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Unit-Contingent Electricity Swap and Asymmetric Information About Plant Outage

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Ross School of Business Working Paper
Working Paper No. 1120
December 2008

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December 17, 2008

This paper analyzes a unit-contingent swap contract between a power plant and an electricity distributor. Under such a contract the distributor pays the power plant a fixed price if the plant is operational and nothing if plant outage occurs. Pricing a unit-contingent swap is complicated by the fact that the power plant's operational status is its private information and is costly for the distributor to observe. The difference between the electricity spot price and the unit-contingent swap price provides an incentive for the plant to misreport its status and earn profit at the distributor's expense. To prevent misreporting the distributor may inspect the plant and levy penalties if misreporting is discovered. We model the Bayesian inspection game between the plant and the distributor, characterize the structure of the optimal penalties the distributor should impose, and compute the corresponding unit-contingent swap prices. We show that the seemingly undesirable misreporting behavior of the plant, when controlled well, can actually benefit both the plant and the distributor because misreporting could serve as a risk-allocation mechanism between the two parties.

Key words: unit-contingent electricity swap, asymmetric information, Bayesian equilibrium, risk allocation

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1. Introduction

We study a problem of an electricity distributor who considers purchasing electrical power provided by a power plant. The transactions between the distributor and the power plant are governed by a *unit-contingent swap* contract. It is a swap, because the distributor pays the power plant a fixed price per megawatt hour (MWh) generated by the plant, thereby the plant transfers the price risk to the distributor. It is unit-contingent, because the distributor pays the power plant only when the power generating unit is operational.

Unit-contingent swaps are widely used in the electricity industry as a risk-management tool for power plants. A unit-contingent swap is more valuable than a firm contract (e.g., a forward or a regular swap contract) with the same price, because under a firm contract the plant bears the outage risk: When an outage occurs, the plant has to buy electricity from the spot market (possibly incurring a loss) to meet the firm contract obligations. The unit-contingent contract protects the power plant against the outage risk and stabilizes its income, which is often a necessary requirement for the plant to obtain financing from banks. Because of this value, the distributor can offer a unit-contingent contract to the plant at a discounted price compared to the firm contract price. Unit-contingent contracts are also good for the economy because they provide incentives for plants to maintain their reliability (only plants that are generating electricity get paid). NorthWestern Energy (2007) provides an example of unit-contingent contracts: NorthWestern Energy owns 222 MW capacity of a coal-fired plant in Montana, of which 97 MW are currently sold to Puget Energy on a unit-contingent basis through 2010 at approximately $56/MWh, 90 MW are sold to Montana regulated customers, and the rest 35 MW are sold to the spot market.

The price discount offered by the unit-contingent contract benefits the distributor, who enters into a firm service contract with its customers and profits on the price difference between the firm contract and the unit-contingent swap. This profit of the distributor is risk-free as long as the power plant is up and running. However, if an outage happens at the plant, the distributor, who must meet obligations on its firm contract, has to purchase electricity on the spot market. Thus, when calculating the unit-contingent contract price, the distributor must trade off between the risk-free profit when the plant is up and the potential losses when the plant is down.

Pricing of the unit-contingent swap contract is complicated by the fact that the distributor cannot directly observe if the plant indeed had an outage. In practice, the power plant informs the distributor one day ahead whether or not its generating unit will be available for every hour the next day. On the operating day, the power plant reports its real-time operating status (if different from the day-ahead information) to the distributor. The swap transaction is made if and only if the unit is *reported* to be on. There is potentially an incentive for the plant to misreport its status: when the spot price is high, the plant that is up could report to the distributor that it is down and sell electricity at the high spot price instead. This hurts the distributor who must cover its firm contract positions by buying at the spot market. Conversely, when the spot price is low, the plant that is down could report to the distributor that it is up, buy electricity on the spot, and sell it to the distributor at the unit-contingent contract price. The distributor is hurt because it could have bought the electricity at the lower price.

To understand why the distributor is unable to observe the plant's true status, we describe how unit-contingent contracts are typically implemented in the electricity market.[1] The transmission system is operated by an Independent System Operator (ISO) or a regional transmission organization. First, everyday before 9 am, the power plant must inform both the ISO and the distributor about its next day's delivery schedule. For example, it intends to deliver 100 MWh to node A (the delivery point specified in the contract) in each hour the next day, except for 3-4 pm when the maintenance crew have to replace an equipment for safety concerns. Second, after receiving the plant's schedule report and before 11 am, the distributor must submit to the ISO demand bids for node A (i.e., how much electricity should be transmitted to node A the next day). For the ease of illustration, let us assume that the distributor needs to satisfy a firm contract of 150 MW the next day, so the distributor will submit a firm demand bid of 150 MWh for each hour. Third, prior to each hour on the operating day, the plant updates the ISO and the distributor about its real-time delivery to node A if different from the day-ahead schedule. If no updates, then the ISO transfers 100MW $\times$ 23h = 2,300MWh (plant is down for one hour) from the plant to the distributor on the operating day. Finally, no later than six days after the operating day, the two parties must enter a financial schedule (typically the plant enters and the distributor approves) to inform the ISO that the energy charges for 2,300MWh will be settled between the parties outside of the market. Thus, the ISO will charge the distributor 50MW $\times$ 23h + 150MWh = 1,300MWh at the market clearing price, and the plant will charge the distributor 2,300MWh at the unit-contingent contract price.

In the above process, the distributor learns about the plant's status only through the plant's report and the financial schedule – both are provided by the plant. The distributor cannot directly

[1] The technical description of this process can be found at http://www.midwestiso.org/publish.

observe whether the plant is indeed down during 3-4 pm and up for the rest of the operating day. The power plant, however, while (mis)reporting the delivery schedule to the distributor as described earlier, may at the same time submit supply offers/demand bids to the ISO to sell/buy electricity to/from the market. Note that the plant does not withhold capacity or output from the market, so the plant's behavior does not violate the ISO's market rules. Monitoring bilateral contracts are beyond the ISO's responsibility, as described in the Midwest ISO's business practice manual.[2] The distributor as an individual market participant does not have the right to access the information recorded in the plant's account without the help from a third party.

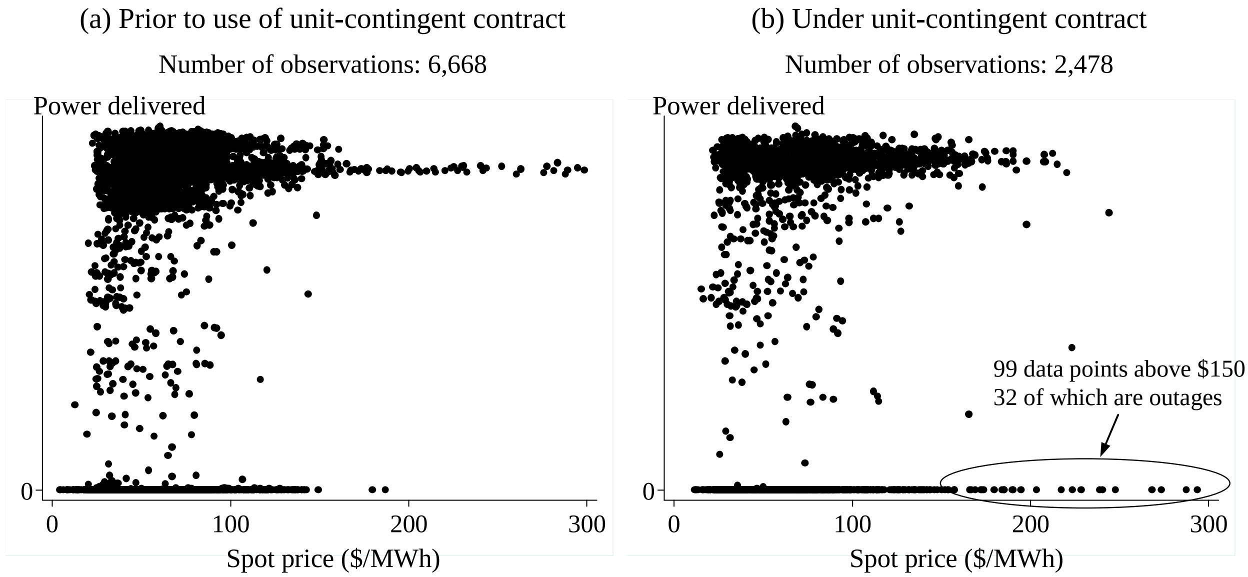
Does the above misreporting problem happen in practice? An empirical investigation is discussed below. Figure 1 shows the power delivery pattern of a coal-fired power generating unit. The data is hourly. Each data point records the actual power delivered to the distributor in a particular hour (vertical axis) and the corresponding spot price in that hour (horizontal axis). Figure 1(a) shows the power delivery pattern before the plant was engaged in the unit-contingent contract. When the spot price was above \$150/MWh, the unit was almost always dispatched and delivering power at its full capacity. (We could not access the information about how the plant was dispatched in that period of time, but most likely the plant was economically dispatched by the ISO.) Figure 1(b), however, reveals that when the same unit was engaged in unit-contingent transactions, much less power was delivered to the distributor when the spot price was high. In fact, when the spot price is above \$150/MWh, outages were reported in almost a third of time. Could the condition of the same unit be worse during high spot prices, or did something else happen?

The highlighted data points in Figure 1(b) represent 32 hours of outage. The average spot price during those outages was \$180/MWh. If the distributor sells power at \$80/MWh via a 100MW firm contract, then those 32 hours of outage had caused a loss of $(\$180-\$80)\times 100\times 32 = \$320,000$ to the distributor.

[2] See Midwest ISO (2008) Section 3.5: "MMM [Market Monitoring and Mitigation] is generally concerned with any market participant behavior that affects the competitiveness of the Energy Markets ... The MMM process, however, is not directly concerned with Internal or External Bilateral Transaction Schedules ..."

Figure 1 Electricity Delivery from a Coal-Fired Power Generating Unit

To disguise the identity of the power plant, we omitted the vertical scale. Disclaimer: We do not claim any conclusive evidence that the plant has misreported; the data presented here is for research purpose only, and should not be used for any other purpose.



It might appear that the distributor would prefer that the plant always truthfully reported its status. To enforce truthful reporting, the distributor could inspect the plant's status at a cost, e.g., cost of hiring a third party to access the plant's internal records and its account at the ISO. If the plant is found to have misreported its status, penalties could be levied. In practice, however, the distributor typically does not conduct inspections to recover potential damages. On the contrary, the distributor is usually satisfied with the aforementioned risk-free profit as a good source of its income, and is willing to tolerate the occasional (if any) plant's misreporting. Why would the distributor not force the plant to always report the truth? Based on our conversations with industry, we surmise that the distributor may be able to keep the contract price low if it tolerates misreporting to a certain extent. But how much misreporting should be tolerated? How low should the contract price be? And more importantly, will the distributor be better off compared to enforcing truthful reporting? These are the key questions to be answered in this paper.

We show that there always exist situations when the plant's misreporting would benefit both the plant and the distributor because misreporting reallocates the risk between the two parties in a way that the unit-contingent swap contract with truthful reporting cannot. Specifically, the following are several important insights derived in this paper.

The misreporting may benefit both the plant and the distributor when the plant is down and the spot price is low. If the plant were truthful, it would earn zero profit. By pretending to be up and buying electricity on the spot market to deliver to the distributor the plant gains positive profit at the distributor's expense. Anticipating this loss, the distributor can lower the unit-contingent contract price upfront. If the plant's utility function is concave, the reduction in the contract price can exceed the expected increase in the plant's profit from misreporting, while keeping the plant's expected utility constant. This benefits the risk-neutral distributor. Furthermore, if the plant were truthful, the distributor would bear the spot price risk when the plant is down. But if the plant pretends to be up, the plant assumes that risk, benefiting the risk-averse distributor.

Reporting down while the plant is up (if spot price is high) will expose the distributor to the spot price risk while the plant also gets a random profit. This type of misreporting, as suspected in Figure 1, increases the uncertainties of both firms' profit and, therefore, is undesirable.

To achieve the best risk allocation, the distributor must design a contract that steers the plant toward the "right misreporting." We show that the optimal penalty scheme has a simple form: when the spot price falls into a certain interval below the contract price, the distributor should tolerates misreporting (i.e., impose zero penalty), but when spot price is not in that interval, misreporting should be banned (i.e., impose high penalty).

In this paper, we keep the unit-contingent contract form intact because this is widely used in practice. Theoretically, if the complete flexibility in designing the contract is allowed, the optimal contract would require a less risk-averse party (typically the distributor) to buy the more risk-averse one (typically the plant). This does not happen in practice, in part because the deregulation of the electricity industry prevents one party from owning too many assets.

The rest of the paper is organized as follows. Section 2 discusses relevant literature. Section 3 models the firms' interaction as a game with asymmetric information. Section 4 solves for the Bayesian equilibrium of the game under any given contract. Section 5 derives the optimal contract.

Numerical examples and the impact of various factors (plant's reliability, spot price volatility, inspection cost, etc.) are analyzed in Section 6. We make a few concluding remarks in Section 7.

2. Related Literature

We apply the theory of Bayesian games originally developed by Harsanyi (1967, 1968a,b) to model the interactions between the power plant and the distributor, as will be detailed in the next section. Bayesian games have been applied to the electricity markets to model the suppliers' bidding processes in which each power plant's marginal cost is its private information. Such a game has been analyzed in various market conditions by Ferrero et al. (1998), Shahidehpour et al. (2002), Correia (2005), and Li and Shahidehpour (2005). Interestingly, a recent study by Hortaçsu and Puller (2008) shows that in the Texas electricity market bidders have good information about their rivals' marginal costs, but they have little information about rivals' contract obligations. The authors analyzed a Bayesian game in which contract positions are private information. In this paper, information asymmetry comes from the fact that the power plant's status cannot be directly observed by the distributor.

