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A METHOD OF TESTING  
FREE SURFACE MODELS IN AN ARTIFICIAL GRAVITY FIELD  
(TO OBTAIN VERY HIGH REYNOLDS NUMBERS  
WHILE SATISFYING FROUDE'S LAW)

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February 1967

## ABSTRACT

It is a well known fact that in a model test by a usual method involving a free surface, both the Reynolds law of similitude and the Froude's law of similitude cannot be satisfied simultaneously.

The proposed method is to test the model in an artificial gravity field so that a very high Reynolds number can be obtained while satisfying the Froude number condition.

The method is not without drawbacks, but there appears to be a number of advantages that are not available in other methods. This method will allow simulation of geosims using a single model.

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## INTRODUCTION

The functional relationship of forces on a body moving in or near the free surface under the influence of gravity and viscosity has the form (Ref. 1) that

$$R = f(\rho, L, v, \mu, g, r_1, r_2, \dots) \quad (1)$$

where  $R$  = force such as drag, lift, etc.

$\rho$  = density of the fluid

$v$  = velocity of the body

$\mu$  = dynamic viscosity of the fluid

$g$  = gravitational constant

$L$  = length

$r_1, r_2, \dots$ : dimensionless ratio defining the shape

By dimensional analysis, the non-dimensional force coefficient can be shown to be

$$C = f(Re, Fr, r_1, r_2, \dots) \quad (2)$$

where  $C = R / \left(\frac{\rho}{2} L^2 v^2\right)$ : force coefficient

$Re = v \cdot L / (\mu / \rho)$ : Reynolds number

$Fr = v / \sqrt{L \cdot g}$ : Froude number

The meaning of equation (2) is that the force coefficients of geometrically similar bodies cannot be expected to agree unless both the Reynolds numbers and the Froude numbers are equal. It is, however, a well known fact that in a usual experimental setup there is no way of making both the Reynolds numbers and the Froude numbers of a model involving a free surface simultaneously equal to those of the full-size prototype because of the conflicting

demands of the Reynolds numbers that the model moves faster and of the Froude numbers that the model moves slower than the prototype, except by testing the model in a fluid having a very much lower coefficient of kinematic viscosity ( $\nu \equiv \mu/\rho$ ). It is also a well known fact that no fluid exists which would permit more than a very moderate reduction in scale from the full size. Thus, it is an accepted conclusion that forces of one body from tests of a similar body of different size cannot be predicted.

Where a free surface model is involved, therefore, the usual method is to satisfy the Froude numbers and try to obtain as high Reynolds numbers as the facility would allow. The end result of this is that either the facility becomes large and expensive or some degree of scale-effect must be tolerated. Today's existing testing facilities dealing with surface models suffer from scale-effect problems of this nature in one form or another, and it is certainly the most important problem in ship model towing tanks (Ref. 2).

The proposed method, to be explained below, is to circumvent this problem by testing a model in an artificially created gravity field so that a very high Reynolds number, if not as high as that of the prototype, can be obtained while satisfying the Froude number condition.

## DESCRIPTION OF PRINCIPLE

For the same Froude number condition, we have

$$(Fr)_p = (Fr)_m \text{ or } \frac{v_p}{\sqrt{L_p \cdot g_p}} = \frac{v_m}{\sqrt{L_m \cdot g_m}} \quad (3)$$

where subscripts p and m denote the prototype and the model respectively. Then, we get

$$v_m = v_p \sqrt{\frac{L_m \cdot g_m}{L_p \cdot g_p}} = v_p \sqrt{\frac{L_m \cdot g_m}{L_p \cdot g}} \quad (4)$$

where  $g_p \equiv g$ : the earth gravitational constant.

For the same Reynolds number condition, we have

$$(Re)_p = (Re)_m \text{ or } \frac{v_p \cdot L_p}{\nu_p} = \frac{v_m \cdot L_m}{\nu_m} \quad (5)$$

where  $\nu \equiv \mu/\rho$ .

