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PREVIRIALIZATION

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ABSTRACT

We report the results of N-body experiments employing Gaussian random initial conditions designed to address the effect of previrialization recently examined by Peebles. The central issue is whether the development of small-scale structure within a collapsing protocluster can significantly retard its collapse. A series of runs with progressively diminished small-scale power indicates that the collapse of objects is insensitive to the degree of clustering on small scales. We conclude that previrialization does not have significant impact on the formation epoch of galaxies and clusters arising from gravitational clustering of initially Gaussian random density fields.

Subject headings: cosmology: theory — galaxies: clustering — large-scale structure of the universe

1. INTRODUCTION

Understanding the dynamical evolution of initially small density inhomogeneities into collapsed, gravitationally bound systems is a key to most theoretical models of galaxy and larger scale structure formation. One of the first attempts at addressing this problem experimentally (Peebles 1970) demonstrated that several fundamental observed characteristics of the Coma Cluster of galaxies could be reproduced by gravitational relaxation of an initially random, cold distribution of point masses. Since that time, much effort has been expended on the N-body gravitational problem, concentrating on comparing the final state of the experiment with observations. Less effort has been directed at illuminating the details of the dynamical relaxation process. Exactly how and when a given dimple in a linear density field will reach its equilibrium state is rather poorly known.

A simple, analytic model which has been tacitly applied over the years is based on spherically symmetric collapse (Gunn & Gott 1972; Peebles 1980) originally proposed by Lemaître (1931). The idea is that Birkhoff's theorem allows one to treat the evolution of a spherical region with an initial mean interior overdensity δ_i at redshift z_i in a universe with average density $\bar{\rho}(z_i)$ as if that sphere were itself an unperturbed piece of a universe with average density $\bar{\rho}(z_i)(1 + \delta_i)$ at that epoch. The evolution of the size of the perturbed region follows the solution of the Friedmann equation employing the perturbed density. For regions locally above the critical density, the evolution is characterized by expansion to a maximum radius followed by recollapse by some factor, at which point the system is assumed to relax to an equilibrium state. The usual assumption that the system contracts by a factor of 2 follows from energy conservation of nonmixing mass shells and is supported by detailed self-similar infall calculations (Bertschinger 1983). It follows from these arguments that the linearly extrapolated perturbation amplitude $\delta_0 = \delta_i(1 + z_i)$ required to collapse at the present epoch is $\delta_0 = 1.69$.

Recently this scenario has been called into question by Peebles (1990). Peebles performed a set of N-body experiments designed to test the spherical model for collapse of perturbations. The focus was the concept of "previrialization" (Davis & Peebles 1977)—the idea that nonradial motions in a developing mass concentration may grow large enough by the epoch of maximum expansion to appreciably prevent further collapse (Cavaliere et al. 1986; Villumsen & Davis 1986). His numerical study involved comparing randomly generated spherical perturbations evolved in two different ways. In one case the perturbation was surrounded by a perfectly homogeneous background, while in the other it was surrounded by 35 similar perturbations laid down in a manner intended to mimic a locally clumpy, but overall homogeneous, background cosmology. By comparing the evolution of systems in the two cases, one could test the hypothesis that clumps born into a lumpy world evolve differently from clumps born into an otherwise homogeneous universe. One might expect this hypothesis to be true if previrialization is important.

Peebles's results led him to conclude that previrialization was very important—nonradial motions served to significantly retard the collapse of developing perturbations. The critical inference drawn from the study was that the minimum perturbation amplitude required to collapse by today should be a factor of 5 larger than the spherical model estimate, $\delta_0 \simeq 8$. Given the observational constraints on the amplitude of initial cosmological perturbations from the *COBE* DMR experiment (Smoot et al. 1992), a revision forcing theories upward by a factor of 5 in amplitude would kill off most interesting models, including the "standard" cold dark matter model.

