Essays on Strategic Voting

by

Katalin I. Bognár

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Doctoral Committee:
Professor Tilman M. Börgers, Chair
Professor Scott Page
Associate Professor Emre Ozdenoren
Professor Hector A. Chade, Arizona State University

to my father

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Abstract

This dissertation consists of three chapters.

The first chapter, which is written jointly with Lones Smith presents a dynamic model of deliberation by two privately informed individuals. Even by assuming the coarsest possible language to communicate information among members, it is shown that the decision is ‘almost instantaneous’ when individuals have identical objectives. Despite the coarse syntax, the model also predicts that information aggregation can be quite effective.

The second chapter asks the question under what circumstances can a static voting mechanism aggregate dispersed information of committee members. I argue that whenever the voters are able to cast multiple votes, the quality of the joint decision increases. However, voting mechanisms are intrinsically additive ways of aggregating private information. This, naturally, is not a binding constraint if the private information is conditionally independent. However, if the ‘meaning’ of the private information depends on other members’ signals, i.e. the signals are conditionally correlated, then the joint decision by voting may be unsatisfactory. I relate this question to a representation problem in utility theory to derive abstract conditions on the joint signal distribution that are necessary and sufficient for efficient voting.

The final chapter proposes a game-theory model to study the relationship of margin and turnout in elections. Common sense suggest that any individual voter is more likely to participate in a closer election. In an equilibrium model, a closer election is a consequence of a shift in the preferences of the electorate. A change in preferences may result in a higher number of voters with strong opinions about the candidates, thus it may directly influence the number of participating voters. I show that a shift in the preferences of the electorate decreases the equilibrium margin and increases the equilibrium turnout, provided that the shift does not decrease the polarization of the preference distribution.

Chapter 1

Introduction

Decision-making in committees is quite common, such as in juries, tenure committees, boards of directors, professional panels of doctors or other experts. In the first two chapters of my dissertation I analyze joint decision-making in committees from a game-theory point of view. The purpose of a deliberation process can be both to reveal the preferences of the committee member, as well as to aggregate the members' private information. I primarily focus on the aspect information aggregation assuming that the committee members have aligned interests.

A seminal result on committee decisions, the Condorcet Jury Theorem, claims that a decision by the majority of a large group is better than the decision made by any of the individual members (Condorcet (1785)). This is simply because a group of people overall possesses more information and hence reaches a better conclusion than any of the individual members. A key assumption of Condorcet's result is that the committee members sincerely express their independent opinions. Contrasting this, the paper by Austen-Smith and Banks (1996) shows that strategic considerations by the committee members can corrupt the jointly made decision, even if the members' interests are aligned.

Intuitively, how well the committee members share their private information depends crucially on the ways in which the members are allowed to convey their information. A simple one-round voting process does not allow for a sophisticated communication. Hence one can expect that the inefficiency result by Austen-Smith and Banks (1996) is sensitive to modeling assumptions. Indeed, in the first two chapters of my dissertation, I suggest two different extensions of the voting model by Austen-Smith and Banks (1996) and I argue that as the committee members are able to communicate in a better way, the joint decision improves. Moreover, I show that whether the full information equivalent decision can be reached by a voting mechanism depends on the structure of the private information of the committee members.

In Chapter 1, I broaden the decision making process into a dynamic mechanism in which the committee members can consecutively state their opinion about which of the decisions they think to be better. The member who is the most insisting determines the final decision. In Chapter 2, I extend the mechanism by allowing multiple votes for each committee members and I show that multiple votes, similarly to the dynamic extension of the mechanism, allows for more sophisticated communication among committee members, hence the overall decision improves.

In the papers of Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1998), Duggan and Martinelli (2001) and others, it is assumed that the committee members have conditionally independent private information. However, the committee members are usually presented the same evidence during deliberation, hence it is plausible to assume that the members private information is correlated. In Chapter 2, I emphasize that, additionally to the insufficient means to communicate private information, the inefficiency of the joint-decision can be inherent in the structure of the joint private information of the committee members. Voting is an intrinsically additive method to aggregate private information. I show that voting is efficient if there is a way to transform the private signals to votes independently of everyone else's signal so that the sum of the votes represent all the relevant information dispersed among the committee members. Given a signal realization, the relevant information is summarized by the likelihood ratio. Whenever the signals are conditionally independent, the log-likelihood ratio of a signal profile, which is a monotone transformation of the likelihood ratio, is the sum of the log-likelihood ratios of the individual signals. Hence, there is a natural candidate for a voting strategy. However, committee members are usually presented the same evidence, therefore, it is plausible to assume that the private signals are conditionally correlated. In this case, the log-likelihood ratio does not have the above described additive property. Naturally, if there is another transformation of the signals to votes that has the above mentioned additive property then voting can be efficient. In Chapter 2, I discuss when such a transformation exists. More precisely, I provide a conditions on the structure of the private information of the committee members which is necessary and sufficient to be able conclude the efficient decision by a voting mechanism.

The final paper addresses how strategic voting affects the outcome of a private value election, namely the relationship between the election margin and the number of voters who participate. Common sense suggests that in closer elections the individual incentives to participate are stronger. A decreasing margin should increase the probability of any individual vote being pivotal. This is because rational voting decisions are exclusively based on the comparison between the benefits from voting if the vote is pivotal and the costs of voting. However, in an equilibrium model, a closer election is a consequence of a shift in the preferences of the electorate. A change in preferences may result in a higher number of voters with strong opinion about the candidates, thus it may directly influence the number of participating voters. In the language of game theory the question thus becomes whether the equilibrium turnout increases if the distribution of voters' preferences changes in way that causes margins to decrease. As noted by Krasa and Polborn (2008, footnote 13) this question is, in fact, an open question in the game theoretic literature on costly voting.

In an equilibrium model of rational voting with a finite population of voters the question that I have posed is difficult to address because of the intricacies of analyzing the comparative statics of the probability with which any vote is pivotal. I propose in this paper a simplified model in which the mapping that relates turnout and margin into voters' perception of their probability of being pivotal is exogenous. I construct this mapping so that it shares some properties with the exact relationship between turnout, margin, and probability of being pivotal in a model with a finite but large population of voters.

Within this model I show that a shift in the preferences of the electorate decreases the equilibrium margin and increases the equilibrium turnout, provided that the shift does not decrease the polarization of the preference distribution. Informally, polarization decreases if voters move from the extreme ends of the preference distribution to the center of the preference distribution. The condition that polarization must not decrease is restrictive. It is, of course, only sufficient. I do not prove that it is necessary, but I do show by means of an example that, if the condition is violated, the result need not be true. This may explain why the literature, such as the study by Blais (2000) cited above, finds exceptions from the rule that decreases in margin lead to higher turnout.

Chapter 2

We Can't Argue Forever

2.1 Introduction

Decision-making in committees is quite common, such as in juries, tenure cases, board of directors, professional panels of doctors or other experts. The purpose of deliberation is to aggregate the members' private information. One critical aspect of joint decision making is its time cost. We assume that individuals wish to make the best decision possible in the least amount of time. Unfortunately, people with different expertise might find it hard to efficiently share their information; they might use different terminology that makes it difficult to understand each other's argument. That is, we wish to analyze costly committee decision-making by like-minded individuals who are unable to simply put their private information 'on the table' as it were. In particular, we assume that individuals can only communicate their information by voting for one of the possible alternatives, i.e. they use the coarsest possible syntax in the situation.

We model deliberation of like-minded committees by a game of two players who need to pick one out of two available actions. Each player has private information on which action is more favorable but they are unable to share this information directly. The process starts with one of the players stating her opinion about the appropriate action to take. The other player can either agree or disagree with this opinion. In the first case the game is over and the corresponding action is executed. In the latter case the players enter a situation in which both of them insist on their own opinions until one of them gives in. Based on their private information players could possibly have different opinions about what to do but before they can act they must reach an agreement. By insisting on her initial choice a player prolongs the debate but also makes it more likely that her own opinion dictates the final decision. Realistically, debating is costly. Therefore, players need to trade off the costs of longer debate and the benefits of prolonging the decision.

Notice that we simplify the concepts of both the language and the argument. On the one hand we assume binary message space. On the other hand, detailed arguments are not considered in the model. The only act that can be convincing for the other player is sticking to a certain opinion.

In our model two individuals with identical preferences but divergent information cannot disagree for a long time on a binary decision, even in the case of the coarsest possible language structure. As the time interval between votes diminishes, the probability that the final decision is realized in any given real time tends to 1. This result about the length of the deliberation has the flavor of the Coase Conjecture.[1] During the deliberation process information is revealed in small bits form period to period. We show that as talking becomes more frequent in the game, almost efficient communication can develop, despite the coarse language. On the other hand, if we take this model to its limit and allow for continuous communication, the juror's incentives to reach a fast decision prevent information sharing in equilibrium, equilibrium simply does not exist. Our paper suggests that it is impossible to reconcile delays in committee decision-making with rational, like-minded individuals with small costs of communication. It should be evident that this result is achieved when we allow for more sophisticated communication. The only way to understand delay is by assuming that jury members entertain conflicting objectives.

We explore the monotone structure in the game and establish existence of equilibria. We think about the deliberation process as a war of attrition game between jurors with different opinions. This interpretation suggests that waiting in real time is necessary to signal the strength of opinion. Our finding that the decision is almost instantaneous reveals that this intuition is not correct.

In a discrete time incomplete information war of attrition game, there are plenty of unwanted asymmetric equilibria with early or immediate concession by one of the players. A similar multiplicity is found in our game; we demonstrate that a whole array of equilibria is present such that the communication between players terminates 'too' early. We characterize the entire set but for our result about efficient information aggregation, we focus on the equilibrium with the most efficient communication. Our results show that as the time between two decision rounds vanishes the decision can be very close to ex-post efficient. At first glance this finding is intuitive. The players' preferences are aligned and it seems that it is in both jurors' interests to find the best decision given available information. On the other hand, in our model the communication of the private information is restricted,

[1] See Gul et al. (1986).

as well as costly. Therefore, the effectiveness of the deliberation is not apparent. Players face a trade-off between refining information, thereby lengthening the decision process and concluding the debate at once with a poor quality decision. With positive waiting costs, players are always willing to sacrifice accuracy of the decision for a shorter debate. Our model shows that this distortion due to the costs is diminishing as the period length vanishes.

Related Literature. Our project sits between two main areas of research. We aim to characterize information exchange *and* decision making in small groups such as juries and panels of experts, etc. Hence, our paper associates with both the literature on communication in environments with uncertainty as well as the theory of strategic voting in small groups. Our title suggests a clear link to the paper by Geanakoplos and Polemarchakis (1982). They propose and discuss a sequential communication process that leads to a common posterior as it is emphasized by Aumann (1976). They fix a finite information partition and show that the repeated communication of the private posteriors leads to an agreement about the probability of a certain event. By contrast, we make no restriction on the information partition, and do not allow agents to fully communicate their posteriors. Rather, we let the players chose their own information partition, essentially, and demand that they communicate in a binary language. While the communication may last arbitrarily long, the players in our game do achieve an arbitrarily fast real-time agreement.

The above literature contends itself in finding that the different players finally agree on a common posterior. We go beyond the setup of Geanakoplos and Polemarchakis (1982) by explicitly modeling the decision that results from the deliberation. For this reason, our paper is related to the work by Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1998) on strategic voting in juries. Feddersen and Pesendorfer (1998) describe decision making as a one-round, and hence costless, voting. Surprisingly, they find that requiring unanimity to convict a defendant can lead to more convictions of innocent defendants than a majority vote. This is because sophisticated jurors do not naively vote to acquit if they believe a priori that the defendant is innocent. Rather, they realize that under the unanimity rule, their votes are only relevant if all the other jurors vote to convict and update their information accordingly. Although, the model we introduce in this paper focuses on two-member panels and we do not address the issue of contrasting voting rules, our results suggest that dynamic mechanisms are more apt than static ones to aggregate the bits of private information possessed by the group members. Few papers, such as Coughlan (2000) and Austen-Smith and Feddersen (2006), intend to model pre-voting communication and reiterate the above result of inferiority of unanimity rule. In their setup, pre-voting communication is cheap-talk: does not influence the outcome of the voting directly and also costless. On the contrary, we see the deliberation as costly repeated voting where the debate can end at any round with a decision implied.

The structure of the chapter is as follows. Section 2.2 describes the model. In Section 2.3, we establish monotonicity of equilibria and argue for existence. Section 2.4 investigates asymptotic properties of equilibria. We show that the decision is almost instantaneous and there are equilibria such that the private information of the juror is 'well' aggregated. In Section 2.5, we conclude our findings.

2.2 Model and Preliminary Results

2.2.1 The Dynamic Voting Game

We model the deliberation process of two jurors trying to reach a verdict about a defendant. We propose the following model of the situation.

Information. There are two states of the world (θ): the defendant is either guilty (G) or innocent (I). The common prior belief of G is 0.5. Both jurors have private information about the state of the world. They observe conditionally iid signals $\sigma^i \in \Sigma \subset \mathbb{R}$. The signal is informative about the state of the world and drawn from a commonly known distribution H^θ given state θ with density h^θ that is finite and strictly positive in the support. We also assume that the signal distribution has the strict monotone likelihood ratio property, namely $\ell(\sigma^i) \equiv \frac{h^I(\sigma^i)}{h^G(\sigma^i)}$ is strictly decreasing in σ^i. Observing a signal σ^i, a juror can update his information using Bayes rule. Her posterior is according to the following equation:

$$p(\sigma^i) = \frac{h^G(\sigma^i)}{h^G(\sigma^i) + h^I(\sigma^i)} = \frac{1}{1+\ell(\sigma^i)}.$$

Notice that the posterior is a function of the signal only. Therefore given the conditional distribution of σ we can derive the conditional distributions F^G and F^I of the private posterior. The strict monotone likelihood ratio assumption ensure that the densities f^G, f^I exist. Notice that the posterior distribution is such that seeing p a juror must really think that guilt has the probability p. Formally,

$$p = \frac{f^G(p)}{f^G(p) + f^I(p)} = \frac{1}{1+\ell(p)}$$

therefore it is necessary such $\ell(p) = \frac{1-p}{p}$. Also, this condition is sufficient since given F^θ

we can consider the private posterior as the signal itself.

In the rest of the paper we summarize the private information immediately by p^i, the private posterior of a juror i about state G.[2] Given that the signals σ^i were conditionally iid, that the jurors' posteriors are conditionally iid drawn from $F^{\theta}(p)$ with common support $supp(F^{\theta}(.)) = [\underline{p}, \overline{p}] \subset (0,1)$ for $\theta = G, I$. A support being a strict subset of the $(0,1)$ means that there is an upper bound on the informativeness of the signals. Finally, all these assumptions are common knowledge among the players. In what follows we refer to the private posterior (p^i) as the type of a juror.

Timing and Strategies. We consider a sequential model of deliberation. In the game that we propose, the jurors communicate their opinion about the necessary verdict in turns. Initially (at time zero), one of the jurors (juror 1) decides on either conviction (C) or acquittal (A). In the next period the other juror (juror 2) has the right to agree or disagree; an agreement ends the game with the verdict that they both support. In case of disagreement the first juror talks again; she can either agree with the other juror (by changing her own opinion) or disagree, etc. In our model jurors cannot choose when to talk, but are forced to take turns in stating their respective opinions. Denote by N_1 the (even) periods when the juror 1 moves and by N_2 the (odd) periods when the juror 2 moves. We can also refer to the jurors by their roles in the debate. These roles are pinned down by the first move. If the first vote is for acquittal then the juror 1 becomes the one who will argue for acquittal in future debate (*A-juror*) and juror 2 is the one who will argue for conviction (*C-juror*). Conversely, if the first vote is for conviction then the roles are switched. Note that just like in a "War of Attrition" game, players of different opinions hold out for a while and then eventually give in. Because of this similarity we refer to the two subgames[3] after the roles A-juror and C-juror are assigned as *the war of attrition subgames*. However, in contrast to a "War of Attrition", here the players share the same preferences over the outcomes, whereas preferences are opposed in a "War of Attrition".

The deliberation stops as soon as there is an agreement between the jurors about the necessary action to take. Moreover, the appropriate verdict is immediately delivered after the agreement. Hence, the only possible histories in the game, beside the null history, are histories of disagreement. Formally, a possible history is a sequence of actions $\{d_t\}_{t=0}^{T}$ such that $d_t \neq d_{t+1}$ where $d_t \in \{A, C\}$. The set of histories is denoted by $\mathscr{H}$, with generic element h^T. The actions at each voting round are: saying either acquit or convict. Therefore a strategy, that prescribes at any point in the game what to do, is a mapping that

[2] An example of valid posterior distributions is: $F^I(p) = 2p - p^2$ and $F^G(p) = p^2$.

