THE UNIVERSITY OF MICHIGAN

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A PROPOSED METHOD FOR THE STRESS ANALYSIS OF PROPELLER BLADES

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INTRODUCTION

In the early stages of propeller design, the engineer must attempt to estimate stresses in his proposed blade design. This operation is quite troublesome since available techniques have been either too simple to be considered accurate or so complicated as to require completion of design before use. The object of this paper is to present a method of preliminary design stress analysis developed by the writer. It is believed that this system will allow a relatively accurate check to be made on blade stresses in the earliest design stages.

Admiral David W. Taylor's stress theory, while relatively simple to use, is based on outmoded ogival sections and is really so overly simplified as to raise doubts as to its accuracy. Later attempts at improvement on Taylor's method have culminated in the vortex theory which is unsuitable for preliminary design purposes. Many designers prefer Taylor's method, even for final analysis, since even the vortex theory is only approximate (mathematics of blade loading being still unknown) and large safety factors are required in any event.

The method presented in this paper is somewhat akin to Taylor's method in that both are based on the blade element theory which assumes a straight line relationship between both thrust and torque and the change of blade radius. Both theories neglect the losses due to the free vortex and blade surface friction, these being small enough to be safely overlooked in preliminary work. In both cases stresses are based on the prismatic beam theory. The method presented here is felt to be an improvement on Taylor's, however, in that centrifugal forces have been introduced as a factor and constants used are appropriate for modern airfoil blade sections. The present development has also resulted in equations which are easier to apply than are Taylor's.

Experimental results, detailed in the Appendix, tend to confirm the theory presented in this paper.

Summary of Propeller Stress Formulae

For a propeller made of bronze, the following are the maximum stresses:

Stress due to torque

$$\sigma_{MQ} = 4.18 \times 10^5 \times \frac{DHP}{N \times Z\ \ell_r t_r^2}$$

Stress due to thrust

$$\sigma_{MT} = 365 \times \frac{DHD \times D}{Z \times V_a \times \ell_r \times t_r^2}$$

Stress due to centrifugal moment

$$\sigma_{MF} = 1.08 \times 10^{-7} \frac{D^4 \times N^2 \times DAR}{Z \times \ell_r \times t_r}$$

Stress due to centrifugal force

$$\sigma_F = 10.3 \times 10^{-7} \times \frac{D^3 \times N^2 \times DAR}{Z \times \ell_r}$$

The total maximum stress

$$\sigma = -\sigma_{MQ} - \sigma_{MT} - \sigma_{MF} + \sigma_F$$

DHP = Developed area ratio = $\frac{\text{Developed Area}}{\text{Disc Area}}$

N = rpm

D = tip diameter of blades (in.)

Z = number of blades

ℓ_r = blade width at 0.2 D

t_r = maximum blade thickness at 0.2 D

1. GENERAL

A propeller rotating behind a ship is driven by the main engines through the shaft, which transfers the torque to the propeller. The main object of the propeller is to convert the torque into thrust with optimum efficiency. This is accomplished by the pitched blades acting like a screw.

The forces acting on the blades are the following:

a. Torque
b. Thrust
c. Centrifugal
d. Vibration

Each of the above forces develop stresses in the blade sections. The maximum stress will, in normally shaped blades, be at the root section where the moment due to the forces is the greatest.

2. THE TORQUE

2.1 THE TORQUE FORCE

The torque or transverse force acts normal to the propeller axis (see Fig. 1). Its magnitude can be calculated as follows:

The work transferred through shaft: $W_S = \dfrac{\text{DHP} \times 33{,}000}{N}$

DHP = Delivered horsepower at propeller's axis

N = rpm.

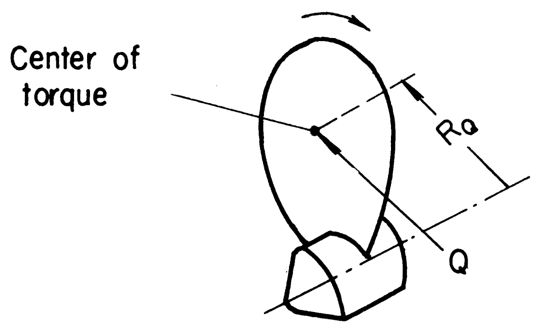

Fig. 1.

The work done by the blades: $W_b = Z \int_{R_o}^{R} dQ\ 2\pi r$

Z = number of blades

R_o = radius at root section

R = radius at tip.

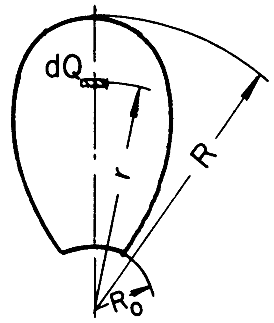

Fig. 2.

Both works are equal; thus

$$\frac{DHP \times 33{,}000}{N} = Z \int_{R_o}^{R} dQ\ 2\pi r$$

$$\frac{DHP \times 33{,}000}{Z \times N \times 2\pi} = \int_{R_o}^{R} dQr\ .$$

It can be assumed that the torque distribution along the blade radius changes linearly with the change of the radius; thus

$$\frac{dQ}{dr} = \text{constant} = K$$

$$dQ = Kdr$$

$$\frac{DHP \times 33{,}000}{Z \times N \times 2\pi} = K\int_{R_o}^{R} rdr$$

$$\frac{DHP \times 5250}{N \times Z} = \frac{K}{2}\left(R^2 - R_o^2\right)$$

$$K = \frac{DHP \times 5250}{N \times Z} \times \frac{2}{R^2 - R_o^2}$$

as

$$dQ = Kdr$$

$$Q = K\int_{R_o}^{R} dr = K(R - R_o)\ .$$

Substitute the value of K

$$Q = \left[\frac{DHP \times 5250}{NZ} \times \frac{2}{R^2 - R_o^2}\right][R - R_o]$$

$$Q = \frac{DHP \times 10{,}500}{N \times Z(R + R_o)} \tag{1}$$

at root section $R_o = 0.2\ R$

$$Q = \frac{DHP \times 10{,}500}{N \times Z \times 1.2\ R} = \frac{DHP \times 10{,}500 \times 2 \times 12}{N \times Z \times 1.2\ D}$$

$$Q = 210{,}000 \frac{DHP}{N \times Z \times D} \,. \tag{2}$$

Q = torque (lb)

N = rpm

Z = number of blades

D = tip diameter (in.)

DHP = delivered horsepower

2.2 CENTER OF TORQUE

The summation of the element torque distributed over the blade face can be presented by a single force concentrated at a point called the center of attack or torque center (see Fig. 1). Its distance from the propeller axis is R_Q.

The magnitude of R_Q can be calculated by dividing the moment torque at any section by the torque itself.

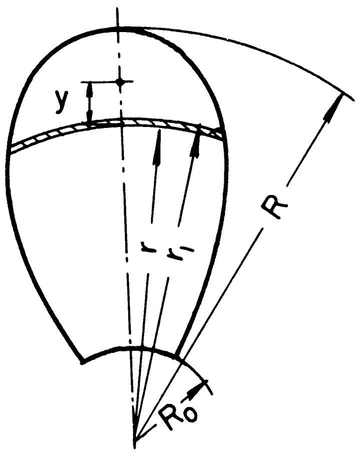

Fig. 3.

