

THE UNIVERSITY OF MICHIGAN  
INDUSTRY PROGRAM OF THE COLLEGE OF ENGINEERING

ANALOG COMPUTER SIMULATION OF A FREE-PISTON ENGINE

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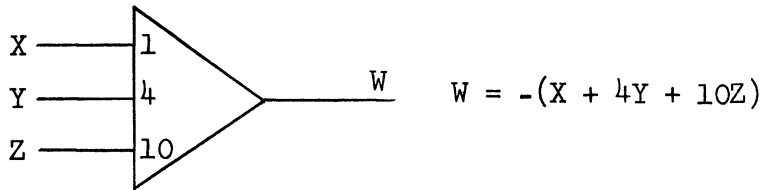
## SYMBOLS

<u>Symbol</u>	<u>Definition</u>
$A^*$	Area of piston, measured in square feet
$A_V$	Area of valve, measured in square feet
$C_P$	Specific heat at constant pressure
$C_V$	Specific heat at constant volume
$d_o$	Valve pressure drops
$F^*$	Force, measured in lbs
$F_f$	Friction force, measured in lbs
HHV	Higher heating value of the fuel, 18,700 Btu/lb
J	Mechanical equivalent of heat, 778 ft.lb/Btu
M	Torque developed by the turbine rotor, measured in lb-ft.
N	Rotor speed of the turbine, measured in rpm
n	Polytropic expansion and compression constant
$P^*$	Total pressure, measured in lbs per square feet
R	Specific gas constant for air, 53.3 ft.lb per degree Rankine
$T^*$	Total temperature, measured in degrees Rankine
$V^*$	Volume of cylinder, measured in cubic feet
$v^*$	Specific volume, measured in cubic feet per lb
$W_F$	Fuel weight, measured in lbs
$W^*$	Gas weight, measured in lbs
$\dot{W}^*$	Gas flow rate, measured in lbs per second
X	Piston displacement, measured in feet
$\dot{X}$	Piston velocity, measured in feet per second
$\ddot{X}$	Piston acceleration, measured in feet per second per second

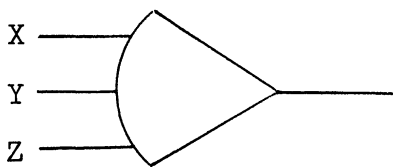
\* Lettered subscripts on these quantities designate various positions within the free-piston engine: C for compressor,  $\alpha$  for reservoir  $\alpha$ , P for power cylinder,  $\beta$  for reservoir  $\beta$ , and B for bounce cylinder.

SYMBOLS (Continued)

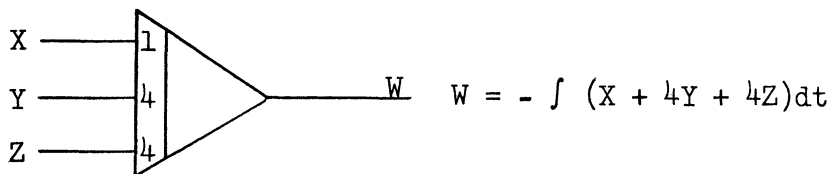
A X indicates "A" volts = 1 unit of X



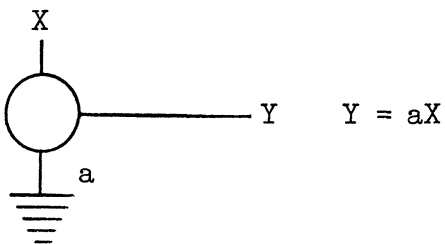
Summing Amplifier



Amplifier with Feedback Removed

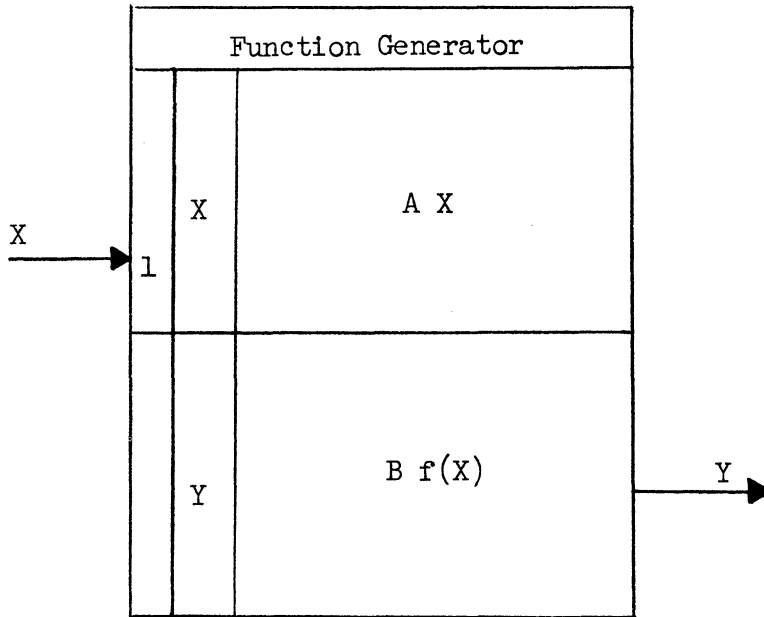


Integrator

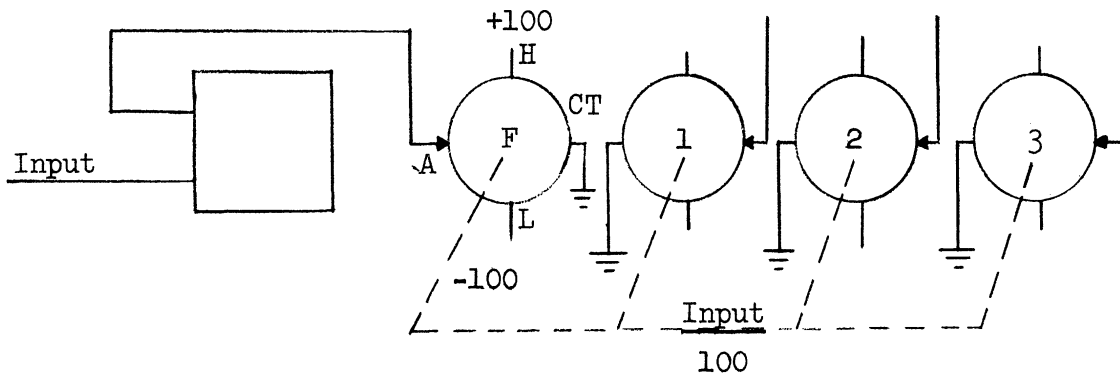


Scale-Factor Potentiometer

SYMBOLS (Continued)



Single-Input Function Table  
 $Y = f(X)$



Servo Multiplier

## ASSUMPTIONS

1. Friction is constant.
2. All compression processes are adiabatic.
3. Effective piston stroke of engine is 75% of total piston travel.
4. Cold scavenge air does not mix with the hot gases in the power cylinder.
5. Specific heats are constant.
6. Approximation flow equations for ideal gases are valid for the engine.



## ABSTRACT

The feasibility of using an electronic analog computer for computing the transient and steady-state performance of a free-piston engine has been demonstrated. The operation of each part of the engine is specified by a series of equations with each particular equation valid for only part of the complete cycle. All the equations are mechanized on the computer with automatic switching arrangements built into the computer circuitry to change equations at the appropriate times.

A steady-state operating point obtained from the simulator, which operates on a thousand-to-one time scale, was compared to hand-computed data.

# I

## SUMMARY

The free-piston engine shows considerable promise as a power plant for automobiles and other vehicles, but its design and the analysis of its operation are extremely difficult. Basically, it consists of a single horizontal cylinder containing two pistons which are moving in opposition to each other. The two pistons meet in the center of the cylinder on the compression stroke and then separate as the burned gases expand. It operates on a two-stroke diesel cycle, and since there is no crankshaft connected to the pistons they are moved during the compression stroke by forces generated by special bounce cylinders whose pistons are attached to the power pistons. In addition to the bounce cylinder, each piston operates an air compressor cylinder. The resulting compressed air is passed into the power cylinder at the end of the power stroke and thus, serves to scavenge the burned gases. The output of the free-piston engine is hot compressed gas, which is used to drive a turbine for converting this gas energy into mechanical energy.

The actions occurring in each part of the engine may be described by a series of equations. These equations change as the engine operation changes from one part of a cycle to another. The simulator operates by solving the appropriate equations simultaneously and by using automatic switching circuits to change from one equation to the next when necessary.

Since an entire cycle of operation of the free-piston engine requires approximately 40 milliseconds, it is necessary to change the time scale when the engine is simulated. A thousand-to-one scale was chosen, so that the 40 milliseconds required for an engine cycle becomes 40 seconds of actual time on the computer.

The computer also connects the actions of the various parts of the engine. It adds the forces acting on the piston and integrates them with respect to time to get piston velocity, which is then again integrated with respect to time to yield piston displacement. Piston displacement is used as an input variable to computing circuits which simulate the action of the air compressor, the bounce cylinders, the power cylinder, and the power turbine. Each of these circuits computes temperatures, pressures, and air flow rates as functions of time and piston displacement.

The operating cycle for any part of the engine has a number of discontinuities, and a fairly complex computer circuit is required for each of the engine parts. For example, the circuit which simulates the power cylinder must first solve a differential equation describing polytropic compression of air, during the compression stroke. Then, temperatures and pressures must be raised abruptly when the fuel ignites, and equations representing expansion at constant pressure must be solved for the portion of the power stroke during which fuel is being injected. After fuel is cut off, the computer for the power cylinder solves a differential equation for polytropic expansion until the exhaust and intake valves open at the end of the stroke. Then, equations describing the expansion of the hot exhaust gases and the intake flow of the scavenge air are solved until the valves are closed and the compression stroke begins again. Thus, simulation of each part of the engine involves solution of a number of different differential equations in a sequence, and the entire computer requires a considerable amount of computer equipment as well as a number of automatic switching circuits for changing equations in each of the engine components.

INTRODUCTION

A free-piston engine, or free-piston gas generator, as it is sometimes called, is a special type of internal combustion engine which has no crankshaft or other power take-off device connected to the pistons. It develops mechanical power by producing large amounts of compressed gas which pass through a gas turbine. The free-piston engine operates on a diesel cycle and can utilize inexpensive diesel-type fuels. Its efficiency is appreciably higher than that of a conventional gas-turbine engine. A further description of the free-piston engine is given in Section V of this report.

Although size and performance computation for a free-piston engine is relatively straightforward<sup>(1)</sup>, computation of performance of the engine for operating conditions other than those specified for the design point is rather difficult. Numerical integration methods must be used, since the engine's operation is specified by a number of non-linear differential equations, with each particular equation valid for only part of the complete cycle. If these equations are mechanized on an electronic analog computer, however, with automatic switching arrangements built into the computer circuitry to shift equations at the proper times, the computer becomes a fairly accurate simulator of the engine. The desired operating information may be obtained by simply adjusting the various coefficient potentiometers of the simulator to the desired values and then operating the simulator. Various changes in engine design, changes in input and load conditions, and various types of control systems may be rapidly and accurately evaluated. In fact, the simulator gives data comparable to that obtained by actually testing a model of the engine. It has the advantages, however, that engine

design changes may be readily tested, more data may be gathered, and there is no risk of damaging an expensive engine if some operating conditions cause instability.

CONCLUSIONS

3.1 FEASIBILITY OF THE COMPUTING METHOD

The feasibility of using an electronic analog computer for simulating the physical actions occurring in a free-piston engine has been adequately demonstrated. Use of the computer to give solutions of the various engine equations with time as the independent variable gives much more useful and complete information regarding the engine's operation than that which would be obtained by the more conventional methods of finding the steady-state operating condition of the engine. In addition, transient operation of the engine may be simulated quite easily, while hand computation of transient response would be very laborious. Any desired refinements in the equations can be added without difficulty, while making such refinements in equations used for hand computation might make the hand-computation process unworkable. In short, use of the analog computer greatly increases the speed at which both steady-state and transient solutions may be obtained, increases the validity of the computation, since many approximations needed to simplify the problem for hand computation may be replaced by the correct equations, and produces a solution with an accuracy comparable to that with which the values of the actual engine, fuel, and air parameters are known. Some characteristics of the computer are listed below.

Accuracy of computer components	
Integrators and summing amplifiers - - - - -	0.01%
Servo multipliers and function generators - - - - -	0.1%
Estimated average over-all computation error - - - -	1%

Set-up time required: (If engine equations to be used and engine constants are known)

Time to construct and scale computer diagram	80 hours
Time to wire computer circuit onto the computer	40 hours
Time required to trouble-shoot and de-bug computer	24 hours

Equipment required:

52	operational amplifiers
12	servo multipliers
1	function generator
8	differential relays
12	4 P D T relays for logical network

### 3.2 USES OF THE COMPUTER

The following paragraphs describe some of the many uses of an analog computer mechanization of a free-piston engine. There are two important differences between a simulated engine and an actual engine, however. The simulated engine is operating on a "slow" time scale, and each engine variable may be readily measured and recorded as a function of time, thus making the test instrumentation considerably simpler than that for an actual engine being operated in a laboratory. In addition, if pressure, temperature, or mechanical limits are exceeded in the simulated engine, the computer merely overloads, while if the same were to occur during the test run of an actual engine, severe damage could result.

#### 3.2.1 Stability Investigation and Control Development

Under some operating conditions, free-piston engines are unstable. If they are in operation with a turbine as a load, and with a fixed value of fuel flow, the amplitude of the piston travel may increase until pressure and mechanical limits are strained, or it may decrease until the travel is insufficient to operate the fuel-injector mechanism or until the compression is too low to cause fuel combustion. Thus, some type of control is necessary

in order to keep the stroke length within satisfactory limits, especially with changing load, such as that imposed by an automobile or other vehicle coupled to the power turbine. This instability of the engine is quite apparent when the engine is simulated on the analog computer. Additional computing equipment may be used to mechanize various control systems, which may be directly connected to the simulated engine in order to determine experimentally the operation of the entire system. An unstable engine and control combination on the computer will, of course, operate in an unstable manner, just as the actual engine would with that particular control.

### 3.2.2 Performance Computation for Various Engine Designs

Performances of engines of a number of different designs may be rapidly measured and compared if the analog computer is used. In many cases, an important fixed mechanical feature of the engine, such as the diameter of a piston, the position of the exhaust and intake valves in the power cylinder, the area of these valves, or even the mass of the piston assembly may be changed in the simulator by merely readjusting a potentiometer, and the simulated engine may then be operated immediately to determine what changes occurred in the operating characteristics.

### 3.2.3 Design by Experimental Modification of Engine Parameters

Since the physical characteristics of the simulated engine may be changed so readily, it is quite feasible to start with a reasonable calculated engine design and then make various changes which improve the performance as obtained from operation of the simulator. This procedure is equivalent to de-bugging an actual engine and making modifications to improve its performance, but it is much less expensive and much more rapid, of course.



### 3.2.4 Derivation of More Realistic Thermodynamic Relationships

The computer may also be used to obtain a better insight into the thermodynamic relationships which actually exist in the engine. If an engine has been built and test data has been collected, a comparison of this test data with corresponding data obtained from the simulated engine may be used to check the validity of the equations used in the simulator. If there are large discrepancies in the two sets of results, some of the equations or constants used in the simulator were probably incorrect. The nature of the discrepancies in results will usually indicate which equations should be suspected. Then corrections may be made, and the resulting corrected equations may be checked by further operation of the simulator.

## 3.3 EXTENSION OF CAPABILITIES AND IMPROVEMENT OF ACCURACY

The mechanization of the free-piston gas generator as described in this report represents the result of a study to determine the feasibility of obtaining engine performance data from analog computing equipment. It uses simplified forms of gas-flow equations and does not simulate any type of engine control, or any power turbine. No special features were added to facilitate data gathering, since the object of the study was to develop a suitable computer mechanization. The following paragraphs describe some modifications which should make the simulator more useful.