[3] Notice that these are non-proper subgames since they do not start with a singleton information set but we believe that using this terminology here will not cause problem.

assigns to any possible history and type one of the possible actions. Formally a strategy is: $s^i : [\underline{p},\overline{p}] \times \mathscr{H} \to \{A,C\}$.[4]

It will be convenient to think about strategies in the war of attrition subgames in two alternative ways. Given assigned roles in the debate, we can define a function $\tau_D : [\underline{p},\overline{p}] \to \mathbb{N}$ that indicates the time when the juror gives up arguing (*stopping time*), where $D \in \{A,C\}$ marks the first vote in the debate. Alternatively, a strategy defines a partition of the type space such that the nth subset consists of all the types who stop arguing at period n. Formally, for all $n \in \mathbb{N}$, $\mathscr{T}_D(n) \subset [\underline{p},\overline{p}]$ stands for the set of the types who stop at period n. Also, it is useful to refer to the set of all the types who have not yet stopped at period n as $\mathscr{T}_{D+}(n)$.

Payoffs. We assume that both jurors have a *common interest* in finding the right verdict, i.e. convict if the defendant is guilty and acquit if the defendant is innocent. As it is standard in the literature, we normalize the payoffs of the correct decisions to zero while convicting an innocent defendant costs ρ and acquitting a guilty defendant costs $(1-\rho)$ for both jurors. We refer to these losses as *terminal costs*. The value ρ can be interpreted as the reasonable doubt to convict a defendant. If jurors attach probability higher than ρ to the event that the defendant is guilty, then they prefer convicting to acquitting. To have a non-trivial problem we assume that the lowest type ex-ante prefers acquittal while the highest type ex ante prefers conviction. Formally, $\rho \in [\underline{p},\overline{p}]$.

The jurors also face a positive *decision cost* that is $\kappa > 0$ per unit of time or $\Delta\kappa$ per time period where the length of a period is Δ.[5] The overall cost is the sum of the terminal and the decision costs. Finally, we assume that jurors are risk neutral and cost minimizing.

Before we derive the explicit formula for the total cost we introduce some further notation. First, let x and y be proxies for the type of juror 1 and juror 2, respectively. Recall that the jurors do not have fully revealing information about the state of the world. Therefore the quality of the verdict is evaluated in terms of all the available information. Recall that $\ell(p) = \frac{f^I(p)}{f^G(p)}$ is the likelihood ratio given p. Then by Bayes Rule, conditional independence, uniform prior and simple algebra the posterior of guilt given the private

[4]Note that considering only pure strategies is not fully justified at this point. To be precise, one would need to define mixed strategies. However, in the proof of Lemma 2.1 we show that given any strategy profile of the opponent, if a certain type of a juror is indifferent between the two actions given a history, then any other types strictly prefer either one or the other action given the same history. Since there are countably many different histories and only two possible actions, the consequence of this observation is that in any best response functions at most countably many type mixes. In a Bayesian game, however, the behavior on a zero-measure subset of the type space does not influence the payoffs. Thus, for any mixed best response strategy, there exists a payoff equivalent pure strategy of a juror. Therefore focusing on pure strategies is without loss of generality.

[5]Deliberation usually happens on a relatively short time horizon so we believe that modeling the consequent time cost as a per unit cost is much more appropriate than using discounting.

signals x and y is:

$$
\begin{aligned}
Prob(G|x,y) &= \frac{f^G(x,y)P(G)}{f^G(x,y)P(G)+f^I(x,y)P(I)} \\
&= \frac{f^G(x)f^G(y)P(G)}{f^G(x)f^G(y)P(G)+f^I(x)f^I(y)P(I)} \\
&= \frac{1}{1+\ell(x)\ell(y)}.
\end{aligned}
$$

Denote by $q(x,y)=\frac{1}{1+\ell(x)\ell(y)}$ this posterior. The expected terminal costs from a verdict given signal realizations (x,y) are: $v_A(x,y)=(1-\rho)q(x,y)$ and $v_C(x,y)=\rho(1-q(x,y))$. Consider the case when the jurors agree on acquittal. This decision is right if the defendant is innocent so the related terminal cost is zero. On the other hand, if the defendant is guilty, an event that has a chance of $q(x,y)$, then the acquittal is the wrong decision and therefore it costs $1-\rho$ to both jurors.

The following technical assumption helps the presentation of our results.

Assumption 2.1. *The information structure in the game is such that:*

$$v_C(\underline{p},\overline{p}) > v_A(\underline{p},\overline{p}) \Longleftrightarrow \rho > q(\underline{p},\overline{p})$$

i.e. in case of extreme opposite signals, acquittal is preferable to conviction.

Next we derive the the payoffs for a given type and an action of a juror. Fix a type realization p^i, the initial vote and a stopping time t for juror i and a strategy for juror j. The payoff relevant uncertainty for the juror i are: (i) the opponent's type and (ii) the state of the world. The opponent's type determines the actual time when she gives up debating which influences both the final verdict and the time of the decision. Given the verdict, the actual terminal cost is contingent on the state of the world. Notice that the total cost related to this strategy is necessarily finite: (i) the terminal cost is by definition less than 1 and (ii) the process concludes the latest at t, so the decision cost is at most $t\Delta\kappa$. Therefore the payoffs in this game are well defined. First we find the ex-post total cost (for a given state of the world and a type realization of the opponent) then we take expectations over the states and finally over the opponent's type. We focus only on the case when the first vote is A and to simplify notation we leave out the subscript referring to this.

- Denote by $V^i(t,p^j,\theta,\mathscr{T})$ the total cost for a juror i who quits the debate at t in the war of attrition subgame if the signal realizations is (p^i,p^j) and the state is θ. Due to the conditional independence of the jurors' signals the value of p^i does not enter this expression. Also, recall that we can represent a strategy as the partition it generates on the type space. Hence, $\mathscr{T}$ refers to the opponent's strategy in the expression.

- Denote by $V^i(t,p^i,p^j,\mathscr{T}) = \sum_\theta V^i(t,p^j,\theta,\mathscr{T})P(\theta|p^i,p^j)$, the expected cost if the state of the world is unknown.
- The total cost for a juror of type p^i who quits the debate at time t in the war of attrition subgame is:

$$V^i(t,p^i,\mathscr{T}) = \int_{\mathscr{T}^j(0)} V^i(t,p^j,p^i,\mathscr{T})\mathrm{d}F(p^j|p^i). \tag{2.1}$$

  where $\mathscr{T}_(0)$ stands for the typeset of juror j that is consistent with an initial A vote. Notice that if $j=2$ this refers the whole type space and for $j=1$ it refers to the types voting A at period zero. Also, $F(p^j|p^i)$ denotes the cdf of the opponent's type given own signal p^i.

By substituting for the total cost and using the proxies x and y for juror i's and j's signal, respectively, we get the followings:

$$\begin{aligned}
V^1(t,x,\mathscr{T}^2) &= \sum_{n\in N_2,t>n>0}\int_{\mathscr{T}^2(n)}[v_A(x,y)+n\Delta\kappa]\,\mathrm{d}F(y|x)+\int_{\mathscr{T}^2_+(t)}[v_C(x,y)+t\Delta\kappa]\,\mathrm{d}F(y|x)\\
V^2(t,y,\mathscr{T}^1) &= \sum_{n\in N_1,t>n>0}\int_{\mathscr{T}^1(n)}[v_C(x,y)+n\Delta\kappa]\,\mathrm{d}F(x|y)+\int_{\mathscr{T}^1_+(t)}[v_A(x,y)+t\Delta\kappa]\,\mathrm{d}F(x|y)
\end{aligned}$$

The first terms in the above equations represent the cost in case the opponent gives in at a period $n<t$ so the juror wins the argument. The second term represents the cost if the juror gives in first, i.e. the opponent has not yet stopped by period t.

For example, the total expected cost for the juror 1 if she is an A-juror with a type x who plans to hold out until t is the following: (i) if the opponent's type y is such that she stops prior to t then the realized verdict is A so the expected terminal cost is $v_A(x,y)$ and the decision cost is according to the opponent stopping time; (ii) if the opponent's type is such that she would only stop after t then the realized verdict is C with an expected terminal cost equal to $v_C(x,y)$ and the deliberation ends at t with a decision cost $t\Delta\kappa$.

Equilibrium Concept. We consider Perfect Bayesian Equilibria of the dynamic game of incomplete information as defined above.

2.2.2 Monotonicity

Next, we explore the monotone structure of the game. Monotonicity allows us to think about the strategies as vectors of the critical types.

We model the committee decision as a war of attrition game preceded by an initial round that decides the sides in the debate. Our intuition in this setup is that, after taking sides, a juror with a more extreme posterior waits longer before agreeing to the verdict favored by the opponent than a juror with a less extreme posterior. Notice that the jurors can choose between learning more about the posterior of the opponent or concluding the debate immediatly. This translates to a trade-off between obtaining information that possibly *improves the decision* and paying the *cost of the decision making*. We think that a juror with more extreme posterior finds it more valuable to refine the decision so is willing to 'pay' more for it.

First, we show that each juror's best response is monotone no matter the opponent's strategy. Results of this flavor usually follow from the single crossing property of the payoff function in action and type. We can phrase this property more intuitively: if players are arranged according their types then if a player 'weakly prefers' an action to an other action that implies that anyone with 'higher' type must strictly prefer the same action compared to the other action.

This is not quite obvious in our story. First, consider a case when a juror prefers to stop at $n+2$ over stopping at n, i.e. her expected cost decreases by switching from n to $n+2$. By doing this, she changes the outcome of the voting with some probability and by assumption that change is favorable for her. On one hand, the same change in the outcome is more 'valuable' for a more extreme juror; on the other hand a more extreme juror might find it less likely that the change actually happens. Therefore, the overall effect on the expected cost is unclear. Fortunately, on balance we found a favorable effect.

A delay improves the quality of the verdict in one state of the world and worsens it in the other state of the world. For example, insisting on conviction adds to the probability that the final verdict is convict which is correct if the defendant is guilty but is incorrect if the defendant is innocent. Considering the necessarily increasing decision cost, a delay in the above example *might decrease* the total cost if the defendant is guilty and *definitely increases* total cost if the defendant is innocent. A juror finds it profitable to insist on convict if (i) the drop in terminal cost is worthwhile the waiting costs in case of a guilty defendant and if (ii) she is convinced enough of the guilt of the defendant. If both conditions hold then all the types who are even more optimistic about guilt prefer to delay as well. Therefore a type with a higher posterior will optimally hold out for at least as long as a one with a lower posterior. The key assumption for the monotonicity argument is that the signals are affiliated, a condition which in this setup follows from assuming conditional iid signals.[6]

[6]The definition for affiliation is the following: for any $x' > x$ and $y' > y$, $f(x',y')f(x,y) \geq f(x',y)f(x,y')$. By conditional independence of the signals we can write: $f(x,y) = f^G(x)f^G(y)P(G) +$

The same reasoning is not valid for the period 0 decision. Holding out in the war of attrition subgame simply means that a juror repeats her vote, say for convict, and by doing so she increases the chance of that particular verdict. At period 0, on the other hand, whether a certain vote does increase the chance of the same verdict is unclear. Voting for convict instead of acquit at period 0 does not necessary make a convict verdict more likely since the final decision depends on the opponent's strategy as well. It is possible, although quite unintuitive, that the juror 2 considers a convict vote by the juror 1 at period 0 as a signal supporting innocence. Therefore she insists more on acquit than she would do in case of acquit as a first vote. This behavior of the juror 2 would induce the juror 1 to follow the unintuitive monotone strategy such as opening the debate with a convict vote if she is a low type and with an acquit vote is she is a high type. Finally, to finish up the argument, the above unintuitive best response of the juror 1 makes juror 2's belief consistent. Although we believe that the asymptotic properties of these irregular equilibria are aligned with the ones of the intuitive equilibria, we simply exclude them from the analysis.

Next we define our monotonicity concept and present the formal statement and proof. The definition describes that jurors with more extreme signal hold out longer or to put differently, all types who stop at a certain period must have a more extreme signal than the ones stopping prior to that period.

Definition 2.1 (Monotone Strategy)**.** *A strategy is* monotone *if more extreme types hold out longer. Formally, for a C-juror if*

$$\tau_C(p') \geq \tau_C(p) \quad \text{iff} \quad p' > p$$

and for an A-juror if

$$\tau_A(p') \geq \tau_A(p) \quad \text{iff} \quad p' < p.$$

The following statements ensure that we can refer equilibrium strategies by the vector of critical types that are *indifferent* between stopping and waiting an additional round.

Lemma 2.1 (Monotonicity)**.**

1. *In the war of attrition subgame, for any strategy of juror j, the best response of juror i is monotone.*
2. *All equilibria in a war of attrition subgame are such that if there are types who give in at time t then there are either types who give in at time t' for all $t' < t$ or the verdict is delivered with certainty by t.*

$f^I(x)f^I(y)P(I) = f^G(x)f^G(y)P(G)[1+\ell(x)\ell(y)\ell_0]$. Therefore the signals are affiliated if $[1+\ell(x')\ell(y')\ell_0][1+\ell(x)\ell(y)\ell_0] \geq [1+\ell(x')\ell(y)\ell_0][1+\ell(x)\ell(y')\ell_0]$ equivalently $\ell(x')\ell(y')+\ell(x)\ell(y) \geq \ell(x')\ell(y)+\ell(x)\ell(y')$ and equivalently $[\ell(x)-\ell(x')][\ell(y)-\ell(y')] \geq 0$ which is an identity in our setup knowing that $\ell(p) = \frac{1-p}{p}$ so that $p' > p \Rightarrow \ell(p) > \ell(p')$.

Proof. 1. Without loss of generality, we give the proof for an A initial vote and to simplify notation, we abandon the subscript referring to this. Recall that the total cost for juror i with a type p^i who quits at t is:

$$V^i(t, p^i, \mathscr{T}) = \int V^i(t, p^j, p^i, \mathscr{T}) \mathrm{d}F(p^j | p^i).$$

where $\mathscr{T}$ refers to the opponent's fixed strategy. We show our result for a juror 2 who insists on conviction. All the other cases are similar. We assume a fixed strategy of the opponent characterized by the partition $\mathscr{T}$ but to simplify notation we will not have it apparent in the following expressions. Also, to reduce the number of confusing typos we replace the notation p^i and p^j and refer to the players private posterior as x and y for juror 1 and 2 respectively. Next, we make the dependence of the total cost on the state of the world more explicit:

$$\begin{aligned} V^2(t,y) &= \sum_\theta \int_{\mathscr{T}(0)} V^2(t,x,\theta,y) P(x,\theta|y) \mathrm{d}x \\ &= \sum_\theta \left(\int_{\mathscr{T}(0)} V^2(t,x,\theta,y) \mathrm{d}F^\theta(x) \right) P(\theta|y) \end{aligned} \tag{2.2}$$

where the second equality is due to the fact that the signals are iid conditional on the state. Also, $V^2(t,x,\theta,y)$ does not depend on y so from now on we only use $V^2(t,x,\theta)$. Then the change in the total cost if a juror holds out until t' instead of t where $t' > t$ can be decomposed in the following way:

$$\begin{aligned} V^2(t',y) - V^2(t,y) &= \sum_\theta \left(\int_{\mathscr{T}(0)} (V^2(t',x,\theta,y) - V^2(t,x,\theta,y)) \mathrm{d}F^\theta(x) \right) P(\theta|y) \\ &= \left(\int_{\mathscr{T}(0)} (V^2(t',x,I) - V^2(t,x,I)) \mathrm{d}F^I(x) \right) (1-y) \\ &+ \left(\int_{\mathscr{T}(0)} (V^2(t',x,G) - V^2(t,x,G)) \mathrm{d}F^G(x) \right) y. \end{aligned}$$

We show that if a type y prefers stopping at t' to stopping at t then all types $y' > y$ also prefers stopping at t' to stopping at t. Namely if

$$V^2(t',y) - V^2(t,y) \leq 0$$

i.e. the expected cost decreases when quitting is delayed to a later period, then for all $y' > y$

$$V^2(t',y') - V^2(t,y') < 0.$$

A delay by a C juror increases expected cost if the state is I: the decision cost rises with some probability and also the incorrect, convict verdict is made with higher chance. Formally this means that:

$$\int_{\mathscr{T}(0)}(V^2(t',x,I)-V^2(t,x,I))\mathrm{d}F^I(x)>0.$$

By assumption, the difference in the total cost for y is non-positive, therefore

$$\int_{\mathscr{T}(0)}(V^2(t',x,G)-V^2(t,x,G))\mathrm{d}F^G(x)\leq 0.$$

In this way, we can conclude that the cost difference decreases in the own posterior if that posterior is high enough. If a delay is profitable for some types then it is profitable for all the more extreme types, hence stronger types optimally hold out at least as long as weaker types. Notice that this result does not rely on any special property of the other's strategy. Therefore any best response in a war of attrition subgame is monotone in the above defined sense.

