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A NOTE ON THE HEAT REQUIRED FOR THERMAL DE-ICING

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SUMMARY

It is suggested that the conventional heat balance written at a plane neglecting the thickness of an accreted ice layer may have to be discarded when heat is applied intermittently. The point of view to be adopted in analyzing thermal effects outside the skin of an iced wing depends on the nature of the criterion for ice separation. The usual criterion (a uniform 32°F temperature at the surface) is questioned, and for the purpose of discussion the point of view is taken that a finite thickness of ice must be melted. The general equations of conduction are written in that case. A special instance of that problem is solved wherein the wing surface temperature is known and uniform, the temperature at infinity is below 32°F and constant, and the heat flux is unidimensional. The ice-water boundary is shown to recede from the surface as the square root of time and at a rate which is increased very little by an increase of the surface temperature. It is shown that large temperature gradients exist in the ice-water layer.

NOMENCLATURE

c	=	specific heat, cal/gram °C
k	=	thermal conductivity; cal/sec-cm-°C
L	=	latent heat of fusion, cal/gram
x,y,z	=	space coordinates, cm
ρ	=	density, gm/cm^3
α	=	thermal diffusivity, cm^2/sec
t	=	time, sec
T	=	temperature, °C
K	=	constant defined by equation 10

INTRODUCTION

In attempting to determine the heat requirements for thermal de-icing, researchers have so far considered a heat balance at the surface of the solid to be de-iced with the assumption that the ice layer was thin enough to be neglected. The results of an analysis of the conduction problem which is raised if the thickness of ice is considered finite, however, point to the possibility that the simplifying assumption may well be questioned. One is led to the following query:

Does an accurate criterion for the separation of the ice from a heated surface exist?

If not, experimental evidence is needed to determine whether a surface temperature of 32°F at the inner surface (solid-ice interface) is sufficient, or if a finite (appreciable) thickness of ice must be melted before ice will separate from the surface. It is obvious that the answer will depend on the shape and surface characteristics of the solid. It seems probable that for a curved surface with a varying radius of curvature, more is required than merely to bring the whole surface to a 32°F temperature. The reason this question is important is as follows: It takes very little expenditure of energy to raise the ice interface temperature to 32°F. In that case most of the heat provided will be used to raise the temperature of the structure to be heated and the metal parts neighboring the heat ducts or elements. The problem then consists primarily in approximating the heat flow and temperature distribution inside the wing or heated part and out to the surface.

However, if in addition an appreciable thickness of ice must be melted, preliminary analysis indicates the possibility of large temperature gradients within the melted ice layer which would change the heat requirements considerably. In addition, the point of view one must adopt in the latter case is quite different. The problem from the surface outward must be considered as a heat conduction problem in a slab of ice, or in general in two layers, one of water and one of ice, with an interface whose ordinate is a function of time and with appropriate boundary conditions at the boundary layer. The heat balance can no longer be written <u>at the surface</u>, and temperature and heat-transfer transients become important. In this case, i.e., if what need be known is the rate of melting of ice as a function of time, the important parameters seem to be merely surface heat transfer and initial temperature of the ice before the heating cycle begins.

Furthermore, it may very well be that the boundary conditions (at the boundary layer) do not affect the conduction problem during the short "on" cycle appreciably, and that they can therefore be simplified.

While we have begun to consider this conduction problem, and some results are stated below, we believe there is a need to study the physics of ice removal experimentally as soon as possible. A few simple tests might indicate if, for an airfoil, a finite layer of ice must be melted, while much more elaborate tests would be required to obtain quantitative information on that point. It seems worth while for us to conduct the simpler kind of tests in the near future, while the more elaborate ones would be entrusted to some other groups.

A PRELIMINARY SURVEY OF THE HEAT-CONDUCTION PROBLEM

We assume that the removal of ice by de-icing (as contrasted to anti-icing) involves the conduction problem mentioned above. We believe that even if a surface temperature of 32°F were sufficient to rid a surface of ice, the temperature distribution throughout the ice layer should be considered. The conduction problem must be stated differently for two different cases.

