10 Experimental Recordings

Chapter XI

SWITCHED SERIES CAPACITORS (FAULT STUDY AND RELAYING)

Introduction

In Chapter IX, an approximate analysis of switched series capacitors was presented. Chapter X related to the experimental demonstration of this technique. In this chapter some problems which will arise in the application of switched capacitors to practical systems will be investigated. These problems include the effect of a second fault while the capacitor is in circuit and the effect of the capacitor insertion on distance relaying.

System Description

Refer to figure 11.1 which is rather a generalized circuit although similar to the one set up in the laboratory. There are three lines in parallel which connect the synchronous machine to the infinite bus. Circuit no. 1 has negligible reactance as was the case in the laboratory set up, in order to get a good transient. Circuits no. 2 and 3, each consist of two sections. Each section has a reactance of 0.16 p.u. There are two Y-Δ transformers at the two ends of the line, which have solidly grounded neutrals. A capacitor of 0.1 p.u. reactance, which has a circuit breaker in parallel with it, is located at the center of the line, as shown in the figure. Each section has two circuit breakers at its ends which have distance relays on them.

It is admitted that since circuit no. 1 has been assigned a reactance of zero, this cannot be called a practical system.

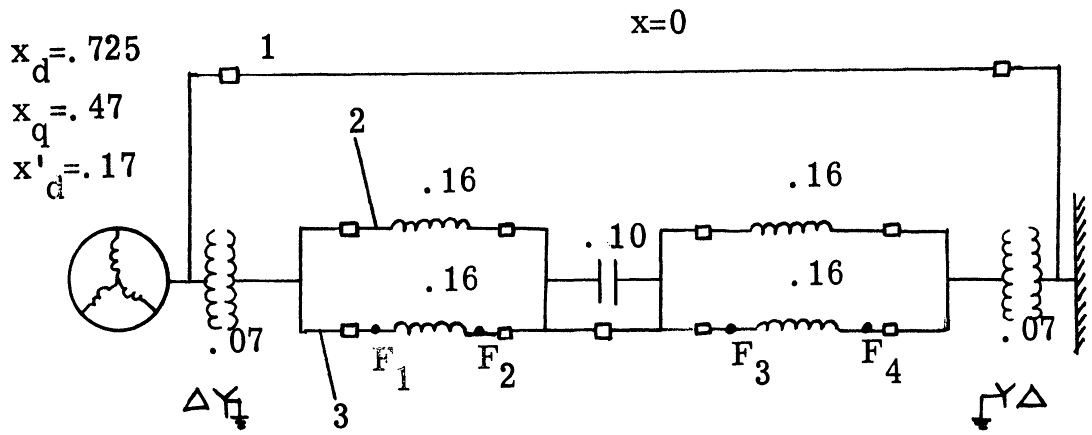


Fig. 11.1 A Generalized Circuit

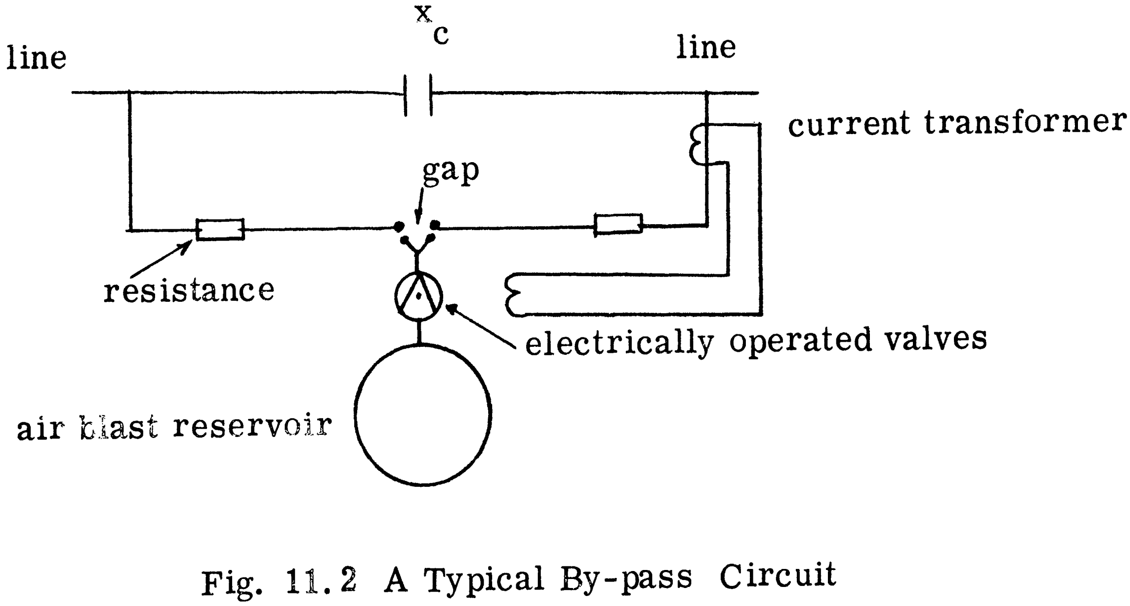


Fig. 11.2 A Typical By-pass Circuit

It was, however, selected in order to have as much similarity to the laboratory set up as possible.

The reactances of the generator are shown in the figure. The total line reactance is the same as in the experiment.

Since the parameters of the system are the same as the system in the laboratory had, the initial conditions for per unit power transfer are also the same.

Initially all these lines are in operation. Due to a fault line no. 1 is relayed out and the capacitor is inserted into the circuit. While the system is oscillating, there is another fault which is removed by switching out the corresponding line section.

The initial steady state conditions are the following. These are shown in figure 10.8.

$$I = 1.00 \angle 0°$$

$$E'_q = 0.98$$

$$E_q = 1.21$$

$$v_b = v_m = 1.00 \angle 0°$$

$$\delta = 25°$$

Position and Type of Fault

In figure 11.1, F_1, F_2, F_3 and F_4 are four different locations at which the fault will be assumed in order to get an overall picture of its effect. A three phase fault will be considered in the investigation since this results in the worst condition from the stability standpoint.

The following table shows the fault current through the capacitor for different locations.

Fault Location	Current through the Capacitor
F_1	7.70 pu
F_2	20.00
F_3	4.45
F_4	3.27

From the above table, it is obvious that as soon as a fault occurs at any of the locations F_1 through F_4, the capacitor will be bypassed due to excessive voltage across it. The fault will result in switching out of a line section by action of the associated relays and the breaker, which will take about 3 cycles. The capacitor will be fully reinserted in the transmission circuit within one half cycle after the overvoltage condition is removed.

Capacitor Protection

Should a second fault occur while the capacitor is in circuit, the current through the capacitor will be many times normal. It would, therefore, be necessary to bypass the capacitor to protect it against overvoltage. It is important that the capacitor bypass circuit be opened immediately after the capacitor voltage is reduced below the critical value.

A typical protective bypass circuit is shown in figure 11. 2. This has been designed for use with fixed series capacitors[22] and can equally well be utilized in the case of switched series capacitors. The breakdown voltage of the bypass circuit can be adjusted in the range of 2. 5 to 3. 0 times the rated voltage of the capacitor. To protect against moderate overvoltages, additional relay equipment has to be provided. Flow of current in the gap circuit causes operation of the electrically operated airvalves and the air-blast reservior discharges a stream of air into the gap conducting area. After establishment of sufficient air flow, the arc in the gap space is quenched at each current zero and gap flashover occurs again on the next half-cycle of high voltage. Thus when the cause of the capacitor overvoltage is removed the gap arc is quenched at the first current zero thereafter and no restrike occurs. The capacitor is fully reinserted in the transmission circuit within one-half cycle after the overvoltage condition is removed.

Analysis of the Effect of Fault on Power Transfer

Power output from the synchronous generator to the infinite bus is given by the following equation

$$P_u = \frac{E'_q v_b}{x'_d + x_e} \sin \delta + \frac{v_b^2 (x'_d - x_q)}{2(x_q + x_e)(x'_d + x_e)} \sin 2\delta$$

$$E'_q = .98$$

$$v_b = 1.00$$

$\delta_o = 25^0$

$x'_d = .17$

$x_q = .47$

The transient proceeds as follows:

Initially all three lines were in operation and therefore x_e, the external reactance was equal to zero (note this is the same condition as in the laboratory). Therefore

$$P_u = \frac{.98 \times 1.0}{.17} \sin 25^0 - \frac{1\,(.30)}{2 \times .47 \times .17} \sin 50^0 = 1.00 \text{ pu}$$

As soon as a fault occurs on line no. 1, it is switched out and the capacitor is switched in. Therefore

$$x_e = 0.07 + 0.08 - 0.10 + 0.08 + 0.07 = .20 \text{ pu}$$

Assume that while the capacitor is in the circuit, a second fault occurs at any of the locations F_1 through F_4, then the power output during the period of the fault is reduced to zero. The faulted section will be relayed out in 3 cycles.

For half a cycle, following the relaying out of the faulted section, the capacitor remains bypassed. Therefore the value of x_e during this half cycle = .07 + .16 + .08 + .07 = .38. Next the capacitor is inserted and therefore x_e = .38 - .1 = .28. The machine continues to swing until the oscillations are damped out.

The capacitor is inserted when $\frac{d\delta}{dt}$ is positive and removed when it is negative. This technique requires a sophisticated control equipment to switch the capacitor in and out of the circuit but this is a superior method of using a capacitor as compared to the method used in the laboratory where the capacitor was switched out after a certain number of cycles.

Computer Study

A computer program was written to simulate the above system. The value of H used in the study was .995 as obtained in the experimental investigation.