To analyze a specific mechanism (unit-contingent swap) in this paper, we find the equilibrium of the Bayesian game directly instead of invoking a mechanism design approach (as in Myerson 1981, 1979, Guesnerie and Laffont 1984). In the Contract Theory, there are papers that are similar to ours but do not have a restriction on a specific contract. For example, Laffont and Martimort (2002, Section 3.6) discuss an adverse selection problem with audits and costly state verification. The costly audit allows the principal to detect an untruthful agent's report and impose penalties. Even with audits and punishments, the Revelation Principal still applies and the punishment allows the principal to reduce agent's incentive to lie and, hence, reduces informational rents. The adverse selection problems with costly state verification were applied to insurance and taxation by Mookherjee and Png (1989), Reinganum and Wilde (1985). Sometimes the burden for paying for the costly state verification lies with the agent as discussed in Bolton and Dewatripont (2005, Section 5.3). Examples of this approach are papers by Townsend (1979) and Gale and Hellwig (1985) who study the optimal debt contract between a financier and an entrepreneur.

This paper is focused on managing the incentive problem inherent in the unit-contingent contracts. Thus, we assume that the distributor does not engage in sophisticated risk management using options and other derivatives. The incentive problem of the unit-contingent contract still persists when the distributor employs other risk management tools. A review of various electricity derivative contracts can be found in Deng and Oren (2006).

3. The Power Purchase Game

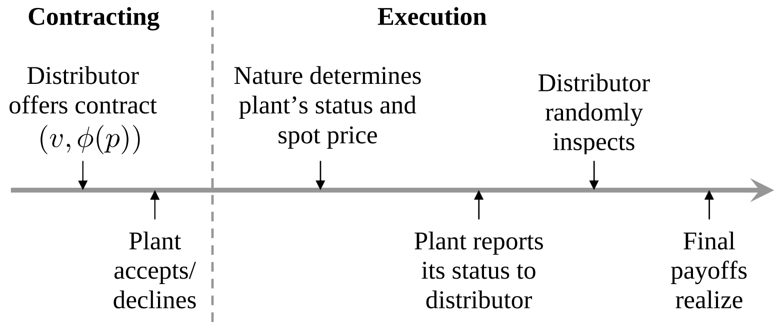
A distributor buys electricity from a plant on a unit-contingent basis. We assume that the plant has two states: UP and DOWN (denoted in capital letters for visual convenience). In the UP state the plant produces power at its full capacity, which we normalize to be one unit, and the production cost is denoted as c. In the DOWN state the plant cannot produce anything, and does not incur any production cost. For simplicity, we assume the maintenance cost is negligible.

We model the transactions between the distributor and the plant as a two-stage game with observed actions and incomplete information. Figure 2 illustrates the timeline of the game. The two stages are: contracting and execution. During the contracting stage, the distributor determines the unit-contingent contract parameters: (v, ϕ), where $v > 0$ is the unit-contingent contract price (i.e., the payment from the distributor to the plant if the plant is delivering electricity to the distributor; otherwise, no payment is made), and $\phi \geq 0$ is the penalty paid by the plant to the distributor if the plant is found to have misreported its status. The penalty $\phi = \phi(p)$ can depend on the electricity spot price, p, realized in the execution stage. This allows the penalty to reflect the distributor's losses due to the plant's misreporting.[3] The plant then accepts or rejects the unit-contingent swap contract, and the game moves into the execution stage.

We assume that the execution stage is one period (see Section 7 for discussion of repeated interactions in the execution stage). At the beginning of the execution stage, the Nature determines

[3] In the current practice, unit-contingent contracts often specify a non-performance penalty equal to the delivery shortfalls multiplied by either v or p and further multiplied by a penalty factor (varies from 2 to 3). This non-performance penalty implies that the penalty for misreporting down (when the plant is actually up) will be significant – at least twice as much as the energy cost. We allow the misreporting penalty to depend on the spot price in a general form, but in Section 5, we will show that the optimal penalty scheme has a very simple structure.

Figure 2 **Timeline of the Power Purchase Game**



the spot price and the plant's status. It is *common knowledge* that the Nature picks a spot price, p, from a continuous distribution with a probability density function $g(p)$, and that the Nature lets the plant be UP with probability $\gamma \in (0,1)$ and DOWN with probability $1-\gamma$. The plant's status is its *private information*, while the realized spot price is public information. We assume that the power generating unit is small so that its status has negligible influence on the spot price distribution.[4] The incentive problem still exists if the spot price is correlated with the plant's status, but the analysis will become much more complicated. In the model, we assume that the plant knows the spot price realization when reporting. This is a reasonable assumption because the spot price uncertainty considered by the two parties when signing the contract is mostly resolved when the plant reports its status (reports by 9 am, and the day-ahead market closes at 11 am). Furthermore, on the operating day when the real-time price uncertainty is mostly resolved, the plant can update its delivery schedule in real-time. In the remainder of the execution stage game, the distributor receives the plant's report, and then either inspects at inspection cost $k > 0$ or does not inspect the plant. The final payoffs are realized at the end of the game.

We assume that the distributor serves one unit constant load, and its service contract is a firm contract with a fixed price $f > v$. The constant load assumption is not restrictive. If the load is

[4] More precisely, we assume the impact of the plant on the energy component of the spot price is negligible. For example, Midwest ISO clears about 100,000MWh every hour on the spot market during the peak time, whereas a typical unit-contingent contract deals with less than a few hundreds megawatts. When an outage occurs at the plant, the ISO can contain the outage by dispatching the next available generator on the network. The increase in the energy component of the spot price will be very small. However, when transmission congestion prevents the outage from being mitigated by relatively inexpensive generating units on the network, the congestion cost component of the local spot price may increase significantly. In practice, however, for the ease of contract evaluation, unit-contingent swaps are often swaps between a fixed price and the spot price at a major hub (e.g., Michigan Hub), which is less affected by regional outages.

random, the incentive problem still remains and its effect on the distributor's profit remains the same.[5] In practice, when the firm contract is a forward or futures contract (e.g., California Oregon Border (COB) futures contract traded on NYMEX), the load is indeed constant.

Table 1 Profit in the Execution Stage Game

In each formula cell, the top expression is the plant's profit, and the bottom expression is the distributor's profit. p stands for any realization of spot price.

		Nature determines plant's status			
		UP		DOWN	
		Plant reports		**Plant reports**	
		UP	DOWN	UP	DOWN
Distributor	Inspect	$v-c$ $f-v-k$	$v-c-\phi(p)$ $f-v-k+\phi(p)$	$-\phi(p)$ $f-p-k+\phi(p)$	0 $f-p-k$
	Do Not Inspect	$v-c$ $f-v$	$p-c$ $f-p$	$v-p$ $f-v$	0 $f-p$

Given the moves of Nature, the plant, and the distributor, the profit (or loss) of both firms are summarized in Table 1. For example, the second formula column describes the situation when the plant is UP but reports DOWN. Note that the plant has incentive to do so only if the spot price is higher than the unit-contingent contract price. If the distributor does not inspect, the plant earns a profit of $p-c$ (selling its output on the spot market), which is higher than $v-c$ the plant would have received by reporting truthfully. The distributor's profit is $f-p$ (buying on the spot market to cover its firm contract), which is lower than $f-v$ the distributor would have received if the plant reported truthfully. Thus, the distributor has an incentive to verify plant's report. If the distributor inspects (at cost k) and the plant is found to have misreported, then the plant must compensate the distributor for the loss of $p-v$ and pay the additional penalty $\phi(p)$ according to the contract. Thus, the plant's profit will be $v-c-\phi(p)$ and the distributor's profit will be $f-v-k+\phi(p)$. Similarly, the third formula column in Table 1 describes the situation when the plant is DOWN but reports UP (it has incentive to do so only if $p<v$).

[5] To see this, notice that with a random load the distributor uses the spot market to satisfy the shortfalls or sell extra power. When the plant reports DOWN, the distributor has more shortfalls to cover or less extra power to sell, thus its profit decreases by $p-v$ no matter the load is random or not.

The spot price is usually higher than the production cost c of a base-load plant. When it happens that the spot price drops below c, it would be uneconomical for the plant to continue production even if it is UP. In practice, however, the plant workers are typically told to keep the plant running whenever possible.[6] This is what we assume in this paper, and this is why in Table 1 when the plant is UP, a cost of c is always incurred.

The payoffs of the game are expressed in terms of the expected utility over the random profit in Table 1. The utility functions of the plant and the distributor are $U_P(\cdot)$ and $U_D(\cdot)$, respectively. We assume (i) both utility functions are concave and strictly increasing, (ii) $U_P(0)=0$ (normalization), and (iii) $U_D(x)\to\infty$ as $x\to\infty$. The last assumption is not crucial for the results in this paper, but helps with the exposition. The complete characterization of the execution stage game solution is given in the next section.

4. Bayesian Equilibrium of the Execution Stage Game

In this section, we find the Bayesian equilibrium of the game played during the execution stage for any given contract $(v,\phi)\geq 0$ and any given realization of the electricity spot price p. The mixed strategies of the plant and the distributor are characterized by the following probabilities:

x_{UP} is the probability that the plant tells the truth (i.e., reports UP) when it is UP;

x_{DOWN} is the probability that the plant tells the truth (i.e., reports DOWN) when it is DOWN;

y_{UP} is the probability that the distributor inspects if the plants reports UP;

y_{DOWN} is the probability that the distributor inspects if the plants reports DOWN.

How the game is played depends on whether the spot price, p, is smaller or greater than the unit-contingent swap price, v. (When $p=v$, the plant has no incentive to misreport and the distributor will not inspect.) We next find the best response functions and the equilibrium for each case.

Case 1: $p>v$

If the plant is DOWN, truth-telling (reporting that it is DOWN) is the dominating strategy for the plant. Therefore, $x^*_{\mathsf{DOWN}}=1$.

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[6] Workers are not involved in reporting, and are told that the plant receives no income if down. The manager may behave strategically.

If the plant is UP, it obtains $U_P(v-c)$ by reporting UP (truth-telling), and an expected utility of $y_{\mathsf{DOWN}}U_P(v-c-\phi(p))+(1-y_{\mathsf{DOWN}})U_P(p-c)$ by reporting DOWN, where y_{DOWN} is the probability that the distributor inspects the plant. Reporting DOWN is preferred by the plant if

$$y_{\mathsf{DOWN}} < \frac{U_P(p-c)-U_P(v-c)}{U_P(p-c)-U_P(v-c-\phi(p))} \stackrel{\text{def}}{=} \widehat{y}_{\mathsf{DOWN}} \in (0,1], \tag{1}$$

where $\widehat{y}_{\mathsf{DOWN}} \in (0,1]$ because $p>v$ and $\phi(p)\geq 0$. Thus, the plant's optimal response expressed as the truth-telling probabilities x_{UP} and x_{DOWN}, given the inspection probability y_{DOWN}, is

$$\text{When UP, } \begin{cases} x^*_{\mathsf{UP}}=0 \text{ (report DOWN)}, & \text{if } y_{\mathsf{DOWN}}<\widehat{y}_{\mathsf{DOWN}}; \\ x^*_{\mathsf{UP}}\in[0,1], & \text{if } y_{\mathsf{DOWN}}=\widehat{y}_{\mathsf{DOWN}}; \\ x^*_{\mathsf{UP}}=1 \text{ (report UP)}, & \text{if } y_{\mathsf{DOWN}}>\widehat{y}_{\mathsf{DOWN}}; \end{cases} \tag{2}$$
$$\text{When DOWN, } x^*_{\mathsf{DOWN}}=1 \text{ (report DOWN)}.$$

Next, we analyze the distributor's best response, given the plant's truth-telling probability. If the plant reports UP, the distributor knows that the plant is indeed UP (because pretending to be UP while DOWN is a dominated strategy for the plant when $p>v$), so the distributor will not inspect: $y^*_{\mathsf{UP}}=0$.

If the plant reports DOWN, the distributor forms a belief about the plant's actual status according to the Bayes rule:

$$\mathsf{P}\{\text{plant is UP} \mid \text{plant reports DOWN}\} = \frac{(1-x_{\mathsf{UP}})\gamma}{(1-x_{\mathsf{UP}})\gamma+(1-\gamma)} \stackrel{\text{def}}{=} \alpha(x_{\mathsf{UP}}) \in [0,\gamma].$$

Based on the above belief about misreporting, if the distributor inspects, its expected utility is

$$\alpha(x_{\mathsf{UP}})U_D(f-v-k+\phi(p))+\big(1-\alpha(x_{\mathsf{UP}})\big)U_D(f-p-k).$$

Without inspection, the distributor can obtain $U_D(f-p)$. Thus, the distributor will not inspect if its belief about misreporting is below a certain level:

$$\alpha(x_{\mathsf{UP}}) < \frac{U_D(f-p)-U_D(f-p-k)}{U_D(f-v-k+\phi(p))-U_D(f-p-k)} \stackrel{\text{def}}{=} \widehat{\alpha} \in (0,\infty). \tag{3}$$

Because $\alpha(x_{\mathsf{UP}})\leq\gamma$, if $\widehat{\alpha}>\gamma$, then, regardless of the plant's strategy x_{UP}, inequality (3) holds and the distributor does not inspect. The condition $\widehat{\alpha}>\gamma$ is equivalent to $\phi(p)<\widehat{\phi}(p)$, where

$$\widehat{\phi}(p) \stackrel{\text{def}}{=} v+k-f+U_D^{-1}\left(\frac{1}{\gamma}U_D(f-p)-\frac{1-\gamma}{\gamma}U_D(f-p-k)\right), \qquad \text{for } p>v. \tag{4}$$

In other words, regardless of the plant's strategy, a penalty payment that is below $\widehat{\phi}(p)$ will not provide sufficient incentive for the distributor to inspect at cost $k>0$. If $\phi(p)\geq\widehat{\phi}(p)$, then the distributor's inspection decision will depend on the plant's truth-telling probability (note that if $\widehat{\phi}(p)\leq 0$ then $\phi(p)\geq\widehat{\phi}(p)$ always holds since the penalty payment is non-negative). The distributor will not inspect if (3) holds, which is equivalent to $x_{\mathsf{UP}}>\alpha^{-1}(\widehat{\alpha})$, or

$$x_{\mathsf{UP}}>1-\frac{(1-\gamma)\widehat{\alpha}}{\gamma(1-\widehat{\alpha})} \stackrel{\text{def}}{=} \widehat{x}_{\mathsf{UP}}, \tag{5}$$

where $\widehat{x}_{\mathsf{UP}}\in[0,1)$ because $\widehat{\alpha}\in(0,\gamma]$ when $\phi(p)\geq\widehat{\phi}(p)$.

