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SOME DETAILS OF THE TRANSITION TO TURBULENT FLOW IN POISEUILLE FLOW IN A TUBE

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12. Abstract: Measurements of velocity fluctuations, Reynolds stresses and shearing stresses at the wall in the transition region of a tube are presented. The measurements are made in a tube at a Reynolds number of 6000 behind three disturbance generators placed in the fully developed laminar flow 620 diameters from the entrance. The results show the way in which some of the statistical details of the transition depend on the nature of the disturbance generated. The Reynolds stresses and the shearing stress at the wall can reach very high values during the early stages of transition. Implications are pointed out regarding possible causes for the high temperature recovery factor during transition in high speed flow over surfaces.

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INTRODUCTION

Attempts to establish some of the physical details of boundary layer transition, whether theoretical or experimental, must contend with its statistical nature as well as with the many factors influencing its appearance. Further, the small boundary layer thickness at transition makes for difficulties in measuring some of the details, such as the build-up of the Reynolds stresses from zero in the laminar layer to the value characteristic of the turbulent layer.

For this reason we were led to the utilization of Poiseuille flow in a tube for the study, first, of stability of the flow to small disturbances and, second, of transition to turbulent flow. The radius of the tube used (0.625") is far greater than the thickness of any practical laminar layer at transition. Also, it is relatively easy to introduce disturbances and to observe their rate of growth or decay with distance downstream.

The earlier experiments showed [1] that, in agreement with the results of linear theory, [2] the fully developed laminar flow is stable to small disturbances. The highest Reynolds number of the study was 12,000 and the superimposed disturbances had fundamental frequencies up to 150 c.p.s., though the observed damping of higher harmonics extends the frequency range for stability well beyond the above limit.

The same apparatus was used to observe the break-up of the regular disturbances and the development of transition as well as for the further study of transition in the current investigation. It was demonstrated[3] that the flow in the tube is unstable to disturbances whose amplitude exceeds a well-defined threshold value and that the threshold amplitude decreases markedly with increasing Reynolds number. These results indicate when the amplitude of the impressed disturbance exceeds the threshold value the destabilizing effect of the non-linear terms governs the behavior of the flow. Stuart[4] investigated theoretically the corresponding theoretical problem for two-dimensional Poiseuille flow and for flow between rotating cylinders.

Our first efforts were directed toward extending the previous study of the break-down of the regular disturbances by means of oscillograms of u, v, w at various distances downstream of the disturbance. These efforts were abandoned for the following reasons. 1. We were not able to generate sufficiently clean axially-symmetric disturbances to obtain reproducible records of the growth of the three-dimensional components or of the development of the turbulent bursts. 2. The apparatus did not permit the simultaneous records of at least two velocity components to permit visualization of the nature of the break-down. 3. The stable disturbances observed in the tube (amplitudes below the threshold value) represent the superposition of the many stable modes predicted by theory.[2] When the disturbance amplitude is increased, the threshold is exceeded and transition follows. Therefore, in an experimental study of tube flow, one is observing either a flow in which linear effects predominate or one in which non-linear effects govern. It was found that a very small range of disturbance amplitude separated these two conditions.[1] In the laminar boundary layer, on the other hand, there is one unstable mode at Reynolds numbers above the minimum critical value. The growth of this mode has been observed and some systematic results obtained relative to the development of the three-dimensional modes.[5]

Other experimental studies of Poiseuille flow[6,7] have served to demonstrate some features of the transition process. These studies are concerned mainly with the appearance and growth of turbulent spots.

The assistance of Messrs. Stanley Wallis and Charles Wooldridge in taking some of the measurements is gratefully acknowledged.

APPARATUS AND METHOD

The investigation was conducted in the same tube used in the previous investigations[1, 3]. It is 1.25'' in diameter and 70' long. Air is supplied by a storage reservoir and passes through a flow meter and settling chamber containing many screens.

The tube was carefully aligned by means of a theodolite but, as in the previous investigations, it was possible to obtain approximately axially-symmetric laminar flow only after installing a heating coil in the upper half of the settling chamber. *

Hot-wire measurements were made in the transition region downstream of the two kinds of fixed disturbance generators shown in Fig. 1. The ring disturbances were of .04'' wire suspended at 0.45a and 0.75a from the center of the tube. The step disturbances were of various heights from 0.1a to 0.36a.

The hot-wire measurements included traverses of u' and v', the root-mean-square values of the axial and radial components of the fluctuations, respectively, and $\overline{uv}$ the Reynolds stress. The hot-wire was 0.0002'' platinum and the probes were of the straight (for u') and of the X type (for v' and $\overline{uv}$). A straight wire was used for determining the velocity gradient in the immediate vicinity of the wall in the transition region. All fluctuation measurements utilized an amplifier with compensation net-work and a flat frequency response from 2 to 20000 c.p.s.

A hot-wire probe assembly for radial traverses is shown in Fig. 2. After previous calibration by means of a traveling microscope the radial position

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* Reshotko[8] has shown that by taking extraordinary precautions in the alignment of a tube he could obtain very nearly axially-symmetric velocity distributions.

of the hot-wire can be adjusted to within $\pm 0.001''$ by means of the micrometer screw. The radial traverses with the X wire yielded the quantities u', v', U and the correlation coefficient $R(u', v') = \overline{uv}/u'v'$ versus r/a. The measurements of the velocity distribution in the immediate vicinity of the wall were used to calculate the friction velocity $U_\tau = \sqrt{\tau_w/\rho}$, where τ_w is the shearing stress at the wall and ρ is the air density.

All except one set of measurements to be described later were made with a symmetrical fully-developed laminar profile prior to the insertion of the disturbance generator. The point of maximum velocity was invariably in the center of the tube in the horizontal plane but below the center in the vertical plane. The application of a small amount of heat by means of the electric heating coil in the upper ahalf of the settling chamber was sufficient to move the maximum velocity point to the axis of the tube.

RESULTS AND DISCUSSION

Measurements of u', v', uv, and τ_w were made in the transition region for the three types of disturbances shown in Fig. 1. The disturbances were placed at 620 diameters downstream of the entrance where the velocity distribution was found to be parabolic at Reynolds numbers up to at least 12000. The Reynolds number ($U_{mean}\, d/\nu$) of the current investigation was 6000.

Measurements of Friction Velocity at the Wall

The first measurements were made with the step disturbance shown in Fig. 1 with various values of step height. The hot-wire placed 50 diameters downstream of the disturbance indicated whether the flow was laminar or turbulent. For each step height the flow velocity was increased until turbulent flow was indicated at the hot-wire position. The lowest Reynolds number, is plotted versus step height in Fig. 3. The curve indicates an asymptote near Re = 2000, in agreement with many previous investigations. The experimental points shown are the lowest for which the turbulent flow was continuous at the measuring station, 50 diameters downstream of the disturbance; at the lower Reynolds numbers the steps produced bursts which persisted as far as the hot-wire location. Rotta[7] and Lindgren[6] showed that at Reynolds numbers below 2000 the duration of the bursts decreased with distance downstream. The results shown in the figure support the previous results[3] indicating the existence of a threshold disturbance which must be exceeded before transition will occur.

Preliminary to the measurements of the shearing stress at the wall it was necessary to investigate the effect of the hot-wire temperature on the indicated velocity very near the surface. Heat conduction across the steep temperature gradient between the wire and the wall will tend to cool the wire and the indicated air velocity will therefore be higher than that actually existing. To test this effect traverses near the wall were made with three "overheating ratios" a_w, where

$$a_w = \frac{R - R_o}{R_o}$$

R is the resistance of the wire at the working temperature and R_o is its resistance at room temperature. The results are shown in Fig. 4. The results for $a_w = 1$ are obviously high but the fact that those for $a_w = 1/4$ and $1/2$ are parallel was interpreted to mean that the use of either of these values should give an accurate value for the shearing stress at the wall.* The indicated velocities clearly diverge from the linear relation in the immediate vicinity of the wall. As a result of these measurements the shearing stresses at the wall were measured with an overheating ratio of 1/4.

Plots of friction velocity $U_\tau = \sqrt{\tau_w/\rho}$ for four conditions at a Reynolds number of 6000 are shown in Fig 5. The friction velocity found by Nikuradse[9] is 0.60 ft/sec whereas our measurements show values of .48 to .53 ft/sec after about 15 diameters downstream of the disturbance. The reason for the discrepancy is not apparent but it is probably attributable to errors in the interpretation of the hot-wire readings near the wall.

For the annular ring disturbances the friction velocity reaches a peak dependent on the position of the ring. Later results (Figs. 12 and 16) show that these high values occur at ring positions for which the Reynolds stress behind the

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* The fact that the straight lines do not converge to zero at y = o is interpreted as due to a small error in determining the zero position and to slight curvature of the wire. The curvature will increase with increasing wire temperature.

disturbances reaches values far above those in the fully-developed flow.

For the step disturbance, if precautions were taken to obtain an axially-symmetric laminar flow (by applying heat in the settling chamber), transition occured in turbulent bursts with high fluctuations. At $x/d < 10$ the bursts were so irregular that we were unable to obtain reliable measurements either of the fluctuations in the main stream or of the friction velocity at the wall. At $x/d > 10$ the bursts had grown in the stream direction to the extent that measurements were possible. Two points at $x/d = 10$ and 24 are shown. At $x/d = 10$ the bursts still existed, but the measured $U_\tau = 0.61$ was higher than that in the fully developed flow. No measurements of the intermittency factor were made for the reason that their interpretation would be quite difficult considering the fact that between bursts the profile is probably quite different from the undisturbed laminar distribution. In measurements in tubes[6,7] and in boundary layers[10] in which the bursts or turbulent spots are not caused by surface irregularities it is assumed that the laminar distribution obtains between bursts. Consequently, under those circumstances, some conclusions can be drawn regarding the velocity distributions during the bursts from the measured intermittency factor and the measured mean velocity profile in the transition region.

When no precautions were taken to obtain an axially-symmetric laminar flow the transition was relatively steady, and measurements of the friction velocity could be made at all values of x/d. The curve shown in Fig. 5 reaches a peak of about 0.65 ft/sec at about 11 diameters.