The program makes the following simulation: The generator is operating in the steady-state supplying a power equal to 1.00. A fault occurs which results in switching out of line no. 1 and insertion of the capacitor. A second fault occurs immediately which drops the output to zero for a period of 3 cycles. At the end of 3 cycles, the fault is removed by switching out a line section. Since the capacitor will be out for another half cycle, the value of x_e during the period immediately following the clearance of the fault $= .07 + .08 + .16 + .07 = .38$. Another half cycle afterwards, the capacitor is reinserted into the circuit and the value of $x_e = .38 - .1 = .28$. The capacitor remains in circuit when $\frac{d\delta}{dt}$ is positive and is out of circuit when $\frac{d\delta}{dt}$ is negative. The enclosed swing curve shows that the system is stable. It may be pointed out that the second fault was assumed to occur at the very start of the first swing which makes it rather a special case. In actual practice, the second fault may occur any time during the swing which fact should be considered when designing real systems.

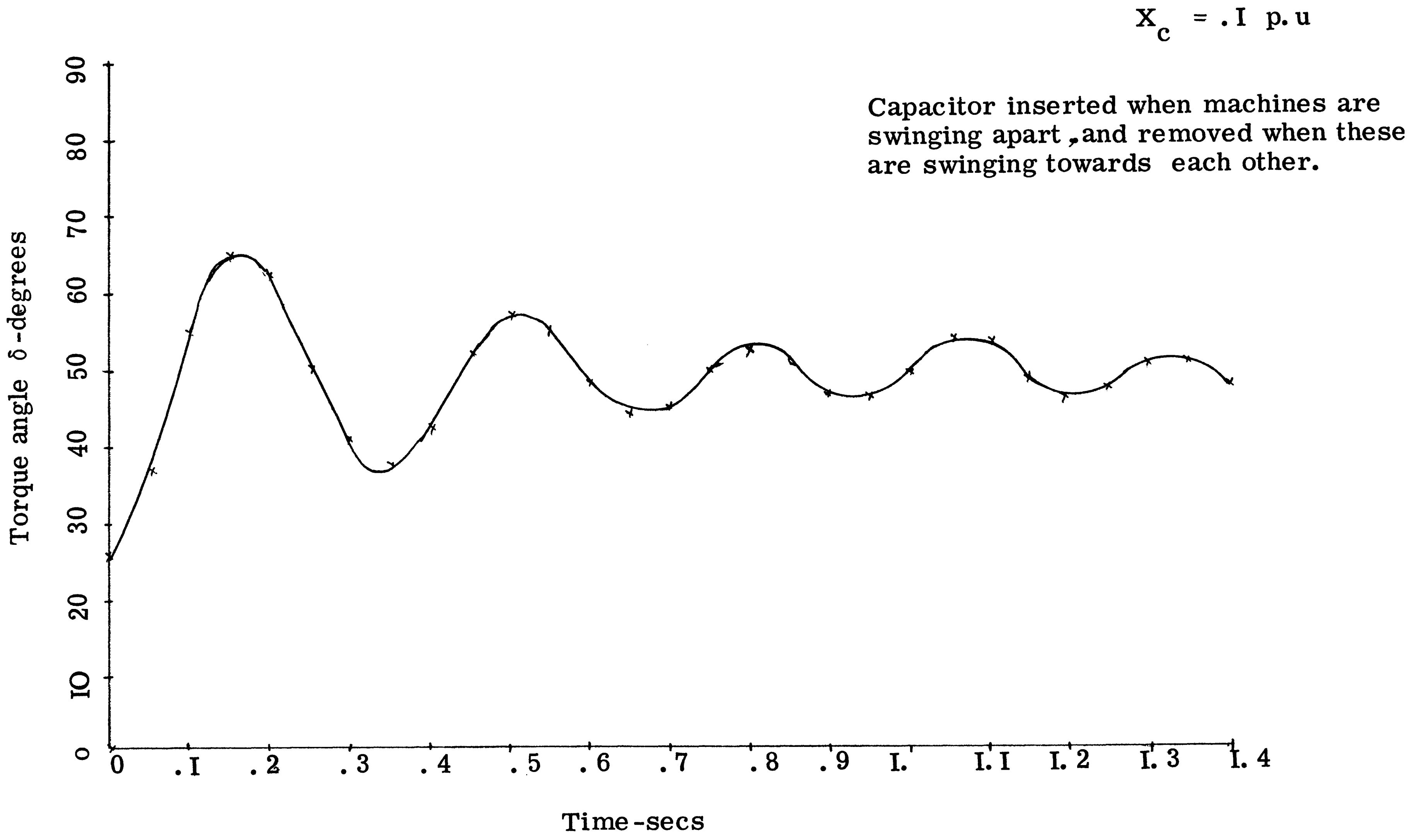


Fig. 11.3 Swing Curve with Capacitor Insertion

Effect of Switched Series Capacitors on Distance Relaying

Refer to figure 11.4(a) which shows a fault at the location F_1. Since the capacitive reactance is subtracted from the total inductive reactance of the lines 1 and 2, the fault appears much closer to relay A which may cause operation of this relay. Thus instead of opening the faulty section 3, the unfaulted section 1 may be switched out.

In practice, due to the action of the bypass circuit, the capacitor will generally remain shorted out during a fault. In most cases, therefore, the operation of the distance relays will not be affected by capacitor insertion during fault conditions. An exception will be a "line-to-ground" fault where the fault current may not be high enough to short out the capacitor. In such a case the relaying will have to be properly designed to take care of the effect of the switched capacitor.

A capacitor insertion can have substantial effect on distance relaying during machine swinging. It would therefore be desirable to study the effect of capacitor insertion on the impedance "seen by the distance relay" during a swing.

Consider figure 11.4 (a) again which can be simplified as shown in figure 11.4 (b), if the parallel direct connection is omitted.

The following analysis is based on the assumption that the voltage behind the transient reactance remains constant, otherwise the calculations will become unduly complicated.

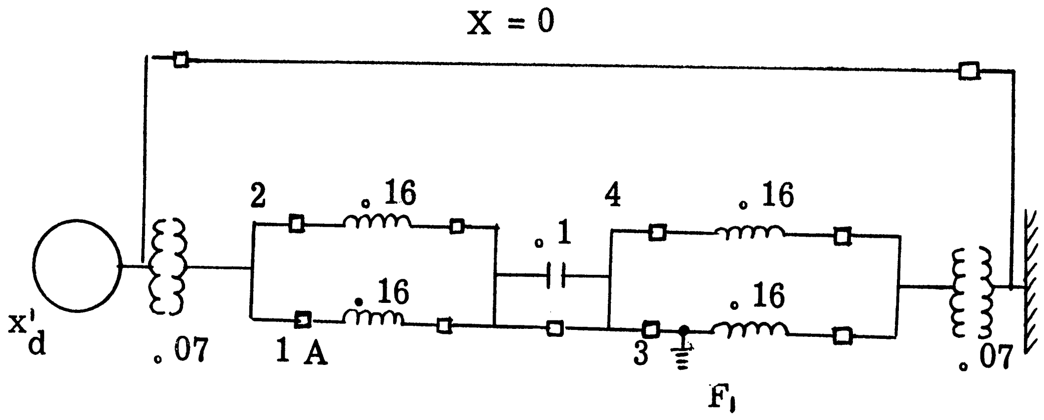


Fig. 11.4 (a) Circuit for Distance Relay Study

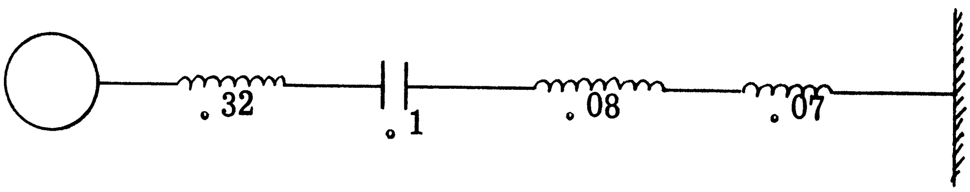


Fig. 11.4 (b) A Simplified Circuit

To investigate the problem, the following discussion will be helpful[23].

Consider figure 11.5, which shows a simple two machine system. Here V_A and V_B are the voltages behind the transient reactances which are assumed constant in magnitude but varying in phase during swings or out-of-step conditions. V_A leads V_B by the variable angle δ. The current anywhere in the series circuit is

$$\bar{I} = \frac{V_A\underline{|\delta} - V_B}{\bar{Z}}$$

where Z is the impedance of the connecting circuit including the transient reactance of the two machines. The total impedance Z is divided by the relay location M into two parts, mZ and (1-m) Z where m is a real number less than 1. At point M the voltage V is then

$$\bar{V} = (1-m)\ V_A\underline{|\delta} + mV_B$$

The impedance "seen" by the relays at M is

$$\bar{Z}_r = \frac{\bar{V}}{\bar{I}} = \frac{(1-m)\ V_A\underline{|\delta} + mV_B}{V_A\underline{|\delta} - V_B}\ \bar{Z}$$

