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Abstract

This paper shows how lifelong survival-contingent payouts can enhance investor wellbeing in the context of a portfolio choice model which integrates uninsurable labor income and asymmetric mortality expectations. Our model generates optimal asset location patterns indicating how much to hold in liquid versus illiquid survival-contingent payouts over the lifetime, and also asset allocation paths, showing how to invest in stocks versus bonds. We confirm that the investor will gradually move money out of her liquid saving into survival-contingent assets to retirement and beyond, thereby enhancing her welfare by as much as 50 percent. The results are also robust to the introduction of uninsurable consumption shocks in housing expenses, income flows during the worklife and retirement, sudden changes in health status, and medical expenses.

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1 Introduction

This paper presents a dynamic model of rational consumption and portfolio choice over the life cycle, in which uninsurable labor income and asymmetric mortality expectations are permitted to shape both the asset allocation and location decisions. While prior studies have examined how mortality risk can influence investment decisions, this is the first analysis to include real-world life-contingent products that hedge the mortality risk in a realistic calibrated life cycle framework.[1] Specifically, we model financial contracts that permit the investor to trade off asset illiquidity in exchange for an extra return known as the 'survival credit;' at the same time, the individual is permitted to allocate her entire investment menu. Confirming previous findings, we show that the fraction of wealth invested in risky assets will optimally decline with age. But we also show that the investor will gradually move her money out of liquid saving into an illiquid survival-contingent payout account, to take advantage of the survival credit up to and beyond her retirement date. This strategy can enhance welfare by as much as 50 percent.

Solving household financial decision-making problems such as these is complex, inasmuch as they involve long time horizons, stochastic investment opportunity sets, shocks to consumption, and other uncertainties.[2] Recent work has evaluated how asset illiquidity can shape investor behavior, mainly focusing on housing and non-tradable labor income.[3] Less attention has been devoted to examining financial

[1]Prior work includes Dus, Maurer, and Mitchell (2005); Gerrard, Haberman, and Vigna (2004); Horneff, Maurer, Mitchell, and Dus (2008); Kaplan (2006), Kapur and Orszag (1999); Kingston and Thorp (2005); Milevsky (1998); Milevsky and Young (2002); Milevsky, Moore, and Young (2006); Mitchell, Poterba, Warshawsky, and Brown (1999); and Stabile (2003). A handful of researchers compare variable payout life-contingent products to other asset classes on a 'stand-alone' basis: for instance Blake, Cairns, and Dowd (2003) show that an equity-linked variable annuity would appeal to many retirees, as compared to either a simple phased withdrawal plan or a fixed payout annuity. Feldstein and Ranguelova (2001) assume full annuitization at the beginning of retirement with an equity exposure of 60 percent. Brown, Mitchell, and Poterba (2001) demonstrate that variable payout annuities should be invested in inflation-indexed bonds, though they find that pure equity-linked annuities can generate greater utility than real annuities for a broad range of risk aversion parameters. Using Monte Carlo simulation, Milevsky (2002) analyzes the risk/return profile of variable payout annuity payout streams and compares them to fixed and escalating annuities. He concludes that variable payout annuities may hedge inflation better than escalating annuities. Koijen, Nijman, and Werker (2006) focus on the annuity risk during the accumulation period when full annuitization is required at age 65. Interestingly, they report that inflation and interest rate risk have only a marginal impact on the welfare effects; for this reason we do not model inflation and interest rate risk separately in what follows. None of these previous studies focuses on the asset location versus allocation pattern over the entire lifetime, allowing for income, health, and other consumption shocks, as here.

[2]For instance interest rate risk is examined in Brennan and Xia (2000) and Wachter (2003); inflation risk is analyzed by Campbell and Viceira (2001) and Brennan and Xia (2002). Changing risk premiums are considered in Brandt (1999), Campbell and Viceira (1999), Wachter (2002), and Campbell, Chan, and Viceira (2003). The long run implications of stock market volatility have been addressed by Chacko and Viceira (2006).

[3]Relevant prior work includes Cocco, Gomes, and Maenhout (2005); Cocco (2005); Yao and Zhang

products that offer investors the opportunity to give up liquidity in exchange for a survival-contingent premium known as the survival credit. One exception is Richard (1975) who elegantly modeled longevity insurance; however that work did not take into account irreversibility of the longevity product purchase, labor income risk, or borrowing constraints.

In what follows, we define a lifelong survival-contingent product as a financial contract between an insured person and an insurer which, in exchange for an initial premium, pays a regular periodic benefit as long as the purchaser is alive (Brown et al., 2001). The essential attraction of such a payout benefit is that, despite uncertainty about one's remaining lifetime, the investor cannot outlive her assets because she pools longevity risk across all purchasers in the pool (Mitchell et al. 1999). The provider invests the premiums in a portfolio of riskless and risky assets which may be selected and directed by the buyer, and then pays to the retiree an annual stream of income for life. As members of the pool die, the forfeited funds are reallocated among survivors in the pool; this generates the survival credit that rises with age. Of course, buying the payout annuity introduces illiquidity to the investor's asset portfolio, as the initial premium cannot be recovered after purchase.

Theoretical groundwork on annuitization dates back as far as Yaari's (1965) seminal study. Yaari suggested that a rational retiree lacking a bequest motive would annuitize all her assets. In his framework, the investor is exposed only to mortality risk (other sources of risk due to interest rates, stocks, and inflation are omitted). In an important recent extension of that work, Davidoff, Brown, and Diamond (2005) conclude that a retiree will still fully annuitize financial wealth in the presence of a complete market if there is no bequest motive, when the net return on the annuity is greater than that of the reference asset. Partial annuitization could be optimal if the assumption about complete markets is relaxed, or if the investor has a bequest motive. We extend prior literature by endogenizing the annuitization decision and asset allocation of variable payout annuities in a dynamic portfolio choice framework.

Introducing such survival-contingent products into a life cycle framework implies that the investor now must make both asset location and allocation decisions, deciding not only how much of the risky and riskless asset to buy, but also how and when to move into irreversible life-contingent payout products over the lifetime. To this end, we derive an optimal endogenous asset allocation and gradual annuitization strategy for a risk-averse retiree facing a stochastic lifetime with uncertain labor income, who can hold her wealth in riskless bonds or risky stocks. Our paper extends previous work by augmenting the investor's asset menu to include so-called variable

(2005); Damon, Spatt, and Zhang (2001, 2004); and Gomes, Michaelides, and Polkovnichenko (2006).

payout life annuities where payments vary with stock returns. Such annuity purchases are irrevocable, but the investor can optimally rebalance both her liquid and her illiquid portfolios. In this way, we endogenously derive the optimal dynamic asset allocation path over the life cycle, taking into account the potential to gain from the equity premium as well as the survival credit. Sensitivity analysis integrates bequests and loads, as well as uninsurable income shocks (during the work life and after retirement), housing, and medical expenses. Also we analyze the impact of sudden deteriorations in health status.

Our findings may be summarized as follows. The investor will optimally begin purchasing these survival-contingent payout annuities at least by the middle of her worklife, and she will continue to do so until (in expectation) she is fully annuitized in her late 70s. The investor will also hold a large fraction of equities in both her liquid and illiquid accounts when young, while the fraction in equity falls with age; this pattern is consistent with previous studies which have not incorporated the value-added of life contingent holdings. Adding the survival-contingent asset is shown to have a large positive impact on welfare. When the product is priced fairly, a 40-year old investor lacking any bequest motive would be willing to give up as much as half of her liquid wealth to gain access to the life-contingent product. Even with a moderate bequest motive, she would be willing to give up almost one-third of her wealth to gain access to the survival-contingent benefit. In other words, variable payout annuities provide a considerably higher standard of living, and those who survive capture not only the equity premium but also share in the survival credit. Our results are robust to uninsurable shocks in housing expenses, income, and medical expenses, as well as a sudden severe deterioration in health status. The contribution of this article is to solve a realistically calibrated life cycle model of consumption, portfolio location, and portfolio allocation with illiquid variable life annuities while taking into account important uninsurable risk factors. One study which comes closest to ours is Horneff, Maurer, Mitchell, and Stamos (2007), which evaluates the role of a life contingent asset in a model of asset location and allocation. Nevertheless, that work limits its attention to decisionmaking only during the retirement period.[4] We contribute to the literature here by including the entire life cycle - from age 20 forward - and assess how labor income uncertainty and access to the survival credit drives key decisions of interest. We also permit gradual timing of the purchase of these survival-contingent assets as well as the asset allocation of both the liquid and

[4]For a review of prior literature see Horneff, Maurer, Mitchell, and Dus (2008). One related study by Horneff, Maurer, and Stamos (2008a) derives an optimal annuitization strategy when an investor is limited to holding bonds in her life contingent product; this generates a co-called constant payout or fixed annuity. But that work does not allow equities in the annuity portfolio, as we do here.

illiquid portfolios.[5] In what follows, we briefly describe our model. Next, we analyze the asset location with and without product loads. We describe the pattern of asset allocation. Subsequently we conduct a sensitivity analysis and examine the welfare gains from expanding the asset space. A final section concludes.