In summary, the distributor's best response expressed as inspection probabilities y_{UP} and y_{DOWN}, given the the plant's strategy x_{UP}, is

$$\begin{aligned} &\text{When plant reports UP, } y^*_{\mathsf{UP}}=0 \text{ (do not inspect);}\\ &\text{When plant reports DOWN, } \begin{cases} \text{If } \phi(p)<\widehat{\phi}(p), \text{ then } y^*_{\mathsf{DOWN}}=0 & \text{(do not inspect);}\\ \text{If } \phi(p)\geq\widehat{\phi}(p), \text{ then} & \\ \quad y^*_{\mathsf{DOWN}}=1 \text{ (inspect)}, & \text{if } x_{\mathsf{UP}}<\widehat{x}_{\mathsf{UP}};\\ \quad y^*_{\mathsf{DOWN}}\in[0,1], & \text{if } x_{\mathsf{UP}}=\widehat{x}_{\mathsf{UP}};\\ \quad y^*_{\mathsf{DOWN}}=0 \text{ (do not inspect)}, & \text{if } x_{\mathsf{UP}}>\widehat{x}_{\mathsf{UP}}. \end{cases} \end{aligned} \tag{6}$$

Combining the best response functions in (2) and (6), we can find the Bayesian equilibrium as summarized in the following proposition. Proofs of all propositions in this paper are included the online appendix.

PROPOSITION 1. *When the realized spot price is above the contract price, $p>v$, the Bayesian equilibrium of the contract execution stage game is as follows:*

(i) If the penalty is below the threshold, $\phi(p)\leq\widehat{\phi}(p)$, we have a pooling equilibrium where the plant always reports DOWN and the distributor does not inspect:

$$x^*_{\mathsf{UP}}=0, \qquad x^*_{\mathsf{DOWN}}=1, \qquad y^*_{\mathsf{UP}}=0, \qquad y^*_{\mathsf{DOWN}}=0,$$

and the equilibrium utilities of the plant and the distributor are:

$$\mathsf{E}[U_P \mid p] \;=\; \gamma U_P(p-c), \qquad \mathsf{E}[U_D \mid p] \;=\; U_D(f-p). \tag{7}$$

(ii) If the penalty is above the threshold, $\phi(p) > \widehat{\phi}(p)$, we have a mixed strategy equilibrium:

$$x^*_{\mathsf{UP}} = \widehat{x}_{\mathsf{UP}}, \qquad x^*_{\mathsf{DOWN}} = 1, \qquad y^*_{\mathsf{UP}} = 0, \qquad y^*_{\mathsf{DOWN}} = \widehat{y}_{\mathsf{DOWN}},$$

where $\widehat{x}_{\mathsf{UP}}$ is defined in (5) and $\widehat{y}_{\mathsf{DOWN}}$ is defined in (1). The firms' equilibrium utilities are:

$$\mathsf{E}[U_P \mid p] \;=\; \gamma U_P(v-c), \qquad \mathsf{E}[U_D \mid p] \;=\; \gamma \widehat{x}_{\mathsf{UP}} U_D(f-v) + (1-\gamma \widehat{x}_{\mathsf{UP}}) U_D(f-p). \tag{8}$$

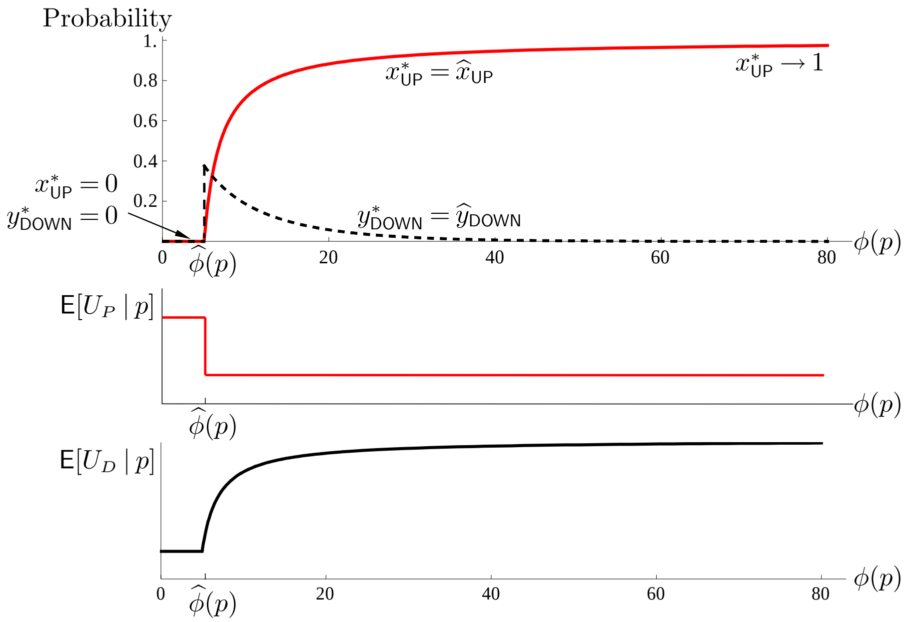
Figure 3 illustrates a few useful properties of this equilibrium. When the plant is UP, the equilibrium probability of truthful reporting is increasing in the level of the penalty: When $\phi(p) \le \widehat{\phi}(p)$, $x^*_{\mathsf{UP}} = 0$; when $\phi(p) > \widehat{\phi}(p)$, $x^*_{\mathsf{UP}} = \widehat{x}_{\mathsf{UP}}$ increases in $\phi(p)$ and approaches to one as $\phi(p) \to \infty$.

The distributor may conduct inspection if and only if the plant reports DOWN and $\phi(p) > \widehat{\phi}(p)$, and the equilibrium probability of inspection, $y^*_{\mathsf{DOWN}} = \widehat{y}_{\mathsf{DOWN}}$, decreases in $\phi(p)$ and approaches zero when $\phi(p) \to \infty$. Intuitively, a very large penalty with a very small inspection probability can effectively deter misreporting.

When the penalty $\phi(p)$ increases, the plant's expected utility decreases (in a step-function fash-

Figure 3 Effect of Penalty $\phi(p)$ on Equilibrium Strategies and Utilities: The Case of $p > v$ and $\widehat{\phi}(p) > 0$

For the case of $p < v$, the effect of $\phi(p)$ is qualitatively the same as shown below, except that x^*_{UP} and y^*_{DOWN} should be replaced by x^*_{DOWN} and y^*_{UP}, respectively.
Parameters: $\gamma = 0.8,\ k = 8,\ v = 70, p = 75,\ U_D(x) = x,\ \widehat{\phi}(p) = v - p + k/\gamma$



ion) and the distributor's expected utility increases.

Case 2: $p < v$

The analysis of this case is parallel to Case 1. We relegate the analysis to the online appendix, and summarize the equilibrium below after giving a few definitions in parallel with Case 1. Define

$$\begin{aligned}
\widehat{x}_{\mathsf{DOWN}} &\stackrel{\text{def}}{=} 1-\frac{\gamma\widehat{\beta}}{(1-\gamma)(1-\widehat{\beta})}, \qquad \text{where} \quad \widehat{\beta} \stackrel{\text{def}}{=} \frac{U_D(f-v)-U_D(f-v-k)}{U_D(f-p-k+\phi(p))-U_D(f-v-k)} \\
\widehat{y}_{\mathsf{UP}} &\stackrel{\text{def}}{=} \frac{U_P(v-p)}{U_P(v-p)-U_P(-\phi(p))}, \\
\widehat{\phi}(p) &\stackrel{\text{def}}{=} p+k-f+U_D^{-1}\left(\frac{1}{1-\gamma}U_D(f-v)-\frac{\gamma}{1-\gamma}U_D(f-v-k)\right), \qquad \text{for } p<v. \qquad (9)
\end{aligned}$$

The Bayesian equilibrium is summarized in the following proposition.

PROPOSITION 2. *When the realized spot price is below the contract price, $p < v$, the Bayesian equilibrium of the contract execution stage game is as follows:*

(i) If the penalty is below the threshold, $\phi(p) \le \widehat{\phi}(p)$, we have a pooling equilibrium where the plant always reports UP and the distributor does not inspect:

$$x^*_{\mathsf{UP}} = 1, \qquad x^*_{\mathsf{DOWN}} = 0, \qquad y^*_{\mathsf{UP}} = 0, \qquad y^*_{\mathsf{DOWN}} = 0,$$

and the equilibrium utilities of the plant and the distributor are:

$$\mathsf{E}[U_P \mid p] = \gamma U_P(v-c)+(1-\gamma)U_P(v-p), \qquad \mathsf{E}[U_D \mid p] = U_D(f-v); \qquad (10)$$

(ii) If the penalty is above the threshold, $\phi(p) > \widehat{\phi}(p)$, we have a mixed strategy equilibrium:

$$x^*_{\mathsf{UP}} = 1, \qquad x^*_{\mathsf{DOWN}} = \widehat{x}_{\mathsf{DOWN}}, \qquad y^*_{\mathsf{UP}} = \widehat{y}_{\mathsf{UP}}, \qquad y^*_{\mathsf{DOWN}} = 0,$$

and the equilibrium utilities of the plant and the distributor are:

$$\mathsf{E}[U_P \mid p] = \gamma U_P(v-c), \quad \mathsf{E}[U_D \mid p] = (1-(1-\gamma)\widehat{x}_{\mathsf{DOWN}})U_D(f-v)+(1-\gamma)\widehat{x}_{\mathsf{DOWN}}U_D(f-p). \quad (11)$$

5. Optimal Contract

Anticipating the equilibrium in the execution stage game, the distributor now chooses the contract parameters $(v, \phi(p))$ to maximize its expected utility while maintaining the plant's reservation utility, denoted as $\underline{U_P}$. We assume $\underline{U_P} > 0$ because $U_P(0) = 0$.

$$\max_{v,\phi(p)} \ \mathsf{E}[U_D] \tag{12}$$
$$s.t. \ \ \mathsf{E}[U_P] \ \geq \ \underline{U_P}.$$

Because we are looking for the optimal features of a specific type of contract – the unit-contingent swap – this precludes us from applying the general mechanism design results, such as the revelation principle. We find the optimal contract based on the explicit solution to the Bayesian game derived in the previous section.

5.1. Unit-Contingent Truth-Telling Contracts

We first analyze the unit-contingent contracts that enforce truth-telling. From the analysis in Section 4, we know that to enforce truth-telling the penalty for misreporting needs to be large (infinite in theory, but as shown in Figure 3, reasonably large penalty is sufficient for practical purpose). When penalty $\phi(p)=\infty$ for all values of the spot price p, the equilibrium truth-telling probabilities $\widehat{x}_{\mathsf{UP}}$ and $\widehat{x}_{\mathsf{DOWN}}$ are equal to one. Therefore, both (8) and (11) imply that the firms' expected utilities (unconditioned on p) are:

$$\mathsf{E}[U_P]=\gamma U_P(v-c), \qquad \mathsf{E}[U_D]=\gamma U_D(f-v)+(1-\gamma)\mathsf{E}U_D(f-p).$$

When the contract price v increases, the plant's expected utility $\mathsf{E}[U_P]$ increases while the distributor's expected utility $\mathsf{E}[U_D]$ decreases. Therefore, the optimal unit-contingent truth-telling contract is $(v_0,\phi(\cdot)=\infty)$, where v_0 is the unique solution to

$$\gamma U_P(v_0-c)=\underline{U_P}. \tag{13}$$

That is, v_0 as the lowest contract price that the plant is willing to accept if truth-telling is enforced. Clearly, $v_0>c$ because $\underline{U_P}>0$ and $U_P(0)=0$. This contract is an important benchmark, as shown in the following lemma.

Lemma 1. *Among all unit-contingent contracts with $v \geq v_0$ (not necessarily enforcing truth-telling), the optimal contract is $(v_0,\phi(\cdot)=\infty)$, under which the distributor's expected utility is*

$$\mathsf{E}[U_D]=\gamma U_D(f-v_0)+(1-\gamma)\mathsf{E}U_D(f-p). \tag{14}$$

Proof. For any fixed $v \geq v_0$, increasing the penalty $\phi(p)$ to infinity will keep the plant satisfied: $\mathsf{E}[U_P] = \gamma U_P(v-c) \geq \underline{U_P}$, while maximizing the distributor's expected utility (refer to Figure 3). Because the distributor's expected utility decreases in v, the minimum possible v, which is v_0, is optimal. □

5.2. The Sub-Optimality of the Unit-Contingent Truth-Telling Contracts

A truth-telling unit-contingent swap contract would be optimal if we restrict the contract price $v \geq v_0$, but is there a better contract with $v < v_0$? To make the contract price $v < v_0$ acceptable to the plant, the distributor needs to reduce the penalty to some extent, that is, does not enforce truth-telling. A natural question is whether such a mechanism would be any better than the one enforcing truth-telling. This section proves the existence of such a superior contract. An important implication of this result is that the seemingly undesirable actions of the plant could actually serve as a tacit risk-allocation mechanism between the plant and the distributor.

We consider a contract $(v, \phi(p))$ of the following form:

$$v < v_0, \qquad \phi(p) = \begin{cases} 0, & p \in [v-\delta, v], \\ \infty, & \text{otherwise}, \end{cases} \tag{15}$$

where v is slightly below v_0 and δ is small (we assume $\delta \in (0, k)$, where k is the inspection cost). Under the contract in (15), when the spot price $p \in [v-\delta, v]$, the equilibrium in Proposition 2(i) is played.[7] In this equilibrium, facing zero penalty for misreporting, the plant would report UP when it is DOWN, obtaining a profit of $v - p$. Because of this benefit, the plant is willing to accept a contract price $v < v_0$. When $p \notin [v-\delta, v]$, the equilibrium is that in Proposition 1(ii) or that in Proposition 2(ii). In this equilibrium, due to infinite penalty, the plant is truthful: $\widehat{x}_{\mathsf{UP}} = \widehat{x}_{\mathsf{DOWN}} = 1$.