Radial Distributions for Step Disturbance

Figures 6 through 9 show measurements of $u'/U_{\tau\infty}$, $v'/U_{\tau\infty}$, $\tau/\tau_{w\infty}$ and U/U_{mean}, respectively for a step height h/a = 0.12 and Re = 6000. In Fig. 8 $\tau = -\rho\overline{uv} + \mu\,\partial U/\partial y$ is the total shearing stress and τ_w is the measured shearing stress at the wall.* The values of $U_{\tau\infty}$ used were those measured at the wall in the fully developed turbulent flow. In Fig. 9 the parabolic laminar profile is shown for comparison.

As was mentioned above, for $x/d < 10$ the turbulent bursts caused such large intermittent fluctuations that accurate measurements were not possible. Even at $x/d = 10$, u', v' and $\overline{uv}$ reach extremely high values. There is a rapid decay of the fluctuations in the region $10 < x/d < 16$ to values in the vicinity of those measured by Laufer[11] in fully developed turbulent flow at a Reynolds number of 50,000. The shearing stress (Fig. 8) is very high at $x/d = 10$ but falls appreciably below the theoretical linear relation for fully-developed turbulent flow at $x/d = 16$, 24 and 48.

The low values of the measured shearing stresses is attributable at least in part to the fact that the X hot-wire averages over a radial distance of about 0.1 inches or about 1/6 of the radius. The effect of the finite scale of the turbulence will be to cause the measured values of the Reynolds stress $\overline{uv}$ to decrease as the dimensions of the measuring instrument increase.

The mean velocity measurements shown in Fig. 9 show that the velocity distribution at $x/d = 24$, is practically fully developed. Between $x/d = 10$ and 16 the distribution develops appreciable departure from axial symmetry.

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* Throughout the flow, except in the immediate vicinity of the wall, $\mu\,\partial U/\partial y$ is not greater than $0.1\,(-\rho\overline{uv})$. Therefore distributions of the Reynolds stresses $(-\rho\overline{uv})$ are not far different from those of the total shearing stress. The two terms will be used interchangeably.

Radial Distributions for Annular Ring Disturbance

Figs. 10 through 13 and 14 through 17 show, respectively, measurements of u'/U_{τ_∞}, v'/U_{τ_∞}, τ/τ_{w_∞} and U/U_{mean} for the ring disturbance at r/a = .45 and .75 at various x/d locations.

In general the turbulent wake builds up behind the disturbance and spreads as we proceed downstream until it fills the tube and gradually establishes the distributions characteristic of the fully developed turbulent layer. The distributions at x/d = 24 appear to be fully developed within the experimental accuracy.

In Figs. 12 and 16 the distributions of τ/τ_{w_∞}, where $\tau_{w_\infty} = \rho U_{\tau_\infty}^2$ are extended to the wall by means of the measured values of U_τ shown in Fig. 5.

As with the measurements with the step disturbance shown in Figs. 6 through 9 the finite dimensions of the instrument probably cause the measured values to be lower than the actual. In Fig. 12 at x/d = 24 the shearing stress distribution on one side is nearly that required by theory but on the other side it is low over most of the traverse. In Fig. 16 a similar behavior is noted.

One can form a qualitative picture of the build-up of the flow that accounts for the high values of τ at the wall. The high Reynolds stresses in the wake behind the disturbance elements tend to flatten the velocity profile in this region. The flattening is seen in the mean velocity profiles of Figs. 13 and 17; one actually sees a dip in the profile at one diameter behind the element for r/a = 0.45 (Fig. 13). The flattening must result in an increase in the velocity gradient at the wall and hence in the wall shearing stress. When the disturbance is placed at 0.75 a instead of at 0.45 a the region of high Reynolds stress occurs nearer the wall and the resulting flattening of the velocity profile would cause a relatively higher shearing stress at the wall. This behavior is demonstrated in Fig. 5.

Apparently this process does not continue indefinitely because for the step disturbance, the peak in U_{τ} is much lower than it is for the other two disturbances shown in Fig. 5. It would be interesting to fill in the gap by placing the ring at various radial distances between .75a and a.

Discussion of Corresponding Boundary Layer Phenomena

Whether high wall stresses and high Reynolds stresses, such as are reported here for transition in a tube, occur during transition in boundary layers can only be inferred from other measurements.

For instance, many investigations of high speed flow over bodies have reported higher temperature recovery factors[12] in the transition region than in either the laminar or turbulent boundary layer regions. This behavior may be rationalized if high Reynolds stresses exist in the transition region. These high Reynolds stresses indicate that the transport of air from outside the boundary layer to the vicinity of the wall will be a relatively efficient process in the sense that the reduction in stagnation temperature through the turbulent portion of the transition layer will tend to be less than for the lower Reynolds stresses in the developed turbulent layer. Consequently, the recovery temperature, that is, the stagnation temperature at the insulated wall, will be nearer the stagnation temperature of the outside stream than it is in the developed turbulent layer.

High values of the wall shearing stress, such as were observed here for transition in a tube, could lead to high rates of heat transfer in the transition region of a boundary layer. The fact that the peak wall shearing stress depends upon the manner in which transition in the tube is excited (Fig. 5) indicates that high heat transfer rates may be controlled if the character of the disturbance causing transition can be controlled.

Conclusions:

1. The transition Reynolds number for fully developed laminar flow in a tube decreases as the magnitude of the superimposed disturbance increases.

2. The nature of the transition process is strongly dependent on the nature of the disturbance generated.

3. When the disturbance generator is placed in the laminar flow away from the wall high shearing stresses at the wall are measured. The maximum value of these stresses depends on the radial distance from the disturbance generator to the wall.

4. The measurements provide some clues to the cause of the high rate of heat transfer and high recovery factor occuring in the transition region in high speed flow.

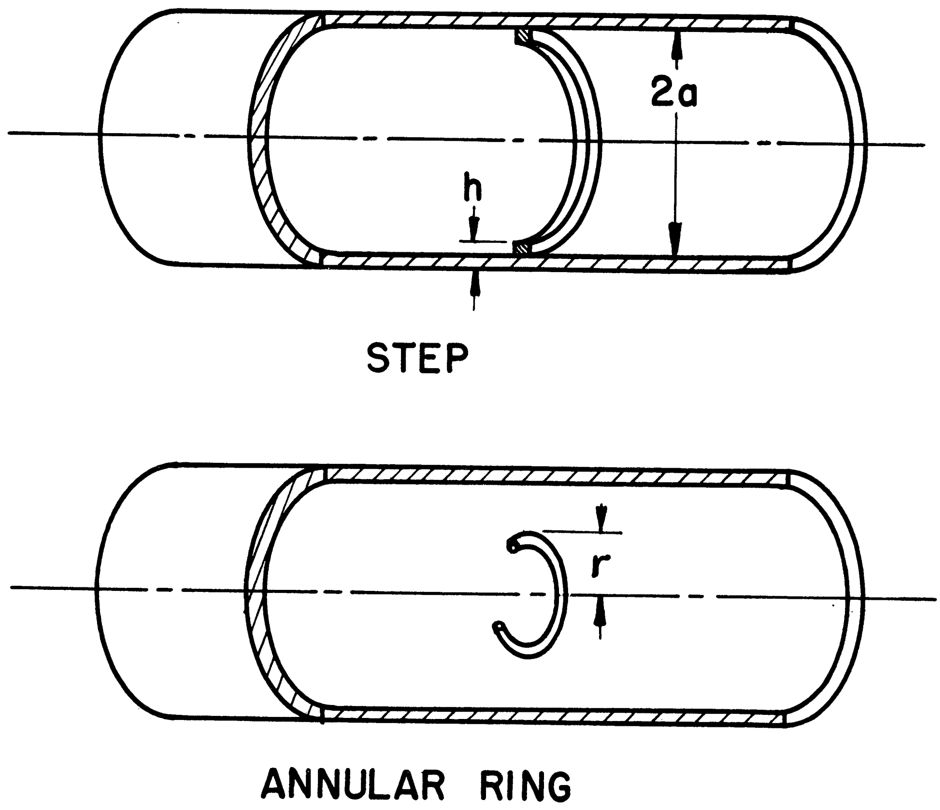

Fig. 1 Two types of disturbance generators used.