2 A Dynamic Asset Allocation and Location Model

2.1 Preference

We employ a time discrete model with $t \in \{0, ..., T+1\}$, where t determines the investor's adult age (computed as actual age minus 19). T is the investor's maximum possible age. Individual preferences are characterized through the $CRRA$ utility function defined over a single non-durable consumption good; the value function V_t is recursively defined as:

$$\begin{aligned} V_t &= \tfrac{C_t^{1-\rho}}{1-\rho} + \beta E_t\left[p_t^s V_{t+1} + (1-p_t^s)k\tfrac{B_{t+1}^{1-\rho}}{1-\rho}\right] \quad \text{and} \\ V_T &= \tfrac{C_T^{1-\rho}}{1-\rho} + \beta E_T\left[k\tfrac{(B_{T+1})^{1-\rho}}{1-\rho}\right] \end{aligned} \tag{1}$$

Here β is the time preference discount factor, k is the strength of the bequest motive, and ρ is the level of risk aversion. C_t is the amount of wealth consumed in period t and B_{t+1} is the level of bequest in $t+1$ if the investor dies between t and $t+1$. The individual has subjective probabilities p_t^s that she survives until $t+1$ given that she is alive in t. Below, we permit her subjective survival probability to potentially differ from the objective survival table assumed by the insurer. As the investor gains utility from consumption and from leaving estate an additional motive for liquid wealth is induced. Having a bequest motive $k > 0$ means that the investor will always keep some wealth liquid (not annuitized), in order to be able to bequeath the desired amount of wealth to potential heirs.

2.2 Labor Income

Several studies have recently highlighted the importance of including labor income as a non-tradable asset[6]; in particular, stochastic labor income is shown to create demand for buffer stock saving early in life. Including stochastic income into our

[5]See Charupat and Milevsky (2002), Koijen, Nijman, and Werker (2006), Browne, Milevsky, and Salisbury (2003), Milevsky and Young (2006), and Milevsky and Young (2007). None of these endogenize the asset location and allocation decisions dynamically, as in the present paper. Browne et al. (2003) assess welfare losses from a stylized case where only fixed/equity linked annuities can be exchanged for each other.

[6]See Bodie, Merton, Samuelson (1992), Heaton and Lucas (1997), Viceira (2001), and Cocco, Gomes, and Maenhout (2005).

analysis is important in explaining the trade-off between the inflexibility created by the survival-contingent benefit versus the return-enhancing survival credit when labor income is uncertain. We assume that the individual earns uninsurable real labor income Y_t during the accumulation phase ($t \leq K$), where K is the retirement adult age. This risky labor income follows the process (as in Cocco et al., 2005):

$$\begin{aligned} Y_t &= \exp(f(t))P_tU_t, & (2) \\ P_t &= P_{t-1}N_t, & (3) \end{aligned}$$

where $f(t)$ is a deterministic function of age to recover the hump-shaped income profile observed empirically. P_t is a permanent component with innovation N_t and U_t is a transitory shock. The logarithms of N_t and U_t are normally distributed with means zero and with volatilities σ_N, σ_U respectively. The shocks are assumed to be uncorrelated. After retirement ($t > K$), we assume that the individual will receive a constant pension benefit payment of $Y_t = \zeta \exp(f(K))P_K$, where ζ is the constant replacement ratio[7].

2.3 Capital and Payout Annuity Market Parameters

The individual can invest via direct investments in the two financial assets: riskless bonds and risky stocks. The real bond gross return is denoted by R_f, and the real risky stock return in t is R_t. The risky log-return lnR_t is also normally distributed with expected return μ and volatility σ. The term $\phi_n(\phi_u)$ denotes the correlation between the stock returns and the permanent (transitory) income shocks.

Capital market securities can be either accessed via liquid savings or the illiquid annuities. But in contrast to direct stock or bond investments, annuities cannot be sold by the individual, which makes them irreversible and creates illiquidity for purchasers. Turning to the variable annuity, this is an insurance contract between an annuitant and an insurer; the purchaser receives a pre-specified number of fund units n_t conditional on survival in each period $t > 0$. When the price of a fund unit at time t is Z_t^a, the survival-contingent income received from this annuity is $\hat{P}_t = n_t Z_t^a, t \in (0, ..., T)$. To receive this income stream, the annuitant must pay the

[7]Here we focus on asset allocation decisions; future work might determine the retirement age K and labor supply endogenously.

insurer an initial premium A, computed according to [8]:

$$A_t = (1+\delta) Z_t^a(t) n_{t+1}(t) \sum_{s=t+1}^{T} \frac{p^a(t,s)}{(1+AIR)^{s-t-1}}, \tag{4}$$

where δ is the expense factor, $p^a(t,s) = \prod_t^{s-1} p_t^a$ is the cumulative conditional survival probability for an individual age t to survive until age s, and AIR is the so-called assumed interest rate. The single period conditional probability p_t^a may be permitted to differ from the individual's subjective survival probability, p_t^s, if we wish to model asymmetric mortality beliefs (as below). The AIR[9] determines how the number of fund units evolves over time, according to $n_t = n_1 \left(\frac{1}{(1+AIR)}\right)^{t-1}$. One can think of the AIR as the deterministic shrinkage rate for the number of fund units the individual is supposed to receive. [10]

The evolutionary equation for the price of the fund unit can be written as follows:

$$Z_{t+1} = Z_t R_{t+1}^a, \tag{5}$$

where $R_{t+1}^a = (R_f + \pi_t^a(R_{t+1} - R_f))$ is the growth rate of the underlying fund and where π^a is the stock fraction inside the variable annuity. The investment return will be random when the fund is invested in risky stocks. Accordingly, the income evolution of a single annuity purchased previously can be recursively expressed as:

$$\hat{P}_{t+1} = \frac{\hat{P}_t R_{t+1}^a}{1+AIR}. \tag{6}$$

This formulation shows that the annuity payout evolves according to a multiplicative random walk: it rises when the fund return $R_{t+1}^a > 1+AIR$, decreases when $R_{t+1}^a < 1+AIR$, and is constant when R_{t+1}^a equals the AIR. Sellers do not generally permit changing the AIR after the annuity is purchased, implying that the annuity market is incomplete. Nevertheless, the investor can still purchase new annuities throughout the lifecycle, in order to align the income profile of all purchased annuities to her

[8] This expression shows that the discount rate is higher than the simple market return since $[p^a(t,s)]^{-1} > 1$; this additional return increment is referred to as the survival credit. It arises from allocating deceased members' remaining assets among the surviving member of the insurance pool

[9] The assumed interest rate could in practice be time dependent but, in keeping with prior studies and industry practice, here we assume it is constant. For instance, a 4 percent AIR is widespread in the US insurance industry (c.f. the Vanguard and TIAA-CREF variable payout annuity websites); furthermore, the US National Association of Insurance Commissioners (NAIC) stipulates that the AIR may not exceed a nominal 5 percent.