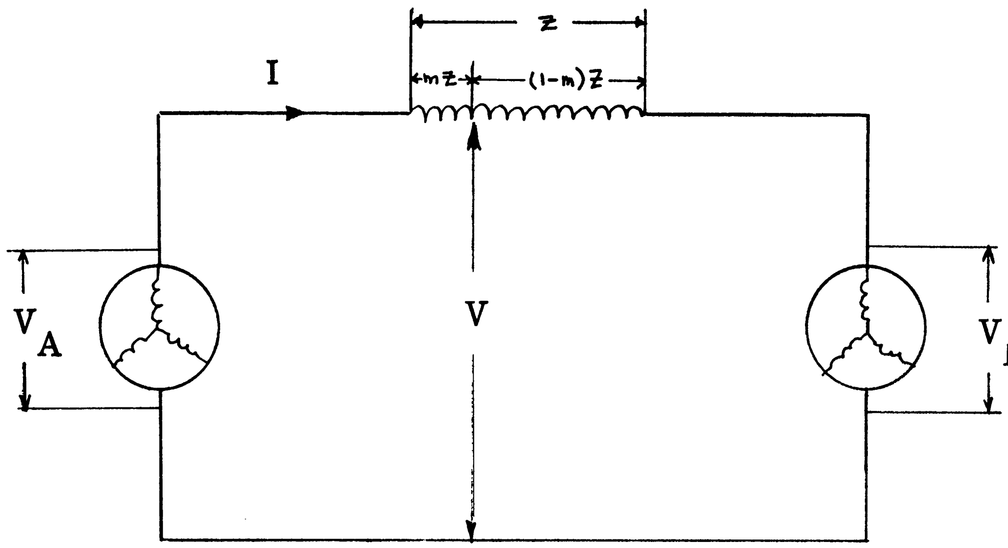


Fig. 11. 5 A Simple Two Machine System

$$\frac{\bar{Z}_r}{\bar{Z}} = \frac{(1-m)\dfrac{V_A}{V_B}\angle\delta + m}{\dfrac{V_A}{V_B}\angle\delta - 1} = \frac{k(1-m)\angle\delta + m}{k\angle\delta - 1}$$

where k is the magnitude of V_A/V_B.

$$= \frac{k(1-m)\angle\delta + m}{k\angle\delta - 1} = \frac{k(1-m) + m\angle-\delta}{k - \angle-\delta}$$

In the system under study $k = 1.01$ and if it can be taken as equal to 1, the above equation simplifies to

$$\frac{\bar{Z}_r}{\bar{Z}} = \frac{(1-m) + m\angle-\delta}{1 - \angle-\delta} = \frac{-m(1-\angle-\delta) + 1}{1 - \angle-\delta}$$

$$= -m + \frac{1}{1-\angle-\delta} = \left(\frac{1}{2} - m\right) - j\frac{1}{2}\cot\frac{\delta}{2} \qquad (1)$$

This equation represents a vertical line, because the real part is constant while the imaginary part varies as a function of δ. If both sides are multiplied by Z, lengths are multiplied by the magnitude of Z and the line is rotated counterclockwise through the impedance angle θ.

In the post-fault steady-state condition the capacitor will be out of the circuit and equation (1) will hold. However when the capacitor is inserted the value of m will change. The new value of m can be found as follows:

Let $m = m_1$ when the capacitor is inserted

Total Impedance $= Z - X_c$

Then $m\ Z = m_1 Z_1$

$$= m_1\ (Z - X_c)$$

or $\quad m_1 = \frac{Z}{Z-X_c}\ m$

Thus the value of m_1 increases by an amount $\frac{Z}{Z-X_c}$.

Applying the above analysis to the system of Figure 11.5(a) the following impedance is "seen" by the relay at location A, without capacitor insertion.

$$\frac{Z_r}{Z} = \left[\left(\frac{1}{2} - m\right) - j\ \frac{1}{2} \cot \frac{\delta}{2} \right]$$

$$m = \frac{.17 + .07}{.17 + .07 + .08 + .08 + .07} = \frac{.24}{.47} = .51$$

Thus the locus of $\frac{Z_r}{Z}$ is a vertical line at a distance of $\frac{1}{2} - .51 = -.01$ from the origin. The ordinates of points on this line representing various values of δ are given by

$$y = -\frac{1}{2} \cot \frac{\delta}{2} .$$

Table Table 1 shown calculation of y for 30° increments, from 0° to 180°.

Table 1

δ	y
0	∞
25°	-2.320
30°	-1.866
60°	-.866
90°	-.500
120°	-.289
150°	-.134
180°	0

Since the resistance of the line is neglected, $Z = .47 \underline{|90°}$. Hence the vertical line is turned counterclockwise by 90^0 and since only one half current flows in the protected line, the loci is multiplied by $.47 \times 2 = .94$. The loci is shown in Figure 11.16. As soon as the capacitor is inserted, the value of m becomes $m_1 =$

$$m \frac{Z}{Z - X_c} = .51 \times \frac{.47}{.47 - .1} = .51 \times \frac{.47}{.37} = .646$$

and $Z_1 = .47 - .1 = .37$. Therefore the loci jumps from A to B.

Assume that the first zone impedance element of the relay at breaker 1 reaches to 80% of the length of the line which it protects or to $.80 \times .16 = .128$. Let the second zone be set at .256 and the third at .384. The tripping characteristics of these elements are concentric circles of radii .128, .256 and .384 with centers at the origin. Let the directional element have maximum torque at a phase angle of 45^0. Its characteristic is a straight line through the origin and perpendicular to the line of maximum torque. The characteristic is shown in Figure 11.6.

From the figure it is observed that although insertion of the capacitor adversely affects the performance characteristics of distance relay no. 1, for the assumed conditions of the problem, it creates no serious difficulty as can be seen from the following discussion.

It has been noted earlier that the value of δ varies from 25^0 to 90^0. The initial locus takes it into the zone Z_2. The new locus keeps it in the same zone. Therefore under conditions of "capacitor in" or "capacitor out" the worst condition is the same. The diagram shows that instantaneous tripping of this particular relay will not occur during swinging or out-of-step operation, but that delayed tripping will occur if the angle δ lies close enough to $90°$ for a sufficient time to close the contacts.

The above study demonstrates that while planning systems involving switched series capacitors care must be exercised to ensure that the protective relaying is not adversely affected.

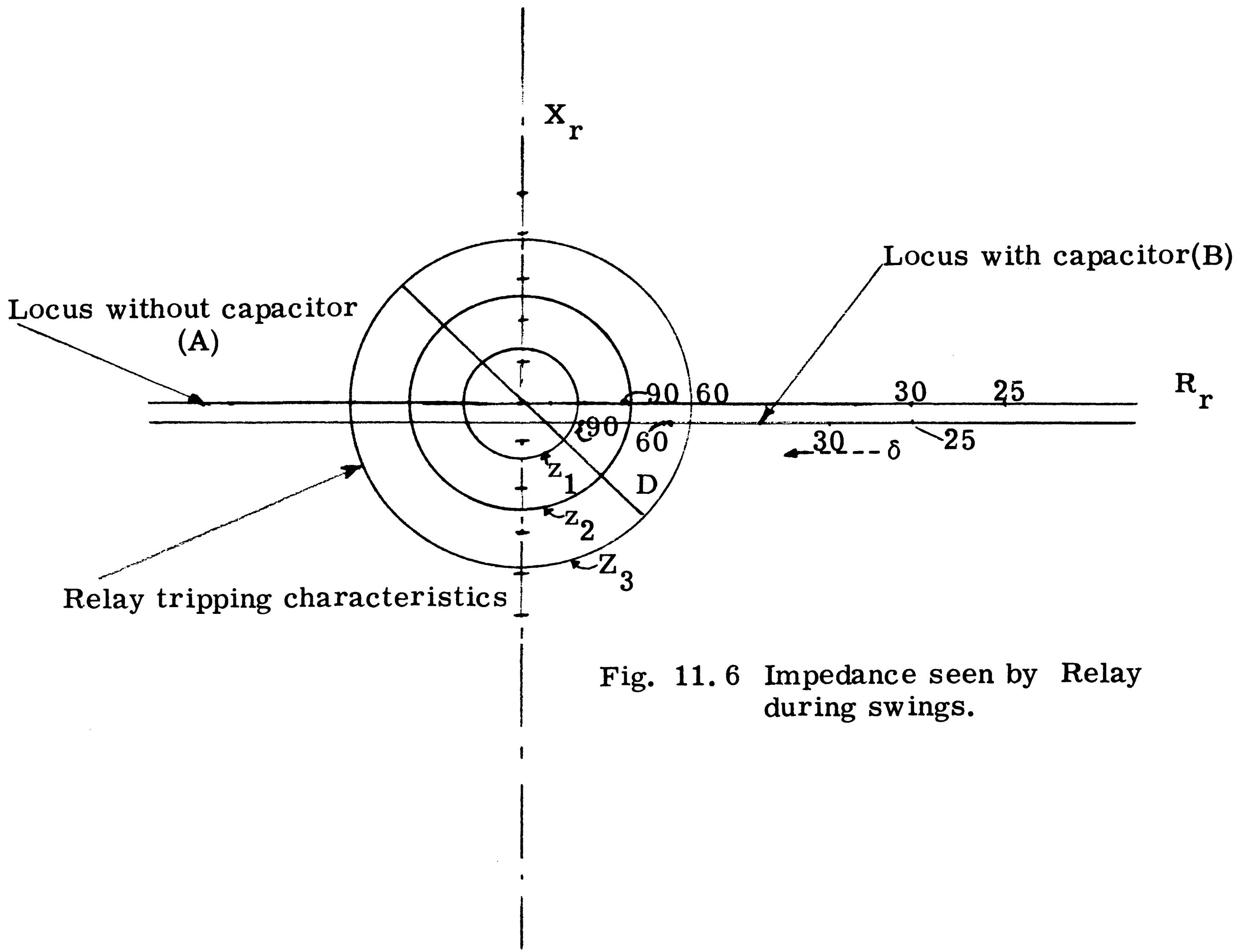


Fig. 11.6 Impedance seen by Relay during swings.

Chapter XII

CONCLUSIONS

Three methods of improving transient stability of a synchronous generator have been investigated.

The first method involves the use of signals proportional to the derivatives of the torque angle δ. It has been demonstrated that the transient stability of a synchronous generator is substantially improved if the derivative signals of proper magnitudes are used in the excitation system. A mathematical method has been demonstrated which develops formulas for amplification factors in various feedback channels. The values of line constants of the excitation system can be substituted in appropriate formulas to determine the magnitudes of these signals. Formulas have also been derived which takes saturation into account.