(a) No Melting Occurs

In this case, the temperature T at any point is given by

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) \quad . \qquad (1)$$

where $\alpha = k/\rho c$ = diffusivity;
k = thermal conductivity,
ρ = density of ice (all of this for ice), and
c = specific heat, with the following conditions:

1. Initial Conditions:

Limit of $T = f(x,y,z)$ = initial temperature distribution.
$t \rightarrow o$

Boundary conditions at the solid surface:

$$\text{either } T_o = f_1(t)$$
$$\text{or } \frac{\partial T_o}{\partial \eta} = f_2(t) \quad ,$$

where η is distance perpendicular to the solid surface.

2. At the edge of the ice layer $\partial v/\partial \eta$ will be a function of the boundary-layer characteristics. Since the boundary-layer characteristics are dependent on $\partial v/\partial \eta$ at the edge of the ice layer, the conduction problem and the boundary-layer problem (involving convection, fusion, sublimation, addition of kinetic energy, etc.) must be solved simultaneously. However, it may be possible to approximate boundary-layer behavior a priori and to impose a boundary condition at the edge of the boundary layer to solve the conduction problem and to verify by iteration whether the imposed conditions were realistic. It may even be permissible to simplify this boundary condition further in the cases where this condition has no critical influence on the solution of the problem of conduction in the ice slab.

(b) Melting Occurs

In this case, the temperature T_1 at any point in the thin water layer is given by

$$\frac{\partial T_1}{\partial t} = \alpha\left(\frac{\partial^2 T_1}{\partial x^2} + \frac{\partial^2 T_1}{\partial y^2} + \frac{\partial^2 T_1}{\partial z^2}\right),$$

where the properties are now those of water. The temperature T_2 in the ice layer is given by:

$$\frac{\partial T_2}{\partial t} = \alpha\left(\frac{\partial^2 T_2}{\partial x^2} + \frac{\partial^2 T_2}{\partial y^2} + \frac{\partial^2 T_2}{\partial z^2}\right);$$

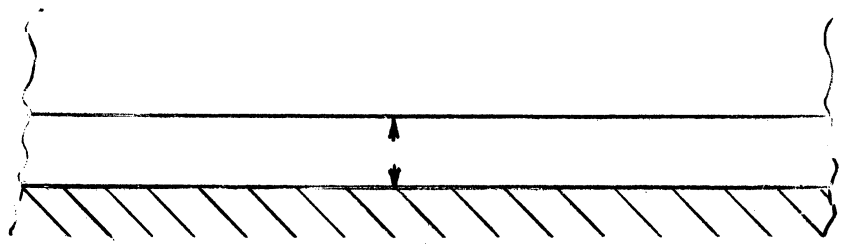
the properties here are those of ice. The initial condition and conditions on the solid surface and at the edge of the boundary layer are given as before, but in addition the condition which has to be met at the interface between water and ice is

$$k_1 \frac{\partial T_1}{\partial \eta} - k_2 \frac{\partial T_2}{\partial \eta} = L\rho \frac{dx}{d\eta},$$

where η is the direction perpendicular to the interface, L is the latent heat of fusion, ρ is the density, and X is the coordinate of the water-ice interface.

A Particular Case*

A solution is now given of a particular case of (b). The solid surface is an infinite flat plate; the problem is one-dimensional (infinite slabs of water and ice); the temperature at infinity in the x direction is constant as is the temperature on the solid surface. The initial conditions are as yet undetermined.