[10] The fixed annuity can be defined as similar to the usual annuity factor whereby the riskless discount factor is replaced by the AIR $\sum_{s=1}^{T} \frac{p^a(t,t+s)}{(1+AIR)^{s-1}}$. In the case where the fund invests only in riskless bonds, we obtain the classical result for constant payout annuities: $A_0 = \sum_{s=1}^{T} \frac{p^a(t,t+s)}{(R_f^t)^s}$

preferences.[11]

2.4 Wealth Accumulation

The household is assumed to decide annually how to spread her cash on hand, W_t, across bonds, stocks, variable payout annuities, and consumption. Her budget constraint is:

$$W_t = S_t + A_t + C_t, \tag{7}$$

where S_t represents liquid saving comprised by her liquid bond and stock investments; A_t is the amount that the investor pays for annuity premiums in the current period; and C_t represents consumption. Her cash on hand one period later is given by:

$$W_{t+1} = S_t R^s_{t+1} + L_{t+1} + Y_{t+1}, \tag{8}$$

where $R^S_{t+1} = (R_f + \pi^s_t(R_{t+1} - R_f))$ is growth rate of liquid saving; π^s_t denotes the fraction of liquid saving S_t invested in risky stocks; L_{t+1} is the sum of annuity payments which the investor receives from all previously purchased annuities; and Y_{t+1} represents labor income. The sum of all payments from previous annuities purchased in $u \in 0, 1, ..., t$ is:

$$L_{t+1} = \sum_{u=0}^{t} Z_{t+1}(u) n_{t+1}(u) \tag{9}$$

The price process of fund units of the annuity purchased at $t = u$ can be written as:

$$Z_{t+1}(u) = Z_t(u)(R_f + \pi^a_t(R_{t+1} - R_f)), Z_u(u) = 1; \tag{10}$$

where π^a_t is the stock fraction at date t inside the purchased annuities. Substituting (10) and (4) into (9) yields the recursive definition of the payout evolution:

$$L_{t+1} = \left[\frac{L_t}{(1+AIR)} + A_t \left(\sum_{s=t+1}^{T} \frac{p^a(t,s)}{(1+AIR)^{s-t-1}} \right)^{-1} \right] (R_f + \pi^a_t(R_{t+1} - R_f)), \tag{11}$$

with $Z_u(u) = 1$;

The recursive intertemporal budget restriction can be obtained as follows by substituting (7) and (11) into (8):

[11] More discussion of the role of the AIR on payout profiles appears in Horneff, Maurer, Mitchell, and Stamos (2007).

$$W_{t+1} = \left[(W_t - C_t - A_t) + \left(\frac{L_t}{1+AIR} + A_t\left(\sum_{s=t+1}^{T}\frac{p^a(t,s)}{(1+AIR)^{s-t-1}}\right)^{-1}\right)\right]R_f +$$
$$\left[(W_t - C_t - A_t)\pi_t^s + \left(\frac{L_t}{1+AIR} + A_t\left(\sum_{s=t+1}^{T}\frac{p^a(t,s)}{(1+AIR)^{s-t-1}}\right)^{-1}\right)\pi_t^a\right](R_{t+1} - R_f))$$
$$+Y_{t+1} \tag{12}$$

If the retiree were to die at $t+1$, her estate remaining would be given by $B_{t+1} = S_t(R_f + \pi_t^S(R_{t+1} - R_f)$. Furthermore, and consistent with the real world, retirees are precluded from borrowing against future labor, pension, and annuity income, by imposing the following non-negativity restrictions:

$$S_t, A_t, \pi_t^a, (1-\pi_t^a), \pi_t^s, (1-\pi_t^s) \geq 0 \tag{13}$$

2.5 Numerical Solution of the Optimization Problem

In what follows, we normalize by permanent income in order to reduce the complexity of the problem by one state variable. We note the normalized variables by the lower-case letters of the variables already introduced:

$$w_t = s_t + a_t + c_t \tag{14}$$

$$s_t, a_t \geq 0 \tag{15}$$

$$w_{t+1} = \left[s_t + \left(\frac{l_t}{1+AIR} + a_t\left(\sum_{s=t+1}^{T}\frac{p^a(t,s)}{(1+AIR)^{s-t-1}}\right)^{-1}\right)\right]R_f(N_{t+1})^{-1} + \qquad \text{if } \; t < K$$
$$\left[s_t\pi_t^s + \left(\frac{l_t}{1+AIR} + a_t\left(\sum_{s=t+1}^{T}\frac{p^a(t,s)}{(1+AIR)^{s-t-1}}\right)^{-1}\right)\pi_t^a\right](R_{t+1} - R_f)(N_{t+1})^{-1}$$
$$+exp(f(t+1))U_{t+1}$$

$$w_{t+1} = \left[s_t + \left(\frac{l_t}{1+AIR} + a_t\left(\sum_{s=t}^{T}\frac{p^a(t,s)}{(1+AIR)^{s-t-1}}\right)^{-1}\right)\right]R_f + \qquad \text{if } \; t \geq K$$
$$\left[s_t\pi_t^s + \left(\frac{l_t}{1+AIR} + a_t\left(\sum_{s=t+1}^{T}\frac{p^a(t,s)}{(1+AIR)^{s-t-1}}\right)^{-1}\right)\pi_t^a\right](R_{t+1} - R_f)$$
$$+\zeta exp(f(K)) \tag{16}$$

The optimization problem is then given by:

$$\max_{\{c_t, a_t, \pi_t^s, \pi_t^a\}_t^T} v_0, \tag{17}$$

where v_0 is the normalized value of utility from future consumption and the optimization problem is subject to the restrictions listed above. Since analytical solutions to this kind of problem do not exist, we solve the problem in a three-dimensional state space $\{w, l, t\}$ by backward induction (see the Technical Appendix). Although we assume CRRA preferences, cash on hand w cannot be omitted as a state variable because illiquid annuities are included in the analysis. It is also necessary to include the sum of current annuity payouts l as a state variable, because once purchased, annuities can no longer be sold. Finally, the optimal policy depends on the retiree's age because this influences the price of newly purchased life annuities as well as the present value of her remaining lifetime income.

2.6 Calibration

The individual lifespan is modeled from age 20 to age 100 ($T = 81$); retirement begins at age 65 ($K = 46$). As a result, the worklife can be, at most, 45 years long; the maximum length of the retirement phase is 36 years. Preference parameters are set to values standard in the life-cycle literature, including a coefficient of relative risk aversion ρ of 5, a discount factor $\beta = 0.96$, and initially, a zero bequest motive ($k = 0$). In sensitivity analyses we do allow positive bequest preferences ($k = 2$) as empirical evidence on bequest motives is ambiguous (Bernheim et al., 1985; Hurd, 1987). Parameter values of labor and pension income processes are set in accord with Cocco, Gomes, and Maenhout (2005); our base case sets the deterministic labor income function $f(t)$ and volatility parameters for transitory and permanent labor income shocks ($\sigma_u = 0.30$ and $\sigma_n = 0.1$) to represent households with high school but no college education. The replacement ratio for Social Security pensions (but exclusive of voluntary annuitization) is set at 68.2 percent. Mean equity returns are set at $\mu = 4.41$ percent and volatility $\sigma_s = 16.86$ percent, equivalent to an expected return of 6 percent and standard deviation of 18 percent; the correlation between stock returns and permanent (transitory) income shocks ϕ_n (ϕ_u) is zero. The assumed interest rate (AIR) is set to a real 2 percent, as is the case in practice. With respect to the additional costs of buying annuities, the base case sets the load at zero and assumes that annuities are priced actuarially fairly by equating conditional survival probabilities of the investor and the insurer (as per the 2000 Population Basic mortality table). In extensions, we permit positive loads with the expense factor δ set to 2.38 percent (in line with industry leaders such as Vanguard); and we implement asymmetric mortality distributions by using the 1996 US Annuity 2000 Aggregate Basic for annuity pricing and the 2000 Population Basic mortality table to compute the investor's expected utility.

3 Asset Location and Allocation

3.1 Optimal Asset Location

In what follows, we first consider no-load survival-contingent products. In such a setting, we can isolate how stochastic labor income creates the need for liquid assets early in the life cycle - even without a bequest motive ($k = 0$). We show how the individual's need for such precautionary saving can influence her demand for annuitization, and how it delays the date at which full annuitization occurs. After deriving the optimal policy for the no-bequest case, we then show how introducing a moderate bequest motive ($k = 2$) induces further demand for liquid saving near the end of life. Subsequently, we discuss how positive loads and asymmetric mortality probabilities alter these predictions.

No Loads. To illustrate the range of asset location and allocation strategies, we next plot the optimal policies by age and cash on hand. Figure (1) illustrates how the individual would act at each age, assuming she receives no payouts from previously-purchased annuities; subsequently we allow for gradual annuitization. [12] Panel (a) of Figure (1) shows that the investor with no bequest motive will optimally purchase zero load life annuities at all ages with her net cash on hand. Because of the need for precautionary saving to offset adverse stochastic labor income shocks, the individual will also invest some portion of her liquid assets until about age 65. After that, all cash will be used to purchase life annuities, since she faces less labor income uncertainty and the survival credit grows at older ages. It is also of interest to note that wealthier people optimally devote more of their cash on hand to annuities. Given their higher wealth levels, less liquidity is needed to protect against labor income shocks; further, their higher annuity payouts compensate them for this illiquidity.