2. We need to show that in equilibrium if $\mathscr{T}^i(t)\neq\emptyset$ then either $\forall t'<t$, $\mathscr{T}^i(t')\neq\emptyset$ or there exists an $t''<t$ such that $\mathscr{T}^j_+(t'')=\emptyset$.
We prove this by contradiction. We assume that $\mathscr{T}^i(t)\neq\emptyset$ and $\mathscr{T}^i(t-1)=\emptyset$ and show that this is impossible unless $\mathscr{T}^j_+(t-1)=\emptyset$. This will be sufficient to prove our statement that is stronger since we can repeat this step again and again.
Consider the value difference as a juror holds out until t instead of $t-2$. For all types who prefer stopping at t compared to $t-2$ this value difference must be negative, i.e. the total costs must decrease. By our assumption, there are types like that.
Recall the value difference conditional on the state. We can decompose this difference to the sum of the change in terminal cost and in the decision cost. Without loss of generality we again assume a juror 2 who insists on conviction.

$$\begin{aligned}\int_{\mathscr{T}^1(0)}(V^2(t,x,\theta)-V^2(t-2,x,\theta))\mathrm{d}F^\theta(x) &= \\ \int_{\mathscr{T}^1(t-1)}(u(C,\theta)-u(A,\theta)+\Delta\kappa)\mathrm{d}F^\theta(x) &+ \int_{\mathscr{T}^1_+(t-1)}2\Delta\kappa\mathrm{d}F^\theta(x).\end{aligned}$$

Notice that if the set $\mathscr{T}^1(t-1)$ is empty and the set $\mathscr{T}^1_+(t-1)$ is non-empty then the conditional value differences are both strictly positive and there is no type y such that the appropriate convex combination of those will lead to a negative value. Therefore no type can prefer stopping at t over stopping at $t-2$. □

Corollary 2.1. *All equilibria in a war of attrition subgame are in monotone strategies.*

We have demonstrated that all equilibria in the war of attrition subgame are monotone in the sense that if a type y weakly prefers to hold out with her convict vote instead of giving in at t then any types $y'>y$ strictly prefer to hold out with a convict vote at period t. Similarly, if a type x weakly prefers to hold out with her acquit vote instead of giving in at t then any types $x'<x$ strictly prefer to hold out at period t. Thus any best response

strategy can be represented by the vector of indifferent types.

A similar argument can show that the juror 1 at period zero follows a cutoff strategy in equilibrium. Given the strategies in the war of attrition subgames, we can calculate the payoff difference if the juror 1 votes for convict at period zero instead of acquit. Again, this payoff difference can be written as the weighted sum of the payoff differences given the state of the world. Since the conditional payoff differences do not depend on the juror 1's type, we can conclude that if a type x is indifferent then all types $x' < x$ strictly prefer one action while all types $x' > x$ strictly prefer the other action. However, here we cannot pin down the signs of the conditional payoff differences. Those depend on the actual strategies in each subgame. As we argued before, it may happen that all types $x' > x$ actually prefer to vote for acquit while all types $x' < x$ prefer to vote for convict, hence the period 0 strategy is counterintuitive.

In the remaining part of the paper, we only concentrate on equilibria such that the period 0 behavior is 'regular'.

Assumption 2.2. *We restrict attention to a subset of all Perfect Bayesian Equilibria such that at time zero the strategy is intuitive in the sense that higher types are voting for conviction.*

Corollary 2.2 (Vector Representation of the Strategies)**.** *All equilibria satisfying the intuitive assumption above, are monotone and can be referred to by the vector of critical types.*

Denote the vector of critical types by $\mathbf{p}$ such that the nth element, p_n is the indifferent type between giving in to the other's opinion at the nth period or holding out for two more periods in the hope of changing the decision. Moreover, p_0 is the type who obtains the same payoff saying A or C at period 0.

2.3 Equilibrium Analysis

In this part we demonstrate that there are multiple equilibria in the game if the jurors can move frequently enough, i.e. the period-length Δ is small enough. We start the analysis with providing a few examples of those equilibria. The way we suggest a sequence of equilibria is informative in terms of characterizing the entire set. During the deliberation, the jurors communicate the strength of their opinion and at the same time they learn about the opponent's type. Unfortunately, this communication can fail too soon along the debate, since either of the jurors prefers to rush into a decision if she believes that the other will stop reacting to information in the future rounds. In what follows, we explain the worst case scenario and then we suggest a possible way to improve on the communication such that it results in a 'better' equilibrium. After presenting the examples we turn to a general treatment of equilibria. We show that the jurors necessarily reach a verdict after finitely many rounds of communication.

2.3.1 Examples of Equilibria

Period-1 Equilibrium. First, we establish an equilibrium such that the decision is made with certainty by period 1. In this equilibrium, in period zero the juror 1 proposes her favored verdict based on her own information and insists on this verdict forever. Given this stubborn behavior the best response of juror 2 is to give in immediately as disagreeing would be a waste of time only. We can determine this equilibrium by finding the type (x_0) of juror 1 who is indifferent between voting for A and C in period 0. Figure 2.1 illustrates this equilibrium.

The above strategies can be formalized in the following way: $x_0 = x_n$ for all $n \in N_1$ and $y_1 = \underline{p}$ in the C-subgame and $y_1 = \overline{p}$ in the A-subgame. Given the strategy $\mathbf{x}$ of the juror, the strategy of the juror 2 is optimal since any further delay only increases the decision cost but does not change the verdict. The total cost for juror 1 with private posterior x is: $\int_{\underline{p}}^{\overline{p}} (1-\rho) q(x,y) \mathrm{d}F(y|x) + \Delta\kappa$ if she votes A and it is $\int_{\underline{p}}^{\overline{p}} \rho(1-q(x,y)) \mathrm{d}F(y|x) + \Delta\kappa$ if she votes C. Therefore, the critical type x_0 is defined by

$$\int_{\underline{p}}^{\overline{p}} (\rho - q(x_0,y)) \mathrm{d}F(y|x_0) = 0. \tag{2.3}$$

As the game is without delay, the value of x_0 is independent of the actual period length.

Lemma 2.2. *A unique period 1 equilibrium exists.*

Proof. I have shown the indifference condition for the juror 1 since it gives the idea how I approach further equilibria. However, there is an easier way to find a period 1 equilibrium. Notice that juror 2's action has no information content. Thus, for a juror 1 of type x, announcing acquit has the expected cost of $(1-\rho)x + \Delta\kappa$ while announcing convict has the expected cost of $\rho(1-x) + \Delta\kappa$. Hence, in the period-1 equilibrium the indifferent juror 1 has a signal equal to ρ and uniqueness naturally follows. □

Period-2 Equilibrium. Notice that the period-1 equilibrium is unfit to aggregate private information, i.e. the signal of the juror 2 is not at all considered for the final decision. We argue that this equilibrium can be 'refined' into a period-2 equilibrium if the period-length is sufficiently small. The necessity of an upper bound on the period-length for this result is quite intuitive. The jurors cannot possibly improve on the terminal cost by more than $\max\{q, 1-q\} < 1$. Therefore no one is willing to sacrifice more than 1 in decision costs to get a possibly better decision. Hence, if $\Delta > 1/\kappa$ then it is never optimal for the juror 2 to disagree. Next, we assume that the period-length is sufficiently short and we consider a refined strategy of the juror 2 such that types with high enough posteriors do not agree with a juror 1 supporting acquit but rather vote convict and insist on conviction forever. At the same time, the juror 1's strategy can be modified such that all the types who initially voted acquit can be convinced to switch to convict if the opponent is insisting on that decision. With appropriate critical types these strategies form an equilibrium. Figure 2.2 illustrates this example.

The above strategies can be formalized in the following way. For the juror 1, x_0 is the type who is indifferent between voting for A and C at period 0 and all $x_n = x_0$ in the C-subgame and all $x_n = \underline{p}$ in the A-subgame. For the juror 2, y_1 is the type who is indifferent between giving in and holding out at period 1 and all $y_n = \underline{p}$ in the C-subgame and all $y_n = y_1$ in the A-subgame. The values x_0 and y_1 form an equilibrium if they satisfy the following system of equations:

$$
\begin{aligned}
x_0: \quad & \int_{\underline{p}}^{y_1} (\rho - q(x_0, y))\mathrm{d}F(y|x_0) &= \quad & \Delta\kappa(1 - F(y_1|x_0)) \\
y_1: \quad & \int_{\underline{p}}^{x_0} (q(x, y_1) - \rho)\mathrm{d}F(x|y_1) &= \quad & \Delta\kappa F(x_0|y_1)
\end{aligned}
$$

The first equation characterizes the type of the juror 1 who is indifferent between voting for A or C in the initial period. If juror 2 follows the strategy described above then a C vote by juror 1 at period 0 triggers an immediate agreement so the convict verdict is delivered in period 1. On the other hand, an A vote at period 0 triggers (i) an agreement on acquittal in period 1 with a certain probability (equal to the probability that juror 2 has a lower posterior than y_1) and (ii) an agreement on conviction in period 2 with the complementary probability. Therefore, if juror 1 switches from C to A, (i) her action might change the verdict from convict to acquit if the juror 2 happens to have a posterior lower than y_1 and (ii) her action possibly extends the debate by one period if the juror 2 happens to have a posterior higher than y_1. We refer to the drop in the total cost due to the refined decision as the *marginal gain* while the additional decision cost due to the prolonged debate is the *marginal loss* associated to this choice.

Note that the critical type x_0 in this example is higher than the similar critical type in the period-1 equilibrium. This is due to the fact that the juror 2 insists on a convict verdict in cases of strong signals, hence reduces the risk of acquitting a guilty defendant. As a consequence the juror 1 does not need to be so concerned about this scenario as in the period-1 equilibrium.

We can interpret the second equation in a similar way. By switching from A to C, juror 2 (i) changes the final verdict if juror 1's type is lower than x_0 and (ii) extends the debate by one period if juror 1's type is lower than x_0.

Finally, we show that a Period 2 equilibrium exists for low enough Δ.[7]

Lemma 2.3. *A Period-2 equilibrium exists for sufficiently low period-length.*

Proof. First, we rewrite the indifference conditions.

$$\begin{aligned} x_0: \quad \rho F^I(y_1)(1-x_0) - (1-\rho)F^G(y_1)x_0 &= \Delta\kappa(1 - F(y_1|x_0)) \\ y_1: \quad (1-\rho)F^G(x_0)y_1 - \rho F^I(x_0)(1-y_1) &= \Delta\kappa F(x_0|y_1). \end{aligned}$$

Then, using the fact that $F(p^i|p^j) = F^G(p^i)p^j + F^I(p^i)(1-p^j)$ we can further modify the system above:

$$\begin{aligned} x_0: \quad (\rho+\Delta\kappa)F^I(y_1)(1-x_0) - (1-(\rho+\Delta\kappa)F^G(y_1)x_0 &= \Delta\kappa \\ y_1: \quad (1-(\rho+\Delta\kappa))F^G(x_0)y_1 - (\rho+\Delta\kappa)F^I(x_0)(1-y_1) &= 0. \end{aligned}$$

The first equation implicitly defines the best response of juror 1 given a strategy of the juror 2 (BR_x) while the second equation implicitly defines the best response of juror 2 given a strategy of the juror 1 (BR_y). The best responses are continuous. To prove existence, we show that (i) $BR_x(\overline{p}) > BR_y^{-1}(\overline{p})$ and (ii) if we define x and y as the 'fixed points' of the best responses, i.e. $BR_x(x) = x$ and $BR_y(y) = y$ then $x < y$. Then, by the continuity of the best response functions the existence of a period-2 equilibrium is implied.

If $y_1 = \overline{p}$ then the indifferent type of juror 1 is $x_0 = \rho$. Also to make $y_1 = \overline{p}$ to be the critical type of juror 2, x_0 must satisfy: $\ell(p < x_0)\ell(\overline{p}) = \ell(\rho+\Delta\kappa)$, where $\ell(p < x) \equiv \frac{F^I(x)}{F^G(x)}$. By Assumption 2.1 for low enough $\Delta\kappa$, an x_0 like this exists and is less than ρ.

Define x and y such that

$$\begin{aligned} (\rho+\Delta\kappa)F^I(x)(1-x) - (1-(\rho+\Delta\kappa)F^G(x)x &= \Delta\kappa \\ (1-(\rho+\Delta\kappa))F^G(y)y - (\rho+\Delta\kappa)F^I(y)(1-y) &= 0 \end{aligned}$$

[7] Unfortunately, we were not able to show the uniqueness of this type of equilibrium.

Dividing the equations by $F^G(x)x$ and $F^G(y)y$ respectively and summing them up we get that

$$(\rho+\Delta\kappa)(\ell(p<x)\ell(x)-\ell(p<y)\ell(y))=\frac{\Delta\kappa}{F^G(x)x}>0$$

Therefore $\ell(p<x)\ell(x)-\ell(p<y)\ell(y)>0$ and this implies that $x<y$ since the likelihood ratio is decreasing. $\square$

Period-3 Equilibrium. For short enough period-length we can further refine the period-2 equilibrium. Notice that in the period-1 equilibrium, the juror 1 declares her opinion and insists on it forever. In the period-2 equilibrium there is a similar persistence in the behavior of the juror 2. Once she casts a vote for conviction she insists to it forever. To construct a period-3 equilibrium, we make the juror 2's behavior more cooperative and instead we assume that there are certain types of juror 1 who hold on their initial vote forever. In particular, we assume that a juror 2 might disagree in period 1 but definitely gives in at period 3 latest. At the same time a juror 1 will not change her opinion after period 2. These strategies can be formalized by determining the critical values: x_2, x_0 and y_1. The following conditions specify those values (yet different from the values of x_0 and y_1 in the above examples).

$$\begin{aligned}
\int_{y_1}^{\overline{p}}(\rho-q(x_2,y))\mathrm{d}F(y|x_2) &= \Delta\kappa(1-F(y_1|x_2))\\
\int_{x_2}^{x_0}(q(x,y_1)-\rho)\mathrm{d}F(x|y_1) &= \Delta\kappa[F(x_0|y_1)+F(x_2|y_1)]\\
\int_{\underline{p}}^{y_1}(\rho-q(x_0,y))\mathrm{d}F(y|x_0) &= \Delta\kappa(1-F((y_1|x_0))
\end{aligned}$$

One can interpret these equations in a similar way than the indifference conditions before. They ensure that the additional gain from switching ones vote at a certain period is equal to the extra costs due the switch. Figure 2.3 illustrates the period-3 equilibria.

Our examples suggest that for short enough period-length, there can be equilibria in which the verdict is delivered by the nth period. Namely, at least one of the juror's strategy is such that by n all the types are quitting the debate. A strategy of this kind can be optimal (i) if the juror believes that there is no way to convince the opponent, i.e. she is simply stubborn or (ii) if the juror is already very much convinced that the opponent's opinion is correct. There is a normative difference between these two situations. In the first scenario, although both jurors are aware that there is information out there that should be considered (given the cost of communication) they do not do it. Simply put, one juror's strategy is insensitive to information (*stubborn*) and therefore the other juror does not provide information just agrees (*accommodate*) and thereby concludes the debate. In the second scenario, all the information that is profitable to consider is considered.

Moreover, starting from an equilibrium in which communication collapses too early, it is possible to introduce additional informative rounds in the following way. First, we relax the stubbornness of the appropriate juror, then we allow for best response iteration. This process may converge to a new equilibrium such that the number of rounds are increasing. This argument holds in the war of attrition subgame starting with am A vote or with s C vote or even in both subgames at the same time. Refining an equilibrium is possible again and again until we reach detailed enough communication. Next, we formalize the above intuition about the *finite monotone strategies*. Let $\mathbf{x}$ and $\mathbf{y}$ denote a strategy of juror 1 and 2, respectively. A certain *type* of juror is stubborn if that particular type insists on one of the actions all along the game and there is no history convincing enough to change her vote. This term is defined for a realization of the signal so it is possible that some types of a juror behave stubborn while others do not. We call a *strategy* stubborn if there are types who cannot be convinced. We classify stubborn strategies according to the time period when they become insensitive to information provided by the opponent. A strategy is accommodating if at some period all the types give in to the opponent. We classify accommodating strategies according to their 'length' as well.

Definition 2.2 (Stubborn / Accommodating Strategy)**.** *We define the following finite strategies in the game:*

- *We call a monotone strategy n-*accommodating *if any type of the juror insist at most until period n. Formally, if $p_{n-2} < p_n = p_{n+2} \cdots = \overline{p}$ (or $\underline{p}$).*
- *We call a monotone strategy m-*stubborn *if the juror never changes her opinion after period m. Formally if $p_{m-4} < p_{m-2} = p_m = p_{m+2} \cdots < \overline{p}$ (or $\underline{p}$).*

Using these strategies we can characterize the finite equilibria of the dynamic voting game.

Definition 2.3 (Finite Equilibria)**.** *We call an equilibrium* period-n equilibrium *if one player plays an $(n+1)$-stubborn strategy while the other plays n-accommodating strategy. A period-n equilibrium produces verdict by period n with certainty.*

Although in general, we refer to the strategies as an infinite dimensional vector of the critical types, when talking about the m-stubborn or the n-accommodating strategies, we only mention the distinct critical types.