### 3.3.1 Addition of a Fuel Computer

The simulated engine was made to follow an ideal diesel cycle by holding the pressure in the power cylinder constant from point 3 to point 4 of the cycle. A computer circuit for calculating fuel was not assembled, since it was not needed for the main engine simulation, but such a computer circuit can easily be added. The equations solved by the fuel computer are

given on the block diagram, Figure 5.6, and are discussed in Section 5.2.2. The fuel computer does not change the nature of the power cylinder cycle; it only calculates the amount of fuel used per cycle.

### 3.3.2 Simulation of Fuel Injection

A more realistic power-cylinder cycle would probably be obtained if the action of the fuel injector and the burning of fuel were simulated in detail, and the resulting temperature and pressure were used as inputs to the power cylinder computer during the period of fuel injection. Simulation of the fuel injector should be no particular problem, but it might be difficult to simulate the dynamics of combustion<sup>(2)</sup>.

### 3.3.3 Addition of a Power Turbine

A turbine may be simulated on an electronic analog computer through the use of dual-input function generators which represent the turbine's operating characteristics<sup>(3)</sup>. This would facilitate investigation of the engine's performance with varying static and dynamic loads on the turbine shaft. For example, the longitudinal dynamics of an automobile could be added as a load on the turbine shaft, although the time required for a solution might be prohibitive because of the time scale used in the engine computer.

### 3.3.4 Automatic Cycling and Monitoring for Steady-State Operation

At present, the computer goes on "hold" whenever point 1 or point 5 in the power-cylinder cycle is reached. This is necessary to allow time for the switching which is performed at these points. When this occurs, it is necessary to manually press the "operate" button on the computer to initiate computation of the next part of the cycle. If a suitable time-delay and relay circuit is added, the computer can be made to switch back automatically to "operate" a few seconds after it has switched to "hold", and thus it can be made to continue to compute cycles indefinitely.

It has been observed that usually ten or more cycles must occur before the computer attains a "steady-state" condition, in which each cycle is identical with its predecessor. A very sensitive indication of the difference between one cycle and the next may be obtained by monitoring the amount of air which is compressed in the power cylinder each time. If an automatic monitoring circuit were added which compared these quantities of air from one cycle to the next, it could be arranged to actuate a relay and thus start the data recording equipment when steady-state operation had been attained. An arrangement like this would be particularly useful if a number of separate steady-state points were needed, since it would eliminate the need for human operators to monitor the computer.

### 3.3.5 Addition of Controls

Since most free-piston engines, especially those with variable loads, use some type of control to keep the stroke and frequency within practical limits, it will probably be necessary to add a simulated control to the engine simulator in many cases. This should present no difficulty, since almost any type of control for which the operating "rules" are known can be mechanized.

### 3.3.6 Improvement of Blow-Down Computation

Even though the computer is operating at 1/1000 of real time, the time during which blow-down in the power cylinder occurs is so short that the servo multipliers associated with this computation cannot follow accurately. More accurate computation of this part of the cycle could be obtained either by using electronic multipliers in the critical positions, or by an automatic switching circuit which changed the computer's time scale to 1/10,000 real time for the period from point 5 to point 5' in the cycle.

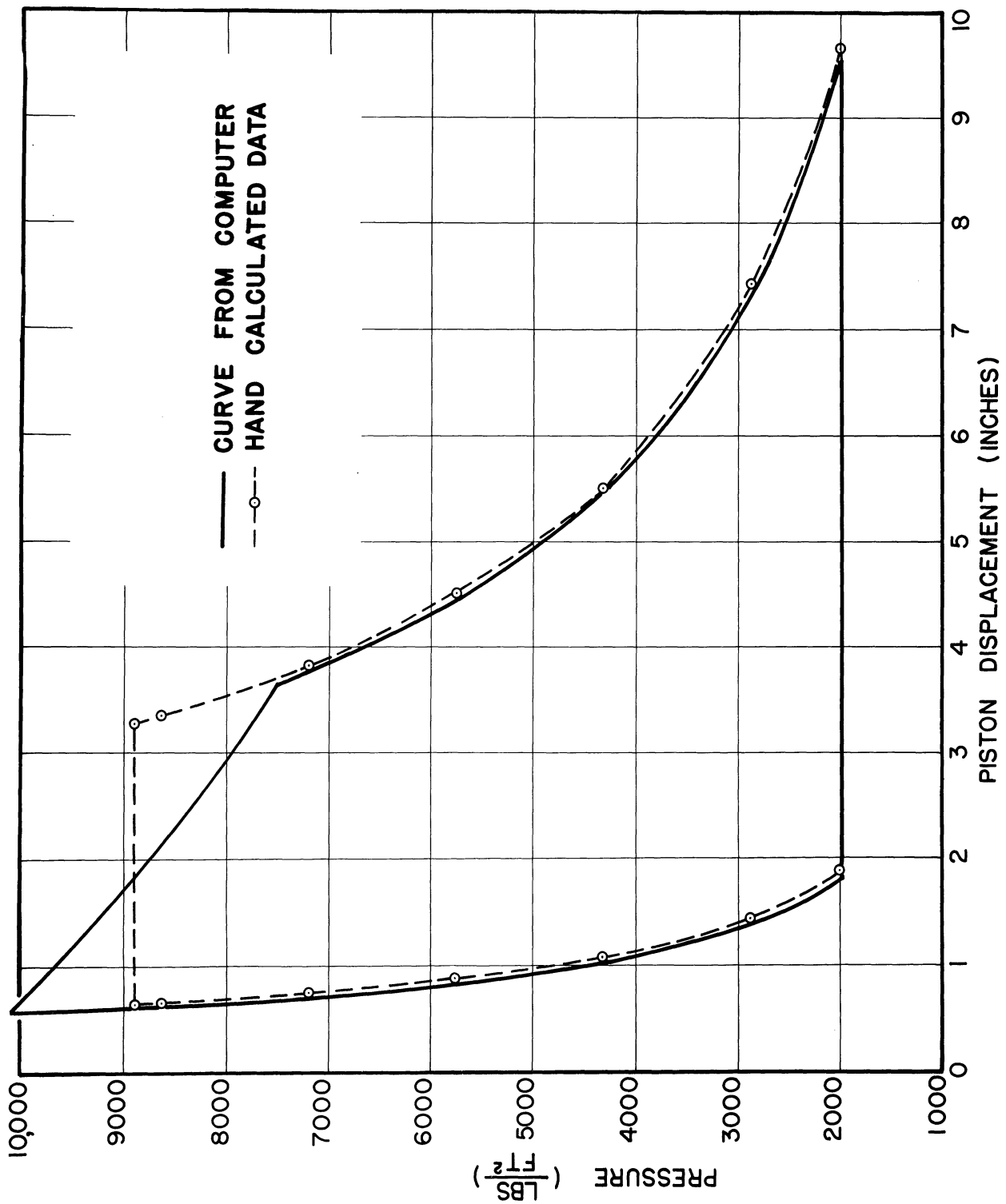
RESULTS

All engine specifications used in setting up the simulator were obtained from Reference 1, in which a sample size and performance calculation was performed. This article also contained a large amount of hand-computed performance data which could be compared with simulator results.

The simulator was operated dynamically, and the air flow rate from reservoir  $\beta$  and the coulomb friction of the piston assembly were adjusted to make the power piston clearance displacement (or the position of inner dead center) correspond to that for the design engine. Steady-state operation was achieved by operating the computer continuously until each cycle was identical with the preceding one. Then, the curves were obtained from servo-driven X-Y plotters connected directly to the appropriate variables in the computer, and the numerical data given in the table was obtained by making voltage measurements at appropriate places. It was found necessary to keep the simulator in operation and repeating the cycle continuously in order to minimize drift effects.

4.1 COMPRESSOR PV DIAGRAM

Figure 4.1 shows a comparison of compressor PV diagrams. The diagram drawn with the solid line was obtained directly from an X-Y plotter connected to the simulator. The diagram shown in dashed lines was plotted from hand-computed points given in Reference 1. The design engine had a compressor piston clearance of .63 inches, but the simulated engine had a compressor piston clearance of .582 inches. The latter clearance was chosen for the simulated engine to make the compressor piston clearance the same as that for the power piston, and thus permit the use of one servo multiplier to multiply by  $\frac{\dot{V}}{V}$  for both compressor and power cylinder. Consequently,



4.1 Comparison of Compressor Cycles

the dashed curve is displaced slightly further to the right than the computer curve because of differences in compressor clearances, and not because of computation error.

In the engine design computations, the assumption was that the compressor was exhausting into a constant pressure reservoir; therefore, the top of the dashed curve is flat. In the simulator, however, the compressor worked into a finite volume reservoir of 4 square feet, so when the air started to flow from the compressor to the reservoir, the pressure continued to rise, but not as rapidly as for the compression of air in the compressor cylinder alone.

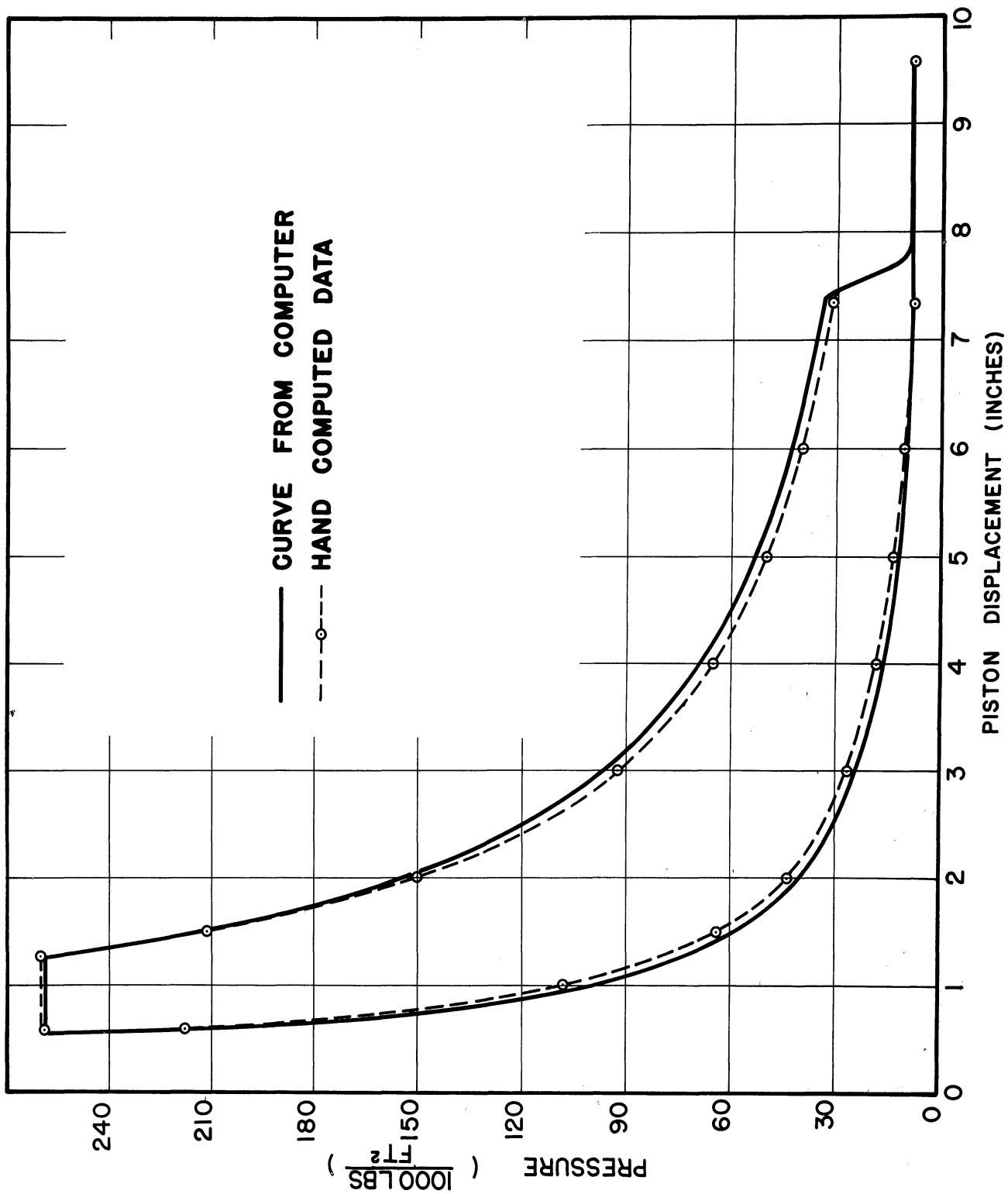
#### 4.2 POWER CYLINDER PV DIAGRAMS

Figure 4.2 is a comparison of the power cylinder PV diagrams for the two engines. The diagram for the design engine falls within the diagram produced by the simulator, and therefore encloses less area. The reason for the discrepancy has not been determined. The extra area enclosed by the simulator diagram represents extra power, and this could explain the need for greater friction to stabilize the simulator version of the engine.

#### 4.3 COMPARISON OF RESULTS

The following table shows further comparisons between the simulated engine and the design engine.

	<u>Hand Computation</u>	<u>Analog Computer</u>
Stroke Length	9 inches	9 inches
Frequency	26.7 CPS	27.3 CPS
Air Flow	.1875 lb/cyl/cycle	.2113 lb/cyl/cycle
Engine Air/Cycle	.035 lb/cycle	.034 lb/cycle
Scavenge Ratio	53.3	62.1
Friction Force	987 lbs	1976 lbs
Gas Power Developed	1000 hp	1280 hp



4.2 Comparison of Power Cylinder Cycles

Probably the greatest discrepancies occur in the developed gas horsepower of the two engines and in the need for considerably greater friction in the simulator engine. Adjustable friction in an actual engine would be considered somewhat ridiculous, but it provided a convenient means to absorb the difference between the power generated by the power cylinder and that absorbed by the compressor, in the case of the simulated engine. The discrepancies are probably directly traceable to the discrepancies in the two PV diagrams for the power cylinders.