The center of torque about an arbitrary radius r is equal to

$$y = \frac{\int_{r_1}^{R} dQ(r-r_1)}{\int_{r_1}^{R} dQ} \tag{3}$$

by substituting the value of the elementary torque

$$dQ = Kdr \; .$$

The numerator will be

$$\int_{r_1}^{R} dQ(r-r_1) = K\int_{r_1}^{R} (r-r_1)dr$$

$$= K\left[\frac{r^2}{2} - r_1 r\right]_{r_1}^{R} = K\left[\left(\frac{R^2}{2} - r_1 R\right) - \left(\frac{r_1{}^2}{2} - r_1{}^2\right)\right]$$

$$= K\left[\frac{R^2}{2} - r_1 R + \frac{r_1{}^2}{2}\right] = \frac{K}{2}(R-r_1)^2$$

Developing the denominator

$$\int_{r_1}^{R} dQ = K\int_{r_1}^{R} dr = K(R-r_1) \; .$$

Substitute both values in Equation 3

$$y = \frac{K/2(R-r_1)^2}{K(R-r_1)} = \frac{R-r_1}{2}$$

$$y = \frac{R-r_1}{2} \tag{4}$$

at root section

$$r_1 = R_o = 0.2\ R$$

$$y = \frac{1}{2}(R-0.2\ R) = 0.4\ R$$

$$y = 0.2\ D \; .$$

The value of R_Q

$$R_Q = 0.4\ R + 0.2\ R = 0.6\ R$$

$$R_Q = 0.6\ R = 0.3\ D \; . \tag{5}$$

2.3 MOMENT DUE TO TORQUE

The torque moment at root section will be

$$M_Q = Q \times y ;$$

by substituting the value of y in Equation

$$M_Q = 210{,}000 \times \frac{DHP}{N \times Z \times D} \times 0.2\ D$$

$$M_Q = \frac{42{,}000 \times DHP}{N \times Z} \quad . \qquad (6)$$

3. THRUST

3.1 GENERAL

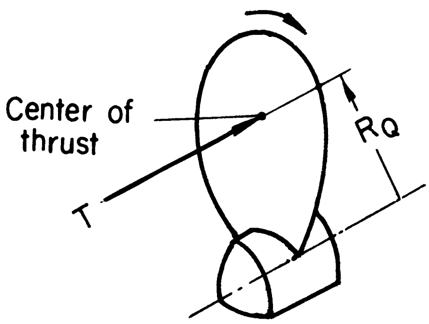

Fig. 4.

It was mentioned that the propeller converts the torque into thrust. The thrust has to overcome the resistance or the tow rope horsepower (TRHP) of the ship's hull. By the interaction of the propeller and the hull on augmentation or a thrust decuction is involved, and as a result the stream line pressure behind the ship is reduced, thus only part of the thrust developed by the propeller is transferred to overcome the hull resistance.

3.2 THE MAGNITUDE OF THE THRUST

The relation between the hull resistance and thrust is expressed as follows.

$$R = T(1-t) \tag{7}$$

The thrust being a function of the horsepower and speed of ship is expressed as

$$T = \frac{33{,}000 \text{ TRHP}}{101.33\, V_K(1-t)} \tag{8}$$

R = hull resistance

T = thrust developed by the propeller

(1-t) = thrust deduction

V_K = speed of ship in knots .

The thrust can be expressed in terms of propeller's speed and horsepower, where

$$V_a = V_K(1-w)$$

$$V_K = \frac{V_a}{(1-w)}$$

$$\text{TRHP} = \text{DHP} \times \text{P.C.}$$

Substitute in Equation 8

$$T = \frac{33{,}000 \times \text{DHP}}{101.33\, V_a} \times \frac{\text{P.C.}(1-w)}{(1-t)} \tag{9}$$

V_a = speed of advance of the propeller (knots)

w = percentage of wake

P.C. = propulsive coefficient

DHP = delivered hp at the propeller .

The following average numerical values are used for vessels with one shaft

$$\left.\begin{array}{l} 1-t = 0.80 \\ 1-w = 0.70 \\ \text{P.C.} = 0.75 \end{array}\right\} \text{For low-speed ships} \qquad \left.\begin{array}{l} 1-t = 0.95 \\ 1-w = 0.80 \\ \text{P.C.} = 0.65 \end{array}\right\} \text{For high-speed ships}$$

$$\frac{\text{P.C.} \times (1-w)}{(1-t)} = \frac{0.75 \times 0.70}{0.80} = 0.656$$

$$\frac{\text{P.C.} \times (1-w)}{(1-t)} = \frac{0.65 \times 0.80}{0.95} = 0.547$$

$$T = \frac{33{,}000 \times \text{DHP}}{Z \times 101.33 \times V_a} \times 0.656 \, .$$

For slow rpm

$$T = 214 \frac{DHP}{Z\ V_a} . \qquad (10)$$

For high rpm

$$T = 178 \frac{DHP}{Z\ V_a} .$$

3.3 THE CENTER OF THRUST

The following calculations were based on the assumption that the thrust is linearly related to the radius (see Fig. 5).

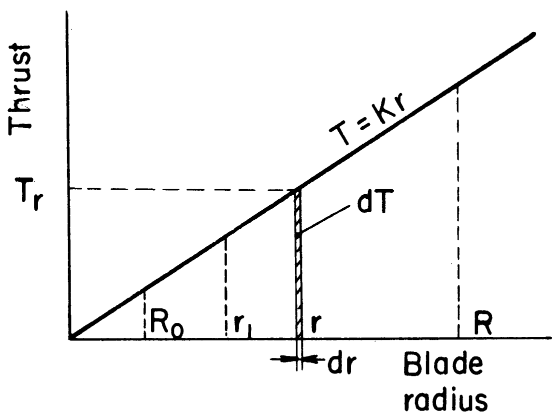

Fig. 5.

The elementary thrust

$$dT = T_r dr .$$

The total thrust

$$T = \int_{R_o}^{R} T_r dr . \qquad (11)$$

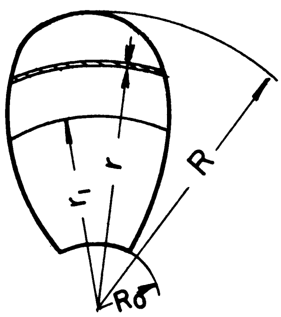

Fig. 6.

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$$M_1 = \frac{2T}{R^2-R_o^2}\left[\frac{R^3}{3} - \frac{R^2 r_1}{2} - \frac{r_1^2}{3} + \frac{r_1^3}{2}\right]$$

$$M_1 = \frac{T}{R^2-R_o^2}\left[\frac{2R^3 - 3R^2 r_1 + r_1^3}{3}\right]$$

$$M_1 = \frac{T}{3(R^2-R_o^2)}[2R^3 - 3R^2 r_1 + r_1^3]$$

at root section where $r_1 = R_o$

$$M = \frac{T}{3(R+R_o)}(2R^2 - RR_o - R_o^2) \quad . \tag{16}$$

The thrust moment about the root section can be expressed as the total thrust concentrated at the center of thrust multiplied by the lever arm to the root section.

$$M = T(R_T-R_o) \quad . \tag{17}$$

By equating Equations 16 and 17

$$R_T - R_o = \frac{2R^2 - RR_o - R_o^2}{3(R+R_o)}$$

$$R_T = R_o + \frac{2R^2 - RR_o - R_o^2}{3(R+R_o)} \tag{18}$$

of root section

$$R_o = 0.2\ R$$

$$R_T = R_o + \frac{2R^2 - 0.2\ R^2 - 0.04\ R^2}{3.6\ R}$$

$$R_T = 0.69\ R \ . \tag{19}$$

3.4 THE MOMENT DUE TO THRUST

$$M = T(R_T - R_o)$$

$$M = T(0.69\ R - 0.2\ R)$$

$$M = T \times 0.49\ R$$

Substitute the value of T from Equation 10

$$M_T = 214 \frac{DHP}{Z\ V_a} \times 0.49\ R$$

Low rpm $$M_T = 52.4 \frac{DHP \times D}{Z \times V_a} \quad (20)$$

High rpm $$M_T = 43.6 \frac{DHP \times D}{Z \times V_a}$$

4. CENTRIFUGAL FORCE

4.1 THE EQUATION OF THE CENTRIFUGAL FORCE

$$F = \frac{mv^2}{r} = mw^2r \quad (21)$$

For a given speed w is constant; thus the force will change with the change of r

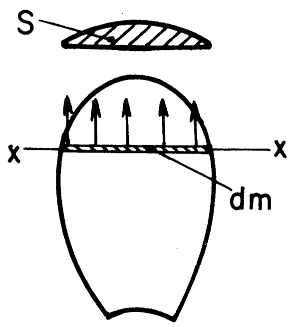


Fig. 7.

The forces at section x-x equal to the centrifugal force due to the mass beyond the section.

The elementary mass

$$dm = \frac{\gamma}{g} S\ dr \quad (22)$$

where

γ = specific weight

S = area of section .

The total force

$$F = \frac{\gamma}{g} \int_{R_o}^{R} Sw^2 \, rdr \qquad (23)$$

The expression of S as a function of r is very complicated and without approximations it can hardly be derived. This is due to the variation of the blade section thickness and shape from tip to root and also the shape of the blade contour.

The simplest expression to use will be the ratio of

$$\frac{\text{mean thickness}}{\text{maximum thickness of hub}} = \frac{tm}{t_r} .$$

This ratio was experimentally found to be 0.428 for most of the propeller blades.