Such dire circumstances should be considered carefully. The issue of previrialization deserves further study. Indeed, Peebles (1990) invites such a comparison in his paper, saying, "Previrialization can and should be studied in greater detail in conventional N-body models." In this *Letter* we present results of a set of conventional N-body experiments following the gravitational clustering of initially Gaussian random fields. The set is designed to test the critical component of previrialization directly—the idea that small-scale power leads to subclustering which significantly retards the collapse of structure on larger scales. We perform a suite of five simulations drawn from a common realization of a scale-free power spectrum with spectral index $n = -1$. The simulations differ in that small-scale power is truncated in Fourier space above some critical wavenumber k_c, with k_c systematically varied between experiments. This procedure sets up a series of runs with the same power and phases in large-scale modes, differing only in the amount of power on small scales. An identical procedure was

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used recently by Little, Weinberg, & Park (1991) in a study which did not directly address the question of previrialization.

We present our procedure and results in the following section, and our conclusions and discussion are given in § 3.

2. METHOD AND RESULTS

We perform our *N*-body simulations using a P3M code (Efstathiou & Eastwood 1981) to evolve distributions of 64^3 particles. A set of initial density perturbations is realized by random sampling on a 64^3 mesh in Fourier space a power spectrum $P(k) \sim k^{-1}$. We normalized the spectrum by defining the rms fluctuations at the end of the run to be unity on the scale of one-eighth the size of the box. This normalization is generous enough to allow clusters with up to 4000 particles to form by the end of the simulation, but small enough that structure is still linear on scales close to the fundamental mode. The full realization has power on modes ranging from a wavenumber $k = 1$ (the fundamental) up to $k = 32$ (the Nyquist frequency). This realization is used directly to produce our basis, or control run. To produce a set of initial conditions with reduced power on small scales, the spectrum is sharply cut off above some critical wavenumber k_c in each Cartesian dimension in Fourier space. We have done four simulations with $k_c = 16$, 8, 4, 2, in addition to the run with the full power spectrum. The run with $k_c = 16$ produces structure so similar to the control run that we do not include it in our analysis below.

Figure 1 (Plate L1) gives a qualitative look at the results of the simulations. Projections showing a slice of dimensions $0.4 \times 0.4 \times 0.2$ (in units in which the box size $L = 1$) are shown at $z = 3$, 1, and 0 for $k_c = 8$, 4, and 2 and the control run. The most striking visual impression is the expected reduction of clustering on small scales. At $z = 1$ the small-scale structure is progressively eliminated as k_c is decreased from right to left in the figure. For runs with $k_c < 8$, the initial grid of particles is still visible. On the other hand, the large-scale behavior at $z = 0$ is relatively unaffected. The largest clusters which form in this slice are in the same locations and are about the same size for all the runs except $k_c = 2$, in which only the largest group appears. For this run there is power sufficient to form only the largest cluster in the region. A similar result was found in the study of Little et al. (1991), employing exactly the same technique. If small-scale structure were effective in inhibiting collapse, one would expect clusters to evolve more slowly in simulations with the full power spectrum, so that, overall, fewer large clusters would have formed by a given epoch. One would expect that strong previrialization would leave a visual impression in Figure 1 of clusters "evaporating" when viewed from right to left along a given row in the figure.

There is danger in placing too much emphasis on results based on individual objects in Gaussian random fields. Addition of extra power can act either to focus or to decohere a region in the density field, thereby altering its dynamical evolution. This is the reason there are noticeable differences in the positions and appearances of the large clusters in Figure 1. It is better to examine statistical properties such as the group multiplicity function. The qualitative signal expected if previrialization is effective is that the fraction of mass in large groups should decrease as k_c increases.