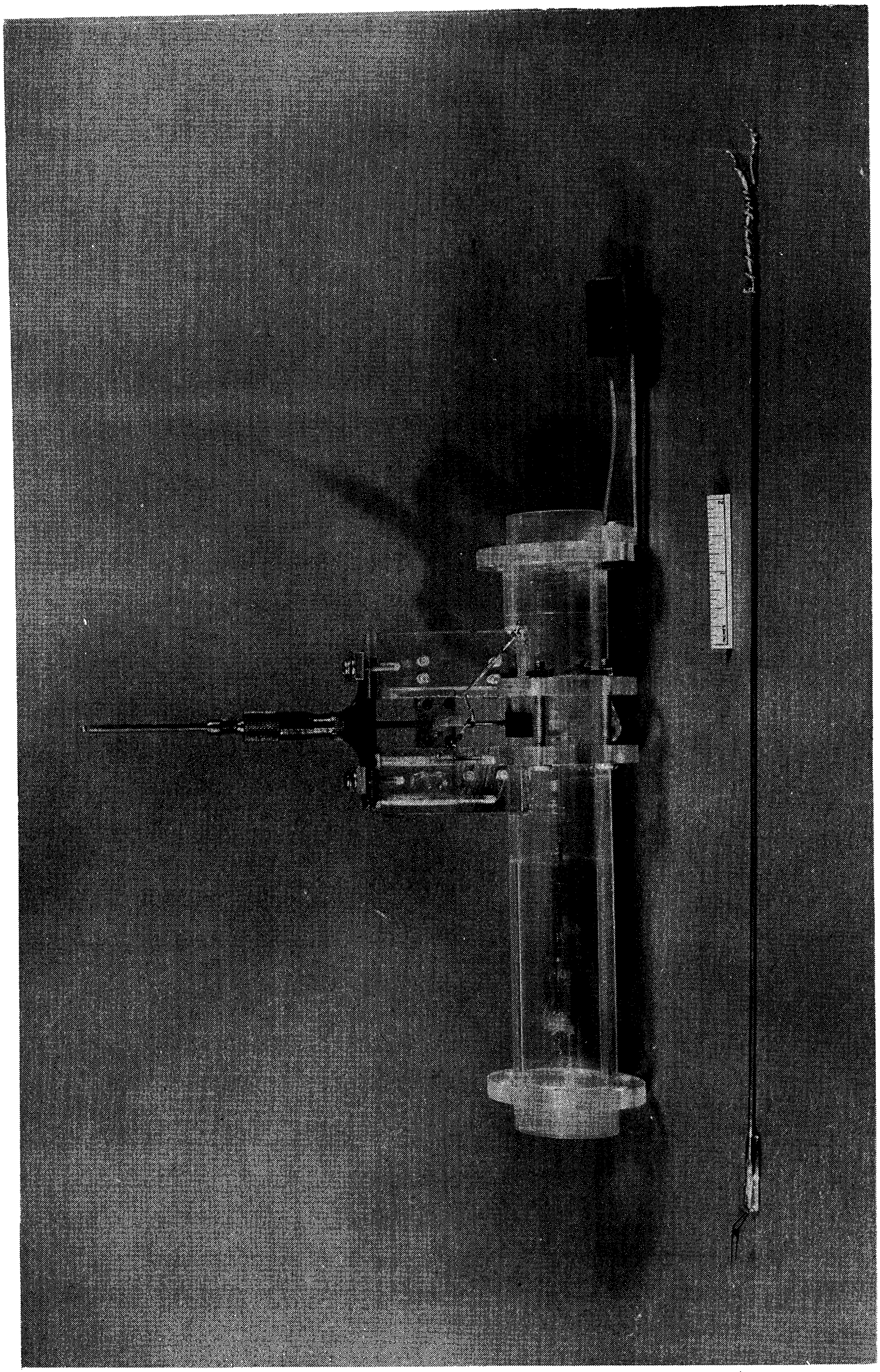

Figure 2. Hot-wire probe for radial traverses.

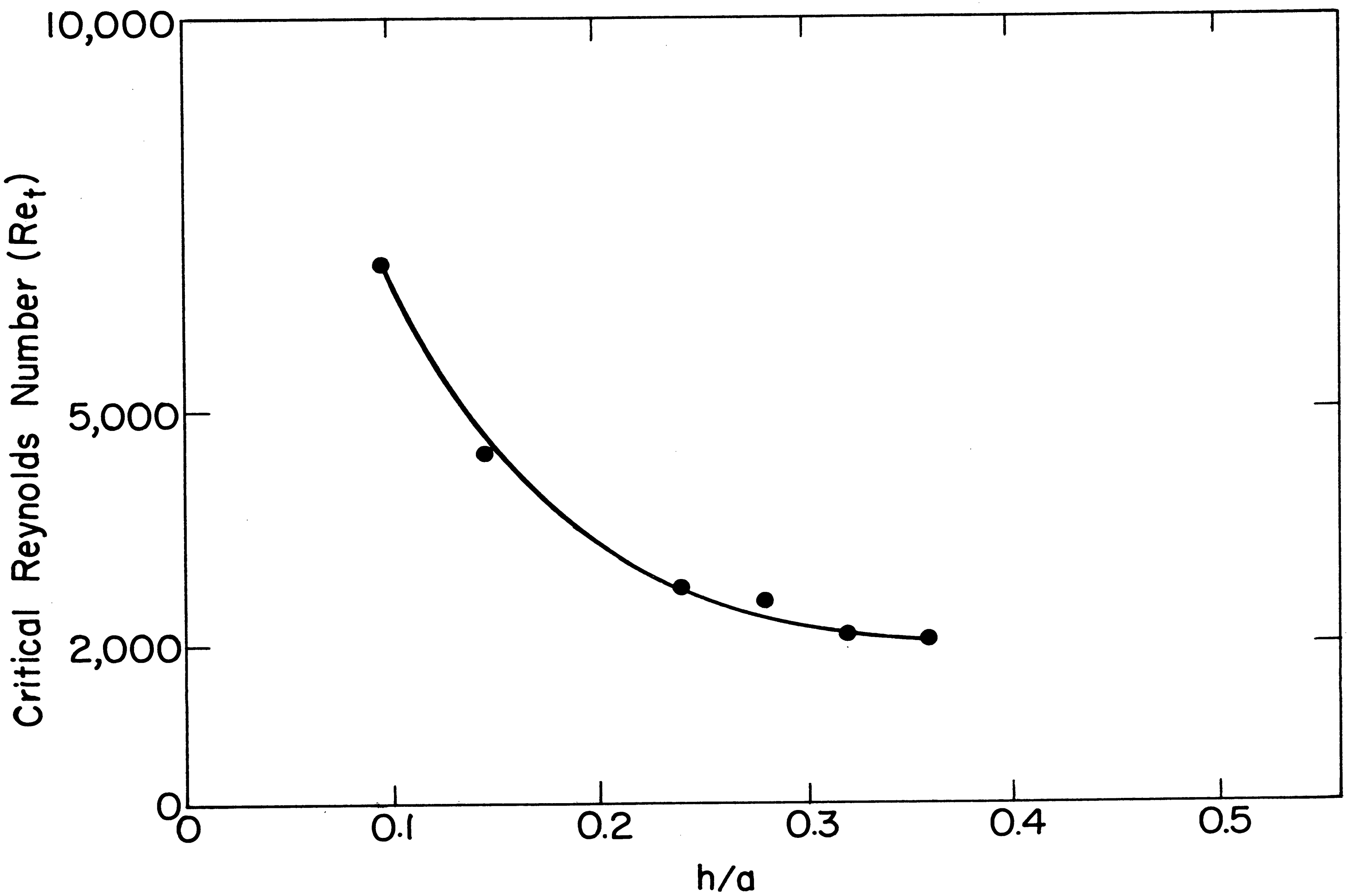

Fig. 3 Critical Reynolds number for transition as function of step height h/a.

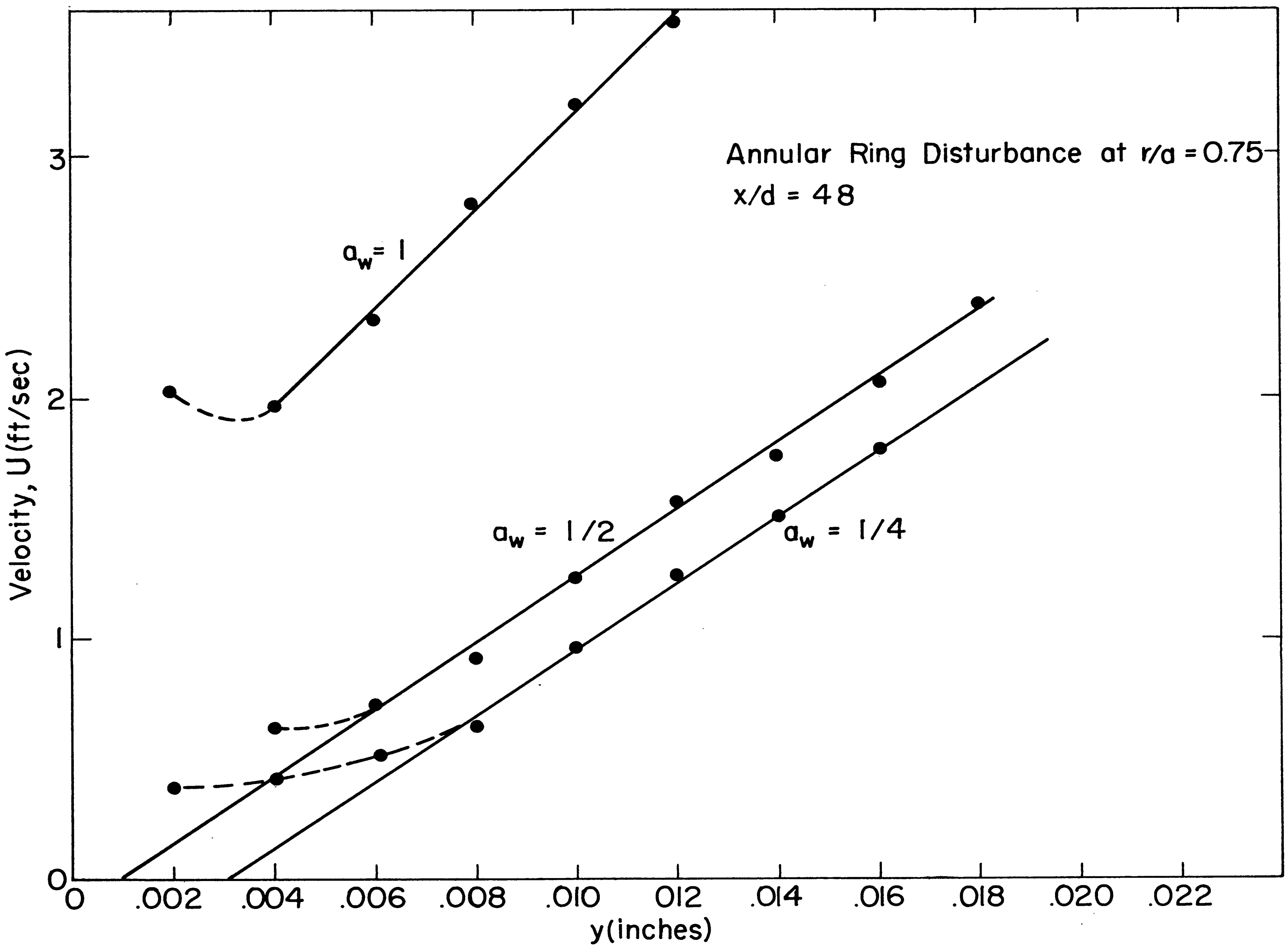

Fig. 4 Effect of overheating ratio of wire on velocity measurements near wall.