2.3.2 Properties of the Equilibria

A type y of juror 2 is indifferent at period n, such that $n \in N_2$ if

$$\int_{x_{n+1}}^{x_{n-1}} (q(x,y_n) - \rho)\mathrm{d}F_{n-1}(x|y_n) = \Delta\kappa + \Delta\kappa F_{n-1}(x_{n+1}|y_n).$$

where the $F_{n-1}(p^1|p^2) \equiv F(p^1|p^2, p^1 < x_{n-1})$. This equation contrasts the additional gains and losses related to a further delay at a certain period n. A delay has an effect on the final verdict if and only if the opponent happens to be one of the types who are just about to give in. On the flip side, the decision cost increases for sure by a delay. Given the signal realizations x, y, the gain from changing the verdict can be represented as the difference between the reasonable doubt and the public posterior. Particularly, the gain if the verdict changes form A to C is: $v_A(x,y) - v_C(x,y) = q(x,y) - \rho$ while if the verdict changes from C to A, the it is: $v_C(x,y) - v_A(x,y) = \rho - q(x,y)$. The total gain for a juror is the expected value of this function with respect to the opponent's type, x. The increase in the decision cost is more straightforward. It is $\Delta\kappa$ for sure and an additional $\Delta\kappa$ if the opponent does not give in immediately.

A sequence of similar indifference conditions characterize the equilibria. We argue that the equilibrium strategies must be 'embedded' in the sense that types whose private information offsets each other should give in in consecutive periods. Intuitively, there is no reason to hold out if it is sure that the opponent has an extreme enough signal to make her preferred decision better overall. The flip side of this coin is that costly deliberation induces jurors to stop before a 'sufficient' amount of information is acquired for doing so. To maintain the equilibrium, each juror should provide enough incentive for the opponent not to give in too early. This intuition is formalized below.

Definition 2.4 (Signal Strength)**.** *A signal x is* at least as A-strong as *y or $x \succeq_A y$, if $q(x,y) \leq \rho$. In other words, if x is A-stronger than y then the jurors prefer acquittal ex-post. Similarly, a signal y is* at least as C-strong *as x or $y \succeq_C x$, if $q(x,y) \geq \rho$. In other words, if y is C-stronger than x then the jurors prefer conviction ex-post.*[8]

We show that in equilibrium the cutoff type at period n cannot be stronger, in the sense of the definition above, than the one in period $n+1$. Not having this, allows for profitable deviation, the indifferent type at n will strictly prefer to agree with the other juror. She is compelled to do so since waiting until $n+2$ increases both the terminal and the decision cost for her, thus it is strictly dominated. The following lemma proves this.

[8]Notice that equivalence in this ordering gives us the types those are offsetting each other.

Lemma 2.4. *For any Δ, in equilibrium $x_n \succ_A y_{n-1}$ $\forall n \in N_1$ and $y_n \succ_C x_{n-1}$ $\forall n \in N_2$.*

Proof. (by contradiction) We only prove that $y_n \succ_C x_{n-1}$, the other case is similar. Assume $y_n \preceq_C x_{n-1}$ where $n \in N_2$ and show that a type y_n strictly prefers stopping at n to $n+2$. Consider the indifference condition:

$$\int_{x_{n+1}}^{x_{n-1}} [q(x, y_n) - \rho] f(x|y_n) \mathrm{d}x = \Delta\kappa \left[F(x_{n+1}|y_n) + F(x_{n-1}|y_n)\right]$$

By monotonicity, $y_n \preceq_C x_{n-1}$ implies that $y_n \prec_C x_{n+1}$, so for any $x \in [x_{n-1}, x_{n+1}]$, $q(x, y_n) \leq \rho$. Therefore, the LHS is strictly negative, so y_n strictly prefers stopping at n. $\square$

Next, we show that the marginal gain for the indifferent C-juror is proportional to $|x_{n-1}^{\Delta} - x_{n+1}^{\Delta}|^2$, while the marginal cost is proportional to Δ. Hence, the indifference condition forces $|x_{n-1}^{\Delta} - x_{n+1}^{\Delta}|$ to approach zero slower than Δ. In the following, we need to index the critical types with the period-length.

Lemma 2.5. *For all $\alpha > 0$ there exists $\Delta_\alpha > 0$ such that $\frac{|x_{n-1}^{\Delta} - x_{n+1}^{\Delta}|}{\Delta} > \alpha$ for all $\Delta < \Delta_\alpha$, if $n \in N_2$.*

Proof. Let us start the proof with defining the type that offsets the highest posterior.

Definition 2.5. *For a given Δ, define ζ such that*

$$q(\zeta, \overline{p}) - \rho = 0. \tag{2.4}$$

By the Assumption 2.1, the value $\zeta > \underline{p}$ exists.

By Lemma 2.4, we know that in equilibrium $x_{n+1}^{\Delta} \succeq_A y_n^{\Delta} \succeq_C x_{n-1}^{\Delta}$, $\forall n \in N_2$. So there exists $\xi_n^{\Delta} \in (x_{n+1}^{\Delta}, x_{n-1}^{\Delta})$ such that $q(\xi_n^{\Delta}, y_n^{\Delta}) = \rho$. By the definition of y_n^{Δ}, the following must hold:

$$\int_{x_{n+1}^{\Delta}}^{x_{n-1}^{\Delta}} \left[q(x, y_n^{\Delta}) - \rho\right] f(x|y_n^{\Delta}) \mathrm{d}x = \Delta\kappa \left[F(x_{n-1}^{\Delta}|y_n^{\Delta}) + F(x_{n+1}^{\Delta}|y_n^{\Delta})\right]. \tag{2.5}$$

The LHS of (2.5) gives us the marginal gain from waiting, i.e. how much the terminal cost decreases in expectation while the RHS of (2.5) describes the expected increase in decision cost. Next, we compare them and an find upper bound for the LHS and a lower bound for the RHS. Notice that for low x, $q(x, y_n^{\Delta}) < \rho$. So the terminal cost actually increases by the delay and the marginal gain is negative, while for high x, the opposite holds. It turns out to be convenient to separate these effects. Using the definition of ξ_n^Δ, the marginal gain $\int_{x_{n+1}^\Delta}^{x_{n-1}^\Delta}\left[q(x,y_n^\Delta)-\rho\right]f(x|y_n^\Delta)\mathrm{d}x$ equals:

$$
\begin{aligned}
&\int_{x_{n+1}^\Delta}^{\xi_n^\Delta}\left[q(x,y_n^\Delta)-q(\xi_n^\Delta,y_n^\Delta)\right]f(x|y_n^\Delta)\mathrm{d}x \quad + \quad \int_{\xi_n^\Delta}^{x_{n-1}^\Delta}\left[q(x,y_n^\Delta)-q(\xi_n^\Delta,y_n^\Delta)\right]f(x|y_n^\Delta)\mathrm{d}x \\
&= \int_{\xi_n^\Delta}^{x_{n-1}^\Delta}\left[\int_{\xi_n^\Delta}^{x} q_x(\xi,y_n^\Delta)d\xi\right]f(x|y_n^\Delta)\mathrm{d}x \quad - \quad \int_{x_{n+1}^\Delta}^{\xi_n^\Delta}\left[\int_{x}^{\xi_n^\Delta} q_x(\xi,y_n^\Delta)d\xi\right]f(x|y_n^\Delta)\mathrm{d}x \\
&< \int_{\xi_n^\Delta}^{x_{n-1}^\Delta}\left[\int_{\xi_n^\Delta}^{x} q_x(\xi,y_n^\Delta)d\xi\right]f(x|y_n^\Delta)\mathrm{d}x \quad < \quad \int_{\xi_n^\Delta}^{x_{n-1}^\Delta}\bar{q}_x(x_{n-1}^\Delta-\xi_n^\Delta)\bar{f}\mathrm{d}x \\
&\quad < \bar{q}_x\bar{f}(x_{n-1}^\Delta-\xi_n^\Delta)^2
\end{aligned}
$$

where first we used $q_x(x,y)>0$ and then that $q_x(x,y)$ and $f(x|y)$ are bounded above for $x,y\in[\underline{p},\overline{p}]$, with bounds $\bar{q}_x$ and $\bar{f}$, respectively.

Also, for the marginal cost (RHS of (2.5))

$$\Delta\kappa\left[F(x_{n-1}^\Delta|y_n^\Delta)+F(x_{n+1}^\Delta|y_n^\Delta)\right]>\Delta\kappa 2\underline{F}$$

where $\underline{F}$ is a lower bound for $F(x|y)$, this exists since $x_t^\Delta>\zeta$.

Therefore, we can conclude that the upper bound for the marginal benefit is bigger than the lower bound for the marginal costs.

$$\bar{q}_x\bar{f}(x_{n-1}^\Delta-\xi_n^\Delta)^2>\Delta\kappa 2\underline{F}.$$

Hence, we find that

$$\frac{|x_{n-1}^\Delta-x_{n+1}^\Delta|}{\Delta}>\frac{|x_{n-1}^\Delta-\xi_n^\Delta|}{\Delta}>\frac{A}{\sqrt{\Delta}}, \tag{2.6}$$

where $A^2=\frac{2\kappa\underline{F}}{\bar{f}\bar{q}_x}>0$ and the first inequality follows by Lemma 2.4. □

Lemma 2.5 shows that in equilibrium two consecutive critical types are well apart. Next we provide an alternative approach for the existence of equilibria and using the fact that the game is common valued and that all equilibria are finite.

Theorem 2.1. *A strategy profile that minimizes the expected costs is a Bayes - Nash Equilibrium and such a profile exists for any Δ.*

Proof. PART 1: In any common value game a welfare maximizing strategy is necessary equilibrium. Assume on the contrary that given a welfare maximizing profile, there is an alternative strategy of a player that leads to higher payoff for her. Then, the common welfare increased and that is a contradiction.

PART 2: By Lemma 2.5, for any Δ there exists a constant A such that the distance between two consecutive critical type is at least $A\sqrt{\Delta}$. Hence, the number of the deliberative rounds is necessary less than $\frac{1}{A\sqrt{\Delta}}$. Therefor we restrict attention to finite strategies when searching for a cost minimizing strategy profile. Finite strategies can be represented by finite dimensional vectors. Hence, any cost minimizing profile is a solution of a finite dimensional minimization problem and such as it necessarily exists. □

2.4 Asymptotic Properties

After describing the equilibrium set, we show the asymptotic properties of the equilibria as the period-length approaches zero. Our results discuss the real time length of the debate as well as the quality of the decision made as the communication gets more frequent. We are interested (i) if a real time waiting is necessary to signal the strength of ones opinion and (ii) whether the deliberation process sufficiently aggregates private information. Our conclusion is that the jurors can signal their information effectively during a very short period of time. This implies that even in the equilibrium in which the deliberation takes the longest time the verdict is delivered quite fast. We also demonstrate that there is an equilibrium that leads to a decision close to the ex-post efficient one.

Earlier we conceptualized a strategy as a partition implied on the signal space. By the monotonicity of any equilibrium strategies, the subsets in the partition are intervals bounded by the consecutive indifferent types. Intuitively, the frequency of those critical types displays the detailedness of the communication in equilibrium. Therefore, how fine does the equilibrium partition become as the period-length vanishes reveals our asymptotic results. We argue that as Δ goes to zero the partition might become detailed but at a lower rate than Δ.

2.4.1 The Length of the Deliberation

We aim to estimate the length of deliberation, i.e. the length of time that elapses before the conclusion is made. We find, in spirit of the Coase Conjecture, that as the time-interval shrinks the delay vanishes too. Formally, we demonstrate that by any real time T, the decision has been made by this time if the period length is small enough.

Proposition 2.1. *For any $T > 0$ there exists a period-length $\Delta_T > 0$ such that the jurors reach a verdict with certainty by T whenever the period length $\Delta < \Delta_T$.*

Proof. By Lemma 2.5, there exists a constant A such that the distance between consecutive indifference types is at least $A\sqrt{\Delta}$, hence the number of the deliberative rounds is necessarily less than $\frac{1}{A\sqrt{\Delta}}$. Thus the total time to verdict is less than $\frac{\sqrt{\Delta}}{A}$. Consequently, for a period-length shorter than T^2A^2 the verdict is reached in T real time. □

2.4.2 The Efficiency of the Verdict

In this section, we evaluate the deliberation process in terms of how well it aggregates the private information that is available to the jurors. As it is shown earlier in the paper, there are multiple equilibria in the game. We emphasized that in most of them the communication terminates too soon. For certain signal realizations the jurors rush into the decision without being sufficiently convinced about the correctness of the final verdict because they believe that arguing is just a waste of time. Certainly, those equilibria do not lead to good information aggregation.

We argue that, at the same time, there exists an equilibrium that aggregates information quite effectively. This equilibrium is special in the sense that it is without communication break-down, i.e. at the time of giving in the highest possible type is quite convinced that the opponent votes for the correct decision.

Importantly, this feature forces all the critical types to be very close to be convinced at the point of giving in. Due to the aligned interests of the jurors this ensures that the decision is likely to be very good.

We call a decision ex-post efficient if the jurors conclude the verdict they would choose had all the private information been public.

Definition 2.6 (Efficiency)**.** *The verdict $d(x,y)$ is* ex-post efficient *if*

$$d(x,y) = \begin{cases} A & \text{if} \quad q(x,y) < \rho \\ A \text{ or } C & \text{if} \quad q(x,y) = \rho \\ C & \text{if} \quad q(x,y) > \rho. \end{cases}$$

Note that the ex-post efficient decision may still be mistaken since the true state of the world is not known to the jurors. However, this is a risk that the jurors cannot avoid since their information is never fully revealing. Also, for every signal realization, except from a zero-measure subset, there is a unique ex-post decision. Hence, the sets of ex-post and ex-ante decision rules coincide. We show that the verdict can be asymptotically efficient as the period-length vanishes, in the sense that the probability of the signal realizations for which the verdict is not efficient goes to zero. In another way, the expected welfare loss due to insufficient information aggregation goes to zero. However, this does not mean that for every possible type realizations the verdict is necessarily the efficient one.

The following lemma is technical and helps in the proof of the main theorem of this section.

Lemma 2.6. *There exists a constant A so that for all X,Y such that $X=[\underline{x},\overline{x}]$ and $Y=[\underline{y},\overline{y}]$ and $X,Y\subset[\underline{p},\overline{p}]$:*

$$\int_{X\times Y} f(x,y)dxdy \le A\int_X f(x)dx\int_Y f(y)dy.$$

Proof. Recall that the conditional densities are bounded. Denote by k and K these bounds, i.e.for all $p\in[\underline{p},\overline{p}]$, $f^\theta(p)\in[k,K]$. Then,

$$\int_{X\times Y} f(x,y)\mathrm{d}x\mathrm{d}y=\int_{X\times Y}\left(\sum_\theta f^\theta(x)f^\theta(y)P(\theta)\right)\mathrm{d}x\mathrm{d}y\le K^2(\overline{x}-\underline{x})(\overline{y}-\underline{y}).$$

Also,

$$\int_X f(x)\mathrm{d}x\int_Y f(y)\mathrm{d}y=\int_X\left(\sum_\theta f^\theta(x)P(\theta)\right)\mathrm{d}x\int_Y\left(\sum_\theta f^\theta(y)P(\theta)\right)\mathrm{d}y\ge k^2(\overline{x}-\underline{x})(\overline{y}-\underline{y}).$$

Therefore,

$$\int_{X\times Y} f(x,y)\mathrm{d}x\mathrm{d}y\le\frac{K^2}{k^2}\int_X f(x)\mathrm{d}x\int_Y f(y)\mathrm{d}y.$$

□

Finally, we state the result about the quality of the decision reached in the equilibria that takes the longest time.

Proposition 2.2. *For all $\mu>0$, there exists a period-length Δ_μ such that the expected welfare loss in the decision process is less than μ if $\Delta\le\Delta_\mu$.*

Proof. We suggest a feasible strategy profile that generates a verdict that is already quite close to the ex-post efficient one. Therefore, there must be an equilibrium profile that is at least as good. Given Δ define a strategy $\mathbf{x}$ for the juror 1 in the following way. For all $n\le\frac{1}{\Delta}$ and $n\in N_1$, let $F(x_n)=n\Delta$ and define ξ_n such that $q(x_n,\xi_n)=\rho$. Then, define the juror 2's strategy $\mathbf{y}$ such that for all $n\in N_2$, $F(y_n)=\frac{F(\xi_{n-1})+F(\xi_{n+1})}{2}$ and define ξ_n such that $q(\xi_n,y_n)=\rho$. Consider the outcome function generated by the strategies $\mathbf{x}$ and $\mathbf{y}$.

By Lemma 2.6, the probability of an inefficient verdict is not more than

$$\frac{K^2}{k^2}\sum_n\left(\frac{(F(y_n)-F(\xi_{n-1}))(F(\xi_n)-F(x_{n-1}))}{2}+\frac{(F(\xi_n)-F(y_{n-1}))(F(x_n)-F(\xi_{n-1}))}{2}\right)$$
$$\le \frac{\Delta}{2}\frac{K^2}{k^2}.$$

Since the loss due to a mistaken decision is certainly less than 1, the welfare loss is bounded by $\frac{\Delta}{2}\frac{K^2}{k^2}$ which approaches zero as $\Delta \to 0$. □

2.5 Conclusion

In this paper we proposed a dynamic model of deliberation in a panel of two like-minded jurors. We assumed positive time costs of the decision making. We explored the monotone properties of the game and described the set of equilibria. Our main contribution was to show that in a deliberation process with positive time-cost the verdict is almost instantaneous and approximately information efficient as the period-length vanishes. This result suggests that dynamic models of group decision making are able to properly aggregate members' private information.

