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COMPUTER APPROACH TO ECONOMIC POWER SYSTEM SCHEDULING

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TO NORA AND NEHAL

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LIST OF SYMBOLS

A(n)	Coefficients of the input fuel cost function.
B_{ij}	The Pi representation B constant of the transmission line joining bus i with bus j.
B_{mn}	The loss formula B constant.
C(i)	Total fuel input cost to station i.
Cost	Total fuel input cost.
D_{in}	The Pi representation D constant of the transmission line joining bus i with bus j.
E_i	Magnitude of the voltage at bus i.
E_r	Magnitude of receiving end voltage.
E_s	Magnitude of sending end voltage.
F_i	Cost function at station i relating input fuel cost and generated power output.
f_n	Intercept of incremental fuel cost curve.
F_{nn}	Slope of incremental fuel cost curve.
F_n	Fuel input to plant n.
K_i	Incremental fuel rate at station i.
K(n)	Incremental fuel rate at station i.
L_n	Penalty factor.
NB	Number of busses.
P_{gi}	Generated power at bus i.
P_i	Sum of all powers flowing away from bus i.

$P_{input(i)}$	Fuel input cost at generator i.
P_{ij}	Transmitted power from bus i to bus j.
P_L	Total transmission power losses.
P_{L_i}	Local load power at bus i.
P_r	Receiving end power.
P_s	Sending end power.
Q_{ij}	Transmitted reactive power from bus i to bus j.
VINC	Voltage increments.
$V_{max(i)}$	Maximum voltage allowed at bus i.
$V_{min(i)}$	Minimum voltage allowed at bus i.
X_{ij}	Defined to be $= K_i \frac{E_i E_i}{B_{ij}} \sin(\beta_{ij} + \theta_{ij})$
Y_i	Total capacitive admittance at bus i.
Z	Transmission line lumped transfer impedance.
Z_i	Defined to be equal to $\frac{\partial \text{ cost}}{\partial \theta_i}$
α	Angle of the Pi representation A constant.
β_{ij}	Angle of the Pi representation B constant of the transmission line joining bus i and j.
θ_i	Voltage phase angle at bus i.
λ	Incremental cost of received power.

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ABSTRACT

COMPUTER APPROACH TO ECONOMIC
POWER SYSTEM SCHEDULING

by
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One of the important considerations in the power system industry is the problem of economic scheduling. This involves many cost factors such as those due to transmission losses , labor , supplies and maintenance. The most important of these is the fuel input cost , since it is related so closely to the transmission losses and constitutes most of the cost in power generation. Consequently a great deal of research work is being done to minimize it.

This thesis uses an approach to the power system economic problem that avoids the imposition of any severe restrictions on the system loads. This approach also shows the distribution of power for each generator in the system and the corresponding fuel cost. It also provides a way of choosing the values of shunt capacitors for each bus in the system when they are available.

In this approach voltage phase angles are considered the optimization variables in obtaining the lowest possible fuel cost. The approach also takes care of two important cases:

1) When the voltages can be varied. In this case the best voltage profile as well as the corresponding power schedule can be

determined.

2) When shunt capacitors are used , the computer program gives the values of these capacitors as well as the required generated power of each generator in the system.

With this approach Synchronous condensers will be used at intermediate busses to keep voltages adjusted to the load requirements.

A comparison test has been made to compare the results of this approach with those of some other generally used methods.

Chapter I

Introduction

One of the important questions in the power system industry is the economic problem; this problem involves many cost factors such as those due to power losses, fuel, labor, supplies and maintenance.

The most important of these is the fuel cost since it constitutes most of the cost in power generation and consequently most of the research work was done to minimize it.

As the power systems expand the determination of the optimum generation schedule for the system becomes more complex, and greater accuracy is required since a percentage saving represents more actual dollar saving on existing large systems.

In the early days, power system generation was done in an isolated form; each generator supplying part of the load. Then as the system load increased, other generators were added in parallel at the same location.

As loads increased and became relatively large, reliability of the system became more important and power stations began to interconnect their lines so that if an outage in one occurred the load could be picked up by spare capacity in the others.

With this interconnection it became apparent that in some instances it might be cheaper for a power company to buy power from a neighboring company than to generate it. For instance, when the load peaks for two stations occur at different times, both stations may reduce their generation costs by exchanging power. Again interconnection is more economical in the case of hydro and steam engines.

When one deals with an electric power system having several generators, with a combined capacity exceeding the existing load, there are infinite combinations of generator loadings which can be used to supply the load and the transmission losses. At least one combination minimizes the total fuel input while supplying the given load. Various methods have been developed and used to achieve this goal, each with different degrees of effectiveness. In any case, where strategy has been used in the assigning of loads to achieve a cheaper input cost, the term "economic dispatch" is applied.

To deal with the economic dispatch problem, the transmission losses must be considered. These losses are the penalty paid for the transmission of power from one place to another which has a relatively higher generation cost. Different stations have different cost rates for the same specific loads. Therefore it is sometimes more profitable to transmit power to the load from a higher efficiency generator located further away than from a relatively low efficiency generator close by.

In dealing with an interconnected system, we should realize that in today's complex systems one can have generating stations of various types (steam, hydro, etc.), and that each has different characteristics and different operating conditions which must be considered in power scheduling.

The use of the digital computer in the last few years has enabled the power industry to solve its problems rapidly. It has also improved the accuracy and speed of scheduling in economic dispatch problems by eliminating tedious man-power calculations.

1-1 History of the Problem:

Stahl and Steinberg [21, 22] both showed that if transmission losses were negligible, economic dispatch could be obtained when all generators operate at the same incremental cost. Incremental cost for generators is defined as the ratio of input fuel cost of the boiler-turbine-generator system to the small increase of generator output power.

Before 1942 it was very difficult to solve the economic dispatch problem power taking into consideration transmission losses because the calculations for each disptaching operation were tedious and time consuming. Around the year 1942 considerable research looking to the determination of accurate transmission loss formulas was initiated. In that year E. E. George [9] developed a superposition method which required operating each plant in turn to carry the whole system load. Then when the individual loads were reduced to stay within the plant rating, the voltage drops were sufficient to bring the substation voltage below normal, thus reducing the loads and distorting the line currents. The reduction of the loads to fractional values was required by the necessity of maintaining reasonably accurate and readable values of power flow in the most lightly loaded lines. The power factor and voltage corrections were made on the basis of the average system condition. This method was reasonably accurate in longhand computation but was only useful then because during war time it became hard to get calculating boards and thus other alternatives had to be considered.

During and after World War II a number of methods which include the transmission losses in power scheduling were developed. One of these methods [9] described a procedure for combining the incremental fuel cost and incremental transmission losses in an effort to predict optimum scheduling. This approach did increase the usefulness of the loss formulae because it used the partial derivatives in terms of some constants called the B constants as one of the important components in the loading equations. At this time an a.c. calculating board was being used as digital and analog computers were not then in general use.

About 1951, Kirchmayer [17] started investigating the problem and gave particular attention to the determination of the loss coefficients. He not only developed many methods of his own but also gathered and coordinated most of the previously existing methods. These were later published in a book.

At the same time Kron [19] presented a method of applying tensor analysis to power systems.

In 1953 the use of digital computers in the solution of the power system problems started coming into its own and naturally the first applications were to already existing methods. The next improvement was to make many of the calculations on a small digital computer, eliminating much of the human manhours necessary for the tedious longhand arithmetical work .

The American Gas and Electric Company installed an incremental transmission loss computer in its Columbus production and coordination office especially for the use of the system load dispatcher. This computer calculated the incremental transmission losses and the penalty factors for the various system operating conditions. The coordinated operation of this computer and the incremental cost slide rule together furnished a flexible and accurate method for taking into account the various and rapidly changing system conditions in the plant and on the transmission system.

Around 1957 rapid progress was made by shortening the computer time considerably, but the real power dispatch problems were developed in the first place for longhand calculations and were not designed to make full use of the versatility of a computer. An iterative method of calculating generation schedules was also introduced [2]. For a given load the computer was programmed to calculate the incremental cost of received power, the total transmission losses, the fuel input, the penalty factors and the received load along with the allocation and summation of generation. This was probably the first iterative solution on a large scale and it was reasonably successful.

In 1954 Brownlee [3] introduced the voltage phase angle method which was a completely new approach and was very useful for checking loss calculations. The industry also developed automatic and economic automation schemes whereby system frequency, net interchange and economic allocation of generation for a given area are simultaneously and automatically maintained. These devices offered important savings as they:

1. Improved the fuel economy by making the optimum scheduling more accurate than would be possible by the manual operation that existed before.
2. Saved many man-hours by eliminating certain manual procedures.

Early, Watson and Smith [8] introduced a new constant , which has been added to the B constant method because it was found that this method lacked enough accuracy when the load did not vary in the same ratio between substations at different times.

In 1956 Early adopted the use of all the power flow studies easily available in solving the resultant equations by the method of least squares.

Kirchmayer[15] collected some of the methods introduced to improve George's B constants applications. A simple explanation of the basic principles supplied a much needed summary for those engineers who had not been able to keep up with the increasing mathematical and procedural complications of loss formulas.

In 1960 George presented another paper [10] describing a new method for calculating a transmission loss formula. This method was very simple, but for large systems it required a large number of highly accurate power flow studies made on a digital computer. The method uses the results from the power flow studies to construct a set of simultaneous equations, which can then be solved on the digital computer by a standard matrix inversion to obtain the B constants for the loss formula.

Recently, some papers were published that treated the dispatch problem not only for real power flow but also for reactive power flow. Some of these papers attacked the problem by using an iterative method which relies on the computer for simplification, others used linear programming techniques.

In 1967 Dopazo, Stagg and Watson [7] published a paper in which they attempted to dispatch the real power of the system by the Lagrangian method and the reactive power by the gradient method. Alternate real and reactive power requirements for economic operation were computed until the total production cost was minimized within the limitations of the system constraints.

In 1968 Garner, Wood and Maliszewski [27] presented a paper in which they considered the reactive power effect on the dispatching problem by using a linear programming approach. The method they used for solving such a problem, with a digital AC load flow program is that of "cut and try." The paper also presented a technique whereby minimum Kilovar additions for a network may be planned on a systematic basis to meet a variety of possible operating conditions. The combined use of linear programming and nonlinear AC load flow computations provided them with an effective tool for such planning.

1-2 Statement of the Problem

In the last few years, a number of ideas that make use of the availability and speed of the modern computers have been developed in response to the demand for more accurate methods of handling the dispatch problems involving existing large and interconnected systems.

For this purpose, it is desired to formulate a mathematical model for the systems real power dispatch assuming constant voltages for all buses.

It is also necessary to investigate the different ways of improving the input fuel cost by changing the voltage profiles and/or the reactive power by any practical means.

Brownlee's idea of using voltage angles will be a good tool in the formulation of the dispatching mathematical model for the system.

Chapter II

Literature Review and Background

Most of the methods of approach to the dispatching problem and its economics were collected and published in a book by Kirchmayer (15) in 1958. The book coordinates these methods so that they can be easily understood, and presents an extensive bibliography of the subject.

For the incremental fuel rate, it is given by the following definition:

$$\text{incremental fuel rate} = \frac{\Delta \text{ input}}{\Delta \text{ output}} \text{ BTU/Kw-Hr}$$

That is, the incremental fuel rate is equal to a small change in the input divided by the corresponding small change of the output. As Δ quantities become progressively smaller, it is seen that the:

$$\text{incremental fuel rate} = \frac{d\,(\text{input})}{d\,(\text{output})} \text{ BTU/Kw-Hr}$$

The units associated with the incremental fuel rate are BTU per Kw-hr and are the same as the heat-rate units. The incremental fuel rate is converted to incremental fuel cost by multiplying the former in BTU per Kw-hr by the fuel cost in cents per million BTU.

The incremental production cost of a given unit is made up of the incremental fuel cost plus the incremental cost of such items as labor, supplies, maintenance and water. In order to be able to carry out an analysis it is necessary to express the cost of these production items as a function of the instantaneous output. Since no method exists at the present time that expresses the cost of labor , supplies and maintenance accurately as a function of output , arbitrary methods of determining the incremental cost factors are used. The most common of these is one that assumes the above costs to be a fixed percentage of the incremental fuel costs. In many systems , for the purpose of scheduling generation , the incremental production cost is assumed to be equal to the incremental fuel cost. This method will be used in this thesis and an approach will be developed to minimize it.

In his discussion of optimum scheduling , Kirchmayer started by neglecting transmission losses.

If : F_n = input to unit n in dollars per hour.

And F_t = total input to system in dollars per hour.

Then

$$F_t = \sum_n F_n$$

Subject to the constraint :

$$\sum_n P_n = P_r$$

Where :

P_r = total received load.

P_n = output of unit n .

Kirchmayer then showed that for the total input cost F_t to be minimum the following condition must be satisfied:

$$\frac{dF_n}{dP_n} = \lambda$$

where

$\frac{dF_n}{dP_n}$ = incremental production cost of unit n in dollars per Kw-hr.

λ = incremental cost of received power in dollars per Kw-hr.

The value of λ must be chosen so that $\sum P_n = P_r$. In other words, the minimum input in dollars per hour for a given total load is obtained when all generating units are operated at the same incremental cost. Increasing λ results in an increase in total generation while decreasing λ results in a decrease in total generation. As a result he noted that the total cost varies slowly with changes from the minimum cost point.

This is but one method of scheduling generation. Other methods which are still used are:

1. Base loading to capacity:

   The turbine generators are successively loaded to capacity in the order of their efficiencies. This is particularly used when most of the generation is in the same area, and becomes less accurate as transmission distances increase.

2. Base loading to most efficient load:

   The turbine-generator units are successively loaded, in ascending order of their heat rates, to their most efficient loads. When all units are operating at their most efficient loads they are loaded to capacity in the same order.

3. Proportional to capacity

The loads on the units are scheduled in proportion to their rated capacity.

None of the above methods takes into account the transmission losses, and since in modern systems these are a major factor, Kirchmayer then went on to investigate these losses and their effects.

In general all sources of generation are not located at the same bus but are connected by means of a transmission network to the various loads, so that some plants will be more favorably located with respect to the loads than others. Also if the criterion of equal incremental production costs is applied there will be transmission of power from low cost areas to high cost areas. It will thus be necessary for optimum economic operation to recognize that transmission losses occur in this operation, and to modify the incremental production costs of all plants to take these losses into account. But it was very tedious to calculate the system transmission losses until George (9) introduced his transmission loss formula.

In order to develop a transmission loss formula certain assumptions had to be made. The transmission losses thus could be approximated by means of a transmission loss formula

$$P_L = \sum P_m B_{mn} P_n \qquad \text{........... (2-1)}$$

where

B_{mn} = loss formula coefficients

P_m = source powers

The loss-formula coefficients may be considered as equivalent transmission loss circuits from each generating source to the hypothetical load.

The assumptions involved in deriving a loss formula of this kind are that:

1. The equivalent load current at any bus remains a constant complex fraction of the total equivalent load current. Nonconforming loads may be treated as negative sources in the formula or in special cases may be handled by a loss formula that includes linear terms and a constant term in addition to the quadratic terms ($P_m B_{mn} P_n$).
2. The generator bus voltages remain constant.
3. The generator bus angles remain constant.
4. The source reactive power may be approximated by the sum of a component which varies with the system load and a component which varies with the source output. [Discrepancies naturally arise due to changes in the above assumptions as the load changes.]

George's loss formula was the basis for many later methods which use it to schedule power systems. Kirchmayer and Stagg (18) used this loss formula to introduce new methods such as the Penalty Factor and the Exact and Average Incremental Rate methods for dispatch power (Appendix I).

George (10) originated the B-constant method and brought it up to date. He also proposed a new method which he designed specifically for the digital computer. This method is based on a principle which can not be used with data from an A-C calculating board power flow study of losses because of limitations of accuracy in A-C board results. This method is extremely straight forward and economical especially for small and medium sized systems. The procedure is the following.

1. Assume a system with three sources. (It will have six B constants.) Set up six power flow studies with different loadings used on every source in each study. Read all the line currents.
2. Calculate the losses for each of the six power flow studies by the I^2R (current-resistance) method, using the currents obtained from the studies just mentioned, and resistance values obtained from the impedance diagram.
3. Set up a system of six simultaneous equations for six unknowns. Place the total calculated losses on the right hand side of equation (2-1).

4. Solve the equations simultaneously by any method, preferably that of matrix inversion on a digital computer. The results should be the B constants.

George and E. D. Early did some work on this method and discovered it to be highly accurate for the given power flow studies and also that the B constants so derived would always fit the given generating conditions exactly, but would not fit other equally valid generating schedules. This method was developed as far back as 1953 but discarded then as too many of the equations were "ill conditioned." However, since then, methods have been developed for solving this type of equation and, therefore, the objection is no longer valid. "Ill conditioned" equations are equations that involve small differences between large quantities in their solution, thus giving rise to gross errors. The Eigenvalue theory can simplify this considerably. The objection to this system is that too many load flow studies are needed for large systems.

In 1954 Kirchmayer (5) introduced a paper in which a new method had been developed for the practical use of voltage phase angles. The procedure involved is:

1. To relate incremental transmission losses between two plants to simple functions of voltage phase angles and system reactance resistance ratios, requiring no separate consideration of:

a) magnitudes of station voltage levels.

b) transfer impedance between plants.

c) ratios of reactive power to real power.

d) system load distribution or generation of other plants.

2. To coordinate incremental fuel costs and incremental transmission losses by comparisons between pairs of plants.

3. To determine changes in system losses due to substantial exchanges in generation between the plants.

4. To examine the influence of loop interconnections on the transmission losses of each area.

5. To evaluate the power loss influence of proposed new transmission lines and alternative plans for generating unit additions.

The Kirchmayer approach involves the least assumptions and is most accurate due to the fact that he avoided the B constant's tedious restrictions.

The two bus system explained in his paper is shown in fig. (2-1). The voltage magnitude is to remain constant and the reactive power flows in such a manner as to keep the voltage constant.