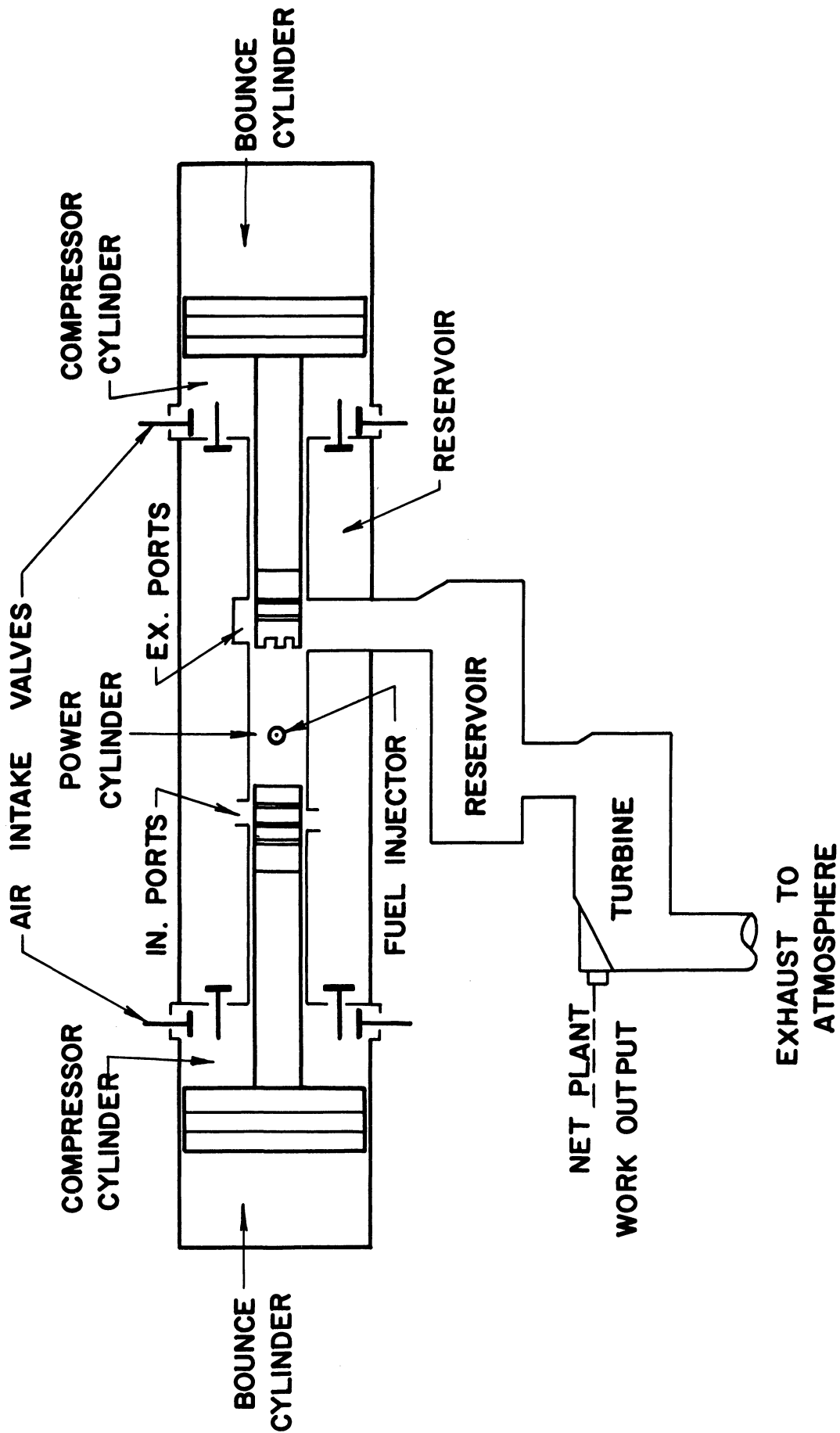


METHOD OF SIMULATION5.1 DESCRIPTION OF THE FREE-PISTON ENGINE

A free-piston engine differs from other types of internal-combustion engines in that its pistons are not mechanically coupled to a crankshaft. The output of such an engine is a flow of hot compressed gas, which is usually passed through a gas turbine to develop mechanical power.

A diagrammatic representation of a free-piston gas generator connected through a reservoir to a gas turbine is shown in Figure 5.1. The gas generator contains two horizontally-opposed piston assemblies. The inner end of each assembly is terminated in a power piston which operates in the small inner power cylinder. At the other end of each piston assembly is a large piston which separates the compressor cylinder from the bounce cylinder. The bounce cylinders contain no valves and function as air springs.

The operation of the engine is best explained by tracing its action through one cycle of operation. Assume that the two piston assemblies have been moved inward, thus compressing the air in the power cylinder and pumping air into the reservoir space from the compressor. Fuel is injected through the fuel injector. Spontaneous combustion of the fuel results in a pressure rise in the power cylinder, and with continued injection of fuel, combustion is maintained and the resulting pressure causes the cylinder assemblies to move outward. The fuel injection is cut off at some specified point, but the piston assemblies continue to move outward, thus compressing the air in the bounce cylinders. When the pistons in the power cylinder uncover the exhaust and intake ports for that cylinder, the heated air in the power cylinder is exhausted into the reservoir connected to the turbine inlet. Air from the reservoir surrounding the power cylinder flows into the



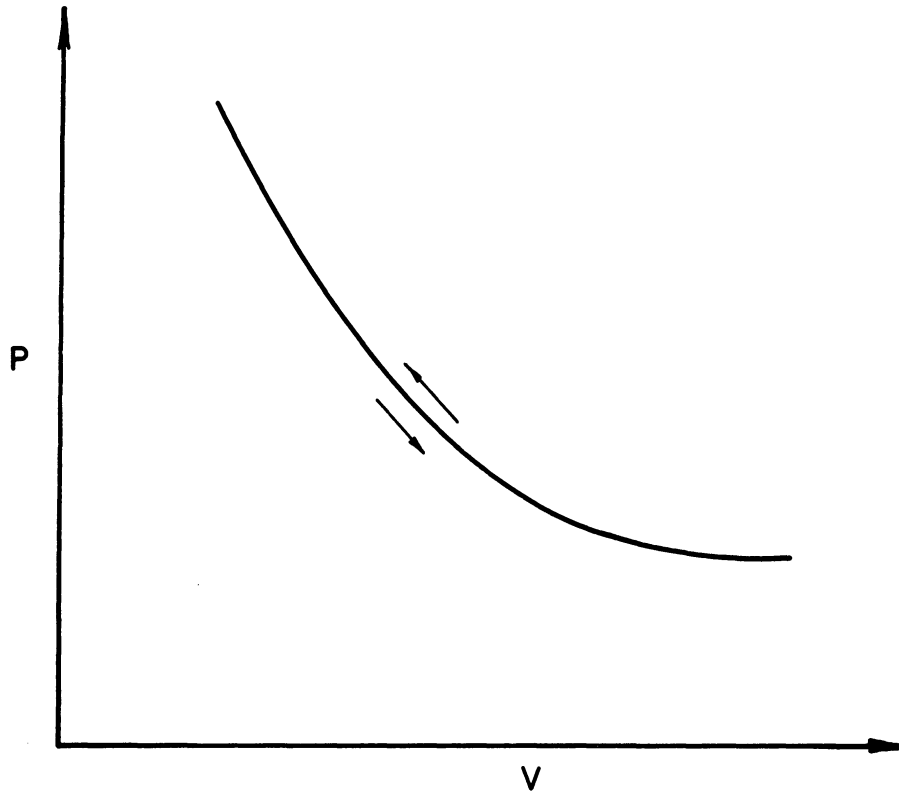
5.1 Diagram of Free-Piston Engine

power cylinder intake port and out through the exhaust port, thus serving as scavenge air to remove the hot gases from the power cylinder. As the pistons continue their outward travel, the pressure in the bounce cylinders builds up, thus causing the pistons to decelerate and to finally reverse their direction of travel. The bounce cylinders impart a large inward velocity to the piston mechanisms, and the resulting momentum is sufficient to bring the power cylinders together for the compression stroke, while the air compressor cylinders are pumping compressed air into the reservoir which surrounds the power cylinder. At the end of the compression stroke, fuel is injected, the fuel ignites, and the cycle is repeated.

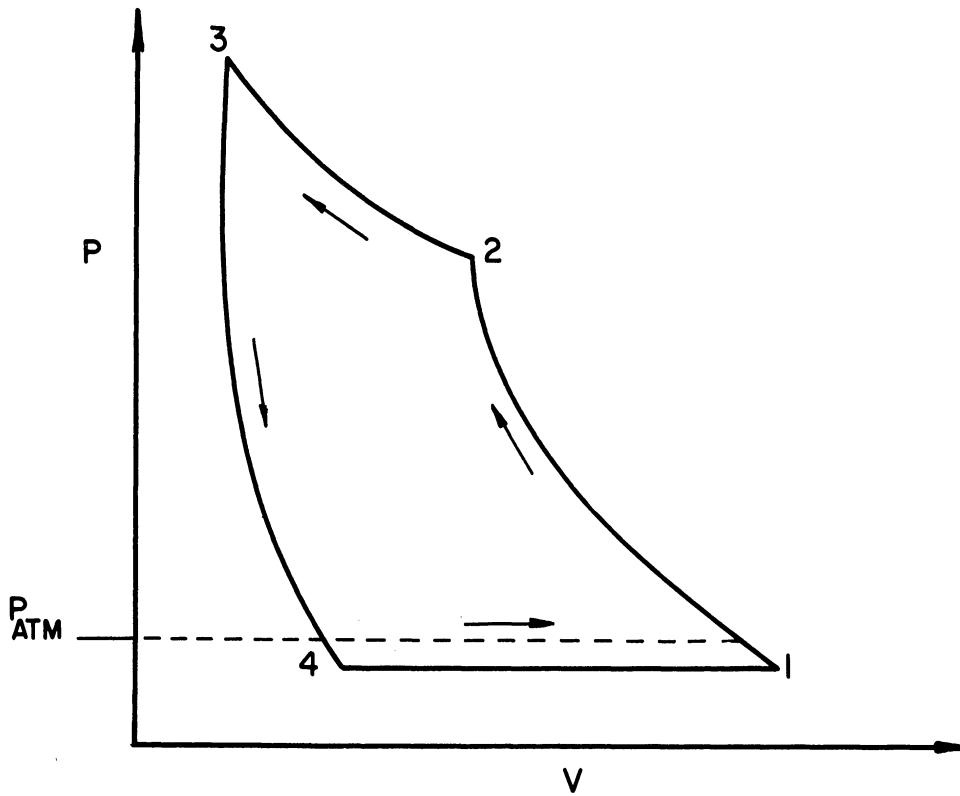
The operation of each engine cylinder may be described more accurately by the use of curves showing pressure versus volume for that cylinder.

The curve of pressure versus volume for the bounce cylinder is shown in Figure 5.2. The curve is single-valued for each value of volume, since it is assumed that no heat is added to or subtracted from the cylinder. The pressure, and hence the force exerted by the bounce cylinder, rises sharply as the volume decreases.

The operating cycle of the compressor cylinder is shown graphically in Figure 5.3. At point 1, the volume of the air compressor cylinder is at maximum, and the compression stroke is beginning. At point 2, the pressure has reached that of the reservoir and the valve between the compressor and this reservoir opens, allowing air to flow. The curve from point 2 to point 3 shows the pressure rise in the compressor cylinder and the reservoir as the compressor cylinder volume continues to decrease. At point 3, the compressor piston reverses its direction, and the remaining air in the compressor cylinder expands along the curve from 3 to 4. At point 4, the



5.2 PV Diagram for Bounce Cylinder



5.3 PV Diagram for Compressor Cylinder

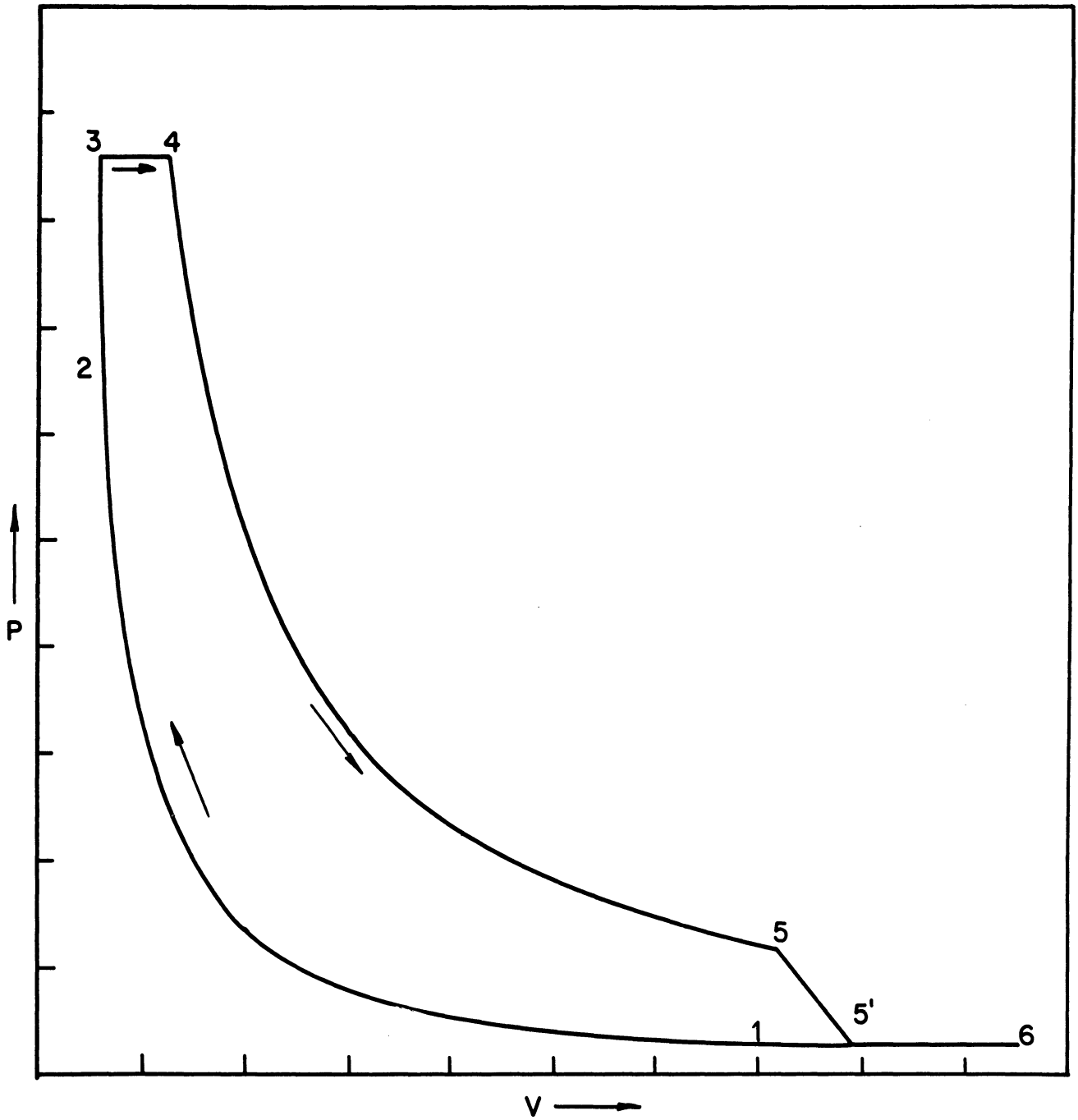
pressure has dropped slightly below that of the atmosphere, thus causing the intake valve to open and air to flow into the compressor cylinder and maintain the pressure constant from 4 to 1.

The power cylinder operates on a diesel cycle, as shown in Figure 5.4. At point 1 on the curve, the intake and exhaust valves are closed and the gas in the power cylinder is compressed up to point 2. Some fuel is injected shortly before point 2, but firing does not occur until point 2 is reached. At point 2, the burning of the fuel causes the pressure to rise abruptly to point 3. Then, as the pistons travel outward, sufficient fuel flow from the injector is maintained to cause the pressure to remain constant until fuel cut-off occurs at point 4. The gases then expand and the pressure drops until point 5 is reached. At this point, the exhaust and intake ports are opened, and the pressure in the power cylinder drops very rapidly as the hot gases flow into the exhaust reservoir. The pressure then remains relatively constant at some value between the pressure of the inlet reservoir and that of the exhaust reservoir, as the scavenge air flows through the system, until the intake and exhaust ports are again closed when point 1 is reached.