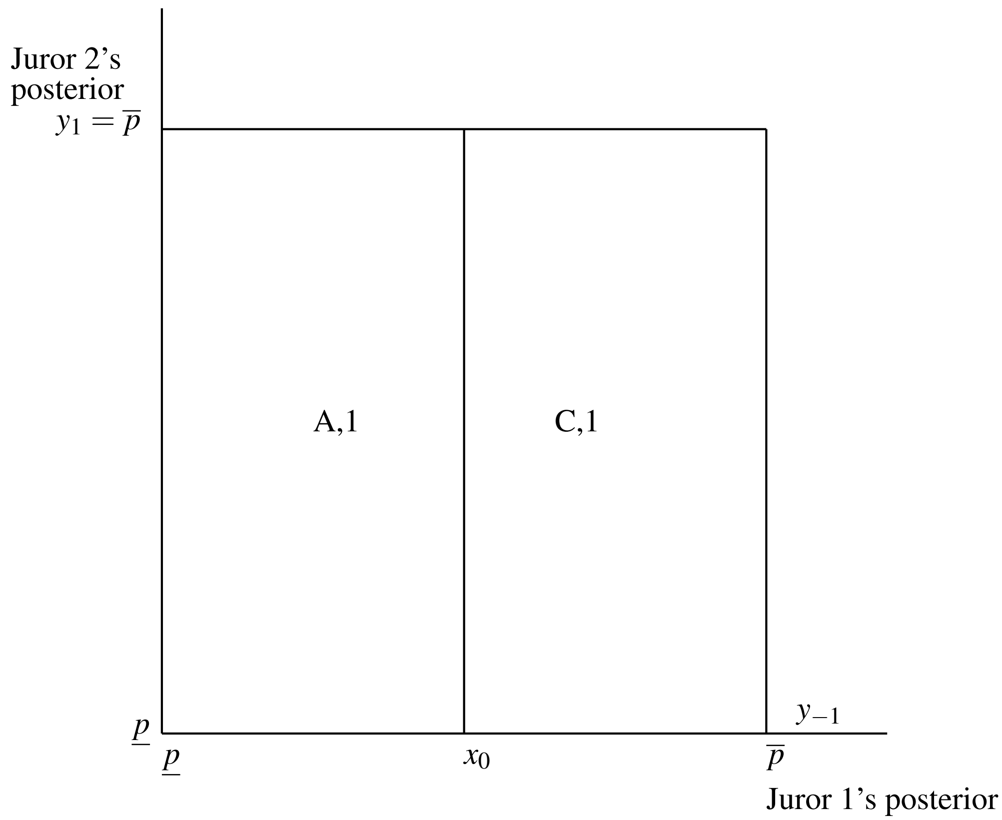


Figure 2.1 Period-1 Equilibrium. We indicate the type of the juror 1 on the horizontal axis while the type of the juror 2 is measured on the vertical axis. We can represent any signal realization as a point in the coordinates. The cutoff types are shown on the axis. The realized verdict and the time of the decision are marked as well. This figure shows an equilibrium in which only juror 1's information matters for the final verdict. She follows a strategy such that type x_0 is the critical type at the initial round. All the types that are lower than x_0 say acquit no matter what the history is while all higher than x_0 say convict no matter what the history is. Any type of juror 2 gives in immediately. We think about this situation as a very poor way of aggregating the private information available for the jurors. Juror 1 states her opinion based on her own signal and is not willing to 'listen' to juror 2. As a best response, juror 2 does not reveal information but terminates the game immediately. Therefore, juror 1's strategy is best response. (y_1 and y_{-1} are the critical types for the second juror if she is a C- or an A-juror, respectively). Notice that here $y_1 =$ "highest possible type" exactly means that all the possible types of juror 2 give in for acquit immediately if juror 1 voted for acquit initially. We can interpret $y_{-1} =$ lowest possible type similarly, i.e. all the possible types of juror 2 gives in for convict if juror 1 voted so.

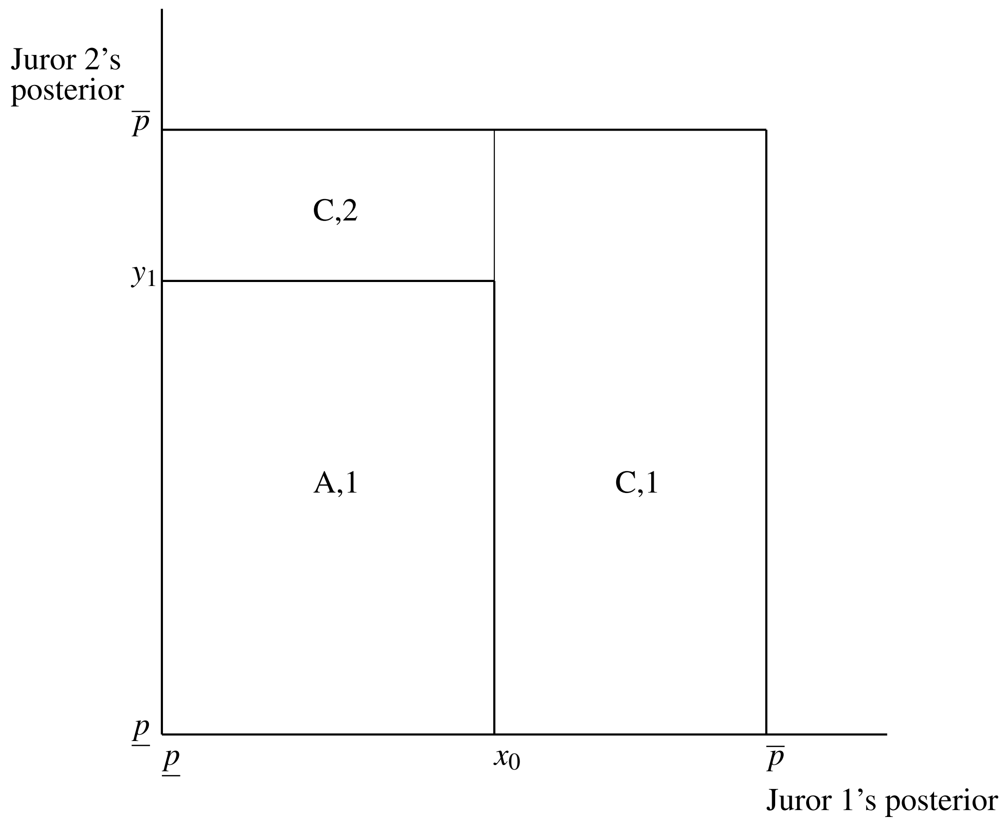


Figure 2.2 Period-2 Equilibrium. This figure shows an equilibrium in which the communication is improved. In this new equilibrium the strategy of juror 2 is more informative than before in the sense that not all the types give in to acquit immediately at period 1. The type y_1 of juror 2 is critical, all the types who find it more likely than y_1 that the defendant is guilty insist on convict. The type x_0 is again a critical type for the juror 1 and separates the types who say acquit at period 0 from the ones who say convict. Also, all the types of juror 1 who voted for acquit initially switch to convict if the opponent insists on convict.

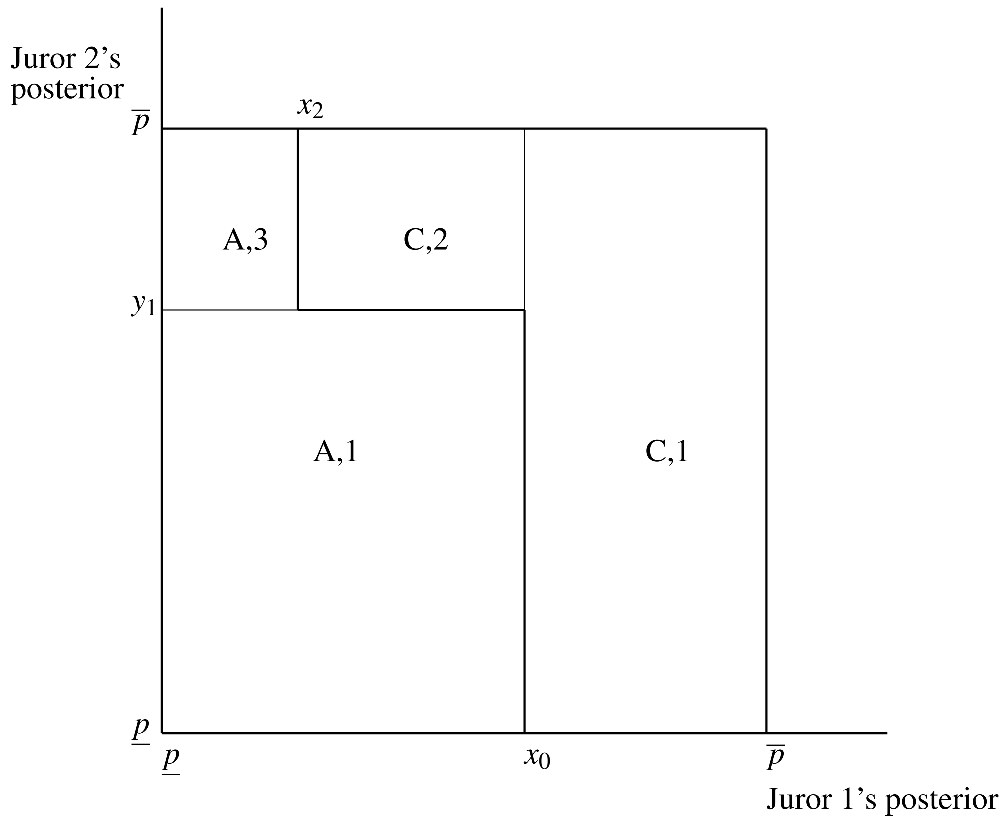


Figure 2.3 Period-3 Equilibrium. This figure shows yet another improvement of the previous equilibrium. Here, the juror 1's strategy reveals more information than before. Type x_0 is the critical type at period 0 and all the types in the range $[x_2, x_0]$ can be convinced by the action of the juror 2. Furthermore, all the types who are more extreme then x_2 insists on A. Concerning the juror 2, some types (ones with higher posterior than y_1) hold out in period 1 and give in at period 3 if the other juror has not yet done so.

Chapter 3

Committee Decision with Multiple Votes

3.1 Introduction

When can a voting mechanism correctly aggregate the dispersed information of the committee members? The papers by Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1998) illustrate how a voting procedure can induce committee members to vote insincerely. With this behavior, the committee members obscure and hence fail to convey their, otherwise valuable, private information through the decision process. Thus, the final verdict can be corrupted.

In this paper, I suggest a variation of the voting procedure discussed in the papers above. I allow more votes for the committee members. I show that whether the efficient decision rule arises in equilibrium depends on the structure of the committee members' private information. When the members have conditionally independent signals and are allowed sufficient number of votes, then the joint decision will be efficient. On the other hand, for a conditionally correlated signal distribution the above result may not be true. The main result of the paper introduces a necessary and sufficient condition on the signal distribution so that voting results in efficient decision.[1]

Why would a committee members ever vote against her information in a common interest setting? Consider a situation where each committee member receives a conditionally i.i.d. binary signal about the guilt of a defendant. The strength of the signals and the common preference is such that all jurors would prefer conviction if more than half of the private signals suggest guilt and similarly, acquittal if more than half of the private signals

[1] A significant part of the committee decision literature is concerned about the efficiency of information aggregation when the number of the committee members participating in the decision increases. However, assuming conditionally independent private information, more members means a refined information structure such that in the limit, the true state reveals. In this paper, I follow a different approach by keeping the structure of the private information fixed.

suggest innocence. However, the voting rule is such that acquittal is the status quo and a conviction requires a 2/3 majority. Now, if all committee members but one, say i, are voting sincerely, then the vote of i is pivotal only if approximately 2/3 of the others have voted for conviction, hence received guilty signal. In this case, the preferred verdict is conviction irrespective of i's signal. Thus, i should vote for conviction even if her signal suggests innocence.[2] The dilemma of the committee members is that they would like to transmit their information to the voting procedure but the latter is not flexible enough to let them do so accurately as two votes for conviction are required to compensate one vote for acquittal. However, the problem disappears when a simple majority is required for either conviction or acquittal.

Now, consider a variant of this problem such that the signals for innocence and guilty are not equally informative, and the commonly preferred verdict is acquittal only if at least 2/3 of the signals point to innocence. In this case the simple majority rule works badly: If everybody else votes sincerely, the remaining committee member i is pivotal only if approximately half of the signals are for guilt. In which case, acquittal is the preferred verdict and i will vote for that alternative even if her signal suggest guilt. In this case the 2/3 majority rule for would incentivize sincere voting and lead to perfect information aggregation. The take-away from this variant is that a good voting procedure should let committee members report the intensity of their information accurately: In the original example they should be able to cast votes for acquittal and conviction that are weighted equally. In the variant they should be able to cast votes for acquittal that count for less than votes for conviction.

A possible way to address this issue would thus be to tailor the majority requirement of a voting rule to the information structure of the committee members. This is problematic for two reasons. First, we should think of voting mechanisms as widely applicable rules, like those outlined in a constitution, rather than specific mechanisms that need to be tailored to the situation at hand. Second, optimizing over the majority rule reaches its limits as soon as the signals of committee members can take on more than two possible values. Obviously, a committee member needs at least as many possible actions as she has possible signals to enable an efficient verdict (if each pair of two signals can be pivotal for some realizations of other signals).

The latter observation points to the approach of this paper: Allowing committee members to cast multiple votes rather than a single vote. If each committee member can cast two, say, instead of one vote the above problems can be resolved by casting only one vote

[2]Note that this insincere behavior of i is in everybody's interest as all agents and society are trying to maximize the same preferences.

for conviction if one's signal suggests guilt but both votes for acquittal if one's signal is suggests innocence in the original example and vice versa in the variant.

Formally, I extend the voting model in Austen-Smith and Banks (1996) in the following ways: (i) I allow committee members to cast multiple votes to express the intensity of their private information and (ii) I allow for conditional correlation of the private signals.[3] Focusing on efficient equilibria, I show the following:

- Increasing the number of votes available to the members improves the verdict for any non-unanimous voting rule (Propositions 3.2 and 3.3).

Whether or not the efficient decision is reached as the number of the votes becomes big, depends on the correlation of the underlying private information.

- If the private information of the committee members is conditionally independent then the efficient decision is possible with sufficiently many votes (Proposition 3.6).
- If the private information of the committee members is correlated then the efficiency of the verdict is not guaranteed. I give an intuitive necessary condition for efficient voting and a more abstract condition that is necessary as well as sufficient.
  The former essentially requires that each two signals s^i, s'^i of a committee member (and each two signal sub-profiles for groups of committee members) are uniformly ordered in the sense that there are no two realizations of others' signals such that in one case the verdict should be acquittal for s^i and conviction for s'^i and vice versa in the other case. The latter abstract condition requires additionally that this order extends to an irreflexive partial order on the formal sums of signal profiles.

The intuition for the first result was given above: introducing multiple votes allows committee members to increase the accuracy of their vote, align the effect of their action on the joint decision to the information content of their signals.

Note that voting is an intrinsically additive method to aggregate private information. It is efficient if there is a way to transform the private signals to votes independently of everyone else's signal so that the sum of the votes represent all the relevant information dispersed among the committee members. Given a signal realization, the relevant information is summarized by the likelihood ratio. Whenever the signals are conditionally independent, the log-likelihood ratio of a signal profile, which is a monotone transformation of the likelihood ratio, is the sum of the log-likelihood ratios of the individual signals. Hence, there is a natural candidate for a voting strategy. However, committee members are usually presented the same evidence, therefore, it is plausible to assume that the private signals are conditionally correlated. In this case, the log-likelihood ratio does not have the above described additive property. Naturally, if there is another transformation of the signals to

[3] Note that it is often the case that the committee members observe the same evidence, hence correlation among their private information can be quite natural.

votes that has the above mentioned additive property then voting can be efficient. In the paper, I discuss when such a transformation exists.

The necessary condition above is straightforward: Committee member i must cast a certain number of votes $d^i(s^i)$ and $d^i(s'^i)$ (counting votes for conviction positively and votes for acquittal negatively) depending on her signal and as one of these numbers is greater than each other the efficient verdict cannot be reached if the above condition is not satisfied. It is tempting to think that this condition is sufficient as well as necessary for efficient voting. This is indeed the case for committees with two members. However, I give a counterexample with three voters where the condition is satisfied but efficient voting is not possible. Fortunately, I can draw a parallel to a mathematically identical question in utility theory. In this way, I can apply a theorem by Krantz et al. (1971) to derive the necessary and sufficient condition for efficient voting.

