4. REGULAR FEATURES

Theorem Museum —

One purpose of a museum is to display to the public concepts of an enduring character in some sort of hands–on manner that will promote grasp and retention of that concept. When the display also piques the interest of the observer, so much the better.

This particular feature is motivated by a variety of sources. About ten years ago, William E. Arlinghaus and I submitted a proposal to *The Mathematical Intelligencer* for a museum exhibit, based on constructing a giant Rubik's (trademarked name) Cube, to teach people elements of group theory by carrying them physically (in Ferris wheel fashion) through group theoretic motions while riding inside the cube. At the same time, I also submitted another proposal to the same journal for another museum exhibit to be called "The Garden of Shadows." This was to be an outdoor display based on using the sun as a point source of light at "infinite" distance to physically demonstrate a number of theorems from projective geometry.

A number of years later, I came to know fine artist David Barr who specializes in large outdoor sculpture. Bill Arlinghaus and John Nystuen are continuing participants at my IMaGe meetings; over the years others have joined us, and one of the most regular is David Barr. Often, we have, as a group, discussed various aspects of using outdoor sculpture to educate the public as well as colleagues. John Nystuen suggested that we build an actual, physical Theorem Museum, dedicated to Theorems that could be portrayed in sculpture (similar to the *Intelligencer* proposals). Barr informs us that interest in this sort of idea is well–established in the world of Art: Swiss artist Max Bill, and other Western European painters and sculptors, create art determined by mathematical equations of various sorts. Here, we are suggesting that it is the theorem, itself, that is art. This feature is therefore the written groundwork for such a museum. If you have a favorite theorem, and can suggest how to express it physically using artistic media, you might want to consider submitting it to *Solstice* for this section. Theorems that can be so envisioned may also be ones that are easiest to mold to fit other real– world phenomena. Projective geometry is a highly general geometry that is perfectly symmetric in its statements. The reason for this is that "parallel" lines meet in "ideal" points, lying on an "ideal" line, at infinity. Thus, in the projective plane, as in the Euclidean plane, two points determine a line; however, in the projective plane a dual statement (that is NOT true in the Euclidean plane) that two lines determine a point is also true. Duality in language results in symmetry of form. Here is a remarkable theorem from projective geometry (see reference for proof).

Desargues's Two Triangle Theorem.

Given two triangles, PQR and $P'Q'R'$ such that PP', QQ', and RR' are concurrent at point O. It follows that the intersection points of corresponding sides of the two triangles are collinear. That is, suppose that corresponding sides PQ and $P'Q'$ intersect at point L, that QR and $Q'R'$ intersect at point M, and that PR and $P'R'$ intersect at point N. Then, the points L, M, and N all lie along a single straight line (please draw your own figure from these directions).

From a geographic viewpoint, this says that if a rigid tetrahedron were built of metal rods with apex at point O, that any two triangles that fit perfectly inside this structure would have this property. One triangle "projects" from a point (as for example in gnomonic