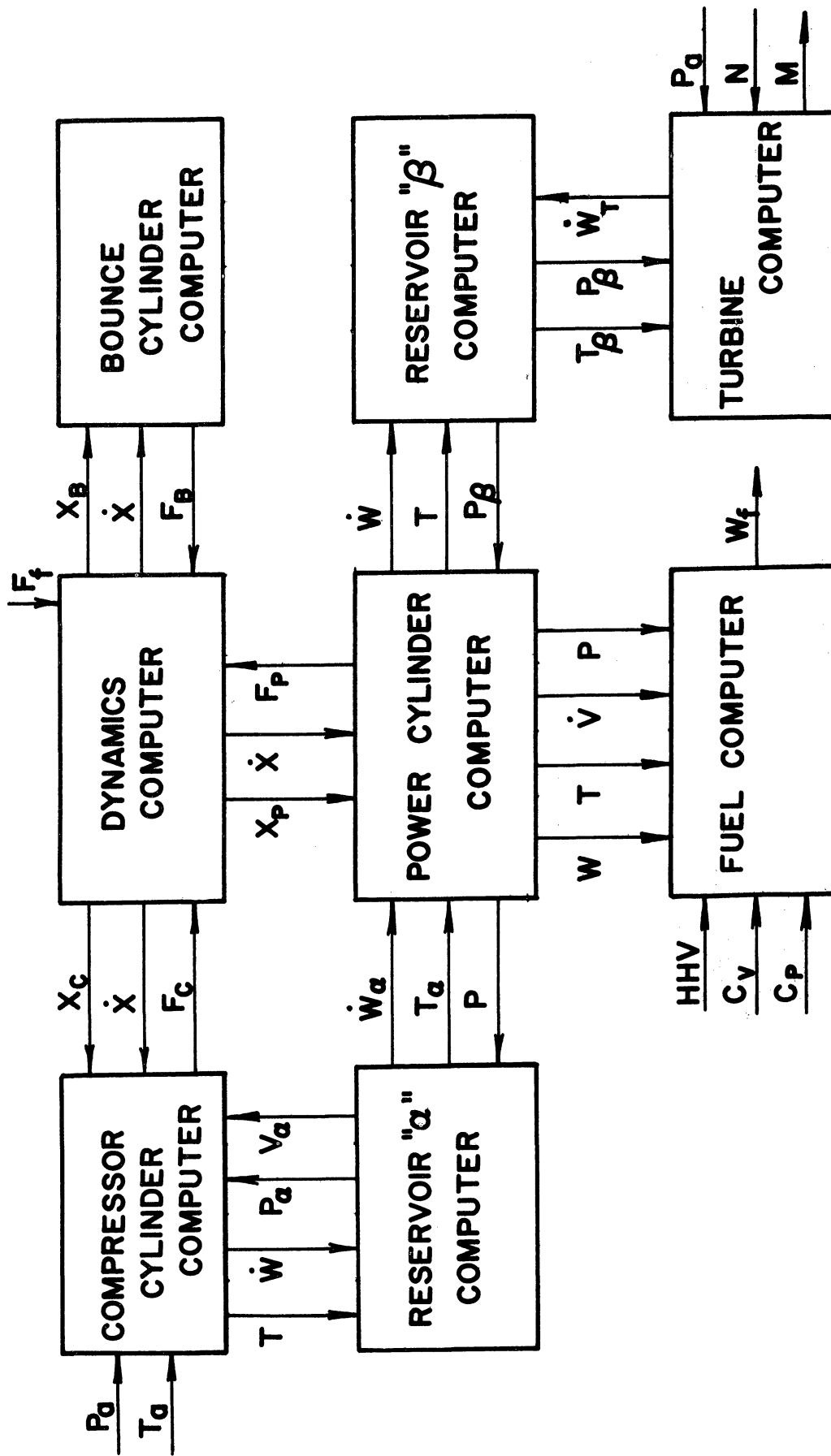
## 5.2 FREE-PISTON ENGINE COMPUTER

### 5.2.1 Information Flow Diagram

The information flow diagram for the free-piston engine is shown in Figure 5.5. Except for the dynamics computer, each block represents a physical part of the engine. In the dynamics computer, the forces exerted on the piston by the pressures in the compressor cylinder, the power cylinder, and the bounce cylinder are summed. The friction force, which opposes the motion of the piston, is also an input to this computer and is added to these forces. The instantaneous acceleration of the piston is then obtained



5.4 PV Diagram for Power Cylinder



5.5 Information Flow Diagram for Free-Piston Engine Computer

by dividing the sum of the forces by the mass of the piston assembly. This acceleration is integrated with respect to time to give the velocity of the piston, which in turn is integrated to give the position of the piston. Both the velocity and the position of the piston assembly are outputs from the dynamics computer and are inputs to the compressor, power, and bounce computers.

There is an interchange of temperature, pressure, and air weight flow information between the blocks which represent engine components. In general, this information exchange is intermittent. For example, air flow information is passed from reservoir  $\alpha$  to the power cylinder only at times during which the intake ports to the power cylinder are open and the gas pressure in the power cylinder is below that of reservoir  $\alpha$ . Air-flow information is passed from the compressor to reservoir  $\alpha$ , then to the power cylinder, and finally to reservoir  $\beta$  before entering the turbine. The temperature information flows in this same direction through the same computers. The pressure information flows in the opposite direction through this same chain of computers. The pressure downstream influences computation upstream. The turbine computer accepts the temperature and pressure information and feeds air flow rate information back to reservoir  $\beta$ . The turbine computer also uses atmospheric pressure and turbine rotor speed information to compute the amount of torque developed. The fuel computer computes the amount of fuel used in the power cylinder per cycle. It receives pressure, temperature, piston velocity, and the air weight information from the power cylinder. Other inputs to the fuel computer are the higher heating value of the fuel, the specific heat at constant volume,  $C_v$ , and the specific heat at constant pressure,  $C_p$ . The bounce cylinder computer exchanges information only with the dynamics computer. It accepts position and velocity as inputs, and generates force as an output.



### 5.2.2 Block Diagram of the Free-Piston Engine Simulator

The block diagram is similar to the information flow diagram which was described in the previous section, but it includes the equations in each block. The block diagram is shown in Figure 5.6. The equations inside each block indicate the mathematical relation between inputs and outputs for that block and thus describe the operation of that particular block during the cycle.

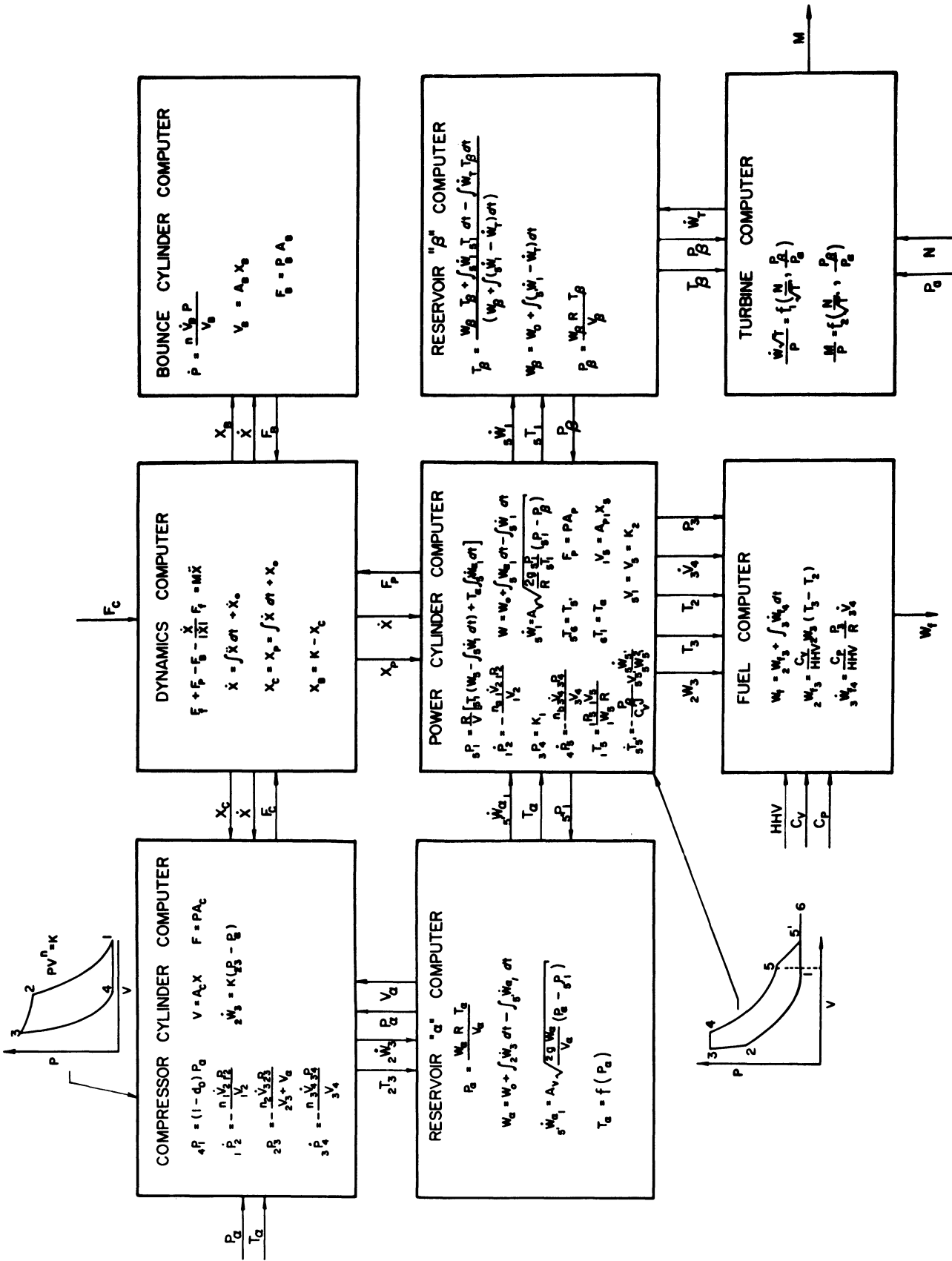
#### Dynamics Computer

In the dynamics computer, the force on the power piston is added to the force on the compressor piston; the sum of these forces is subtracted from the opposing force on the bounce piston to obtain the actual force on the piston assembly. The frictional force is always of such a nature as to oppose the motion of the piston. The first equation in the dynamics computer block equates the sum of the forces to piston assembly mass times acceleration. The next two equations describe the integration of acceleration and velocity to obtain velocity and position. The last equation relates bounce piston position to compressor piston position.

#### Compressor Cylinder Computer

The PV diagram immediately above this block shows graphically one cycle of the compressor operation. The points 1, 2, and 3 are on the compression stroke, or instroke, while 3, 4, and 1 are on the expansion stroke, or outstroke. These numbers are used as subscripts on the variables in the equations to indicate the portion of the cycle over which the equation is valid. For example, the notation  ${}_1P_2$  means the pressure at points along the curve from 1 to 2 of the cycle.

On the compression stroke (1 to 2), the equation  $PV^n = K$  describes the gas relationships, but the computer solves a differential equation



5.6 Free-Piston Engine Computer--Block Diagram

whose solution is this desired relationship. The computer equation is  $\dot{P} = \frac{n\dot{V}P}{V}$ , and its derivation is given in Appendix I. Position 2 on the cycle is reached when the pressure in the compressor is slightly higher than the back pressure exerted by reservoir  $\alpha$  on the other side of the compressor's exhaust valve. At this point, the valve between the compressor and the reservoir opens. The compression of gas continues in the compressor, but the volume used in the  $PV^n$  relationship is now the volume of the compressor cylinder plus the total volume of the reservoir. The time derivative of pressure in the compressor for the period 2 to 3 is then given by:

$${}_2\dot{P}_3 = - \frac{n_2 \dot{V}_3 {}_2P_3}{2V_3 + V_\alpha} .$$

The rate of flow of air between the compressor and reservoir is determined by the difference in pressure between these two parts multiplied by a constant. The value of the constant is arbitrary since this computation is only approximate, but the value is great enough to cause the two pressures to be very nearly equal. The expansion of the gas from 3 to 4 is described by the same differential equation as that for the compression from 1 to 2. The pressure at position 4 becomes less than the atmospheric pressure, so the valve on the intake duct opens and air is drawn into the compressor during the interval from 4 to 1. The compressor volume is obtained by multiplying the area of the piston in the compressor by the piston position. The force on the piston is computed by the multiplication of the piston area and the continuous pressure information.

#### Reservoir $\alpha$

The pressure in this reservoir is obtained by computing  $\frac{WRT}{V}$ , where  $V$  is the volume of the reservoir, and  $T$  is the temperature of the gas

entering the reservoir from the compressor. The weight of gas in the reservoir is equal at all times to the initial weight of gas plus the time integral of the rate of air weight flow entering the reservoir from the compressor, minus the time integral of the rate of air weight flow leaving the reservoir to scavenge the power cylinder. The flow rate of gas between reservoir  $\alpha$  and the power cylinder is determined by the approximation formula for flow when the pressures are very nearly equal:

$${}_{5'}\dot{W}_{\alpha 1} = A \sqrt{\frac{2g W_{\alpha}}{V_{\alpha}} (P_{\alpha} - {}_{5'}P_1)} .$$

### Power Cylinder

The PV diagram is shown in the lower left corner of Figure 5.6. The numbers on the cycle are used in the equations as subscripts to refer to the corresponding points on the cycle.

The equation  $\dot{P} = - \frac{n\dot{V}P}{V}$  is used in the power cylinder for the compression stroke, or instroke (1 to 2). At point 2 on the cycle, the pressure rises abruptly to a fixed value, which is held constant from point 3 to point 4. On the outstroke, from point 4 to point 5, the equation  $\dot{P} = - \frac{n\dot{V}P}{V}$  is used again, but the polytropic expansion coefficient,  $n$ , has a smaller value to compensate for fuel afterburning effects on the expanding gases.

During the period from 5 to 1, when the intake and exhaust ports are open, several modes of gas flow are encountered. For example, from 5 to 5', the hot gases in the power cylinder are rapidly expanding and flowing into reservoir  $\beta$ . At 5', the pressure in the power cylinder has just dropped below that of reservoir  $\alpha$ , and cold air from this reservoir flows into the power cylinder to displace the remaining hot gases during

the interval from 5' to 6. From point 6 (the time at which the last of the hot gas has left the power cylinder) until point 1 (when the power cylinder ports are again closed), air flows into the power cylinder from reservoir  $\alpha$  and out of the power cylinder into reservoir  $\beta$ .

In order to simplify computation, it is assumed that no air flows between reservoir  $\alpha$  and the power cylinder until the pressure in the power cylinder is lower than that in the reservoir. Also, it is assumed that the cold air from reservoir  $\alpha$  does not mix with the hot gas in the power cylinder, and that a boundary which separates the entering cold gas from the retreating hot gas moves through the power cylinder to the exhaust port in the interval from 5' to 6. A third assumption is that the volume of the power cylinder does not change from point 5 to point 1.

One basic quantity which is computed in the power cylinder block is the weight of gas in the power cylinder at all times. This is obtained by integrating with respect to time, the incoming air flow rate minus the outgoing air flow rate. The outgoing air flow rate is given by the equation:

$$\dot{W}_1 = A \sqrt{\frac{2g}{R} \frac{5P_1}{5T_1} \left( 5P_1 - P_\beta \right)}.$$

This equation is an approximation which is valid for small differences in pressure only. Thus, it is not correct for the "blow-down" period from 5 to 5', since during most of this period the pressure difference is quite large. The actual gas flow would be somewhat less than that specified by the equation. The computer mechanization partially corrects for this discrepancy by limiting the value of air weight flow during blow-down. From 5' to 1, the pressure differences are such that the equation is reasonably correct.

Incoming air during the period from 5' to 1 is computed in the reservoir  $\alpha$  block by a similar equation.

Temperature during the portion of the cycle from 1 to 5 is computed by the formula  $T = \frac{PV}{WR}$ . The temperature from 5 to 5' is computed by integrating  $\dot{T}$  from the equation

$$5\dot{T}_{5'} = - \frac{P_{\beta}}{C_{VJ}} 5V_{5'} \frac{5\dot{W}_{5'}}{5W_{5'}^2}$$

(which is derived in Appendix I) and adding the result to  $T_5$ . Temperature from 5' to 6 is assumed to be constant, and the temperature from 6 to 1 is assumed to be the temperature of the gas entering from reservoir  $\alpha$ .

The computation of power cylinder pressure for the interval from point 5 to point 1 is based on the equation  $P = \frac{WRT}{V}$ .

