ABSTRACT

Combinatorial Compressive Sampling with Applications

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We simplify and improve the deterministic Compressed Sensing (CS) results of Cormode and Muthukrishnan (CM). A simple relaxation of our deterministic CS technique then generates a new randomized CS result similar to those derived by CM. Finally, our CS techniques are applied to two computational problems of wide interest: The calculation of a periodic function's Fourier transform, and matrix multiplication. Short descriptions of our results follow.

(i) Sublinear-Time Sparse Fourier Transforms: Suppose $f : [0, 2\pi] \rightarrow \mathbb{C}$ is k-sparse in frequency (e.g., f is an exact superposition of k sinusoids with frequencies in $[1 - \frac{N}{2}, \frac{N}{2}]$). Then we may recover f in $O(k^2 \cdot \log^4(N))$ time by deterministically sampling it at $O(k^2 \cdot \log^3(N))$ points. If succeeding with high probability is sufficient, we may sample f at $O(k \cdot \log^4(N))$ points and then reconstruct it in $O(k \cdot \log^5(N))$ time via a randomized version of our deterministic Fourier algorithm. If N is much larger than k, both algorithms run in sublinear-time in the sense that they will outrun any procedure which samples f at least N times (e.g., both algorithms are

faster than a fast Fourier transform).

In addition to developing new sublinear-time Fourier methods we have implemented previously existing sublinear-time Fourier algorithms. The resulting implementations, called AAFFT 0.5/0.9, are empirically evaluated. The results are promising: AAFFT 0.9 outperforms standard FFTs (e.g., FFTW 3.1) on signals containing about 10^2 energetic frequencies spread over a bandwidth of 10^6 or more. Furthermore, AAFFT utilizes significantly less memory than a standard FFT on large signals since it only needs to sample a fraction of the input signal's entries.

(ii) Fast matrix multiplication: Suppose both A and B are dense $N \times N$ matrices. It is conjectured that $A \cdot B$ can be computed in $O(N^{2+\epsilon})$-time. If $A \cdot B$ is known to be $O(N^{2.9462})$-sparse/compressible in each column (e.g., each column of $A \cdot B$ contains only a few non-zero entries) we show that $A \cdot B$ may be calculated in $O(N^{2+\epsilon})$-time. Thus, we generalize previous rapid rectangular matrix multiplication results due to D. Coppersmith.