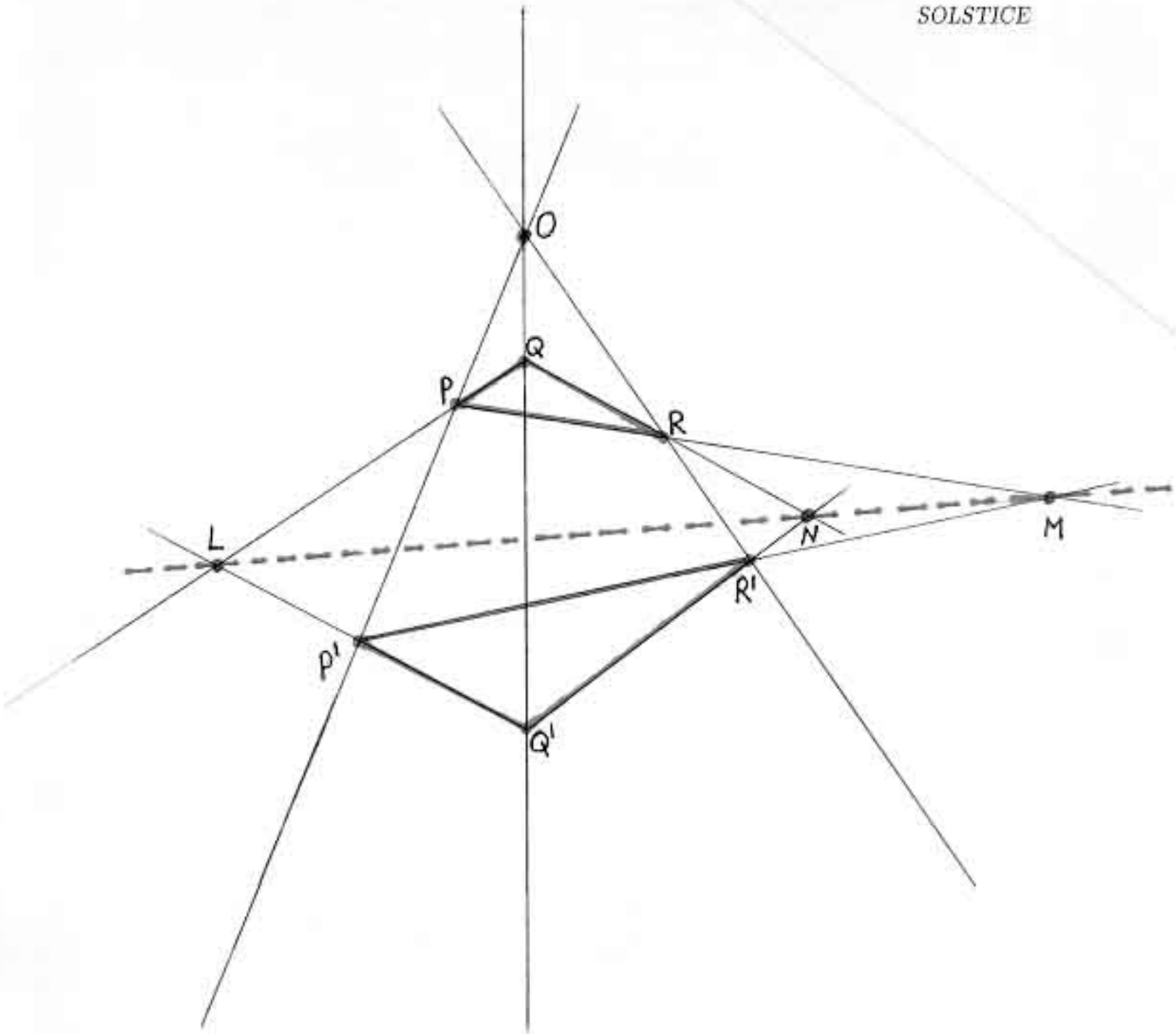


Figure to accompany Desargues's Two Triangle Theorem

or stereographic map projection) to the other. This might suggest a way to deform cells of a triangulation of a region of the earth into one another in such a way that this Desargues's line serves as some sort of an invariant of the deformation.

This observation might then make one wonder what sorts of geometries exist that do not obey Desargues's Theorem. There is a whole class of "Combinatorial geometries" or finite

projective planes that do not.

References

Coxeter, H. S. M. *Introduction to Geometry*, New York: Wiley, 1961.

Coxeter, H. S. M. *Projective Geometry*, Toronto: University of Toronto Press, 1974.