Using the equilibrium utilities given in (8), (10), and (11), the expected utilities of the plant and the distributor under the contract (15) can be expressed as:

$$\mathsf{E}[U_P] = \gamma U_P(v-c) + (1-\gamma)\int_{v-\delta}^{v} U_P(v-p) g(p) dp, \tag{16}$$

[7] To see this, we need to show $\phi(p) \leq \widehat{\phi}(p)$. Note that $\widehat{\phi}(p)$ defined in (9) satisfies $\dfrac{U_D(f-v) - U_D(f-v-k)}{U_D(f-p-k+\widehat{\phi}(p)) - U_D(f-v-k)} = 1-\gamma < 1$, which implies that $\widehat{\phi}(p) > p + k - v$. Since $p \geq v - \delta$ and $\delta < k$, we have $\widehat{\phi}(p) > k - \delta > 0 = \phi(p)$.

$$\begin{aligned}\mathsf{E}[U_D] &= \gamma U_D(f-v) + (1-\gamma)\int_{p\notin[v-\delta,v]} U_D(f-p)g(p)dp + (1-\gamma)\int_{v-\delta}^{v} U_D(f-v)g(p)dp,\\ &= \Big[\gamma U_D(f-v) + (1-\gamma)\mathsf{E}U_D(f-p)\Big] - (1-\gamma)\int_{v-\delta}^{v}\big[U_D(f-p) - U_D(f-v)\big]g(p)dp, \quad (17)\end{aligned}$$

where $g(p)$ is the probability density function of the spot price. In (16), the first term $\gamma U_P(v-c)$ is the plant's expected utility if it is always truthful, and the second term is the additional utility the plant can obtain by misreporting its status when $p \in [v-\delta, v]$. In (17), the first term in the brackets is the distributor's expected utility when the plant always reports the truth, and the second term accounts for the distributor's utility loss due to the plant's misreporting.

In the following proposition, we show that under fairly general conditions there exist parameters v and δ such that the plant's expected utility in (16) is above the reservation utility $\underline{U_P}$ and the distributor's expected utility in (17) is even higher than that under the truth-telling contracts.

PROPOSITION 3. *If either the plant or the distributor or both are strictly risk-averse, then there exists a unit-contingent contract of the form (15) that strictly dominates the optimal truth-telling enforcing contract* $(v_0, \phi(\cdot) = \infty)$.

Proposition 3 suggests that whenever risk is a concern of at least one party, a contract of the form (15) is able to allocate the risk between the plant and the distributor in a more efficient manner. If both firms were risk-neutral, improving both firms' expected profit is not possible, because the sum of the firms' profit is always $f-c$ when the plant is UP and $f-p$ when the plant is DOWN.

Let us intuitively explain Proposition 3. Consider a contract price v slightly below v_0, the lowest price that the plant is willing to accept if truth-telling is enforced. This reduces the plant's profit when the plant is UP, resulting in a utility loss of $\gamma U_P(v_0-c) - \gamma U_P(v-c)$. To compensate for this loss, the distributor eliminates the misreporting penalty when the spot price is slightly below the contract price: $p \in [v-\delta, v]$. When the spot price falls in this range, the plant that is DOWN can still gain a small profit of $v-p$ by pretending to be UP and buying electricity on the spot market to deliver to the distributor. This extra benefit can offset the loss, i.e., $(1-\gamma)\int_{v-\delta}^{v} U_P(v-p)g(p)dp = \gamma U_P(v_0-c) - \gamma U_P(v-c)$, thereby keeping the plant's expected utility unchanged. If the plant is risk-averse, the expected value of this benefit, $(1-\gamma)\int_{v-\delta}^{v}(v-p)g(p)dp$, will be smaller than the expected loss $\gamma(v_0-v)$. Hence, the distributor's expected profit increases.

If the plant is risk-neutral and the distributor is risk-averse, then the above expected benefit to the plant (loss to the distributor) will be exactly equal to the expected loss to the plant (benefit to the distributor). The distributor's expected profit will not change, but the distributor now enjoys more certain profit: The distributor obtains a fixed profit of $f-v$ with a higher probability than under the truth-telling enforcing contract. In particular, when $p\in[v-\delta,v]$ the distributor obtains $f-v$ no matter whether the plant is UP or DOWN.

5.3. The Optimal Unit-Contingent Contract

We now proceed to find the optimal unit-contingent contract. First, we make the following observation based on Figure 3.

LEMMA 2. *A unit-contingent contract* $(v,\phi(\cdot))$ *with penalty* $\phi(\cdot)<\infty$ *is strictly dominated by* $(v,\phi^\dagger(\cdot))$*, where* $\phi^\dagger(p)=0$ *if* $\phi(p)\le\widehat{\phi}(p)$*, and* $\phi^\dagger(p)>\phi(p)$ *if* $\phi(p)>\widehat{\phi}(p)$*.*

The proof is omitted. From Figure 3, it can be seen that $(v,\phi^\dagger(\cdot))$ defined in Lemma 2 will not change the plant's expected utility, but increase the distributor's expected utility. Intuitively, any penalty level below the threshold $\widehat{\phi}(p)$ induces misreporting and no inspection, so does the zero penalty. When the penalty is above the threshold, a higher penalty results in higher truth-telling probability, benefiting the distributor. An important implication of Lemma 2 is that without loss of optimality, we can restrict our attention to only two penalty values: zero and infinity. (In practice, a large penalty would be sufficient, because the truth-telling probability approaches to one quickly as penalty increases, as shown in Figure 3.)

With the above observation, choosing a contract $(v,\phi(\cdot))$ is equivalent to choosing a contract price v and a price set L within which the penalty is zero and beyond which the penalty is infinite. For the need of the analysis, we break this price set L into two subsets as follows:

$$L_1 \stackrel{\text{def}}{=} \{\ p\in[0,v]:\phi(p)=0\ \},\qquad L_2 \stackrel{\text{def}}{=} \{\ p\in(v,\infty):\phi(p)=0\ \}.$$

From Lemma 2, zero penalty is never optimal when the threshold $\widehat{\phi}(p) < 0$. Thus, the above subsets must be contained in the region where the threshold is non-negative:

$$L_1 \subseteq S_1(v) \stackrel{\text{def}}{=} \{\ p \in [0, v] : \widehat{\phi}(p) \geq 0\ \}, \qquad L_2 \subseteq S_2(v) \stackrel{\text{def}}{=} \{\ p \in (v, \infty) : \widehat{\phi}(p) \geq 0\ \}.$$

In the rest of the paper, we will refer to L_1 and L_2 as *zero-penalty regions* and $S_1(v)$ and $S_2(v)$ as *zero-penalty feasible regions* or simply *feasible regions*. We omit the dependence of S_1 and S_2 on v when no confusion will arise.

Using Proposition 1 and 2, we can derive the firms' expected utilities as follows:[8]

$$\mathsf{E}[U_P] = \gamma U_P(v-c) + (1-\gamma)\int_{L_1} U_P(v-p)g(p)dp + \gamma \int_{L_2} \big[U_P(p-c) - U_P(v-c)\big]g(p)dp \tag{18}$$

$$\begin{aligned}\mathsf{E}[U_D] = \Big[\gamma U_D(f-v) + (1-\gamma)\mathsf{E}U_D(f-p)\Big] - (1-\gamma)\int_{L_1} \big[U_D(f-p) - U_D(f-v)\big]g(p)dp \\ -\gamma \int_{L_2} \big[U_D(f-v) - U_D(f-p)\big]g(p)dp\end{aligned} \tag{19}$$

In (18), the first term $\gamma U_P(v-c)$ is the plant's expected utility if it always reports the true status, and the rest two terms are the additional utility the plant can get by misreporting its status in the zero-penalty regions L_1 and L_2. In (19), the first term in the brackets is the distributor's expected utility when the plant is truthful, and the remaining two integrals account for the utility loss due to the plant's misreporting behavior.

Using (18) and (19), the distributor's problem in (12) now becomes:

$$\begin{aligned} \max_{\{v \leq v_0,\ L_i \subseteq S_i(v),\, i=1,2\}} \quad & \mathsf{E}[U_D] \\ s.t. \quad & \mathsf{E}[U_P] \ \geq\ \underline{U_P}. \end{aligned} \tag{20}$$

Note that the contract price v cannot be too low, otherwise the plant's reservation utility level will not be satisfied even if the plant is allowed to misreport over the entire feasible region $S_1 \cup S_2$.

---

[8] To derive (18) and (19), notice the following. When $p \in L_1$, the equilibrium utilities are given in (10):

$$\mathsf{E}[U_P \mid p] \ =\ \gamma U_P(v-c) + (1-\gamma)U_P(v-p), \qquad \mathsf{E}[U_D \mid p] \ =\ U_D(f-v).$$

When $p \in L_2$, the equilibrium utilities are given in (7):

$$\mathsf{E}[U_P \mid p] \ =\ \gamma U_P(p-c), \qquad \mathsf{E}[U_D \mid p] \ =\ U_D(f-p).$$

When $p \notin (L_1 \cup L_2)$, it is optimal for the distributor to set penalty $\phi(p) = \infty$. Both (8) and (11) imply that:

$$\mathsf{E}[U_P \mid p] \ =\ \gamma U_P(v-c), \qquad \mathsf{E}[U_D \mid p] \ =\ \gamma U_D(f-v) + (1-\gamma)U_D(f-p).$$

Integrating over the above three regions and rearranging terms, we have the expressions (18) and (19).

We define a feasible contract price v as satisfying the feasibility inequality:

$$\gamma U_P(v-c)+(1-\gamma)\int_{S_1(v)} U_P(v-p)g(p)dp+\gamma\int_{S_2(v)} \big[U_P(p-c)-U_P(v-c)\big]g(p)dp \;\geq\; \underline{U_P}. \quad (21)$$

Solving the optimal contract problem in (20) is complicated in general, but we are able to identify some structures of the optimal contract to reduce the search space significantly. The following Proposition 4 presents a general structure of L_2^*. A simple structure of L_1^* can be obtained when one of the parties is risk-neutral, as discussed in Proposition 5. The structural results in these propositions are stated for any fixed v, and thus they also hold under the optimal contract price v^*.

PROPOSITION 4. *Consider solving (20) for any fixed contract price $v \leq v_0$, satisfying the feasibility condition (21). If either the plant or the distributor or both are strictly risk-averse, then there exists p^* such that the optimal zero-penalty region $L_2^* = [v, p^*] \cap S_2$. (In case $p^* \leq v$, $L_2^* = \varnothing$.)*

Proposition 4 says that the optimal zero-penalty region L_2^*, if non-empty, must be contained in the leftmost part of the feasible region S_2. That is, if the distributor chooses to induce the plant to misreport when $p > v$, then p should be as small as possible. Intuitively, when the spot price falls in the zero-penalty region L_2, the plant will pretend to be DOWN when it is UP, sell to the spot market to make more profit. A risk-averse plant would prefer low and frequent gains (corresponding to low spot prices) over high and infrequent gains (corresponding to high spot prices) with the same expected value. The distributor has to buy from the spot, incurring a loss. A risk-averse distributor would prefer to avoid the risk of large and infrequent losses (corresponding to high spot prices) in favor of low and frequent losses (corresponding to low spot prices) with the same expected value.

The optimal L_1^* and its relation to L_2^* is complicated in general, but a simple structure can be proven if one of the two parties is risk-neutral, as summarized in Proposition 5.

PROPOSITION 5. *Consider solving (20) for any fixed contract price $v \leq v_0$, satisfying (21).*

(i) If the distributor is risk-neutral and the plant is risk-averse, then there exist p_1^* and p_2^* such that $L_1^* = [p_1^*, v] \cap S_1$ and $L_2^* = [v, p_2^*] \cap S_2$.

(ii) If the plant is risk-neutral and the distributor is risk-averse, then there exists p^* such that $L_i^* = [0, p^*] \cap S_i$, $i = 1, 2$. In particular, if $L_2^* \neq \varnothing$, then $L_1^* = S_1$.

Proposition 5(i) shows that the optimal zero-penalty region L_1^* resides in the rightmost part of the feasible region S_1. That is, if the distributor chooses to induce the plant to misreport when $p < v$ (the plant reports UP when DOWN to gain a profit of $v - p$), then p should be as large as possible. The reason is similar to that discussed following Proposition 4: The risk-averse plant prefers low and frequent gains (now corresponding to high spot prices in the region $p < v$) over high and infrequent gains with the same expected value. Thus, by inducing the plant to misreport when the spot price is high in the region $p < v$, the distributor can lower the plant's expected gains due to misreporting, thereby improving the distributor's expected profit while keeping the plant's expected utility constant. The argument for L_2^* is similar.

In contrast, Proposition 5(ii) shows that a risk-averse distributor who is contracting with a risk-neutral plant will try to move the zero-penalty region L_1 to the left of the feasible region S_1. Intuitively, by inducing the plant to report UP when DOWN, the distributor essentially sacrifices a random profit of $v - p$ in return for a lower contract price v and a lower variance in its profit (note that the risk-neutral plant's expected profit is held constant, so is the distributor's expected profit). To minimize the variance, the risk-averse distributor prefers sacrificing higher and infrequent profit (corresponding to low spot prices) over sacrificing lower and frequent profit. To explain the last statement in Proposition 5(ii), observe that the plant's misreporting when $p < v$ reduces the distributor's risk, while misreporting when $p > v$ increases the distributor's risk. Therefore, the risk-averse distributor should always induce misreporting for $p < v$ before allowing it for $p > v$.

The structural results in Proposition 4 and 5 provide us with a convenient method of finding the optimal zero-penalty region for any feasible contract price v, enabling us then to numerically search for the optimal contract price v^*. We illustrate this procedure in the next section.

6. Numerical Examples

We present numerical results in this section to illustrate our theoretical results and discuss additional insights. We focus on the situation with a risk-averse plant and a risk-neutral distributor.