We identify groups of particles using a friends-of-friends algorithm in which particles are linked to each other if they are separated by less than some fraction η of the mean interparticle separation. We choose a linking parameter $\eta = 0.15$ which

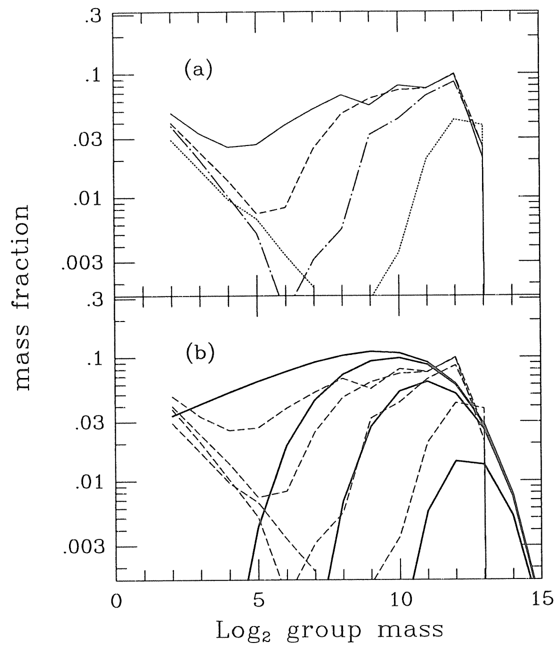

FIG. 2.—Fraction of mass in groups as a function of group mass. The integrated fraction in logarithmic bins a factor of 2 wide is shown. (*a*) Experimental results for the control run (*solid line*), $k_c = 8$ (*dashed line*), $k_c = 4$ (*dot-dash lines*), and $k_c = 2$ (*dotted lines*). The fact that the amount of mass in the largest groups does not decline as small-scale power is added is evidence against previrialization. (*b*) Predictions from the Press-Schechter formula, eq. (2) (*solid lines*), compared with the experimental results (*dashed lines*).

picks out groups with overdensity $\rho/\bar{\rho} \sim \eta^{-3} = 300$. This choice is motivated by a desire to locate collapsed groups in virial equilibrium. The largest clusters contain about 4000 particles, enough that two-body relaxation is negligible and also sufficient to allow calculation of internal group profiles.

The group multiplicity functions are shown in Figure 2. We plot the fraction of mass in groups binned in logarithmic bins a factor of 2 wide against group mass. Note in Figure 2*a* that the fraction of mass in the largest clusters is the same for all the runs. With power eliminated on scales $x \lesssim k_c^{-1}/2$, it is reasonable to expect that this will have the effect of eliminating objects on mass scales smaller than a critical value

$$M_c \simeq \bar{\rho} x^3 = 32768 k_c^{-3} , \quad (1)$$

where the second expression is in simulation units in which $\bar{\rho} = 64^3$. For runs with $k_c = 8$, 4, and 2, one expects $\log_2 M_c \simeq$ 6, 9, and 12, respectively. An analytic expression for the abundance of groups in Gaussian models is given by the formula derived by Press & Schechter (1974), recently rederived more rigorously by Bond et al. (1991). Note that the derivation of the formula relies on the spherical collapse model discussed in § 1. For spectral index $n = -1$, the Press-Schechter model leads to the following expectation for the fraction of material in groups as a function of mass:

$$\frac{Mn(M)}{M_{\rm tot}} d\ln M = 0.45\left(\frac{M}{M_0}\right)^{1/3} \times \exp\left[-1.41\left(\frac{M}{M_0}\right)^{2/3} - \frac{M_c}{M}\right] d\ln M , \quad (2)$$

where $M_0 = 4\pi 8^3/3 = 2145$ is the normalization mass and $M_{\rm tot} = 64^3$ is the total mass in our experiments. The exponen-

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tial cutoff on mass scales below M_c is our approximation to the effects of eliminating small-scale power. The integral of this expression in bins a factor of 2 wide is shown in Figure 2*b* along with the simulation results. Inspection of the figure shows reasonably good agreement between the analytic model and the numerical experiments. In particular, the mass scale of the low-mass cutoff scales as expected between the $k_c = 8$ and $k_c = 4$ cases. The noticeably poorer agreement for the run with $k_c = 2$ is likely due to the fact that this run has so few independent waves that it is not a good representation of a Gaussian random field. With no cutoff, the Press-Schechter formula tends to overestimate the abundance of objects below M_0. This behavior was previously seen in the $n = -1$ models examined in the self-similar clustering study of Efstathiou et al. (1988) as well as in Bond et al. (1991).