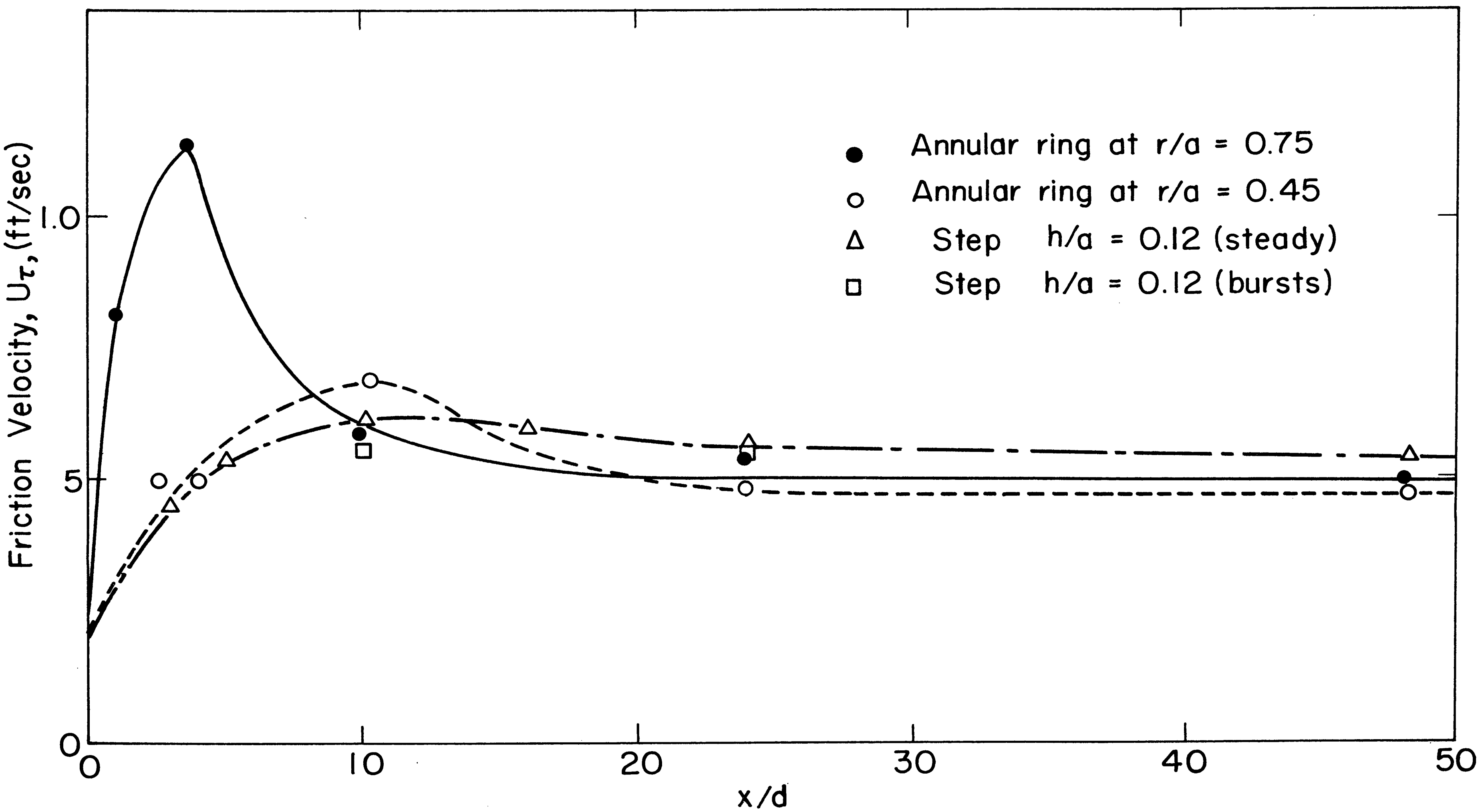

Fig. 5 Measurements of friction velocity in transition region behind ring and step disturbances.

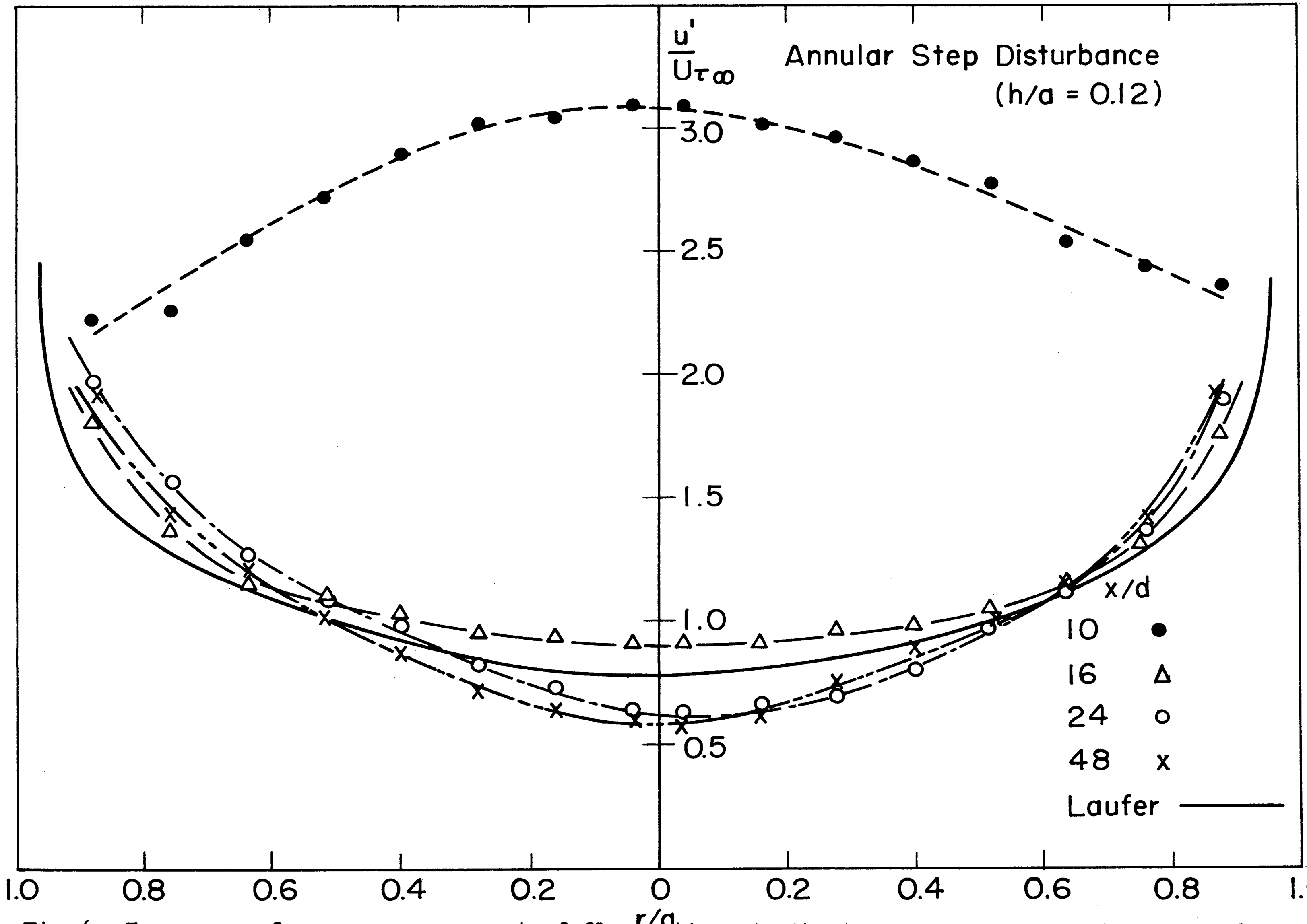

Fig. 6 Traverses of r.m.s x component of fluctuations in the transition region behind step disturbances at Re=6000. Comparison with Laufer[11] at Re=50,000

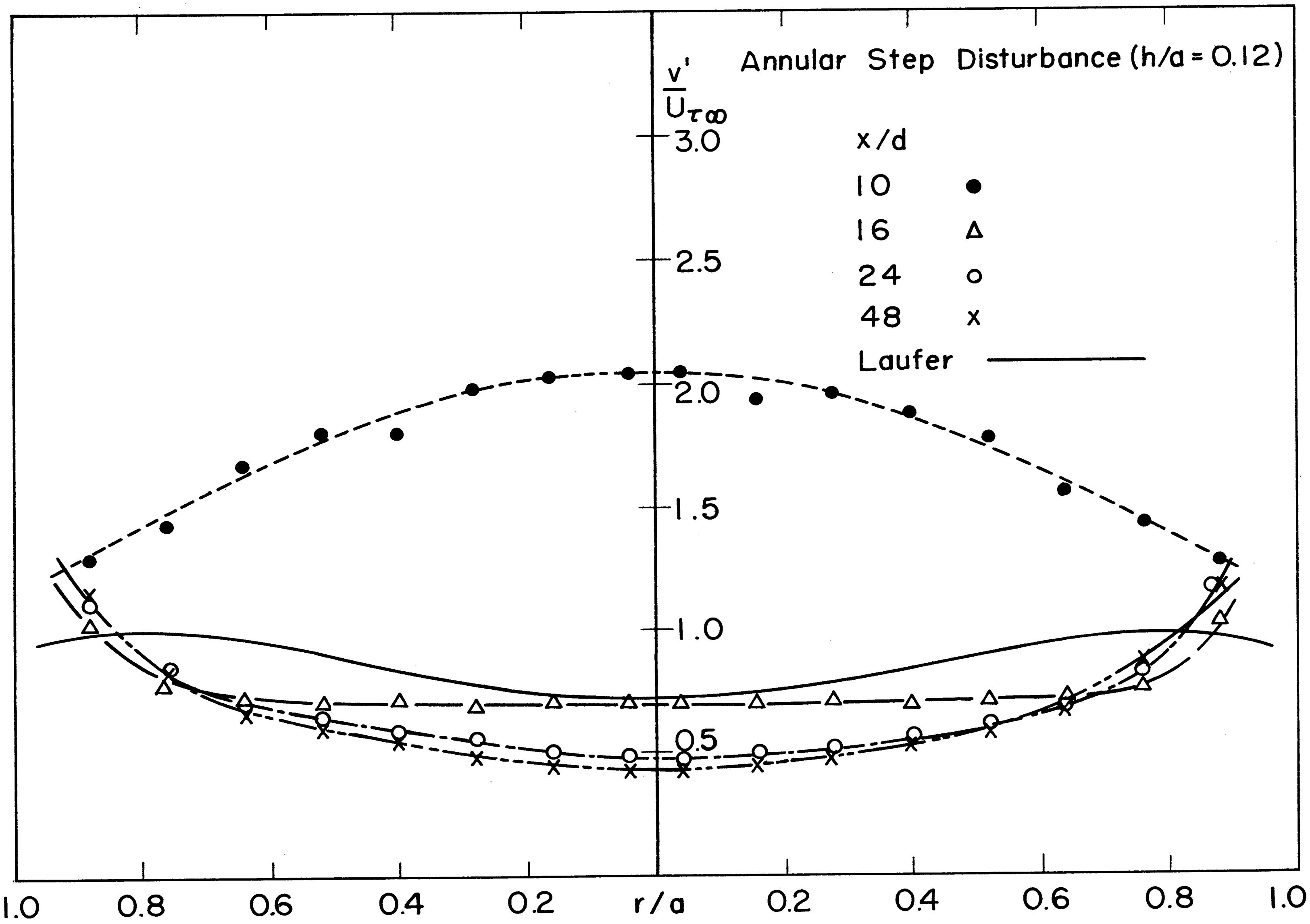

Fig. 7 Traverses of r.m.s. r component of fluctuations in the transition region behind step disturbances at Re = 6000. Comparison with Laufer[11] at Re = 50,000.