The power angle equations may be written as:

$$P_1 = \frac{V_1^2}{Z_{11}} \sin \alpha_{11} + \frac{V_1 V_2}{Z_{12}} \sin (\theta_{12} - \alpha_{12})$$

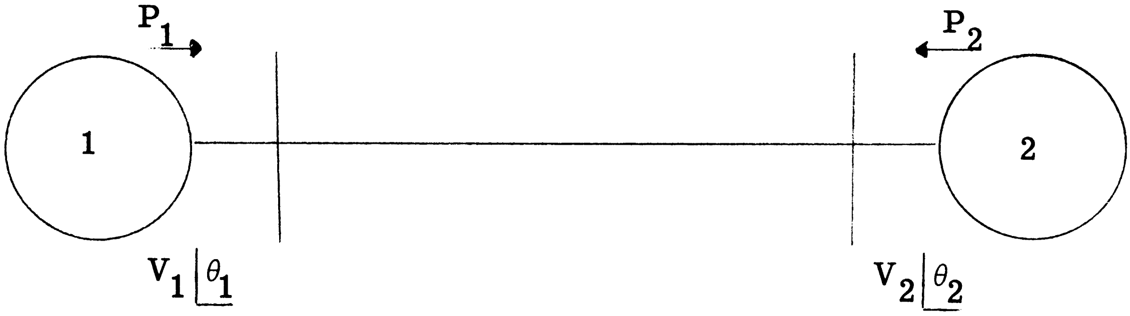


Figure 2-1 Two machine system

$$P_2 = \frac{V_2^2}{Z_{22}} \sin \alpha_{22} + \frac{V_2 V_1}{Z_{21}} \sin (\theta_{21} - \alpha_{21})$$

where

P_i = power at source i

$Z_{ii}\underline{|\alpha_{ii}}$ and $Z_{ij}\underline{|\alpha_{ij}}$ = Driving point and transfer impedances

V_i = voltage at Bus i

θ_i = angle of voltage at source i

$\theta_{ij} = \theta_i - \theta_j$

It is intended to calculate the change in losses involved when the generation is swung between sources 1 and 2 by increasing the output of source 1 and decreasing that of source 2.

The transmission losses are given by:

$P_L = P_1 + P_2$

$$= \frac{V_1^2}{Z_{11}} \sin \alpha_{11} + \frac{V_1 V_2}{Z_{12}} \sin (\theta_{12} - \alpha_{12}) + \frac{V_2^2}{Z_{22}} \sin \alpha_{22}$$

$$+ \frac{V_2 V_1}{Z_{21}} \sin (\theta_{21} - \alpha_{21})$$

Assume that the system has changed to a new condition in which the angle θ_{12} between V_1 and V_2 increases to θ'_{12} then

$$P'_L = P'_1 + P'_2$$

The changes in total losses are then obtained from the following equation:

$$\Delta P_L = P'_L - P_L$$

The incremental loss which is required is obtained by dividing the change in loss by the change in generation of a given source when swinging generation between that one source and the other source. In this manner the value can be calculated as:

$$\frac{dP_{L_{1,2}}}{dP_1} = \frac{-2 \tan \theta_{12}}{\dfrac{x_{12}}{R_{12}} - \tan \theta_{12}}$$

Kirchmayer then applied this further to many machines but pointed out that this could lead to a cumbersome expression that might force him to make a number of mathematical assumptions which would invalidate the method's accuracy.

In 1961 Carpentier (6) of Electricité de France introduced some work on the optimum control of power using non linear programming techniques. He used the theorem of Kuhn and Tucker (Appendix II) to optimize the fuel input cost function.

In his paper he considered the system power's function as his cost function and then tried to optimize it according to the given physical restrictions.

The Carpentier method can be summarized thus: Consider an electric power system of N nodes. There may be a production P_i, Q_i and a consumption C_i, D_i of active and reactive power, respectively, at the i^{th} node, which is further characterized by its voltage V_i and phase angle θ_i. For pure production nodes, $C_i = D_i = 0$, and for pure consumption nodes, $P_i = Q_i = 0$. A frequent intermediate case is $P_i = 0$ and $Q_i \neq 0$.

The total productions $\sum_i P_i$ and $\sum_i Q_i$, must equal the total consumptions $\sum_i C_i$ and $\sum_i D_i$ plus the respective transmission losses. The cost J (in dollars per unit time) of operating the systems depends only on the active powers P_i, the reactive powers Q_i being free of cost once the equipment required for their production (e.g., capacitors) has been installed. Thus,

$$J = F(P_1, \ldots, P_N)$$

If, in a given power system, the P_i, C_i, Q_i and D_i are specified then the voltages V_i and the phase angles θ_i between these voltages and V_1 (whose phase angle is arbitrarily set equal to zero) are fixed. The related variables S_i, P_i, Q_i, V_i, and θ_i are subject to upper (M) and lower (m) constraints of the form

$$P_i^2 + Q_i^2 \leq (S_i^M)^2 \qquad 2\text{-a}$$

$$P_i > P_i^m \qquad 2\text{-b}$$

$$Q_i < Q_i^M \qquad 2\text{-c}$$

$$Q_i > Q_i^m \qquad \text{2-d}$$

$$V_i < V_i^M \qquad \text{2-e}$$

$$V_i > V_i^m \qquad \text{2-f}$$

where:

S_i^M Maximum permissible apparent production at node i.

V_i^m, V_i^M max. and min. permissible voltage at node i.

The problem of power system optimization then consists of minimizing J, subject to two sets of constraints:

1. Satisfaction of the inequality constraints imposed upon P_i, Q_i and V_i in accordance with (2).
2. Satisfaction of the network relations that must exist between the variables P_i, Q_i and V_i, θ_i.

For convenience of analysis (but not of programming) the network (power-flow equations) may be written as

$$P_i - C_i = I_i = \sum_{\alpha} \frac{V_i V_\alpha}{Z_{i\alpha}} \sin(\theta_i - \theta_\alpha - \delta_{i\alpha}) + \sum_{\alpha} \frac{V_i^2 \sin \delta_{i\alpha}}{Z_{i\alpha}}$$

$$Q_i - D_i = - \sum_{\alpha} \frac{V_i V_\alpha}{Z_{i\alpha}} \cos(\theta_i - \theta_\alpha - \theta_{i\alpha}) + \sum_{\alpha} \frac{V_i^2}{Z_{i\alpha}} \cos \delta_{i\alpha} - Y_{ii} V_i^2$$

The stated problem is one of static optimization in the presence of constraints. The optimization conditions provided by the theorem of Kuhn and Tucker constitute a practical solution which can be implemented for real-time control.

Chapter III

Theoretical Approach

3-1 Introduction

In order to handle the power dispatching problem, a mathematical model of the power network must be developed. This mathematical model must be suitable for convenient handling in the computer.

In this model the transmission lines will be represented by equivalent PIES and the fuel input to each generator will be expressed as a function of its output power.

For the initial solution, the voltage profile in the system will be considered known and an attempt will be made to find the optimum generation schedule while allowing the generator phase angles to vary. When the optimum real power generation has been obtained the reactive power pattern will be changed either by changing the voltage magnitudes within the permissible range or adding shunt capacitors so as to achieve the most economic solution. It must be noted that the saving obtained by adding capacitors must exceed the cost of the capacitors.

This chapter will include the mathematical model of the system without adding any shunt capacitors, a method of optimizing the cost and a computer program which will handle the equations for the optimum cost. This program will have the phase angles as the only variables if we consider the voltage magnitudes to be constant or varied in a very small insignificant range.

Otherwise, the program--if voltage can vary in a wide range--can detect the best voltage profile for a better optimum solution.

Adding shunt capacitors will give us another dimension in optimizing the cost. This will be discussed in chapter 5.

3-2 Mathematical Model

Any power system network consists mainly of generators, inter-connected transmission lines and loads. If we assume that we have an n terminal network, there will be n(n-1)/2 transfer impedances. These transfer impedances can be represented as the Z's shown in fig. 3-1. The elements Y_1, Y_2 are the shunt admittances. In general Y_1 and Y_2 are not equal.

The power flowing through the transmission lines can be expressed in terms of the ABCD transmission line constants. A conversion from the constants of Fig. 3-1 is given by the following equations:

$$A = 1 + ZY_2 = A \angle \alpha$$

$$B = Z = B \angle \beta$$

$$C = (Y_1 + Y_2) + ZY_1 Y_2 = C \angle \gamma$$

$$D = 1 + ZY_1 = D \angle \Delta$$

If we let E_s and E_r represent the sending end and receiving end voltages for a transmission line, respectively, then we can write the power in terms of these voltages as follows:

$$P_s = \frac{-E_s E_r}{B} \cos(\beta + \theta) + \frac{D E_s^2}{B} \cos(\beta - \Delta) \quad \text{..........} \quad (3.1)$$

$$P_r = \frac{E_s E_r}{B} \cos(\beta - \theta) - \frac{AE_r^2}{B} \cos(\beta - \alpha) \quad \ldots\ldots\ldots\ldots (3.2)$$

where θ is the angle by which E_s leads E_r. E_s and E_r are assumed to be held constant.

Any terminal in the power network can be represented as shown in Fig. (3-2) and a power flow equation can be written for this terminal. P_{L_i} represents the local load at station i. P_{g_i} the generated power of generator i. P_{ij} the power transmitted from terminal i to terminal j.

The cost of operating this plant depends on a number of factors, such as overhead, maintenance, depreciation, labor, and interest. There are also variable charges due to the type of fuel used, its cost, and the efficiency of the plant. The relationship therefore between the plant output and the cost of the fuel input will not in general be linear.

The output-input cost function is best approximated by a power series function since it can be handled easily. The function will be accurate at the data points given and close to accurate at intermediate points. The more points given, naturally, the more accurate the representation. The fuel input cost for generator i will be denoted by C(i).

Considering the transmission line connecting terminal i with terminal n, the power transmitted can be written in the following form:

$$P_{in} = -\frac{E_i E_n}{B_{in}} \cos(\beta_{in} + \theta_{in}) + \frac{D_{in} E_i^2}{B_{in}} \cos(\beta_{in} - \Delta_{in}) \quad \cdots (3\text{-}3)$$

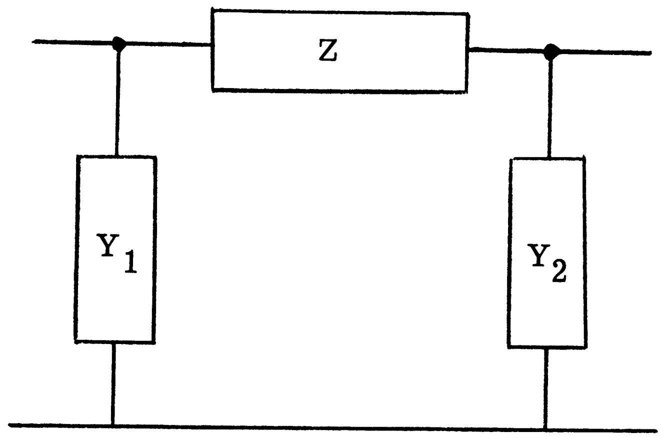


Fig. 3-1

Pi representation

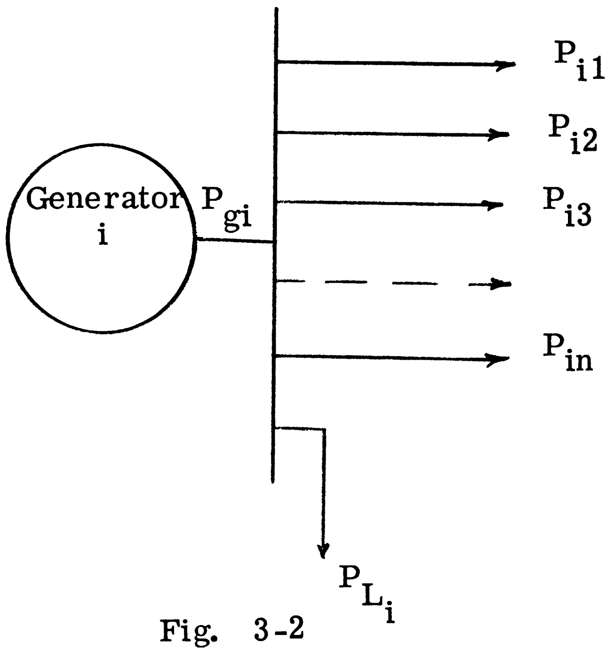


Fig. 3-2

where

θ_{in} is defined as $\theta_i - \theta_n$

It is clear that $\theta_{ni} = -\theta_{in}$

It should be also noted that the impedance angle $\beta_{in} = \beta_{ni}$.

In Fig.(3-2) designate P_i to be equal to the sum of all power flows away from station i. i. e.

$$P_i = \sum_{m=1}^{n} P_{im} \qquad \text{.....................} \quad (3.\ 4)$$

where P_{im} is defined before as the power flowing away from station i to station m.

Having defined the local load at station i as P_{L_i}, therefore:

$$P_{gi} = P_{Li} + P_i \qquad \text{.....................} \quad (3.\ 5)$$

To compute the total input cost let us recall that (C (I)) the input cost of station I is related to P_{gi} by a functional relationship which can be expressed as

$$C(I) = F_i [P_{gi}] \qquad \text{.....................} \quad (3.\ 6)$$

$$= F_i[P_{Li} + P_i]$$

The total input cost for the system is given by the sum of all C's. i. e.

$$\text{Cost} = \text{Total input cost} = \sum_{i=1}^{n} C(i) \qquad \text{.....................} \quad (3.\ 7)$$

Where :

n is the number of stations.

Our objective is to make this cost a minimum by varying the voltage phase angles. In other words the total cost can be minimized by controlling the phase angles to the condition of:

$$\frac{\partial(\text{cost})}{\partial\theta_j} = 0 \qquad \text{for } j=1, \ldots, n \qquad \ldots\ldots\ldots\ldots\ldots \quad (3.8)$$

which means that the partial derivative to each variable must be equal to zero.

To develop the expression $\frac{\partial(\text{cost})}{\partial\theta_j}$ we have:

$$\frac{\partial(\text{cost})}{\partial\,\theta_j} = \frac{\partial}{\partial\theta_j} \sum_{i=1}^{n} F_i\,(P_{gi})$$

$$= \frac{\partial}{\partial\,\theta_j} \sum_{i=1}^{n} F_i\,(P_{Li} + P_i)$$

Since $P_{gi} = P_{Li} + P_i$ at any junction point i.

$$\frac{\partial(\text{cost})}{\partial\,\theta_j} = \sum_{i=1}^{n} \frac{\partial}{\partial\,\theta_j} F_i\,(P_{Li} + P_i) \qquad \ldots\ldots\ldots\ldots \quad (3.9)$$

But F_i is a function of P_{gi} which in turn is a function of θ_j, then:

$$\frac{\partial F_i}{\partial\,\theta_j} = \frac{\partial F_i}{\partial P_{gi}} \; \frac{P_{gi}}{\partial\,\theta_j}$$

$$= \frac{\partial F_i}{\partial P_{gi}} \; \frac{\partial P_i}{\partial\,\theta_j} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.10)$$

Since P_{Li} is independent of θ_j.

Now:

From equation (3. 9) and equation (3. 10) we can have:

$$\frac{\partial(\text{cost})}{\partial \theta_j} = \sum_{i=1}^{n} \frac{\partial F_i}{\partial P_{gi}} \cdot \frac{\partial P_i}{\partial \theta_j}$$

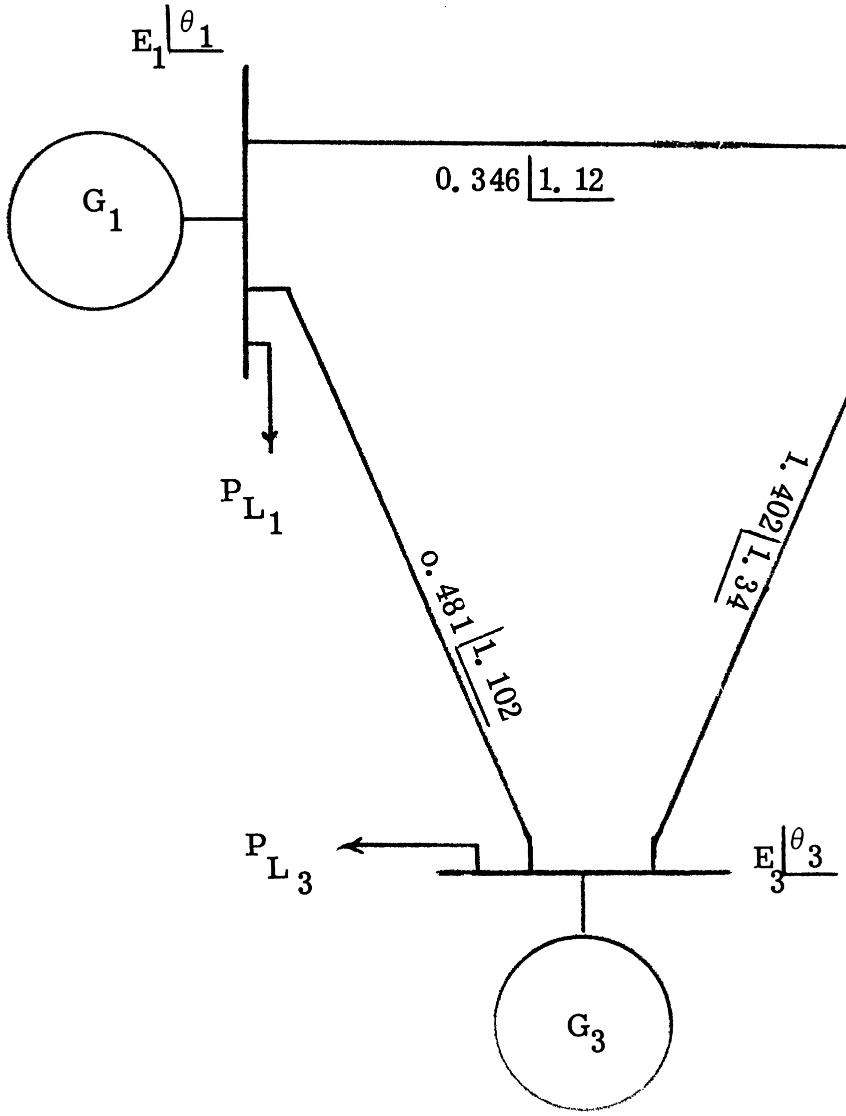
$$= \sum_{i=1}^{n} K_i \cdot \frac{\partial P_i}{\partial \theta_j} \quad \ldots\ldots \text{(3-9a)}$$

where K_i represents the quantity $\frac{\partial F_i}{\partial P_{gi}}$.

It is worthwhile to note that the incremental production rate of station i is expressed in general as:

$$\frac{\partial C(i)}{\partial P_{gi}} = \frac{\partial F_i (P_{gi})}{\partial P_{gi}} = K_i \quad \ldots\ldots\ldots\ldots \text{(3-11)}$$

Since $C(i) = F_i (P_{gi})$



Three Machine System

Fig. 3-3

Now it is necessary to express $\frac{\partial (\text{cost})}{\partial \theta_i}$ in terms of our variable angles so that a technique can be established to detect the minimum cost.