The second method involves the use of Phase-shifting transformers. A mathematical analysis of the effect of these transformers, on the transient stability, has been made. It has been demonstrated that this method should have considerable effect on improving the transient stability.

The third method uses switched series capacitors. By experiment and by theory the feasibility of this method has been demonstrated. Many technical problems associated with the use of the series capacitors have been thoroughly investigated.

In conclusion, it appears that there is considerable scope for improving the system stability during transient conditions by all these methods. While switched series capacitors and phase-shifting transformers are strong measures, due to economy these may have limited applications. There will, however, be situations where these will be the most economical techniques of controlling transient instability. On the other hand, excitation control continuously contributes to damping. With the development of static excitation systems, which have low time constants and much higher voltage limits, there is great potential for improving transient stability by using derivative signals.

REFERENCES

1. American Institute of Electrical Engineers, American Standard Definitions of Electrical Terms, 35.20.200 and 35.20.203, New York, 1942.

2. Glavitsch, J., "Theoretical Investigations into the Steady-State Stability of Synchronous Machines", The Brown Boveri Review, vol. 49, no. 3/4, 1962.

3. Langer, P. and Johansson, K. E., "Influence of Load-Angle-Dependent Signals on the Voltage Regulation of Synchronous Machines", Proc. Int. Conf. Large Electric Systems (C.I.G.R.E.), Paris, 18th Convention, 1960, vol. III, paper no. 315.

4. Dineley, J. L., Morris, A. J. and Preece, C., "Optimized Transient Stability from Excitation Control of Synchronous Generators", I.E.E.E. Trans. (Power Apparatus and Systems) vol. PAS-87, no. 8, August 1968.

5. Dandeno, P. L., Karas, A. N., McClymont, K. R. and Watson, W., "Effect of High Speed Rectifier Excitation Systems on Generator Stability Limits", I.E.E.E. Trans. (Power Apparatus and Systems) vol. PAS-87, no. 1, January 1968.

6. Kimbark, E. W., "Improvement of System Stability by Switched Series Capacitors", I.E.E.E. Trans. (Power Apparatus and Systems) vol. PAS-85, no. 2, February 1966.

7. Park, R. H., "Two-Reaction Theory of Synchronous Machines - Part I, Generalized Method of Analysis", A.I.E.E. Trans., vol. 48, July 1929.

8. Olive, D. W., "New Techniques for the Calculation of Dynamic Stability", I.E.E.E. P.I.C.A. Conference, May 1965.

9. Kron, G., "A Super Regulator Concelling the Transient Reactance of Synchronous Machines", Matrix and Tensor Quarterly 1955, 5, p. 71.

10. Thaler, G. J. and Pastel, M. P., "Analysis and Design of Nonlinear Feedback Control Systems", McGraw-Hill Book Co. Inc., 1962.

11. Baba, J., Hayashi, S., Yamada, I., Haneda, H. and Ishiguro, F., "Sensitivity Analysis of Power System Stability", I.E.E.E. P.I.C.A. Conference, May 1967.

12. Smith, O. J. M., "Power System Transient Control by Capacitor Switching", I.E.E.E. Trans. (Power Apparatus and Systems) vol. PAS-88, no. 1, January 1969.

13. Breuer, G. D., Rustebakke, H. M., Gibley, R. A. and Simmons, H. O. Jr., "The Use of Series Capacitors to Obtain Maximum EHV Transmission Capability", I.E.E.E. Trans. (Power Apparatus and Systems) vol. 83, no. 11, November 1964.

14. Mittelstadt, W. A., "Four Methods of Power System Damping", I.E.E.E. Trans. (Power Apparatus and Systems) vol. PAS-87, no. 5, May 1968.

15. Stagg, G. W., Gabrielle, A. F., Moore, D. R. and Hohenstein, F., "Calculation of Transient Stability Problems Using a High Speed Digital Computer", I.E.E.E. Trans. (Power Apparatus and Systems) vol. 72, August 1959.

16. Kimbark, E. W., "Power System Stability", vol. 1, John Wiley and Sons, Inc., 1967.

17. Kimbark, E. W., "Power System Stability - Synchronous Machines", Dover Publications, Inc., New York 1968.

18. Crary, S. B., "Power System Stability", vol. II, Transient Stability, New York: John Wiley and Sons, Inc., 1962.

19. Aldred, A. S., Shackshaft, G., "The Effect of a Voltage Regulator on the Steady State and Transient Stability of a Synchronous Generator", I.E.E.E. Trans., vol. 105, part A, August 1958.

20. R. Fairfield, "Posicast Switched Series Capacitors to stablize a transmission System," M. S. thesis, Un. of California. June 1966

21. Richard, C. R., Stemler, G. E., "A Fast Response Instrument for Measurement of Power System Phase Angles," IEEE Trans-PAS-87, no. 1, January 1968.

22. Harder, E. L., Barkle, J. E., Ferguson, R. W., "Series Capacitors During Faults and Reclosing," AIEE Trans., vol. 70, 1951.

23. Kimbark, E. W., "Power System Stability," vol. 2, John Wiley and Sons, Inc., 1967.

Appendix I

Mathematical Representation of a Synchronous Generator

The analysis to be presented here employs the concept of an ideal synchronous machine and is based on the classical work of Park.

An ideal synchronous machine has no saturation, hysteresis or eddy currents and all of its fields are sinusoidally distributed. The assumption of sinusoidal field distribution removes the harmonics of the airgap mmf and flux waves, from consideration. Under most operating conditions, these harmonics have a secondary effect on machine behavior, hence this assumption is justified.

The neglect of magnetic saturation is a serious approximation, particularly in steady state stability study. Hence the technique by which magnetic saturation can be taken into account will be discussed.

Magnetic hysteresis is negligible for materials used in the construction of modern machines and will be ignored here.

In the discussion to follow, a salient pole rotor is analyzed, rather than a nonsalient pole. The reason for this is that a hydroset is usually a slow speed set and hence has a salient pole rotor. Since this research deals with the analysis of a single machine connected with a large system by means of a long transmission system, it is more representative of a remote hydro-station connected to a load center by means of a transmission line.

It would be appropriate here to briefly describe the construction of a typical synchronous machine. The salient-pole rotor has two axes of symmetry. One, called the direct axis, is identical to the polar axis, the other is coincident with the interpolar axis and is called the quadrature axis. By definition, the positive quadrature axis is taken ahead of the positive direct axis in the direction of rotation.

The stator has three, distributed phase windings, the positive axes of which are equally spaced about the stator periphery at intervals of $2\pi/3$ electrical radians. The three phases are lettered abc to indicate the order in which their positive axes are encountered in skirting the stator periphery in the direction of rotor rotation. A schematic diagram of an idealized, two-pole synchronous machine is given in figure A1-1. The angle θ is the electrical angle by which the rotor reference axis is advanced on the stator reference axis in the direction of rotation.

To develop the equations of motion for an idealized, three-phase, salient pole synchronous alternator, linear system of coupled networks will be used which is the usual method of treatment for such a study. Saturation will be taken into account later on by means of certain factors known as saturation factors. A set of inductance coefficients will be defined and the principal of superposition used in determining the total flux linkages of each of the several machine windings.

Double subscript notation will be used to identify the inductance coefficients, like subscipts designate a self-inductance coefficient, while unlike subscripts indicate the mutual inductance between the windings indicated by the subscripts.