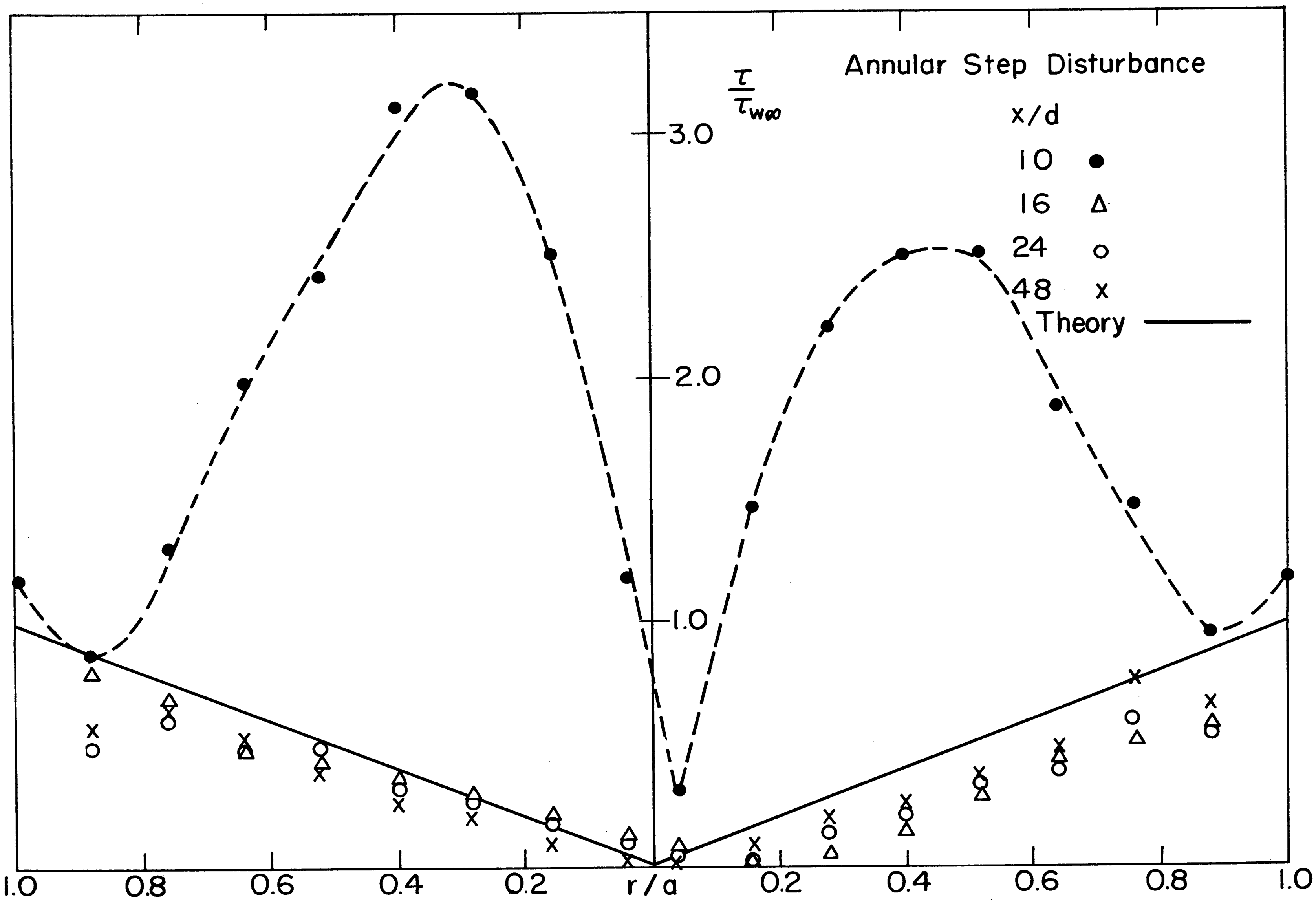

Fig. 8 Traverses of total shearing stress in the transition region behind step disturbances at Re=6000. Comparison with Theory

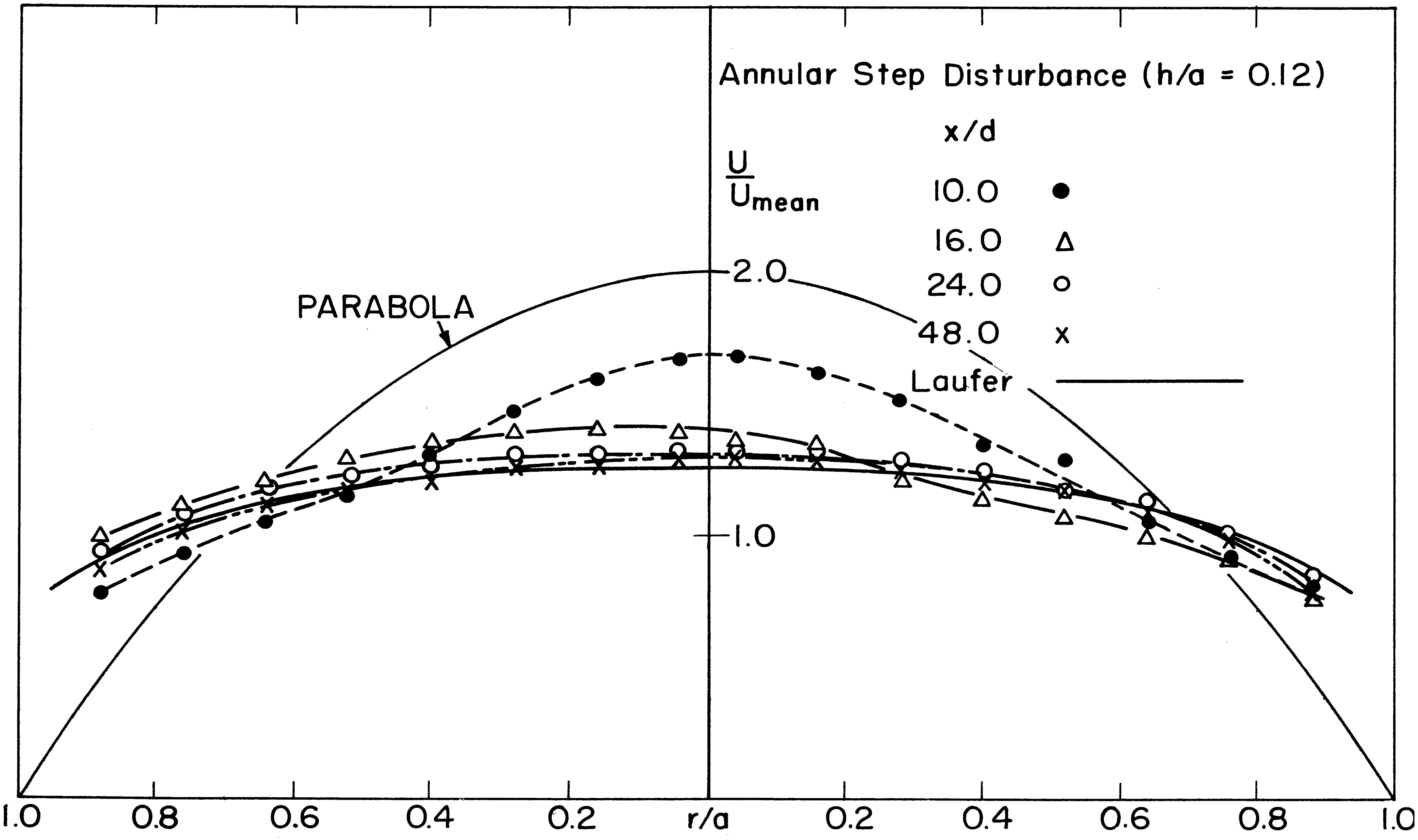

Fig. 9 Traverses of mean velocity in the transition region behind step disturbance at Re = 6,000. Comparison with Laufer[11] at Re=50,000