One specific mechanism by which substructure might inhibit cluster formation is contained in the moment equation of motion

$$\frac{\partial}{\partial t} n\langle v_r \rangle = -ng - \frac{\partial}{\partial r} n\langle v_r^2 \rangle + n \frac{\langle v_t^2 \rangle - 2\langle v_r^2 \rangle}{r} \qquad (3)$$

for a spherically symmetric mass distribution with density $n(r)$, gravitational acceleration $g(r)$, radial velocity $v_r(r)$, and tangential velocity $v_t(r)$. Collapse and subsequent virialization could be inhibited if the inward gravitational acceleration were offset by an increased tangential velocity dispersion in the third term. It is perhaps reasonable to expect that increased substructure will amplify the tangential term as it degrades the spherical symmetry of the mass distribution.

To address this question, we compared the internal profiles of the largest groups found in each of the runs. For each cluster, the density and the radial and tangential velocity dispersions were calculated in radial bins of 250 particles, starting from the center of mass. Bins extend to a radius R_δ, defined as the radius at which the mean interior density falls to a factor of 1.5 above the background density. The profiles of the five most massive groups in each run are then averaged in log (R/R_δ) bins. Results for the control run and the $k_c = 8$ and 4 runs are shown in Figure 3.

The most striking impression from Figure 3 is the lack of any significant differences in the mean profiles of the different runs. For $0.01 < R/R_\delta < 0.1$, the density scales as $\rho \sim R^{-2}$ and the velocity dispersion is nearly constant to within $\sim$20%. In this virial regime there is no net radial flow. Exterior to this is a region of infall extending approximately through the range $0.1 < R/R_\delta < 0.4$. In the infall regime the velocity is primarily radial, directed inward. Farther out in radius, the effects of the Hubble expansion and external mass distributions take over. The ratio $v_t^2 r/GM(r)$ of the tangential dispersion term to the gravitational acceleration term is plotted in Figure 3*d*. The ratio in the infall regime is 0.1 and is approximately the same for each run. Substructure apparently is not providing a dispersion significant enough to inhibit collapse. The explanation is that the dominant contribution to v_t^2 is coming not from small-scale lumps but from bulk flow of material (clumpy or not) onto sheets and filaments extending outward from and draining onto the cluster. Power to form these filaments and sheets comes from scales comparable to that required to form the cluster itself, for it is precisely at the intersections of these sheets and filaments that rich clusters form in Gaussian random fields. The addition of small-scale clumpiness does not act to significantly retard or decohere these larger scale flows.

3. CONCLUSIONS AND DISCUSSION

We find no evidence in support of previrialization from N-body experiments employing Gaussian random initial conditions. The major piece of evidence supporting this conclusion is the fact that the abundance of rich clusters formed at the end of our experiments was insensitive to the amount of small-scale power present in the initial conditions. If previrialization had prevented the collapse of the largest systems, one would have expected a signal of decreasing abundance of rich clusters with increasing small-scale power. This signal was not seen. Our result is in accord with the conclusions of Little et al. (1991). In