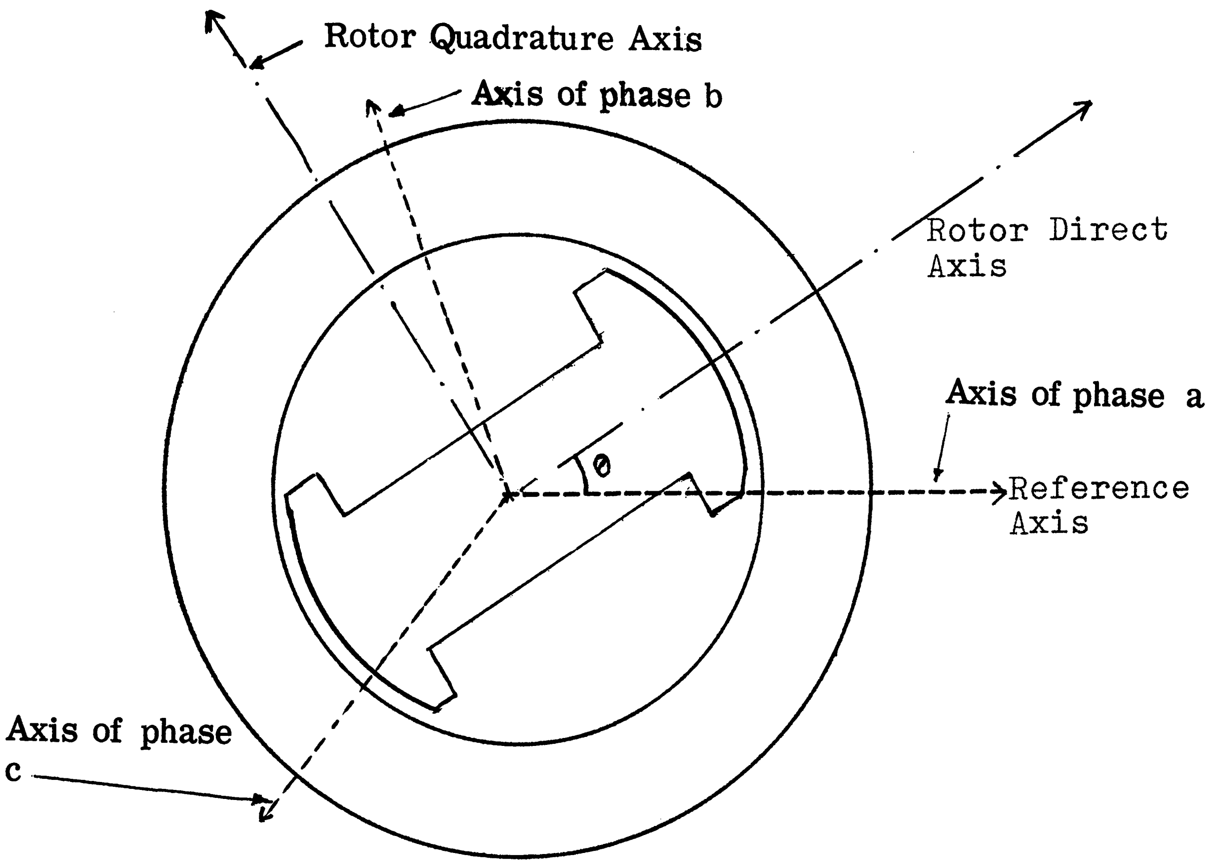


Figure A-1. 1 Two Pole Synchronous Machine

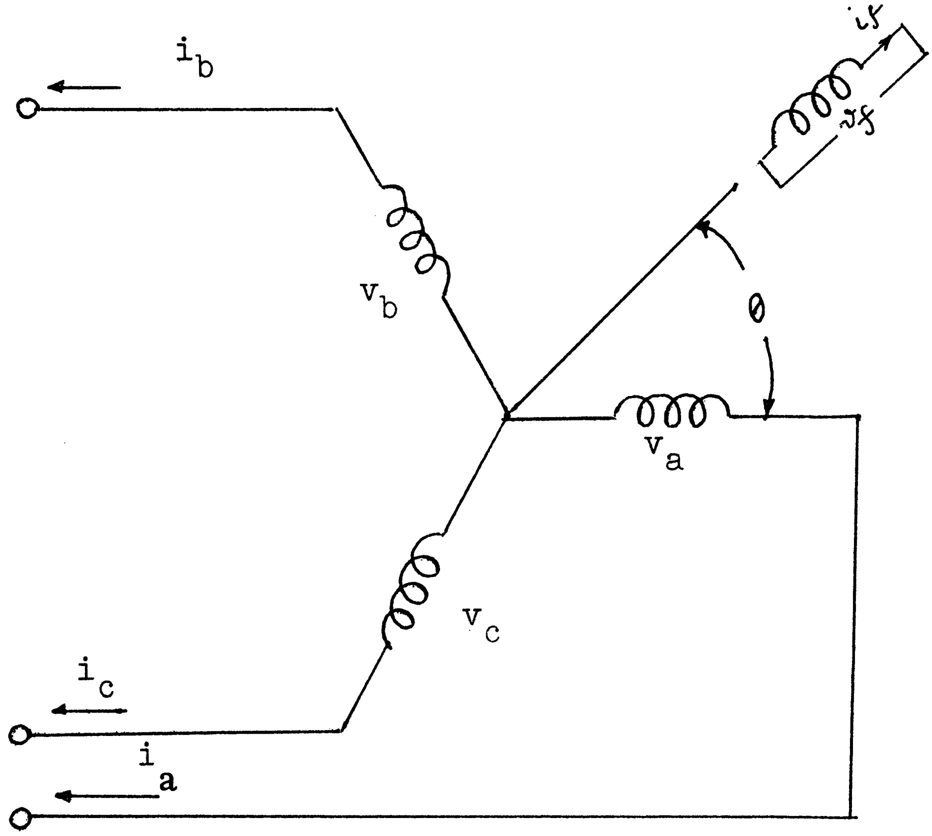


Fig. A-1.2 Y-Connected Synchronous Machine Windings

The induced voltage in a coil is calculated as

$$v(t) = \frac{d\,\psi(t)}{dt} \qquad (1)$$

where $\psi(t)$ is the coil's flux linkage. If current flows in the windings,

$$v_a(t) = ri_a(t) + \frac{d\,\psi_a(t)}{dt}$$

$$v_b(t) = ri_b(t) + \frac{d\,\psi_b(t)}{dt} \qquad (2)$$

$$v_c(t) = ri_c(t) + \frac{d\,\psi_c(t)}{dt}$$

$$v_f(t) = r_f\, i_f(t) + \frac{d\,\psi_f(t)}{dt}$$

Since saturation is not as yet taken into account, flux linkages can be assumed to be proportional to mmf's and superposition is applicable. Dropping t for convenience,

$$\psi_a = L_{aa}\, i_a + L_{ab}\, i_b + L_{ac}\, i_c + L_{af}\, i_f$$

$$\psi_b = L_{ba}\, i_a + L_{bb}\, i_b + L_{bc}\, i_c + L_{bf}\, i_f$$

$$\psi_c = L_{ca}\, i_a + L_{cb}\, i_b + L_{cc}\, i_c + L_{cf}\, i_f \qquad (3)$$

$$\psi_f = L_{fa}\, i_a + L_{fb}\, i_b + L_{fc}\, i_c + L_{ff}\, i_f$$

L_{aa}, L_{bb}, L_{cc} are self inductances, L_{ab}, L_{ac}, etc. are mutual inductances.

L_{aa} will have a maximum value when the direct axis is aligned with the axis of coil a and L_{ab} will have a maximum value when the rotor has moved 30° clockwise from the center line of coil a. L_{aa} has a minimum value when the quadrature axis is aligned with the axis of coil a. The variation of L_{aa} between its extreme values is sinusoidal and can be expressed as

$$L_{aa} = L_s + L_m \cos 2\theta$$

where θ is measured from the center line of coil a.

$$L_{bb} = L_s + L_m \cos 2(\theta - 120^\circ)$$

$$= L_s + L_m \cos(2\theta + 120^\circ)$$

$$L_{cc} = L_s + L_m \cos(2\theta - 120^\circ)$$

The expressions for mutual inductances are

$$L_{ab} = L_{ba} = -[M_s + L_m \cos 2(\theta + 30^\circ])$$

$$= -M_s + L_m \cos(2\theta - 120^\circ)$$

$$L_{bc} = L_{cb} = -M_s + L_m \cos 2\theta$$

$$L_{ca} = L_{ac} = -M_s + L_m \cos(2\theta + 120^\circ)$$

$$L_{af} = L_{fa} = M_f \cos \theta$$

$$L_{bf} = L_{fb} = M_f \cos(\theta - 120^\circ)$$

$$L_{cf} = L_{fc} = M_f \cos(\theta + 120^\circ)$$

Since the rotor mmf is driving flux across a constant air gap, L_{ff} is a constant.

These expressions for the various inductances can be substituted in equation (3) to calculate flux linkages in terms of currents. The constant velocity of the rotor is taken into account by writing $\theta(t) = \omega t + \theta_o$. The equation for v_a is

$$
\begin{aligned}
v_a = {} & [r - 2\omega L_m \sin 2(\omega t + \theta_o)]\, i_a - [2\omega L_m \sin(2\omega t + 2\theta_o - 120^\circ)]\, i_b \\
& - [2\omega L_m \sin(2\omega t + 2\theta_o + 120^\circ)]\, i_c - [\omega M_f \sin(\omega t + \theta_o)]\, i_f \\
& + [L_s + L_m \cos 2(\omega t + \theta_o)] \frac{di_a}{dt} + [M_s + L_m \cos(2\omega t + 2\theta_o - 120^\circ)] \frac{di_b}{dt} \\
& + [M_s + L_m \cos(2\omega t + 2\theta_o + 120^\circ)] \frac{di_c}{dt} + [M_f \cos(\omega t + \theta_o)] \frac{di_f}{dt}
\end{aligned}
$$

Similar differential equations are obtained for v_b, v_c and v_f. It is obviously very difficult to solve these equations.

d-q Transformations

It will simplify the calculations if the flux linkages due to the stator coils are resolved along the direct and quadrature axes of the machine. See Figure A1.1. Using the following Park's transformation

$$
\begin{aligned}
\psi_d &= \frac{2}{3} [\psi_a \cos\theta + \psi_b \sin(\theta - 120^\circ) + \psi_c \cos(\theta + 120^\circ)] \\
\psi_q &= -\frac{2}{3} [\psi_a \sin\theta + \psi_b \sin(\theta - 120^\circ) + \psi_c \sin(\theta + 120^\circ)] \qquad (4) \\
\psi_o &= \frac{1}{3} (\psi_a + \psi_b + \psi_c)
\end{aligned}
$$

or

$$\psi_a = \psi_d \cos\theta - \psi_q \sin\theta + \psi_o$$

$$\psi_b = \psi_d \cos(\theta - 120^\circ) - \psi_q \sin(\theta - 120^\circ) + \psi_o$$

$$\psi_c = \psi_d \cos(\theta + 120) - \psi_q \sin(\theta + 120^\circ) + \psi_o \quad (5)$$

Similarly

$$v_a = v_d \cos\theta - v_q \sin\theta + v_o$$

$$v_b = v_d \cos(\theta - 120^\circ) - v_q \sin(\theta - 120^\circ) + v_o$$

$$v_c = v_d \cos(\theta + 120) - v_q \sin(\theta + 120^\circ) + v_o$$

$$i_a = i_d \cos\theta - i_q \sin\theta + i_o$$

$$i_b = i_d \cos(\theta - 120^\circ) - i_q \sin(\theta - 120^\circ) + i_o$$

$$i_c = i_d \cos(\theta + 120^\circ) - i_q \sin(\theta + 120^\circ) + i_o$$

Substituting expressions for ψ_a, ψ_b, and ψ_c in (4), the result is

$$\left.\begin{aligned}
\psi_d &= (L_s + M_s + \tfrac{3}{2} L_m)\, i_d + M_f\, i_f \\
\psi_q &= (L_s + M_s - \tfrac{3}{2} L_m)\, i_q \\
\psi_o &= (L_s - 2M_s)\, i_o \\
\psi_f &= \tfrac{3}{2} M_f\, i_d + L_{ff}\, i_f
\end{aligned}\right\} \quad (6)$$