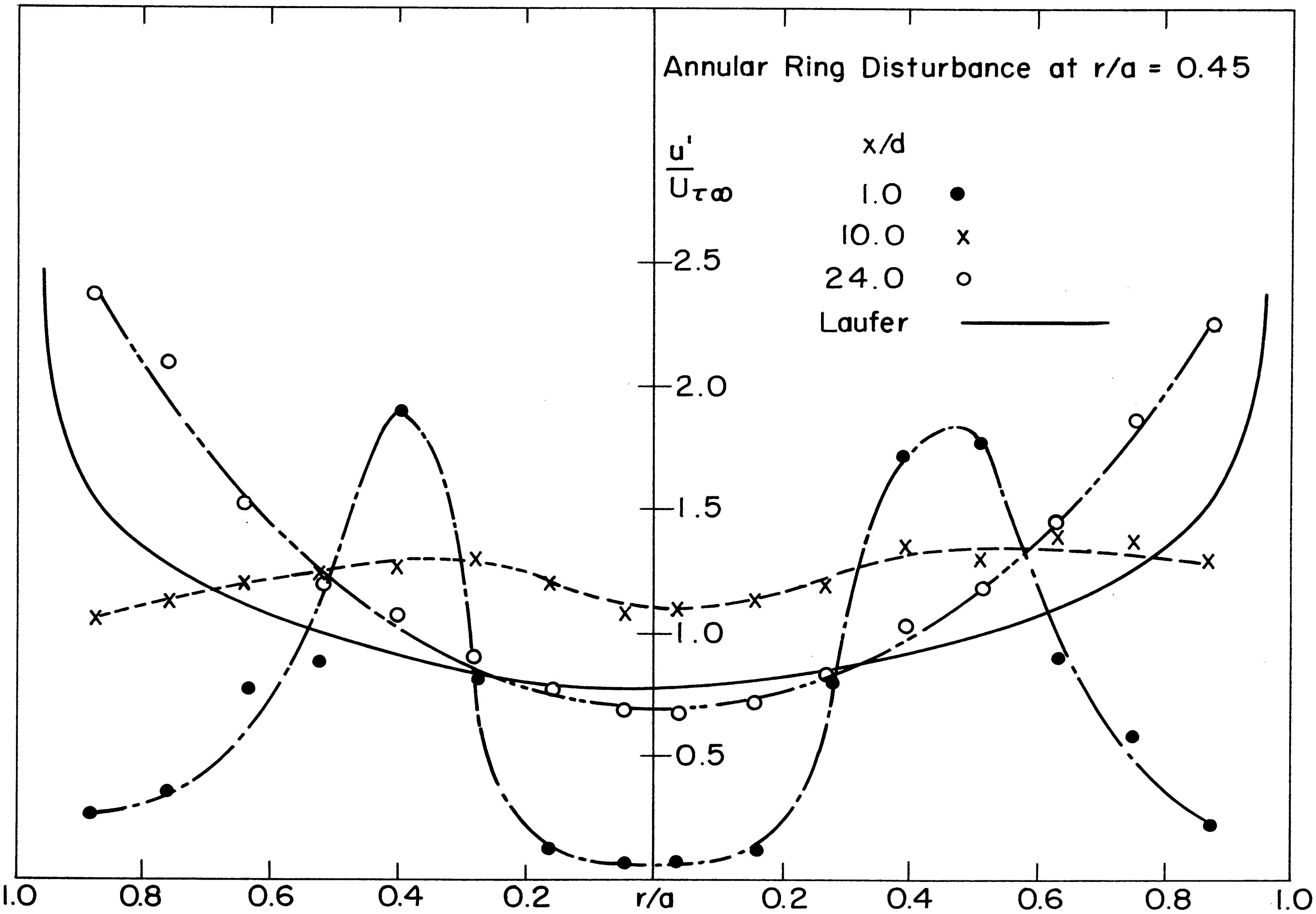

Fig. 10 Traverses of r.m.s. x component of fluctuations in the transition region behind annular ring at 0.45 at Re=6000. Comparism with Laufer[11] at Re=50,000

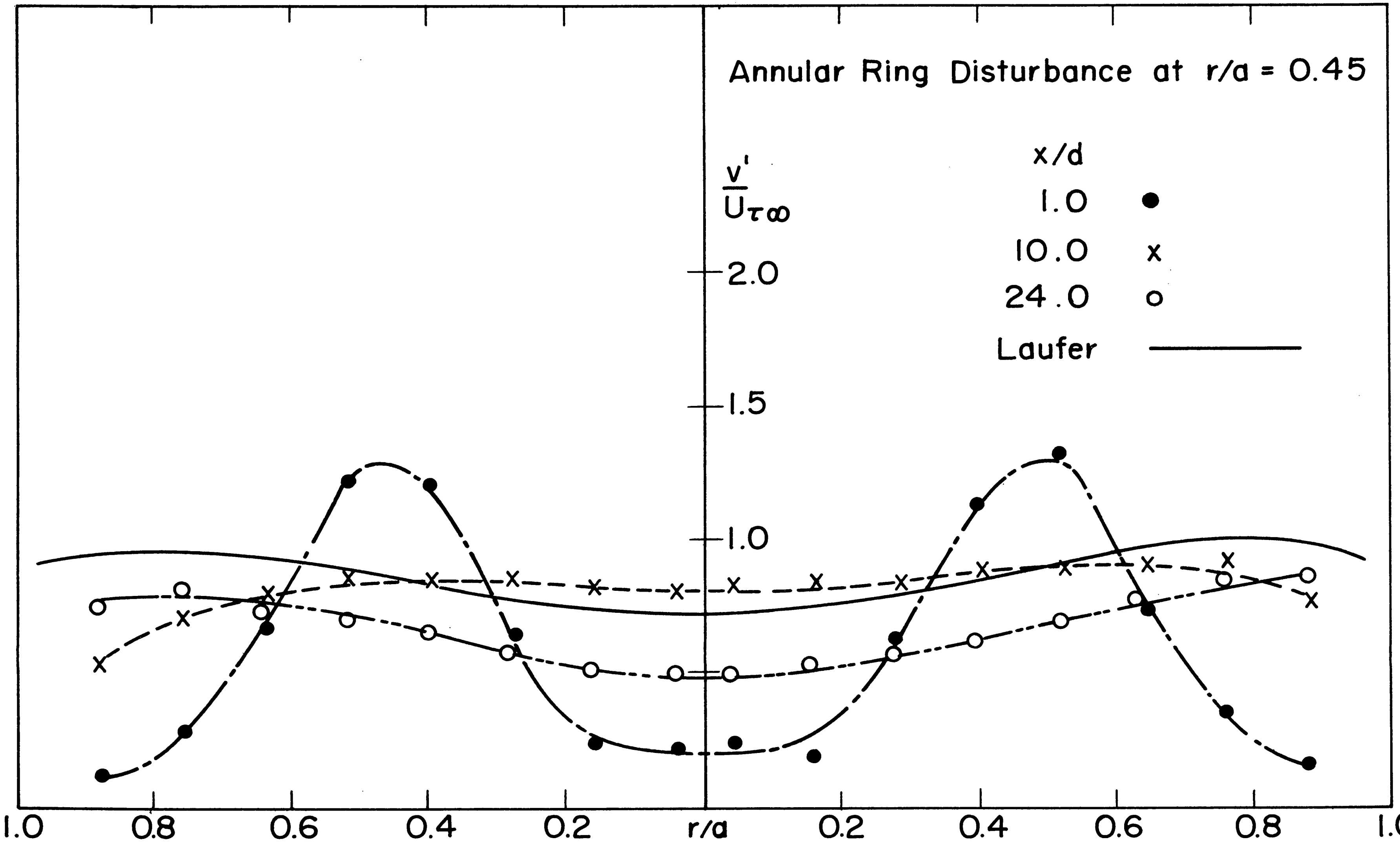

Fig. 11 Traverses of r.m.s r component of fluctuations jin the transition region behind annular ring at 0. + 5a at Re = 6000. Comparison with Laufer[11] at Re = 50,000.

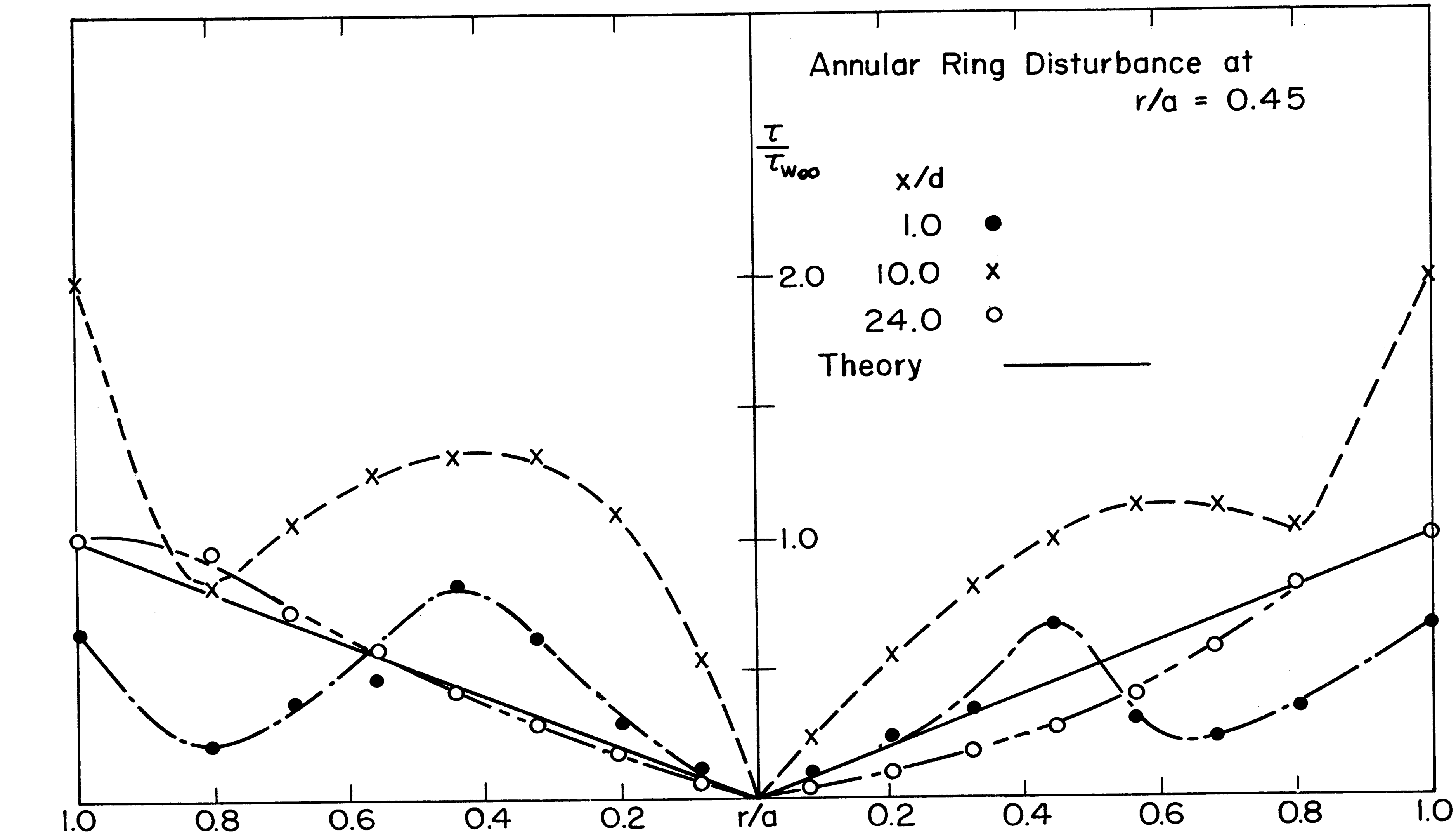

Fig. 12 Traverses of total shearing stress in the transition region behind annular ring at 0. 45a at Re = 6000. Comparison with Theory.