#### Reservoir $\beta$

The pressure in this reservoir is computed by  $P = \frac{WRT}{V}$ . The volume is constant, while  $W$  and  $T$  vary in magnitude. The weight of air in the reservoir is obtained by integrating the rate of air flow being furnished to the turbine continuously, and subtracting this amount of air from the amount of air initially in the reservoir and the amount of air added each time the ports are opened between this reservoir and the power cylinder. The temperature is computed by adding amounts of air multiplied by their respective temperatures, and then dividing this sum by the total air in reservoir  $\beta$ .

#### Bounce Cylinder Computer

The three equations in the bounce-cylinder block are obvious and need no explanation. (See Figure 5.6).

## Fuel Computer

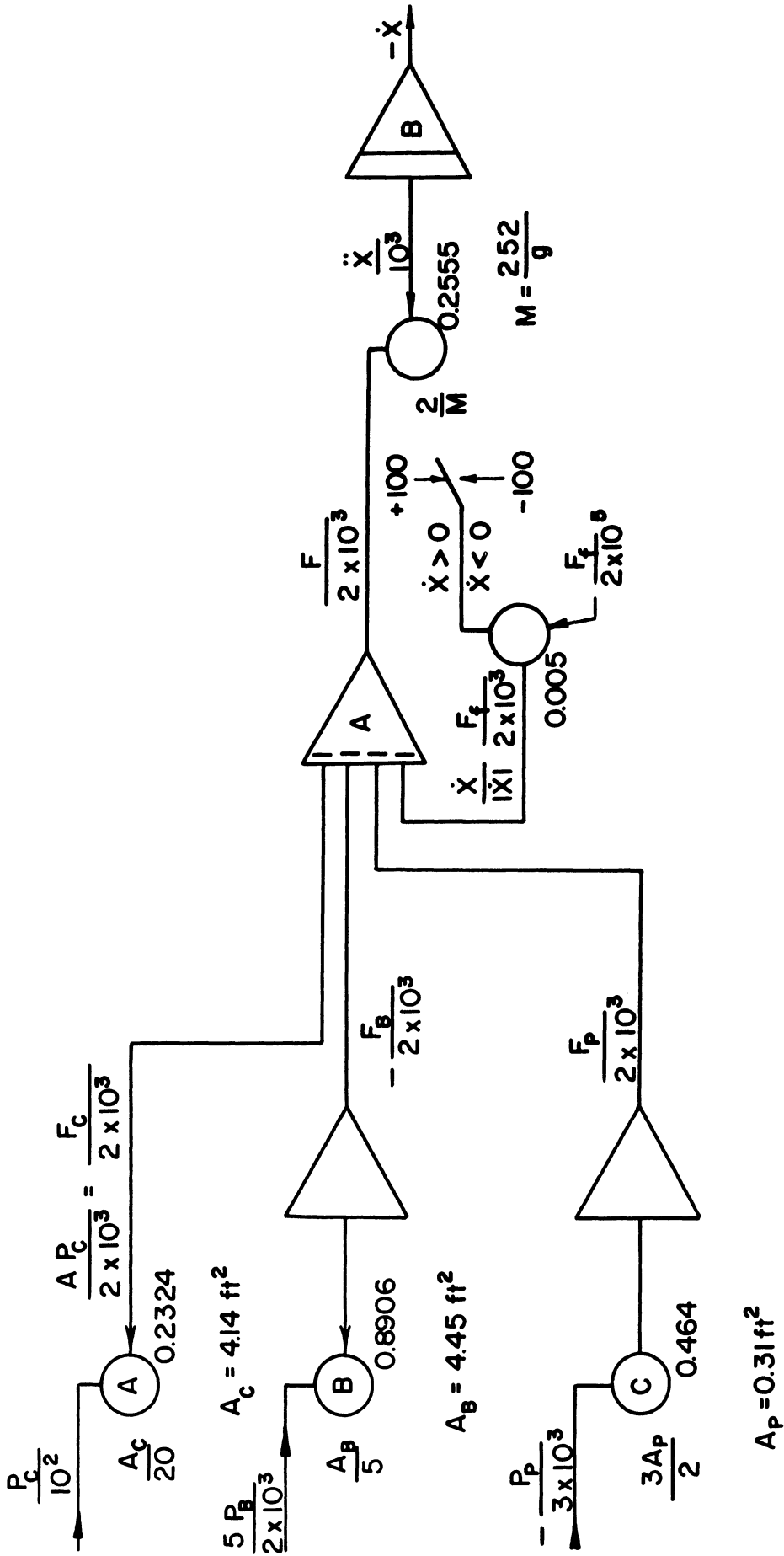
Since the pressure in the power cylinder was found to follow an ideal diesel cycle, there was no need for fuel-flow computation. The amount of fuel used per cycle is of interest, however, so the diagram includes a fuel computer block. The numerical subscripts on the variables in the fuel computer block refer to the positions on the power cylinder cycle. The amount of fuel used to cause the pressure to rise abruptly from 2 to 3 is calculated by taking the product of the difference in temperature from 2 to 3, the air weight in the power cylinder, and the specific heat for constant volume, and then dividing this product by the higher heating value of the fuel. The rate of fuel flow in going from 3 to 4 is calculated by dividing the product of  $2P_3$ ,  $C_p$ , and  $2\dot{V}_3$  by the product of the gas constant,  $R$ , and the higher heating value of the fuel. The fuel used per cycle is then computed by adding the integral of fuel flow rate from 3 to 4 to the fuel necessary to cause the pressure rise from 2 to 3.

### 5.3 COMPUTER ROAD MAPS

The following section contains computer diagrams, or "road maps", which show the actual computing circuit used. The symbols used in these diagrams are the conventional ones and are explained on page iv. The lines in the diagrams are labeled in accordance with the voltage which appears on that line. For example, the notation  $\frac{\dot{V}}{20}$  appearing next to a line indicates that the voltage on that line is equal to  $\frac{\dot{V}}{20}$ . If  $\dot{V}$  were 200 feet per second, the voltage would be 10 volts.

#### 5.3.1 Dynamics Computer

The three inputs to the dynamics computer, as seen in Figure 5.7, are the pressures exerted on the piston in the compressor, bounce, and power cylinders. Each of these pressures is multiplied by its respective



5.7 Road Map of the Dynamics Computer



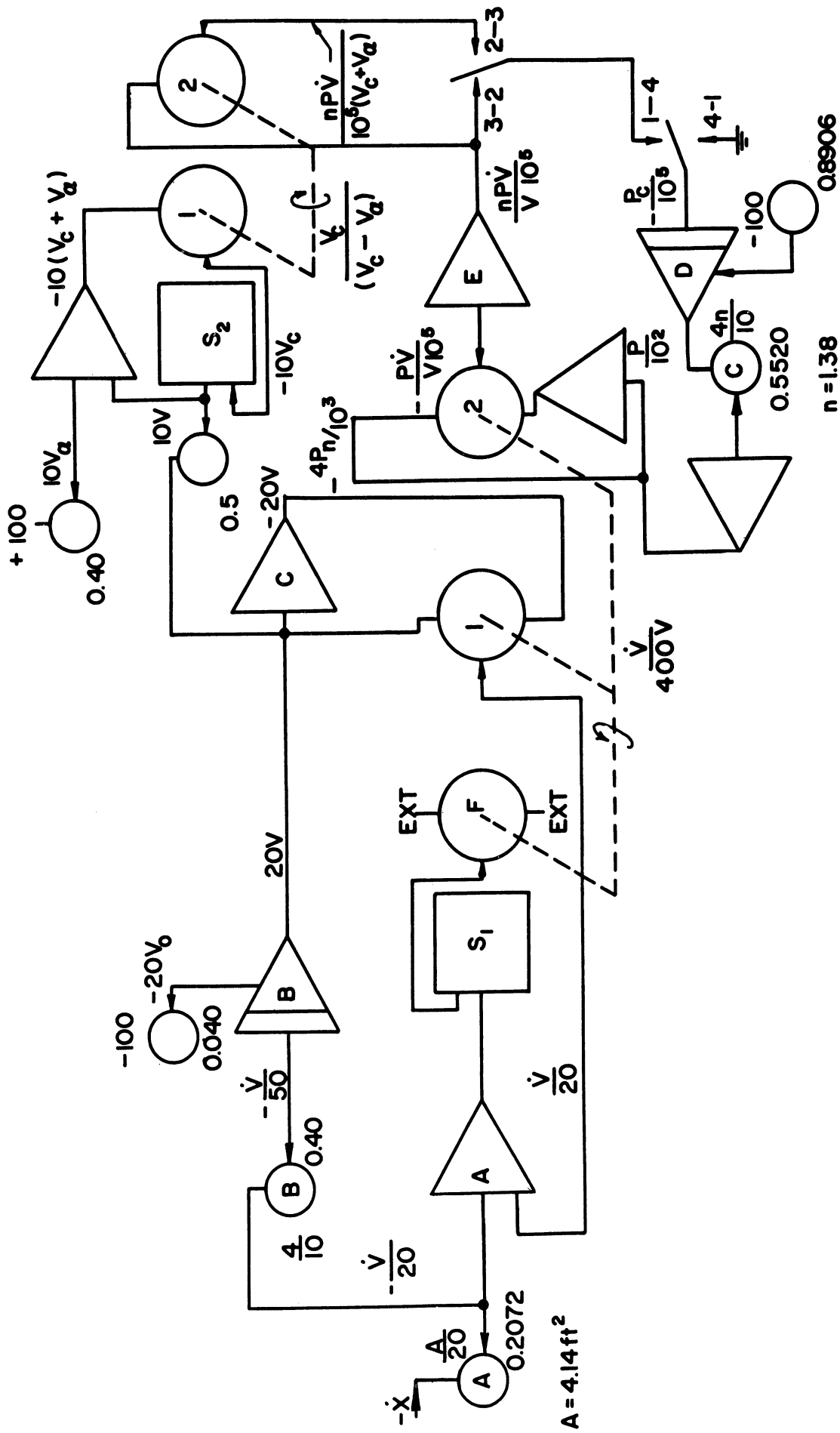
piston area by a coefficient potentiometer A, B, or C. The forces are summed on amplifier A, along with the frictional force. The frictional force is considered constant, and the polarity of the voltage which represents this force is determined by a set of relay contacts. The arm of the relay changes position when direction of piston travel reverses. The polarity of the voltage on the relay arm is positive for the outstroke, when  $\dot{X}$  is positive, and is negative for the instroke, when  $\dot{X}$  is negative.

The voltage which represents the sum of the forces is fed to the top of a potentiometer which is set to a value of  $\frac{2}{M}$  to give  $\frac{\ddot{X}}{10^3}$  as its output. This acceleration voltage is integrated by integrator B. The time scale is such that the integrator has a gain of  $10^3$ , so that its output is  $-\dot{X}$ . Choice of time scale for this simulation is discussed in Section 5.5.

Although the block diagram, Figure 5.6, shows the integration of  $\dot{X}$  to give X in the dynamics computer, this was actually accomplished in the compressor computer by first multiplying  $\dot{X}$  by the compressor piston area to obtain  $\dot{V}$ , and then integrating  $\dot{V}$  to obtain V. For this reason, the integration of  $\dot{X}$  to obtain X is not shown in the dynamics computer diagram.

### 5.3.2 Compressor Cylinder Computer

Piston velocity is an input to the compressor cylinder computer, which is shown in Figure 5.8. Potentiometer A multiplies this velocity by the compressor piston area to give  $-\frac{\dot{V}}{20}$ , which is then fed to the amplifier A and to the top of potentiometer B. The output of potentiometer B is  $-\frac{\dot{V}}{50}$ . This is integrated by integrator B, with a gain of 1000 (due to the time scaling), to give 20V at the output of this integrator. The output of integrator B and the output of inverter C are fed to the high and low



5.8 Road Map of the Compressor Cylinder Computer

ends of potentiometer 1, which is on the shaft of servo multiplier  $S_1$ . The arm of this potentiometer is connected to an input of amplifier A. Both inputs to amplifier A are the same quantity  $\frac{\dot{V}}{20}$ , but they are of opposite polarity if the servo shaft is at  $\frac{\dot{V}}{400V}$ . If the shaft is not at this position, the output of amplifier A produces a voltage which acts as an error signal to the servo amplifier  $S_1$  and causes the servo shaft to rotate toward  $\frac{\dot{V}}{400V}$ . The follow-up potentiometer on the shaft has no voltage on either end. This circuit implicitly solves for the quantity  $\frac{\dot{V}}{V}$  on the servo shaft.

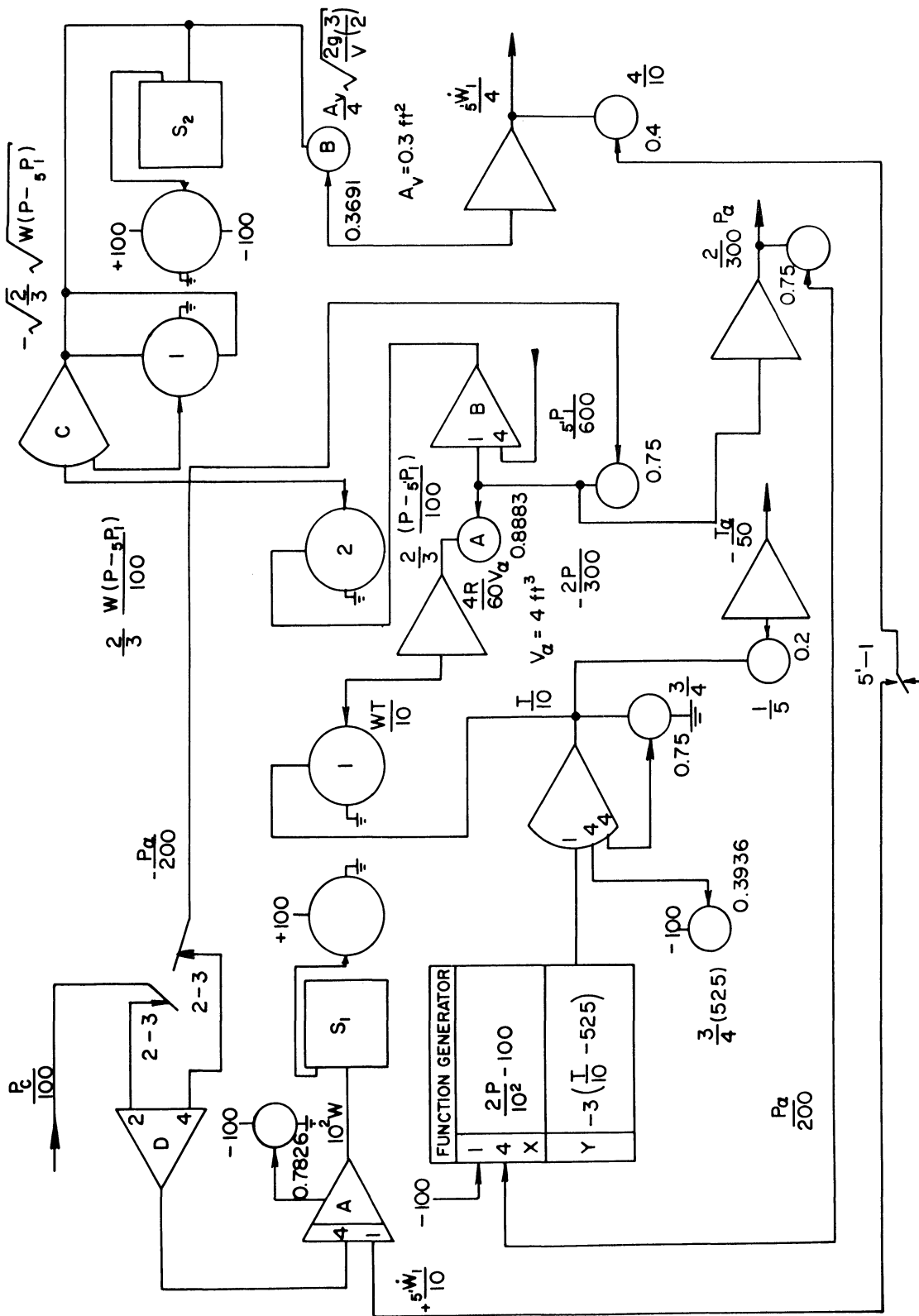
Integrator D's output is  $\frac{P}{10^2}$ ; this is multiplied by  $\frac{4n}{10}$  on potentiometer C to give  $\frac{4Pn}{10^3}$ . The two inverter amplifiers drive opposite ends of potentiometer 2 on the servo shaft,  $S_1$ , so that the arm of this potentiometer gives  $+\frac{nP\dot{V}}{V10^5}$  after amplifier E. This output is connected to one contact of a relay, while the other relay contact is connected to the arm of the servo potentiometer 2 on  $S_2$ . The arm of this servo potentiometer produces  $\frac{nP\dot{V}}{10^5(V_C + V_\alpha)}$ . This quantity is obtained by multiplying  $\frac{nP\dot{V}}{10^5V}$  on the output of amplifier E by  $\frac{V_C}{V_C + V_\alpha}$ , which is the shaft position of servo  $S_2$ . Servo  $S_2$  operates implicitly to produce  $\frac{V_C}{V_C + V_\alpha}$  as a shaft position. The arm of the relay selects the output of amplifier E from 3 to 2 on the compressor cycle and then selects the arm of potentiometer 2 on servo  $S_2$  from 2 to 3. The arm of this relay goes to a contact on a second relay. The arm of this second relay provides the input to integrator D. It connects to the arm of the first relay from 1 to 4 and connects to ground from 4 to 1. These two sets of relay contacts provide the four modes of operation for the compressor cycle. The circuits used to control the relays are described in Section 5.4.