Let $L_d = L_s + M_s + \frac{3}{2} L_m$. This may be called the direct axis synchronous inductance.

$$L_q = L_s + M_s - \frac{3}{2} L_m$$

This is called the quadrature axis synchronous inductance.

$$L_o = L_s - 2M_s,$$

this is called the zero sequence inductance. Then

$$\psi_d = L_d\, i_d + M_f\, i_f$$

$$\psi_q = L_q\, i_q$$

$$\psi_o = L_o\, i_o$$

$$\psi_f = \frac{3}{2} M_f\, i_d + L_{ff}\, i_f$$

Transforming the voltage equations to the d-q reference frame.

$$v_d = \frac{2}{3}[\, v_a \cos\theta + v_b \cos(\theta - 120^\circ) + v_c \cos(\theta + 120^\circ)]$$

$$= \frac{2}{3}[(ri_a + \frac{d\,\psi_a}{dt})\cos\theta + (ri_b + \frac{d\,\psi_b}{dt})\cos(\theta - 120^\circ) + (ri_c + \frac{d\,\psi_c}{dt})\cos(\theta + 120^\circ)]$$

$$= \frac{2}{3} r[i_a \cos\theta + i_b \cos(\theta - 120^\circ) + i_c \cos(\theta + 120^\circ)] + \frac{2}{3}[\frac{d\,\psi_a}{dt}\cos\theta + \frac{d\,\psi_b}{dt}\cos(\theta - 120^\circ) + \frac{d\,\psi_c}{dt}\cos(\theta + 120^\circ)]$$

$$v_d = ri_d + \frac{d\,\psi_d}{dt} - \omega\,\psi_q \qquad (8)$$

Similarly

$$v_q = ri_q + \frac{d\,\psi_q}{dt} + \omega\psi_d \qquad (9)$$

$$v_o = ri_o + \frac{d\,\psi_o}{dt}. \qquad (10)$$

Note that $-\omega\,\psi_q$ and $\omega\,\psi_d$ terms are large as compared to ri and $\frac{d\psi}{dt}$ terms and correspond to the voltages induced as the air gap flux wave sweeps past the stationary stator coils.

The same equations can represent a generator if the sign of the currents is changed, which will give the following set of equations

$$v_d = -ri_d + \frac{d\,\psi_d}{dt} - \omega\,\psi_q \tag{12}$$

$$v_q = -ri_q + \frac{d\,\psi_q}{dt} + \omega\,\psi_d \tag{13}$$

$$v_f = r_f\, i_f + \frac{d}{dt}\,\psi_f \tag{14}$$

$$\psi_d = -L_d\, i_d + M_f\, i_f \tag{15}$$

$$\psi_f = -\frac{3}{2} M_f\, i_d + L_{ff}\, i_f \tag{16}$$

$$\psi_q = -L_q\, i_q \tag{17}$$

For a balanced system,

$$v_d = v \sin\delta \tag{18}$$

$$v_q = v \cos\delta \tag{19}$$

where δ is the angle between the voltage v and the quadrature axis.

Solving equations (15)-(17) for currents and substituting in (12)-(14)

$$i_d = \frac{M_f}{L_{ff}\, L'_d}\,\psi_f - \frac{\psi_d}{L_d'} \tag{20}$$

$$i_f = \frac{Ld}{L_{ff}\, L'_d}\,\psi_f - \frac{3}{2}\,\frac{M_f}{L_{ff} L'_d}\,\psi_d$$

where $$L'_d = L_d - \frac{3}{2}\,\frac{M_f^{\,2}}{L_{ff}}$$

is called the direct axis transient inductance.

$$v_d = \frac{d\psi_d}{dt} - \frac{r M_f}{L_{ff} L'_d} \psi_f + \frac{r \psi_d}{L'_d} - \omega \psi_q \tag{21}$$

$$v_q = \frac{d\psi_q}{dt} + \frac{r}{L_q} \psi_q + \omega \psi_d \tag{22}$$

$$v_f = \frac{d\psi_f}{dt} + \frac{\gamma_f}{L_{ff}} \frac{L_d}{L_d'} \psi_f - \frac{3}{2} \frac{r_f}{L_{ff} L'_d} M_f \psi_d \tag{23}$$

If the derivatives of ψ_d and ψ_q are neglected, since these are small compared to the $\omega\psi$ terms, this reduces (21) - (23) to

$$v_d = - \omega \psi_q - ri_d$$

$$v_q = \omega \psi_d - ri_q$$

$$\frac{d\psi_f}{dt} = v_f - \frac{r_f}{L_{ff}} \frac{L_d}{L_d'} \psi_f + \frac{3}{2} \frac{r_f}{L_{ff}} \frac{M_f}{L_d'} \psi_d$$

Let $\frac{L_{ff}}{r_f} = T'_{do}$ = open circuit field time constant, then

$$\frac{d\psi_f}{dt} = v_f - \frac{L_d}{L_d' T'_{do}} \psi_f + \frac{3}{2} \frac{M_f}{L'_d T'_{do}} \psi_d$$

Let $\omega M_f i_f = E_q$

Multiply equation (14) by $\frac{\omega M_f}{L_{ff}}$

$$\frac{\omega M_f}{L_{ff}} v_f = \frac{\omega M_f}{L_{ff}} r_f i_f + \frac{d}{dt} \left(\frac{\omega M_f}{L_{ff}} \psi_f \right)$$

Let $E_q' = \frac{\omega M_f}{L_{ff}} \psi_f$.

$$\frac{\omega M_f}{L_{ff}} v_f = \frac{\omega M_f}{L_{ff}} \frac{r_f}{r_f} v_f = \frac{E_{ex}}{T_{do}}, \text{ where}$$

E_{ex} is the open circuit armature voltage.

Hence $$\frac{d}{dt} E_q' = \frac{1}{T_{do}'} (E_{ex} - E_q)$$

where $\overline{E}_q = \bar{v} + \mathbf{r}\overline{I} + j\omega L_d \bar{i}_d + j \omega L_q \bar{i}_q$

from equation (2 0)

$$i_d = \frac{M_f}{L_{ff} \; L'd} \psi_f - \frac{\psi_d}{L'_d}$$

$$\text{or} \quad \omega L'_d i_d = \frac{\omega M_f}{L_{ff}} \psi_f - \omega\psi_d$$

$$\text{or} \quad \omega L'_d i_d = E_q' - v_q - ri_q$$

$$\text{Hence } E_q' = v_q + \omega L'_d i_d + ri_q \tag{24}$$

Since $v_q = -ri_q - \omega L_d i_d + E_q$

Substituting in (24) gives

$$E_q' = -ri_q - \omega (L_d - L_d') i_d + E_q + ri_q$$

But $E_q = E_{qd} + \omega(L_d - L_q) i_d$

Therefore $E_q' = E_{qd} - \omega(L_q - L_d')i_d$

or in phasor form

$$\bar{E}_q' = \bar{E}_{qd} - j\omega(L_q - L_d')\bar{i}_d$$

If stator resistance is neglected, the equations can be summarized as follows:

$$\frac{d E_q'}{dt} = \frac{1}{T_{do}'} (E_{ex} - E_q) \tag{25}$$

$$E_q = E_{qd} + \omega(L_d - L_q) i_d \tag{26}$$

$$\bar{E}_{qd} = \bar{v} + j\omega L_q \bar{I} \tag{27}$$

$$E_q' = E_{qd} - \omega (L_q - L_d') i_d \tag{28}$$

$$\bar{E}_q = \bar{v} + j\omega L_d \bar{i}_d + j \omega L_q \bar{i}_q \tag{29}$$

The phasor diagram corresponding to above equations is shown on the next page. In the diagram, v_b is the busbar voltage, v_m the machine terminal voltage and x_e the external reactance.