For a given system, our minimum criteria is:

$$\frac{\partial \text{ cost}}{\partial \theta_i} = 0 \qquad i=1, 2, \ldots, n$$

where

$$\frac{\partial \text{ cost}}{\partial \theta_i} = \sum_{i=1}^{n} K_i \frac{\partial P_j}{\partial \theta_i} = 0 \qquad \ldots\ldots\ldots\ldots\ldots (3.12)$$

So in a 3 Machine System n = 3, and then we have:

$$\frac{\partial P_1}{\partial \theta_1} = \frac{E_1 E_2}{B_{12}} \sin(\beta_{12} + \theta_{12}) + \frac{E_1 E_3}{B_{13}} \sin(\beta_{13} + \theta_{13}) \qquad \ldots\ldots\ldots\ldots (3.13)$$

$$\frac{\partial P_2}{\partial \theta_1} = \frac{-E_2 E_1}{B_{21}} \sin(\beta_{21} + \theta_{21}) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.14)$$

$$\frac{\partial P_3}{\partial \theta_1} = \frac{-E_3 E_1}{B_{31}} \sin(\beta_{31} + \theta_{31}) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.15)$$

A similar set of equations can be derived for θ_2 and θ_3. It is noticeable that all terms are negative except when j = i, i.e. P_1 and θ_1 in the above set of equations.

So for θ_2 we have:

$$\frac{\partial P_1}{\partial \theta_2} = - \frac{E_1 E_2}{B_{12}} \sin(\beta_{12} + \theta_{12}) \qquad \ldots\ldots\ldots\ldots (3.16)$$

$$\frac{\partial P_2}{\partial \theta_2} = \frac{E_2 E_1}{B_{21}} \sin(\beta_{21} + \theta_{21}) + \frac{E_2 E_3}{B_{23}} \sin(\beta_{23} + \theta_{23})$$

$$\frac{\partial P_3}{\partial \theta_2} = - \frac{E_3 E_2}{B_{32}} \sin(\beta_{32} + \theta_{32})$$

Finally for θ_3 we have:

$$\frac{\partial P_1}{\partial \theta_3} = - \frac{E_1 E_3}{B_{13}} \sin(\beta_{13} + \theta_{13}) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.\ 17)$$

$$\frac{\partial P_2}{\partial \theta_3} = - \frac{E_2 E_3}{B_{23}} \sin(\beta_{23} + \theta_{23})$$

$$\frac{\partial P_3}{\partial \theta_3} = \frac{E_3 E_1}{B_{31}} \sin(\beta_{31} + \theta_{31}) + \frac{E_3 E_2}{B_{32}} \sin(\beta_{32} + \theta_{32})$$

It is now possible to set out all the derivatives for total cost as:

$$\frac{\partial\, cost}{\partial \theta_1} = 0 = K_1 \left[\frac{E_1 E_2}{B_{12}} \sin(\beta_{12} + \theta_{12}) + \frac{E_1 E_3}{B_{13}} \sin(\beta_{13} + \theta_{13}) \right] \quad (3.\ 18)$$

$$- K_2 \frac{E_2 E_1}{B_{21}} \sin(\beta_{21} + \theta_{21})$$

$$- K_3 \frac{E_3 E_1}{B_{31}} \sin(\beta_{31} + \theta_{31})$$

$$\frac{\partial\, cost}{\partial \theta_2} = 0 = - K_1 \frac{E_1 E_2}{B_{21}} \sin(\beta_{21} + \theta_{12}) \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.\ 19)$$

$$+ K_2 \left[\frac{E_2 E_1}{B_{12}} \sin(\beta_{21} + \theta_{21}) + \frac{E_2 E_3}{B_{23}} \sin(\beta_{23} + \theta_{23}) \right]$$

$$- K_3 \frac{E_3 E_2}{B_{32}} \sin(\beta_{32} + \theta_{32})$$

$$\frac{\partial \, cost}{\partial \theta_3} = 0 = -K_1 \frac{E_1 E_3}{B_{13}} \sin(\beta_{13} + \theta_{13}) \qquad (3.20)$$

$$- K_2 \frac{E_2 E_3}{B_{23}} \sin(\beta_{23} + \theta_{23})$$

$$+ K_3 \left[\frac{E_3 E_1}{B_{31}} \sin(\beta_{31} + \theta_{31}) + \frac{E_3 E_2}{B_{32}} \sin(\beta_{32} + \theta_{32}) \right]$$

It is necessary to solve these equations simultaneously to get the desired minimum value of the input cost.

To simplify our expressions let us define:

$$X_{12} = K_1 \frac{E_1 E_2}{B_{12}} \sin(\beta_{12} + \theta_{12}) \qquad (3.21)$$

$$X_{13} = K_1 \frac{E_1 E_3}{B_{13}} \sin(\beta_{13} + \theta_{13})$$

$$X_{21} = K_2 \frac{E_2 E_1}{B_{21}} \sin(\beta_{21} + \theta_{21})$$

$$X_{23} = K_2 \frac{E_2 E_3}{B_{23}} \sin(\beta_{23} + \theta_{23})$$

$$X_{31} = K_3 \frac{E_3 E_1}{B_{31}} \sin(\beta_{31} + \theta_{31})$$

$$X_{32} = K_3 \frac{E_3 E_2}{B_{32}} \sin(\beta_{32} + \theta_{32})$$

Rewriting our expression of $\frac{\partial \, cost}{\partial \theta_i}$ again using these notations we get

$$\frac{\partial \, cost}{\partial \theta_1} = (X_{12} + X_{13}) - X_{21} - X_{31} = 0 \qquad (3.22)$$

$$\frac{\partial \, cost}{\partial \theta_2} = - X_{12} + (X_{21} + X_{23}) - X_{32} = 0$$

$$\frac{\partial \, cost}{\partial \, \theta_3} = - X_{13} - X_{23} + (X_{31} + X_{32}) = 0$$

For convenience we will define the terms Z_i's as :

$$Z_1 = \frac{\partial \, cost}{\partial \, \theta_1} = (X_{12} - X_{21}) + (X_{13} - X_{31}) = 0 \qquad \text{.........} (3.23)$$

$$Z_2 = \frac{\partial \, cost}{\partial \, \theta_2} = (X_{21} - X_{12}) + (X_{23} - X_{32}) = 0$$

$$Z_3 = \frac{\partial \, cost}{\partial \, \theta_3} = (X_{31} - X_{13}) + (X_{32} - X_{23}) = 0$$

It can be seen from these equations that the number of unknowns (Phase angles differences) are one less than the number of equations. This means that an infinite number of solutions exist.

As it would be very difficult to start with these equations and work backward for θ_1, θ_2 and θ_3 we will start by assuming the angles and then work forward. We will then consider the deviation of the quantities of each equation from zero and correct our angles accordingly.

To summarize, first for the entire system determine the transfer impedances among all possible pairs of generator terminals. This can be done by using a network analyzer or by a computer program selected from those available for this purpose. Next determine the bus impedances to be applied to these terminals and network. Then assign initial values of voltage phase angles to the system. These may be chosen arbitrarily. The voltage magnitudes of the system are presumed known.

From these voltage angles, the transferred powers through all transmission lines can be calculated. Summing all powers going away from each junction together with the local load will show the required generation at that terminal. The corresponding cost can be evaluated using the given input cost function for that generator. The total system cost is then obtained by summing the cost of all stations.

At this point a check must be made to the values of the Z's and if their values are not close to zero, the voltage angles must be incremented and the new values of angles will be considered for a new iteration.

A stop can be made when all the Z's are close to a specified small value ϵ which can be assumed to be about 0.1 or so.

Generalization for More Than Three Machines

The solution for a large system with N machines is the application of equation (3. 23) which can be extended to the general form:

$$\frac{\partial \text{cost}}{\partial \theta_j} = \sum_{\substack{i=1 \\ i \neq j}}^{n} (X_{ji} - X_{ij}) = 0 \quad \ldots\ldots\ldots\ldots\ldots (3.24)$$

Where :

$$X_{ij} = K_j \frac{E_j E_i}{B_{ji}} \sin(\beta_{ji} + \theta_{ji}) \quad \ldots\ldots\ldots\ldots (3.25)$$

The computer flow chart will be that shown in Fig. (3-4) and the method of solution will be similar to that of the three machines. Synchronous condensers will be located at the system's intermediate busses to control its voltages. These condensers will be considered as normal generators with high input fuel cost rate.

3-3 Computer Solution to the N machine

The computer approach to the problem is best illustrated by a flow chart which is shown in Fig. (3-4). First, we have to enter the data which consists of the voltage magnitudes and phase angles, the local loads and the impedance magnitudes and angles. The voltage angles which are the variables in the solution are not really necessary as the procedure is capable of arriving at the solution no matter what set of voltage angles it starts with. However, the number of iterations is lessened somewhat if the first trials are in the neighborhood of the final answers. The local loads will be determined by system requirements and will be known at the start of the problem. The impedance magnitudes and angles will also be known. A separate computer program can be used for this purpose. It could also be arranged that this program can be added to the main program as a subroutine to evaluate these impedances if the voltages were to be varied in the system. Then the impedances would be recalculated automatically and the corrected versions can be used. This refinement can be added with very little extra trouble.

The next step is for the procedureto compute the angular difference between each and every pair of plants. It will then store these for further use. It then computes the transferred powers P_{ij} and stores them. By adding each local load P_L to the sum of transferred powers P_i, it will compute P_{gi}. From these we can evaluate the cost of each generator which is given as a function of generated powers.

The incremental rates K_i's can be calculated as these are functions of the generated powers alone. The next step is the evaluation of the X_{ij} factors and these are obtained from equations (3. 21). Then these values can be added as in equations (3. 23).

In general the values for the left hand sides of equations (3. 23) will not be zeros. For convenience we will designate:

$$Z_1 = \frac{\partial(\text{cost})}{\partial \theta_1}$$

$$Z_2 = \frac{\partial(\text{cost})}{\partial \theta_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3. 26)}$$

$$\vdots$$

$$Z_n = \frac{\partial(\text{cost})}{\partial \theta_n}$$

However, the sum $Z_1 + \ldots + Z_n$ will be always zero. This can be proved by adding the set of equations 3-23 together . This means that some of them are positive while the others are negative. The procedure can sense their sign and then correct the corresponding angle. This correction must be done in the opposite sign to that of Z_i. Thus if Z_i is positive, θ_i must be decreased and if it is negative, θ_i must be increased.

Now a critical decision has to be made. If an over correction is made then on the next iteration, then Z_i will be farther from zero.

On the other hand, if the correction is too small, then too many iterations will be necessary to arrive at the solution. The choice of an accelerating factor is of great importance.

Assuming that the correct correctionis applied, then the values of $Z_1, Z_2, \ldots Z_n$ will be close to zero. A stop to the iteration process can be made when either the magnitude of the Z's is close enough to zero or the decrease in total power cost becomes negligible due to each succeeding correction. It is better to watch the Z'smagnitude until it gets lower than a predetermined small value, this will be sufficient indication of the closeness of the solution, since less calculations will be needed.

At this stage, the computer can furnish the values of the voltage phase angles θ_i which give the minimum input cost to the system while holding the voltages constant in magnitude. However, this will not be an absolute minimum and in order to find this or as close to it as practical, a further program is necessary in which the permissable change in the system voltage magnitudes can be taken into consideration. (see flow chart no. 3-5).

The difference here is that the computer will iterate through a number of successive voltage profiles optimizing the cost for each profile and then compare this cost with the previous voltage profile cost. If the new profile has a lower cost the program will store this voltage profile and its corresponding results in place of the old lowest cost case. Otherwise it will keep the old results.

The voltage magnitude of each bus and the variation permissable to its magnitude vary from one system to the other and even in the same system it can vary differently depending on many factors. But anyhow, we will be given the minimum and maximum value permitted for each bus for the solution.

If the requirements of the system are very tight requiring the voltage magnitude to be almost constant on all busses then the program may seem unnecessary.

The choice of the accelerating factor is at present a trial and error method. The factor for this work was chosen as 0.0045 as this value seemed to give satisfactory results on different systems.

Further investigations into this factor might well be rewarding as it is the key to the whole solution and a trial and error method of arriving at a key is never satisfactory.

The decision on how close Z_i should be to zero is important. Obviously it will need an almost infinite number of iterations to reach zero but a practical limit is reached much before that. This predetermined value (ϵ) can be determined by experience with the system.

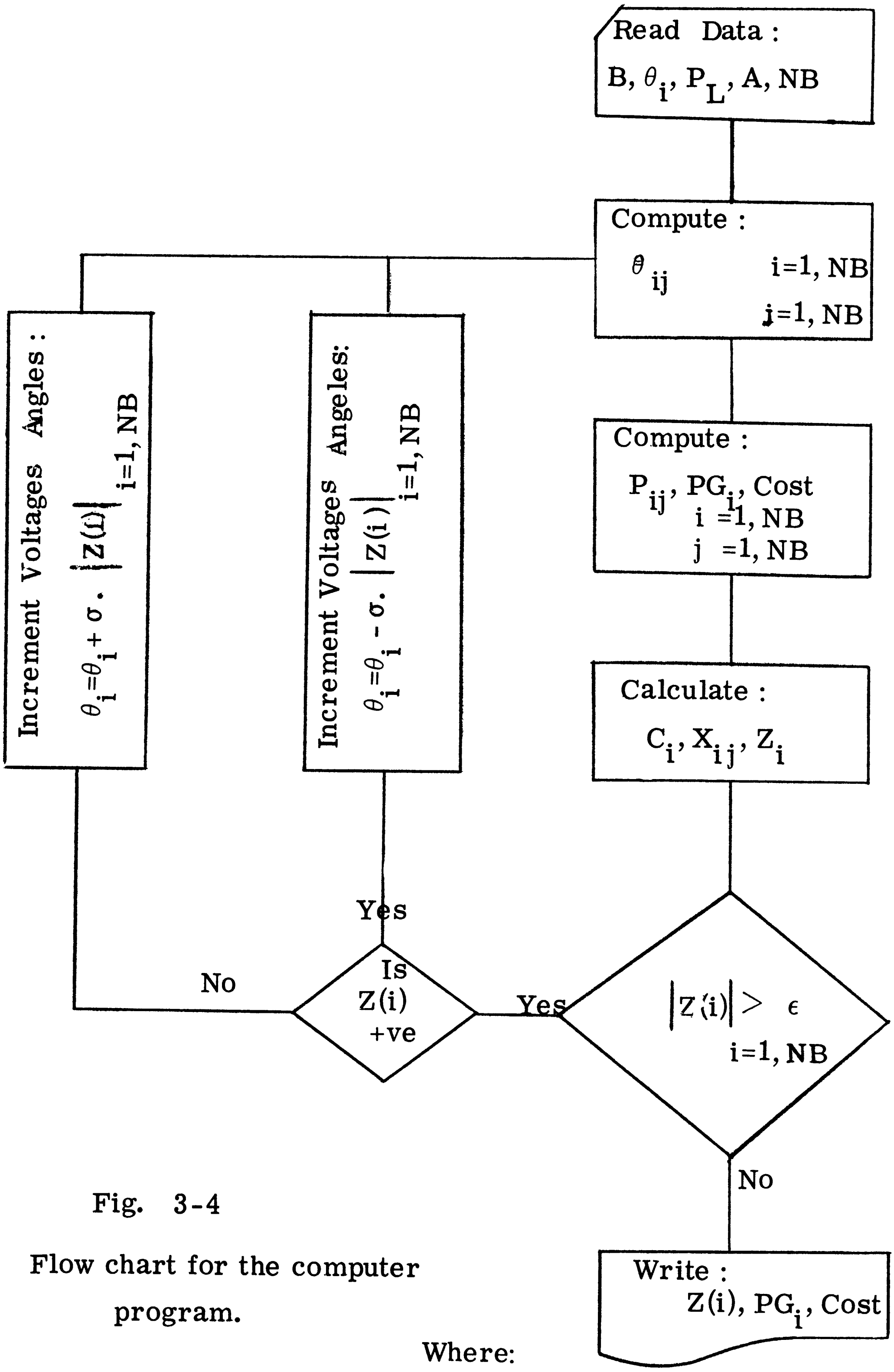


Fig. 3-4

Flow chart for the computer program.

Where:

B = Transmission line Constants.
A = Coefficients array of the input fuel cost's functions.
σ = Accelerating Factor.

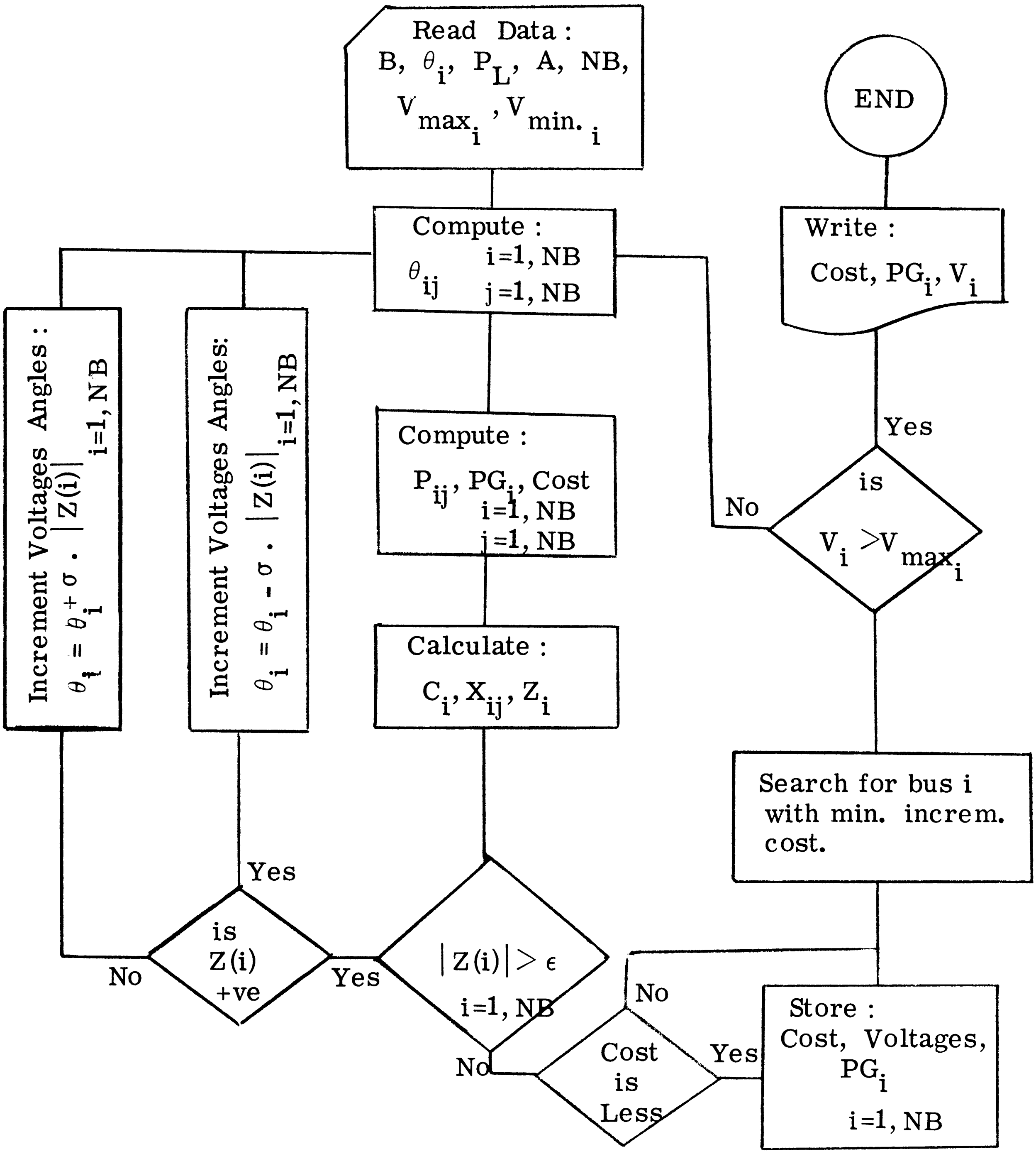


Fig. 3-5
Flow chart for variable voltage
computer program.