### 5.3.3 Reservoir $\alpha$ Computer

The only function generator in the entire simulator circuit is used in the reservoir  $\alpha$  computer, which is shown in Figure 5.9. This function generator produces temperature as a function of pressure; the data for the relationship was obtained from the Keenan and Kaye gas tables<sup>(4)</sup>. The temperature output of the function generator passes through a circuit to change its scaling and is applied to the top of potentiometer 1 of servo  $S_1$ . The output of integrator A represents the total amount of air in the reservoir, and it positions the shaft of servo  $S_1$ . The product,  $WT$ , is then inverted in sign and multiplied by  $\frac{4R}{60V_\alpha}$  by potentiometer A. The resulting quantity, the reservoir pressure, is fed back to the input of the function generator to provide the necessary input for this device. The flow rate for air leaving reservoir  $\alpha$  and going to the power cylinder during 5' to 1 of the power cylinder cycle is computed by first using amplifier B to take the difference between  $P_\alpha$  and the power cylinder pressure. This pressure difference is multiplied by  $W$  by potentiometer 2 of servo  $S_1$ , and the square root of this product is computed by the circuit composed of high gain amplifier C and servo  $S_2$ . This square root then passes through potentiometer B, which is set at  $\frac{A}{4} \sqrt{\frac{2g}{V} \left(\frac{3}{2}\right)}$ , so that the arm of the potentiometer produces the quantity :

$$- \frac{A}{4} \sqrt{\frac{2gW}{V} (P_\alpha - 5P_1)}, \text{ or } - \frac{5\dot{W}_1}{4} .$$

This outward flow rate is fed to the power cylinder and is also used as an input on integrator A, so that the reduction of air in the reservoir during this flow can be computed. The flow of air into reservoir  $\alpha$  from the compressor during the period from 2 to 3 of the compressor cycle is obtained from amplifier D, whose output is  $\frac{P_\alpha - P_C}{50}$ . This voltage, feeding a 4 gain on integrator A, keeps the output of A at a level such that  $P_\alpha$  is only slightly less than  $P_C$  from 2 to 3 on the compressor cycle.

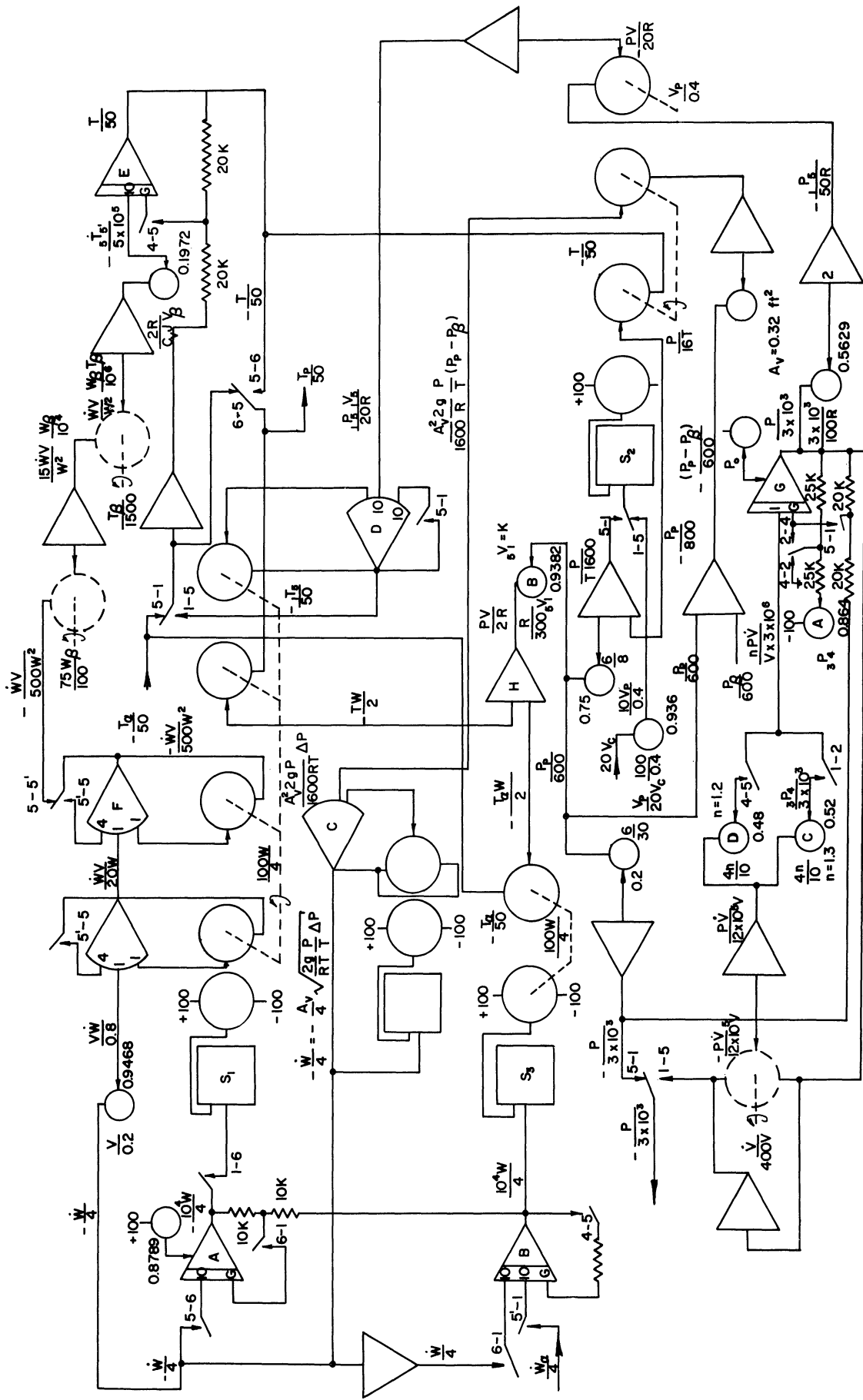


5.9 Road Map of the Reservoir & Computer

#### 5.3.4 Power Cylinder Computer

The power cylinder computer is the most complicated part of the engine simulator because of its many modes of operation in the diesel cycle. This computer requires numerous equations and rather involved switching circuits. No attempt will be made to give a complete description of the mechanization in the text of this report, although all details are given in Figure 5.10. Briefly, the main operating principles are as follows:

In Figure 5.10, there are two integrators, A and B, the outputs of which are labeled identically, except for algebraic signs. The output of integrator A represents the total amount of air in the power cylinder from 1 to 5' on the cycle. This is constant from 1 to 5 and sharply decreases from 5 to 5', due to blow-off. During the period from 1 to 5', the output of integrator B remains constant at zero. From 5' to 6, integrator B integrates the flow rate of cool air entering the power cylinder from reservoir  $\alpha$ , and thus, its output represents the amount of cool air in the cylinder, while integrator A integrates the outward flow of air to reservoir  $\beta$ , thus, producing at its output the amount of hot gas remaining in the power cylinder. At point 6, the output of integrator A is zero, since all of the hot gas has flowed out of the power cylinder. The voltage representing the outward flow of gas is then switched to the input of integrator B and this rate, subtracted from the flow rate of the incoming gas, determines the rate of change of gas weight in the power cylinder. Since integrator B is integrating the algebraic sum of these rates, its output represents the actual weight of gas in the power cylinder from 6 to 1. During the interval from 6 to 1, this output is transferred through an external initial condition circuit to integrator A as shown in the diagram. Then, at point 1, the initial condition contacts are broken and the output



5.10 Road Map of Power Cylinder Computer

of integrator A is connected into the computer circuit to provide the correct value of weight of air in the power cylinder for the period from 1 to 5, and so the process is repeated. During the period from 4 to 5, a resistor is connected between the output and grid of integrator B, thus discharging it in preparation for its participation in the computation during the next 5 to 1 period.

Power cylinder temperature for the period from 1 to 5 is computed by the formula  $T = \frac{PV}{WR}$ , and it appears as the output of amplifier D. Through the proper switching arrangement, it is transferred to integrator E as an initial condition at point 5. From 5 to 5', the rate of temperature change is computed by the formula:

$$\dot{T} = \frac{P_{\beta}}{C_V J} \quad 5V_{5'} \quad \frac{5W_{5'}}{5W_{5'}^2} \quad .$$

The computation path for this starts with the output of amplifier C, which is  $\frac{\dot{W}}{I}$ , proceeds through the two dividing circuits associated with servo S<sub>1</sub>, and then goes from the output of amplifier F through a series of amplifiers and potentiometers (including two servo potentiometers from the reservoir β computer), and finally into the input of integrator E. At 5', the contacts on the output of amplifier F open, thus keeping any voltage from reaching integrator E and holding this integrator at a constant voltage. Then, at point 6, the line for temperature of the power cylinder is switched to the temperature of the incoming air from reservoir α.

During the period from 1 to 5, pressure is continuously produced by the output of integrator G. Except for the interval from 3 to 5, when fixed peak pressure is set in by the potentiometer A by the external initial condition circuit, the pressure is determined by solving the differential equation for the polytropic process. The polytropic exponent "n" is equal



to 1.3 for the compression process and is made equal to 1.2 for the expansion process in order to compensate for afterburning of the fuel after fuel cut-off (point 4). These two values of the coefficient are set on potentiometers C and D, respectively, and automatic switching circuits select the output of the appropriate potentiometer as needed. From 5 to 1, pressure is computed by the formula:

$$P_P = \frac{R}{V} (T_\alpha W_{\text{cold gas}} + T_P W_{\text{hot gas}}),$$

and it is placed on integrator G through an external initial condition circuit.

Servo  $S_2$  is time-shared. From 5 to 1, its shaft is positioned at  $-\frac{P}{T}$ , while from 1 to 5, its shaft is positioned to the volume of the power cylinder.

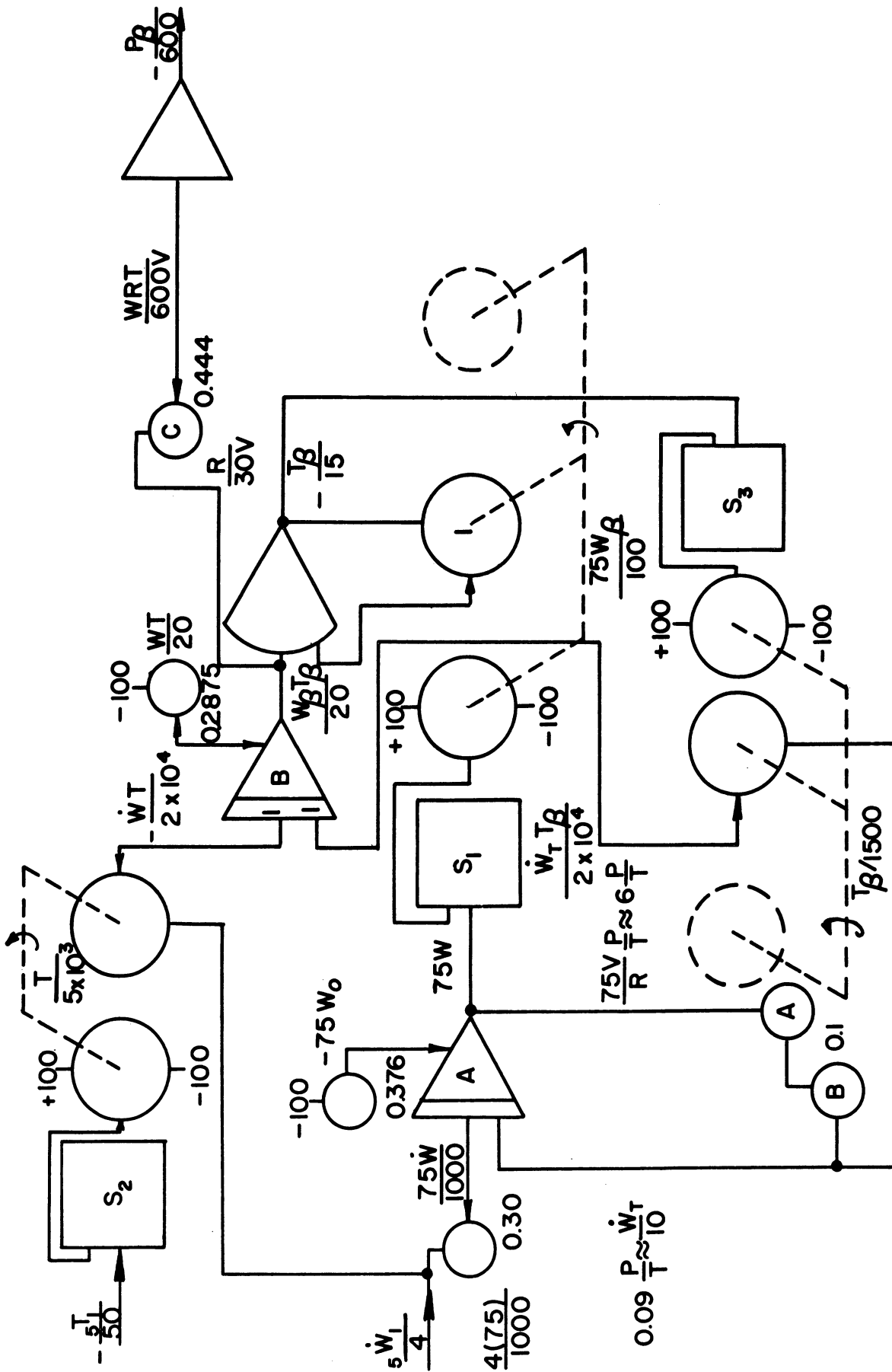
#### 5.3.5 Reservoir $\beta$ Computer

This computer circuit is shown in Figure 5.11. It is built around integrator A, whose output voltage represents the weight of air in the reservoir. Inputs to integrator A are the incoming air flow from the power cylinder during the period from 5 to 1, and the outgoing air to the turbine. Since a turbine computer was not mechanized for this simulation, the outgoing flow was approximated. In a turbine, corrected air weight flow  $\frac{\dot{W}\sqrt{T_\beta}}{P_\beta}$  is a function of  $\frac{N}{\sqrt{T}}$  and  $\frac{P_\beta}{P_a}$ . If the turbine is operating at constant shaft speed and under choked conditions, the corrected air weight flow is constant. Then,

$$\frac{\dot{W}\sqrt{T}}{P} = K,$$

or

$$\dot{W} = K \frac{P}{\sqrt{T}}.$$



5.11 Road Map of the Reservoir  $\beta$  Computer

The output of integrator A is  $W_{\beta} = K \frac{P}{T}$ . The air flow out was made equal to a constant times  $\frac{P}{T}$ , by using the two cascaded potentiometers, A and B, to connect the output of integrator A back to one of its inputs. This gives a fair approximation if T is almost constant.