The problem is stated:

$$\frac{\partial^2 T_1}{\partial x^2} - \frac{1}{\alpha_1}\frac{\partial T_1}{\partial t} = o \qquad o < x < X \tag{2}$$

$$T_1 = T_o \qquad x = o \tag{3}$$

$$\frac{\partial^2 T_2}{\partial x^2} - \frac{1}{\alpha_2}\frac{\partial T_2}{\partial t} = o \qquad x > X \tag{4}$$

$$T_2 \longrightarrow - T_\infty \qquad x \longrightarrow \infty \tag{5}$$

At the ice-water boundary: $T_1 = T_2 = o; \; x = X$; (6)

and

$$k_1 \frac{\partial T_1}{\partial x} - k_2 \frac{\partial T_2}{\partial x} = - L\rho_1 \frac{dx}{dt} \quad . \tag{7}$$

A solution of the type

$$T_1 = T_o + A \text{ erf. } \frac{x}{2\sqrt{\alpha_1 t}} \tag{8}$$

* See Carslaw and Jaeger, Conduction of Heat in Solids, p. 71.

A solution of the type

$$T_2 = - T_\infty + B \operatorname{erfc}. \frac{x}{2\sqrt{\alpha_2 t}} \qquad (9)$$

satisfies (4) and (5). At $x = X$, (6) requires:

$$A \operatorname{erf} \frac{X}{2\sqrt{\alpha_1 t}} = - T_o \qquad B \operatorname{erfc} \frac{X}{2\sqrt{\alpha_2 t}} = + T_\infty. \qquad (10)$$

X is a function of time, and since (10) must be true at any time $t > o$,

$$X = Kt^{1/2} , \qquad (11)$$

where K is as yet undetermined. K is found from a numerical solution of (7), which becomes

$$\frac{T_o k_1 e^{-k^2/4\alpha_1}}{\sqrt{\pi \alpha_1} \operatorname{erf} \left[K/2(\alpha_1)^{1/2}\right]} - \frac{T_\infty k_2 e^{-k^2/4\alpha_2}}{\sqrt{\pi \alpha_2} \operatorname{erfc} \left[K/2(\alpha_2)^{1/2}\right]} = - \frac{L\rho_1 K}{2} .$$

For water and ice, in cgs units,

$k_1 = .00144$ cal/sec-cm-°C $\qquad k_2 = .0053$ cal/sec-cm-°C

$\alpha_1 = .00144$ cm^2/sec $\qquad \alpha_2 = .0115$ cm^2/sec

$L\rho_1 = 73.5$ cal/cm^3

K is plotted in Fig. 1 as a function of surface temperature (T_o) and temperature at infinity (T_∞).

From K, A and B can be found in (10) and the problem is solved. (The initial condition is found to be a uniform temperature, $- T_\infty$.) It is seen that in order to increase substantially the rate of melting at constant surface temperature, T_o, must be increased very rapidly. For a temperature at infinity of - 10°C, a surface temperature of 50°C will melt 0.037 cms in 1 second and 0.074 cms in 4 seconds, but a surface temperature of 100°C will melt only 0.047 cms in 1 second and 0.094 cms in 4 seconds. It can be visualized from this example that during the first few seconds of the

heating cycle very great temperature gradients exist within very thin layers of melted ice. The heat-transfer rate at the surface corresponding to this case varies as $1/\sqrt{t}$.

Because the ice-water boundary acts as a moving heat sink, it is certainly not true that for the case of a heated solid boundary the temperature within the ice layer can be considered uniform. Simple solutions of the type shown above are not valid if the boundary conditions are other than the ones we specified, and before more elaborate and tedious means of solution are attempted it seems desirable to know the answer to the questions raised in the first part of this report.