To illustrate how bequests might alter the analysis, Panel (b) indicates results for $k = 2$. As before, we assume that the individual receives no payouts from previously-purchased annuities. Now she will need to keep more money in liquid form, in case she experiences labor income shocks or early death. Consequently she will optimally reserve a larger portion of her net cash on hand to meet these goals (panel b). Nevertheless, similar to the no bequest case, partial annuitization is still optimal, and in fact it could begin as early as age 20 if her cash on hand is high enough. Interestingly, the annuitization fraction still exceeds 70 percent of net cash on hand for the wealthiest individual, but it is a decreasing function of age because the urgency of the bequest motive at older ages offsets the rising survival credit.

[12] Accordingly, Figure (1) cleanly illustrates the demand for de novo liquid versus illiquid annuity investments. The blank area in the lower right corner of panel (a) of Figure (1) represents the region of the state space in which it is optimal to consume 100 percent of cash on hand.

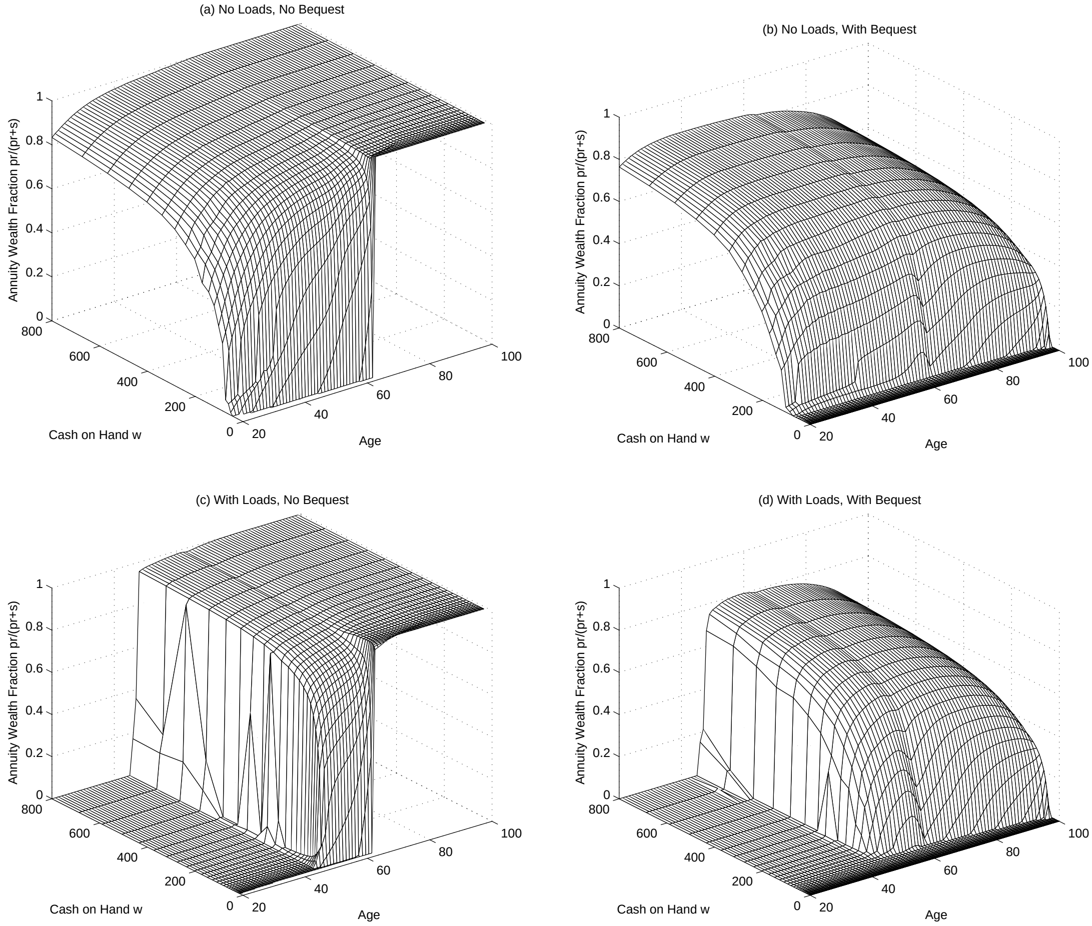

Figure 1: Illustrative Optimal Dynamic Asset Location Outcomes Assuming No Loads vs. Loads (top-bottom), No Bequest vs. Moderate Bequest Cases (left-right). These figures represent optimal policies for annuity purchases, as a function of cash on hand (w) and age; no prior annuitization is assumed ($l = 0$). For instance, in Panel (a), a 20-year old with no bequest motive and with cash on hand $w = 200$ would spend 70 percent of her net cash on annuities, and the rest on liquid investments. Panel (b) shows that the 20-year old individual having a moderate bequest motive ($k = 2$) and the same cash on hand ($w = 200$) would annuitize 40 percent. In Panel (c), the individual at age 20 with no bequest motive and with cash on hand w = 200 would spend zero percent of her wealth on annuities and devote all to financial investments. Panel (d) shows that this individual would also hold 100 percent liquid investments. Note: Calculations are based on backward optimization of the value function given in (1). The base case individual has CRRA utility with $\rho = 5$, $\beta = 0.96$; for the computations without loads: survival probabilities are taken from the corresponding population mortality table to calculate utility and price annuities. Loads for annuities are set to zero; for the computations with loads: Survival probabilities are taken from the US 1996 Annuity 2000 mortality table for females to price annuities and from the corresponding population mortality table to calculate utility. Annuity loads are set to 2.38 percent. Yearly real stock returns are i.i.d. log-normal distributed with mean 6 percent and standard deviation 18 percent. The real interest rate and AIR are set to 2 percent.

At low wealth levels, by contrast, the annuitization fraction rises until the worker retires; after that date, the pattern is complicated by the offsetting effects of the rising survival credit, on the one hand, and the desire to leave a bequest, on the other.

Loads. Next we explore how adding loads and mortality asymmetries influences optimal location choices. In particular, we assume that the annuity provider charges a front load of 2.38 percent to account for administration, mortality changes, and reserves.[13] Further, we acknowledge that the annuity provider is aware of the fact that healthier-than-average people are more inclined to purchase annuities (McCarthy and Mitchell, 2002). This is implemented by taking survival probabilities from the US female 1996 Annuity 2000 mortality table to price annuities and from the corresponding population mortality table to calculate utility. Results appear in Figure (1) panel (c) and panel (d) for $l = 0$ (no preexisting annuities) and assuming no bequest motive (left side) versus a moderate bequest rationale (right hand side). In particular, adding loads and asymmetric information induces the individual to defer annuitization to around the age of 50. That is, she waits until the survival credit is high enough to overcome the implicit and explicit costs related to the annuity purchase. Nevertheless, an individual without a bequest motive will fully annuitize after age 65, whereas she would only move to about a 60 percent annuitization strategy if she had sufficient wealth and a moderate bequest motive (panels c versus d).

3.2 Optimal Asset Allocation and the Impact of Human Capital

We next turn to a discussion of how the uncertain human capital influences the optimal allocation and location patterns. Figure (2) shows illustrative optimal stock fractions in the combined annuity and financial wealth portfolio for the no load, no bequest case. Panel (a) provides the stock fraction as a function of age and cash on hand (w), and it can be seen that he stock fraction falls with the level of cash on hand (w). The rationale is that bonds are perceived as a closer substitute for human capital than stocks. In turn, the decline in human capital over time is compensated for by reducing the stock fraction in order to purchase bonds.

Thus far, we have assumed that the individual has no income from pre-existing annuities ($l = 0$); next we allow for gradual annuitization. This means that she can purchase new annuities as long as the budget constraint permits it, in which case her asset allocation decision will depend on how much annuity income the individual is already receiving. Accordingly another dimension must be added to

[13] This corresponds to the industry average according to Vanguard.