Related Literature. The quality of the decisions that are made by groups of decision makers has interested researchers for a long time. A seminal result on committee decisions, the Condorcet Jury Theorem, claims that a decision by the majority of a large group is better than the decision made by any of the individual members Condorcet (1785). Moreover, if the committee is big enough, it can outperform the decision of even a highly competent individual. The early formal arguments to support this statement are of *statistical* nature. For Condorcet, committees are groups of people with limited decision skills (probability with which the member makes the right decision), who sincerely report their independent opinion. The sincere behavior and the independence of the occasional mistakes by the members allow the use of Law of Large Numbers to show that (i) the group is less likely to conclude a mistaken decision than any individual member and that (ii) the decision by the majority is almost certainly good when the committee is big.

The model of joint decision situations that I use in this paper builds on the work of Austen-Smith and Banks (1996). They consider privately informed committee members, such that the private information is conditionally independent and identical. They spell out the optimization problem of an individual committee member.[4] They show that in equilibrium, the individuals with binary signals may conclude the inappropriate verdict depending on the voting rule. This inefficiency is connected to insincere voting on the side of the committee members. Moreover, rational voters do not vote sincerely whenever the signal is more than binary and in committees with conflicting interests.

[4]The literature on 'statistical voting' is silent on the origin of the voters' decision skills, whether the limited competence is due to a cognitive constraint or to lack of information. Miller (1986) links a mistaken vote to insufficient *information* and Ladha (1993) refers to a Bayesian updating process prior to voting. However, neither papers explicitly uses a state space and a signal distribution.

Further papers elaborate on the model by Austen-Smith and Banks (1996). Theorem 3.1 builds on the result of McLennan (1998) linking Bayes-Nash equilibria of the game to the cost minimizing voting profiles. Feddersen and Pesendorfer (1998) use the concept of pivotal voting to explain the behavior of the rational committee members. Their work demonstrates that a strategic individual considers not only her own information but the information content of the event that her vote is decisive in the process (pivotal voting). They show that the equilibrium verdict can be mistaken and this problem is the most severe if unanimity is required. Feddersen and Pesendorfer (1998), Coughlan (2000) and Duggan and Martinelli (2001) establish limit behavior of voting mechanisms as the committee becomes big, a question that is not in the focus of my work. All the papers above assume that only a single vote is available for each committee member and compare the performance of the different voting rules.

A few paper considers multiple votes for the committee members. Casella (2005) discusses a repeated joint-decision problem of individuals with independent private preferences. In her model, committee members may have multiple votes available in a decision round since they are able to transfer votes inter-temporally. She concludes that in this mechanism voters preference intensity can be expressed hence the quality of the decision is improved compared to a one vote / one decision problem mechanism. However, she investigates a situation with conflicting interests, where the goal is aggregating private preferences rather than private information. Li et al. (2001) considers a two-member committee with conditionally independent private information. They show that whenever the interests of the members are aligned, efficient decision is possible if the number of votes is sufficiently big. However, with conflicting interests, in the equilibrium the information is garbled, in other words, there is no equilibrium in which the private information is fully revealed by any strategies. They state that with conflicting interests more votes can improve the decision but there are bounds on the quality of the verdict as the number of votes increases.

The independent work of Chakraborty and Ghosh (2003) is the closest to this paper. They establish that with perfectly divisible votes and conditionally independent signals efficient information aggregation is possible. However, their main focus is the limit properties of voting mechanisms as the number of committee members increases.

The paper is organized as follows. In Section 3.2, I introduce the formal model. In Section 3.3, I argue that increasing the number of the votes that are available for the committee members improves the joint decision. In Section 3.4, I state a condition on the joint signal distribution so that the private information can be effectively aggregated by a voting mechanism. Section 3.5 concludes the paper.

3.2 A Model of Committee Decision

Next, I introduce the model of a committee decision situation and define voting procedures. Juries are well-known examples of groups of people with the task of reaching a joint decision. Therefore, in the paper I use the terminology of a jury situation.

3.2.1 The Joint Decision Problem

Private Information. There are N jurors who have to come up with a joint decision. $\mathscr{N}$ denotes the set of jurors. There are two possible states of the world: innocence and guilt ($\theta \in \Theta = \{I, G\}$). The jurors' prior probability of each state is 0.5. Each juror i is endowed with a private signal $s^i \in S^i$ about the true state of the world. I assume that S^i is finite. The signal space is $S = S^1 \times S^2 \times \cdots \times S^N$ and a signal profile is $(s^1, s^2, \ldots, s^N) = s \in S$. The probability and conditional probability of the signal profiles are $P(s)$ and $P^\theta(s)$, respectively.

I assume that no signal profile is fully revealing. Hence for any signal profile the likelihood ratio $\ell_s \equiv \frac{P^G(s)}{P^I(s)}$ exists and is non-zero. It then follows that there is an upper limit on the informativeness of any signal profile.

Payoffs. There are two possible decisions to make: acquit or convict ($\omega \in \{A, C\}$). The jurors' *common* preference is characterized by a parameter $q \in [0,1]$ such that q is the cost of convicting an innocent defendant and $1-q$ is the cost of acquitting a guilty defendant. The cost of reaching the appropriate verdict is zero. Formally, denote the payoff of decision ω in state θ by $u(\omega|\theta)$. Then $u(C|I) = -q$, $u(A|G) = -(1-q)$ and $u(C|G) = u(A|I) = 0$.

The *ex-post cost* of a decision ω if the signal profile is s is:

$$c(\omega, s) = \sum_\theta -u(\omega|\theta) P(\theta|s)$$

where $P(\theta|s)$ is the probability of the state θ when s is realized. Consider an *outcome function* $\Omega : S \to \{A, C\}$ that maps any signal profile into a decision. Then we can define the following costs:

- The ex-post cost for a signal realization s is $c(\Omega(s), s)$.
- The *interim cost* for a juror i with a signal s^i is:

$$\mathscr{C}^i(\Omega, s^i) \equiv \mathbb{E}_{(s^{-i}|s^i)} c(\Omega(s^{-i}, s^i), (s^{-i}, s^i)).$$

- The *ex-ante cost* of the mechanism with outcome function Ω is:

$$\mathscr{C}(\Omega) \equiv \mathbb{E}_s c(\Omega(s), s).$$

Efficiency. The jurors' goal is to acquit whenever the defendant is innocent and to convict whenever the defendant is guilty. However, no matter what the mechanism is, the decision they reach can never be the right one with certainty, simply because there is no fully informative signal profile. Given a signal realization s, the *efficient verdict* is $\arg\min_\omega c(\omega, s)$. An *efficient outcome function* is cost minimizing for any realized signal profile, i.e. it is ex-post efficient for any signal realizations.

Next, I show that the likelihood ratio of a signal realization is a convenient measure to characterize the efficient decision. Acquitting is costly only if the state is G and in this case it costs $1-q$. Similarly, conviction is costly if the state is I and then it costs q. Hence, for a signal profile s, the cost of acquittal is: $c(A,s) = (1-q)P(G|s)$ while the cost of conviction is: $c(C,s) = qP(I|s)$. Therefore, the efficient decision rule is:

$$\Omega^e(s) = \begin{cases} C & \text{if} \quad (1-q)P(G|s) \geq qP(I|s) \\ A & \text{otherwise.} \end{cases}$$

Intuitively, acquittal is better if the available information suggests that the state is rather I than G, and if acquitting a guilty defendant is not too costly compared to convicting an innocent defendant. Using that $\frac{P(G|s)}{P(I|s)} = \frac{P^G(s)}{P^I(s)}$ by the Bayes rule and the uniform prior assumption, the above decision rule can be conveniently rephrased as a threshold problem:

$$\Omega^e(s) = \begin{cases} C & \text{if} \quad \ell_s \geq \ell_q \\ A & \text{otherwise.} \end{cases} \tag{3.1}$$

where $\ell_q \equiv \frac{q}{1-q}$ is the relative costs of the mistaken decisions and $\ell_s = \frac{P^G(s)}{P^I(s)}$ is the likelihood ratio of a signal profile s. This latter ratio expresses how likely guilt is relatively to innocence, given the signals. To summarize, if the likelihood ratio exceeds the ratio of the costs of mistaken decisions then conviction is the better verdict.

3.2.2 Voting Game (V, α)

Voting Procedure (V, α). In this paper, I focus on voting procedures to mediate the joint decision problem. I assume that the jurors reach a verdict in a one-shot game. Each juror is endowed with $V \in \mathbb{N}$ votes that she can fully or partly cast to either of the two possible decisions. The joint decision is convict if and only if a certain majority of the cast votes is for convict. The voting rule can be described by a parameter $\alpha \in [0,1]$, so that the decision is convict if at least a proportion α of the cast votes is convict, i.e. if α(# votes cast for convict) $\geq$ (# votes cast for convict and acquit) then the joint verdict is convict.[5]

Strategies. A juror can determine the number of the votes she casts and which decision she supports. Her choice depends on her private signal about the state of the world. Formally, a strategy of player i is $v^i = (v_A^i, v_C^i) : S^i \to \{0,1,2,\ldots,V\}^2$ such that either $v_A^i(s^i) = 0$ or $v_C^i(s^i) = 0$.[6] To simplify the analysis I alter the action space in two ways. I sign votes for acquittal as negative and votes for conviction as positive. Second, I normalize a juror's vote with the total number of votes that is available for her. Assume that the voting rule is simple majority, i.e. $\alpha = 1/2$, then define

$$X_V^{1/2} \equiv \left\{ \frac{k}{2V} | k \in \{1,2,\ldots V\} \right\}.$$

Then there is a one-to-one correspondence between the intuitive action set and $X_V^{1/2}$. For any α-majority rules, I can generalize the method above and define:

$$X_V^{\alpha} \equiv \left\{ -\alpha \frac{k}{V} | k = \{1,2\ldots,V\} \right\} \cup \left\{ (1-\alpha) \frac{k}{V} | k = \{1,2\ldots,V\} \right\}.$$

Then a *normalized strategy* is $d^i : S^i \to X_V^{\alpha}$ such that

$$d^i(s^i) \equiv \frac{(1-\alpha) v_C^i(s^i) - \alpha v_A^i(s^i)}{V}. \tag{3.2}$$

[5] For a given number of votes, V, any voting rule $\alpha < \frac{1}{NV}$ requires unanimity for acquittal, and similarly, any voting rule $\alpha \geq \frac{NV-1}{NV}$ requires unanimity for conviction. In the limit, as the number of votes increases, $\alpha = 1$ refers to the unanimity rule for conviction, however, $\alpha = 0$ does not mean unanimity for acquittal. This asymmetry follows from defining the tie breaking rule in an asymmetric manner. Recall, that if exactly α proportion of the cast votes is convict, the joint verdict is convict. It is possible to make the tie breaking rule dependent on the actual level of α and hence restore $\alpha = 0$ as a unanimous rule for continuous votes. This change would not alter my results except Proposition 3.5 in which I would need to make sure that I only compare voting rules that have the same tie breaking rule.

[6] Notice that by formalizing the strategies in this way, I do not allow for abstention from voting. This assumption is standard in the models of Austen-Smith and Banks (1996), McLennan (1998) and Feddersen and Pesendorfer (1998), however others especially papers on elections allow for abstention. Assuming that both $v_A^i(s^i) = 0$ and $v_C^i(s^i) = 0$ is possible, I can easily formalize my model in a way that makes it possible for jurors to stay neutral. This modification would not change any of the result, however it would make it somewhat harder to present concise examples illustrating the advantages of introducing more votes into the voting procedure.

Denote by $\Delta^i_{(V,\alpha)}$ the set of *feasible normalized voting strategies* for the juror i and $\Delta_{(V,\alpha)}$ the profiles of feasible normalized voting strategies.

This representation is convenient since the sum of d^i translates easily into the verdict. That is convict if $\sum_{\mathscr{N}} d^i(s^i) \geq 0$ and acquit otherwise, no matter the voting rule. However, the strategy space will depend on the voting rule.

I refer to the joint decision problem with a voting procedure as a *voting game*. Finally, denote by Ω_d the outcome function that is generated by a strategy profile d. An outcome function Ω is feasible in the voting game (V,α) if there is a feasible strategy profile that generates it.

Equilibrium Concept. I consider Bayes-Nash Equilibria of the voting game. I say that voting is efficient in a voting procedure if some equilibrium strategy profile in the actual voting game implements the efficient decision rule.

3.3 Committee Decisions with Finite Votes

I am interested in how well a voting procedure can aggregate the private information of the committee members. The papers by Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1998) emphasize possible inefficiencies in the process. In this section, I argue that allowing more votes can improve the joint verdict for all voting rules except the unanimous rule. In all voting games there are potentially multiple equilibria. An important limitation of my analysis is that I state my results only with respect of one of those equilibria, namely the one with the highest welfare.[7] I neither investigate the effect of allowing more votes on equilibria with low welfare nor the question whether new equilibria with high ex-ante costs arise as the number of available votes increases. I cannot refer to any intuitive selection mechanisms that can support my treatment, however, given that the equilibrium I focus on is the one with the highest welfare a designer would want players to coordinate on that.

The first set of results shows that whenever the jurors have common interests, the equilibrium with the lowest ex-ante costs can be found by solving a constrained minimization problem. This result is convenient for two reasons. First, it simplifies the process of finding a particular equilibrium of the game. Second, it makes it easy to compare equilibria of games that only differ in the number of available votes. I argue that having more votes translates into a relaxed constraints of the optimization. This leads to the second set of

[7]The ordering of the equilibria according to the ex-ante costs is possible since the players have common interest.

results: allowing more votes for each juror raises expected welfare in equilibrium. I close the section with examples that illustrates this point.

3.3.1 Constrained Efficiency in Voting Games

Definition 3.1 (Constrained Efficiency)**.** *Given a voting procedure (V,α), I refer to an outcome function Ω as* constrained efficient *if it is feasible with the voting procedure (V,α) and generates the lowest ex-ante costs among the feasible outcome functions in the voting game (V,α).*

Next, note that there are finitely many signal profiles thus finitely many different outcome functions, which makes the feasible set finite as well given any voting game (V,α). Thus the infimum of the ex-ante costs over the set of feasible outcome functions is always attained and we have the following result.

Proposition 3.1. *A constrained efficient outcome function exists in any voting game (V,α).*

The last result in this section shows that a voting profile that generates the constrained efficient outcome function is a Bayes-Nash Equilibrium in the voting game. It is easy to establish that if a deviation from the constrained efficient profile by juror i results in a lower interim cost for the juror then the original voting profile could not have been constrained efficient.

Theorem 3.1 (McLennan, 1998)**.** *If the voting profile d_e is such that*

$$d_e = \arg\min_{d \in \Delta_{(V,\alpha)}} \mathscr{C}(\Omega_d)$$

then d_e is a Bayes-Nash Equilibrium in the voting game.

Proof. Assume that all but juror i follow the prescribed strategy, d_e^{-i}. If there exists d^i and s^i such that $\mathscr{C}^i(\Omega_{(d_e^{-i},d^i)}, s^i) < \mathscr{C}^i(\Omega_{d_e}, s^i)$ then $\mathscr{C}(\Omega_{(d_e^{-i},d'^i)}) < \mathscr{C}(\Omega_{d_e})$. Contradiction. □

Corollary 3.1. *In any voting game (V,α), an equilibrium exists.*

3.3.2 More Votes Are Better

In this section, I show that a better joint decision can be reached when more votes are available for the jurors. I have argued before that the constrained efficient equilibria are solutions of a constrained minimization problem. By allowing more votes, the constraint of the optimization problem is relaxed. Hence, the ex-ante cost in equilibrium cannot increase. To illustrate the point, I provide some examples after the formal results.

Proposition 3.2. *Consider a jury of size N and a voting game* (V,α). *Denote by* d_V *and* d_{V+1} *the constrained efficient equilibria of the voting games* (V,α) *and* $(V+1,\alpha)$, *respectively. Then* $\mathscr{C}(\Omega_{d_V}) \geq \mathscr{C}(\Omega_{d_{V+1}})$.

Proof. I argue that the feasible outcome set in the voting game $(V+1,\alpha)$ contains the one in the voting game (V,α). For any strategy profile in $\Delta_{(V,\alpha)}$ there is a strategy profile that is feasible in the voting game $(V+1,\alpha)$ and generates the same outcome. Consider a strategy profile $d_V \in \Delta_{(V,\alpha)}$ and define $d'_V \in \Delta_{(V+1,\alpha)}$ such that $d'^i_V(s^i) = d^i_V(s^i)\frac{V}{V+1}$. Then the set $\{s|\sum_{\mathscr{N}} d^i_V(s^i) \geq 0\}$ is identical to $\{s|\sum_{\mathscr{N}} d'^i_V(s^i) \geq 0\}$ since by definition $\sum_{\mathscr{N}} d'^i_V(s^i) = \frac{V}{V+1}\sum_{\mathscr{N}} d^i_V(s^i)$. Hence, the outcome functions implemented by d_V and d'_V are identical.