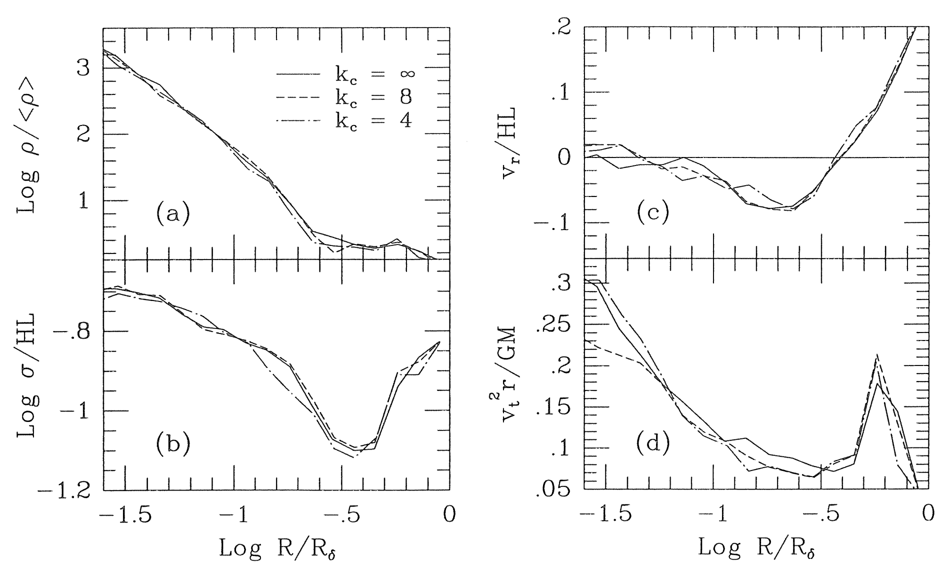

FIG. 3.—Average radial profiles of (*a*) the local overdensity, (*b*) the velocity dispersion, (*c*) the radial velocity, and (*d*) the ratio $v_t^2 r/GM$ for the five largest groups in the control experiment (*solid lines*) and runs with $k_c = 8$ (*dashed lines*) and $k_c = 4$(*dash-dot lines*). Velocities are normalized to the Hubble velocity across the box, and R_δ is the radius at which the mean interior density reaches 1.5 times the background value. No significant differences can be seen in the profiles of runs with different cutoffs.

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a similar study, they showed that structure on scales just entering the nonlinear regime was insensitive to the character of the power on smaller scales. Note that we have performed other sets of smaller (32^3 particle) simulations using power spectra with indices $n = -2$ and 0 as well as open ($\Omega_0 = 0.1$) universes, with the same qualitative result.

The alert reader will naturally be wondering why two different N-body studies came to opposite conclusions. After all, the study of Peebles (1990) seemed to demonstrate a significant effect from previrialization. The explanation of the discrepancy between the two approaches hinges on what we feel is the defining issue of previrialization, which can be posed as the question, *Does small-scale structure affect the evolution of clustering on larger scales?* Consider that we are interested in clusters of a particular mass M_c. Our numerical experiments were designed to study the role that power on scales $M < M_c$ might play in the collapse of these clusters. Peebles's numerical study targeted the effect on clusters coming from scales $M > M_c$. The nonradial motions and delayed collapse in Peebles's work came about because of external torques from the inhomogeneous surroundings, not from substructure within the test cluster itself. By focusing on the role of external torques in cluster evolution, the study did not directly test the question of whether small-scale structure inhibits collapse on larger scales. One might expect that the shear generated by torques from the nonuniform surroundings would serve to delay collapse of a perturbation, as was seen in his numerical experiments.

Regardless of any discrepancy with previous work, the reader is owed an explanation for the absence of any effect of previrialization in these experiments. A heuristic explanation supported by the work of Little et al. (1991), Frenk et al. (1988), and this study is that, statistically speaking, formation of structure on a mass scale M is driven directly by power in wave modes $k \simeq (\bar{\rho}/M)^{1/3}$. In fact, this was one of the principal conclusions of the study by Little et al. (1991), who emphasized the robustness of the formation of filaments and sheets to changes in small-scale power. Since rich clusters form at the intersections of sheets and filaments, the concurrence of their result with ours is not surprising. The simulations of Frenk et al. (1988) had demonstrated a link between collapsed objects of mass M and peaks in the initial density field smoothed on the appropriate mass scale M. This correspondence would seem unlikely if objects were sensitive to small-scale clustering, since the filtering process involved in locating peaks removes all small-scale power.

An encouraging aspect of these results is that observational data on clusters of galaxies can have direct impact on theories of large-scale structure; there is no need to add an extra obscuring layer between theory and observation.