The approximate weight of blade:

$$G = \gamma \times 144 \times A \times 0.428 \frac{t_r}{Z} \qquad (24)$$

G = weight of one blade (lb)
A = developed area of blade (sq ft)
γ = specific weight of material (lb per cu in.)

For bronze γ = 0.315
For cast steel γ = 0.284

t_r = maximum thickness of blade section at root
R_G = radius of center of blade weight (in.)
V_G = the velocity of the center of blade (fpm).

By experimental test the approximate distance of center of the blade's mass was found to be

$$R_G = 0.485 \, R .$$

The velocity of the center

$$V_G = \frac{2\pi \, R_G \, N}{60}$$

$$F = \frac{V_G^2}{R_G} \times \frac{\text{weight of one blade}}{g} . \qquad (25)$$

By substituting the values of V_G and G in Equation 25

$$F = \frac{\gamma D N^2 A t_r}{2350 Z} \qquad (26)$$

D = tip diameter (in.)

A = developed area (sq ft) .

4.2 MOMENT DUE TO CENTRIFUGAL FORCE

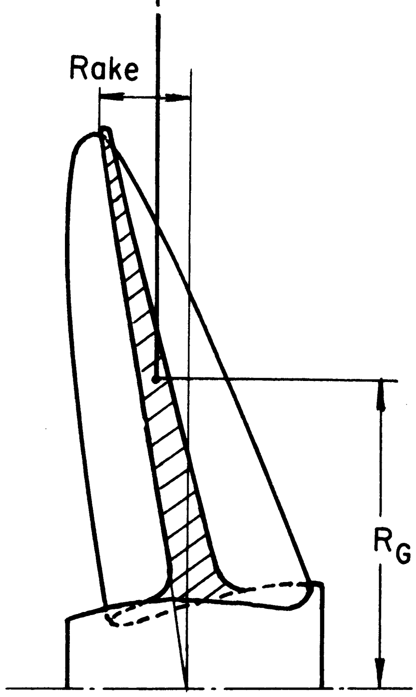


Fig. 8.

Most of the new built propellers are raked at a certain angle. It is acceptable to use an angle of 5 degrees for the average type of propeller. For a 5-degree angle the lever arm b (see Fig. 8) will be

$$b = r \times t_g\, 5° \times 0.485$$

$$b = 0.0875 \times 0.485 \times r$$

$$b = 0.0212\, D\ .$$

The moment due to centrifugal force

$$M = F \times b\ ;$$

using F from Equation 26

$$M_F = \frac{\gamma D^2 N^2 A\, t_r}{111{,}000 \times Z}\ .$$

Substituting the value of the developed area ratio (DAR)

$$DAR = \frac{\text{developed area}}{\text{disc area}}$$

$$DAR = \frac{A}{\pi D^2/4}$$

$$A = \frac{DAR \times \pi D^2}{4 \times 144} .$$

Substitute the value of A.

$$M_F = \frac{\gamma D^2 \; N \; t_r}{111{,}000 \times Z} \times \frac{DAR \times \pi D^2}{4 \times 144}$$

$$M_F = \frac{\gamma D^2 N \times DAR \times t_r}{2.03 \times 10^7 \times Z} \qquad (27)$$

5. VIBRATION

Stresses due to vibration of propeller blades are known to exist. In a well shaped and balanced propeller the vibrations are considerably reduced. The critical vibration will rise when the frequency of the blades is equal to the natural frequency of the hull. At high modes the effect of the resonance is much more destructive.

There are two sources of vibration; either by unbalanced blades or by the shock of the blades entering and leaving the wake region. The vibration frequency due to the unbalanced blade equals the number of revolutions per minute. The frequency of the second type is equal to the number of blades multiplied by the rpm and an integer number.

The vibration may be either flexural, where the blade vibrations are perpendicular to its surface, or torsional, where the blade twists around the root section.

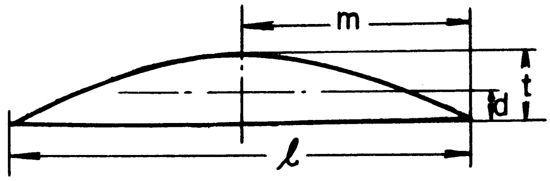
The calculation of the stress due to vibration, which is a fatigue stress, is laborious due to the many factors involved; in general they will not exceed the stresses due to the moment. When a propeller is well designed and perfectly balanced, and its natural frequencies such that they will not be in phase with those of the hull, especially at high modes, no vibration stress problem will exist.

6. PROPERTIES OF BLADE SECTION

Figure 9 shows three types of blade section of which the aerofoil type is the most commonly used.

$\frac{d}{t} = 0.4$

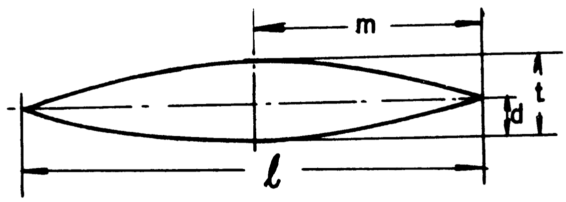
$\frac{m}{\ell} = 0.5$



(a) Ogival

$\frac{d}{t} = 0.5$

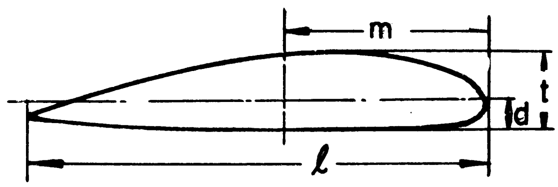
$\frac{m}{\ell} = 0.5$



(b) Symmetrical

$\frac{d}{t} = 0.44$

$\frac{m}{\ell} = 0.46$



(c) Aerofoil

Fig. 9.

The areas and moment of resistance of cross sections were experimentally tested. The constants tabulated in Table I are average values which are very close to actual conditions.

The area of the blade section can be expressed as

$$S = K_a \ell t \qquad (28)$$

as

S = area (in.^{2})
t = thickness of section (in.)
ℓ = width of section (in.) .

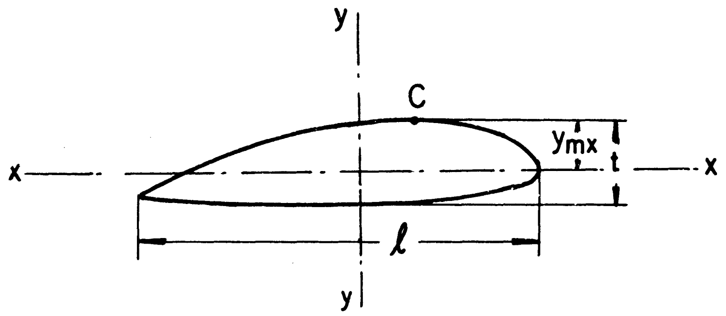


Fig. 10.

The moment of inertia about xx axis

$$I_{xx} = K_n \ell t^3 \quad . \qquad (29)$$

The moment of inertia about yy axis

$$I_{yy} = K_p \ell^3 t \quad . \qquad (30)$$

The constants K_n and K_p are tabulated in Table I. The xx and yy axes are parallel and normal to the face of the blade. These axes are also called the principal axes.

TABLE I

Type of Section	K_a	K_n	K_p
Ogival	0.67	0.046	0.033
Aerofoil	0.71	0.046	0.39

7. MOMENT DIAGRAM

The three moments acting at different centers of attack are shown in Fig. 11.

Figure 12 illustrates the components of the moments on xx and yy axes.

My is the sum of the moment on yy axis.
Mx is the sum of the moment on xx axis.

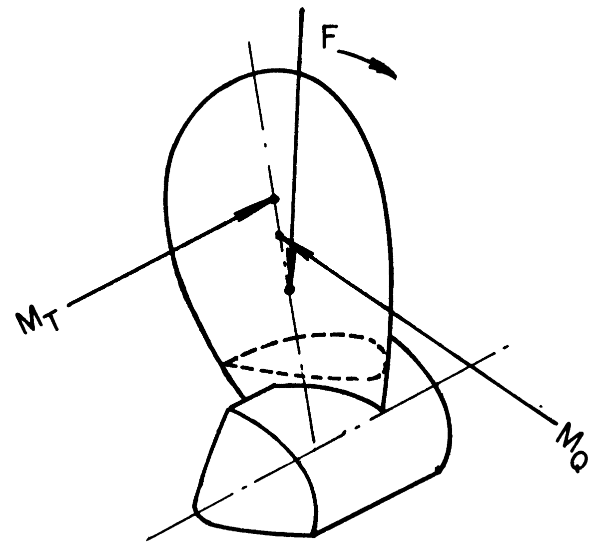


Fig. 11.