Therefore, the constrained efficient outcome function is at least as good in the voting game $(V+1,\alpha)$ as in the voting game (V,α). □

An additional vote may improve the decision if the voting rule is not too extreme. On the other hand, if the required majority is too strong, i.e. a single acquit vote of a single player can determine the verdict, then an additional vote is not useful. Whether or not this happens depends on the parameters of the model: the number of jurors involved in the decision process and the number of votes available for each juror. However, when unanimity is required, increasing the number of votes does not improve the equilibrium verdict. All voting games $(V,1)$ (voting games $(V,0)$) are equivalent in the sense that they generate the same equilibrium outcome.

Proposition 3.3. *If* Ω *is a feasible outcome function in the voting game* $(V,1)$ *then* Ω *is a feasible outcome function in the voting game* $(1,1)$. *Hence, in unanimous voting games, allowing more votes never strictly improves the verdict.*

Proof. Denote by d_V an equilibrium strategy profile in the voting game $(V,1)$. For voting games such that unanimity is required for conviction, the set of normalized votes is $X^1_V \subset [-1,0]$. Define the following strategy profile d_1 in the voting game $(1,1)$:

$$d^i_1(s^i) = \begin{cases} 0 & \text{if} \quad d^i_V(s^i) = 0 \\ -1 & \text{otherwise.} \end{cases}$$

The profile d_1 is valid in the voting game $(1,1)$ since for every i it maps to $\{-1,0\}$. Second, the profile d_1 induces the same outcome function as the profile d_V. In a voting game $(V,1)$ for a signal realization s the verdict $\Omega_d(s) = C$ if and only if for all i, $d^i(s^i) = 0$. Therefore, for all $s \in S$, $\Omega_{d_1}(s) = \Omega_{d_V}(s)$. □

Notice that if the feasible votes are in $[-1,0]$ then the only action supporting conviction is to assign all the available votes to convict. Therefore, even though multiple votes are available, there is no instrument to express signal strength in these procedures. This implies that the unanimity rule rarely allows for efficient information aggregation. The jurors are only able to express a binary partition of their signal space and that is sufficient only for very restricted information structures.

Finally, I give two illustrative examples. The first example shows that even with binary signals, a single vote can be insufficient for efficient voting if the voting rule is simple majority but the signals are not symmetric. In case of such biased signals, the information carried by one guilty signal does not cancel the information carried by one innocent signal. However, due to simple majority, casting the single available vote to acquit exactly balances a convict vote by an opponent. This implies that the intuitive strategy profile in which all the jurors vote according to their own signal is not an equilibrium strategy profile.[8] Thus, in the equilibrium, which necessarily exists, a juror's action is non-responsive to her information with some probability. Therefore, the equilibrium cannot implement efficient voting.

As I discussed it in the introduction, there are two ways to address this problem. One solution is to adjust the voting rule so that the relative strength of the two kind of votes are aligned to the relative strength of the opposing signals. However, if one follows this route, then the adequate voting rule depends on the details across distributions. There is no rule that generally works well. A more robust solution for the problem is to allow more votes for the jurors. Having multiple votes to allocate for either of the two possible decisions enables the jurors to refine the effect of their actions on the final verdict. One can conclude that allowing multiple vote is a robust way to improve the quality of the joint decision while the optimal α would have to be tailored to the problem at hand.

The second example shows that for more than two signals, multiple votes are necessary to express different intensities of information carried by different signals.

Example 1. Consider a joint decision problem of $N = 5$ jurors, each with a preference parameter $q = 0.5$ ($\ell_q = 1$) and conditionally iid binary signals with the following distribution:

[8] The other fully informative profile in which each juror vote against her signal cannot be an equilibrium either.

	$P^I(s^i)$	$P^G(s^i)$	ℓ_{s^i}
s_1	$4/7$	$1/7$	$1/4$
s_2	$3/7$	$6/7$	2

The efficient decision rule in this joint decision problem is:

$$\Omega^e(s) = \begin{cases} C & \text{if} \quad \#\{i|s^i = s_2\} \geq 4 \\ A & \text{otherwise.} \end{cases}$$

In this example, the signals are initially biased in the sense that one innocent signal and one guilty signal do not balance out, i.e. in case of the same number of innocent and guilty signals the committee has a strict preference for acquittal. In other words, more than the simple majority of the signals have to be s_2 for a convict verdict to be efficient. If the voting procedure is such that $V = 1$ and $\alpha = 1/2$ then there is *no equilibrium that implements the efficient decision rule.* First, a mixed strategy equilibrium cannot lead to the efficient outcome. For any signal profile, the efficient verdict is unique, either convict or acquit. However mixed strategies lead to a probabilistic outcome, that means inefficiency with positive probability. Second, the only strategy profile that may implement the efficient decision rule must be informative for all jurors and also, must assign a C vote, $(1/2)$ to s_2 and an A vote $(-1/2)$ to s_1. To see this consider the situation when the others received one s_1 and three s_2 overall. Then the jurors information is decisive, hence her vote must be decisive, it has to push the joint decision to be convict if her signal is s_2 and to be acquit if her signal is s_1. The only possible pure strategy of such is $(d^i(s_1), d^i(s_2)) = (-1/2, 1/2)$. However, this is not an equilibrium in this voting game since if all the others follow this strategy then the juror wants to vote acquit no matter. Her vote only counts if exactly two of the others received s_1 but then even if her signal is s_2, the efficient verdict is acquittal.

Notice that there is an equilibrium that implements the efficient decision rule if the voting procedure is such that $V = 1$ and $\alpha = 2/3$. The following strategies lead to the efficient outcome:[9]

$\alpha = 2/3$	$v_A(s^i)$	$v_C(s^i)$	$d^i(s^i)$
s_1	1	0	$-2/3$
s_2	0	1	$1/3$

It is easy to check that the verdict is C exactly if at least 4 out of the 5 members received s_2.

[9]In this example I show the strategies with both the 'natural' votes and the normalize votes to help the understanding.

Next, I demonstrate that allowing multiple votes for the jurors, can also improve on information aggregation. I show that with the voting procedure $(2, 1/2)$, the efficient decision rule in the above joint decision problem can be implemented as an equilibrium outcome. Consider the following voting profile:

$\alpha = 1/2$	$v_A(s^i)$	$v_C(s^i)$	$d^i(s^i)$
s_1	2	0	$-1/2$
s_2	0	1	$1/4$

If two jurors received s_1 and three jurors received s_2 then the sum of the votes are $(4, 3)$, for acquit and convict respectively, and the verdict is A according to simple majority. (The sum of the weighted votes is $-1/4 < 0$.) If one juror received s_1 and four jurors received s_2 then the sum of the votes are $(2, 4)$ so the verdict is C. (The sum of the vote difference is $1/2 > 0$.) Hence, the above voting profile implements the efficient decision rule and by the Theorem 3.1 it is an equilibrium in the voting game $(2, 1/2)$.

Example 2. Consider the joint decision problem of $N = 3$ jurors with preferences $q = 0.5$ ($\ell_q = 1$). Assume that the jurors have conditionally iid signals according to the following distribution:

	$P^I(s^i)$	$P^G(s^i)$	ℓ_{s^i}
s_1	$5/12$	$1/12$	$1/5$
s_2	$4/12$	$2/12$	$1/2$
s_3	$2/12$	$4/12$	2
s_4	$1/12$	$5/12$	5

If the voting procedure is such that $V = 1$ and $\alpha = 1/2$ then the efficient decision rule cannot be implemented. The argument in the previous example, such that mixed strategies cannot lead to the efficient decision rule, is equally valid here. Then notice that there are only two different actions to take: vote for convict ($1/2$), vote for acquit ($-1/2$). However, each juror can have four different signals, a strong and a weak signal supporting acquittal and a strong and a weak signal supporting conviction. Thus, in any pure strategy of a juror, at least two signals trigger the same action.

Next, I show that any two signals of a juror can be decisive in the sense that one of the signals with a possible realization of the others information suggests acquittal while the one signal with the same realization of the others suggests conviction. If the two signals of the juror support different alternatives then, whenever the two others received s_2 and s_3, the signal of the juror is decisive. If the two signals of the juror support the same alternative but with different strength then, whenever the others both have the opposite but weak signals, the signal of the juror is decisive

Nevertheless, if the juror's action is the same for the signals, the verdict is the same as well given any realization of the other. Therefore the resulting outcome cannot be efficient. To implement the ex-post decision rule it is important to follow different actions for different signals, which is impossible if every juror only has one vote.

If at least 3 votes are available for the jurors, then there is an equilibrium voting profile which implements the efficient decision rule, see for example, the strategy below.

	$v_A(s^i)$	$v_C(s^i)$	$d^i(s^i)$
s_1	3	0	$-1/2$
s_2	1	0	$-1/6$
s_3	0	1	$1/6$
s_4	0	3	$1/2$

3.4 Committee Decisions with Continuous Votes

In the previous section, I argued that allowing the committee members to cast multiple votes enables them to communicate their information in a more accurate way. As the number of the votes grows, an obstacle is removed from the way of information aggregation. It is reasonable to ask if there are limits to this improvement. Can the jurors always conclude the efficient verdict if there are sufficiently many votes available? It turns out that the answer to this question is sensitive to the underlying signal structure.[10]

Focusing on voting mechanisms to reach a joint verdict restricts the feasible outcome functions in the joint decision problem. Simultaneous voting does not allow the jurors' strategies to depend on each other's signals. As a consequence, it is possible that the ex-post efficient outcome rule cannot be implemented by a voting game, i.e. voting is not efficient. Consider, for example, a situation with two jurors and binary signals, such that matching signals suggest one decision while opposite signals suggest the other decision. In this case it is, indeed, impossible to always reach the efficient verdict by simultaneous voting such that the individual's vote only depends on the juror's own signal. In this situation not even increasing the number of available votes help. Example 3 formalizes this argument.

[10]When answering the above question, I focus on the constrained efficient equilibria in the voting games, similarly to the treatment in Section 3.3.

First, I define the limit game where each committee member can cast a continuous rather than a discrete number of votes. Proposition 3.4 shows that this game is not just the limit of a sequence of discrete voting games with growing number of votes but that for large enough V every decision function that is implementable in the limit voting game is also implementable in a discrete game with V or more votes. Then, I show that whenever the private signals of the jurors are conditionally independent, then one can construct an equilibrium voting profile in the limit game that leads to the efficient verdict for any signal realizations. Hence, having conditionally independent private information is sufficient for efficient voting. Using the intuition in the previous paragraph, I provide examples of correlated signals such that the private information cannot be perfectly aggregated. Then, I give a necessary and sufficient condition on the joint signal distribution for the voting to be efficient.

Definition 3.2 (Voting Game (∞, α))**.** *There are N jurors to make a joint decision. The private information and the preferences of the jurors are as characterized earlier. Define the voting procedure (∞, α) in the following way.*

- *A pure* strategy *for a juror i is a function mapping from the signal space to the interval $[-\alpha, 1-\alpha]$, formally $d^i : S^i \to [-\alpha, 1-\alpha]$.*
- *The outcome $\Omega_d(s)$ is convict if $\sum_{\mathcal{N}} d^i(s^i) \geq 0$ and it is acquit otherwise.*

I intend refer to the voting procedure (∞, α) as the limit of the procedures with finite votes as the number of the votes increases. Next, I show that if the number of the available votes is high enough then a decision rule is feasible in the limit game if and only if it is feasible in the finite games. This fact conveniently implies that (i) a constrained efficient decision rule exists in the limit game since it exists in the finite games by Proposition 3.1 and (ii) there is a $\overline{V}$ such that for all $V > \bar{V}$ the constrained efficient decision rule is the same as the one in the limit game. Therefore the limit game is informative about voting games with high but finite number of votes.

Proposition 3.4 (Properties of the Limit Game)**.** *For a given voting rule α there is a finite number $\overline{V}$ such that for all $V > \overline{V}$ the set of feasible decision rules in the voting game (V, α) is identical to the set of feasible decision rules in the voting game (∞, α).*

The difficulty lays in showing that every decision rule that is feasible in the limit game is feasible in the finite game. Notice, that an action that is available in the limit may be impossible in the finite game. However, if V high enough then for any voting strategy in the infinite game, one can define voting strategies in the finite game that are sufficiently close. Thus, for any signal profile the sums of the individual votes are close to each other, especially the sign of the sums are the same. Therefore the two voting profiles implements the same outcome function.

Proof. Since, the action set in the continuous game contains the action set of any finite game $X^\alpha_V \subset X^\alpha_\infty$, the set of the feasible decision rules in the limit game clearly contains the set of the feasible decision rules in any voting game (V, α). Next, fix a decision rule Ω that is implementable with the voting procedure (∞, α) and denote by d the voting profile that implements it. Note that if V votes are available for the jurors and the voting rule is α then the distance between two consecutive elements, x and x' of the action set, X^α_V is exactly $\frac{\alpha}{V}$ if $x, x' < 0$ and is exactly $\frac{\alpha}{V}$ if $x, x' > 0$. In any case there is a feasible action in any interval $\mathscr{I} \subset [-\alpha, 1-\alpha]$ of length $\frac{1}{V}$.

Next, due to the finite signal space, I can find the smallest margin by which acquittal is chosen with d, denote this margin by

$$m_\Omega \equiv \min\left\{ |\sum_{\mathscr{N}} d^i(s^i)| | s \in \Omega^{-1}(A) \right\}.$$

Fix a V such that $V > \frac{N}{m_\Omega}$ and define

$$d^i_V(s^i) = \min\left\{\delta | \delta \in X^\alpha_V, \delta \geq d^i(s^i)\right\}.$$

For every i, $d^i(s^i) + \frac{m_\Omega}{N} \geq d^i_V(s^i) \geq d^i(s^i)$. Therefore, $\sum_{\mathscr{N}} d^i(s^i) + m_\Omega \geq \sum_{\mathscr{N}} d^i_V(s^i) \geq \sum_{\mathscr{N}} d^i(s^i)$ which implies that $\sum_{\mathscr{N}} d^i_V(s^i) \leq 0 \iff \sum_{\mathscr{N}} d^i(s^i) \leq 0$. $\square$

The next result shows that with continuous votes the voting rule does not influence the efficiency of the voting mechanism.

Proposition 3.5 (Neutrality of the Voting Rule)**.** *If a decision rule Ω is feasible in a voting procedure (∞, α') then it is feasible with a voting procedure (∞, α''), where $\alpha', \alpha'' \in (0,1)$.*

Proof. Assume that a decision rule Ω is implemented in the voting game (∞, α') by the voting profile d'. If $\alpha' > \alpha''$, then $\frac{\alpha''}{\alpha'} d'$ is a feasible strategy profile in the voting game (∞, α'') and implements Ω while if $\alpha' < \alpha''$ then $\frac{1-\alpha''}{1-\alpha'} d'$ is a feasible strategy profile in the voting game (∞, α'') and implements Ω. $\square$

3.4.1 Conditionally Independent Signals - A Sufficient Condition for Efficient Voting

Next, I argue that the efficiency of a voting mechanism, as a method of reaching a verdict, hinges critically on the structure of the information possessed by the jurors.

If the jurors have conditionally independent information then it is possible to construct a voting profile that is feasible in the limit game and that generates the efficient verdict in the joint decision problem. Hence it is an equilibrium in the limit game. By Proposition 3.4 voting games with sufficiently many votes then have efficient equilibria as well.

Whenever the private information is conditionally independent across the committee members, it is possible to disentangle the information content of any individual signal from the rest of the signals. Formally, the log-likelihood ratio of any signal profile is the sum of the log-likelihood ratios of the individual signals. Hence, one can construct an efficient strategy such that the vote for any signal is a linear function of the log-likelihood ratio of the signal. The following theorem formalizes this argument.[11]

Proposition 3.6 (Efficient Voting Profile)**.** *Consider a joint decision problem such that the jurors have conditionally independent private information. In the voting game* (∞, α) *the efficient decision rule is implemented by the voting profile* d_o *such that a juror* i *with signal* s^i *casts*

$$d_o^i(s^i) = a\left(\log \ell_{s^i} - b^i\right) \tag{3.3}$$

where $\sum_i b^i = \log \ell_q$ *and* $a < \frac{\min\{\alpha, 1-\alpha\}}{\max_{i,s^i} |\log \ell_{s^i} - b^i|}$.[12] *Hence, the voting profile* d_o *is an equilibrium.*

If the committee members are more concerned with, say, convicting an innocent defendant than they are with acquitting a guilty defendant, i.e. $\log \ell_q > 0$, then the equilibrium votes should reflect this by concluding acquit whenever the ex-post probability of guilt and innocence are equal. The terms b^i take care of this. Second, a feasible strategy requires that $d^i(s^i) \in [-\alpha, 1-\alpha]$. The constant a rescales the value of $(\log \ell_{s^i} - b^i)$ making sure that it falls into this range.