Chapter IV

Experimental Results

4-1 Data Preparation

In order to prepare the system for solution the transfer impedances and admittances must be determined as well as the voltage profile. The older method of determining the transfer impedances was on the network analyzer and this would be quite satisfactory if a limited accuracy is desired. If more accuracy is needed, then the computer itself can be used for determining the transfer impedances. The methods for doing this have been discussed in reference (11).

The transfer impedance value between each pair of busses must be supplied to the computer. If there is no connection between any two specific busses, then the value ∞ must be assigned as the corresponding transfer impedance there.

The local loads will also need to be known as they are not included in the calculations of the partial derivatives. They are present in the incremental rate calculations.

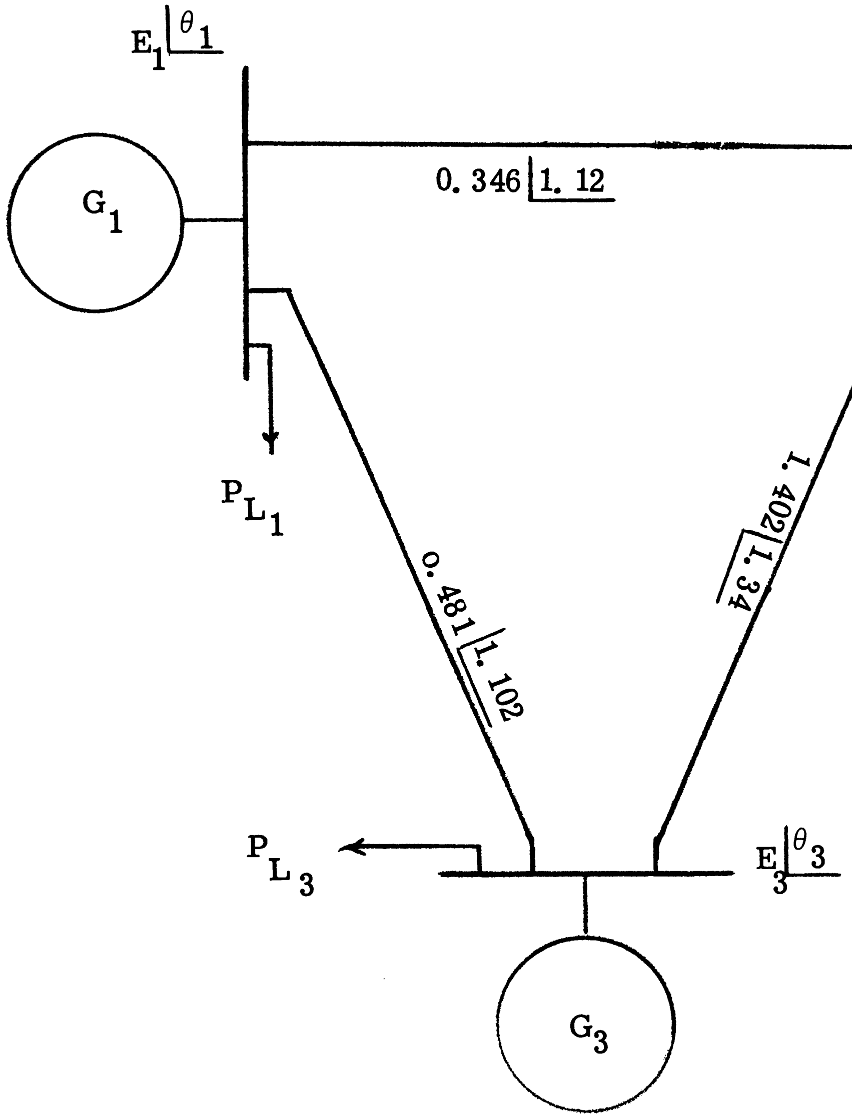
The fuel input data must also be calculated in order to solve the problem and to do this the input cost of each plant must be known as a function of its output power. In general these curves are nonlinear and can be approximated by either a series of straight lines or a power series. The power series method is more satisfactory especially for computer use.

The power series method is adopted in this work. In general, there will be as many terms as data points for its determination, including a constant term to represent the input at zero output. The accuracy of this power-cost representation depends upon the number of points given and so on the order of the polynomial to represent it. As the magnitude of the terms diminishes with increasing order of power output, it will be generally sufficient to take only those significant terms (usually three or four is enough to represent the function). For greater accuracy in this representation the whole curve can be programmed in the computer which can be done with very little difficulty.

Once the input output curve has been represented, the incremental rate of that plant can be determined by differentiating this curve. The power series method makes this somewhat easier. In the power series method the incremental rate curve of any plant will then always be another curve of order one less than input-output curve.

The system chosen for this thesis is shown in Fig. (4-1). The input-output and incremental rate curves are shown in Fig. (4-2).

It is clear from the curves that while plant 1 has the highest no load input, it has the lowest incremental rate curve. This proves



Three Machine System

Fig. (4-1).

that not too much information can be got from the input curves on how to distribute the load and the incremental rate curve is of the utmost importance in calculating the load sharing.

The expressions for input as function of plant output for the three plants used in the example are:

$$C(1) = 2.28 + 0.52\, P_{g_1} + 0.380\, P_{g_1}^2 + 0.04\, P_{g_1}^3$$

$$C(2) = 1.59 + 0.75\, P_{g_2} + 0.44\, P_{g_2}^2 + 0.03\, P_{g_2}^3$$

$$C(3) = 1.04 + 1.16\, P_{g_3} + 0.84\, P_{g_3}^2 + 0.01\, P_{g_3}^3$$

From these equations we can derive the incremental rate of each plant as functions of output power by differentiation.

$$K_i = \frac{\partial\, C(i)}{\partial\, P_{gi}}$$

so we get:

$$K_1 = 0.52 + 0.76\, P_{g_1} + 0.12\, P_{g_1}^2$$

$$K_2 = 0.75 + 0.88\, P_{g_2} + 0.09\, P_{g_2}^2$$

$$K_3 = 1.16 + 1.68\, P_{g_3} + 0.03\, P_{g_3}^2$$

4-2 Computer Program Solution :

The solution to the problem by the computer is in two parts. First , when the system loads require a constant voltage magnitudes the computer program will calculate the best optimum input cost by chosing the appropriate set of voltage phase angles. The program will also compute the corresponding power

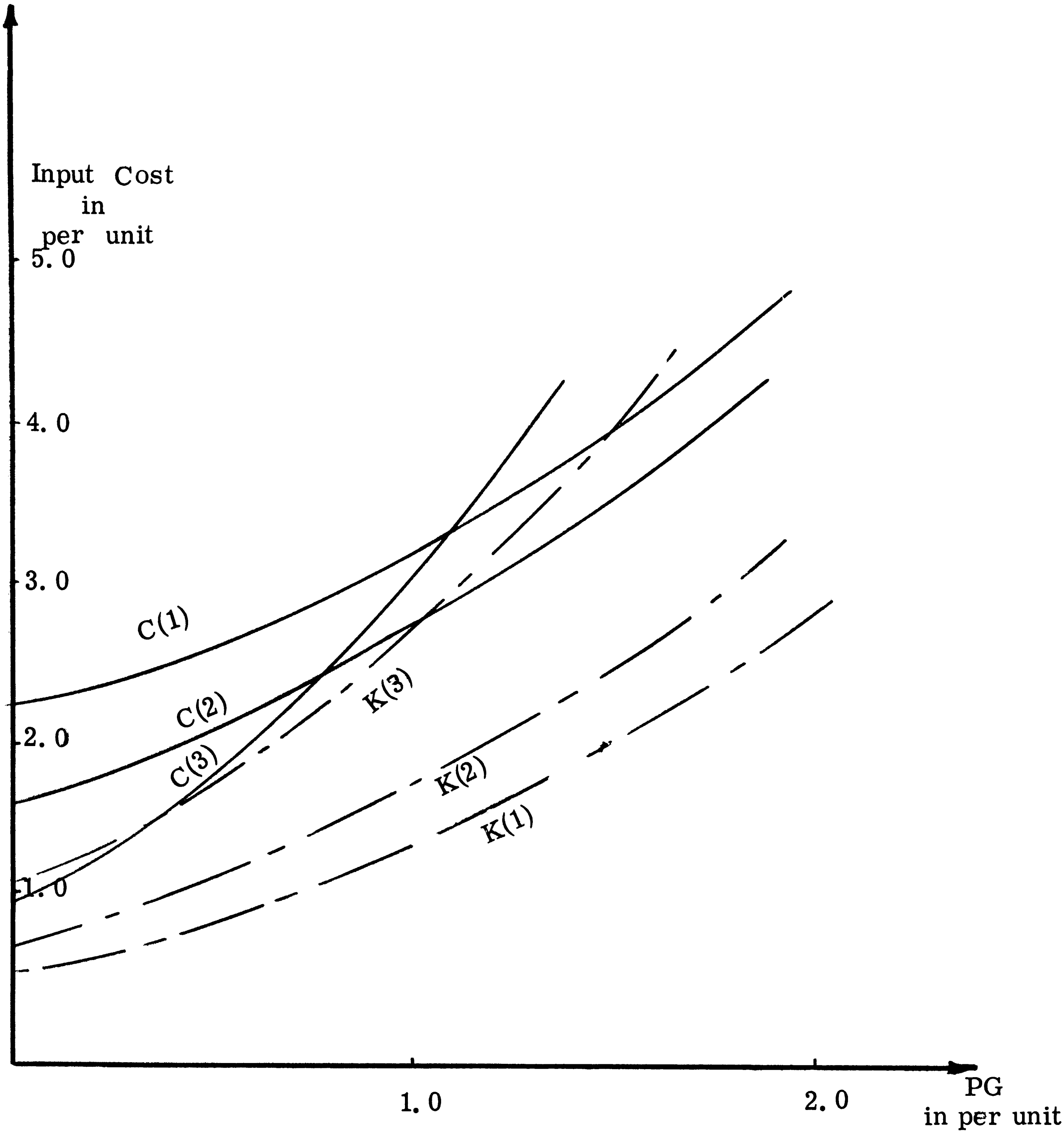


Fig. 4-2

K(n) is the incremental fuel input cost to station n.
C(n) is the total input fuel cost to station n.

distribution. If voltage magnitudes can vary in a wide range, we can reiterate this program for different voltage profiles searching for a better minimum cost. One of these profiles will have the absolute minimum cost but this may not be practicable in terms of system physical realities, hence the calculations of a number of such profiles with the one to be used being left for an operator to determine. This choice is usually based on both the physical capabilities of the system as well as experience.

Second, a better minimum can be tried by changing the eactive power profile by adding shunt capacitors as described in Chapter 5. The saving gained by using these capacitors must exceed the capacitor cost, of course.

The flow chart for the first part is given in Figure (3-4) for those systems having constant voltages or those whose voltages can't vary in a wide range.

Another flow chart is given in Figure (3-5) for these systems where voltages can vary in a wide range so a voltage profile can be selected.

In this program the input data will consist of the voltage magnitudes for the generators, the transfer impedances in both magnitude and angle, the local loads at each station and an initial set of phase angles for the procedure to start from. The voltage data will be the magnitudes of the voltages at the generator busses and these should be known. The transfer impedances are presumed known. The initial

set of voltage phase angles is not critical. The program can start with any angle values, but it is quicker if it can be supplied values close to the actual ones.

The next step is to calculate the phase angles differences between each pair of machines, then the procedurewill calculate the various power flows between each pair of generators. The summation of the power flows from any one generator will give the total power flow away from that generator. Addition of the local load to this will give the total power output from any one generator. The computer will now calculate the input cost for each generator using the input-output curves given before. Adding all these input costs will give the total power input to the system. It will then proceed to calculate incremental rates for each generator using equation (3-11).

Now it is possible to calculate the various X_{ij}, then grouping them together to form the Z's defined in Chapter III, when Z_1 is defined as the partial derivative of the total cost with respect to angle θ_i. In the final solution these Z's must approach zero or sufficiently close to be neglected.

In the program, incrementing the phase angle is done either by adding or subtracting Z_i multiplied by a constant to accelerate the iteration. This process will continue until all Z's are close enough to zero.

The computer time spent on calculation is normally less than the time taken for the printer to write results, accordingly while the

machine can print out all data, valuable time will be wasted in printing them. So, inspite of the fact that we have many results in this thesis to show the details of the program yet in practical application only the result of the final iteration must be written.

Finally, if some or all of the voltages are permitted to vary in a wide range, then different profiles can be tested for better minimum cost.

4-3 Results for the Three Machine System

The results of the example given in Chapter III are given in Table 1. For this example all voltages are constant and assumed to be 1.0 p.u. and the local load assumed to be equal to 0.5 p.u. at each generator.

In order to illustrate that the values of phase angles used to start the iteration can be any value whatsoever, three different runs for the same example were carried on. The results are shown in Table 1, 2, and 3. It is clear from the results that we can start with any angles and we get the same results.

The same system is run again with each local load doubled, and the corresponding results in Table 5 show that the new power sharing on generators is not doubled. This is because incremental values change with power change, and also because the increase in the transmission line losses is not linear.

The next step in the results is to investigate the effect of the voltage variation on the cost. It is understood that this will be only tried when wide change in voltage is permissible in the system. The same example is used allowing voltages to vary from 0. 95 p. u. up to 1. 05 p. u. and the corresponding results are shown for different voltage profiles. We can see that we get a better minimum than the one we had before when voltages were constants.

4-4 Systems with Intermediate Load Busses

In this work, systems with intermediate busses can be handled in the same way as in the three-bus system given before. Synchronous condensers will be used on those intermediate busses to keep the voltage at the required value. The synchronous condensers will be treated as over excited generators with high generating cost, so they share a very small amount of load.

Two power systems have been run. In the first a six-bus system with one intermediate bus; while in the second, a twelve-bus system with two intermediate busses are considered.

4-4a Twelve-bus System:

The twelve-bus system considered is shown in fig. (4. 3). The total and incremental cost of each station is shown in section (4-5b) later while the transmission impedances are shown on the schematic diagram of the system. The results are shown in Table 6.

4-5a Three Bus System Data and Results

I - The following data is provided :

$$P_{L_1} = 0.5 \quad \text{p.u.}$$

$$P_{L_2} = 0.5 \quad \text{p.u.}$$

$$P_{L_3} = 0.5 \quad \text{p.u.}$$

Where :

P_{L_n} is defined as the local load at station n .

The total input cost functions are :

$$P_{input(1)} = 2.28 + 0.52\ PG(1) + 0.38\ PG(1)^2 + 0.04\ PG(1)^3$$

$$P_{input(2)} = 1.58 + 0.75\ PG(2) + 0.44\ PG(2)^2 + 0.03\ PG(2)^3$$

$$P_{input(3)} = 1.04 + 1.16\ PG(3) + 0.84\ PG(3)^2 + 0.01\ PG(3)^3$$

Where :

$P_{input(n)}$ is defined as the fuel input cost to station n.

The transmission lines impedances are as follows :

$$B_{12} = 0.346 \angle 1.12$$

$$B_{23} = 1.402 \angle 1.34$$

$$B_{31} = 0.481 \angle 1.102$$

Where :

B_{ij} is the impedance of the transmission line connecting bus i and bus j.

The incremental fuel input cost functions are :

$$K(1) = 0.52 + 0.76\ PG(1) + 0.12\ PG(1)$$

$$K(2) = 0.75 + 0.88\ PG(2) + 0.09\ PG(2)$$

$$K(3) = 1.16 + 1.68\ PG(3) + 0.03\ PG(3)$$

Where :

K(n) is the incremental fuel input cost at station n.

II- Results:

Three runs for the same system were run with different angles assumed. The results shown in the following tables shows that same results would be achieved.

The economical schedule for the power among the three generators is tabulated in these figures in terms of per unit.

In table 5 , each load in the three bus system has been doubled and results in this table shows that generated powers and cost are not doubled.

An attempt was made to study the case when voltages are permitted to vary in a given range (± 5 %) . Results for the three bus system are shown in table 4.

set of voltage phase angles is not critical. The program can start with any angle values, but it is quicker if it can be supplied values close to the actual ones.

The next step is to calculate the phase angles differences between each pair of machines, then the procedurewill calculate the various power flows between each pair of generators. The summation of the power flows from any one generator will give the total power flow away from that generator. Addition of the local load to this will give the total power output from any one generator. The computer will now calculate the input cost for each generator using the input-output curves given before. Adding all these input costs will give the total power input to the system. It will then proceed to calculate incremental rates for each generator using equation (3-11).

Now it is possible to calculate the various X_{ij}, then grouping them together to form the Z's defined in Chapter III, when Z_1 is defined as the partial derivative of the total cost with respect to angle θ_i. In the final solution these Z's must approach zero or sufficiently close to be neglected.

In the program, incrementing the phase angle is done either by adding or subtracting Z_i multiplied by a constant to accelerate the iteration. This process will continue until all Z's are close enough to zero.

The computer time spent on calculation is normally less than the time taken for the printer to write results, accordingly while the

	1^{st} iteration	3^{d}	5^{th}	7^{th}	9^{t}	11^{th}	13^{th}
θ_1	0.21230	0.22885	0.23845	0.24399	0.24718	0.24900	0.24959
θ_2	0.20536	0.21030	0.21182	0.21215	0.21211	0.21199	0.21193
θ_3	0.18234	0.16085	0.14973	0.14386	0.14071	0.13901	0.13848
PG_1	0.50000	0.63184	0.71119	0.75807	0.78541	0.80122	0.81030
PG_2	0.50000	0.55987	0.57912	0.58400	0.58421	0.58321	0.58215
PG_3	0.50000	0.31188	0.21818	0.17001	0.14471	0.13123	0.12398
Z_1	-2.73335	-1.59095	-0.92211	-0.53150	-0.30491	-0.17428	-0.09935
Z_2	-1.19027	-0.40046	-0.11593	-0.01919	0.00923	0.01413	0.01203
Z_3	3.92362	1.99141	1.03804	0.55069	0.29568	0.16015	0.08732
Cost	6.54999	6.40723	6.36732	6.35556	6.35198	6135088	6.35053

Table 2.