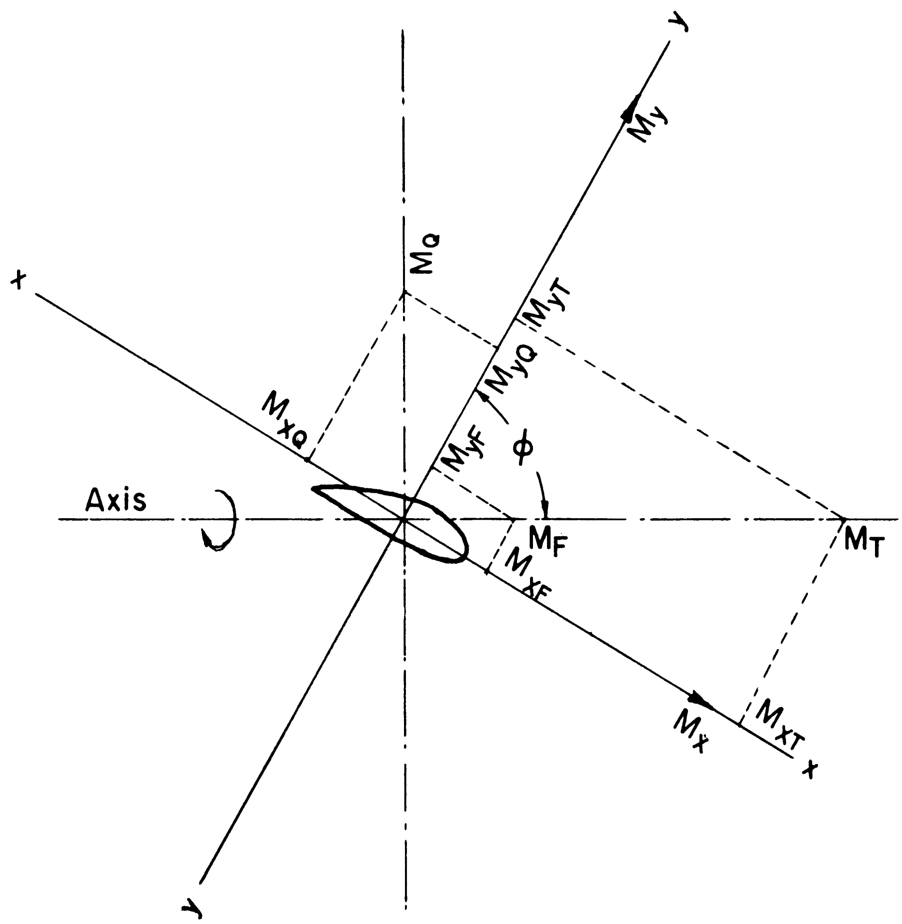


Fig. 12.

The angles can be expressed by the pitch.

$$\cos \phi = \frac{2\pi r}{\sqrt{(2\pi r)^2 + p^2}}$$

at root section

$$\cos \phi = \frac{1}{\sqrt{1 + [P_r^2/(4\pi R_o)^2]}} = \frac{1}{\sqrt{1 + (P_r/\pi D_o)^2}}$$

$$\sin \phi = \frac{P_r/\pi D_o}{\sqrt{1 + (P_r/\pi D_o)^2}}$$

D_o = diameter at root section (in.)

P_r = pitch at root section (in.)

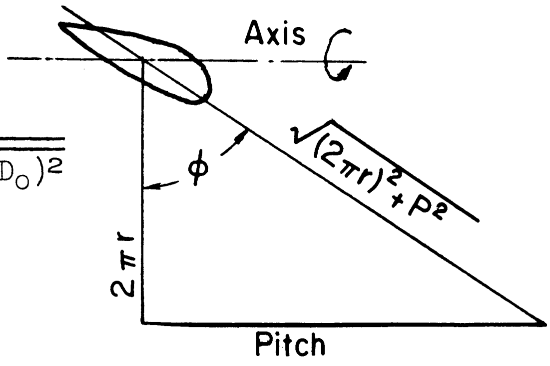


Fig. 13.

The average ratio for $P_r/D_o = 0.45$ will result in a pitch angle at root ϕ = 55 degrees.

$$\cos \phi = 0.57358$$

$$\sin \phi = 0.81915$$

8. STRESSES AT ROOT SECTION

8.1 STRESS DUE TO TORQUE

Since the moment of resistance of the blade section about yy axis is much greater than that about xx axis, it can be proved that the maximum stress will appear at c (see Fig. 10) which is the point of the greatest distance from the xx axis.

The stress equation for a beam can be applied.

$$\sigma = \frac{M \times y}{I_{xx}}$$

From Fig. 12 the component of the torque along yy axis is

$$My_Q = M_Q \sin \phi$$

$$My_Q = 0.81915 \, M_Q$$

The stress at c will be

$$\sigma_Q = \frac{My_Q \times y_{max}}{I_{xx}} .$$

From Equation 6

$$M_Q = \frac{42{,}000 \text{ DHP}}{N \times Z} \quad .$$

From Equation 30

$$I_{xx} = K_n \ell t_r^3 = 0.046 \ell t_r^3 \quad .$$

From Fig. 9

$$y_{max} = 0.56 \ t_r \quad .$$

Thus stress at root section

$$\sigma_{MQ} = \frac{42{,}000 \times \text{DHP}}{N \times Z} \times 0.81915 \times \frac{0.56 \ t_r}{0.046 \ \ell_r t_r^3}$$

$$\sigma_{MQ} = 4.18 \times 10^5 \times \frac{\text{DHP}}{N \ Z \ \ell_r t_r^2} \qquad (31)$$

(ℓ_r and t_r expressed in inches).

8.2 STRESS DUE TO THRUST

The moment due to thrust from Equation 20:

$$M_T = 52.4 \frac{\text{DHP} \times D}{Z \times V_a} \quad .$$

The component of this moment on yy axis:

$$My_T = M_T \cos \phi \ .$$

$$My_T = 52.4 \times 0.57358 \times \frac{\text{DHP} \times D}{Z}$$

$$My_T = 30 \frac{\text{DHP}}{Z \times V_a}$$

The maximum stress at point c will be

$$\sigma_{MT} = \frac{My_T \times y_{max}}{I_{xx}}$$

$$\sigma_{MT} = \frac{30 \times \text{DHP} \times 0.56 \ t_r}{Z \times V_a \times 0.046 \ \ell_r t_r^3} \quad .$$

Low rpm $\quad \sigma_{MT} = 365 \times \frac{DHP \times D}{Z \times V_a \, \ell_r t_r^2}$ (32)

High rpm $\quad \sigma_{MT} = 304 \frac{DHP \times D}{Z \times V_a \times \ell_r t_r^2}$

(D, ℓ_r, and t_r expressed in inches)

8.3 STRESS DUE TO CENTRIFUGAL MOMENT

The moment due to the force and rake arm was expressed in Equation 27.

$$M_F = \frac{\gamma D^4 N^2 \times DAR \times t_r}{2.03 \times 10^7 \times Z}$$

The component on yy axis:

$$My_F = M_F \cos \phi$$

$$My_F = 2.82 \times 10^{-8} \frac{\gamma D^4 N^2 \times DAR \times t_r}{Z} \quad .$$

The stress at point c:

$$\sigma_{MF} = \frac{My_F \times y_{max}}{I_{xx}}$$

$$\sigma_{MF} = 3.44 \times 10^{-7} \frac{\gamma D^4 N^2 \, DAR}{Z \times \ell_r \times t_r} \quad . \qquad (33)$$

For bronze $\gamma = 0.315$

(Low rpm) $\quad \sigma_{MF} = 1.08 \times 10^{-7} \frac{D^4 N^2 \, DAR}{Z \times \ell_r \times t_r}$ (34)

(High rpm) $\quad \sigma_{MF} = 7.8 \times 10 \quad \frac{D^4 N^2 \, DAR}{Z \times \ell_r \times t_r}$.

For cast steel $\gamma = 0.284$

(Low rpm) $\quad \sigma_{MF} = 9.8 \times 10^{-8} \frac{D^4 N^2 \, DAR}{Z \times \ell_r \times t_r}$ (35)

(High rpm) $\quad \sigma_{MF} = 7 \times 10^{-8} \frac{D^4 N^2 \, DAR}{Z \times \ell_r \times t_r}$.