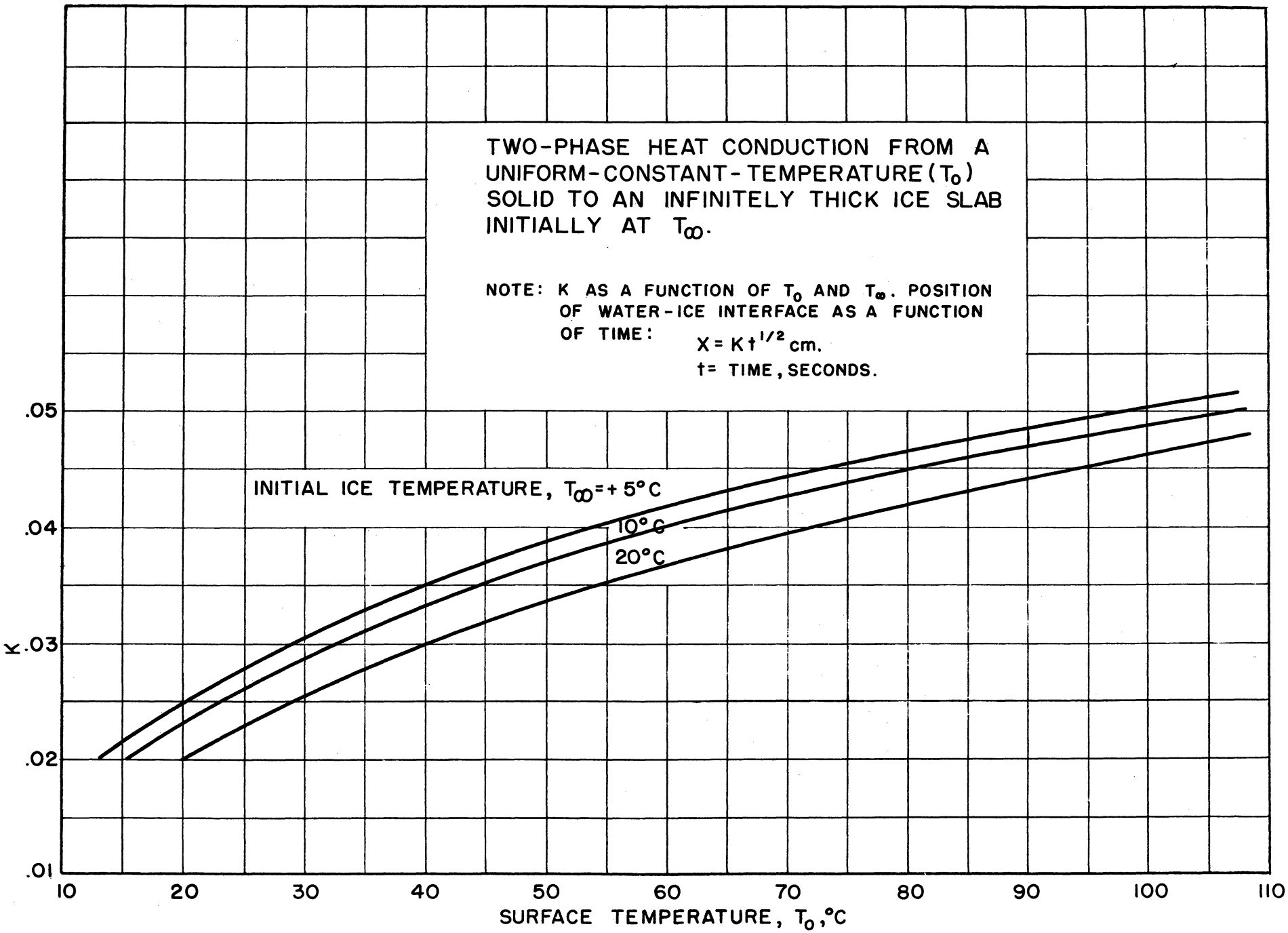

TWO-PHASE HEAT CONDUCTION FROM A UNIFORM-CONSTANT-TEMPERATURE (T_0) SOLID TO AN INFINITELY THICK ICE SLAB INITIALLY AT T_∞.
NOTE: K AS A FUNCTION OF T_0 AND T_∞. POSITION OF WATER-ICE INTERFACE AS A FUNCTION OF TIME: $X = K t^{1/2}$ cm.
t = TIME, SECONDS.
INITIAL ICE TEMPERATURE, T_∞ = + 5°C
10°C
20°C
K
.05
.04
.03
.02
.01
10
20
30
40
50
60
70
80
90
100
110
SURFACE TEMPERATURE, T_0, °C

FIG. 1