So Optimum Power Generation is $PG_1 = 0.81030$

$PG_2 = 0.58215$

$PG_3 = 0.12398$

Angles Assumed are : $\theta_1 = 0.2$

$\theta_2 = 0.2$

$\theta_3 = 0.2$

Fuel input cost is = 6.35053

	1^{st} iteration	3^{d}	5^{th}	7^{th}	9^{th}	11^{th}	13^{th}
θ_1	1.01230	1.02884	1.03845	1.04399	1.04717	1.04900	1.04959
θ_2	1.00536	1.01030	1.01182	1.01215	1.01211	1.01198	1.01192
θ_3	0.98234	0.96085	0.94973	0.94386	0.94071	0.93901	0.93848
PG_1	0.50000	0.63184	0.71119	0.75807	0.78540	0.80121	0.81030
PG_2	0.50000	0.55987	0.57912	0.58400	0.58421	0.58321	0.58215
PG_3	0.50000	0.31188	0.21819	0.17001	0.14472	0.13124	0.12399
Z_1	-2.73335	-1.59097	-0.92212	-0.53151	-0.30495	-0.17432	-0.09936
Z_2	-1.19027	-0.40045	-0.11596	-0.01922	0.00920	0.01410	0.01198
Z_3	3.92362	0.31188	1.03808	0.55073	0.29575	0.16024	0.08739
Cost	6.54999	6.40723	6.36733	6.35556	6.35199	6.35088	6.35054

Table 3.

So Optimum Power Generation is $PG_1 = 0.81030$ Angles Assumed are : $\theta_1 = 1.0$

$PG_2 = 0.58215$ $\theta_2 = 1.0$

$PG_3 = 0.12398$ $\theta_3 = 1.0$

Fuel input Cost is = 6.35054

Input fuel cost = 6. 34628

Station	Max. Voltage allowed	Min. Voltage allowed	Voltage increments in volt per unit	Voltage profile for optimum input cost	Generated Power in per unit.
1	1. 05	0. 95	0. 01	1. 05	0. 82245
2	1. 05	0. 95	0. 01	1. 05	0. 57635
3	1. 05	0. 95	0. 01	1. 02	0. 11546

Table 4

Data and results for the three machine system with voltage can be varied within ± 5 %

Input fuel cost = 8. 73244

Station	Generated power in per unit	Local load per unit
1	1. 36093	1. 0
2	1. 15357	1. 0
3	0. 51097	1. 0

Table no. 5

Generation Schedule for double load case

4-5b Twelve Buss System Data and Results :

I- The following data is provided :

$P_{L_1} = 0.5$	$P_{L_4} = 0.5$	$P_{L_7} = 0.5$	$P_{L_{10}} = 0.5$
$P_{L_2} = 0.5$	$P_{L_5} = 0.5$	$P_{L_8} = 0.5$	$P_{L_{11}} = 0.5$
$P_{L_3} = 0.5$	$P_{L_6} = 0.5$	$P_{L_9} = 0.5$	$P_{L_{12}} = 0.5$

Where :

P_{L_N} is the local load at bus N .

The total input cost functions are :

$$P_{input\ (1)} = 2.28 + 0.520\ PG(1) + 0.38\ PG(1)^2$$

$$P_{input\ (2)} = 1.80 + 0.420\ PG(2) + 0.37\ PG(2)^2$$

$$P_{input\ (3)} = 1.70 + 0.500\ PG(3) + 0.36\ PG(3)^2$$

$$P_{input\ (4)} = 1.90 + 0.300\ PG(4) + 0.38\ PG(4)^2$$

$$P^*_{input\ (5)} = 1.00 + 0.964\ PG(5) + 0.50\ PG(5)^2$$

$$P_{input\ (6)} = 1.75 + 0.300\ PG(6) + 0.37\ PG(6)^2$$

$$P_{input\ (7)} = 1.60 + 0.500\ PG(7) + 0.36\ PG(7)^2$$

$$P_{input\ (8)} = 1.70 + 0.450\ PG(8) + 0.32\ PG(8)^2$$

$$P_{input\ (9)} = 1.75 + 0.400\ PG(9) + 0.36\ PG(9)^2$$

$$P^*_{input\ (10)} = 1.00 + 1.550\ PG(10) + 0.50\ PG(10)^2$$

$$P_{input}(11) = 1.75 + 0.800\ PG(11) + 0.35\ PG(11)^2$$
$$P_{input}(12) = 1.75 + 0.600\ PG(12) + 0.36\ PG(12)^2$$

Where

PG(n) is the power generated at station n , and

$P_{input}(n)$ is the fuel input cost at station n .

The incremental fuel cost functions are

$$K(1) = 0.520 + 0.76\ PG(1)$$
$$K(2) = 0.420 + 0.74\ PG(2)$$
$$K(3) = 0.500 + 0.72\ PG(3)$$
$$K(4) = 0.300 + 0.76\ PG(4)$$
$$\overset{*}{K}(5) = 0.964 + 1.00\ PG(5)$$
$$K(6) = 0.300 + 0.74\ PG(6)$$
$$K(7) = 0.500 + 0.72\ PG(7)$$
$$K(8) = 0.450 + 0.64\ PG(8)$$
$$K(9) = 0.400 + 0.72\ PG(9)$$
$$\overset{*}{K}(10) = 0.550 + 1.00\ PG(10)$$
$$K(11) = 0.800 + 0.70\ PG(11)$$
$$K(12) = 0.600 + 0.72\ PG(12)$$

Where:

K(n) is the incremental fuel cost function at station n .

* Symbol given to show that this information belongs to a Synchronous condenser.

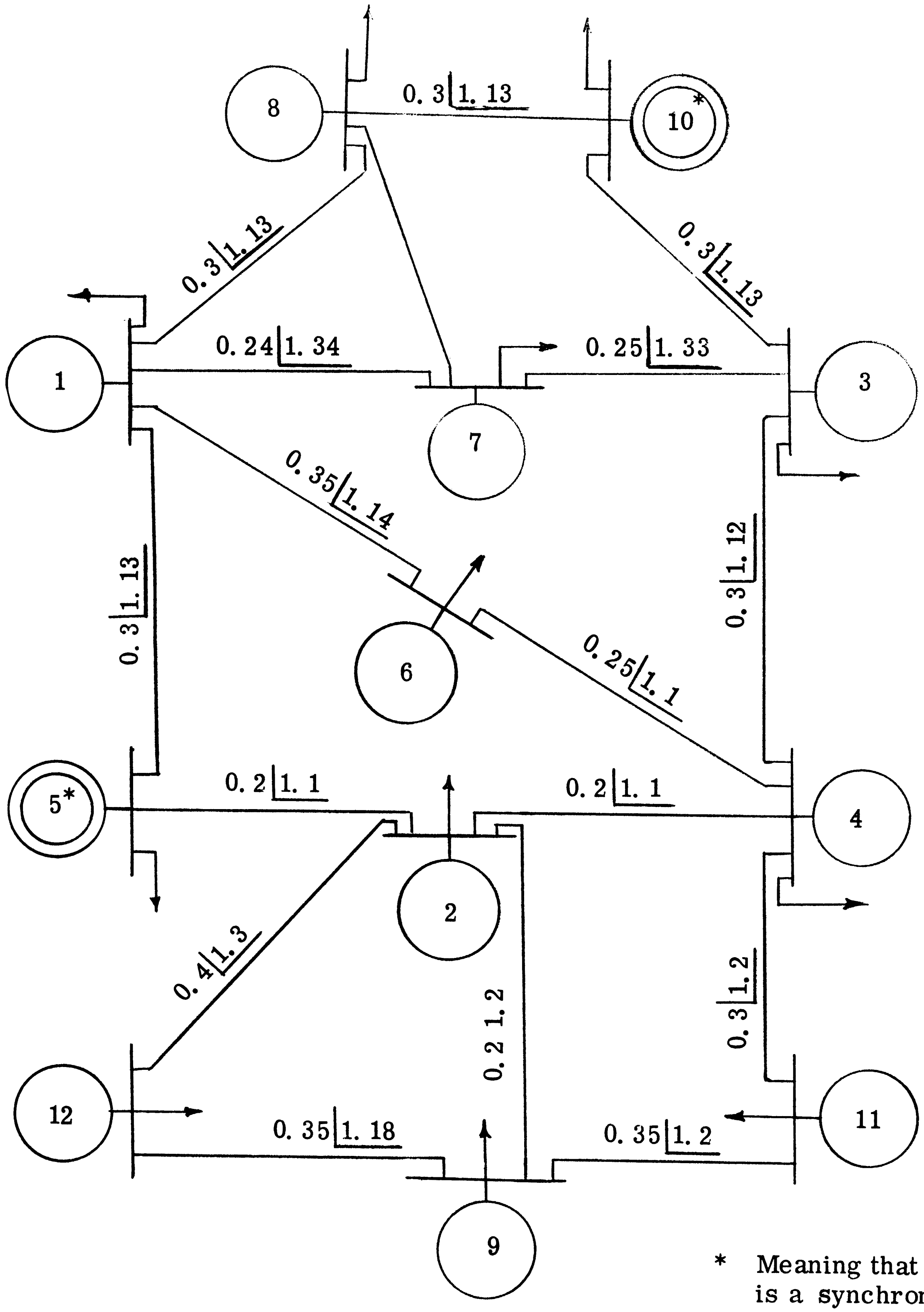
The impedance matrix is given as follows :

$$
B = \begin{matrix}
\infty & \infty & \infty & \infty & .30 & .35 & .24 & .30 & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & .20 & .20 & \infty & \infty & \infty & .20 & \infty & \infty & .40 \\
\infty & \infty & \infty & .30 & \infty & \infty & .25 & \infty & \infty & .30 & \infty & \infty \\
\infty & .20 & .30 & \infty & \infty & .25 & \infty & \infty & \infty & \infty & .30 & \infty \\
.30 & .20 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
.35 & \infty & \infty & .25 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\
.24 & \infty & .25 & \infty & \infty & \infty & \infty & .25 & \infty & \infty & \infty & \infty \\
.30 & \infty & \infty & \infty & \infty & \infty & .25 & \infty & \infty & .30 & \infty & \infty \\
\infty & .20 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & .35 & .35 \\
\infty & \infty & .30 & \infty & \infty & \infty & \infty & .30 & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & .30 & \infty & \infty & \infty & \infty & .35 & \infty & \infty & \infty \\
\infty & .40 & \infty & \infty & \infty & \infty & \infty & \infty & .35 & \infty & \infty & \infty
\end{matrix}
$$

The impedance angle matrix is given by :

β =

0. 00	0. 00	0. 00	0. 00	1. 12	1. 14	1. 34	1. 13	0. 00	0. 00	0. 00	0. 00
0. 00	0. 00	0. 00	1. 10	1. 10	0. 00	0. 00	0. 00	1. 20	0. 00	0. 00	1. 30
0. 00	0. 00	0. 00	1. 12	0. 00	0. 00	1. 33	0. 00	0. 00	1. 13	0. 00	0. 00
0. 00	1. 10	1. 12	0. 00	0. 00	1. 10	0. 00	0. 00	0. 00	0. 00	1. 20	0. 00
1. 12	1. 10	0. 00	0. 00	0. 00	0. 00	0. 00	0. 00	0. 00	0. 00	0. 00	0. 00
1. 14	0. 00	0. 00	1. 10	0. 00	0. 00	0. 00	0. 00	0. 00	0. 00	0. 00	0. 00
1. 34	0. 00	1. 33	0. 00	0. 00	0. 00	0. 00	0. 25	0. 00	0. 00	0. 00	0. 00
1. 13	0. 00	0. 00	0. 00	0. 00	0. 00	1. 20	0. 00	0. 00	1. 13	0. 00	0. 00
0. 00	1. 20	0. 00	0. 00	0. 00	0. 00	0. 00	0. 00	0. 00	0. 00	1. 20	1. 18
0. 00	0. 00	1. 13	0. 00	0. 00	0. 00	0. 00	0. 30	0. 00	0. 00	0. 00	0. 00
0. 00	0. 00	0. 00	1. 20	0. 00	0. 00	0. 00	0. 00	1. 20	0. 00	0. 00	0. 00
0. 00	1. 30	0. 00	0. 00	0. 00	0. 00	0. 00	0. 00	1. 18	0. 00	0. 00	0. 00



* Meaning that this is a synchronous condenser.

Fig. 4-3 Twelve bus system diagram

II - Results for the 12 bus system:

The results for the 12 bus system are shown in table 6. Where the economic distribution of powers among the generating stations are in terms of per unit.

In running this 12 bus system program, the total computer time used (C. P. U.) was equal to 6.186 seconds including 2.779 seconds for loading the program. So the net time used is only 3.407 seconds, which is fairly good.

These programs were run on the University of Michigan computer, which is an IBM System -360-67 model under MTS (Michigan Terminal System).

Station no.	Generated Power	Z
1	0. 4849	0. 02442
2	0. 6729	-0. 04522
3	0. 8787	0. 05433
4	0. 8397	-0. 02274
5*	0. 0030	-0. 02290
6	0. 7796	-0. 01443
7	0. 8004	0. 05253
8	0. 1868	0. 06645
9	0. 6976	-0. 06303
10*	0. 0005	-0. 09792
11	0. 2116	-0. 05945
12	0. 4253	-0. 06775

Table number 6

Chapter V

Reactive Power Approach

In optimizing the cost of a power system, a consideration of the options and restrictions of the system must be noted. In some systems - especially small ones - voltage can vary in a relatively wide range (about $\pm$ 5%) while in a large interconnected system with many restrictions, voltage cannot be varied in such magnitude and can be considered almost constant.

The reactive power profile of the system depends on many factors; voltage magnitude is one of them. So by changing the voltage magnitude profile of the system - if possible - an optimum cost can be achieved.

In Chapter III, an extension of the mathematical model and a computer program were done on permissable voltage profiles ranging from 0.95 p.u. to 1.05 p.u. It is clear from the results that some savings can be achieved. Changing these voltages can be done in different ways, such as using transformers with switched tap settings or by changing generators excitation.

The reactive power profile can be also changed by injecting some reactive power into the system using shunt capacitors. These capacitors can change the reactive power carried by the transmission lines and so can be used as a means of optimization of the input cost.

5-1 Shunt Capacitors

Most of the power systems use shunt capacitors as a means of controlling the voltage. In this thesis shunt capacitors will be considered as a means of controlling the reactive power to secure the optimum value as far as the input cost is concerned.

The function of a shunt capacitor applied as a single unit or in groups of units is to supply lagging Kilo vars to the system at the point where it is connected. A shunt capacitor has the same effect as an over excited synchonous condensor or generator. It supplies Kilo vars or current in such a way as to decrease the reactive power carried by the transmission line, resulting in the decrease of the transmission line current and in turn decreasing the transmission line losses.

Shunt capacitors applied to a power system have several effects, one or more of which may be the reason for their applications. These can be stated as:

(1) Reduce lagging component of circuit current.

(2) Increase voltage level at the load.

(3) Improve voltage regulation, if the capacitors are properly switched.

(4) Reduce I^2R loss in the system because of reduction in current.

(5) Reduce I^2X loss in the system because of reduction in current.

optimum result.

(6) Increase power factor of the source generators.

(7) Decrease KVA loading on the source generators and circuits to relieve an overloaded condition or release capacity for additional load growth.

5-2 Mathematical Model

Considering the mathematical model for the system described in Chapter III and the system circuit, we can state the active and reactive powers as:

$$P_{in} = - \frac{E_i E_n}{B_{in}} \text{Cos} (\beta_{in} + \theta_{in}) + \frac{D_{in} E_i^2}{B_{in}} \text{Cos} (\beta_{in} - \Delta_{in})$$

$$Q_{in} = - \frac{E_i E_n}{B_{in}} \text{Sin} (\beta_{in} + \theta_{in}) + \frac{D_{in} E_{in}^2}{B_{in}} \text{Sin} (\beta_{in} - \Delta_{in})$$

where

P_{in} = active power transmitted from bus i to bus n.

Q_{in} = reactive power transmitted from bus i to bus n.

$D_{in} \angle \Delta_{in}$ = D constant for transmission line joining bus i with bus j.

Y_i = Admittance of the shunt capacitance added at bus i.

The admittance value Y_i at bus i is related to $D_{in} \angle \Delta_{in}$ as follows:

$$D_{in} \angle \Delta_{in} = 1 + Z_{in} Y_i$$

For large systems, voltage magnitudes are required to be maintained at almost constant values and the only way to change reactive power is by the addition of shunt capacitors. The size of these capacitors must be proportional to the reactive power. The mathematical model and the computer program introduced before can be changed to include the addition of shunt capacitors.

The strategy of selecting the proper shunt capacitor can be stated as follows: For bus i of the system we can:

(1) Consider each transmission line connecting bus i and j. The reactive power Q_{ij} can be calculated.

(2) Check on this reactive power sign can be made, to show if it is a lagging or leading power with respect to bus i.

(3) If this reactive power is a lagging one, then we can increase the capacitance value Y(i) at bus i by an amount proportional to this lagging reactive power, where Y(i) is the admittance of the capacitors to be added at bus i.

(4) Check other busses and repeat step 2 and 3 until all busses are checked.

(5) Check the reactive power carried by each generator and fulfill the restrictions given on each generator.

At the end of each iteration, the total input cost can be compared to the previous one and the better result can be stored as a nearer optimum result.