The *base scenario* is defined as follows. The plant is UP with probability $\gamma = 0.8$ (for real coal-fired plants, γ varies from plant to plant, but it is usually above 0.7, and can sometimes be as high as 0.99). The production cost is $c = 30$ (average operating cost of fossil steam engines is \$29.59/MWh, reported by Energy Information Administration 2007). The plant's utility function is $U_P(x) = \log\left(\frac{x+x_0}{x_0}\right)$, where $x_0 = 20$. The distributor is risk-neutral, its inspection cost is $k = 4$, and its firm contract price is $f = 80$ (PJM futures for 2008 summer electricity were traded around \$80 in 2005; data available from Bloomberg). The spot price p is log-normally distributed and $\log(p)$ has a mean of 4.2 and a standard deviation of 0.5; hence, $\mathsf{E}[p] = 75.57$ (the logarithm of the day-ahead spot price at the PJM hub in July-August 2008 has a mean of 4.171 and a standard deviation of 0.506, estimated using the hourly price data from Platts). Assume $\underline{U_P} = 0.879$, which implies that the lowest contract price the plant is willing to accept if truth-telling is enforced is $v_0 = 70$, satisfying $\gamma U_P(v_0 - c) = \underline{U_P}$.

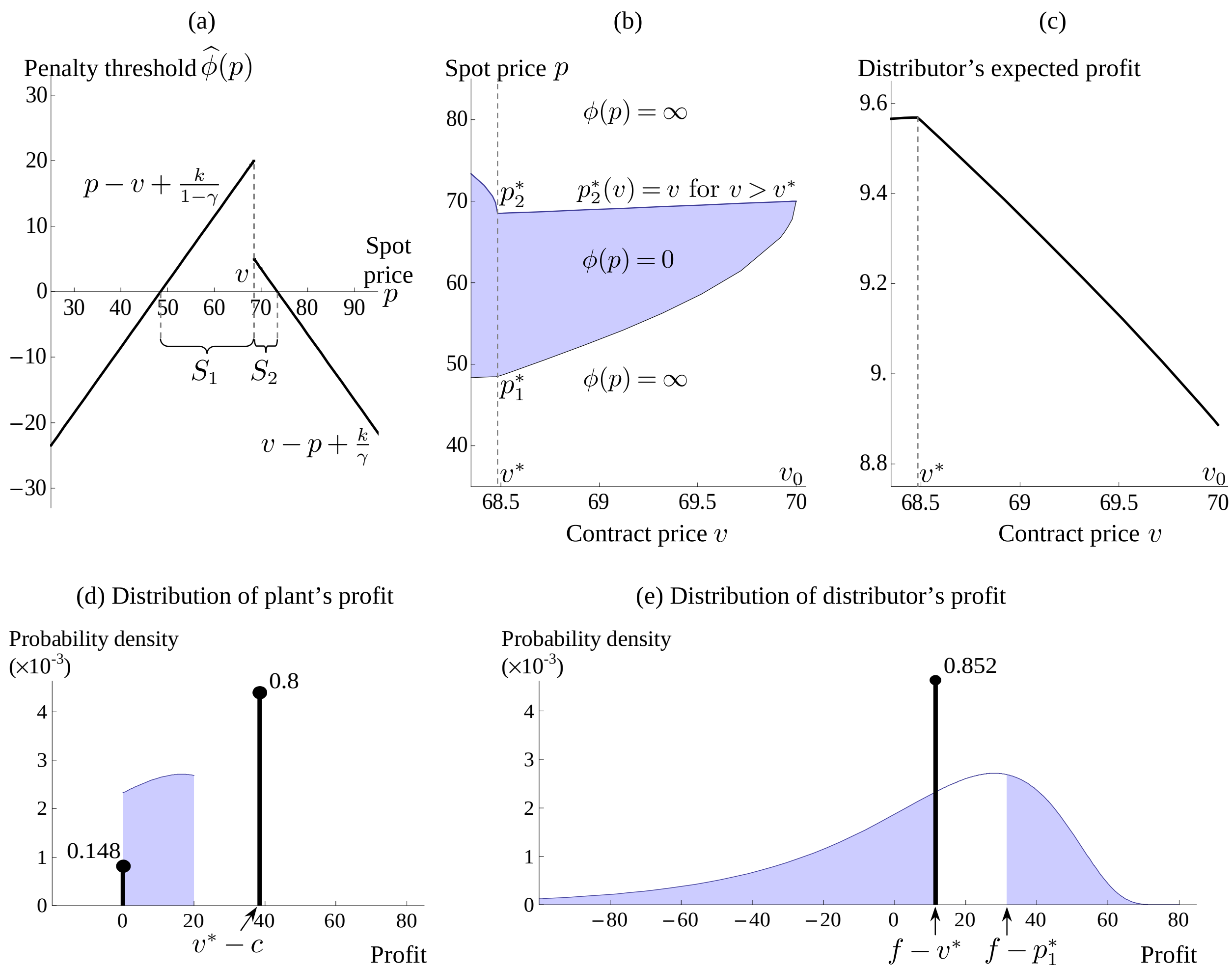
Figure 4(a) shows the zero-penalty feasible region $S_1 \cup S_2$, as defined by the positive part of the penalty threshold function $\widehat{\phi}(p)$. Using the structural results in Proposition 4 and Proposition 5, we search for the optimal zero-penalty region for various fixed contract price v, as shown in panel (b), and compute the corresponding expected profit for the distributor, shown in panel (c). Under the truth-telling contract with $v_0 = 70$ and $\phi(p) = \infty$, the distributor's expected profit is 8.887. Under the optimal contract with $v^* = 68.483$, $L_1^* = [48.483, v^*]$ and $L_2^* = \varnothing$, the distributor can achieve the highest expected profit of 9.569.

Let us further compare the firms' profit distributions under the truth-telling contract and the optimal contract. Under the truth-telling contract ($v_0 = 70, \phi(\cdot) = \infty$), with probability $\gamma = 0.8$ the plant is UP and obtains a profit of $v_0 - c = 40$; with probability $1 - \gamma = 0.2$ the plant is DOWN and receives zero profit. Thus, the plant's profit has a mean of 32 and a variance of 256. For the distributor, with probability $\gamma = 0.8$, its profit is $f - v_0 = 10$ and with probability 0.2, it obtains a random profit of $f - p$. Hence, the distributor's profit has a mean of 8.887 and a variance of 329.3.

Under the optimal contract (v^*, L_1^*, L_2^*), the firms' profit distributions are shown in Figures 4(d) and (e). Because misreporting is induced under the optimal contract, the plant gets zero profit only if it is DOWN and $p \notin [p_1^*, v^*]$. The plant obtains a profit of $v^* - p$, shown as shaded area in panel (d), when $p \in [p_1^*, v^*]$. This shaded area is exactly what the distributor has sacrificed, shown

Figure 4 An Example of Optimal Contract with Misreporting

Panel (a) shows the penalty threshold function $\widehat{\phi}(p)$ as defined in eqs. (4) and (9). Because $U_D(\cdot)$ is linear, $\widehat{\phi}(p)$ is piecewise linear. When threshold $\widehat{\phi}(p) < 0$, zero penalty cannot induce misreporting as a pure strategy. Zero penalty is only effective in the region $S_1 \cup S_2$, where $\widehat{\phi}(p) \geq 0$.
Panel (b) illustrates Proposition 5(i) by showing that the zero penalty region is an interval for any fixed v. In addition, it shows that the lower the contract price the wider the zero penalty region. The lowest possible contract price is $v = 68.35$, for which the zero penalty region covers the entire feasible region $S_1 \cup S_2$.
Panel (c) shows the value of the objective function (19) as a function of the contract price v with L_1 and L_2 optimized for each v. The optimal contract price is $v^* = 68.483$.
Panel (d) shows the plant's profit distribution under the optimal contract. It contains two point masses and a continuous distribution (the vertical axis is for the continuous distribution). The point mass 0.8 is the probability that the plant gains $v^* - c$, which is the probability of being UP. The point mass 0.148 is the probability of zero profit, which is the probability that the plant is DOWN and the spot price is outside the zero penalty region. The shaded area represents the plant's extra profit from misreporting its status. Its area is 0.052.
Panel (e) shows the distributor's profit distribution under the optimal contract. The point mass 0.852 is the probability that the distributor obtains $f - v^*$, which equals the probability that the plant reports UP. The curves represents the distribution of $f - p$, the distributor's profit when the plant reports DOWN. The white area under the curve is the profit loss due to the plant's misreporting.



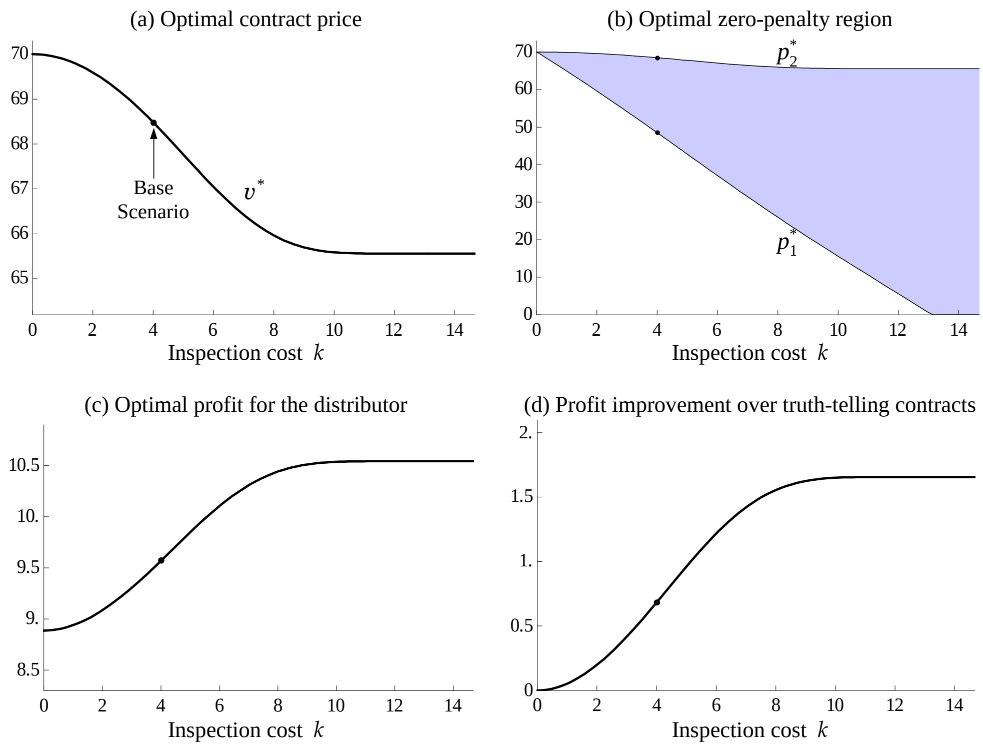
as white area in panel (e). In return for the benefit, they agrees on $v^* = 68.483 < v_0 = 70$. The plant's expected utility is the same as under the truth-telling contract. The plant's expected profit is now 31.318 (decreased by 2.1%) with a variance of 211.1 (a 17.5% reduction). The distributor's expected profit is improved to 9.569 (7.7% higher). Furthermore, although the distributor does not care about variance in this case, its profit variance is reduced to 323.5 (a 1.8% reduction). In fact, we could choose a slightly higher contract price (between v^* and v_0) such that both firms' expected profit is the same as in the truth-telling enforcing contract, but both have lower variance.

Figure 4(b) deserves more attention. Observe that when v decreases from v_0 to v^*, only L_1 expands while L_2 remains empty. When v drops below v^*, L_2 starts to expand, but the distributor's expected profit drops. This is evidence that misreporting when $p < v$ benefits the distributor while misreporting when $p > v$ hurts. Intuitively, when $p > v$ and the plant pretends to be DOWN, it sells output at spot price while the distributor has to buy from the spot. This introduces unnecessary spot price risk into both parties' profit distribution. Thus, this type of misreporting is undesirable and should be discouraged. However, we have found numerical examples where the optimal contract involves a non-empty L_2^*. The reason is that allowing misreporting for some $p > v$ would lead to an even lower contract price v, which changes $S_1(v)$ and may improve both firms' risk profiles. But this effect is relatively weak and can only be observed in few examples.

Next, we study the effects of a few key model parameters: the distributor's inspection cost, the plant's reliability, and the spot price volatility.

Figure 5 shows the effect of the inspection cost. Observe that as the inspection cost increases, the optimal unit-contingent contract price decreases (panel (a)), the misreporting region widens (panel (b)), the distributor's profit increases (panel (c)), and the distributor's profit improvement over the truth-telling contract increases (panel (d)). Why does the distributor's profit increase in the inspection cost k? An increase in k does not affect the distributor directly, because under the optimal contract, inspection probability is zero when the penalty is either zero or infinity (see Figure 3). But with a higher inspection cost k, the penalty threshold $\widehat{\phi}$ increases, and the zero-penalty feasible region $S_1 \cup S_2$ expands (see Figure 4(a)). A wider feasible region provides the distributor with more flexibility in using misreporting as a risk-allocation tool. Thus, the distributor's profit increases. When the inspection cost k is already high, further increase in k does not significantly increase the probability that the spot price falls in the feasible region and, therefore, the distributor's profit only increases up to a limit.