Solving for  $v_m$ , we get

$$v_m = v_p \frac{L_p \cdot \nu_m}{L_m \cdot \nu_p} \quad (6)$$

From (4) and (6), we get

$$g_m = g \left( \frac{L_p}{L_m} \right)^3 \left( \frac{\nu_m}{\nu_p} \right)^2 \quad (7)$$

$$\approx \lambda^3 g \quad ; \quad \lambda \equiv \frac{L_p}{L_m} \quad (8)$$

When the result of (7) and (8) is substituted into (4), we get

$$v_m = v_p \cdot \lambda \cdot \left( \frac{\nu_m}{\nu_p} \right) \approx \lambda \cdot v_p \quad (9)$$

Thus, by running the model at a speed approximately scale ratio  $\lambda$  times faster than the prototype in an artificial gravity

field of approximately  $\lambda^3$  times the earth gravitational field, in principle it is possible to satisfy both Reynolds numbers and Froude numbers simultaneously. Although there are some practical constructional limitations on the maximum model speed and the high gravitational field that can be obtained, it remains a technical possibility.

A more practical solution may be to relax the requirement of Reynolds numbers, thus reducing the model speed. The following examples show that with moderate model speeds, what may be said, sufficiently high Reynolds numbers can be obtained for small models.

Example 1

Consider a hydrofoil of 10-foot chord length running at 100 ft/sec and a 1.0-foot model of it in a gravitational field of 1000 g's, then, we get

$$v_m \approx v_p \sqrt{\frac{1}{10} \cdot \frac{1000g}{g}} = 1000 \text{ ft/sec.}$$

The prototype Reynolds number is

$$(Re)_p = \left( \frac{vL}{\nu} \right)_p = \frac{100 \times 10}{10^{-5}} \approx 1.0 \times 10^8$$

and the model Reynolds number is

$$(Re)_m = \frac{1000 \times 1.0}{10^{-5}} \approx 1.0 \times 10^8$$

Although this satisfies the Froude's and Reynolds' law of similitudes exactly, the model speed of 1000 ft/sec appears excessive. Supposing, therefore,  $(Re)_m$  of  $1 \times 10^7$  is sufficient, we have by solving backward,

$$v_m = 100 \text{ ft/sec}$$

$$g_m = 10 \text{ g's.}$$

(The model speed of approximately 200 ft/sec is considered to be attainable.)

$(Re)_m$  of this order can be obtained with a 26-inch chord model running at about 46 ft/sec by a usual method in one g-field, which might require a towing tank of approximately 700 ft in length.

### Example 2

Consider a surface vessel of 1000-foot size running at 50 ft/sec and a 3.3-foot model of it in a gravity field of 3000 g's, then, we get

$$v_m = v_p \sqrt{\frac{3.3}{1000} \cdot \frac{3000g}{g}} \approx 158 \text{ ft/sec.}$$

The Reynolds numbers are

$$(Re)_p = 1000 \times 50 / 10^{-5} \approx 5 \times 10^9$$

$$(Re)_m = 3.3 \times 158 / 10^{-5} \approx 5.2 \times 10^7,$$

the latter model Reynolds number being what one would have obtained from a 47.5-foot model by a usual manner. To run such a model might require a tank of approximately 1000-foot long, 50-foot wide and 25-foot deep.



## DESCRIPTION OF DEVICE

Accelerating elevators, rockets and the like can provide up to approximately 10 g's for a short period of time, but providing such g's and any sort of control is not simple. The only practical means of obtaining controlled high g-field is by means of a high speed centrifuge. Commercially available small centrifuges (costing less than \$100.00) can provide up to 12,000 g's, and the unique whirling tank at the Grumman Engineering Corporation (Ref. 4) has about 1200 g's at the maximum speed. Incidentally it is interesting to note that the highest known g's ever attained are about 58,000,000! (Ref. 3).