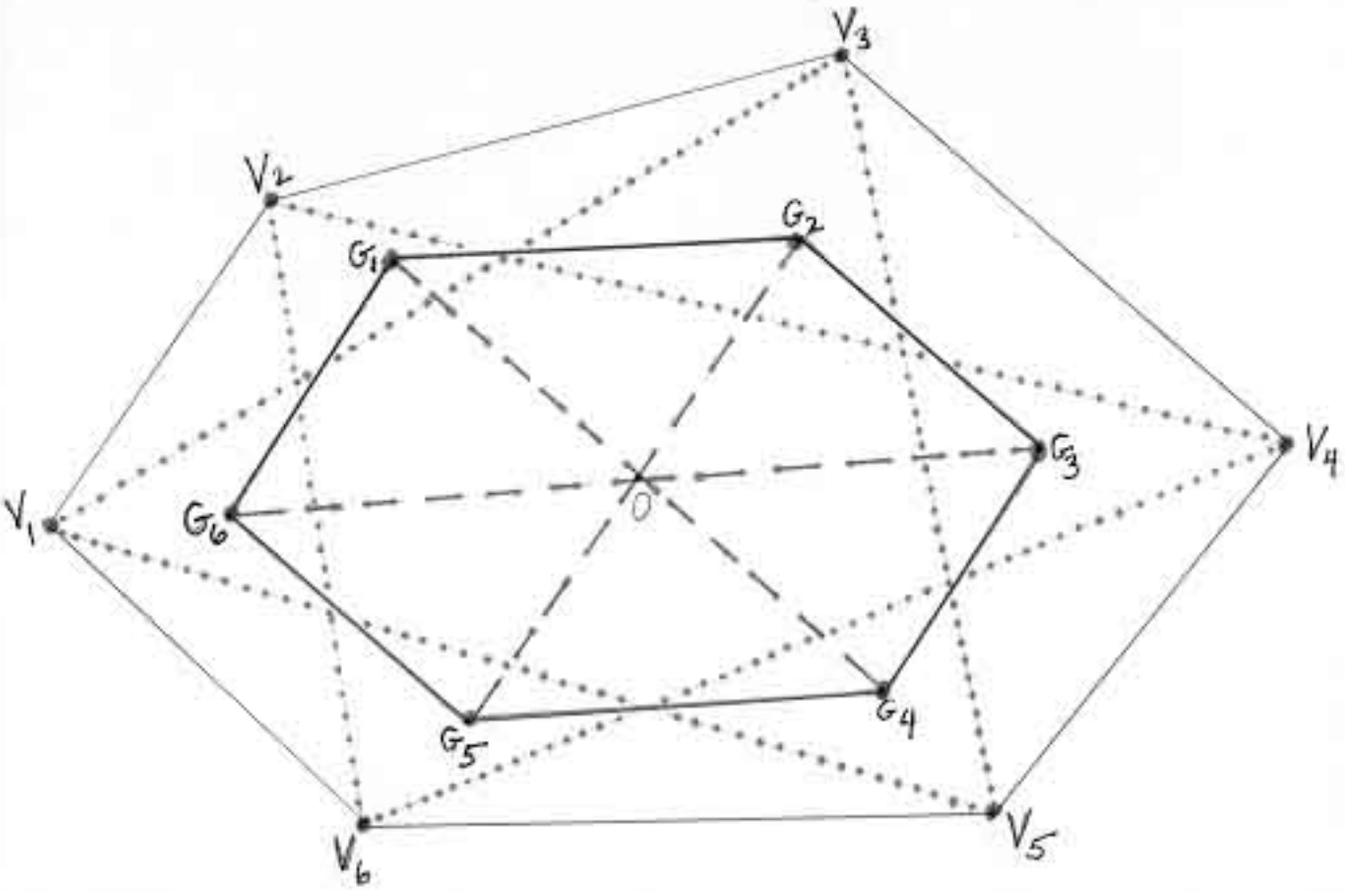


Figure to accompany construction of centrally symmetric hexagon.

Construction Zone —

One possible direction for application of Desargues's Theorem is to deform one tesselation of a region into another, leaving something invariant. Another related issue with tesselations is to try to regularize a tesselation composed of irregularly shaped cells. The following construction shows how to derive a centrally symmetric hexagon from an arbitrary convex hexagon. Given an arbitrary convex hexagon, $V_1V_2V_3V_4V_5V_6$. Join alternate vertices to inscribe a six-pointed star within this hexagon—that is, draw lines V_1V_3, V_2V_4, V_3V_5, V_4V_6, V_5V_1, V_6V_2 (it is suggested that you do so on a separate sheet of paper, at this point).