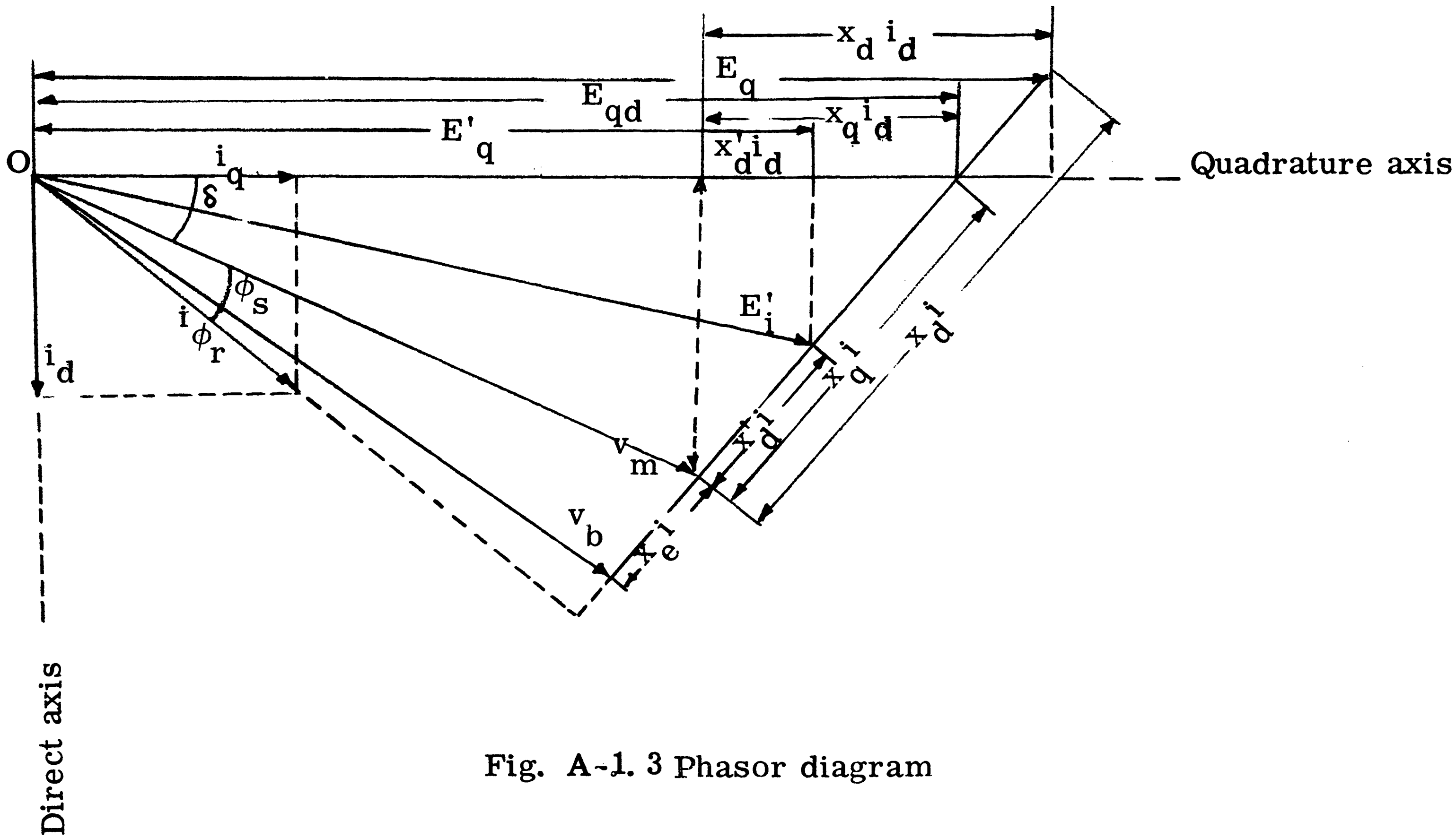


Fig. A-1. 3 Phasor diagram

APPENDIX II

Computer Programs

Symbols Used- Program nos 1, 2 and 3

G = Machine Rating

H = Inertia Constant

DLTT = Time Interval

XD = Direct Axis Synchronous Reactance

XQ = Quadrature Axis Synchronous Reactance

XDP = Direct Axis Transient Reactance

XE = External Reactance

TDO = Open Circuit Time Constant

TE = Exciter Time Constant

VFO = Initial Value of Excitation Voltage

VR = Reference Voltage

MUV = Gain in Voltage Channel μ_v

AMUD = A_2 = Gain in $\frac{d\delta}{dt}$ Channel

AMU2D = A_1 = Gain in $\frac{d^2\delta}{dt^2}$ Channel

VB = Busbar Voltage

XC = Input Power in the First Interval

YC = Input Power in the Second and Subsequent Intervals

EEXO = Initial Value of Exciter Voltage Referred to Armature Circuit

EQPO = Initial Value of Voltage Proportional to Field Flux Linkage

PU = Output Power

$$DF = \frac{d\delta}{dt}$$

$$DS = \frac{d^2\delta}{dt^2}$$

$$D(N) = \delta_n$$

V = Machine Terminal Voltage

F = Frequency

Program No. 3

C = A Constant = $M = \frac{GH}{180f}$

DLTT = Time Interval δT

PAI = Accelerating Power in the First Interval

DI = Initial Value of δ

PI = Input Power

THETA = θ the Angle by which the Power Angle Curve is Shifted

Computer Program No. 1

Chapter V : Excitation Systems (Digital Computer Results)

Title : To find transient Stability of a System using auxiliary signals in the excitation system.

```
      DIMENSION D(50)
      NAMELIST/NAME1/F,G,H,DLTT,XD,XQ,XDP,XE,TDO,TE,
     1VFO,VR,MUV,AMUD,AMU2D,VB,XC,YC,EEXO,EQPO
      PU=1.0
      READ(5,NAME1)
      D(1)=50.2
      PI=XC
      EEX=EEXO
      EQP=EQPO
      T=0.
      DF=0.
      N=1
    1 T=T+DLTT
      N=N+1
      DS=(PI-PU)*180.*F/(G*H)
      DDF=DS*DLTT
      DF=DDF+DF
      DD=DF*DLTT
      D(N)=D(N-1)+DD
      WRITE(6,3)T,D(N),EEX
    3 FORMAT(2HT=,F6.3,2X,2HD=,F6.3,2X,4HEEX=,F6.3)
      X=D(N)*3.14159/180.
      AA=VB*VB*(XQ-XDP)*SIN(X)*COS(X)/((XQ+XE)*(XDP+XE))
      BB=(XDP-XQ)/(XQ+XE)
      EQ=(EQP+BB*COS(X))/(1.+BB)
      VD=XQ*VB*SIN(X)/(XQ+XE)
      CC=(XD-XDP)/(XQ-XDP)
      DD=(XD-XQ)/(XQ-XDP)
      VF=CC*EQ-DD*EQP
      VQ=(XD*VB*COS(X)+XE*VF)/(XD+XE)
      V=SQRT(VD*VD+VQ*VQ)
      WRITE(6,10)V
   10 FORMAT(2HV=,F6.3)
      EX=VFO-MUV*(V-VR)+AMUD*DF+AMU2D*DS
      IF(EX+1.)6,6,7
    6 EX=-1.
      GO TO 4
```

continued

```
7 IF(EX-3.5)4,5,5
5 EX=3.5
4 CONTINUE
  AK1=DLTT*(EX-EEX)/TE
  AK2=DLTT*(EX-EEX+.5*AK1)/TE
  AK3=DLTT*(EX-EEX+.5*AK2)/TE
  AK4=DLTT*(EX-EEX+AK3)/TE
  EEX=EEX+(AK1+AK2+AK2+AK3+AK3+AK4)/6.
  DNM=(EEX-VF)*DLTT
  DNMM=(VF/EQP)*DLTT/2.
  DEQP=DNM/(TDO+DNMM)
  EQP=EQP+DEQP
  PU=(EQP*VB*SIN(X)/(XDP+XE))-AA
  PI=YC
  IF(N-50)2,8,8
2 GO TO 1
8 CONTINUE
  END
```

Computer Program No. 2

Chapter VIII : Application of Phase Shifting Transformers.

Title: To compute data for a Swing Curve for a system using Phase Shifting Transformers.

```
      DIMENSION D(25),DLTD(25),PO(25)
      NAMELIST/NAME1/C,DLTT,PAI,DI,PI,DDI,THETA
      READ(5,NAME1)
      D(1)=DI
      PO(1)=PAI
      DLTD(1)=0.
      T=0.
      N=1
      DDIO=DDI
    1  T=T+DLTT
      N=N+1
      DLTD(N)=DLTD(N-1)+PO(N-1)*DLTT*DLTT/C
      DLTD(N)=DLTD(N)+DDIO
      D(N)=D(N-1)+DLTD(N)
      WRITE(6,3)T,D(N)
    3 FORMAT(2HT=,F6.3,5X,2HD=,F10.5)
      X=((D(N)+THETA)*3.1416/180.)
      PO(N)=PI-1.25*SIN(X)
      DDIO=0.
      IF(N-24)1,1,9
    9 CONTINUE
      END
# END OF FILE
```

Computer Program No. 3

Chapter X : Switched Series Capacitors (Experimental Study)

Title: To determine Transient Stability of a system using switched series capacitors. The capacitors are taken out after a fixed number of of cycles.

```
      DIMENSIØN D(30)
      NAMELIST/NAME1/F,G,H,DLTT,XD,XQ,XDP,XEI,TDØ,TE,
     1VFØ,VR,MUV,AMUD,AMU2D,VB,XC,YC,EEXØ,EQPØ,D10,TF
      PU=1.
      READ(5,NAME1)
      D(1)=D10
      PI=YC
      EEX=EEXØ
      EQP=EQPØ
      V=VR
      T=0.
      DF=0.
      N=1
      XE=XEI-XC
    1 T=T+DLTT
      N=N+1
      DS=(PI-PU)*180.*F/(G*H)
      DDF=DS*DLTT
      DF=DDF+DF
      DD=DF*DLTT
      D(N)=D(N-1)+DD
      WRITE(6,3)T,D(N),V
    3 FØRMAT(2HT=,F6.3,2X,2HD=,F6.1,2X,2HV=,F6.2)
      X=D(N)*3.14159/180.
      AA=VB*VB*(XQ-XDP)*SIN(X)*CØS(X)/((XQ+XE)*(XDP+XE))
      BB=(XDP-XQ)/(XQ+XE)
      EQ=(EQP+BB*CØS(X))/(1.+BB)
      VD=XQ*VB*SIN(X)/(XQ+XE)
      CC=(XD-XDP)/(XQ-XDP)
      DD=(XD-XQ)/(XQ-XDP)
      VF=CC*EQ-DD*EQP
      VQ=(XD*VB*CØS(X)+XE*VF)/(XD+XE)
      V=SQRT(VD*VD+VQ*VQ)
      EX=VFØ-MUV*(V-VR)+AMUD*DF+AMU2D*DS
      IF(EX+1.)6,6,7
    6 EX=-1.
      GØ TØ 4
    7 IF(EX-3.5)4,5,5
    5 EX=3.5
```

continued

```
    6 EX=-1.
      GØ TØ 4
    7 IF(EX-3.5)4,5,5
    5 EX=3.5
    4 CØNTINUE
      AK1=DLTT*(EX-EEX)/TE
      AK2=DLTT*(EX-EEX+.5*AK1)/TE
      AK3=DLTT*(EX-EEX+.5*AK2)/TE
      AK4=DLTT*(EX-EEX+AK3)/TE
      EEX=EEX+(AK1+AK2+AK2+AK3+AK3+AK4)/6.
      DNM=(EEX-VF)*DLTT
      DNMM=(VF/EQP)*DLTT/2.
      DEQP=DNM/(TDØ+DNMM)
      EQP=EQP+DEQP
      PU=(EQP*VB*SIN(X)/(XDP+XE))-AA
      IF(DF) 11,11,2
   11 XE=XEI
      GØ TØ 12
    2 XE=XE2
   12 IF(N-30)13,8,8
   13 GØ TØ 1
    8 CØNTINUE
      END
```