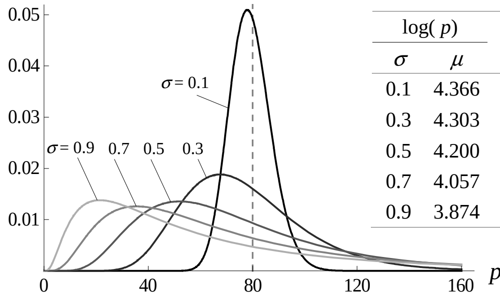
Figure 5 Effect of the Distributor's Inspection Cost



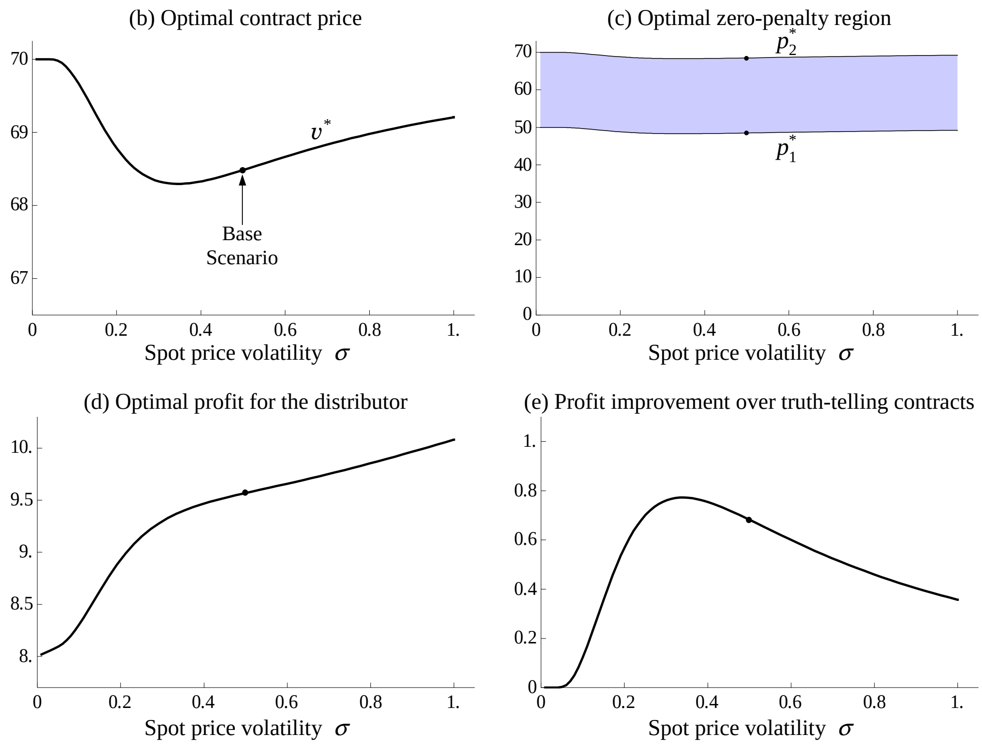
Next, we examine the effect of the spot price volatility on the optimal contract and the distributor's profit. Figure 6(a) shows a series of spot price distributions, which are assumed to be log-normal. The mean and standard deviation of $\log(p)$ are shown as σ and μ in the table along panel (a). The last column of this table lists the probability that the spot price falls in the zero-penalty region, $[p_1^*, p_2^*]$. The base scenario is $\sigma = 0.5$. We assume that the forward premium (forward price minus the average spot price, $f - \mathsf{E}[p]$) increases in the spot price volatility σ. We have experimented with different magnitudes of the forward premium, including zero premium, and found that the distributor's profit in panel (d) is sensitive to the forward premium because the distributor earns the forward premium when the plant reports DOWN, whereas the trends shown in other panels are quite robust.

Figure 6 Effect of the Spot Price Volatility

(a) Spot price distribution



log(p)		spot price p		prob. of
σ	μ	mean	std.dev.	$p \in [p_1^*, p_2^*]$
0.1	4.366	79.09	7.93	0.114
0.3	4.303	77.31	23.72	0.318
0.5	4.200	75.57	40.27	0.259
0.7	4.057	73.86	58.73	0.194
0.9	3.874	72.20	80.65	0.147

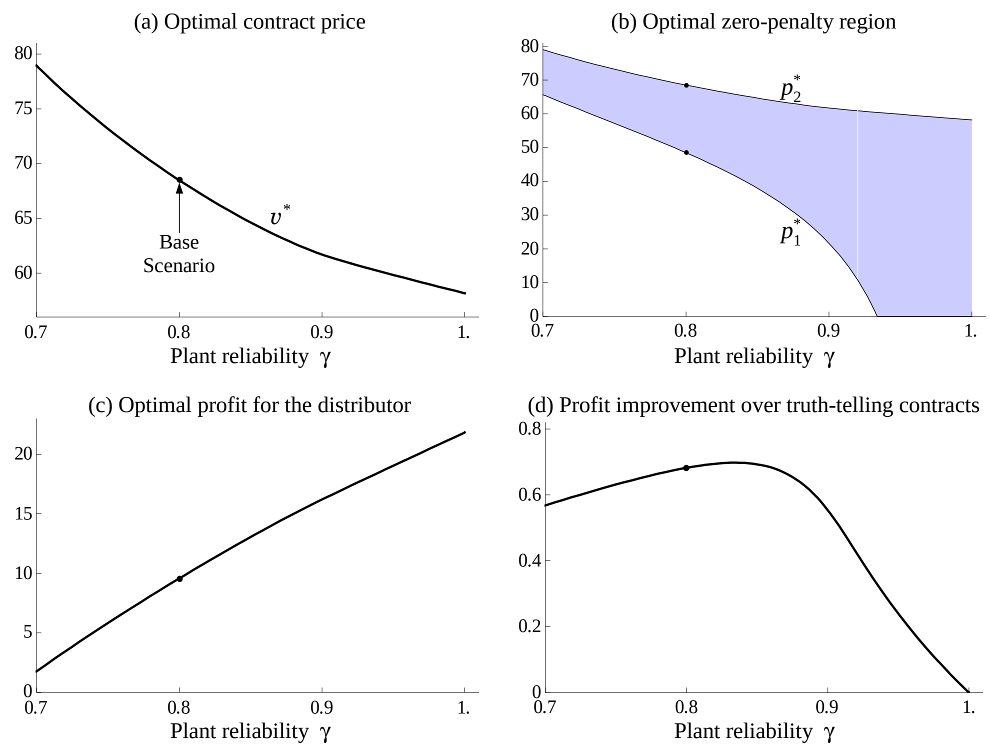


The plant's benefit from misreporting increases in the probability that the spot price falls in the zero-penalty region. Observe from panel (a) that this probability increases as the spot price volatility increases from $\sigma = 0$ to about 0.3. As the plant benefits more from misreporting, the distributor can lower the contract price (corresponding to the decreasing part in panel (b)) and achieve a higher profit improvement over the truth-telling contract (corresponding to the increasing part in panel (e)). However, as σ continues to increase, the spot price becomes too volatile and the probability that it falls within the zero-penalty region decreases. Consequently, the distributor is forced to raise the contract price to keep the plant at its reservation utility and the profit improvement over the truth-telling contract decreases.

Finally, we examine the effect of the plant's reliability, measured by γ, the probability that plant is UP. We first consider the case that the plant's reservation utility $\underline{U_P}$ is fixed and the truth-telling contract price v_0 is defined as in (13): $\underline{U_P} = \gamma U_P(v_0 - c)$. Note that as γ increases, v_0 decreases. All other parameters are the same as in the base scenario. Figure 7 shows that as γ increases, the optimal contract price decreases (panel (a)), the misreporting region widens initially and then shrinks slightly (panel (b)), the distributor's profit increases (panel (c)), and the improvement in the distributor's profit over the truth-telling contract is non-monotone (panel (d)). Intuitively, as the plant's reliability increases, for fixed contract terms, both firms' expected utilities increase. Thus, the distributor can lower the contract price v (panel (a)), still meet the plan't reservation utility, while achieving a higher profit (panel (c)). To explain the non-monotonicity in panel (d), we consider two factors. First, as the plant becomes more reliable, the zero-penalty feasible region increases, providing the distributor with more flexibility in using misreporting as a risk-allocation tool. Second, a more reliable plant is less likely to be DOWN, providing the plant

Figure 7 Effect of Plant's Reliability I

Reservation utility $\underline{U_P}$ is fixed. Truth-telling contract price v_0 satisfies $\underline{U_P} = \gamma U_P(v_0 - c)$.

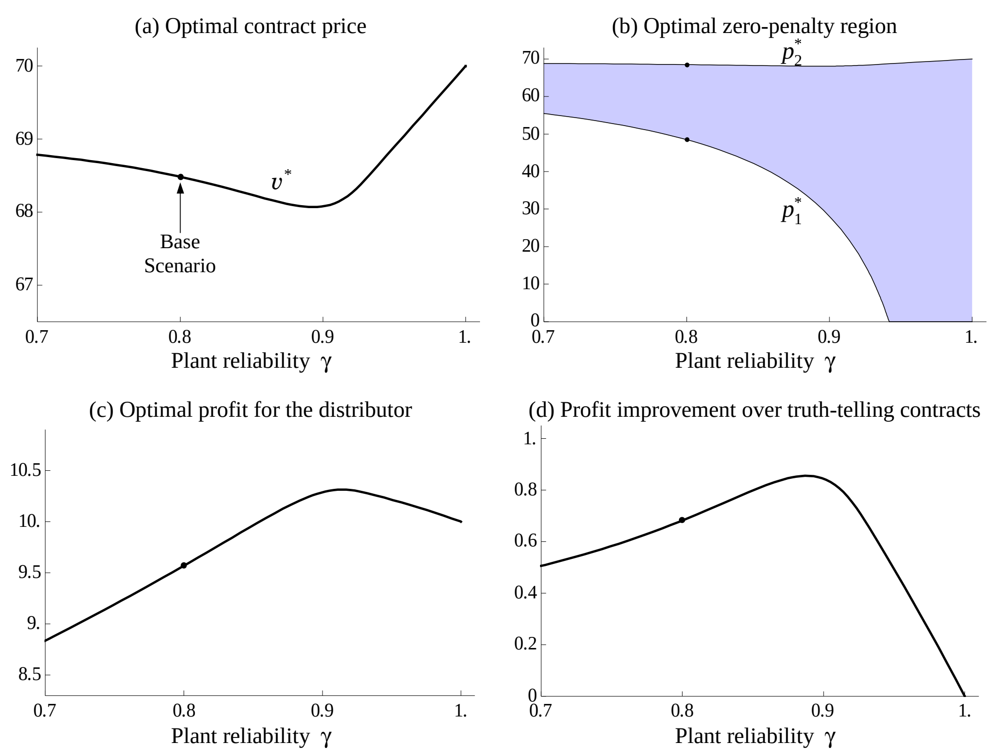


with less opportunity to misreport. These two forces counteract with each other. The first force is more significant for relatively low γ, but it becomes weaker as γ increases, because the benefit of a wider feasible region diminishes. This is why the distributor's profit improvement over the truth-telling contract increases first and then decreases in γ.

One could argue that the plant's reservation utility should depend on its reliability. Figure 8 shows the effects of reliability when the truth telling price v_0 is fixed and the plant's reservation utility satisfies $\underline{U_P} = \gamma U_P(v_0 - c)$. Similar to Figure 7(d), Figure 8(d) shows that the profit improvement over the truth-telling contract increases first and then decreases in the plant's reliability. The reason is essentially the same. Different trends can be observed in Figure 8(a) and (c). For moderate values of γ, the contract price decreases and the distributor's profit increases in γ. However, since $\underline{U_P}$ increases in γ, the distributor has to pay more to a more reliable plant. For high reliability levels, the zero-penalty region cannot be made any larger and the distributor has no choice but to increase the contract price to meet the increasing reservation utility of the plant.

Figure 8 Effect of Plant's Reliability II

The truth-telling contract price is fixed: $v_0 = 70$. The plant's reservation utility satisfies $\underline{U_P} = \gamma U_P(v_0 - c)$.



7. Concluding Remarks

Unit-contingent swap contracts are widely used in the electricity industry to govern relationships between electricity distributors and plants. Because plants possess private information about their true operational status, the contingent feature of the unit-contingent swap contracts may create an opportunity for a plant to misreport its true status. To the best of our knowledge, this paper is the first one to analyze a potential misreporting incentive problem of the unit-contingent contracts. Intuitively, one might expect that the distributor would always prefer plants to submit truthful reports. However, industry practice suggests that the distributors might be experiencing and tolerating misreporting to some extent. Why? One possible answer is that the inspection cost for a distributor is high. Another possible answer, as our analysis suggests, is that misreporting could actually benefit both the plant and the distributor by reshaping their risk profiles and by allowing a more risk-averse company to sell risk to a less risk-averse one. We show that a carefully designed penalty scheme can steer the plant toward "right misreporting."

We use numerical analysis to refine our managerial insights. We observe that as the plant becomes more reliable or as the inspection cost increases, the distributor becomes more tolerant of the plant's misreporting, reflected by the wider zero-penalty region. We observe two ways that the distributor can use to satisfy the plant: a direct benefit through the unit-contingent payment, and an indirect benefit from the plant's misreporting. When the latter benefit increases, the former will decrease (i.e., swap price decreases). And it is the latter benefit from misreporting that helps reshape both firms' risk profiles. The more benefit the plant can obtain from misreporting, the better risk allocation can be achieved between the firms. Furthermore, we find that this benefit is not only affected by the distributor's tolerance, but also directly impacted by spot price volatility and the plant's reliability.

A number of assumptions facilitated our analysis. For example, we assumed that the execution stage lasts only one period. Our insights could be extended to the repeated interactions between the plant and the distributor during the execution stage, as long as these interactions are modeled as a repeated game with the finite number of periods. In practice, the distributor might take advantage of the repeated interactions to learn something about the plant, for example, the plant's true reliability or the plant's utility function. Learning, while interesting, requires a different modeling approach from the one in the paper. In particular, it introduces an interesting strategic behavior on the part of the plant, which might take actions to disguise its true parameter values. Such dynamic games with asymmetric information tend to be quite complex and are left for future research. We assumed a continuous distribution of electricity prices. Because electricity spot prices are occasionally very volatile, some of the previous models used stochastic processes with jumps to describe price movements. We believe that the insights of this paper will not be affected if we used a more general stochastic process for electricity prices, but the analysis would be more complex.

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Proofs of Propositions

Proof of Proposition 1 [Bayesian Equilibrium for $p > v$]. The best response functions have been derived in the paper as shown in (2) and (6). Combining the best response functions, we can derive the Bayesian equilibrium as follows.