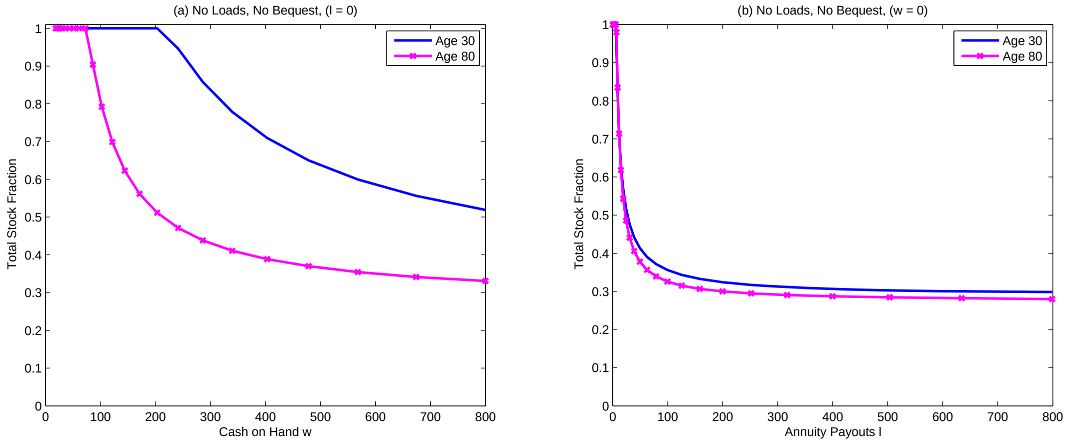

Figure 2: Illustrative Optimal Stock Fractions in the Combined Annuity and Financial Wealth Portfolios. These figures represent the relationship between the individual's age and the optimal stock fraction in her combined annuity and financial asset holdings. The latter is defined as the stock fraction of her expected annuity wealth PV (the present value of the remaining annuity payouts) plus the stock fraction of her financial wealth s, as a percent of financial plus annuity wealth . Panel (a) shows how the total stock fraction varies with age and cash on hand w; panel (b) shows how it varies with age and pre-existing annuity payments l. Note: Calculations are based on backward optimization of the value function given in (1). The base case individual has $CRRA$ utility with $\rho = 5$, $\beta = 0.96$; the computations are done for the no load, no bequest-case: survival probabilities are taken from the corresponding population mortality table to calculate utility and price annuities. Loads for annuities are set to zero. Yearly real stock returns are i.i.d. log-normal distributed with mean 6 percent and standard deviation 18 percent. The real interest rate and AIR are set to 2 percent.

the presentation. Panel (b) of Figure (2) shows the stock fraction as a function of age and payouts from pre-existing annuities (l). The effects are quite similar to panel (a). The more payouts one has, the less one must be invested in equity to achieve the optimal split of augmented wealth. However, the stock fraction drops much faster for the annuity payouts than for the level of cash on hand. Panel (b) also shows the age effect due to the decreasing human capital. The older the individual, the lower is the stock fraction.

3.3 Expected Asset Allocation and Location

Next we conduct a Monte Carlo analysis of 100,000 life cycles to depict the expected evolution of the investments in illiquid stocks and bonds (variable annuities), and liquid equity and bonds, assuming that the individual behaves optimally over her lifetime. Figure (3) traces expected asset allocation patterns for the same four cases, with and without bequest motives and loads. For the no-load-no-bequest case (panel a), the worker holds all her assets in non-annuitized form so as to protect against labor income shocks; her liquid saving is fully invested in stocks. After about age

60, she then shifts her holdings into an illiquid annuitized portfolio, after which time she no longer has liquid wealth. She also begins to shift her optimal asset allocation from illiquid stocks to illiquid bonds, because of her declining human capital. At older ages, she would hold some 40 percent of her annuity portfolio in equities, in expectation. Interestingly, only a few bonds are ever held outside the annuity portfolio. Including a bequest motive, as in Panel (b), we see that once again, the bulk of her portfolio at younger ages is held in equity, but now the share of liquid stocks is higher. Further, after about age 35, she again optimally shifts into liquid bonds in order to safely accumulate her bequest during her early years when her labor income is quite uncertain. Her fraction of wealth held in annuities now rises to age 75, when about three-quarters of total wealth is annuitized. After that point, her fraction of annuitized wealth shrinks, since survival probabilities decrease with age. The fraction of liquid wealth becomes 100 percent again at the very end of the life cycle. Overall, it is interesting that, with or without a bequest, the individual will hold similar cumulative stock holdings in liquid and illiquid wealth. What is different is that without a bequest, almost no bonds are held outside the annuity. Having a bequest motive also leads the individual to invest substantial assets in liquid bonds. Expected asset allocations are displayed in panel (c) and (d) of Figure (3) for the case with loads and mortality asymmetry. The most notable difference is that now liquid bonds play a much larger role in the no-bequest (panel c) and moderate bequest cases (panel b). Having a bequest motive means that liquid bonds will play an important role all the way to the oldest possible age. At age 80, for instance, the individual would be expected to hold about 70 percent of her wealth in a variable annuity which is about two-thirds equities; her remaining non-annuitized wealth would be mainly in bonds.

4 Sensitivity Analysis

We next ask how sensitive the results are to additional liquidity shocks. We use the load, no-bequest case as the benchmark, and we ask how, at ages 30, 50, 60, and 80, the baseline expected asset allocations compare with scenarios that include retirement income shocks, housing expenditures, and health shocks. In the benchmark case, liquidity shocks before retirement are the result of labor income risk (for more detail see the Technical Appendix).

Risky Retirement Income Streams. To analyze the impact of adding risk to the retirement income stream on the need for liquidity reserves, we multiply what was previously a constant retirement income flow $Y_t = \zeta exp(f(K))P_K$ by a transitory

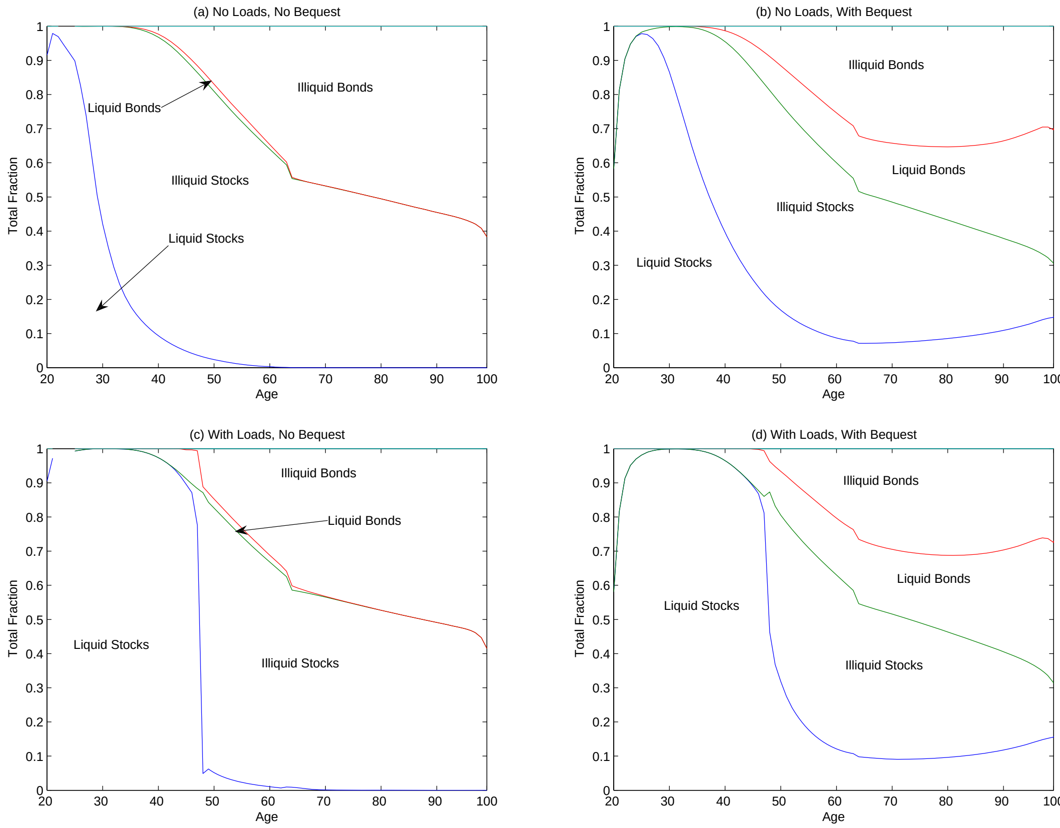

Figure 3: Optimal Expected Asset Allocation Over the Life Cycle Assuming No Loads vs. Loads (top-bottom), No Bequest vs. Moderate Bequest Cases (left-right). Panel (a) plots the expected trajectory for the fraction held in stocks inside the annuity (illiquid) and outside the annuity (liquid) for a female with a maximum life span of age 100, having no initial endowment and no bequest; in panel (b) the individual has a bequest motive ($k = 2$). In Panel (c), the individual has no bequest motive while in panel (d) the individual has a bequest motive of (k=2). In panel (a) and panel (b) annuities have zero loads, while in panel (c) and (d) annuities are loaded. Note: Expected values are computed by simulating 100,000 life-cycle paths based on the optimal policies derived by the numerical optimization. The base case individual has $CRRA$ utility with ρ = 5, β = 0.96; for the computations without loads: survival probabilities are taken from the corresponding population mortality table to calculate utility and price annuities. Loads for annuities are set to zero; for the computations with loads: Survival probabilities are taken from the US 1996 Annuity 2000 mortality table for females to price annuities and from the corresponding population mortality table to calculate utility. Annuity loads are set to 2.38 percent. Yearly real stock returns are i.i.d. log-normal distributed with mean 6 percent and standard deviation 18 percent. The real interest rate and AIR are set to 2 percent.