8.4 STRESS DUE TO CENTRIFUGAL FORCE

The centrifugal force due to the mass of the blade tends to tear off the blade from its root. This force creates a tensile stress over the entire cross-sectional area.

$$\sigma_F = \frac{F}{S_r}$$

S_r = area of blade section at root.

$$S_r = K_a \, \ell_r t_r$$

From Table I

$$K_a = 0.71$$

$$S_r = 0.71 \, \ell_r t_r \quad .$$

From Equation 26

$$F = \frac{\gamma D N^2 \, A \, t_r}{2350 \, Z}$$

or

$$F = 2.32 \times 10^{-6} \, \frac{\gamma D N^2 \times DAR \times t_r}{Z} \tag{36}$$

$$\sigma_F = 3.27 \times 10^{-6} \, \frac{\gamma D^3 N^2 \, DAR}{Z \times \ell_r} \quad . \tag{37}$$

For bronze

$$\sigma_F = 10.3 \times 10^{-7} \, \frac{D^3 N^2 \, DAR}{Z \times \ell_r} \quad . \tag{38}$$

For cast steel

$$\sigma_F = 9.3 \times 10^{-7} \, \frac{D^3 N^2 \, DAR}{Z \times \ell_r} \quad . \tag{39}$$

8.5 MAXIMUM TOTAL STRESS

The maximum stress at the root section will be at point c which is the furthese point from xx axis. As shown in Figs. 11 and 12, the stresses at point c due to torque thrust and centrifugal moment are compressive stresses while the stress at the same point due to the centrifugal pull on the section area is a tensile stress. The total stress at the point will be

$$\sigma_2 = -\sigma_{MQ} - \sigma_{MT} - \sigma_{MF} + \sigma_F .$$

For a bronze propeller and low rpm

$$\sigma = -4.18 \times 10^5 \frac{DHP}{N \times Z \times \ell_r t_r^2} - \frac{365 \times DHP \times D}{Z \times V_a \times \ell_r t_r^2} - 1.08 \times 10^{-7} \frac{D^4 N^2 \, DAR}{Z \, \ell_r t_r} + 10.3 \times 10^{-7} \frac{D^3 N^2 \, DAR}{Z \times \ell_r} . \qquad (40)$$

For a bronze propeller and high rpm

$$\sigma = -4.18 \times 10^5 \frac{DHP}{N \times Z \times \ell_r t_r^2} - 304 \times \frac{DHP \times D}{Z \times V_a \times \ell_r t_r^2} - 7.8 \times 10^{-8} \frac{D^4 N^2 \, DAR}{Z \, \ell_r t_r} + 10.3 \times 10^{-7} \frac{D^3 N^2 \, DAR}{Z \times \ell_r} . \qquad (40)$$

APPENDIX I

STRAIN TEST ON PROPELLER BLADE

Propeller's Characteristics

Diameter = 19" = 1.583 ft
Disc area = 283.53 sq in.
Developed area = 144 sq in.
DAR = 0.508
Number of blades = 3
Tip pitch = 13.35"
Mean pitch = 12.88"
Root pitch = 9.81" at 0.2 D
Pitch ratio (a) = 0.678
Max thickness of blade at root = 0.58"
Width of blade at root = 4.1"

A G.M. diesel engine developing 110 hp (max) at 1840 rpm was selected for driving the propeller with no gear reduction. Using Troost charts for three-bladed screw series type B3-50 the following table was arranged. The speed of advance V_a was arbitrarily chosen.

TABLE II

V_a	$V_a^{2.5}$	δ	B_p	a	ϵ
8.3	200	350	96.3	0.62	43.6
11	400	265	48.2	0.675	52.2
12	500	243	38.5	0.695	55.3
14.55	800	200	24.1	0.74	61.2

$$\delta = \frac{ND}{Va}$$

$$B_p = \frac{N\sqrt{DHD}}{Va^{2.5}}$$

From the curves on Fig. 14, the efficiency and speed corresponding to the propeller pitch ratio, a = 0.678 are :

$$\epsilon = 47.8\%$$
$$V_a = 11.15 \text{ knots}$$

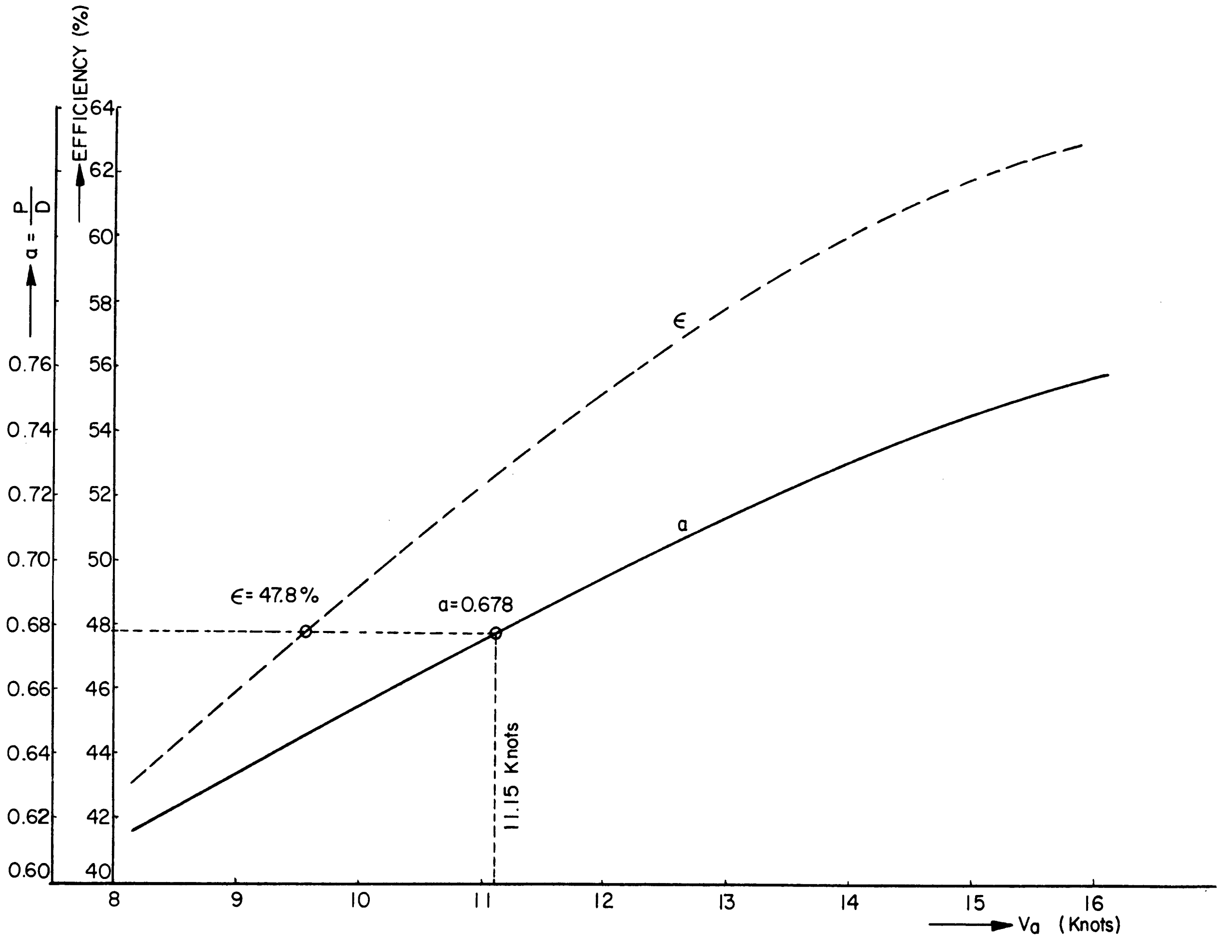


Fig. 14. Efficiency, pitch ratio versus speed of propeller.

THE LOADS APPLIED IN TEST

1. Thrust.—From Equation 10, $T = 178\ DHP/ZV_a$ for high rpm. As the propeller is driven directly by the engine shaft without a gear reduction, we can neglect friction losses in shaft bearings and use the shaft horsepower as the delivered horsepower.

$$T = \frac{178 \times 110}{3 \times 11.15} = \underline{586}\ \text{lb}$$

The center of thrust $R_t = 0.69 \times R = 6.55''$.