The plant truthfully reports its status if it is DOWN, and the distributor never inspects when the plant reports UP:

$$x^*_{\mathsf{DOWN}} = 1, \qquad y^*_{\mathsf{UP}} = 0.$$

If $\phi(p) < \widehat{\phi}(p)$, the plant reports DOWN when it is UP, and the distributor does not inspect:

$$x^*_{\mathsf{UP}} = 0, \qquad y^*_{\mathsf{DOWN}} = 0. \tag{EC.1}$$

If $\phi(p) = \widehat{\phi}(p)$, there are many equilibria: $x^*_{\mathsf{UP}} = 0$, $y^*_{\mathsf{DOWN}} \in [0, \widehat{y}_{\mathsf{DOWN}}]$. The distributor is indifferent among all these equilibria, but the plant would prefer the distributor to play $y^*_{\mathsf{DOWN}} = 0$ (no inspection). Thus, we assume the equilibrium played in this scenario is the same as in (EC.1).

If $\phi(p) > \widehat{\phi}(p)$ and $\phi(p) > 0$, we have a mixed strategy equilibrium:

$$x^*_{\mathsf{UP}} = \widehat{x}_{\mathsf{UP}}, \qquad y^*_{\mathsf{DOWN}} = \widehat{y}_{\mathsf{DOWN}}.$$

If $\phi(p) > \widehat{\phi}(p)$ and $\phi(p) = 0$, there are many equilibria: $x^*_{\mathsf{UP}} \in [0, \widehat{x}_{\mathsf{UP}}]$, $y^*_{\mathsf{DOWN}} = 1$. The plant is indifferent among all these equilibria, but the distributor would prefer the plant to play $x^*_{\mathsf{UP}} = \widehat{x}_{\mathsf{UP}}$ (higher truth-telling probability). Thus, we assume that the equilibrium played in this scenario is

$$x^*_{\mathsf{UP}} = \widehat{x}_{\mathsf{UP}}, \qquad y^*_{\mathsf{DOWN}} = 1 = \widehat{y}_{\mathsf{DOWN}}.$$

The above Bayesian equilibria can be summarized into two cases $\phi(p) \le \widehat{\phi}(p)$ and $\phi(p) > \widehat{\phi}(p)$, which are exactly what Proposition 1 describes.

Next, we prove the firms' expected utilities are those in (7) and (8) for any given spot price realization $p > v$. If $\phi(p) \le \widehat{\phi}(p)$, the plant's profit is zero when DOWN, and it reports DOWN when UP to obtain a profit of $p - c$. Thus, the plant's expected utility is $\gamma U_P(p-c)$ (recall that the plant is UP with probability γ and $U_P(0) = 0$). Since the plant always reports DOWN in this case, the distributor always purchases from the spot and obtains a utility of $U_D(f - p)$.

If $\phi(p) > \widehat{\phi}(p)$, the plant's profit is zero when DOWN. When the plant is UP, it plays a mixed strategy, so its expected utility is equal to the utility it obtains by playing any pure strategy, e.g., reporting UP gives the plant a utility of $U_P(v-c)$. Thus, the plant's expected utility is $\gamma U_P(v-c)$.

When the plant reports UP (which happens with probability $\gamma\widehat{x}_{\mathsf{UP}}$), the distributor does not inspect and gets a profit of $f-v$. When the plant reports DOWN, the distributor plays a mixed strategy, so its expected utility is equal to the utility it obtains by playing any pure strategy, e.g., not conducting inspection gives the distributor a utility of $U_D(f-p)$. Thus, its expected utility is $\gamma\widehat{x}_{\mathsf{UP}}U_D(f-v)+(1-\gamma\widehat{x}_{\mathsf{UP}})U_D(f-p)$, as shown in (8). □

Proof of Proposition 2 [Bayesian Equilibrium for $p<v$]. The analysis is parallel to the $p>v$ case in Section 4.

If the plant is UP, truth-telling is the dominating strategy for the plant. Therefore, $x^*_{\mathsf{UP}}=1$.

If the plant is DOWN, it obtains $U_P(0)=0$ by reporting DOWN (truth-telling), and an expected utility of $y_{\mathsf{UP}}U_P(-\phi(p))+(1-y_{\mathsf{UP}})U_P(v-p)$ by reporting UP, where y_{UP} is the probability that the distributor inspects the plant. Reporting UP is preferred by the plant if

$$y_{\mathsf{UP}} < \frac{U_P(v-p)}{U_P(v-p)-U_P(-\phi(p))} \stackrel{\text{def}}{=} \widehat{y}_{\mathsf{UP}} \in (0,1].$$

Thus, the plant's optimal response expressed as the truth-telling probabilities x_{UP} and x_{DOWN}, given the inspection probability y_{UP}, is

$$\text{When UP, } x^*_{\mathsf{UP}}=1 \text{ (report UP);}$$

$$\text{When DOWN, } \begin{cases} x^*_{\mathsf{DOWN}}=0 \text{ (report UP)}, & \text{if } y_{\mathsf{UP}}<\widehat{y}_{\mathsf{UP}}; \\ x^*_{\mathsf{DOWN}}\in[0,1], & \text{if } y_{\mathsf{UP}}=\widehat{y}_{\mathsf{UP}}; \\ x^*_{\mathsf{DOWN}}=1 \text{ (report DOWN)}, & \text{if } y_{\mathsf{UP}}>\widehat{y}_{\mathsf{UP}}. \end{cases}$$

Next, we analyze the distributor's best response, given the plant's truth-telling probability. If the plant reports DOWN, the distributor knows that the plant is indeed DOWN (because pretending to be DOWN while UP is a dominated strategy for the plant when $p<v$), so the distributor will not inspect: $y^*_{\mathsf{DOWN}}=0$.

If the plant reports UP, the distributor forms a belief about the plant's actual status according to the Bayes rule:

$$\mathsf{P}\big\{\text{plant is DOWN} \mid \text{plant reports UP}\big\} = \frac{(1-x_{\mathsf{DOWN}})(1-\gamma)}{\gamma+(1-x_{\mathsf{DOWN}})(1-\gamma)} \stackrel{\text{def}}{=} \beta(x_{\mathsf{DOWN}}) \in [0, 1-\gamma].$$

Based on the above belief about misreporting, if the distributor inspects, its expected utility is

$$\beta(x_{\mathsf{DOWN}})U_D(f-p-k+\phi(p)) + (1-\beta(x_{\mathsf{DOWN}}))U_D(f-v-k).$$

Without inspection, the distributor can obtain $U_D(f-v)$. Thus, the distributor will not inspect if its belief about misreporting is below a certain level:

$$\beta(x_{\mathsf{DOWN}}) < \frac{U_D(f-v)-U_D(f-v-k)}{U_D(f-p-k+\phi(p))-U_D(f-v-k)} \stackrel{\text{def}}{=} \widehat{\beta} \in (0,\infty). \tag{EC.2}$$

Because $\beta(x_{\mathsf{DOWN}}) \le 1-\gamma$, if $\widehat{\beta} > 1-\gamma$, then, regardless of the plant's strategy x_{DOWN}, inequality (EC.2) holds and the distributor does not inspect. The condition $\widehat{\beta} > 1-\gamma$ is equivalent to $\phi(p) < \widehat{\phi}(p)$, where $\widehat{\phi}(p)$ is defined below and also given in (9) in the paper:

$$\widehat{\phi}(p) \stackrel{\text{def}}{=} p+k-f+U_D^{-1}\left(\frac{1}{1-\gamma}U_D(f-v)-\frac{\gamma}{1-\gamma}U_D(f-v-k)\right), \qquad \text{for } p<v.$$

In other words, regardless of the plant's strategy, a penalty payment that is below $\widehat{\phi}(p)$ will not provide sufficient incentive for the distributor to inspect at cost $k>0$. If $\phi(p) \ge \widehat{\phi}(p)$, then the distributor's inspection decision will depend on the plant's truth-telling probability (note that if $\widehat{\phi}(p) \le 0$ then $\phi(p) \ge \widehat{\phi}(p)$ always holds since the penalty payment is non-negative). The distributor will not inspect if (EC.2) holds, which is equivalent to $x_{\mathsf{DOWN}} > \beta^{-1}(\widehat{\beta})$, or

$$x_{\mathsf{DOWN}} > 1-\frac{\gamma\widehat{\beta}}{(1-\gamma)(1-\widehat{\beta})} \stackrel{\text{def}}{=} \widehat{x}_{\mathsf{DOWN}},$$

where $\widehat{x}_{\mathsf{DOWN}} \in [0,1)$ because $\widehat{\beta} \in (0, 1-\gamma]$ when $\phi(p) \ge \widehat{\phi}(p)$.

In summary, the distributor's best response expressed as inspection probabilities y_{UP} and y_{DOWN}, given the the plant's strategy x_{DOWN}, is

$$\text{When reporting UP,} \begin{cases} \text{If } \phi(p) < \widehat{\phi}(p), \text{ then } y^*_{\mathsf{UP}} = 0; \\ \text{If } \phi(p) \geq \widehat{\phi}(p), \text{ then} \\ \qquad y^*_{\mathsf{UP}} = 1 \text{ (inspect)}, & \text{if } x_{\mathsf{DOWN}} < \widehat{x}_{\mathsf{DOWN}}; \\ \qquad y^*_{\mathsf{UP}} \in [0,1], & \text{if } x_{\mathsf{DOWN}} = \widehat{x}_{\mathsf{DOWN}}; \\ \qquad y^*_{\mathsf{UP}} = 0 \text{ (do not inspect)}, & \text{if } x_{\mathsf{DOWN}} > \widehat{x}_{\mathsf{DOWN}}; \end{cases}$$

$$\text{When reporting DOWN, } y^*_{\mathsf{DOWN}} = 0 \text{ (do not inspect).}$$

Combining the best response functions of the plant and the distributor, we derive the Bayesian equilibrium as follows. The plant truthfully reports its status if it is UP, and the distributor never inspects when the plant reports DOWN:

$$x^*_{\mathsf{UP}} = 1, \qquad y^*_{\mathsf{DOWN}} = 0.$$

If $\phi(p) < \widehat{\phi}(p)$, the plant reports UP when it is DOWN, and the distributor does not inspect:

$$x^*_{\mathsf{DOWN}} = 0, \qquad y^*_{\mathsf{UP}} = 0. \tag{EC.3}$$

If $\phi(p) = \widehat{\phi}(p)$, there are many equilibria: $x^*_{\mathsf{DOWN}} = 0,\ y^*_{\mathsf{UP}} \in [0, \widehat{y}_{\mathsf{UP}}]$. The distributor is indifferent among all these equilibria, while the plant would prefer the distributor to play $y^*_{\mathsf{UP}} = 0$ (no inspection). Thus, we assume the equilibrium played in this scenario is the same as in (EC.3).

If $\phi(p) > \widehat{\phi}(p)$ and $\phi(p) > 0$, we have a mixed strategy equilibrium:

$$x^*_{\mathsf{DOWN}} = \widehat{x}_{\mathsf{DOWN}}, \qquad y^*_{\mathsf{UP}} = \widehat{y}_{\mathsf{UP}}.$$

If $\phi(p) > \widehat{\phi}(p)$ and $\phi(p) = 0$, there are many equilibria: $x^*_{\mathsf{DOWN}} \in [0, \widehat{x}_{\mathsf{DOWN}}],\ y^*_{\mathsf{UP}} = 1$. The plant is indifferent among all these equilibria, but the distributor prefers the plant to play $x^*_{\mathsf{DOWN}} = \widehat{x}_{\mathsf{DOWN}}$ (higher truth-telling probability). Thus, we assume that the equilibrium played in this scenario is

$$x^*_{\mathsf{DOWN}} = \widehat{x}_{\mathsf{DOWN}}, \qquad y^*_{\mathsf{UP}} = 1 = \widehat{y}_{\mathsf{UP}}.$$

Next, we prove the firms' expected utilities are those in (10) and (11) for any given spot price realization $p > v$. If $\phi(p) \leq \widehat{\phi}(p)$, the plant's profit is $v - c$ when UP, and it reports UP when DOWN to obtain a profit of $v - p$. Thus, the plant's expected utility is $\gamma U_P(v-c) + (1-\gamma) U_P(v-p)$. Since the plant always reports UP in this case, the distributor always obtains a utility of $U_D(f - v)$.

If $\phi(p) > \widehat{\phi}(p)$, the plant's profit is $v - c$ when UP. When the plant is DOWN, it plays a mixed strategy, so its expected utility is equal to the utility it obtains by playing any pure strategy, e.g., reporting DOWN gives the plant a utility of $U_P(0) = 0$. Thus, the plant's expected utility is $\gamma U_P(v - c)$.

When the plant reports DOWN (which happens with probability $(1-\gamma)\widehat{x}_{\mathsf{DOWN}}$), the distributor does not inspect and gets a profit of $f - p$. When the plant reports UP, the distributor plays a mixed strategy, so its expected utility is equal to the utility it obtains by playing any pure strategy, e.g., not conducting inspection gives the distributor a utility of $U_D(f - v)$. Thus, its expected utility is $(1-(1-\gamma)\widehat{x}_{\mathsf{DOWN}})U_D(f-v) + (1-\gamma)\widehat{x}_{\mathsf{DOWN}}U_D(f-p)$, as shown in (11). □

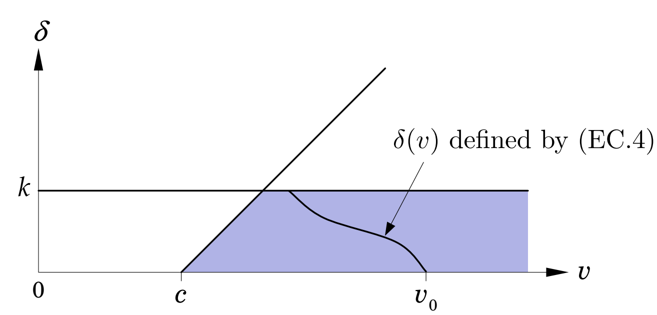
Proof of Proposition 3 [Sub-optimality of the truth-telling contract]. We consider the contract form in (15):

$$v < v_0, \qquad \phi(p) = \begin{cases} 0, & p \in [v-\delta, v], \\ \infty, & \text{otherwise.} \end{cases}$$

We choose the parameters v and δ such that $\delta \in (0, k)$ and $\delta < v - c$, and that the plant's expected utility in (16) is maintained at the reservation level $\underline{U_P} = U_P(v_0 - c)$. That is, v and δ also satisfy:

$$\gamma\big[U_P(v_0 - c) - U_P(v - c)\big] - (1-\gamma)\int_{v-\delta}^{v} U_P(v-p)g(p)dp \;=\; 0. \tag{EC.4}$$

To see the existence of such v and δ, note that (EC.4) defines an implicit function $\delta(v)$ in a neighborhood left to v_0, and $\delta(v_0) = 0$. Observe that left side of (EC.4) is continuously differentiable in (δ, v) and its partial derivative with respect to δ is non-zero. Thus, the implicit function $\delta(v)$ is continuous, and therefore, we can always choose $\delta(v) \in (0, k)$ and $\delta(v) < v - c$, as illustrated in the following figure.