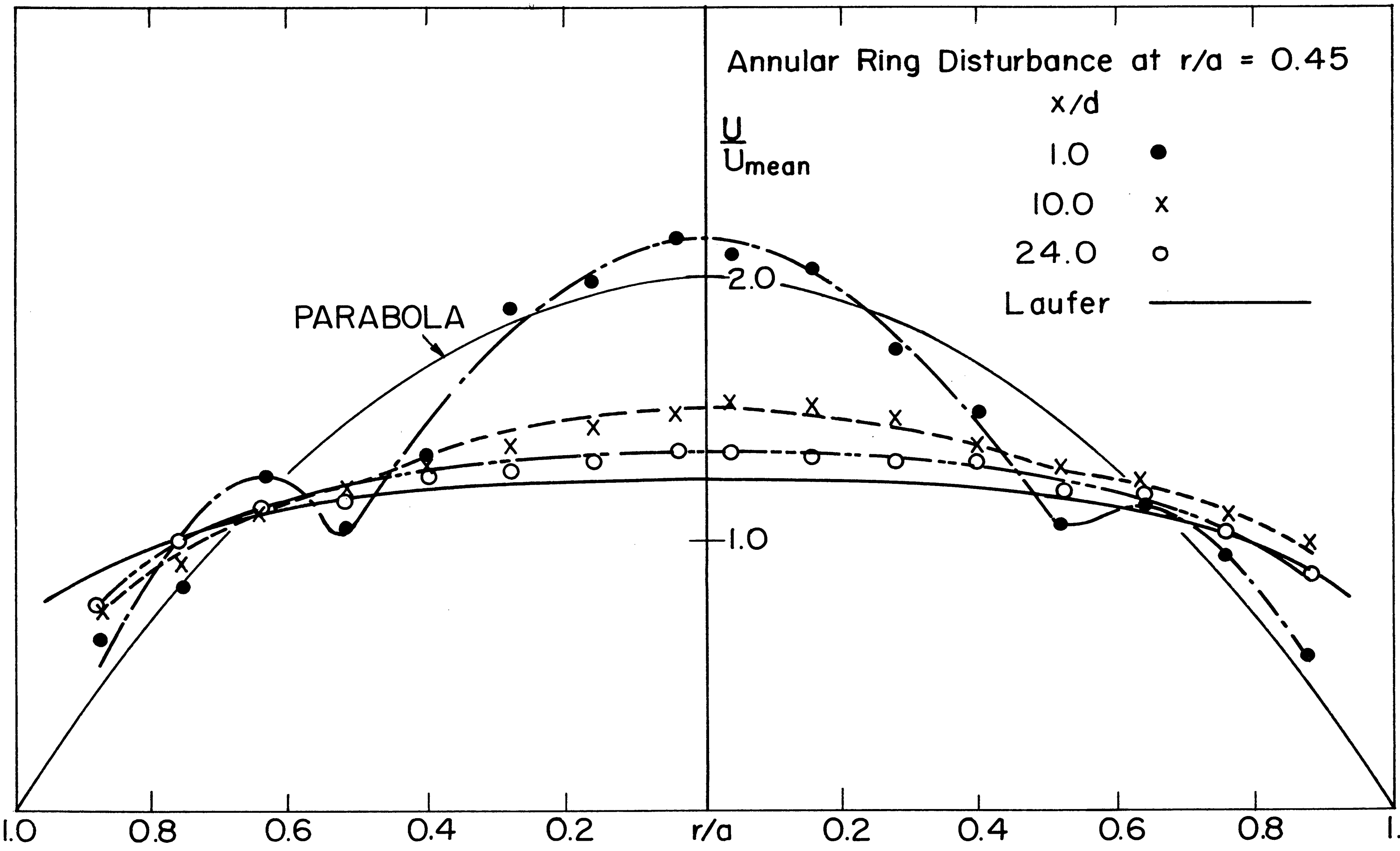

Fig. 13 Traverses of mean velocity in the transition region behind annular ring at 0.45a at Re = 6000. Comparison with Laufer[11] at Re = 50,000.

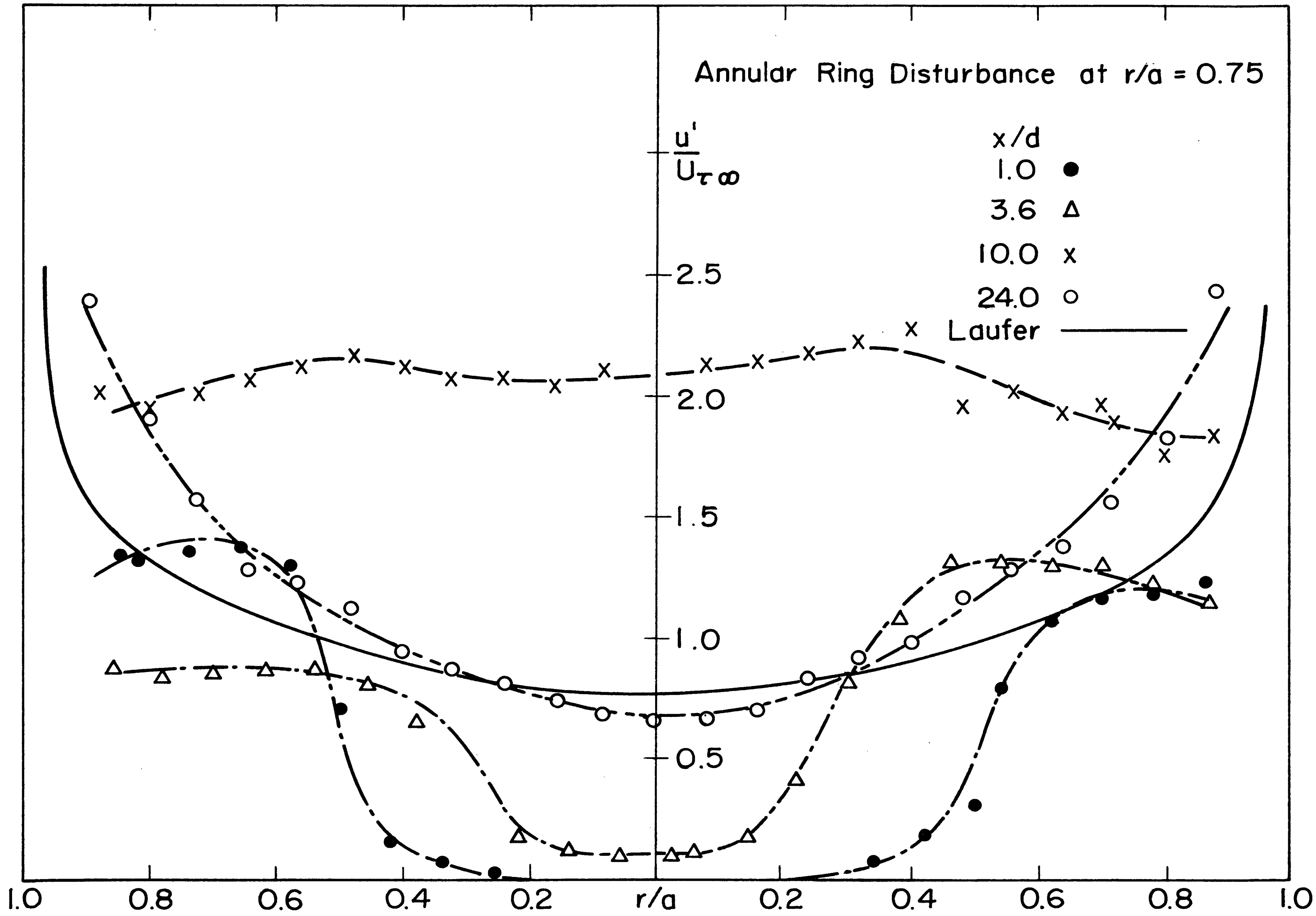

Fig. 14 Traverses of r. m. s. u component of fluctuations in the transition region behind annular ring at 0. 75a at Re = 6000. Comparison with Laufer at Re = 50, 000.