2. Torque.—

$$Q = 210,000 \frac{DHP}{N \times 2 \times D}$$

$$Q = \frac{210,000 \times 110}{1840 \times 3 \times 19} = \underline{220}\ \text{lb}$$

The center of torque $R_Q = 0.6\ R = 5.7''$.

3. Centrifugal Force.—

$$F = \frac{\gamma D N^2 A t_r}{2350\ Z} = \frac{0.315 \times 19 \times 1840^2 \times 1 \times 0.58}{2350 \times 3}$$

$$F = \underline{1665}\ \text{lb}$$

Center of force $R_F = 0.485\ R = 4.61''$.

TEST PROCEDURE

The propeller was secured to a 12" x 12" x 1/2" plate. The plate was fastened to a built-up frame by clamps (Fig. 15). Holes of 1/4" were drilled at the centers of the forces where a bolt was secured so that the weight basket could be suspended. Bars of approximately 50 lb were used as loads. The maximum load which could be handled in the test was 715.5 lb.

The centrifugal load was applied as shown in Figs. 15 and 16. The position of the loaded blade in the torque loading is shown in Fig. 17. The blade was secured horizontally and the load was applied vertically.

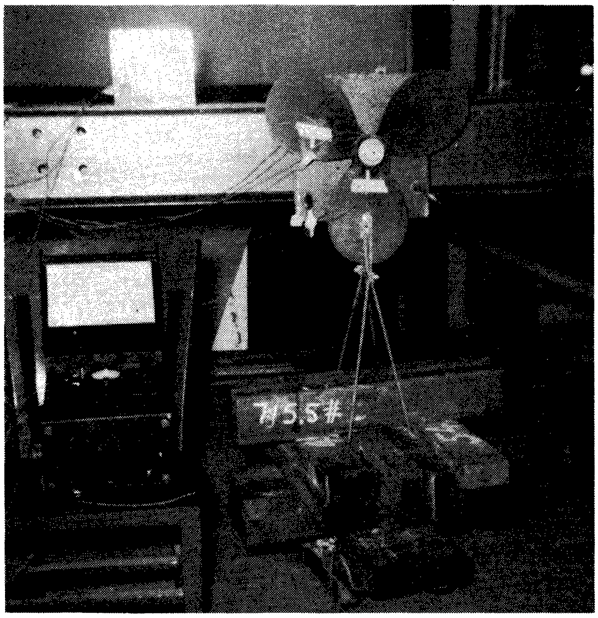


Fig. 15.

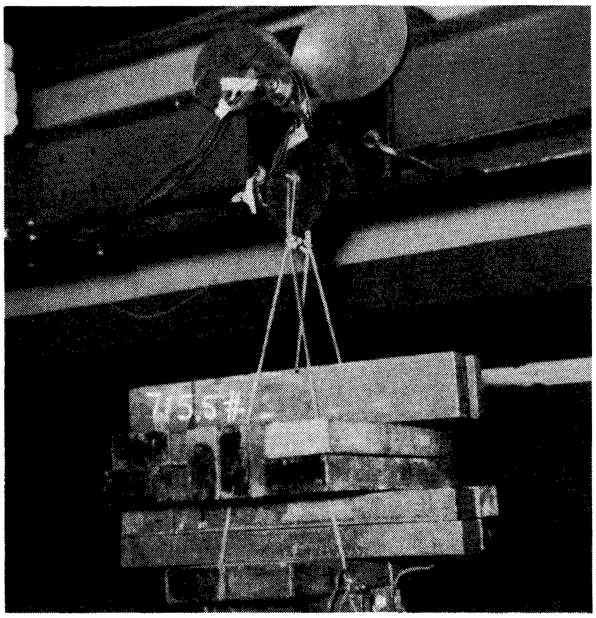


Fig. 16.

For the thrust loading the propeller was turned and secured as shown in Fig. 18. In each test three strain gages of 1/4" and gage factor of 1.91 were used. The gages were tied to the propeller one at each side of the blade at root section at the maximum thickness of blade (center of blade). The third gage, was a dummy gage tied to a free blade for temperature compensation. The wiring, gages, and strain indicator are shown in Fig. 15.

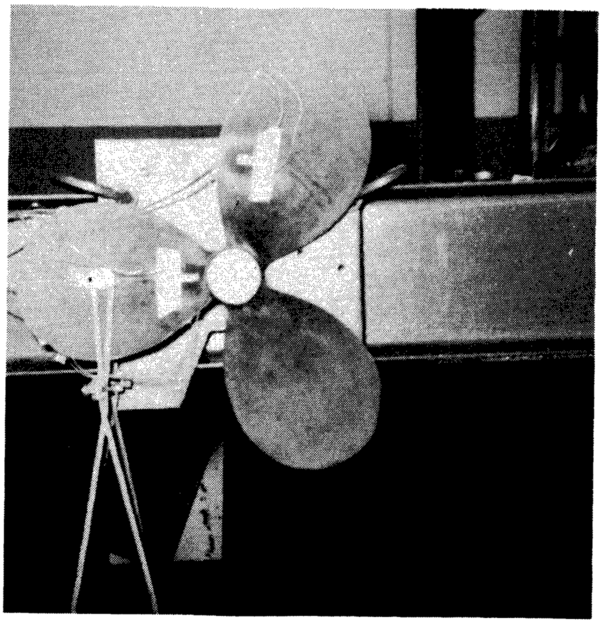


Fig. 17.

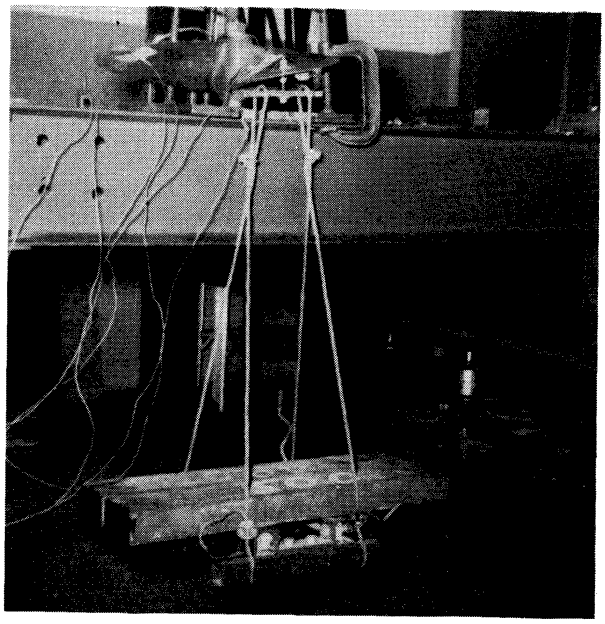


Fig. 18.

The results of the strain indicator readings are tabulated in the following tables.

TORQUE TEST ON PROPELLER BLADE

		Gage No. 1 (Front)				Gage No. 2 (Back)			
Test No.	Load lb	Ref	Indicator Reading μin./in.	Difference μin./in.	Increase in Strain μin./in.	Ref	Indicator Reading μin./in.	Difference uin./in.	Increase in Strain μin./in.
1	0	3	1335	0	0	6	1210	0	0
2	83	3	1170	-165	-165	6	1360	150	150
3	132	3	1080	- 90	-225	6	1445	85	235
4	182.5	3	975	-105	-360	6	1535	90	325
5	220	3	900	- 75	-435	6	1605	70	395
6	283.5	3	775	-125	-560	6	1715	110	505
7	331	3	680	- 95	-655	6	1800	85	590

Note: uin. means microinch.

THRUST TEST ON PROPELLER BLADE

		Gage No. 1 (Front)				Gage No. 2 (Back)			
Test No.	Load lb	Ref	Indicator Reading μin./in.	Difference μin./in.	Increase in Strain μin./in.	Ref	Indicator Reading μin./in.	Difference μin./in.	Increase in Strain μin./in.
1	0	6	1320	0	0	3	1420	0	0
2	112.5	6	1130	-190	-190	3	1577	157	157
3	218	6	935	-195	-385	3	1770	193	350
4	343	6	730	-205	-590	3	1950	180	530
5	440	6	570	-160	-750	*4	1080	150	680
6	541	6	390	-180	-930	4	1240	160	840
7	630	6	245	-145	-1075	4	1375	135	975

*The difference between ref 3 and 4 is 980 μin./in.