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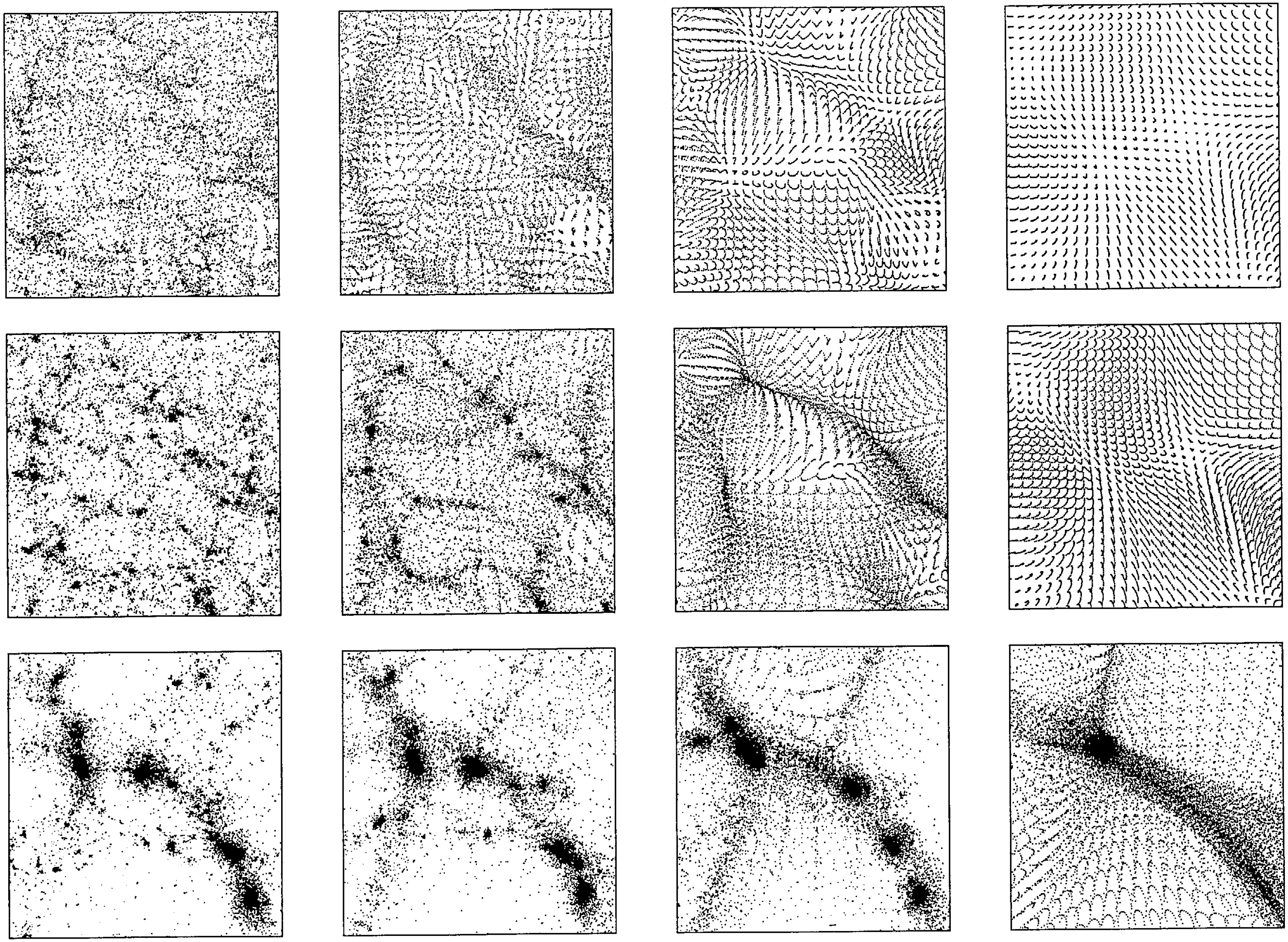

FIG. 1.—Projections of a slice of dimensions $0.4 \times 0.4 \times 0.2$ times the box length. The same slice is shown at three different epochs, $z = 3$ (*top row*), $z = 1$ (*middle row*), and $z = 0$ (*bottom row*), for the runs with (*from left to right*) no cutoff, $k_c = 8$, 4, and 2. At the final epoch the largest clusters appear at roughly the same locations regardless of the amount of small-scale power. The extreme reduction in power for the $k_c = 2$ run eliminates all but the largest cluster in the region.

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