The condition $\delta < k$ is need to ensure that (16) is indeed the expected utility of the plant under the contract (15). This is discussed in the paper after (15).

By the concavity of the utility functions, we have:

$$U_D(f - v_0) \leq U_D(f - v) + U_D'(f - v)(v - v_0), \tag{EC.5}$$

$$U_P(v_0 - c) \leq U_P(v - c) + U_P'(v - c)(v_0 - v), \tag{EC.6}$$

$$U_D(f - p) \leq U_D(f - v) + U_D'(f - v)(v - p), \tag{EC.7}$$

$$\frac{U_P(v - p)}{v - p} \geq U_P'(v - c), \qquad \text{for } p \in (v - \delta, v), \tag{EC.8}$$

where the last inequality follows from $U_P(0) = 0$ and $\delta < v - c$. Then, we have

$$\begin{aligned} U_D(f - v) - U_D(f - v_0) \geq U_D'(f - v)(v_0 - v) \; &= \; U_P'(v - c)(v_0 - v)\frac{U_D'(f - v)}{U_P'(v - c)} \\ &\geq \big[U_P(v_0 - c) - U_P(v - c)\big]\frac{U_D'(f - v)}{U_P'(v - c)} \end{aligned} \tag{EC.9}$$

which follows from (EC.5) and (EC.6), and we also have

$$\begin{aligned} \int_{v-\delta}^{v} \big[U_D(f - p) - U_D(f - v)\big] g(p)dp &\leq \int_{v-\delta}^{v} U_D'(f - v)(v - p)g(p)dp \\ &= \int_{v-\delta}^{v} U_P'(v - c)(v - p)g(p)dp \; \frac{U_D'(f - v)}{U_P'(v - c)} \\ &\leq \int_{v-\delta}^{v} U_P(v - p)g(p)dp \; \frac{U_D'(f - v)}{U_P'(v - c)} \end{aligned} \tag{EC.10}$$

which follows from (EC.7) and (EC.8).

We are now in the position of comparing the distributor's expected utility under the contract (15), as expressed in (17) in the paper, and its expected utility under the truth-telling contract, shown in (14). Taking the difference and employing the relations in (EC.9) and (EC.10), we have

$$\begin{aligned} (17) - (14) &= \gamma\big[U_D(f - v) - U_D(f - v_0)\big] - (1 - \gamma)\int_{v-\delta}^{v} \big[U_D(f - p) - U_D(f - v)\big] g(p)dp \\ &\geq \left[\gamma\big[U_P(v_0 - c) - U_P(v - c)\big] - (1 - \gamma)\int_{v-\delta}^{v} U_P(v - p)g(p)dp\right]\frac{U_D'(f - v)}{U_P'(v - c)} \qquad \text{(EC.11)} \\ &= (\text{EC.4}) \times \frac{U_D'(f - v)}{U_P'(v - c)} \; = \; 0. \end{aligned}$$

Notice that the inequality (EC.11) will be strict as long as one of the inequalities in (EC.5)-(EC.8) is strict. This means that as long as one of the distributor and the plant is strictly risk-averse, by using the contract in (15) the distributor can obtain a strictly higher utility than it would get under a truth-telling contract, while the plant's utility is maintained at its reservation level. □

Proof of Proposition 4 [Structure of L_2^*]. The distributor's problem is rewritten below in (EC.12)-(EC.13), which is to maximize the difference between (19) and (14) (equivalent to maximize (19), but working with the difference is easier for exposition) while maintaining the plant's expected utility in (18) at $\underline{U_P}$:[9]

$$\max_{\{v\le v_0,\ L_i\subseteq S_i(v)\}} \gamma\big[U_D(f-v)-U_D(f-v_0)\big]-(1-\gamma)\int_{L_1}\big[U_D(f-p)-U_D(f-v)\big]g(p)dp \\ -\gamma\int_{L_2}\big[U_D(f-v)-U_D(f-p)\big]g(p)dp \tag{EC.12}$$

$$s.t.\quad \gamma\big[U_P(v_0-c)-U_P(v-c)\big]=(1-\gamma)\int_{L_1}U_P(v-p)g(p)dp \\ +\gamma\int_{L_2}\big[U_P(p-c)-U_P(v-c)\big]g(p)dp \tag{EC.13}$$

Consider adjusting any given $L_2\subseteq S_2$ to improve the objective in (EC.12). Suppose there exist $L_a\subseteq S_2\backslash L_2$ and $L_b\subseteq L_2$ such that L_a and L_b have positive measures, $p_a<p_b$ for any $p_a\in L_a, p_b\in L_b$, and

$$\int_{L_a}\big[U_P(p-c)-U_P(v-c)\big]g(p)dp=\int_{L_b}\big[U_P(p-c)-U_P(v-c)\big]g(p)dp,$$

that is, constraint (EC.13) remains satisfied if we adjust L_2 to include L_a but exclude L_b. We now show that this adjustment improves the objective in (EC.12).

Since L_a and L_b are disjoint, there exists p_o such that $v\le p_a\le p_o\le p_b$, $\forall p_a\in L_a, p_b\in L_b$. By the concavity of the utility functions, we have

$$\frac{U_P(p_b-c)-U_P(v-c)}{p_b-v}\le\frac{U_P(p_o-c)-U_P(v-c)}{p_o-v}\overset{\text{def}}{=}C_P\le\frac{U_P(p_a-c)-U_P(v-c)}{p_a-v},$$
$$\frac{U_D(f-v)-U_D(f-p_a)}{p_a-v}\le\frac{U_D(f-v)-U_D(f-p_o)}{p_o-v}\overset{\text{def}}{=}C_D\le\frac{U_D(f-v)-U_D(f-p_b)}{p_b-v},$$

where the equalities hold only when p_a or p_b coincides with p_o. Hence,

$$\int_{L_a}\big[U_D(f-v)-U_D(f-p)\big]g(p)dp \\ \le\int_{L_a}(p-v)C_D\ g(p)dp \qquad =\int_{L_a}(p-v)C_P\ g(p)dp\frac{C_D}{C_P}$$

[9] The original problem formulation (20) has $\mathsf{E}[U_P]\ge\underline{U_P}$. Requiring it to be equality does not cause any loss of optimality, because if strict inequality holds we can always shrink L_1 or L_2 to improve the objective.

$$
\begin{aligned}
&\leq \int_{L_a} \big[U_P(p-c) - U_P(v-c)\big] g(p)dp \frac{C_D}{C_P} = \int_{L_b} \big[U_P(p-c) - U_P(v-c)\big] g(p)dp \frac{C_D}{C_P} \\
&\leq \int_{L_b} (p-v) C_P \; g(p)dp \frac{C_D}{C_P} \qquad\qquad = \int_{L_b} (p-v) C_D \; g(p)dp \\
&\leq \int_{L_b} \big[U_D(f-v) - U_D(f-p)\big] g(p)dp
\end{aligned}
$$

Notice that if at least of the plant and the distributor is strictly risk-averse, then at least one of the above inequalities will hold strictly. Therefore, we can always improve the objective by replacing L_b with L_a, which in effect shifts L_2 towards the left. Thus, the optimal L_2 should be contained in the leftmost part of S_2. □

Proof of Proposition 5 [Structure of the optimal zero-penalty region].

(i) When $U_D(x) = x$, the objective (EC.12) becomes

$$
\gamma(v_0 - v) - (1-\gamma) \int_{L_1} (v-p) g(p)dp - \gamma \int_{L_2} (p-v) g(p)dp \tag{EC.14}
$$

Consider adjusting any given $L_1 \subseteq S_1$ to improve the objective in (EC.14). Suppose there exist $L_a \subseteq S_1 \backslash L_1$ and $L_b \subseteq L_1$ such that L_a and L_b have positive measures, $p_a > p_b$ for any $p_a \in L_a, p_b \in L_b$, and

$$
\int_{L_a} U_P(v-p) g(p)dp = \int_{L_b} U_P(v-p) g(p)dp,
$$

that is, constraint (EC.13) remains satisfied if we adjust L_1 to include L_a but exclude L_b. We now show that this adjustment improves the objective in (EC.14).

Since L_a and L_b are disjoint, there exists p_o such that $p_b \leq p_o \leq p_a \leq v$, $\forall p_a \in L_a, p_b \in L_b$. Since $U_P(\cdot)$ is strictly concave and $U_P(0) = 0$, we have

$$
\frac{U_P(v-p_b)}{v-p_b} \;\leq\; \frac{U_P(v-p_o)}{v-p_o} \;\stackrel{\text{def}}{=}\; C_3 \;\leq\; \frac{U_P(v-p_a)}{v-p_a},
$$

where the equalities hold only when p_a or p_b coincides with p_o. Hence,

$$
\begin{aligned}
\int_{L_a} (v-p) g(p)dp &= \int_{L_a} \frac{v-p}{U_P(v-p)} U_P(v-p) g(p)dp \\
&< C_3^{-1} \int_{L_a} U_P(v-p) g(p)dp \;=\; C_3^{-1} \int_{L_b} U_P(v-p) g(p)dp \\
&< \int_{L_b} \frac{v-p}{U_P(v-p)} U_P(v-p) g(p)dp
\end{aligned}
$$

$$= \int_{L_b} (v-p)g(p)dp$$

Therefore, we can always improve the objective by replacing L_b with L_a, which in effect shifts L_1 towards the right. Since S_1 is an interval with its right end being at v, the optimal L_1 must be of the form $[p_1^*, v]$, otherwise the above improvement can always be made.

(ii) When $U_P(x) = x$, the constraint (EC.13) becomes

$$\gamma(v_0 - v) = (1-\gamma)\int_{L_1} (v-p)g(p)dp + \gamma \int_{L_2} (p-v)g(p)dp \tag{EC.15}$$

We first consider adjusting any given $L_1 \subseteq S_1$ to improve the objective in (EC.12). Suppose there exist $L_a \subseteq S_1 \backslash L_1$ and $L_b \subseteq L_1$ such that L_a and L_b have positive measures, $p_a < p_b$ for any $p_a \in L_a, p_b \in L_b$, and

$$\int_{L_a} (v-p)g(p)dp = \int_{L_b} (v-p)g(p)dp,$$

that is, constraint (EC.15) remains satisfied if we adjust L_1 to include L_a but exclude L_b. We now show that this adjustment improves the objective in (EC.12).

Since L_a and L_b are disjoint, there exists p_o such that $p_a \le p_o \le p_b \le v$, $\forall p_a \in L_a, p_b \in L_b$. Since $U_D(\cdot)$ is strictly concave, we have

$$\frac{U_D(f-p_a) - U_D(f-v)}{v-p_a} \le \frac{U_D(f-p_o) - U_D(f-v)}{v-p_o} \stackrel{\text{def}}{=} C_1 \le \frac{U_D(f-p_b) - U_D(f-v)}{v-p_b},$$

where the equalities hold only when p_a or p_b coincides with p_o. Hence,

$$\begin{aligned}
\int_{L_a} \big[U_D(f-p) - U_D(f-v)\big] g(p)dp &= \int_{L_a} \frac{U_D(f-p) - U_D(f-v)}{v-p}(v-p)g(p)dp \\
&< C_1 \int_{L_a} (v-p)g(p)dp \;=\; C_1 \int_{L_b} (v-p)g(p)dp \\
&< \int_{L_b} \frac{U_D(f-p) - U_D(f-v)}{v-p}(v-p)g(p)dp \\
&= \int_{L_b} \big[U_D(f-p) - U_D(f-v)\big] g(p)dp
\end{aligned}$$

Therefore, we can always improve the objective by replacing L_b with L_a, which in effect shifts L_1 towards the left.

We next consider shrinking L_2 and expanding L_1 to improve the objective. Suppose there exist $L_a \subseteq S_1 \backslash L_1$ and $L_b \subseteq L_2$ such that L_a and L_b have positive measures, and

$$(1-\gamma)\int_{L_a}(v-p)g(p)dp = \gamma\int_{L_b}(p-v)g(p)dp$$

that is, constraint (EC.15) remains satisfied if we add L_a to L_1 and subtract L_b from L_2. We now show that this adjustment improves the objective in (EC.12).

For any $p_a \in L_a \backslash \{v\}, p_b \in L_b \backslash \{v\}$ (excluding a single point v does not affect the integrals below), we have $p_a < v < p_b$, and due to the strict concavity of $U_D(\cdot)$, we have:

$$\frac{U_D(f-p_a)-U_D(f-v)}{v-p_a} \;<\; U_D'(f-v) \;<\; \frac{U_D(f-v)-U_D(f-p_b)}{p_b-v}$$

Hence,

$$\begin{aligned}
(1-\gamma)\int_{L_a}\big[U_D(f-p)-U_D(f-v)\big]g(p)dp &= (1-\gamma)\int_{L_a}\frac{U_D(f-p)-U_D(f-v)}{v-p}(v-p)g(p)dp \\
&< (1-\gamma)U_D'(f-v)\int_{L_a}(v-p)g(p)dp \\
&= \gamma U_D'(f-v)\int_{L_b}(p-v)g(p)dp \\
&< \gamma\int_{L_b}\frac{U_D(f-v)-U_D(f-p)}{p-v}(p-v)g(p)dp \\
&= \gamma\int_{L_b}\big[U_D(f-v)-U_D(f-p)\big]g(p)dp
\end{aligned}$$

Therefore, we can improve the objective by expanding L_1 and shrinking L_2 until that is not possible any more. Hence, if L_2 has a positive measure, then $L_1 = S_1$. $\square$