log-normal iid. shock:

$$U_{t,t>K} \sim LN\left(-\frac{1}{2}\sigma_R^2, \sigma_R^2\right). \tag{18}$$

We set the volatility of retirement income equal to the volatility during the worklife $\sigma_r = \sigma_u = 0.3$. The results (Table (1), Row 2) show that the individual will hold only slightly more liquid wealth, but the optimal fraction of annuitized wealth is around 95 percent. This result indicates that transitory retirement income shocks can largely be absorbed by the annuity income stream. Also the asset allocation does not differ substantially from that of the benchmark case.

To explore the sensitivity of results to permanent retirement income shocks, we allow for a disastrous permanent retirement income downturn of 75 percent which occurs with a probability ψ of 5 percent in each year, but it can only happen once. [14] Such a shock can be thought of as significant background risk, or it could be conceived of as large medical bills incurred late in the life cycle. Row 3 of Table (1) shows that annuitization rates in this scenario are similar to the benchmark case. What responds, however, is the asset allocation inside the variable annuity: now the bond fraction inside the variable annuity is substantially larger than before. In other words, the individual will adjust for an anticipated bad income draw by increasing the portion of fixed permanent annuity income.

Housing Expenditures. Next we introduce shocks to housing expenses by employing the polynomial function estimated by Gomes and Michaelides (2005) from the Panel Study of Income Dynamics. Specifically, housing expenses h_t, defined as the annual mortgage and rent payments relative to labor income Y, are given by:

$$h_t = max(\hat{A} + \hat{B}_1 \cdot age + \hat{B}_2 \cdot age^2 + \hat{B}_3 \cdot age^3, 0), \tag{19}$$

where $\hat{A}$ = 0.703998, $\hat{B}_1$ = -0.0352276, $\hat{B}_2$ = 0.0007205, and $\hat{B}_3$ = -0.0000049 (we truncate h_t to zero for age = 80). As Row (4) of Table (1) shows, such deterministic housing expenses reduces the annuitization fraction by 2 percentage points compared to the benchmark case. If we add a stochastic component to the housing expenditure consisting of a lognormal shock:

$$H_t \sim LN\left(-\frac{1}{2}\sigma_h^2, \sigma_h^2\right). \tag{20}$$

with $\sigma_h = 0.25$, then disposable income is now given as $(1 - \hat{h}_t)Y_t$ where:

$$\hat{h}_t = h_t H_t \tag{21}$$

[14] This type of sensitivity analysis is similar to that carried out in Cocco, Gomes, and Maenhout (2005) where they analyzed the impact of disastrous events on the asset allocation decision including stocks and bonds.

Expected Asset Allocation: Fraction in Stocks and Bonds held Inside and Outside the Annuity

	Liquid Stock Fraction				Liquid Bond Fraction				Annuity Stock Fraction				Annuity Wealth Fraction			
Age	30	50	60	80	30	50	60	80	30	50	60	80	30	50	60	80
(1) Base Case	100	5.1	1.0	0	0	2.6	2.0	0	0	77.4	65.7	52.6	0	92.3	97.0	100
(2) Pension Shocks (iid.)	100	4.5	1.2	0.5	0	2.7	3.1	4.1	0	77.2	63.7	49.5	0	92.8	95.7	95.4
(3) Disastrous Shock	100	1.3	0	0	0	1.8	0	0	0	69.7	47.4	33.4	0	96.9	100	100
(4) Housing Exp. (det.)	100	6.7	2.1	0	0	3.0	2.9	0	0	78.7	68.8	59.0	0	90.3	95.0	100
(5) Housing Exp. (risky)	100	11.2	4.5	0	0	1.7	1.9	0	0	82.4	79.0	71.1	0	87.1	93.6	100
(6) (2) + (5)	100	10.4	7.2	0	0	1.5	4.2	0	0	82.8	74.4	68.8	0	88.1	88.6	100
(7) Health Shock + (3)	100	1.9	1.0	0.1	0	1.9	0.1	2.0	0	71.2	48.2	35.1	0	96.2	98.9	97.9

Table 1: Expected Asset Allocation: Fraction in Stocks and Bonds held Inside and Outside the Annuity. Figures reported are percentages; expected values are computed based on 100,000 Monte-Carlo simulations by using the optimal policies. All results are for a female having a maximum life-span of age 100, having no initial annuity endowment, a CRRA utility function with $\rho = 5$ and $\beta = 0.96$, and $AIR = 2$ percent. Loads are set to $\delta = 0.0238$ and the retiree faces mortality asymmetries (2000 Population female Basic vs. 1996 US Annuity 2000 Basic female mortality table). In Row (2), a transitory shock with a standard deviation of 30 percent is added to the retirement income. Row (3) includes the possibility of a disastrous permanent income drop of 75 percent of the retirement income with 5 percent probability. Row (4) includes deterministic housing expenses. Row (5) adds to (4) a transitory shock in housing expenses with a standard deviation of 25 percent. Row (6) aggregates the transitory shocks of retirement income and housing expenses. Row (7) adds to the 75 percent permanent income drop in row (3) a health shock increasing the force of mortality 4 times.

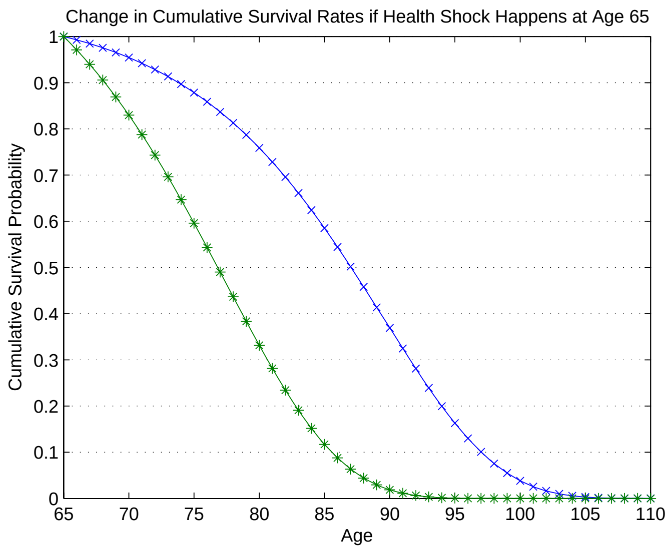

Figure 4: Illustration of Health Shock at Age 65. To model the drop in survival rates, we assume that the individual's force of mortality becomes 4 times the force of mortality implied by the 1996 US Annuitant Mortality table if a health shock occurs.

It is shown in Table (1), Row 5, that the fraction of annuitized wealth is again quite robust to this innovation. In Row (6), we combine both the transitory income and housing shocks (equations 18 and 20). Remarkably, the fraction of annuitized wealth never drops more than 10 percent below that of the benchmark case.

Health Shocks. To implement health shocks, we assume that, in each year during retirement, a sudden decline in the survival rate and a permanent 75 percent drop in retirement benefits occurs with a 5 percent probability. This scenario would be equivalent to a sudden severe deterioration in health status accompanied by a spike in medical or nursing home costs. To model the poorer survival rate, we assume that the individual's force of mortality becomes 4 times that in the 1996 US Annuitant Mortality table (see Figure 4). Thus, survival rates after a health shock can be expressed as $p_t^s = (p_t^a)^4$; if, for instance, a health shock occured at age 65, the remaining expected lifetime would fall from 22 to 12 years. Clearly a lower survival rates means makes annuities less attractive. Nevertheless, the resulting reduction in the fraction of wealth annuitized is remarkably small (Table 1, row 7).