This produces six distinct triangles (of interest here–of course there are more):

$$\triangle V_1V_2V_3; \quad \triangle V_2V_3V_4; \quad \triangle V_3V_4V_5; \quad \triangle V_4V_5V_6; \quad \triangle V_5V_6V_1; \quad \triangle V_6V_1V_2.$$

To find the center of gravity of any triangle, find the point at which the medians are concurrent (the median is the line joining a vertex to the midpoint of the opposite side). This point is the center of gravity. Find the centers of gravity

$$G_1, \quad G_2, \quad G_3, \quad G_4, \quad G_5, \quad G_6$$

of each of the triangles distinguished above (in the order suggested). The hexagon determined by these centers of gravity will be centrally symmetric. That is, opposite sides will be equal in length and parallel to each other:

$$G_1G_2 \parallel G_4G_5; \quad |G_1G_2| = |G_4G_5|;$$

$$G_2G_3 \parallel G_5G_6; \quad |G_2G_3| = |G_5G_6|;$$

$$G_3G_4 \parallel G_6G_1; \quad |G_3G_4| = |G_6G_1|.$$

Another way of visualizing the symmetry is to observe that the three lines joining G_1G_4, G_2G_5, G_3G_6 are concurrent at a single point (call it O). In this way, one might also determine a "center" for this symmetric hexagon which might then serve as a point to which a reference value might be attached for the arbitrary hexagon from which it was derived. This centrally symmetric hexagon is called the Dirichlet region of the arbitrary convex hexagon. This construction can be proved using Euclidean geometry (if requests come in, I'll put it in a later issue).

This feature is based on discussions in

Kasner, Edward, and Newman, James R. "New names for old," in *The World of Mathematics*, edited by James R. Newman, Volume III, 1996-2010. New York: Simon and Schuster, 1956.

Coxeter, H. S. M. *Introduction to Geometry*, New York: Wiley, 1961.

Reference Corner —

Point set theory and topology. A recent pleasant evening spent with Hal Moellering had him questioning me and Bill Arlinghaus as to what might be reasonable, or useful, references from which graduate students in geography could get some sort of grasp of the elements of point set topology. A few references are listed below; send in your favorites and they will be printed next time. Hope that mathematicians as well as geographers will do so. Future topics to include graph theory and number theory as well as others suggested by reader input. Thanks Hal for the idea (generated by your questions) of doing this feature!

Some long-time favorites and classics:

Dugundji, James. *Topology*. Boston: Allyn and Bacon, 1960.

Hall, Dick Wick and Guilford L. Spencer II, *Elementary Topology*, New York: Wiley, 1955.

Halmos, Paul R., *Naïve Set Theory*, Princeton: D. Van Nostrand, 1960.

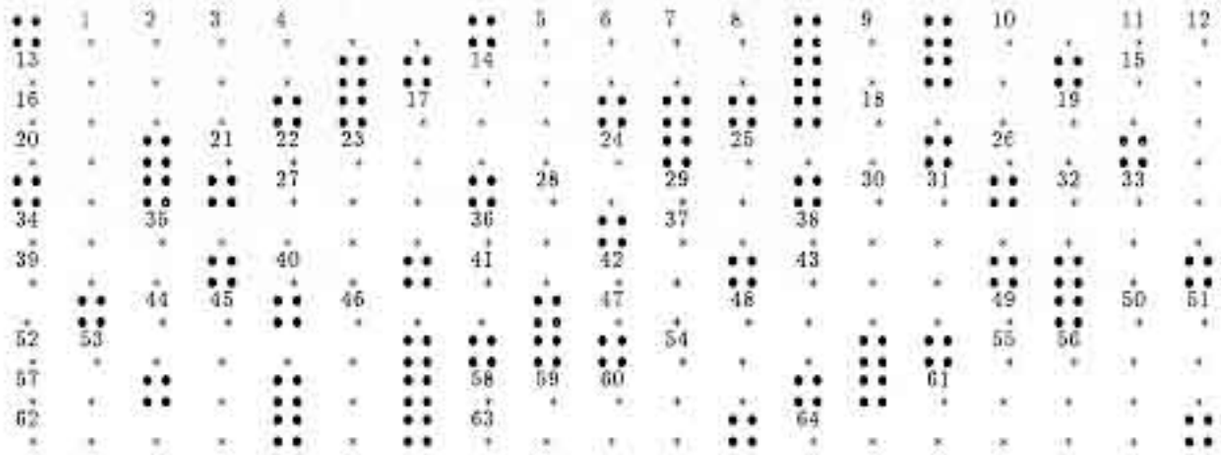
Hausdorff, Felix, sl Mengenlehre, Berlin: Walter de Gruyer, 1935.