CENTRIFUGAL FORCE TEST

		Gage No. 1 (Front)				Gage No. 2 (Back)			
Test No.	Load lb	Ref	Indicator Reading μin./in.	Difference μin./in.	Increase in Strain μin./in.	Ref	Indicator Reading μin./in.	Difference μin./in.	Increase in Strain μin./in.
1	0	8	890	0	0	7	155	0	0
2	214.5	8	850	-40	-40	7	160	5	5
3	311.5	8	835	-15	-55	7	165	5	10
4	412	8	815	-20	-75	7	170	5	15
5	513	8	800	-15	-90	7	175	5	20
6	612	8	785	-15	-105	7	180	5	25
7	715.5	8	765	-20	-125	7	185	5	30

The tabulated values were plotted on Figs. 19, 20, and 21 and the strain corresponding to the required forces are as follows:

	Load lb	Strain μin./in.
Torque	220	435
Thrust	586	1010
Centrifugal force	1665	290

THE STRESS CALCULATED FROM STRAIN

For biaxial strain the stress equation is

$$\sigma = \frac{E(\epsilon_1 + \mu\epsilon_2)}{1 - \mu^2}$$

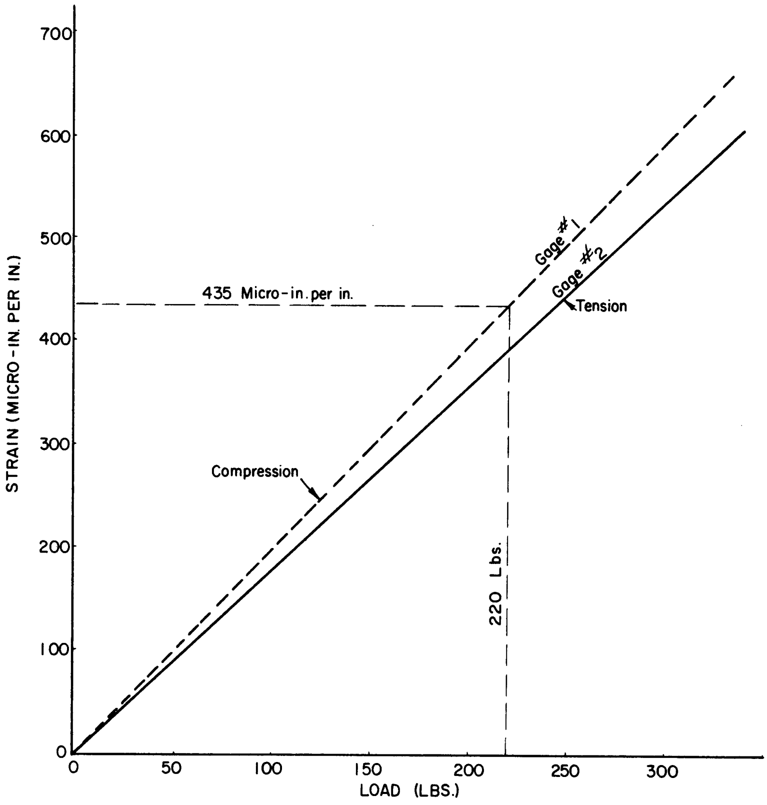


Fig. 19. Strain-load diagram for torque.

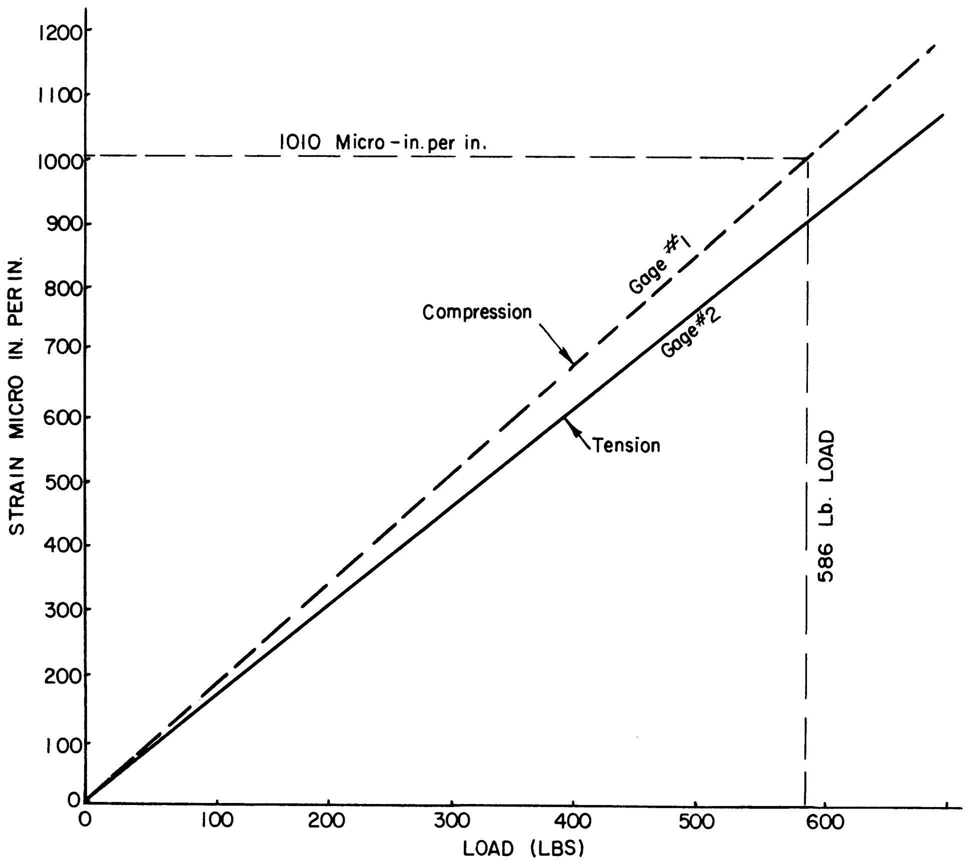


Fig. 20. Strain-load diagram for thrust.