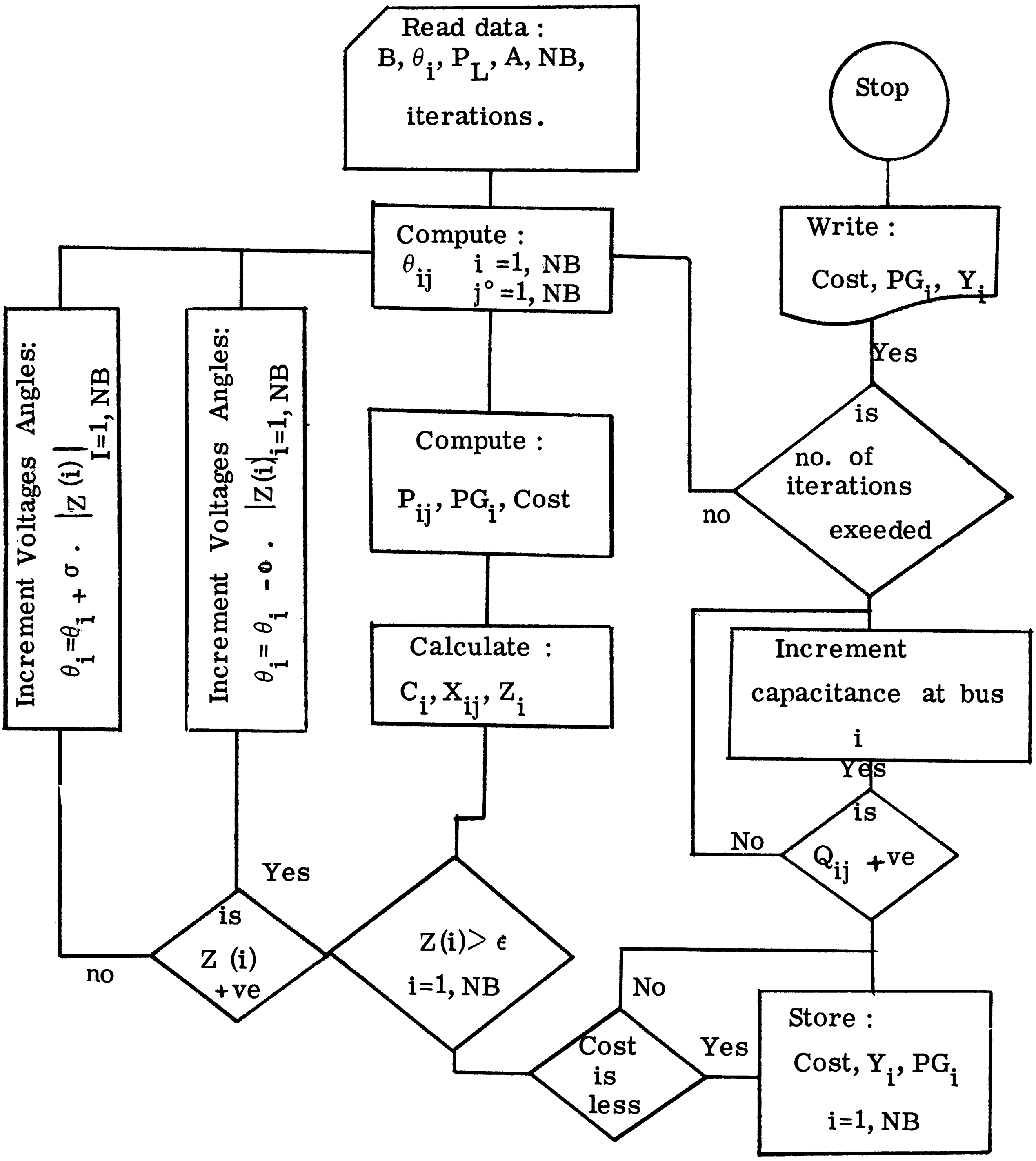


Fig. 5-1

Where :

A=Coefficients array of the input fuel cost's functions.

The computer program introduced for Chapter III has been modified to include the addition of these shunt capacitors. A flow chart is also shown in Fig.(5-1) which describes it.

Results for the three bus system :

The three bus system introduced before is considered and the results obtained by running this computer program show that a better saving can be achieved by applying these capacitors .

Results shows that for the same system with the same loads , the cost of the input fuel is only 6. 33623 instead of 6. 35053.

Chapter VI

Comparison With Some Other Methods

In this chapter, a comparison will be made between the results obtained from this approach and those given by some other methods. The Kirchmayer and Stagg paper (18) has been selected for this comparison.

To determine the optimum generation schedule Kirchmayer and Stagg presented different methods, which can be summarized as:

6-1 Exact Method

In this method a set of nonlinear equations has been derived and the optimum fuel input for a given received load is obtained by solving them.

These equations are:

$$\frac{dF_n}{dP_n} + \lambda \frac{\partial P_L}{\partial P_n} = \lambda \qquad (6.1)$$

where

F_n = fuel input to plant n in dollars per hour.

P_n = output of plant n in mega watts.

$\frac{\partial F_n}{\partial P_n}$ = incremental fuel cost of plant n in dollars per mega watt-hour.

P_L = total transmission losses.

$\frac{\partial P_L}{\partial P_n}$ = incremental transmission loss at plant n in mega watts per mega watt.

λ = incremental cost of received power in dollars per mega watt-hour.

The incremental transmission loss at plant i may be expressed by:

$$\frac{\partial P_L}{\partial P_n} = \sum_m 2 \; B_{nm} P_m$$

The incremental fuel cost of a given plant over a limited range may be represented by:

$$\frac{dF_n}{dP_n} = F_{nn} P_n + f_n$$

F_{nn} = slope of incremental fuel cost curve

f_n = intercept of incremental fuel cost curve.

Then the coordination equation 6. 1 will be:

$$F_{nn} P_n + f_n + \lambda \sum_m 2 \; B_{nm} P_m = \lambda$$

Solutions for different loads are obtained by varying λ.

6-2 Penalty Factor Methods:

If the incremental transmission loss at plant n is charged at a rate corresponding to the incremental fuel cost of plant n, the following equations result:

$$\frac{dF_n}{dP_n} + \frac{dF_n}{dP_n} \frac{\partial P_L}{\partial P_n} = \lambda$$

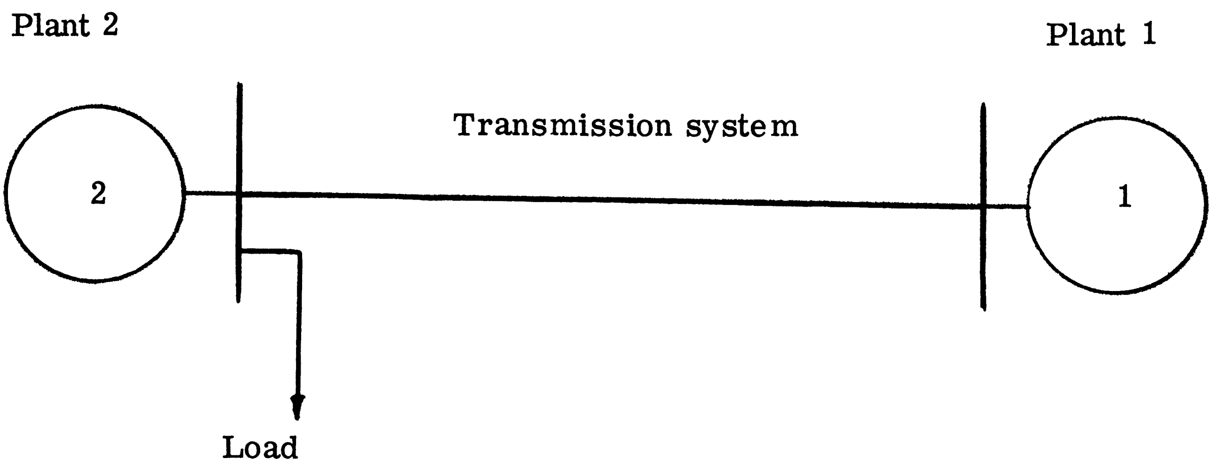


Fig. 6-1

Kirchmayer's two bus system example

$$\frac{dF_n}{dP_n}\left(1 + \frac{\partial P_L}{\partial P_n}\right) = \lambda$$

$$\frac{dF_n}{dP_n} L_n = \lambda$$

where L_n = penalty factor of plant n.

Kirchmayer Simple 2 - bus system is shown in Figure (6-1). This system was a simple representation of the American Gas and electric system and illustrated the relatively high cost generation in the Ohio Division available for Transfer West. These two areas are about 250 miles apart. The incremental fuel cost for both plants are shown in Figure (6-2).

The generation schedules obtained from Kirchmayer's exact solution and the penalty factor method compared to the results obtained using the method of this thesis are shown in Figure(6-3).

Fig. (6. 3) shows the generation schedule on each generator using different methods.

Curves A_1 and A_2 shows the Generation schedule shared by station 1 and 2 respectively using the Penalty Factor method.

Curves B_1 and B_2 shows the generation schedule shared by station 1 and 2 respectively using the the Exact method.

Curves C_1 and C_2 shows the generation schedule shared by station 1 and 2 respectively using this thesis method.

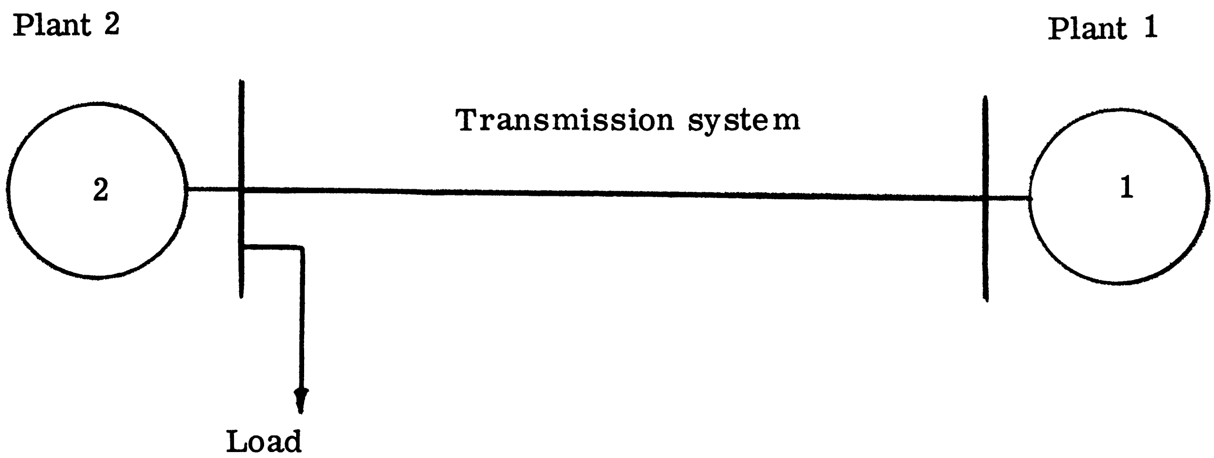


Fig. 6-1

Kirchmayer's two bus system example

Comparing the results obtained by this method and those of Kirchmayer's paper , it is found that these results are more accurate than any method introduced by Kirchmayer. This fact can be understood since this method has no severe restrictions as in the case of the Penalty Factor method or all those based on the loss formula technique.

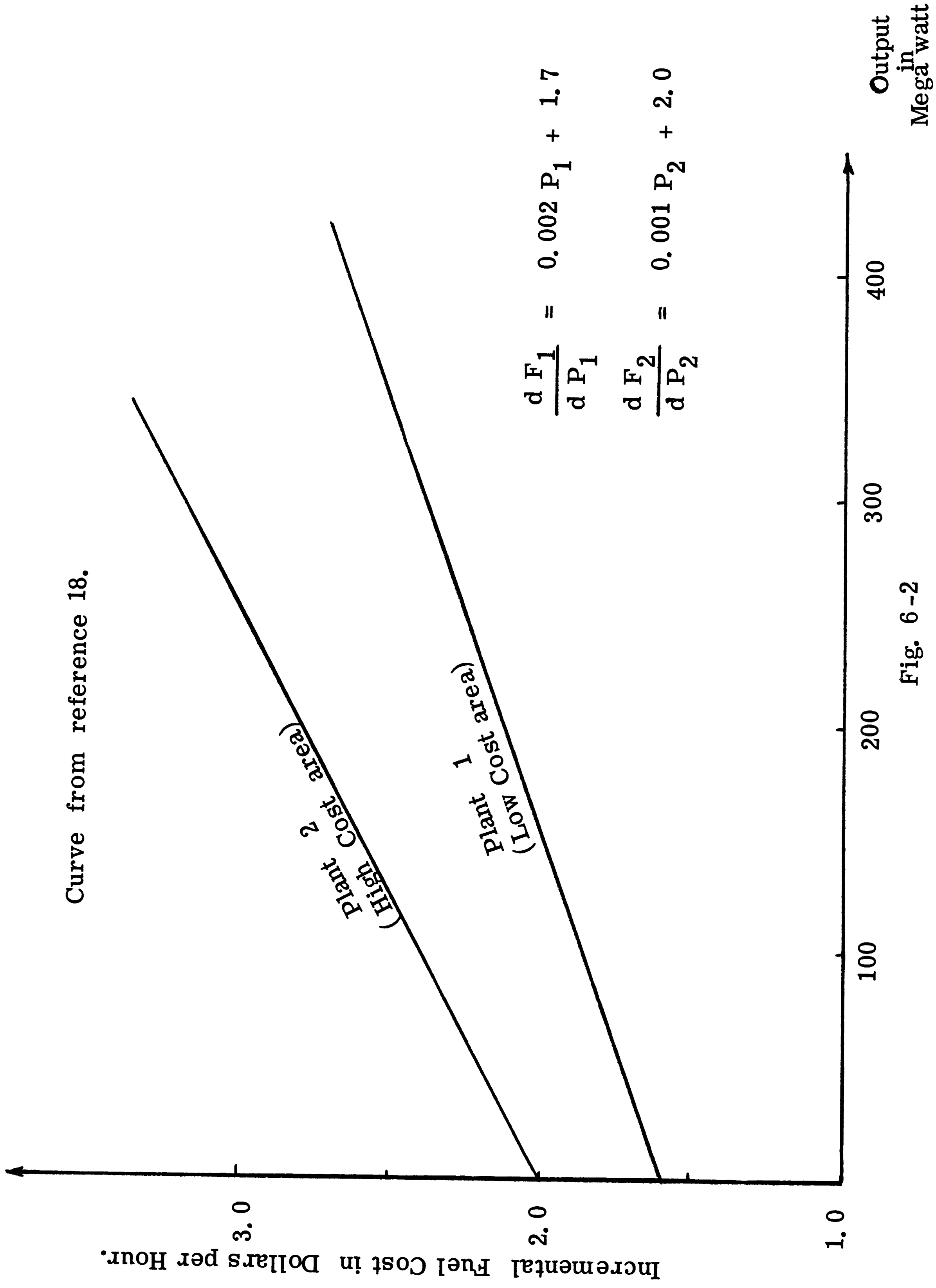

Curve from reference 18.
Plant 2 (High Cost area)
Plant 1 (Low Cost area)
$\frac{dF_1}{dP_1} = 0.002\,P_1 + 1.7$
$\frac{dF_2}{dP_2} = 0.001\,P_2 + 2.0$
Incremental Fuel Cost in Dollars per Hour.
1.0
2.0
3.0
100
200
300
400
Output in Mega watt

Fig. 6-2

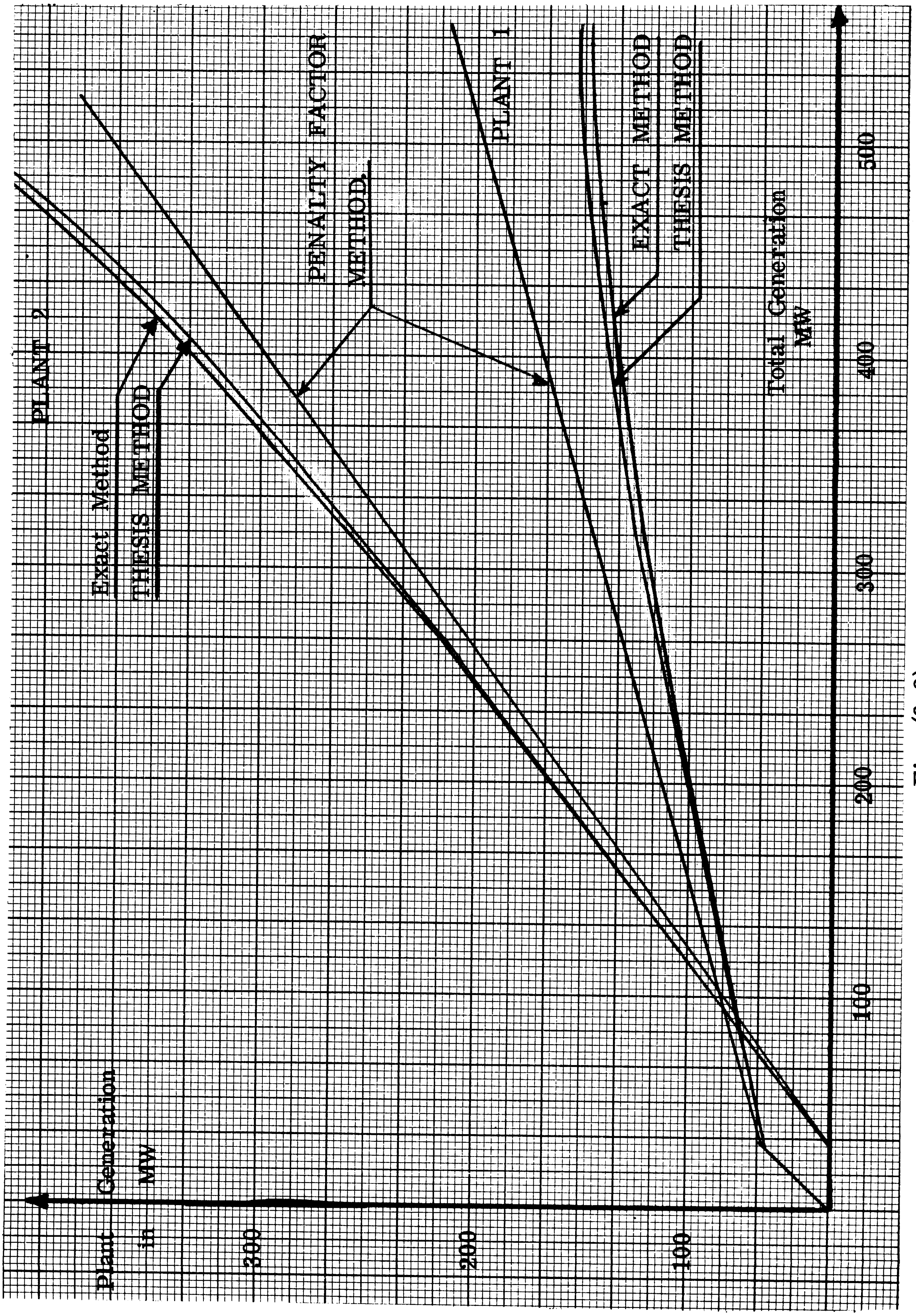

Plant Generation in MW
300
200
100
PLANT 2
Exact Method
THESIS METHOD
PENALTY FACTOR METHOD
PLANT 1
EXACT METHOD
THESIS METHOD
Total Generation MW
100
200
300
400
500

Fig. (6-3)

Chapter VII

Summary and Conclusion

This thesis develops a method of determining the minimum fuel cost for a system of electrical plants which have different fuel economics for each station. The thesis acomplishes this with different computer programs geared to the system requirements. In the case of a system with constant voltages, due to customer requirements, the voltage phase angles are to be considered as the only independent variables in the process of optimization of the fuel input cost for the power system. When a voltage variation range is possible another program can be used, in which these variations are supplied and the optimum cost, generation schedule and final voltage profile can then be determined.