The essentials of a centrifuge type is given in Figure 1. The circular tank T, originally filled partially with a liquid, rotates about its vertical axis at a prescribed constant speed of revolution,  $\omega_1$  being its circular frequency. The free surface F of the liquid is in general describable by a body of revolution about the rotating axis with a generatrix of  $\frac{1}{2} \omega_1^2 r$ , where r is the radial coordinate, but because of large  $\omega_1$  and r, the free surface becomes almost a cylindrical one with a constant radius ( $R_1$ ).

The total gravity is given by a vector sum of g and  $\omega_1^2 r$ , or approximately equal to  $\omega_1^2 r$  since g is relatively small, and varies almost linearly from  $\omega_1^2 R_1$  at  $R_1$  to  $\omega_1^2 R_2$  at  $R_2$ , where  $R_1$  is the inner radius of fluid surface and  $R_2$  is the radius of the tank wall.

The model M is towed by a radial arm A, which rotates at a controlled speed  $\omega_2$  different from the first one. This arrangement provides relative speed of the model  $v_m$  with respect to the fluid, which is rotating at the same speed  $\omega_1$  as the tank T.

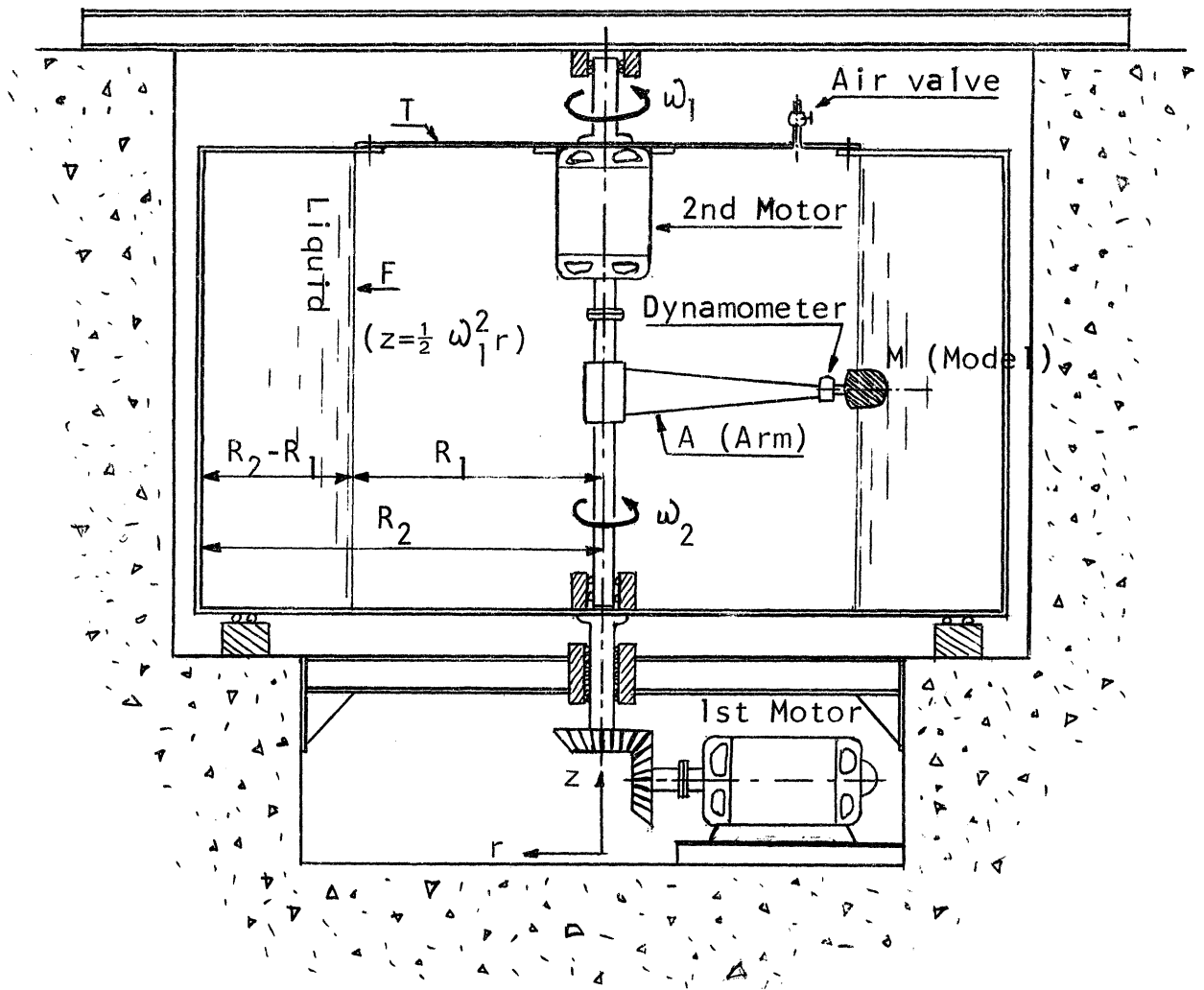


Figure 1. Schematic of Proposed Artificial Gravity Tank

Data from a dynamometer for measuring forces and moments, a viewing device such as a camera, if needed, and other usual measuring gages which are attached to the rotating arm A, can be telemetered electronically to a monitoring console.

The cavitation number is controlled by air pressure in the tank, if so desired. The air pressure should be normally above the atmospheric pressure. And this fact enables a cavitation study to be much more readily accessible.

Liquid other than water may be used, the case being much more feasible than in a regular towing tank or in a circulating water channel.