5 Expected Life Cycle Profiles

Next we turn to an analysis of the expected evolution of key decision variables over the life cycle; results are from a Monte Carlo simulation of 100,000 life cycles.

No Loads. First we assume no loads, and show in Figure (5) that expected consumption rises remarkably steeply with age - in fact, so steeply (in expectation) that the retiree's living standard greatly exceeds that she had during her worklife

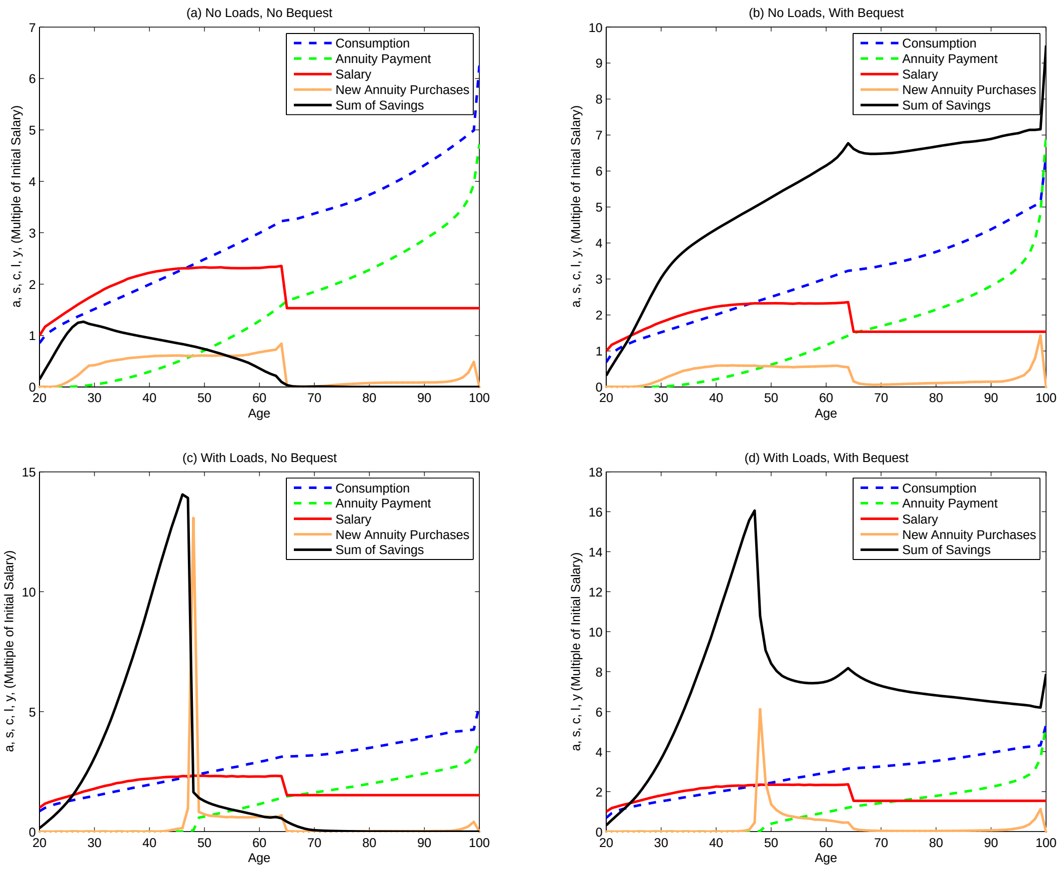

Figure 5: Optimal Expected Consumption, Labor Income, Saving, and Annuity Purchases Over the Life Cycle Assuming No Loads vs. Loads (top-bottom), No Bequest vs. Moderate Bequest Cases (left-right). Panel (a) plots expected trajectories for a female with a maximum life span of age 100, having no initial endowment, in panel (b) the individual has a bequest motive ($k = 2$). In Panel (c), the individual has no bequest motive while in panel (d) the individual has a bequest motive of ($k = 2$). In panel (a) and panel (b) annuities have zero loads, while in panel (c) and (d) annuities are loaded. Note: Expected values are computed by simulating 100,000 life-cycle paths based on the optimal policies derived by the numerical optimization. The base case individual has CRRA utility with $\rho = 5$, $\beta = 0.96$; for the computations without loads: survival probabilities are taken from the corresponding population mortality table to calculate utility and price annuities. Loads for annuities are set to zero; for the computations with loads: Survival probabilities are taken from the US 1996 Annuity 2000 mortality table for females to price annuities and from the corresponding population mortality table to calculate utility. Annuity loads are set to 2.38 percent. Yearly real stock returns are i.i.d. log-normal distributed with mean 6 percent and standard deviation 18 percent. The real interest rate and AIR are set to 2 percent.

(panel a). Having a rising consumption profile might seem counterintuitive given the investor's time preference, but it is driven by the survival credit generating rising payouts from her previously-purchased variable annuities. Of course these are, in turn, counterbalanced by their illiquidity; people cannot borrow against annuity payout streams so they cannot use annuity payouts to smooth their consumption paths earlier in life.

We also see that, when she has no bequest motive, the individual accumulates rather low amounts of precautionary saving to cover transitory income shocks up to age 65 (panel a). She begins buying annuities in her early 20's, and her long-term permanent income risk is diversified by the annuity income. Annuity income also cushions the drop in income when transitioning to retirement. After age 65, her annuity payouts are sufficiently high that no liquid wealth is needed. In the moderate bequest case (panel b), once again pre-retirement consumption is lower than post-retirement consumption which is mainly financed by variable annuities - though now, it is optimal to postpone a first annuity purchase to age 27. The bequest motive drives a higher demand for liquid wealth; now, the first 7 years of the work life are used to build up an estate to bequeath to the heirs, instead of purchasing annuities.

Including Loads. Turning to the impact of loads, it is clear that even without a bequest motive, saving levels now surge to much higher levels. Obviously the uncertainty of labor income cannot explain this difference; rather, the individual now accumulates liquid wealth in order to finance consumption until such time that the survival credit improves enough to compensate her for the additional annuity loads. This occurs around age 50, and later she moves all her saving into annuities. Having a bequest motive moderates the shift to annuities, so as to preserve an estate on the individual's death.

6 Welfare Analysis

To assess how consumers might value access to payout annuity products, compared to the asset universe which includes only stock and bond investments, we now turn to a welfare analysis. Specifically, we compute the equivalent increase in financial wealth that would be needed to boost the investor's total expected utility given no access to annuity products, to the level she could attain with access to the survival-contingent assets.[15]

Table (2) reports the wealth equivalent values of calculations undertaken for every year of life remaining, conditional on survival, for both a 40-year old and an 80-year old female. The equivalent increase in financial wealth needed to compensate the investor if she had no access to a fixed annuity appears in the left column, and for variable annuity products in the right column. The results show, first, that all individuals are substantially better off if they can access survival-contingent products, be they fixed or variable in nature (Rows 1 and 2). Second, the utility

[15] To compare fixed and variable annuities on an equal basis, we set the AIR to 2 percent and derive the respective optimal policies.

	Expected Equivalent Increase in Financial Wealth			
	Age 40		Age 80	
	Fixed Annuity	Variable Annuity	Fixed Annuity	Variable Annuity
1. No Loads, No Bequest	21.0	52.60	22.0	28.4
2. No Loads, With Bequest	17.5	29.2	24.2	35.6
3. With Loads, No Bequest	12.1	38.0	12.0	12.7
4. With Loads, With Bequest	9.7	14.8	16.6	19.8

Table 2: Expected Equivalent Increase in Financial Wealth Needed to Compensate an Individual Lacking Access to Annuity Products. Figures reported are percentages; expected values are computed from 100,000 Monte-Carlo simulations by using the optimal policy. All results are for a female having a maximum life-span of age 100, having no initial annuity endowment, a $CRRA$ utility function with ρ=5 and β=0.96, and $AIR = 2$ percent. Optimal annuitization strategies are compared to optimal savings and withdrawal schemes without any annuities. Lines 1 and 2 assume no annuity loads and no mortality asymmetry; Lines 3 and 4 set δ=0.0238 and mortality asymmetries (2000 Population female Basic vs. 1996 US Annuity 2000 Basic female mortality table). Lines 1 and 3 assume $k = 0$ while Lines 2 and 4 set k=2.

gains are greatest for variable payout annuities. For instance, at age 40, an individual facing realistic insurance loads but having no bequest motive would value a fixed annuity at 12 percent of her wealth, compared to having no annuity access, and a variable annuity at 38 percent (Row 3). If she also has a bequest motive (Row 4), the value of having access to a fixed annuity is less, at 10 percent, but still half as much again, 15 percent, for the variable annuity. Obviously, the investor would be much better off if the insurance were priced fairly. For instance, if a government could mandate no-load annuities, someone with no bequest motive would be willing to pay 21 percent of her wealth to gain access to a fixed annuity and 53 percent for a variable annuity (Row 1). It is also interesting that welfare gains for the 80 year old lacking a bequest motive are lower, compared to the 40 year old, but they still remain substantial: access to a variable annuity product is worth 28 percent of her wealth in the actuarially fair case.