In chapter III, a mathematical model for the power system was developed and an approach to schedule the power between the generators was set up in the following steps:

1. Determine the transfer impedances among all possible pairs of busses for the entire system.
2. Determine local loads required from the system.
3. Determine the fuel input-output characteristic of each plant and store it in the computer in the form of a power series for easy handling.
4. Determine the incremental rate curves for each plant by differentiating the input-output characteristic.

5. Assign initial values of voltage phase angles to the system. These angles may be chosen arbitrarily but for minimum iterations they should be chosen close to the expected solutions.
6. Calculate the phase angle differences between all pairs of generators.
7. Calculate the power transmitted by each transmission line P_{ij}.
8. Add all power transmitted from each station to its local load to get the generated power and then calculate the corresponding fuel cost.
9. Evaluate the terms X_{ij} as defined by equation (3-21).
10. Combine the products X_{ij} as required by equation (3-23) to get the partial derivatives of the cost with respect to the voltage phase angles.
11. For the next iteration change each voltage phase angle by a quantity proptional to the corresponding value of the derivative (Z_i) and in the opposite sense.
12. Repeat the above steps until all derivatives come close to zero. Then print out the generated power schedule. This is the minimum input cost schedule.

Chapter IV covers the results obtained from running the program on three and twelve bus systems. In the former system three runs were made with the same data, but with different starting voltage phase angle assignments.

In the twelve bus system two intermediate busses were included. The time taken in the latter case to run the program through the computer was only 3.407 seconds, which is fairly fast.

In chapter V another dimension was introduced to deal with the problem of fuel cost optimization. In this chapter shunt capacitors were considered in an attempt to reduce the reactive power carried by the transmission lines and consequently the transmission losses and the total fuel cost.

In chapter VI comparison was made between the results obtained from the approach used in this thesis and those given by Kirchmayer and Stagg (18). Results obtained show a great closeness to Kirchmayer's Exact Method introduced in his paper.

In conclusion, it can be said that the approach used in this thesis has many advantages, since no severe restrictions were imposed on the system as in the case of all other methods based on the B constants.

This approach makes use of the up-to-date fast computer ability. Thus when the load varies a complete run for the whole system with the new load requirements can be made in a few seconds and the schedule can be adapted by the dispatcher almost instantaneously.

Appendix I

Determination of Co-ordination Equation

The derivation of these equations follows directly from the method of Lagrangian multipliers. Let

$F_t =$ total fuel input to the system in dollars per hour.

$= \sum F_n$

where

$F_n =$ fuel input to plant n in dollars per hour.

Let

$P_L =$ total transmission losses in mega watts

$= \sum \sum P_n B_{nm} P_m$

where

$P_n =$ loading of plant n

$B_{nm} =$ transmission loss formula coefficients.

It is desired to minimize the total fuel input F_t for a given received load P_R. Let

$P_R =$ given received load

By application of the method of Lagrangian multipliers the equation of constraint is given by:

$$\psi (P_1, P_2, \ldots, P_n) = \sum_n P_n - P_L - P_R = 0$$

This minimum fuel input for a given received load is obtained when:

$$\frac{\partial \phi}{\partial P_n} = 0$$

Where:

$$\phi = F_t - \lambda . \ \psi$$

λ = Lagrangian type of multiplier.

$$\frac{\partial \phi}{\partial P_n} = \frac{\partial F_t}{\partial P_n} - \lambda \frac{\partial \psi}{\partial P_n}$$

Then :

$$\frac{\partial F_t}{\partial P_n} - \lambda \quad \frac{\partial}{\partial P_n} [\sum_n P_n - P_L - P_R] = 0$$

$$\frac{\partial F_t}{\partial P_n} - \lambda \ [1 - \frac{\partial P_L}{\partial P_n}] = 0$$

$$\frac{\partial F_t}{\partial P_n} + \lambda \ \frac{\partial P_L}{\partial P_n} = \lambda$$

But

$$\frac{\partial F_n}{\partial P_n} + \lambda \ \frac{\partial P_L}{\partial P_n} = \lambda$$

So we have

$$\frac{\partial F_n}{\partial P_n} + \lambda \frac{\partial P_L}{\partial P_n} = \lambda$$

These are covered in reference (15).

Computer Programs Index

```
SLIST PRØGRAM
>            DIMENSIØN E(3),DELTA(3),B(3,3),C(3),BETA(3,3),
>           1PINPUT(3),BDELTA(3,3),PG(3),Z(3),P(3,3),X(3,3),
>           2PL(3),A(4,3)
>            NAMELIST/N2/E,DELTA,B,BETA,PL,A,NB
>            READ(5,N2)
>            DØ 277 I=1,NB
>            P(I,I)=PL(I)
>       277  CØNTINUE
>         1  DØ 2 I=1,NB
>            DØ 2 J=1,NB
>         2  BDELTA(I,J)=DELTA(I)-DELTA(J)
>            GØ TØ 19
>         3  DØ 554 I=1,NB
>            C(I)=A(2,I)+2.*A(3,I)*PG(I)+3.*A(4,I)*PG(I)**2
>       554  CØNTINUE
>       102  FØRMAT(2HC=,6F9.4)
>            DØ 4 I=1,NB
>            DØ 4 J=1,NB
>            IF(I-J)5,4,5

>         5  U=SIN(BETA(I,J)+BDELTA(I,J))
>            X(I,J)=(E(I)*E(J)*C(I)/B(I,J))*U
>         4  CØNTINUE
>            DØ 411 I=1,NB
>            Z(I)=0.0
>            DØ 411 J=1,NB
>            IF(I-J)412,411,412
>       412  Z(I)=Z(I)+X(I,J)-X(J,I)
>       411  CØNTINUE
>       103  FØRMAT(2HZ=,6F9.5,/,12H............)
>            DØ 6 I=1,NB
>            IF(ABS(Z(I))-.1)6,6,7
>         6  CØNTINUE
>            GØ TØ 18
>         7  DØ 666 I=1,NB
>            IF(Z(I))12,1,11
>        11  DELTA(I)=DELTA(I)-ABS(Z(I)* .0045)
>            GØ TØ 666
>        12  DELTA(I)=DELTA(I)+ABS(Z(I)* .0045)
>       666  CØNTINUE
>            GØ TØ 1
>        18  CØNTINUE
>       105  FØRMAT(2HE=,6F9.5)
>            GØ TØ 23
>        19  DØ 21 I=1,NB
>            DØ 21 J=1,NB
>            IF(I-J)20,21,20
>        20  W=CØS(BETA(I,J)+BDELTA(I,J))
>            WW=CØS(BETA(I,J))
>            P(I,J)=(-E(I)*E(J)*W)/B(I,J)+E(I)*E(I)*WW/B(I,J)
>        21  CØNTINUE
>            DØ 22 IG=1,NB
>            PG(IG)=.0
```

```
      DØ 22 JG=1,NB
      PG(IG)=PG(IG)+P(IG,JG)
   22 CØNTINUE
  106 FØRMAT(6H PG  =,6F9.5)
      DØ 555 I=1,NB
      PINPUT(I)=A(1,I)+A(2,I)*PG(I)+A(3,I)*PG(I)**2+
     1 A(4,I)*PG(I)**3
  555 CØNTINUE
      PTØTAL=.0
      DØ 299 IP=1,NB
  299 CØNTINUE
  107   FØRMAT(7HPTØTAL=,F10.5)
      GØ TØ 3
   23 WRITE(6,107)PTØTAL
      WRITE(6,106)PG
      WRITE(6,103)Z
      END
# END ØF FILE
#
```

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```
$LIST DATA
>        &N2
>          E=1.,1.,1.,DELTA=.0,.0,.0,
>         B=.0,.346,.481,.346,.0,.402,.481,.402,.0
>         ,BETA=.0,1.12,1.102,1.12,.0,1.34,1.102,1.34,.0,
>         PL=.5,.5,.5,  A=2.28,.52,.38,.04,
>           1.59,.75,.44,.03,
>           1.04,1.16,.84,.01,NB=3 &END
# END ØF FILE
#
```

```
 $RUN NNN 5=DATA 6=*SINK*
#EXECUTION BEGINS
 PTOTAL=   6.35053
  PG  =  0.81030  0.58215  0.12398
 Z= -0.09935  0.01203  0.08732

   STOP      0
#EXECUTION TERMINATED
#SSIG
#OFF AT 20:47.23
#ELAPSED TIME          '01.286  SEC.
#CPU TIME USED           2.933  SEC.
#STORAGE USED          863.843  PAGE-SEC.
#DRUM READS              0
#APPROX. COST OF THIS RUN     $.46
#FILE STORAGE            2 PG-HR.      .00
```

```
$RUN -A 5=DATA  6=*SINK*
#EXECUTIØN BEGINS

 DELTA=   1.01230   1.00536   0.98234
  PG  =   0.50000   0.50000   0.50000
 PTØTAL=    6.54999
 Z= -2.73335 -1.19027  3.92362


 DELTA=   1.02169   1.00850   0.96982
  PG  =   0.57406   0.53784   0.38930
 PTØTAL=    6.45611
 Z= -2.08587 -0.69813  2.78400


 DELTA=   1.02884   1.01030   0.96085
  PG  =   0.63184   0.55987   0.31188
 PTØTAL=    6.40723
 Z= -1.59097 -0.40045  1.99142


 DELTA=   1.03430   1.01130   0.95440
  PG  =   0.67664   0.57235   0.25717
 PTØTAL=    6.38129
 Z= -1.21207 -0.22186  1.43393


 DELTA=   1.03845   1.01182   0.94973
  PG  =   0.71119   0.57912   0.21819
 PTØTAL=    6.36733
 Z= -0.92212 -0.11596  1.03808


 DELTA=   1.04160   1.01206   0.94633
  PG  =   0.73774   0.58253   0.19020
 PTØTAL=    6.35973
 Z= -0.70055 -0.05422  0.75476


 DELTA=   1.04399   1.01215   0.94386
  PG  =   0.75807   0.58400   0.17001
 PTØTAL=    6.35556
 Z= -0.53151 -0.01922  0.55073


 DELTA=   1.04580   1.01215   0.94204
  PG  =   0.77358   0.58438   0.15537
 PTØTAL=    6.35326
 Z= -0.40281 -0.00028  0.40309
```

```
DELTA=  1.04717  1.01211  0.94071
 PG   =  0.78540  0.58421  0.14472
PTØTAL=    6.35199
Z= -0.30495  0.00920  0.29575


DELTA=  1.04821  1.01205  0.93973
 PG   =  0.79439  0.58376  0.13694
PTØTAL=    6.35128
Z= -0.23067  0.01319  0.21747


DELTA=  1.04900  1.01198  0.93901
 PG   =  0.80121  0.58321  0.13124
PTØTAL=    6.35088
Z= -0.17432  0.01410  0.16022


DELTA=  1.04959  1.01192  0.93848
 PG   =  0.80638  0.58266  0.12706
PTØTAL=    6.35066
Z= -0.13166  0.01342  0.11823


PTØTAL=    6.35054
 PG   =  0.81030  0.58215  0.12399
Z= -0.09936  0.01198  0.08739


ITER= 13

   STØP      0
#EXECUTIØN TERMINATED
#$LIST  DATA
>       1       &N2
>       2         E=1.,1.,1., DELTA=1.,1.,1.,
>       3        B=.0,.346,.481,.346,.0,.402,.481,.402,.0
>       4         ,BETA=.0,1.12,1.102,1.12,.0,1.34,1.102,1.34,.0,
>       5         PL=.5,.5,.5, A=2.28, .52,.38,.04,
>       6         1.59,.75,.44,.03,
>       7         1.04,1.16,.84,.01,NB=3 &END
# END ØF FILE
#
```

```
$RUN -A   5=DATA   6=*SINK*
#EXECUTIØN BEGINS

DELTA=  0.01230   0.00536  -0.01766
 PG   =  0.50000   0.50000   0.50000
PTØTAL=    6.54999
Z= -2.73335 -1.19027  3.92362


DELTA=  0.02169   0.00850  -0.03018
 PG   =  0.57407   0.53784   0.38930
PTØTAL=    6.45611
Z= -2.08584 -0.69812  2.78397


DELTA=  0.02885   0.01030  -0.03915
 PG   =  0.63184   0.55987   0.31188
PTØTAL=    6.40723
Z= -1.59095 -0.40046  1.99141


DELTA=  0.03430   0.01130  -0.04560
 PG   =  0.67663   0.57236   0.25717
PTØTAL=    6.38129
Z= -1.21208 -0.22183  1.43392


DELTA=  0.03845   0.01182  -0.05027
 PG   =  0.71119   0.57912   0.21818
PTØTAL=    6.36732
Z= -0.92211 -0.11593  1.03804


DELTA=  0.04160   0.01206  -0.05367
 PG   =  0.73774   0.58253   0.19020
PTØTAL=    6.35973
Z= -0.70055 -0.05417  0.75472


DELTA=  0.04399   0.01215  -0.05614
 PG   =  0.75807   0.58400   0.17001
PTØTAL=    6.35556
Z= -0.53150 -0.01919  0.55069


DELTA=  0.04581   0.01215  -0.05796
 PG   =  0.77358   0.58439   0.15537
PTØTAL=    6.35326
Z= -0.40280 -0.00022  0.40302
```

```
DELTA=  0.04718   0.01211 -0.05929
 PG  =  0.78541   0.58421  0.14471
PTØTAL=    6.35198
Z= -0.30491   0.00923   0.29568


DELTA=  0.04822   0.01205 -0.06027
 PG  =  0.79439   0.58376  0.13693
PTØTAL=    6.35127
Z= -0.23062   0.01323   0.21739


DELTA=  0.04900   0.01199 -0.06099
 PG  =  0.80122   0.58321  0.13123
PTØTAL=    6.35088
Z= -0.17428   0.01414   0.16014


DELTA=  0.04959   0.01193 -0.06152
 PG  =  0.80639   0.58266  0.12705
PTØTAL=    6.35066
Z= -0.13162   0.01345   0.11817


PTØTAL=    6.35053
 PG  =  0.81030   0.58215  0.12398
Z= -0.09935   0.01203   0.08732


ITER= 13

   STØP      0
#EXECUTIØN TERMINATED
#$LIST  DATA
>        1       &N2
>        2         E=1.,1.,1., DELTA=.0,.0,.0,
>        3       B=.0,.346,.481,.346,.0,.402,.481,.402,.0
>        4        ,BETA=.0,1.12,1.102,1.12,.0,1.34,1.102,1.34,.0,
>        5         PL=.5,.5,.5,  A=2.28, .52,.38,.04,
>        6         1.59,.75,.44,.03,
>        7         1.04,1.16,.84,.01,NB=3 &END
# END ØF FILE
#
```

```
$RUN -A 5=DATA  6=*SINK*
#EXECUTION BEGINS
 DELTA=  0.21230  0.20536  0.18234
  PG  =  0.50000  0.50000  0.50000
 PTOTAL=   6.54999
 Z= -2.73335 -1.19027  3.92362


 DELTA=  0.22169  0.20850  0.16982
  PG  =  0.57407  0.53784  0.38930
 PTOTAL=   6.45611
 Z= -2.08584 -0.69812  2.78397


 DELTA=  0.22885  0.21030  0.16085
  PG  =  0.63184  0.55987  0.31188
 PTOTAL=   6.40723
 Z= -1.59095 -0.40046  1.99141


 DELTA=  0.23430  0.21130  0.15440
  PG  =  0.67663  0.57236  0.25717
 PTOTAL=   6.38129
 Z= -1.21208 -0.22183  1.43392


 DELTA=  0.23845  0.21182  0.14973
  PG  =  0.71119  0.57912  0.21818
 PTOTAL=   6.36732
 Z= -0.92211 -0.11593  1.03804


 DELTA=  0.24160  0.21206  0.14633
  PG  =  0.73774  0.58253  0.19020
 PTOTAL=   6.35973
 Z= -0.70055 -0.05417  0.75472


 DELTA=  0.24399  0.21215  0.14386
  PG  =  0.75807  0.58400  0.17001
 PTOTAL=   6.35556
 Z= -0.53150 -0.01919  0.55069


 DELTA=  0.24581  0.21215  0.14204
  PG  =  0.77358  0.58439  0.15537
 PTOTAL=   6.35326
 Z= -0.40280 -0.00022  0.40302
```

```
DELTA=  0.24718  0.21211  0.14071
 PG   =  0.78541  0.58421  0.14471
PTOTAL=   6.35198
Z= -0.30491  0.00923  0.29568


 PG   =  0.79439  0.58376  0.13693
PTOTAL=   6.35127
Z= -0.23062  0.01323  0.21739


DELTA=  0.24900  0.21199  0.13901
 PG   =  0.80122  0.58321  0.13193
PTOTAL=   6.35088
Z= -0.17428  0.01413  0.16015


DELTA=  0.24959  0.21193  0.13848
 PG   =  0.80639  0.58266  0.12705
PTOTAL=   6.35066
Z= -0.13162  0.01345  0.11817


PTOTAL=   6.35053
 PG   =  0.81030  0.58215  0.12399
Z= -0.09935  0.01203  0.08732


ITER= 13

   STOP      0
#EXECUTION TERMINATED
#$LIST DATA
>        1        &N2
>        2         E=1.,1.,1.,   DELTA=.2,.2,.2,
>        3         B=.0,.346,.481,.346,.0,.402,.481,.402,.0
>        4           ,BETA=.0,1.12,1.102,1.12,.0,1.34,1.102,1.34,.0,
>        5          PL=.5,.5,.5,   A=2.28,  .52,.38,.04,
>        6          1.59,.75,.44,.03,
>        7          1.04,1.16,.84,.01,NP=3 &END
# END OF FILE
#
```