Servo  $S_2$  multiplies the incoming air weight flow by its temperature, and the product of the temperature in reservoir  $\beta$  and the outgoing air weight flow. Integrator B integrates the difference and produces  $\frac{W_{\beta} T_{\beta}}{20}$  as its output. This is then divided by  $W_{\beta}$  by potentiometer 1 on servo  $S_1$  and the high-gain amplifier to give  $T_{\beta}$  which, in turn, is used as the input to servo  $S_3$ . Pressure in reservoir  $\beta$  is computed by using potentiometer C to multiply  $\frac{W_{\beta} T_{\beta}}{20}$  by the constant  $\frac{R}{V}$ .

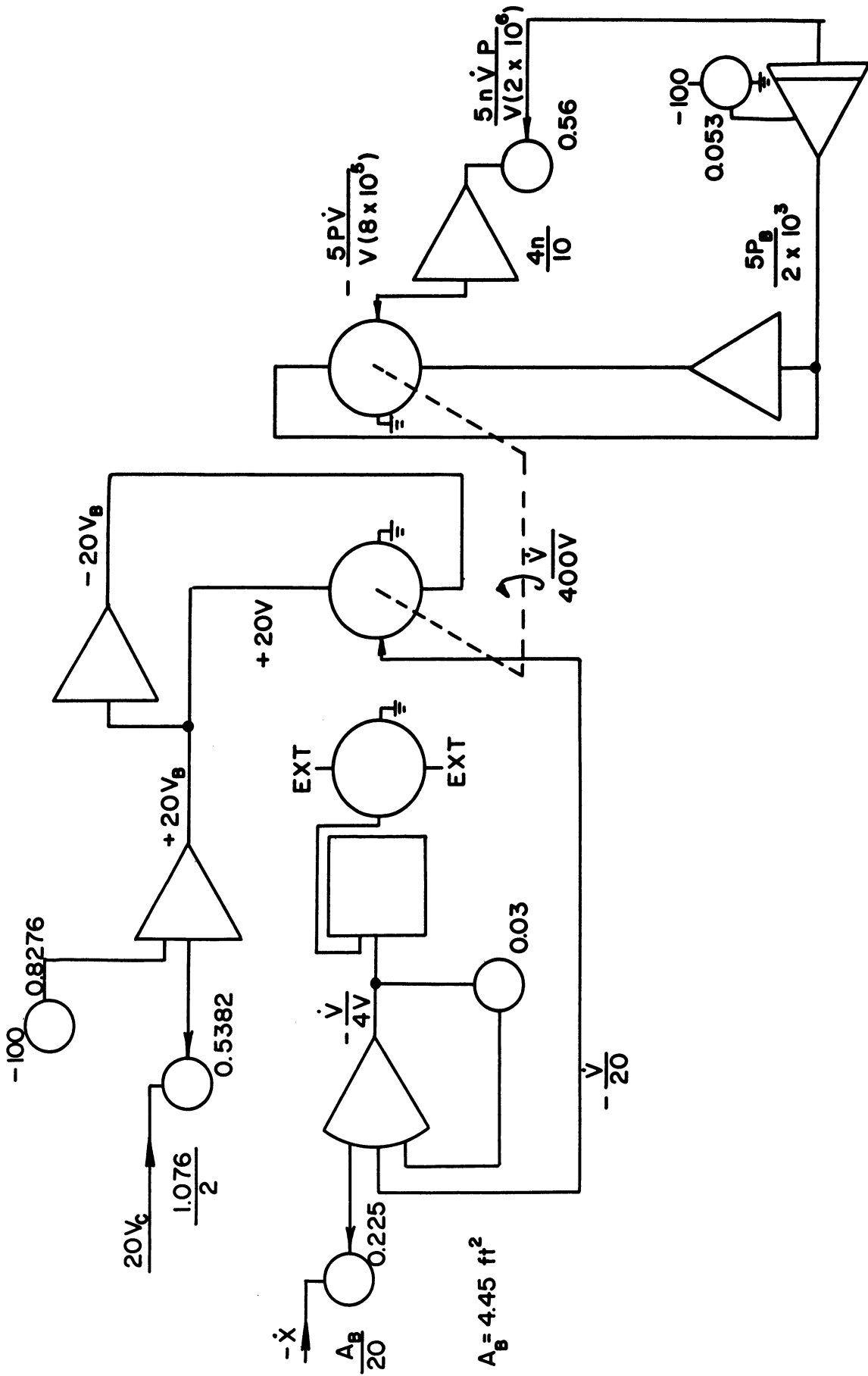
#### 5.3.6 Bounce-Cylinder Computer

The mechanization of the bounce-cylinder computer is similar to that for the compressor cylinder computer, except that no relay switching is required. Its road map is shown in Figure 5.12.

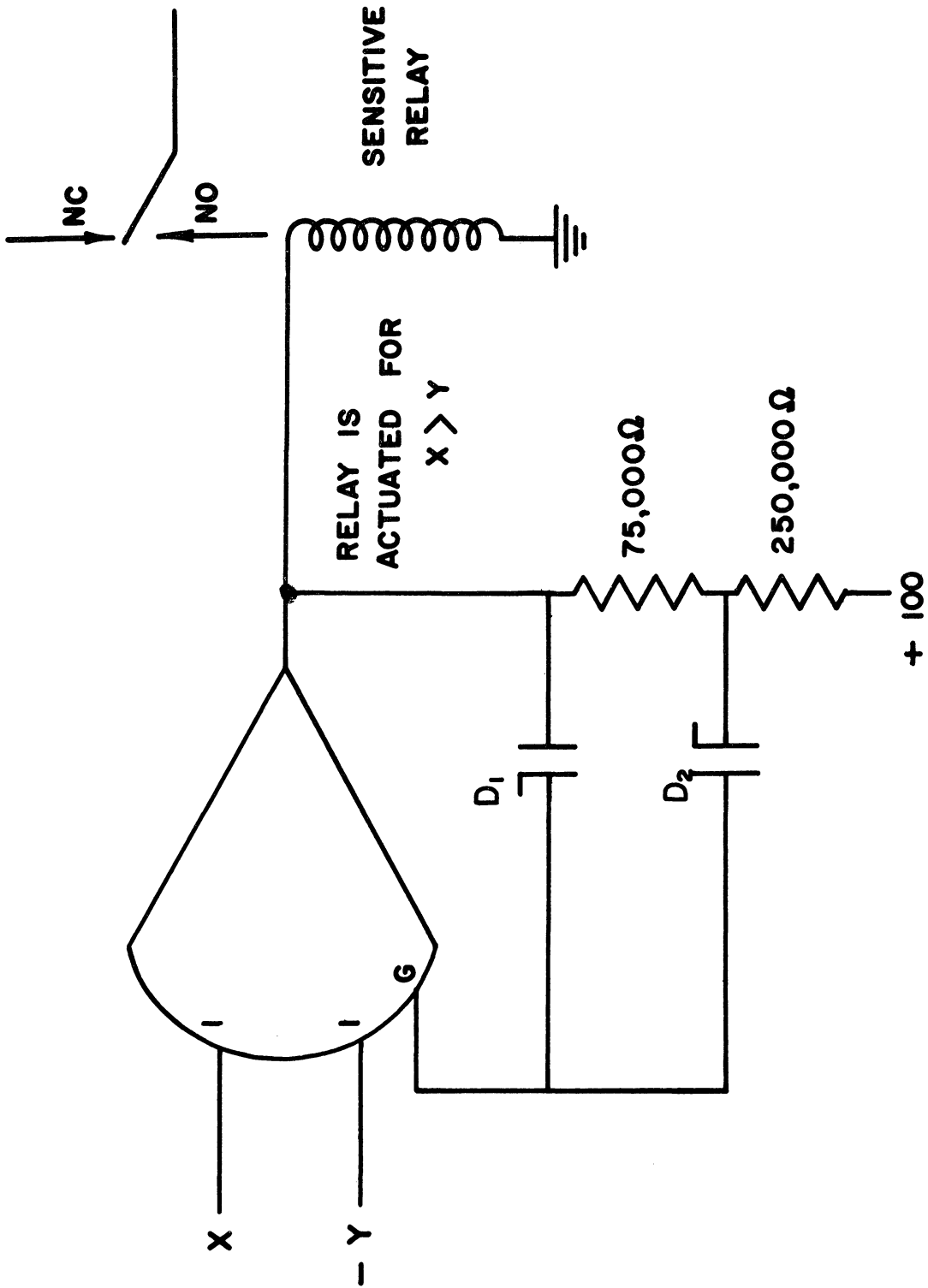
### 5.4 SWITCHING CIRCUITS

A vital part of the free-piston engine simulator is a system of relays which perform automatic switching in the computer circuits for the various engine components when valves are opened, fuel is injected, and other discontinuities occur in the equations. The switching system is based upon the use of sensitive differential relays, devices which operate by comparing two voltages and which close a relay at the instant one voltage becomes greater than the other. These devices are available in commercially-built computing equipment, but they may also be constructed in almost any analog computer laboratory by using vacuum-tube diodes, sensitive relays, and high-gain operational amplifiers.

A differential relay circuit of the type used in the free-piston engine simulator is shown in Figure 5.13. It consists of a high-gain



5.12 Bounce-Cylinder Computer

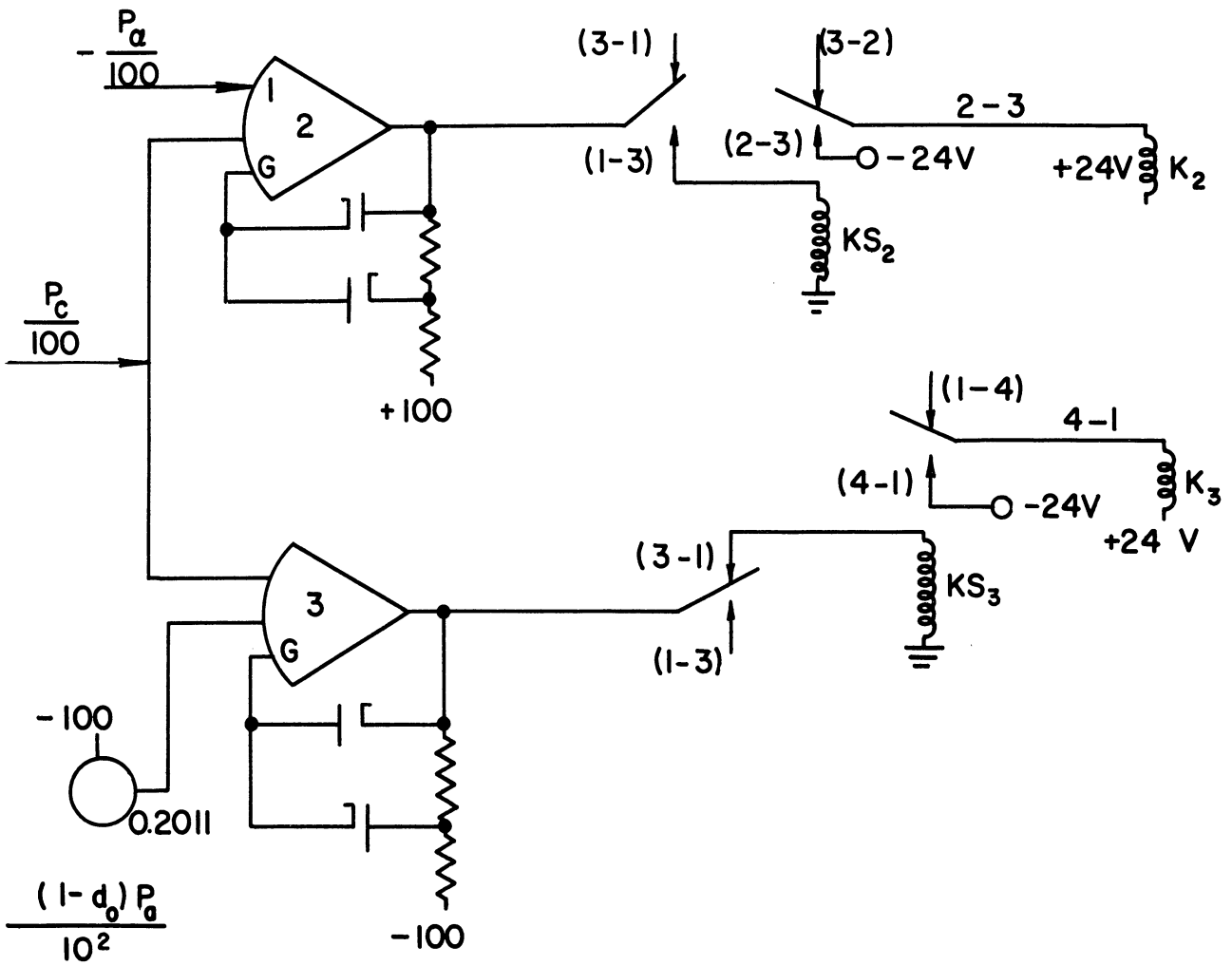
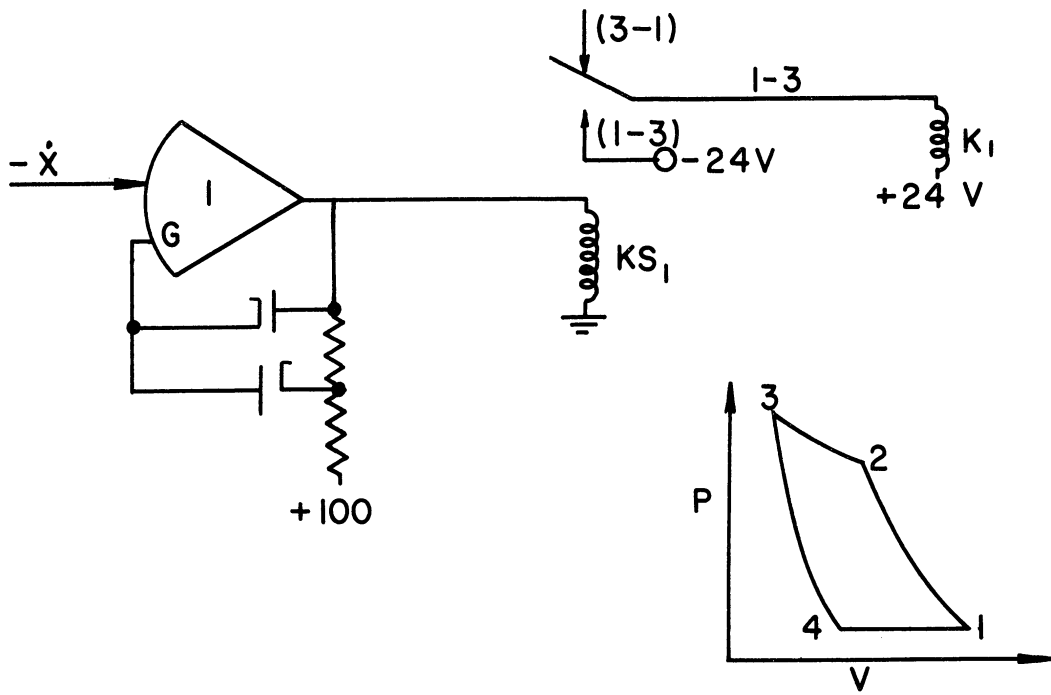


5.13 Differential Relay Circuit

operational amplifier with its output connected to the coil of a sensitive relay, and with a pair of diodes in a biased feedback network. It can have one, two, or even more inputs. The switching occurs when the sum of the inputs passes through zero. If the sum of the inputs is negative, the output tries to become positive, but this positive voltage is carried back to the grid by the diode  $D_1$  and thus cancels out the negative voltage on the grid produced by the inputs. This prevents any appreciable positive voltage from appearing on the output, and the relay is not actuated. If the sum of the inputs is positive, however, the output attempts to become negative. In this case, diode  $D_1$  does not conduct, and the output voltage becomes sufficiently negative to cause diode  $D_2$  to conduct and thus cancel out any tendencies to become even more negative. The magnitude of this negative voltage which is generated at the output of the amplifier depends upon the biasing voltage (in this case +100 volts) and upon the relative resistances of the two resistors. It is of sufficient magnitude to actuate the relay and is relatively independent of the magnitude of the sum of the input voltages to the amplifier.