7 Conclusion

This paper contributes to the portfolio choice literature by integrating survival-contingent payout products in the investor's menu, while incorporating uninsurable shocks to housing, medical expenses, health, and income during the worklife and retirement. We generate optimal asset location patterns, indicating how much to hold in liquid versus illiquid assets over the lifetime, and also asset allocation paths, explaining how to optimally invest in stocks versus bonds. By contrast, prior studies on annuitization strategies have focused mainly on fixed annuities, or products where only nominal bonds are held inside the illiquid annuity investment. Accordingly, these studies have not acknowledged the substantial welfare-enhancing aspect of variable payout annuities. We show that the investor will gradually move money from her liquid saving into annuities as she nears retirement, and indeed she continues to do so until her late 70s. She will also optimally hold a high percentage of equities in both the liquid and illiquid accounts while young, with the fraction in equity falling with age. This pattern is consistent with previous studies which have not integrated the survival credit of life-contingent holdings. Sensitivity analysis shows that uninsurable shocks increase the preference for liquid savings only marginally. In addition, access to annuities allows the individual to capture the equity premium as well as the survival credit, which enhances her welfare substantially. If the product is fairly priced, a 40-year old investor with no bequest motive is willing to give up as much as half of her liquid wealth to gain access to the life-contingent product. Even with a moderate bequest motive, she would be willing to give up almost one-third of her wealth to gain access to annuities. When reasonable loads and asymmetric mortality distributions are included, the utility gains are still worth over one-third of financial wealth. In other words, this payout product provides a considerably higher standard of living for those who survive, as they can benefit not only from the equity premium but they will also share in the survival credit. In future work we hope to extend this analysis in several directions. One possible addition would be to include inflation and interest rate risk while keeping real annuities and including nominal payout annuities as well. It may also be worthwhile to endogenize labor supply. Housing might be modeled endogenously to understand how much the individual saves in housing vis-*á*-vis life annuities. Life insurance could be included in order to understand how annuities are affected by the interference of bequest and life insurance. In general, however, it is clear that introducing survival-contingent payouts into the life cycle framework adds many interesting dimensions to the investor's asset location and allocation decisions.

8 Appendix (A): Technical Details

The irreversibility of the annuity purchase translates into a real option problem. Due to the irreversibility it is necessary to record how much annuities have been purchased to date. We are required to introduce an additional state variable for the base case. In order to reduce the problem by one state variable, we normalize every evolutionary equation by the permanent income component which itself is a state variable. For solving the gradual annuitization problem after omitting the permanent income component, it is necessary to construct a three dimensional state space because the optimal policies still depend on three state variables: normalized cash on hand w_t which represents the level of liquid financial wealth, normalized annuity payouts from previously purchased annuities l_t, and age t. The continuous state variables cash on hand w_t and annuity payouts l_t have to be discretized and the only true discrete state variable left is age t. For most computations, we use a 45x45x81 grid with a log-scale for both the normalized wealth and for normalized annuity payout. Moving backward along the t dimension we calculate the optimal policy and the value of the value function for each grid point. After applying a series of monotonic transformation and normalizing the value function by the permanent income, we can compute the new expectation operator as:

$$\begin{array}{ll} \iiint(-p_t^s(-\nu_{t+1})^{1-\rho} + (1-p_t^s)k\frac{b_{t+1}^{1-\rho}}{1-\rho})N_{t+1}^{1-\rho}\cdot & \text{if} \quad t < K \\ \varphi(N_{t+1}, U_{t+1}, R_{t+1})dN_{t+1}dU_{t+1}dR_{t+1} & \\ \int(-p_t^s(-\nu_{t+1})^{1-\rho} + (1-p_t^s)k\frac{b_{t+1}^{1-\rho}}{1-\rho})\varphi(R_{t+1})dR_{t+1} & \text{if} \quad t \geq K \end{array} \tag{22}$$

The expectation operator is computed by resorting to Gaussian quadrature integration and the optimization is done by numerical constrained minimization. We inter(extra-)polate the policy and value functions for points which do not lie on the grid. Therefore, we compute the policy functions for gradual annuitization $\pi_S(w;l;t)$, $\pi_A(w;l;t)$, $pr(w;l;t)$, $c(w;l;t)$ and the value function $v(w;l;t)$ by cubic-splines interpolation.

For the case of consumption shocks in retirement, we change the computation of the expectation operator to:

$$\begin{array}{ll} \iint(-p_t^s(-\nu_{t+1})^{1-\rho} + (1-p_t^s)k\frac{b_{t+1}^{1-\rho}}{1-\rho})N_{t+1}^{1-\rho}\cdot & \text{if} \quad t \geq K \\ \varphi(U_{t+1}, R_{t+1})dU_{t+1}dR_{t+1} & \end{array} \tag{23}$$

For the housing shocks, we have to alter the expectation operator as follows:

$$\begin{array}{ll} \iiiint(-p_t^s(-\nu_{t+1})^{1-\rho} + (1-p_t^s)k\frac{b_{t+1}^{1-\rho}}{1-\rho})N_{t+1}^{1-\rho}\cdot & \text{if} \quad t < K \\ \varphi(N_{t+1}, U_{t+1}, R_{t+1}, H_{t+1})dN_{t+1}dU_{t+1}dR_{t+1}dH_{t+1} & \\ \iint(-p_t^s(-\nu_{t+1})^{1-\rho} + (1-p_t^s)k\frac{b_{t+1}^{1-\rho}}{1-\rho})\varphi(R_{t+1}, H_{t+1})dR_{t+1}dH_{t+1} & \text{if} \quad t \geq K \end{array} \tag{24}$$

If housing shocks and retirement income shocks are combined the expectation operator for the retirement period changes to

$$\begin{array}{ll} \iiint(-p_t^s(-\nu_{t+1})^{1-\rho} + (1-p_t^s)k\frac{b_{t+1}^{1-\rho}}{1-\rho})\cdot & \text{if} \quad t \geq K \\ \varphi(U_{t+1}, R_{t+1}, H_{t+1})dU_{t+1}dR_{t+1}dH_{t+1} & \end{array} \tag{25}$$

For the medical consumption shock we have to introduce another binary state variable I. The medical consumption shock leads to a permanent reduction in retirement income with a certain probability. In total, we have four state variables: normalized cash on hand, normalized annuity payouts, medical consumption state, and age. The new binary state variable is $(I = 1)$ in case the medical consumption shock has occurred and is zero otherwise. In order to attain the value function from the future period, we use 4 dimensional cubic-splines inter(extra-)polation. Let ψ denote the probability that the medical consumption shock occurs. If no medical consumption shock has occurred previously $(I = 0)$, then the expectation operator in retirement can be written as a weighted sum

$$\begin{array}{l} \psi \int(-p_t^s(-\nu_{t+1})^{1-\rho} + (1-p_t^s)k\frac{b_{t+1}^{1-\rho}}{1-\rho})\varphi(R_{t+1})dR_{t+1} \\ +(1-\psi) \int -p_t^s(-\nu_{t+1})^{1-\rho} + (1-p_t^s)k\frac{b_{t+1}^{1-\rho}}{1-\rho})\varphi(R_{t+1})dR_{t+1} \end{array} \tag{26}$$

If the medical consumption shock has already occurred $(I = 1)$ previously, the expectation operator for retirement period can be stated as in (22). Then the optimal policies and the value function are given by $\pi^a(w; l; I; t)$, $\pi^s(w; l; I; t)$, $a(w; l; I; t)$, $c(w; l; I; t)$, and $v(w; l; I; t)$, respectively. If we add health shocks, we need to replace the subjective survival probabilities in the expectation operator by lower survival probabilities for the case that a medical shock occurs or has occurred.

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