Computer Program No. 4

Chapter XI : Switched Series Capacitors (Fault Study and Relaying)

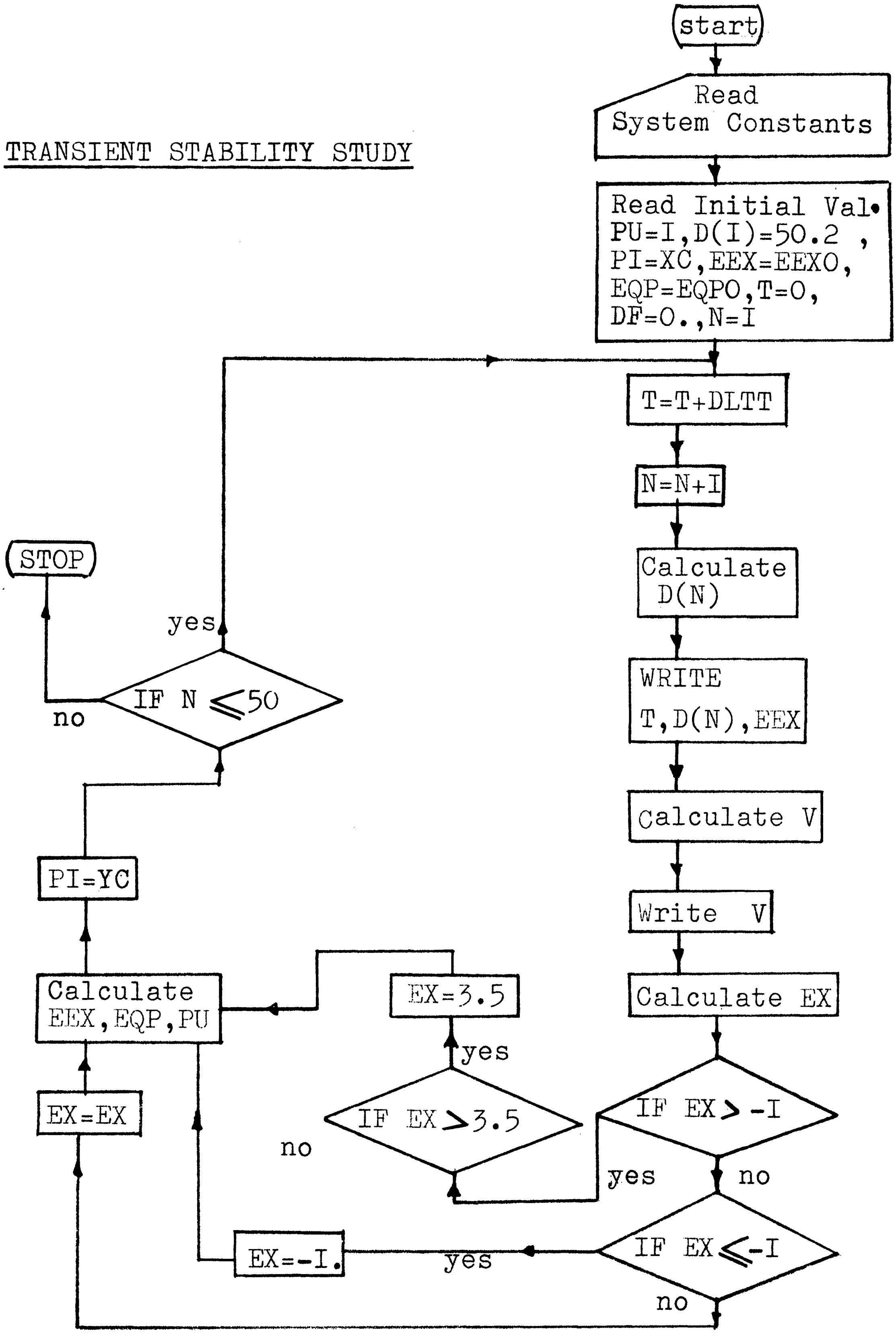
Title : To determine Transient Stability of a system using switched series capacitors. The capacitors are taken out when the torque angle is decreasing and inserted when the torque angle is increasing.

```
      DIMENSIØN D(30)
      NAMELIST/NAME1/F,G,H,DLTT,XD,XQ,XDP,XEI,TDØ,TE,
     1VFØ,VR,MUV,AMUD,AMU2D,VB,XC,YC,EEXØ,EQPØ,XE2,D10
      PU=1.
      READ(5,NAME1)
      D(1)=D10
      PI=YC
      EEX=EEXØ
      EQP=EQPØ
      V=VR
      T=0.
      DF=0.
      N=1
      XE=XEI-XC
    1 T=T+DLTT
      N=N+1
      DS=(PI-PU)*180.*F/(G*H)
      DDF=DS*DLTT
      DF=DDF+DF
      DD=DF*DLTT
      D(N)=D(N-1)+DD
      WRITE(6,3)T,D(N),V
    3 FØRMAT(2HT=,F6.3,2X,2HD=,F6.1,2X,2HV=,F6.2)
      X=D(N)*3.14159/180.
      AA=VB*VB*(XQ-XDP)*SIN(X)*CØS(X)/((XQ+XE)*(XDP+XE))
      BB=(XDP-XQ)/(XQ+XE)
      EQ=(EQP+BB*CØS(X))/(1.+BB)
      VD=XQ*VB*SIN(X)/(XQ+XE)
      CC=(XD-XDP)/(XQ-XDP)
      DD=(XD-XQ)/(XQ-XDP)
      VF=CC*EQ-DD*EQP
      VQ=(XD*VB*CØS(X)+XE*VF)/(XD+XE)
      V=SQRT(VD*VD+VQ*VQ)
      EX=VFØ-MUV*(V-VR)+AMUD*DF+AMU2D*DS
      IF(EX+1.)6,6,7
```

continued

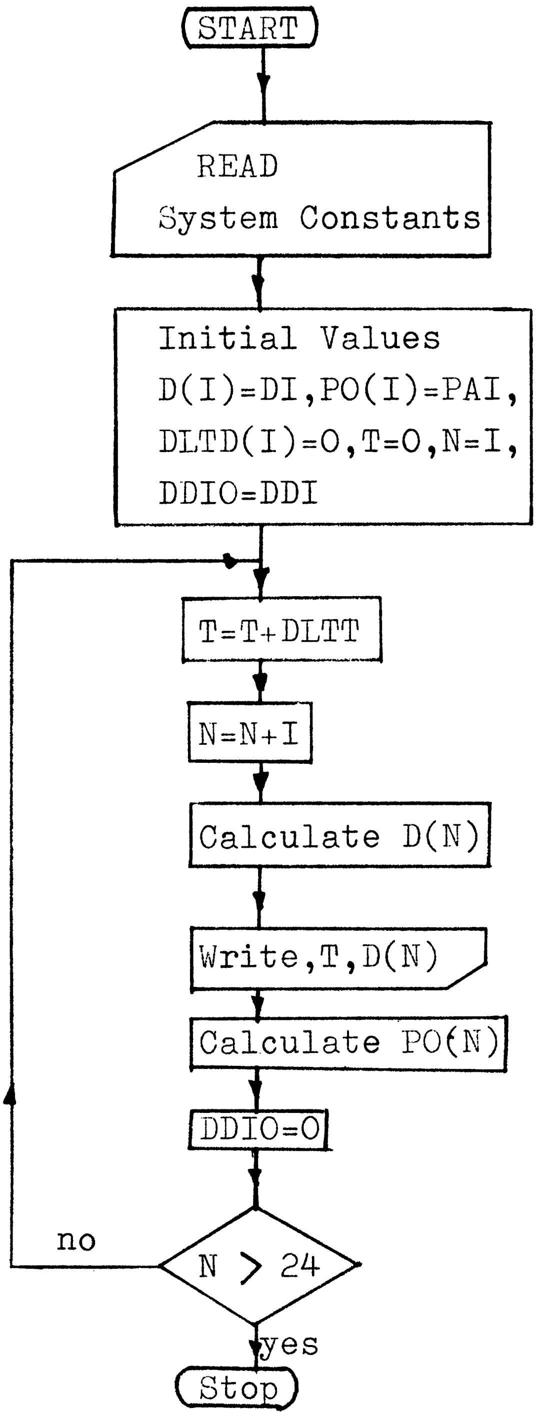
```
    4 CØNTINUE
      AK1=DLTT*(EX-EEX)/TE
      AK2=DLTT*(EX-EEX+.5*AK1)/TE
      AK3=DLTT*(EX-EEX+.5*AK2)/TE
      AK4=DLTT*(EX-EEX+AK3)/TE
      EEX=EEX+(AK1+AK2+AK2+AK3+AK3+AK4)/6.
      DNM=(EEX-VF)*DLTT
      DNMM=(VF/EQP)*DLTT/2.
      DEQP=DNM/(TDØ+DNMM)
      EQP=EQP+DEQP
      PU=(EQP*VB*SIN(X)/(XDP+XE))-AA
      IF(T-TF)2,11,11
   11 XE=XEI
      GØ TØ 12
    2 XE=XEI-XC
   12 IF(N-30)13,8,8
   13 GØ TØ 1
    8 CØNTINUE
      END
```

TRANSIENT STABILITY STUDY



FLOW CHART I FOR THE COMPUTER Programs

Transient Stability Study
using phase shift transformers
in the system



Flow Chart II for Computer Programs