Winter, 1990

SCALE AND DIMENSION: THEIR LOGICAL HARMONY

Sandra Lach Arlinghaus

"Large streams from little fountains flow,
Tall oaks from little acorns grow."
David Everett, *Lines Written for a School Declamation.*

Introduction.

Until recently, the concept of "dimension" was one that brought "integers" to mind to all but a handful of mathematicians [Mandelbrot, 1983]; a point has dimension 0, a line dimension 1, an area dimension 2, and a volume dimension 3 [Nystuen, 1963]. When a fourth dimension is added to these usual spatial dimensions, time can be included, as well. Indeed, much "pure" mathematics takes place in abstract n–dimensional hypercubes, where n is an integer. Geographic maps, globes (and other representations of part or of all of the earth), are traditionally bounded by these integral dimensions, as well; map scale is expressed in discrete, integral units. Often, however, it is the case in geography as it is in mathematics, that a change in scale, or in dimension, runs across a continuum of possible values. In either case, discrete regular steps are usual as benchmarks at which to consider what the continuing process looks like at varying stages of evolution. As fractal geometry suggests, however, this need not be the case.

Within an integral view of scale or dimension, there are logical and perceptual difficulties in jumping from one integral vantage point to another: Edwin Abbott [1955] has commented on this in his classic abstract essay on "Flatland," and more recently, Edward Tufte has done so in the real–world context of "envisioning information" [1989].

Methods for dealing with these dimensional–jump difficulties abound, particularly in the arts [Barratt, 1980]. In a musical context Charles Wuorinen sees composition as a process of fitting "large" musical forms with scaled–down, self–similar, equivalents of these larger components in order to introduce richness of detail to the theme [NY Times, 1990]. Maurits Escher, in his "Circle Limit" series of tilings of the non– Euclidean hyperbolic plane, uses tiles of successively smaller size to suggest a direction of movement—that of falling off an edge or of being engulfed in a central vortex. A gastronomic leap sees a Savarin as self–similar to a Baba au Rhum [Lach, 1974]; indeed, even more broadly, Savarin himself is purported to have said, "You are what you eat." Rupert Brooke (in "The Soldier") captured this notion poetically, in commenting on the possible fate of a soldier in a distant land:

"If I should die, think only this of me;
that there is some corner of a foreign field
that is forever England."

In the end, Brooke's "Soldier" becomes 'place'.

The fractal concept of self–similarity can be employed to suggest one way to resolve difficulties in scale changes as one moves from dimension to dimension. At the theoretical level, symbolic logic classifies logical fallacies that may, or may not, emerge from scale shifts. When self–similarity is viewed in this sort of logic context, the outcome is a "Scale Shift Law." What is presented here are the abstract arguments; it remains to test empirical content against these arguments.

Logical fallacies.

A question of enduring interest in geography, and in other social sciences, is to consider what can be said about information concerning individuals of a group when given information only about characteristics of the group as a whole. When an attribute of the whole is **erroneously** assigned to one or more of its parts, the logic of this assignment falters. In the social scientific literature, this is generally referred to as commission of the so–called "ecological" fallacy; because the symphony played poorly does not necessarily mean that each, or indeed that any, individual musician did so. In this circumstance, it is simply not possible to assign any truth value, derived from principles of symbolic logic, to the quality of the performance of any subset of musicians (based only on the quality of the performance of the whole orchestra) [Engel, 1982].

It is natural, however, to look for a cause for the poor performance, and indeed to consider some "middle" position that asks to what extent the performance of the orchestra is related to the performance of its individual members. It is this sort of search for finding and measuring the extent of relationship that is the hallmark of quantitative social scientific effort, much of which appears to have been guided [Upton, 1990], in varying degree, by an early effort to determine the extent to which race and literacy are related [Robinson, 1950].

A fallacy, in a lexicographic sense might be "a false idea" or it might be of "erroneous character" or "an argument failing to satisfy the conditions of valid or correct inference" [Webster, 1965]. In a formal logic sense, a fallacy is "a 'natural' mistake in reasoning" [Copi, 1986, p. 4] or it is an argument that fails because its premisses do not imply its conclusion; it is an argument whose conclusion **could** be (but is not necessarily) false even if all of its premisses are true [Copi, 1986, p. 90].

Viewed in this manner, the so–called "ecological" fallacy is nothing different; it is merely a restatement of the "fallacy of division" of classical elementary symbolic logic. The fallacy of division is committed by assigning, **erroneously**, the attributes of the whole to one or more of its parts [Copi]. Thus, it may or may not be valid to make an inference about the nature of a part based on the nature of the whole. That is, sometimes the assignment of truth value from whole to part, in jumping across the dimensional scale from whole to part, is a reasonable practice, and sometimes it is not. The key is to determine when this practice is reasonable, when it is not, and when it simply does not apply. Commission of this fallacy is frequently the result of confusing terminology which refers to the whole ("collective" terms) with those which refer only to the parts ("distributive" terms) [Copi, 1986].