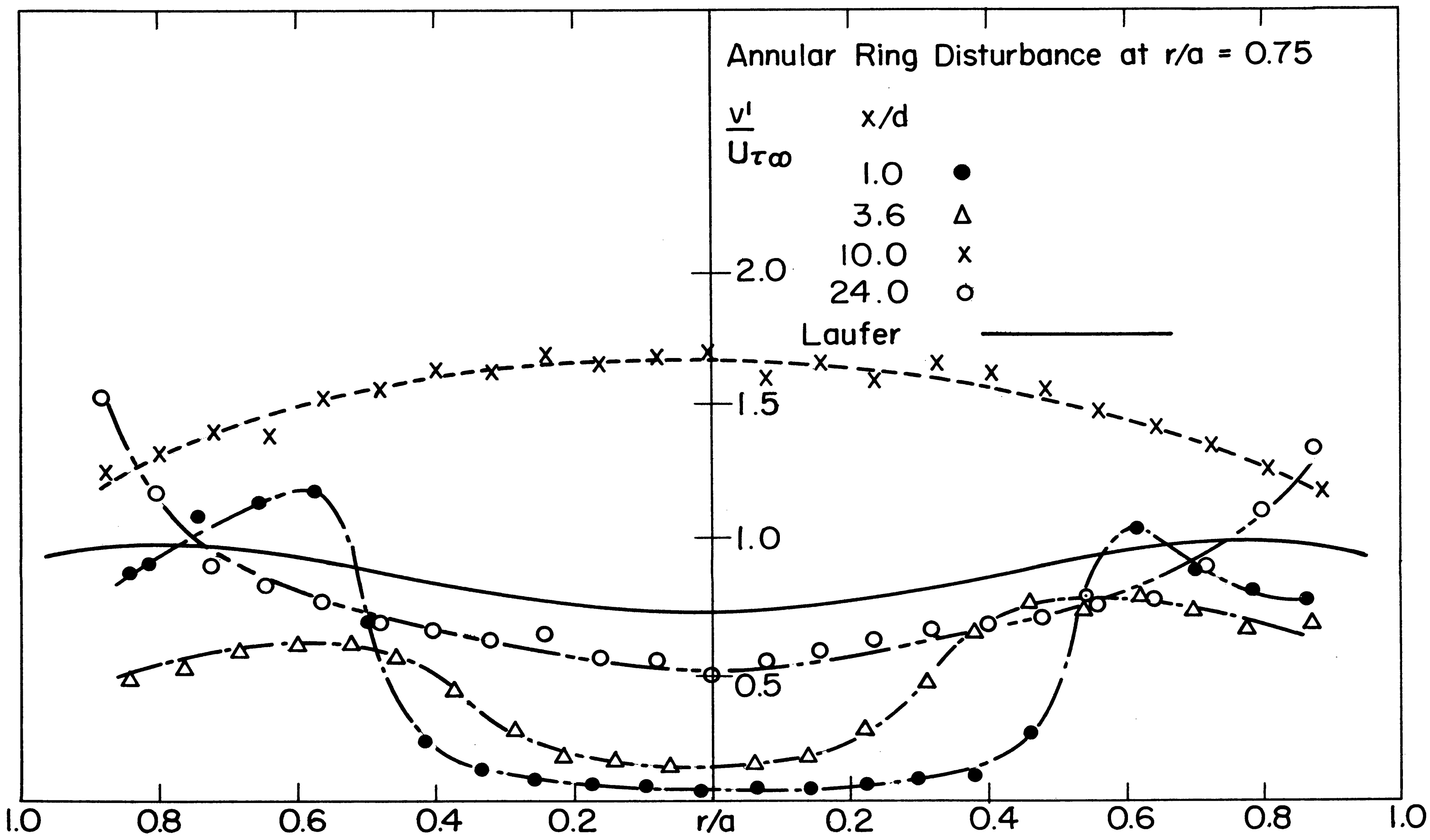

Fig. 15 Traverses of r. m. s. r component of fluctuations in the transition region behind annular ring at 0. 75a at Re = 6000. Comparison with Laufer[11] at Re = 50, 000.

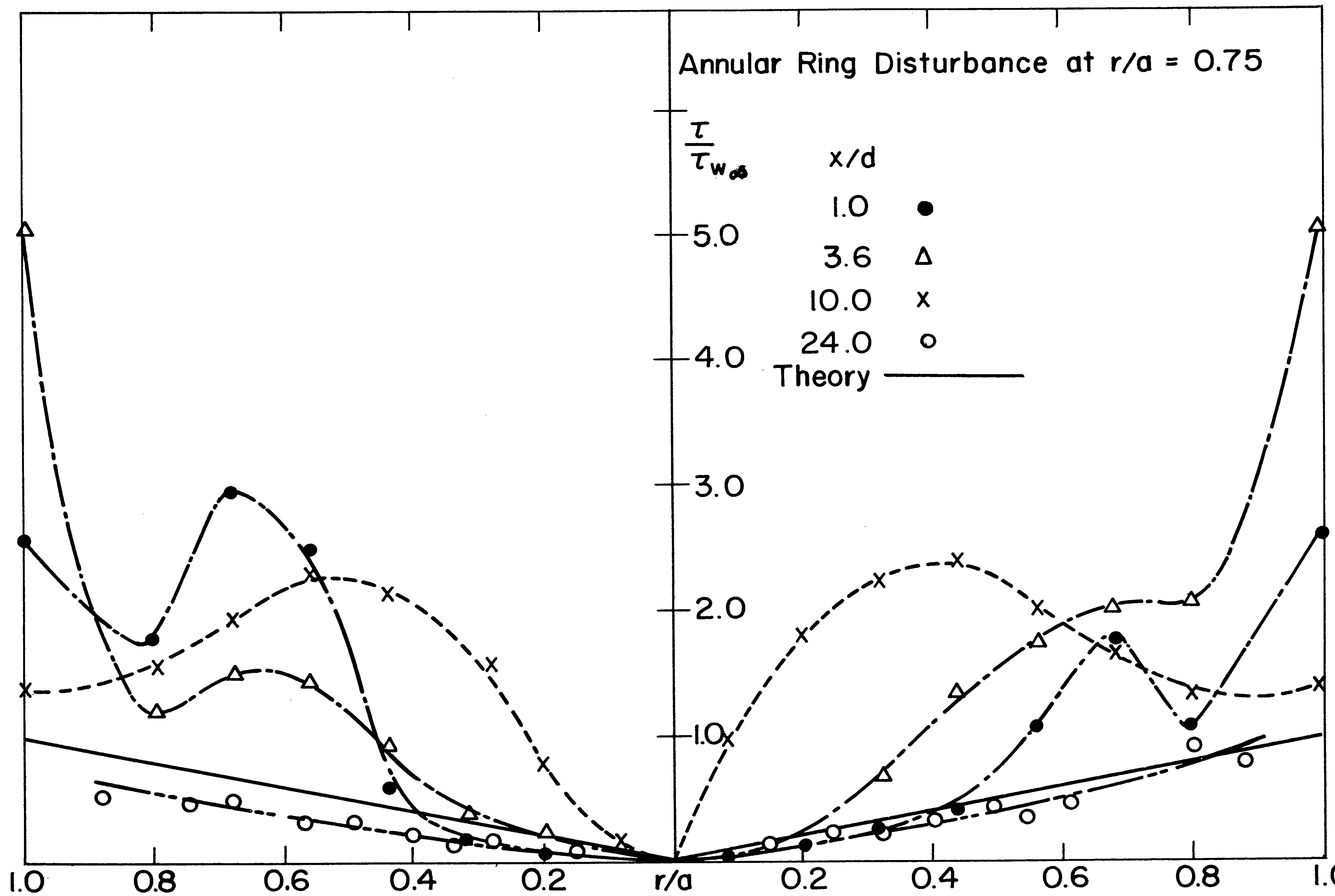

Fig. 16 Traverses of total shearing stress in the transition region behind annular ring at 0. 75a at Re = 6000. Comparison with Theory.

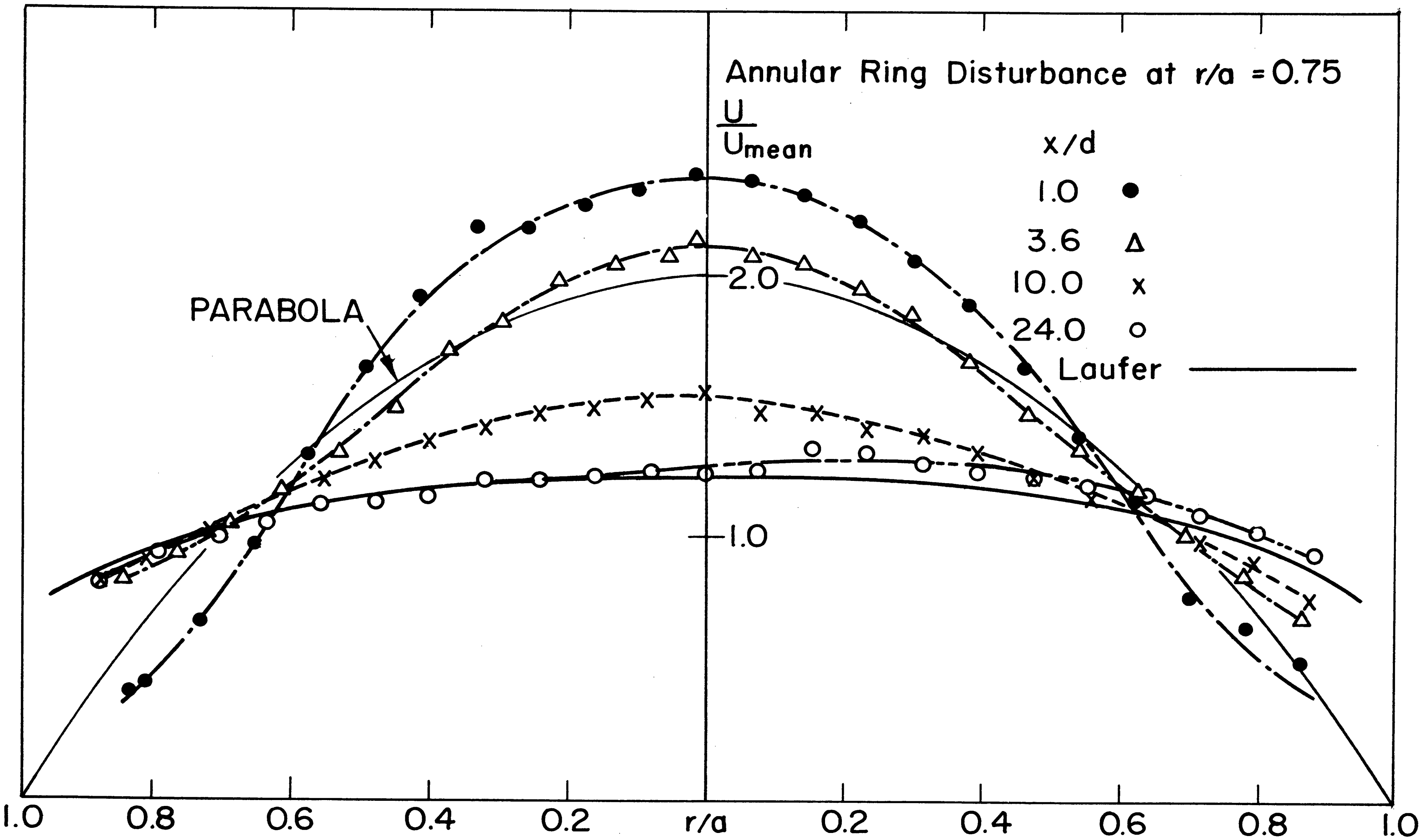

Fig. 17 Traverses of mean velocity in the transition region behind annular ring at 0.75a at Re = 6000. Comparison with Laufer[11] at Re = 50,000.

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