The proposed device resembles the unique Grumman whirling tank (Ref. 4) but there are some differences and it is meaningful to point out these differences. The Grumman tank has a stationary model and the model speed  $v_m$  (in Eq. 4) is obtained by the tank rotational speed alone. The model g-field ( $g_m$  in Eq. 4) is set always equal to  $\lambda \cdot g$  so that the model and the prototype have the same speeds.

By adding the second rotating arm, the proposed device aims to attain higher model speed (up to  $\lambda v_p$ ) thus higher Reynolds numbers. The speed control should be simpler. The g-field need not be changed during the test, thus not affecting the change in free surface form. In the Grumman tank, g-field must change to adjust the speed and this causes the form of free surface to change.

## SUMMARY OF HYDRODYNAMIC SIMILITUDE

The functional relationship corresponding to (1), but including the effect of surface tension, pressure at the free surface and compressibility, is given by

$$R = f(\rho, L, v, \mu, g, \tau, P_{\infty}, e, K, r_1, r_2, \dots) \quad (10)$$

where  $\tau$  = surface tension\*

$P_{\infty}$  = pressure at the free surface

$e$  = vapor pressure of the liquid

$K$  = bulk modulus\*\*

Five non-dimensional numbers (including Fr and Re that have already been considered) are required to express the functional relationship in non-dimensional form. Table I summarizes.

Table I  
List of Non-Dimensional Similitudes

Name	Nomenclature	Definition	Meaning (Ratio of Forces)
Froude Number	Fr	$v/\sqrt{g \cdot L}$	$\frac{\text{Inertia}}{\text{Gravity}}$
Reynolds Number	Re	$v \cdot L / (\mu / \rho)$	$\frac{\text{Inertia}}{\text{Viscosity}}$
Cavitation Number	$\sigma$	$(P_{\infty} - e) / \frac{\rho}{2} v^2$	$\frac{\text{Pressure}}{\text{Inertia}}$
Weber Number	W	$v / \sqrt{\tau / (\rho L)}$	$\frac{\text{Inertia}}{\text{Surface Tension}}$
Cauchy (Mach) Number	M	$v / \sqrt{K / \rho}$	$\frac{\text{Inertia}}{\text{Elasticity}}$

\*  $\tau$  = .00498 lbs/ft at 68°F for water and air

\*\* K = 300,000 psi for water

Table II, Col. 2, summarizes these similitudes when Froude and Reynolds numbers are satisfied for a scale model. Included in the same table is a comparison with those of other methods (Ref. 4).