In the engine simulator, a number of differential relays are used to actuate standard 24-volt 4-pole double-throw relays, which are wired into a logical network so that any desired switching condition may be obtained.

Figure 5.14 shows the relay circuitry used to switch equations in the compressor. The numbers opposite the relay contacts refer to points on the compressor's operating cycle. The differential relay circuit at the top of the figure, involving sensitive relay KS 1, is operated by an input voltage proportional to  $-\dot{X}$  for the compressor piston. When  $-\dot{X}$  becomes positive (corresponding to the instroke of the compressor), the output of the amplifier becomes negative and operates relay KS 1. When  $-\dot{X}$  becomes negative (corresponding to the outstroke), the diode provides a direct feedback



5.14 Switching Circuitry for Compressor-Cylinder Cycle

path around the amplifier, thus preventing generation of sufficient output voltage to trigger the relay. The coil of a 24-volt relay, K 1 is connected between +24 volts and the arm of the sensitive relay KS 1, and -24 volts is connected to the normally-open contact of KS 1. Then, when the piston is moving on the instroke, from point 1 to 2 to 3, KS 1 is actuated and it, in turn, actuates K 1. The normally-open contacts on relay K 1 are closed from 1-3, and the normally-closed contacts are closed from 3-1. These contacts are not shown near the coil of K 1, but two arms are shown connected to the outputs of high-gain amplifiers 2 and 3. The contacts may be identified by the labeling of 3-1 for the normally-closed contacts and 1-3 for the normally-open ones.

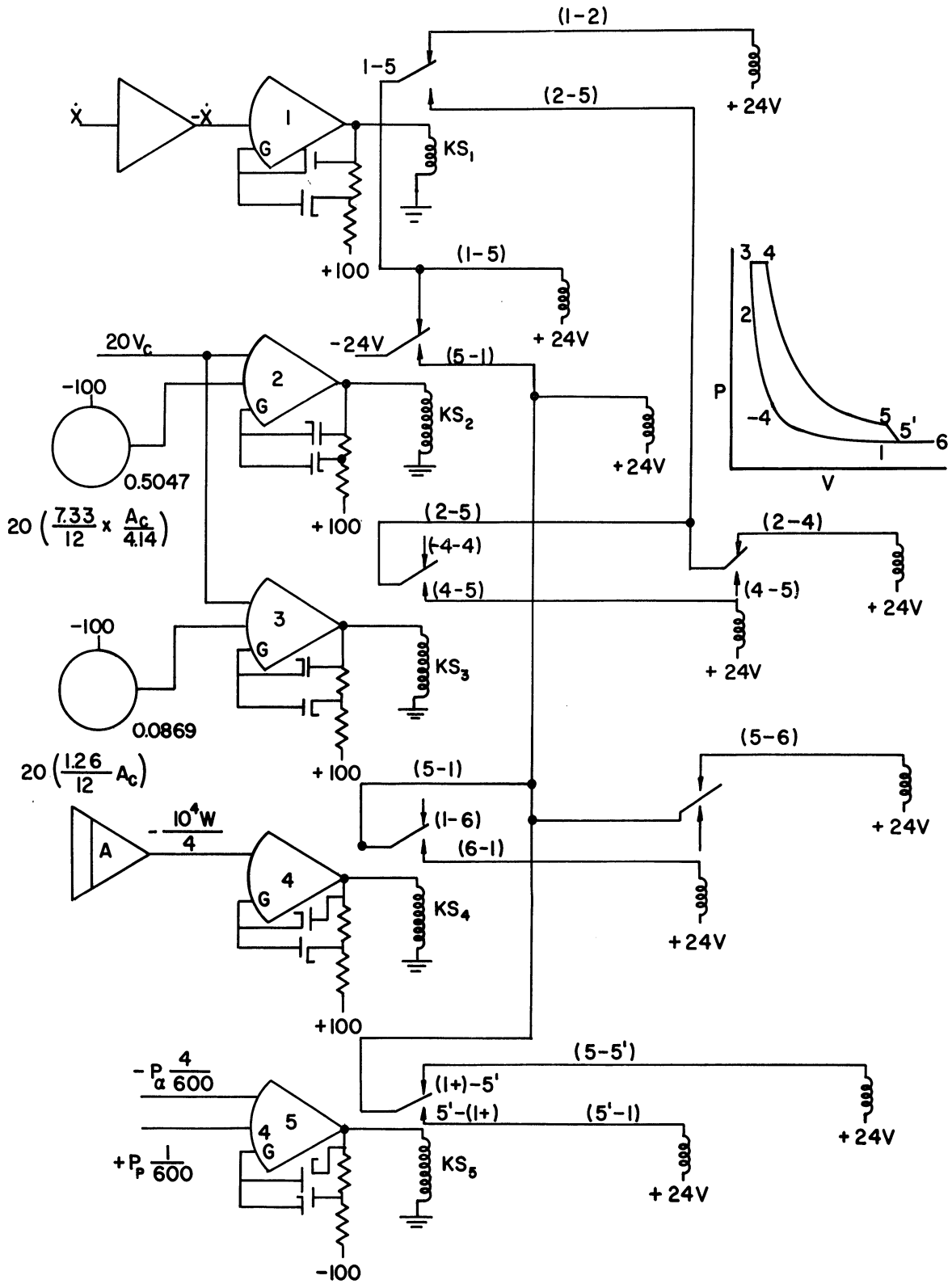
Amplifier 2 produces a voltage whenever the compressor pressure is greater than the reservoir  $\alpha$  pressure. When this occurs on the instroke, the 1-3 contacts are closed and the voltage actuates relay KS 2 which, in turn, supplies voltage to actuate the coil of relay K 2. Thus, relay K 2 is actuated only for the part of the cycle between points 2 and 3.

Amplifier 3 has its feedback diodes reversed, so that it can produce a positive output voltage but not a negative one. Thus, it produces a voltage whenever its input is negative, or whenever the compressor pressure is less than  $(1-d_0)$  times the atmospheric pressure. It, therefore, produces a voltage and actuates relay KS 3 for the period 4-1, and KS 3 in turn, actuates K 3 for the same period.

In the computer circuits, several sets of relay contacts are often required for the same period, so it is necessary to use the sensitive relays, which have only one set of contacts, to actuate one or more 24-volt relays, each of which has 4 sets of contacts.

The relay circuitry for the power cylinder is shown in Figure 5.15. Relay KS 1 is actuated when  $-\dot{X}$  is positive (instroke). Relay KS 2 is actuated when  $20V_C$  is greater than 50.47 volts. This corresponds to a piston displacement  $X$ , which is greater than 7.33 inches, the point where





5.15 Switching Circuit for Power-Cylinder Cycle

the intake and exhaust ports to the power cylinder open, and thus KS 2 is actuated during the part of the power cylinder cycle from 5 to 1. Relay KS 3 is actuated whenever the piston displacement is greater than that for the fuel cut-off point, or 1.26 inches. Relay KS 4 is actuated whenever the voltage representing the cold air in the power cylinder goes to zero (see Section 5.3.4). Relay KS 5 is actuated whenever the pressure in the power cylinder is less than that in reservoir  $\alpha$ .

The contacts of these sensitive relays are connected to a network of 24-volt relays, as shown, to give all of the required contact closures for the intervals in the power cylinder cycle. The label on the line going to the top of each 24-volt relay coil indicates the period during which that particular coil is actuated. The switching is straightforward and hence will not be described in detail.

## 5.5 TIME SCALING

Since the free-piston gas generator operates at 20 to 30 cycles per second, and since most analog computers operate at from 0.1 to 10 radians per second, it is impossible to operate the simulator in "real" time. The operating frequency of the engine to be simulated was about 27 cycles per second. This corresponded to a period of 37 milliseconds. A one thousand-to-one time scale was chosen for the simulator, so that seconds of computer time corresponded to milliseconds of engine time. This scaling is quite satisfactory for the period during which the intake and exhaust valves in the power cylinder are closed (period 1-5). During period 5-1, when blow-down occurs, the pressures and temperatures in the power cylinder drop so rapidly that the computer servo-multipliers do not follow accurately. This difficulty could be remedied by shifting the time scaling to  $\frac{1}{10,000}$  of real time for the period 5-5'.

APPENDIX I

DERIVATION OF EQUATIONS

1. Differential Equation for Polytropic Process

Start with

$$PV^n = K$$

differentiate with respect to time

$$\dot{P}V^n + n\dot{V}V^{n-1}P = 0$$

then,

$$\dot{P} = - \frac{n\dot{V}V^{n-1}P}{V^n}$$

or

$$\dot{P} = - \frac{n\dot{V}P}{V}$$

2. Temperature Change in Gas During Blow-Down

The expression

$$\dot{T}_5 = - \frac{P_\beta}{C_{vJ}} \dot{V}_5 \frac{\dot{W}_5}{W_5^2}$$

is integrated in the power cylinder computer to determine exhaust gas temperature during blow-down. It is derived as follows:

Assuming that the pressure in reservoir  $\beta$  is constant during blow-down, the specific energy of the gas in the cylinder at any time during blow-down is given by a form of the general energy equation

$$E_5 = E_{5'} + \frac{P_\beta}{J} (v_{5'} - v_5)$$

where the last term represents the work involved in expanding against the reservoir  $\beta$  pressure.

The specific volume may be expressed as  $v = \frac{V}{W}$ , and the internal energy is given by  $E_5 \times E_{5'} = C_V(T_5 - T_{5'})$ . Substituting these into the energy equation gives:

$$C_V(T_5 - T_{5'}) = \frac{P_\beta}{J} \left( \frac{V_{5'}}{W_{5'}} - \frac{V_5}{W_5} \right) .$$

Differentiating with respect to time, and considering  ${}_5V_{5'}$  as a constant,

$$- C_V {}_5\dot{T}_{5'} = - \frac{P_\beta}{J} {}_5V_{5'} \frac{{}_5\dot{W}_{5'}}{{}_5W_{5'}^2} .$$

If  $\dot{W}$  is re-defined as the air flow leaving the power cylinder instead of as change of air weight in the power cylinder, it changes in sign. Making this sign change and dividing by  $C_V$  gives:

$${}_5\dot{T}_{5'} = - \frac{P_\beta}{C_V J} {}_5V_{5'} \frac{{}_5\dot{W}_{5'}}{{}_5W_{5'}^2} .$$

### 3. Gas Flow Equations

An approximate form for the ideal velocity of a perfect gas is given in Reference 7 on pages 199 and 200.

$$v_5 = \sqrt{2gRT \left( \frac{P_1 - P_2}{P_1} \right)}$$

This equation is reasonably correct when  $P_1$  is very nearly equal to  $P_2$ . Since  $\dot{W} = \frac{A_V v_5}{v}$  where  $\dot{W}$  is the rate of air flow,  $A_V$  is the cross sectional area of the port, and  $v$  is the specific volume, then

$$\dot{W} = \frac{A}{v} \sqrt{2g \frac{RT}{P} (P_1 - P_2)} .$$

If the substitution  $\frac{RT}{P} = \frac{V}{W} = v$  is made, and if the  $v$  is placed in the denominator under the radical sign, the equation becomes

$$\dot{W} = A\sqrt{\frac{2g}{v} (P_1 - P)} .$$

Substitution of  $\frac{V}{W}$  for  $v$  in this equation gives the following equation which is used for computing the flow rate between reservoir  $\alpha$  and the power cylinder.

$${}_5\dot{W}_1 = A_V\sqrt{2g \frac{W_\alpha}{V_\alpha} (P_\alpha - {}_5P_1)}$$

In this use, the approximation form is very good since the two pressures are very nearly equal.

Next, by substituting  $\frac{P}{RT}$  for  $\frac{1}{v}$  or  $\frac{W}{V}$ , the equation becomes the form used in the power cylinder, namely:

$$\dot{W} = A_V\sqrt{\frac{2g {}_5P_1}{R {}_5T_1} (P_5 - P_\beta)} .$$

## APPENDIX II

### ENGINE SPECIFICATIONS

Specifications for the engine to be simulated were obtained from an article by Professor F. L. Schwartz of The University of Michigan, which gave a sample hand calculation of size and performance of a free-piston engine<sup>(1)</sup>. The method of calculation followed with some modification the method presented by A. L. London and A. K. Oppenheim of Stanford University<sup>(5,6)</sup>. The assumptions used for the simulated engine were the same as those set forth for the hand-computed engine, and these assumptions are listed on page vii. The  $\alpha$  and  $\beta$  reservoirs, which were not specified for the hand-computed engine, were given volumes of 4 cubic feet per cylinder.

<u>Compressor Cylinder</u>	<u>Hand-Computed Engine</u>	<u>Simulated Engine</u>
Polytropic exponent for expansion and compression, $n$	1.38	1.38
Clearance stroke, $S_C$	.63 inches	.58 inches
Intake pressure, $P_0$	14.7 psi	14.7 psi
Intake temperature, $T_0$	540° R	540° R
Delivery pressure, $P_D$	58.8 psi	Variable
Piston Area, $A_C$	4.14 ft <sup>2</sup>	4.14 ft <sup>2</sup>
Stroke position at point 1 on cycle, $S_1$	9.63 inches	9.58 inches
Weight of air for process, ${}_1W_{C_2}$	.2325 lbs	.2325 lbs

<u>Power Cylinder</u>	<u>Hand-Computed Engine</u>	<u>Simulated Engine</u>
Polytropic exponent for compression process, $n$	1.3	1.3
Maximum cylinder pressure, $P_3-P_4$	1800 psi	1800 psi
Polytropic exponent for expansion process, $n$	1.2	1.2
Scavenge ratio, $\frac{W}{W_E}$	5.33	5.33
Piston area, $A_P$	0.31 ft <sup>2</sup>	0.31 ft <sup>2</sup>
Inner dead center	.58 inches	.58 inches

### Bounce Cylinder

Polytropic exponent for compression and expansion, $n$	1.4	1.4
Clearance stroke, $S_C$	1.58 inches	1.58 inches

### General

Piston assembly weight, $W$	252 lbs	252 lbs
Air delivered/cycle	.1875 lbs/ cyl/cycle	.1875 lbs/ cyl/cycle
Net horsepower	1000 hp	1000 hp
Volume of reservoir $\alpha$	Not specified	4 cu ft/cyl
Volume of reservoir $\beta$	Not specified	4 cu ft/cyl

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