The fallacy of division exists within an abstract human system of reasoning based on the Law of the Excluded Middle: in this Law, a statement is true or false—not some of each. There is "black" and "white," but no "gray" in this system. Statistical work that stems from this fallacy seeks, when it rests on finding correlations, relations that blend "black" and "white"—the foundation in "logic" is thus ignored. This fallacy is examined, here, with an eye to understanding the logical circumstances under which such assignment might, or might not, be erroneous (when it applies).

Scale and dimension.

To understand when the assignment of characteristics from whole to part (division), or from part to whole (the fallacy of composition—the string sections played well, therefore the symphony played well), might be erroneous, it is useful to consider what are the fundamental components composing these fallacies. The notion of scale is involved in the consideration of "whole" and "part." When is the individual a "scaled–down" orchestra; or, when is the orchestra a "scaled–up" individual? The notion of dimension is also involved. When does the zero–dimensional musician– point spread out to fill the two–dimensional (or three–or more–dimensional) orchestra; or, when does the higher dimensional orchestra collapse, black–hole–like, into the single performer. The performing soloist can dominate the orchestra; the conductor perhaps does dominate the orchestra; yet, the orchestra itself is composed of numerous single performers who do not dominate.

Self–similarity and scale shift.

Integral dimensions, with discrete spacing separating them, might be viewed as simply a set of positions marking intervals along a continuum of fractional dimensions [Mandelbrot, 1983]. When the discrete set of integral dimensions is replaced by the "dense" set of fractional dimensions (between any two fractional dimensions there is another one), what happens to our various relative vantage points and to scale problems associated with them?

Abstractly, the relationship is not difficult to tie to logic, under the following fundamental assumption.

Fundamental Assumption.

When two views of the same phenomenon at different scales are self–similar one can properly divide or compose these views to shift scale.

The whole can be divided "continuously" through a "dense" stream of fractional dimensions until the part is reached (and in reverse). Self–similarity suggests a sort of dimensional stability of the characteristic or phenomenon in question. One commits the Fallacy of Division ("Ecological" Fallacy) when the attributes (terminological or otherwise) of the whole are assigned to the parts that are **not** self–similar to the whole. One commits the Fallacy of Composition when the attributes of the parts are assigned to a whole that is **not** self–similar to these parts. This notion is evident in the many animated graphic displays of the Mandelbrot (and other) sets in which zooming in on some detail presents some sort of repetitive sequence of views (in the case of self–similarity, this sequence has length 1). More formally, this idea may be cast as a "Law."

Scale Shift Law

Suppose that the attributes of the whole (part) are assigned to the part (whole).

1. If the whole and the part **are not** self– similar, then that assignment **is** erroneous; and, conversely (inversely, actually),
2. If the whole and the part **are** self–similar, then that assignment **is not** erroneous.

This is one way to look at the "part–whole" dichotomy; physicists wonder about splitting the latest "fundamental" particle; philosophers search for fundamental units of the self [Leibniz, monadology, in Thompson, 1956; Nicod, 1969]; topologists worry about what properties a topological subspace can inherit from its containing topological space [Kelley, 1955].

References.

Abbot, Edwin A. (1956) "Flatland." reprinted in *The World of Mathematics*, James R. Newman, editor. New York: Simon and Schuster.

Barratt, Krome (1980) *Logic and Design: The Syntax of Art, Science, and Mathematics.* Westfield, NJ: Eastview Editions, 1980.

Copi, Irving M. (1986) *Introduction to Logic.* Seventh Edition. New York: Macmillan Publishing Company, (first edition, 1953).

Engel, S. Morris (1982) *With Good Reason: An Introduction to Informal Fallacies.* Second Edition. New York: St. Martins Press.

Kelley, John L. (1963) *General Topology.* Princeton: D. Van Nostrand.

Lach, Alma S. (1974) *The Hows and Whys of French Cooking*, Chicago: The University of Chicago Press.

Mandelbrot, Benoit (1983) *The Fractal Geometry of Nature.* San Francisco: Freeman.

Nicod, Jean (1969) *Geometry and Induction: Containing 'Geometry in the Sensible World' and 'The Logical Problem of Induction' with Prefaces by Roy Harrod, Bertrand Russell, and Andre Lalande.* London: Routledge and Kegan Paul, New translation.

Nystuen, John D. (1963) "Identification of some fundamental spatial concepts." *Papers of Michigan Academy of Letters, Sciences, and Arts.* 48: 373-384.

Robinson, W. (1950) Ecological correlations and the behavior of individuals, *American Sociological Review.* 15: 351-357.

Rockwell, John (1990) "Fractals: A Mystery Lingers." Review/Music, *The New York Times*, Thursday, April 26.

Thompson, D'Arcy Wentworth (1956) "On Magnitude." In *The World of Mathematics*, James R. Newman, Editor. New York: Simon and Schuster.

Tufte, Edward (1989) *Envisioning Information.* Cheshire, CT.

Upton, Graham J. G. (1990) "Information from Regional Data," in *Spatial Statistics: Past, Present, and Future*, edited by Daniel A. Griffith. IMaGe Monograph, #12. Ann Arbor: Michigan Document Services.

Webster's Seventh New Collegiate Dictionary (1965) Springfield, MA: G. and C. Merriam Company.