Hocking, John G. and Gail S. Young, *Topology*, Reading: Addison–Wesley, 1961.

Kelley, John L., *General Topology*, Princeton: D. Van Nostrand, 1955.

Landau, Edmund, *Grundlagen der Analysis*, New York: Chelsea, 1946. Third edition, 1960.

Mansfield, Maynard J. *Introduction to Topology*, Princeton: D. Van Nostrand, 1963.



Games and other educational features —

Crossword puzzle.

The focus of this puzzle is on herbs and spices. Spice trade has helped to shape many geographic alignments and spices such as pepper, known from its preservative characteristic, helped make long voyages possible. Puzzles should be fun; they should also stimulate thought and offer some sort of educational value. If you think that this puzzle might be of use to your students in this capacity, feel free to copy it from this page. Think of the asterisks as the blank squares, or as tiles with letter on the other side. Each set of four bullets represents a black square.

ACROSS

1. Plant of the Capsicum family, native to the Americas. Good source of vitamins A and C. Some varieties are native to Tabasco in Mexico.
5. Fruit native to the Americas is the prickly —.
10. Powder made from young sassafras leaves that is essential in making creole gumbo.
13. The "royal" herb – often the dominant herb in Pesto.
14. Bush–bud often seen in Tartare sauce or with an anchovy coiled around it.
15. Hour – abbreviation.
16. College of Liberal —.
17. A fundamental tool of the geographer and of the mathematician.
18. U.S. state – remove one letter from the spice in 47 across to form an anagram of this state name.
20. United States – abbreviation.
21. Jumble of letters in "another."
25. Black, sticky substance.
26. Eastern Uganda – abbreviation.
27. The bran of this grain is much in vogue.
28. Unit on a ruler.

30. He, she, —.
32. Herb sometimes used in fruit cup. Can cause severe allergic reactions. Also: French for street.
34. Along with coriander or cumin, this is a dominant ingredient in many curries.
37. A plant extract from which candies can be made.
39. In humans, the color blue, for this, is a recessive genetic trait.
40. Senior – abbreviation.
41. Spice with flavor close to nutmeg.
43. Chronological or mental —.
44. Association of American Geographers: —G.
46. "A poem should be palpable and mute; As a globed fruit," from Archibald MacLeish's "— Poetica."
47. Often found in Italian sauces.
50. Fifth and sixth letters of the alphabet used in English.
52. Spice often ground freshly and sprinkled on eggnog.
54. Eau de —.
55. Noise a lion might make.
57. First two letters of Spanish for United States.
58. Jumble of the letters in the name of an herb with a licorice flavor.
61. Word that might describe the flavor of a julep (adjectival form).
62. This broadleaf "big onion" is a key ingredient in Vichyssoise.
63. Herb used in many pickled cucumbers.
64. "Spiced-up" multiplication tables might be called "—" tables.