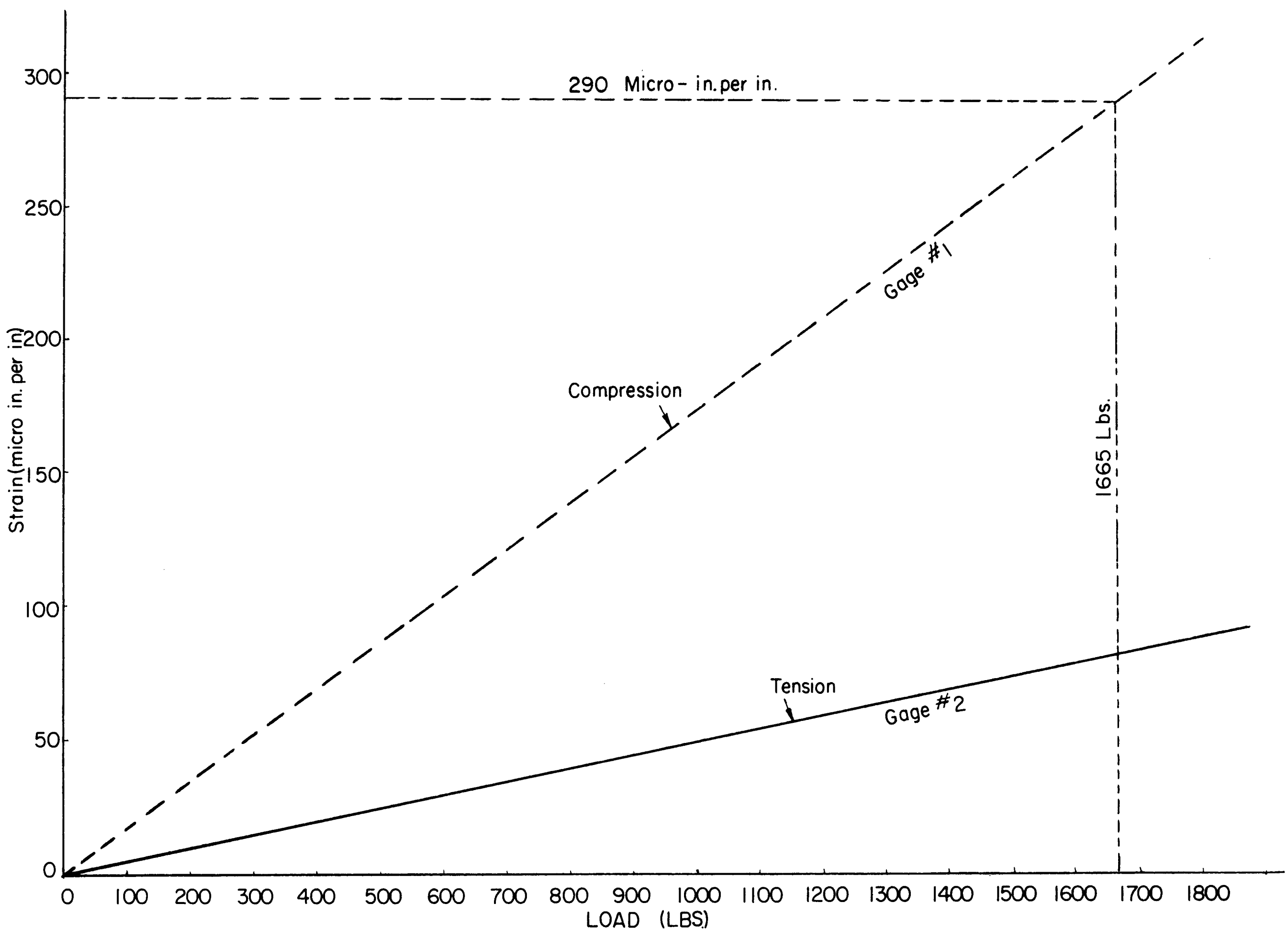


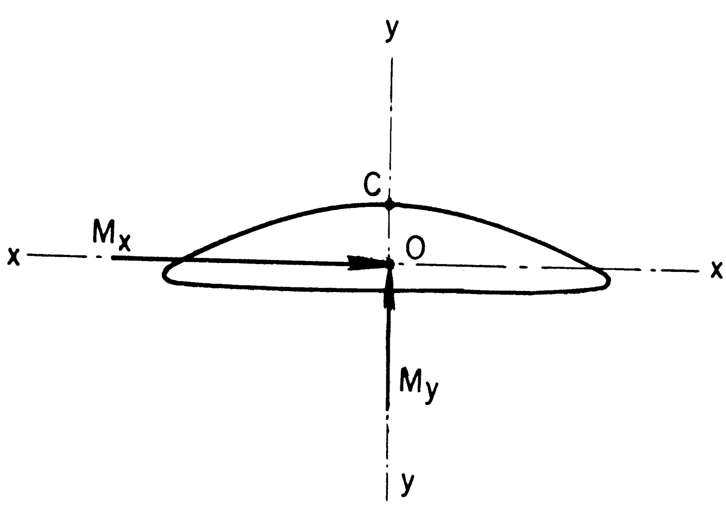
Fig. 21. Strain-load diagram for centrifugal force.

ϵ = strain (in./in.)

σ = stress (psi)

E = modulus of elasticity = 15×10^6 for bronze

μ = Poissons ratio = 0.35 for bronze



For the blade section of the tested propeller which is symmetric about the yy axis no tension or compression stress will exist at point **c** due to the moment Mx along xx axis. The center of attack at O is on the same yy axis as c. Due to the moment My point c will be under the maximum tensile stress which will be uniaxial. The uniaxial stress will be expressed as $\sigma = E\epsilon$

Applying the results of the strain test, the following stresses resulted.

Torque stress $\sigma_Q = 15 \times 10^6 \times 435 \times 10^{-6} = 6520$ psi

Thrust stress $\sigma_T = 15 \times 10^6 \times 1010 \times 10^{-6} \doteq 15{,}150$ psi

Centrifugal stress $\sigma_F = 15 \times 10^6 \times 290 \times 10^{-6} = 4350$ psi

Total stress $= 6520 + 15{,}150 + 4350 = \underline{26{,}020}$ psi.

STRESS CALCULATION BY PROPOSED EQUATIONS

$$\text{Torque } \sigma_{MQ} = \frac{4.18 \times 10^5 \times DHP}{N \times z \times \ell_r \times t_r^2} = \frac{4.18 \times 10^5 \times 110}{1840 \times 3 \times 4.1 \times 0.58^2} = 6{,}040 \text{ psi}$$

$$\text{Thrust } \sigma_{MT} = \frac{304 \times DHP \times D}{Z \times V_a \times \ell_r \times t_r^2} = \frac{304 \times 110 \times 19}{3 \times 11.15 \times 4.1 \times 0.58^2} = 13{,}900 \text{ psi}$$

$$\text{Centrifugal Moment } \sigma_{MF} = \frac{7.8 \times 10^{-8} \times D^4 \times N^2 \times DAR}{Z \times \ell_r \times t_r} = \frac{7.8 \times 10^{-8} \times 19^4 \times 1840^2 \times 0.52}{3 \times 4.1 \times 0.58}$$

$$= 2{,}490 \text{ psi}$$

$$\text{Centrifugal Force } \sigma_F = \frac{10.3 \times 10^{-7} \times D^3 \times N^2 \times DAR}{Z \times \ell_r} = \frac{10.3 \times 10^{-7} \times 19^3 \times 1840^2 \times 0.52}{3 \times 4.1}$$

$$= 1010 \text{ psi}$$

Total stress $\sigma = 6040 + 13{,}900 + 2490 + 1010 = \underline{23{,}440}$ psi

APPENDIX II

STRESS CALCULATION FOR THE FOLLOWING PROPELLER

The following propeller data is required when calculating stress by the proposed equations:

Diameter	$D = 210$ in.
Shaft horsepower	$SHP = 4000$
Number of blades	$Z = 4$
Revolutions per minute	**rpm** $= 87$
Speed of ship	$V_K = 14.5$ knots
Developed area	D.A. $= \cdot 110$ sq ft
Thickness of blade at root	$t_r = 7.12$ in.
Width of blade at root	$\ell_r = 38.3$ in.

CALCULATION

The developed horsepower will be 3850 hp considering about 5% bearing losses.

Speed of advance of propeller $V_a = V_K(1 - w) = 14.5 \times 0.7$

$$V_a = 10.15 \text{ knots}$$

Developed area ratio

$$DAR = \frac{110 \times 4}{\pi \times \left(\frac{210}{12}\right)^2} = 0.458$$

Stress due to torque $\sigma_{MQ} = \dfrac{4.18 \times 10^5 \times 3850}{87 \times 4 \times 38.3 \times 7.12^2} = -2390$ psi

Stress due to thrust

$$\sigma_{MT} = \frac{365 \times 3850 \times 210}{4 \times 10.15 \times 38.3 \times 7.12^2} = -3750 \text{ psi}$$

Stress due to centrifugal moment

$$\sigma_{MF} = \frac{1.08 \times 10^{-7} \times 210^4 \times 87^2 \times 0.458}{4 \times 38.3 \times 7.12} = -660 \text{ psi}$$

Stress due to centrifugal force

$$\sigma_F = \frac{10.3 \times 10^{-7} \times 210^3 \times 87^2 \times 0.458}{4 \times 38.3} = +160 \text{ psi}$$

Sum of stress = - 2390 - 3750 - 660 + 160 = - 6640 psi

CONCLUSIONS FROM TESTING

The strain test as described in Appendix I does not show a complete picture of the stress distribution in the blade. Such a test involves a great number of strain gages. The main object of the test was to estimate the maximum stress by assuming the location and direction of the maximum strain, based on the theory and conditions described in the paper. The results of the strain gage test compared with those calculated by the proposed equations clearly indicate that a stress concentration factor has not been considered in the calculations. This concentration factor is a result of both the single load applied at the center of attack, and the change from the blade section to the root section. These factors explain why the stress measured by strain gages are higher than those calculated. There is no reason to expect the stress results, obtained by the two different methods, to be very close. To the writer they seem too close. From the two examples in Appendices I and II, it can be stated that the proposed equation lead to a fairly accurate prediction of the stress in the blade. In case of high speed vessels, the constants in the equations have been changed. It should also be remembered that these constants change when a twin screw vessel is considered. The difference will be due to valuations in the thrust load, the wake thrust deduction and propulsion coefficients with resulting variations in loadings. For high speed vessels the factors mentioned above will change, particularly the centrifugal force which will have considerable effect on the stress. To decrease the centrifugal stresses, in most high speed wheels, the rake of the blades is decreased. These factors have been considered and the constant in the equation for calculating the stress due to centrifugal force has been reduced in proportion to the decrease in the rake. The torque stress constants are independent of rpm.

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