Table II  
Comparison of Similitudes for Different Methods

(1)	(2) Proposed Device	(3) Grumman Tank	(4) Towing Tank Speed Law	(5) Froude Law
Geometry Ratio to Prototype	$\frac{1}{\lambda}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda}$
Velocity Ratio	$\lambda \frac{V_m}{V_p} \approx \lambda$	1	1	$\lambda^{1/2}$
Gravity Ratio	$\lambda^3 \frac{V_m^2}{V_p^2} \approx \lambda^3$	$\lambda$	1	1
Fr	1	1	$\lambda^{1/2}$	1
Re	1	$\frac{1}{\lambda}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^{3/2}}$
$\sigma$	$\frac{1}{\lambda^2}$	1	1	$\lambda$
W	$\lambda^{1/2}$	$\frac{1}{\lambda^{1/2}}$	$\frac{1}{\lambda^{1/2}}$	$\frac{1}{\lambda}$
M	$\lambda$	1	1	$\frac{1}{\lambda^{1/2}}$

## ADVANTAGES AND DISADVANTAGES OVER CONVENTIONAL METHODS

It should be obvious from Table II that all five non-dimensional numbers cannot be satisfied simultaneously no matter what method is used. Thus, which method is to be used depends basically on which of the five numbers are considered to be more important than others. And from this viewpoint, the present approach must be considered the best. Rows (5th to 9th) in Table II are arranged according to the order of what was considered to be more important.

The cavitation number may be adjusted by having the tank pressurized. Hence, the problem is not difficult to solve. The Weber number is proportional to  $\lambda^{\frac{1}{2}}$ . This means the surface tension effect is deamplified in relation to the inertia effect, and this is perhaps not an objectionable matter. In all other methods listed, viscous force effect and surface tension effect are exaggerated in the model and the elimination of this in the present method must be considered as an advantage. Mach number is not considered to be important as the bulk modulus of water is very large.

Another advantage of the present device over that of the Grumman tank is that the free surface form need not be changed during a test program if  $\omega_1$  is held constant. Also, if the g-field in the tank is varied, the present method allows a scale effect study with respect to Reynolds numbers with a single model. This must be considered to be one of the strongest points of the proposed method. Over the years, a theory has been advanced that there is a specific frictional resistance relation with respect to Reynolds number for a geosim series (geometrically

similar model series) involving free surface. If such a relationship can be found, the extrapolation of model data to prototype would be greatly simplified according to the theory. The drawback of this technique, which is known in naval architecture as Telfer's method of extrapolation (Ref. 5), has been that one must build many models of various sizes and run a large number of experiments in order to find such a relationship.

This is not only costly but also hard to ensure a reliable comparison because of technical difficulties in manufacturing similar models and making tests with a uniform standard. The present device, although it involves certain problems of its own, can establish such a relationship for a great deal less money and certainly with less ambiguity.

The fact that the free surface is cylindrical and that the gravitational field is varying linearly resulting in a stratification of gravity, are the two major shortcomings of the present device. These adverse effects can be lessened by making the radius  $R_1$  of the tank T large (Figure 1). Curved models to fit the curvature of fluid also help in the first effect. It is incidental to note that a test involving stratified fluid can be conducted in the proposed device.

If a model is continuously circulated in a tank, this creates a current in the direction of the model movement. If the model is to attain a high speed, either the tank must be large or a system to control this current must be provided. Such a system can be worked out, however.

Another disadvantage is that the forces involved are large in this device and therefore the structure must be sturdy, or it imposes upper limit on the attainable model speed.

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