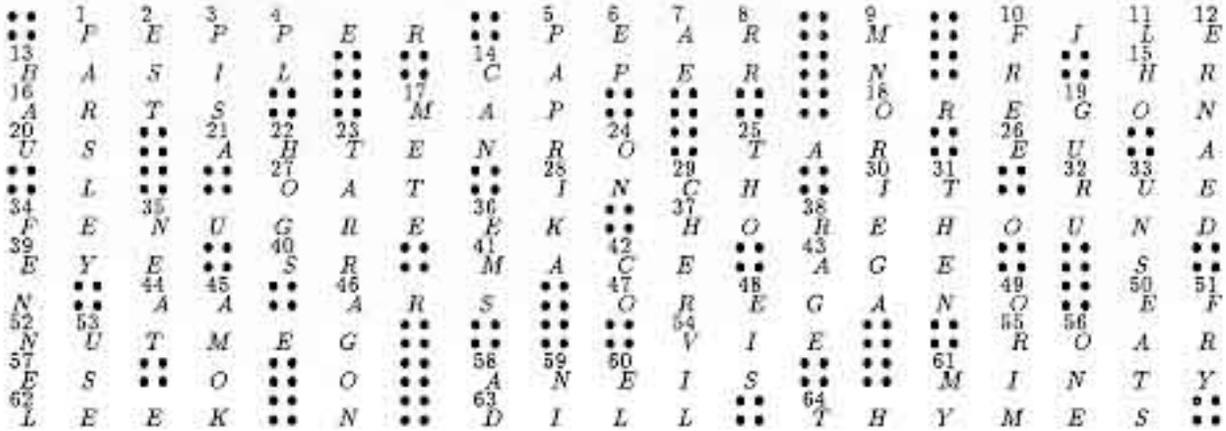
DOWN

1. This herb supposedly has the power to destroy the scent of garlic and onion.
2. East, in French.
3. Italian city – home to Fibonacci.
4. Postal letter (abbreviation)
5. Orangish powder often association with Hungarian dishes.
6. East Prussia (abbreviation).
7. Almost everywhere (mathematical term – abbreviation).
8. Railroad (abbreviation).
9. First initial and last name of former Panamanian leader.
10. A complimentary copy is a — one.
11. Left hand opponent (duplicate bridge term, abbreviation).
12. Jumble of the word "neared."
17. "—s and bounds."
19. Spiritual guide in Hinduism.

22. Poland China is a variety of these.
23. This herb is often held in vinegar because its leaf veins stiffen when dried and do not resoften when cooked. "Estragon" in French.
24. "— A Clear Day"
25. "Though" – some newspapers spell that word in this way.
29. This herb loses most of its flavor when dried: "Pluches de cerfeuil" refers to sprigs of this herb.
31. If/"—": typical manner in which a theorem is stated.
33. Removes from political office.
34. One variety of this herb, often used in conjunction with fat fish and lentils, is known as Florence —.
35. Tidy.
36. Paramedic vans are often marked with these three letters.
38. Uncontrolled anger.
42. Company (abbreviation)
45. Running — (Malay word). To be in a violently frenzied state.
48. Fine German white wine made from grapes harvested after frost: —wein.
49. Oyster Research Institute of Michigan, might be abbreviated thus.
51. Popular description of wok cookery: stir—.
53. Employ.
56. Identity element of the integers under multiplication.
58. Anno Domini (abbreviation)
59. National income (abbreviation)
60. Elevated train (abbreviation) – forms "Loop" in Chicago.
61. Prefix meaning "muscle."

Coming attractions —

Feigenbaum's number
Pascal's theorem from projective geometry
Braikenridge–MacLaurin construction for a conic in the projective plane.



Crossword puzzle solution

5. SAMPLE OF HOW TO DOWNLOAD THE ELECTRONIC FILE

This section shows the exact set of commands that work to download this file on The University of Michigan's Xerox 9700. Because different universities will have different installations of TEX, this is only a rough guideline which *might* be of use to the reader.

This document prints out to be about 50 pages; on UM equipment, there are varying rates at varying times of day. At the minimum rate, the cost to print this out, using TEX, is about six dollars.

ASSUME YOU HAVE SIGNED ON AND ARE AT THE SYSTEM PROMPT, #. # create −t.tex

percent-sign t from pc c:backslash words backslash solstice.tex to mts −t.tex char notab

[this command sends my file, solstice.tex, which I did as a WordStar (subdirectory, "words") ASCII file to the mainframe]

run *tex par=−t.tex

run *dvixer par=−t.dvi

control *print* onesided

run *pagepr scards=−t.xer, par=paper=plain