```
SLIST  VØLT3
>            DIMENSIØN E(3),DELTA(3),B(3,3),C(3),BETA(3,3),
>           1PINPUT(3),BDELTA(3,3),PG(3),Z(3),P(3,3),X(3,3),
>           2PL(3),A(4,3),GEN(3),VFIN(3),VMAX(3),VMIN(3)
>           3,CMIN(3),NBUS(3)
>            NAMELIST/N2/E,DELTA,B,BETA,PL,A,NB,VMAX,VMIN,VINC
>            READ(5,N2)
>            CØST=100.
>      108    FØRMAT(5HITER=,I3)
>            DØ 277 I=1,NB
>            P(I,I)=PL(I)
>      277   CØNTINUE
>      1     DØ 2 I=1,NB
>            DØ 2 J=1,NB
>       2    BDELTA(I,J)=DELTA(I)-DELTA(J)
>            GØ TØ 19
>       3    DØ 554 I=1,NB
>            C(I)=A(2,I)+2.*A(3,I)*PG(I)+3.*A(4,I)*PG(I)**2
>     554    CØNTINUE
>     102    FØRMAT(2HC=,6F9.4)
>            DØ 4 I=1,NB
>            DØ 4 J=1,NB
>            IF(I-J)5,4,5
>      5     U=SIN(BETA(I,J)+BDELTA(I,J))
>            X(I,J)=(E(I)*E(J)*C(I)/B(I,J))*U
>          4 CØNTINUE
>            DØ 411 I=1,NB
>            Z(I)=0.0
>            DØ 411 J=1,NB
>            IF(I-J)412,411,412
>      412   Z(I)=Z(I)+X(I,J)-X(J,I)
>      411   CØNTINUE
>      103   FØRMAT(2HZ=,6F9.5,/,12H............)
>            DØ 6 I=1,NB
>            IF(ABS(Z(I))-.1)6,6,7
>       6    CØNTINUE
>            GØ TØ 18
>      7     DØ 666 I=1,NB
>            IF(Z(I))12,1,11
>      11    DELTA(I)=DELTA(I)-ABS(Z(I)* .0045)
>            GØ TØ 666
>      12    DELTA(I)=DELTA(I)+ABS(Z(I)* .0045)
>     666    CØNTINUE
>            GØ TØ 1
>       18   CØNTINUE
>      105   FØRMAT(2HE=,6F9.5)
>            GØ TØ 23
>      19    DØ 21 I=1,NB
>            DØ 21 J=1,NB
>            IF(I-J)20,21,20
>      20    W=CØS(BETA(I,J)+BDELTA(I,J))
>            WW=CØS(BETA(I,J))
>            P(I,J)=(-E(I)*E(J)*W)/B(I,J)+E(I)*E(I)*WW/B(I,J)
>      21    CØNTINUE
```

set of voltage phase angles is not critical. The program can start with any angle values, but it is quicker if it can be supplied values close to the actual ones.

The next step is to calculate the phase angles differences between each pair of machines, then the procedurewill calculate the various power flows between each pair of generators. The summation of the power flows from any one generator will give the total power flow away from that generator. Addition of the local load to this will give the total power output from any one generator. The computer will now calculate the input cost for each generator using the input-output curves given before. Adding all these input costs will give the total power input to the system. It will then proceed to calculate incremental rates for each generator using equation (3-11).

Now it is possible to calculate the various X_{ij}, then grouping them together to form the Z's defined in Chapter III, when Z_1 is defined as the partial derivative of the total cost with respect to angle θ_i. In the final solution these Z's must approach zero or sufficiently close to be neglected.

In the program, incrementing the phase angle is done either by adding or subtracting Z_i multiplied by a constant to accelerate the iteration. This process will continue until all Z's are close enough to zero.

The computer time spent on calculation is normally less than the time taken for the printer to write results, accordingly while the

```
$LIST DATA
>        &N2
>          E=.95,.95,.95,DELTA=.0,.0,.0,
>        B=.0,.346,.481,.346,.0,.402,.481,.402,.0
>        ,BETA=.0,1.12,1.102,1.12,.0,1.34,1.102,1.34,.0,
>        PL=.5,.5,.5,   A=2.28,.52,.38,.04,
>          1.59,.75,.44,.03,
>          1.04,1.16,.84,.01,NB=3,
>          VMAX=1.05,1.05,1.05, VMIN=.95,.95,.95, VINC=.1 &END
# END ØF FILE
#
```

```
$RUN FØRVØLTAGE 5=DATA  6=*SINK*
#EXECUTIØN BEGINS
 E=  1.05000  1.05000  1.02000
 PTØTAL=    6.34628
  PG  =  0.82245  0.57635  0.11546

   STØP      0
#EXECUTIØN TERMINATED
#$SIG
#ØFF AT 12:52.30
#ELAPSED TIME        87.37   SEC.
#CPU TIME USED        3.413  SEC.
#STØRAGE USED       460.79   PAGE-SEC.
#DRUM READS           0
#APPRØX. CØST ØF THIS RUN     $.44
#FILE STØRAGE            PG-HR.      .00
```

```
 $RUN -Q 5=DATA 6=*SINK*
#EXECUTIØN BEGINS
 DELTA=  0.01601  0.00879 -0.02480
  PG  =  1.00000  1.00000  1.00000
 PTØTAL=   9.08000
 Z= -3.55887 -1.95239  5.51126


 DELTA=  0.02777  0.01399 -0.04176
  PG  =  1.09530  1.06285  0.84411
 PTØTAL=   8.90190
 Z= -2.61326 -1.15652  3.76977


 DELTA=  0.03640  0.01708 -0.05348
  PG  =  1.16712  1.10009  0.73934
 PTØTAL=   8.81644
 Z= -1.91698 -0.68719  2.60417


 DELTA=  0.04272  0.01893 -0.06164
  PG  =  1.22076  1.12214  0.66798
 PTØTAL=   8.77454
 Z= -1.40361 -0.40951  1.81312


 DELTA=  0.04733  0.02003 -0.06736
  PG  =  1.26056  1.13520  0.61886
 PTØTAL=   8.75369
 Z= -1.02556 -0.24443  1.26999


 DELTA=  0.05070  0.02068 -0.07138
  PG  =  1.28992  1.14295  0.58476
 PTØTAL=   8.74321
 Z= -0.74784 -0.14583  0.89367


 DELTA=  0.05315  0.02107 -0.07422
  PG  =  1.31148  1.14752  0.56093
 PTØTAL=   8.73790
 Z= -0.54432 -0.08678  0.63110


 DELTA=  0.05493  0.02130 -0.07623
  PG  =  1.32727  1.15021  0.54420
 PTØTAL=   8.73518
 Z= -0.39557 -0.05132  0.44689
```

```
DELTA=  0.05622  0.02144 -0.07766
 PG  =  1.33879  1.15177  0.53240
PTØTAL=   8.73380
Z= -0.28709 -0.03004  0.31712

DELTA=  0.05716  0.02152 -0.07867
 PG  =  1.34719  1.15266  0.52406
PTØTAL=   8.73309
Z= -0.20810 -0.01732  0.22542

DELTA=  0.05783  0.02156 -0.07939
 PG  =  1.35329  1.15316  0.51815
PTØTAL=   8.73272
Z= -0.15070 -0.00978  0.16047

DELTA=  0.05832  0.02158 -0.07991
 PG  =  1.35772  1.15343  0.51396
PTØTAL=   8.73254
Z= -0.10904 -0.00534  0.11438

PTØTAL=   8.73244
 PG  =  1.36093  1.15357  0.51097
Z= -0.07885 -0.00274  0.08158

ITER= 13

   STØP      0
#EXECUTIØN TERMINATED
```

```
$LIST CAPACITØR
>           DIMENSIØN E(3),DELTA(3),B(3,3),C(3),BETA(3,3),
>          1PINPUT(3),BDELTA(3,3),PG(3),Z(3),P(3,3),X(3,3),
>          3PL(3),A(4,3),GEN(3),Y(3),YF(3),Q(3,3),DEE(3,3),D(3,3)
>           NAMELIST/N2/E,DELTA,B,BETA,PL,A,NB
>           READ(5,N2)
>           CØST=100.
>      108   FØRMAT(5HITER=,I3)
>           DØ 277 I=1,NB
>           P(I,I)=PL(I)
>           Q(I,I)=0.0
>      277  Y(I)=0.0
>           NUM=10
>        1  DØ 2 I=1,NB
>           DØ 2 J=1,NB
>        2  BDELTA(I,J)=DELTA(I)-DELTA(J)
>           GØ TØ 19
>        3  DØ 554 I=1,NB
>           C(I)=A(2,I)+2.*A(3,I)*PG(I)+3.*A(4,I)*PG(I)**2
>      554  CØNTINUE
>           DØ 4 I=1,NB
>           DØ 4 J=1,NB
>           IF(I-J)5,4,5
>        5  V=SIN(BETA(I,J)+BDELTA(I,J))
>           X(I,J)=(E(I)*E(J)*C(I)/B(I,J))*V
>        4  CØNTINUE
>           DØ 411 I=1,NB
>           Z(I)=0.0
>           DØ 411 J=1,NB
>           IF(I-J)412,411,412
>      412  Z(I)=Z(I)+X(I,J)-X(J,I)
>      411  CØNTINUE
>      103  FØRMAT(2HZ=,6F9.5,/,12H............)
>           DØ 6 I=1,NB
>           IF(ABS(Z(I))-.1)6,6,7
>        6  CØNTINUE
>           GØ TØ 18
>        7  DØ 666 I=1,NB
>           IF(Z(I))12,1,11
>       11  DELTA(I)=DELTA(I)-ABS(Z(I)* .0045)
>           GØ TØ 666
>       12  DELTA(I)=DELTA(I)+ABS(Z(I)* .0045)
>      666  CØNTINUE
>           GØ TØ 1
>       18  CØNTINUE
>           GØ TØ 23
>       19  DØ 21 I=1,NB
>           DØ 21 J=1,NB
>           U=(1.+B(I,J)*Y(I)*SIN(BETA(I,J)))**2+(B(I,J)*Y(I)*
>          1 CØS(BETA(I,J)))**2
>           IF(I-J)20,21,20
>       20  W=CØS(BETA(I,J)+BDELTA(I,J))
>           ARG=-B(I,J)*Y(I)*CØS(BETA(I,J))/(1.+B(I,J)*Y(I)*
>          1 SIN(BETA(I,J)))
>           DEE(I,J)=ATAN(ARG)
```

```
>                 D(I,J)=SQRT(U)
>                 WW=D(I,J)*CØS(BETA(I,J)-DEE(I,J))
>                 WO=SIN(BETA(I,J)+BDELTA(I,J))
>                 P(I,J)=(-E(I)*E(J)*W)/B(I,J)+E(I)*E(I)*WW/B(I,J)
>                 WS=D(I,J)*SIN(BETA(I,J)-DEE(I,J))/B(I,J)
>                 Q(I,J)=(-E(I)*E(J)*WO)/B(I,J)+E(I)*E(I)*WS
>            21   CONTINUE
>                 DØ 22 IG=1,NB
>                 PG(IG)=0.0
>                 DØ 22 JG=1,NB
>                  PG(IG)=PG(IG)+P(IG,JG)
>            22   CØNTINUE

>                 DØ 555 I=1,NB
>                 PINPUT(I)=A(1,I)+A(2,I)*PG(I)+A(3,I)*PG(I)**2+
>                1A(4,I)*PG(I)**3
>           555   CØNTINUE
>                 PTØTAL=.0
>                 LØ 299 IP=1,NB
>                 PTØTAL=PTØTAL+PINPUT(IP)
>           299   CØNTINUE
>                 GØ TØ 3
>            23   IF(PTØTAL .GE. CØST)GØ TØ 111
>                 CØST=PTØTAL
>                 DØ 222 I=1,NB
>                 GEN(I)=PG(I)
>           222   YF(I)=Y(I)
>           109   FØRMAT(2HY=,3F10.3)
>           106   FØRMAT(//,3HPG=,3F10.5)
>           107   FØRMAT(5HCØST=,F9.5)
>                 WRITE(6,106)GEN
>                 WRITE(6,109)YF
>                 WRITE(6,107)CØST
>           111   ITER=ITER+1
>                 IF(ITER-NUM)333,444,444
>           333   DØ 331 IA=1,NB
>                 DØ 331 JA=1,NB
>                 IF(IA .EQ. JA)GØ TØ 331
>                 IF(Q(IA,JA))331,331,332
>           332   Y(IA)=Y(IA)+.1*Q(IA,JA)
>           331   CØNTINUE
>                 GØ TØ 1
>           444   CØNTINUE
>                 END
# END ØF FILE
#
```

```
 $RUN -V  5=DATA 6=*SINK*
#EXECUTIØN BEGINS


 PG=    0.82155    0.57045    0.11445
 CØST=   6.33623
 Y=      0.004      0.027      0.0

   STØP      0
#EXECUTIØN TERMINATED
```

```
$LIST 12BUS
      DIMENSION E(12),DELTA(12),B(12,12),C(12),BETA(12,12),
     1PINPUT(12),BDELTA(12,12),PG(12),Z(12),P(12,12),X(12,1
     2,PL(12),A(3,12)
      NAMELIST/N2/E,DELTA,B,BETA,PL,A,NB
      READ(5,N2)
      DO 277 I=1,NB
      P(I,I)=PL(I)
  277 CONTINUE
    1 DO 2 I=1,NB
      DO 2 J=1,NB
    2 BDELTA(I,J)=DELTA(I)-DELTA(J)
      GO TO 19
    3 DO 554 I=1,NB
      C(I)=A(2,I)+2.*A(3,I)*PG(I)
  554 CONTINUE
  102 FORMAT(2HC=,6F9.4)
      DO 4 I=1,NB
      DO 4 J=1,NB
      IF(I-J)5,4,5
    5 U=SIN(BETA(I,J)+BDELTA(I,J))
      X(I,J)=(E(I)*E(J)*C(I)/B(I,J))*U
    4 CONTINUE
      DO 411 I=1,NB
      Z(I)=0.0
      DO 411 J=1,NB
      IF(I-J)412,411,412
  412 Z(I)=Z(I)+X(I,J)-X(J,I)
  411 CONTINUE
  103 FORMAT(2HZ=,6F9.5,/,12H............)
      DO 6 I=1,NB
      IF(ABS(Z(I))-.1)6,6,7
    6 CONTINUE
      GO TO 18
    7 DO 666 I=1,NB
      IF(Z(I))12,1,11
   11 DELTA(I)=DELTA(I)-ABS(Z(I)* .0045)
      GO TO 666
   12 DELTA(I)=DELTA(I)+ABS(Z(I)* .0045)
  666 CONTINUE
      GO TO 1
   18 CONTINUE
  105 FORMAT(2HE=,6F9.5)
      GO TO 23
   19 DO 21 I=1,NB
      DO 21 J=1,NB
      IF(I-J)20,21,20
   20 W=COS(BETA(I,J)+BDELTA(I,J))
      WW=COS(BETA(I,J))
      P(I,J)=(-E(I)*E(J)*W)/B(I,J)+E(I)*E(I)*WW/B(I,J)
   21 CONTINUE
      DO 22 IG=1,NB
      PG(IG)=.0
      DO 22 JG=1,NB
      PG(IG)=PG(IG)+P(IG,JG)
   22 CONTINUE
```

```
>         106  FØRMAT(6H PG   =,6F9.5)
>              DØ 555 I=1,NB
>              PINPUT(I)=A(1,I)+A(2,I)*PG(I)+A(3,I)*PG(I)**2
>         555  CØNTINUE
>              PTØTAL=.0
>              DØ 299 IP=1,NB
>              PTØTAL=PTØTAL+PINPUT(IP)
>          299 CØNTINUE
>          107   FØRMAT(7HPTØTAL=,F10.5)
>              GØ TØ 3
>          23  WRITE(6,107)PTØTAL
>              WRITE(6,106)PG
>              WRITE(6,103)Z
>              END
# END ØF FILE
#
```

```
$LIST DATA12
>         &N2
>          E=12*1.,   DELTA=12*0.0,
>          B= 4*1000.,.3 ,.35 ,.24 ,.3 ,4*1000.,
>             3*1000.,.2 ,.2 ,3*1000.,.2,2*1000.,.4,
>             3*1000.,.3 ,2*1000.,.25,2*1000.,.3,2*1000.,
>             1000.,.2,.3,2*1000.,.25,4*1000.,.3,1000.,
>             .3 ,.2 ,10*1000.,
>             .35,2*1000.,.25,8*1000.,
>             .24,1000.,.25,4*1000.,.25,4*1000.,
>             .3. 5*1000.,.25,2*1000.,.3,2*1000.,
>             1000.,.2,8*1000.,.35,.35,
>             2*1000.,.2,4*1000.,.3,4*1000.,
>             3*1000.,.3,4*1000.,.35,3*1000.,
>             1000.,.4,6*1000.,.35,3*1000.,
>          BETA=4*.0,1.12,1.14,1.34,1.13,4*.0,
>             3*.0,2*1.1,3*.0,1.2,2*.0,1.3,
>             3*.0,1.12,2*.0,1.33,2*.0,1.13,2*.0,
>             .0,1.1,1.12,2*.0,1.1,4*.0,1.2,.0,
>             1.12,1.1,10*.0,
>             1.14,2*.0,1.1,8*.0,
>             1.34,.0,1.33,4*.0,1.2,4*.0,
>             1.13,5*.0,.25,.0,.0,.3,.0,.0,
>             .0,1.2,8*.0,1.2,1.18,
>             .0,.0,1.13,4*.0,1.13,4*.0,
>             3*.0,1.2,4*.0,1.2,3*.0,
>             .0,1.3,6*.0,1.18,3*.0,
>          PL=12*.5,
>          A=    2.28    ,    .52    ,    .38 ,
>                1.8     ,    .42    ,    .37 ,
>                1.7     ,    .5     ,    .36 ,
>                1.9     ,    .3     ,    .38 ,
>               1.       ,   .964        .5 ,
>                1.75    ,    .3     ,    .37 ,
>                1.6     ,    .5     ,    .36 ,
>               1.7  ,     .45    ,    .32 ,
>                1.75    ,    .4     ,    .36 ,
>               1.       ,  1.55    ,    .5 ,
>                1.75    ,    .8     ,    .35 ,
>                1.75    ,    .6     ,    .36 ,
>           NB=12 &END
# END ØF FILE
#
```

```
 $RUN FØR12BUS  5=DATA12  6=*SINK*
#EXECUTIØN BEGINS

 PTØTAL=  24.15683
  PG  =  0.48491  0.67290  0.87870  0.83974  0.00302  0.77965
  PG  =  0.80040  0.18677  0.69756  0.00052  0.21161  0.42529
 Z=  0.02442 -0.04522  0.05433 -0.02275 -0.02293 -0.01442
 ............
 Z=  0.05249  0.06637 -0.06304  0.09788 -0.05941 -0.06774
 ............

   STØP     0
#EXECUTIØN TERMINATED
#$SIG
#ØFF AT 19:51.30
#ELAPSED TIME        220.51   SEC.
#CPU TIME USED         6.186  SEC.
#STØRAGE USED       2294.05   PAGE-SEC.
#DRUM READS           53
#APPRØX. CØST ØF THIS RUN   $1.05

#FILE STØRAGE          3 PG-HR.      .00
```

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