Proof. By Theorem 3.1, if the outcome function implemented by d_o is efficient then it is an equilibrium in the voting game. An outcome function is efficient if the verdict is acquittal whenever the probability of innocence is high enough. Recall equation (3.1) that characterizes the efficient decision rule. With conditionally independent signals it simplifies to:

$$\Omega^e(s) = \begin{cases} C & \text{if} \quad \sum_{\mathcal{N}} \log \ell_{s^i} \geq \log \ell_q. \\ A & \text{otherwise.} \end{cases}$$

Without loss of generality, pick a signal realization, s such that $\sum_{\mathcal{N}} \log \ell_{s^i} \geq \log \ell_q$, hence the efficient verdict for s is conviction. Then according to the strategy in (3.3) the sum

[11] In the independent work of Chakraborty and Ghosh (2003) Theorem 4 demonstrates an equivalent result.

[12] Given that the signal space is finite and there is no perfectly informative signal, the bound on the likelihood ratio exists.

of the votes is $\sum_{\mathscr{N}} d_o^i(s^i) = \kappa \sum_{\mathscr{N}} (\log \ell_{s^i} - a^i) = \kappa (\sum_{\mathscr{N}} (\log \ell_{s^i}) - \log q) \geq 0$. Hence, convict is concluded. For signal realization such that the efficient verdict is acquittal the proof is analogous. $\square$

The construction of the efficient voting profile in Proposition 3.6 suggests that the additivity of the log-likelihood ratio is important for information aggregation by a voting procedure. Conditionally independent signals imply this property.

However, conditional independence is not necessary for additivity of the log-likelihood ratios. It is possible to tweak conditionally independent distributions in a way so that the log-likelihood ratio remains additive. For example, assume that $P(s,\theta)$ is a conditionally independent joint signal distribution and define $\tilde{P}(s,\theta) \equiv \prod_i P(s^i,\theta)\mu(s)$ where μ is not constant and picked appropriately so that $\tilde{P}$ is a probability distribution. The distribution $\tilde{P}$ implies the same likelihood ratios as the distribution P. Thus, signals according to $\tilde{P}$ also allow efficient information aggregation, although $\tilde{P}$ is not conditionally independent.

Below, I show that even additive log-likelihood ratios are not necessary for efficient voting when the signal space is finite.

3.4.2 Ordered Signals - A Necessary Condition for Efficient Voting

In the case of conditionally independent signals and the efficient voting strategies defined in (3.3), the sum of the individual votes is a strictly increasing function of the likelihood ratio of the realized signal profile. It is important to realize that this is not necessary for efficient voting. To reach an efficient outcome it is enough if the sign of sum of the individual votes is positive if the efficient verdict is conviction and is negative if acquittal is the efficient verdict.

Before coming to a necessary and sufficient condition for efficient voting in the next subsection, I first introduce a necessary condition. Namely, for every committee members there needs to be an unambiguous relation between every two signals of her, in the sense that one of the signals always makes conviction more favorable than the other.

Example 3. Consider a decision problem of a two-member committee. Prior to the voting, each member can observe the realization of a binary signal. The values s_j refer to the signal of the first and t_j to the signal of the second juror. The table below represents the efficient decision rule given each of the four possible signal profiles. A '$+$' indicates that the efficient verdict is conviction while a '$-$' refers to realizations such that the efficient verdict is acquittal.

ℓ	t_1	t_2
s_2	-	+
s_1	+	-

For example, the following conditional distributions with the preference parameter $q = 0.5$ generate this decision rule.

P^G	t_1	t_2
s_2	3/14	2/14
s_1	6/14	3/14

P^I	t_1	t_2
s_2	4/14	1/14
s_1	5/14	4/14

ℓ	t_1	t_2
s_2	3/4	2
s_1	6/5	3/4

Next, I show that there is no voting profile that leads to the efficient decision in this case. Assume, to the contrary, that there exist appropriate voting strategies d^1 and d^2. Then for (s_1,t_1) and (s_2,t_2) the sum of the votes must be positive while for (s_2,t_1) and (t_1,s_2) it must be negative. Therefore the following must be true:

$$\begin{aligned} d^1(s_1)+d^2(t_1) &> d^1(s_2)+d^2(t_1) \\ d^1(s_2)+d^2(t_2) &> d^1(s_1)+d^2(t_2). \end{aligned} \tag{3.4}$$

However, there are no numbers $d^1(s_2), d^2(t_2), d^1(s_1)$ and $d^2(t_2)$ that satisfy the above system. Adding up the two strict inequalities leads to a strict inequality with the same expression on both sides, which is a contradiction.

In this example, s_1 is more favorable for conviction than s_2 if the opponent has t_1 and less if the opponent has t_2. This feature makes it impossible to find good voting strategies. The signal s_j must be more or less favorable for conviction than an $s_{j'}$ independently of the opponents' realization.

Does excluding the above pattern always allow for efficient information aggregation? The answer is positive for two-member panels, no matter the number of possible signal values. The following definition generalize the notion that efficient voting fails in the above examples because there is no order on S^i such that Ω^e is monotone in s^i.

Definition 3.3. *For any subset of the committee members, $I \subset N$, denote the signal space by S^I, which is the product of the signal spaces S^i such that $i \in I$. Define a binary relation on S^I by $s^I \succ_I s'^I$ such that $s^I, s'^I \in S^I$ if there exists $t^{-I} \in S^{-I}$ such that $\Omega^e(s^I, t^{-I}) = C$ and $\Omega^e(s'^I, t^{-I}) = A$.*

The idea of this definition is that s^I is better news for conviction that s'^I whenever $s^I \succ_I s'^I$. It follows immediately from this definition that if the voting profile d is to implement the efficient decision rule then it needs to be that $\sum_{i\in I} d^i(s^i) > \sum_{i\in I} d^i(s'^i)$. This points straight to the next definition and the statement afterward.

Definition 3.4 (No flip-flop)**.** *The signal distribution satisfies* no flip-flop *if $\succ_I$ is a non-reflexive binary relation on S^I, i.e. $s^I \succ_I s'^I$ implies that $s'^I \succ_I s^I$ is not true.*

Lemma 3.1. *No flip-flop is necessary for efficient voting, and it is necessary and sufficient for efficient voting for a two-member committee.*

The proof of the second statement is constructive. I argue that for two jurors, no flip-flop allows for an intuitive order on the signals and based on this order one can construct an efficient voting profile.

Proof. PART 1: Whenever the binary relation is irreflexive, i.e. $s^I \succ_I s'^I$ as well as $s'^I \succ_I s^I$, efficient voting requires that $\sum_{i\in I} d^i(s^i) > \sum_{i\in I} d^i(s'^i)$ as well as $\sum_{i\in I} d^i(s'^i) > \sum_{i\in I} d^i(s^i)$, which is a contradiction.

PART 2: I denote by s_j the signals of juror 1 and by t_k the signals of juror 2. For any s_j, I define $T(s_j) = \{t_k | \Omega^e(s_j, t_k) = C\}$. The no flip-flop condition implies that for any s_j and $s_{j'}$ either $T(s_j) \subseteq T(s_{j'})$ or $T(s_{j'}) \subseteq T(s_j)$. Hence, it is possible to order the signals of the first juror such that $s_j \geq s_{j'}$ if $T(s_{j'}) \subseteq T(s_j)$. A similar property is true for the signals of the second juror and hence, there is an order on S^2 as well.

Rename the signals such that the indices now refer to the order in the above defined sense, i.e. such that $T(s_1) \subseteq T(s_2) \cdots \subseteq T(s_J) \subseteq S^2$, where J is the number of the signals in S^1. Find values $\phi^1(s_j)$ such that $\phi^1(s_j) < \phi^1(s_{j+1})$ for all $j \in \{1, 2, \ldots J-1\}$ and $\phi^1(s_1) < 0$ while $\phi^1(s_J) > 0$.

For a $t_k \in T(s_1)$ set $\phi^2(t_k) > -\phi^1(s_1)$. The set $T(s_1)$ includes all the t_k signals of the juror 2 such that the efficient verdict in case of (s_1, t_k) is conviction, and with this constriction the vote does conclude convict since, $\phi^1(s_1) + \phi^2(t_k) > 0$.

For all $j \in \{2, 3 \ldots J\}$, if $t_k \in T(s_{j+1}) \setminus T(s_j)$, set $\phi^2(t_k) \in (-\phi^1(s_{j+1}), -\phi^1(s_j))$. Note that a $t_k \in T(s_{j+1}) \setminus T(s_j)$ requires conviction if the juror 1 receives s_{j+1} but acquittal if the juror 1 receives s_j, and hence this construction ensures that $\phi^1(s^j) + \phi^2(t_k) < 0$ while $\phi^1(s^{j+1}) + \phi^2(t_k) > 0$.

For a $t_k \in S^2 \setminus T(s_J)$, let $\phi^2(t_k) < -\phi^1(s_J)$. Note that a $t_k \in S^2 \setminus T(s_J)$, requires acquittal for s_J which happens since $\phi^2(t_k) + \phi^1(s_J) < 0$.

Thus, we assigned ϕ^2 for every elements of S^2. Finally, depending on the voting rule α in the actual voting game, one can find $a > 0$ to make sure that $d^i \equiv a\phi^i$ is a valid voting strategy, i.e. it maps into $[-\alpha, 1-\alpha]$. $\square$

I have argued that no flip-flop is the necessary and sufficient condition of efficient voting in two members committee. The next section shows that for more than two committee members, further restrictions are needed to ensure efficient voting. But before I discuss the relation of two orders on the individual signal space, the one implied by the efficient decision and the one implied by the likelihood ratio function.

Remark. It is tempting to think that the "no flip-flop" condition implies some sort of monotonicity of the likelihood ratio in the signals. This is not the case. Remember the earlier discussion that full information aggregation is sufficient but not necessary for efficient voting. The following example illustrates this:

P^G	t_1	t_2
s_1	3/7	2/7
s_2	1/7	1/7

P^I	t_1	t_2
s_1	1/7	1/7
s_2	3/7	2/7

ℓ	t_1	t_2
s_1	3	2
s_2	1/3	1/2

There is no order on the signals such that the likelihood ratio function is monotone and still efficient voting is possible for any value of q as for any value of q there is an order on the signals with respect to which the efficient decision rule Ω^e is monotone. The following tables represent the efficient decision for $\ell_q < 1/2$, $\ell_q \in [1/2, 2]$ and $\ell_q > 2$, respectively.

P^G	t_1	t_2
s_1	+	-
s_2	-	-

P^I	t_1	t_2
s_1	+	+
s_2	-	-

ℓ	t_1	t_2
s_1	+	+
s_2	-	+

Hence, the order on the signal space of the second juror, S^2 and hence the efficient vote will depend on q, i.e. if $\ell_q < 1/2$ then $t_2 \succ_1 t_1$ and if $\ell_q > 2$ then $t_1 \succ_1 t_2$ while for any other preference parameter either of the orders work.

3.4.3 A Necessary and Sufficient Condition for Efficient Voting

I start the section with an example that does not violate the flip-flop condition, however, does not allow for efficient voting. I discuss the property that blocks efficient voting in this example and suggest a sequence of conditions that are necessary for the existence of efficient voting profile. Then, I link the problem of efficient voting to a classic problem in utility theory. Finally, I present a necessary and sufficient condition for efficient voting. My formal argument relies on the work of Krantz et al. (1971).

Example 4. Consider a decision problem of a three-member committee. Prior to the voting each member can observe the realization of a private signal that has three possible values. The signals s_j, t_j and z_j refer to the signals of the juror 1, 2 and 3, respectively. The table below represents the efficient decision rule given all the possible signal profiles. Again, a

'$+'$ indicates that the efficient verdict is conviction while a '$-'$ refers to the realizations such that the efficient verdict is acquittal.

z_1

	t_1	t_2	t_3
s_3	-	-	+
s_2	-	-	+
s_1	-	-	-

z_2

	t_1	t_2	t_3
s_3	+	+	+
s_2	-	-	+
s_1	-	-	-

z_3

	t_1	t_2	t_3
s_3	+	+	+
s_2	-	+	+
s_1	-	+	+

To see that this is a valid example, consider the conditional distributions below that generate this decision rule with the preference parameter $q = 0.5$.

z_1

P^G	t_1	t_2	t_3
s_3	$\frac{12}{376}$	$\frac{14}{376}$	$\frac{18}{376}$
s_2	$\frac{4}{376}$	$\frac{10}{376}$	$\frac{16}{376}$
s_1	$\frac{2}{376}$	$\frac{6}{376}$	$\frac{8}{376}$

z_2

P^G	t_1	t_2	t_3
s_3	$\frac{16}{376}$	$\frac{18}{376}$	$\frac{22}{376}$
s_2	$\frac{15}{376}$	$\frac{14}{376}$	$\frac{20}{376}$
s_1	$\frac{16}{376}$	$\frac{15}{376}$	$\frac{15}{376}$

z_3

P^G	t_1	t_2	t_3
s_3	$\frac{15}{376}$	$\frac{15}{376}$	$\frac{15}{376}$
s_2	$\frac{15}{376}$	$\frac{15}{376}$	$\frac{15}{376}$
s_1	$\frac{15}{376}$	$\frac{15}{376}$	$\frac{15}{376}$

z_1

P^I	t_1	t_2	t_3
s_3	$\frac{15}{376}$	$\frac{15}{376}$	$\frac{15}{376}$
s_2	$\frac{15}{376}$	$\frac{15}{376}$	$\frac{15}{376}$
s_1	$\frac{15}{376}$	$\frac{15}{376}$	$\frac{15}{376}$

z_2

P^I	t_1	t_2	t_3
s_3	$\frac{15}{376}$	$\frac{15}{376}$	$\frac{15}{376}$
s_2	$\frac{20}{376}$	$\frac{15}{376}$	$\frac{15}{376}$
s_1	$\frac{22}{376}$	$\frac{18}{376}$	$\frac{16}{376}$

z_3

P^I	t_1	t_2	t_3
s_3	$\frac{8}{376}$	$\frac{6}{376}$	$\frac{2}{376}$
s_2	$\frac{16}{376}$	$\frac{10}{376}$	$\frac{4}{376}$
s_1	$\frac{18}{376}$	$\frac{14}{376}$	$\frac{12}{376}$

z_1

ℓ	t_1	t_2	t_3
s_3	$\frac{12}{15}$	$\frac{14}{15}$	$\frac{18}{15}$
s_2	$\frac{4}{15}$	$\frac{10}{15}$	$\frac{16}{15}$
s_1	$\frac{2}{15}$	$\frac{6}{15}$	$\frac{8}{15}$

z_2

ℓ	t_1	t_2	t_3
s_3	$\frac{16}{15}$	$\frac{18}{15}$	$\frac{22}{15}$
s_2	$\frac{15}{20}$	$\frac{14}{15}$	$\frac{20}{15}$
s_1	$\frac{16}{22}$	$\frac{15}{18}$	$\frac{15}{16}$

z_3

ℓ	t_1	t_2	t_3
s_3	$\frac{15}{8}$	$\frac{15}{6}$	$\frac{15}{2}$
s_2	$\frac{15}{16}$	$\frac{15}{10}$	$\frac{15}{4}$
s_1	$\frac{15}{18}$	$\frac{15}{14}$	$\frac{15}{12}$

One can check that there is no flip-flop in this example. However, efficient voting is still impossible. Assume, on the contrary, that there exist good voting strategies: d^1,d^2 and d^3. Then the sum of the votes for the signal profiles (s_2,t_3,z_1),(s_3,t_1,z_2) and (s_1,t_2,z_3) has to be positive while for (s_3,t_2,z_1),(s_1,t_3,z_2) and (s_2,t_1,z_3) it has to be negative. Therefore the following system of inequalities must have a solution.

$$\begin{aligned} d^1(s_2)+d^2(t_3)+d^3(z_1) &> d^1(s_3)+d^2(t_2)+d^3(z_1) \\ d^1(s_3)+d^2(t_1)+d^3(z_2) &> d^1(s_1)+d^2(t_3)+d^3(z_2) \\ d^1(s_1)+d^2(t_2)+d^3(z_3) &> d^1(s_2)+d^2(t_1)+d^3(z_3). \end{aligned} \tag{3.5}$$

However, one can see that there are no numbers $d^1(s_j), d^2(t_j)$ and $d^3(z_j)$ that satisfy the system. Adding up the three lines again, leads to a strict inequality with the same expression on both sides.

What goes wrong here? One can directly compare two signal sub-profiles if there is a profile of all the other jurors, such that the two signal sub-profiles are decisive. No flip-flopping occurs whenever directly comparison is not possible or if it is possible and the order is consistent. However, there are implicit ways of comparing signals.

Consider two pairs of signals (s_L, t_H) and (s_H, t_L) so that $s_H \succ_i s_L$ and $t_H \succ_j t_L$ and $(s_L, t_H) \succ_{i,j} (s_H, t_L)$. Then one can conclude that t_H signal of juror j relatively to t_L is stronger than the signal s_H is relatively to s_L, when comparing (s_H, t_L) to (s_L, t_H), juror i's information becomes more favorable for acquittal while juror j's information becomes more favorable for conviction. When these two effects are aggregated the one for conviction dominates. Now, consider an additional signal for both jurors, s_M and t_M such that $s_H \succ_i s_M$ and $s_M \succ_i s_L$ and also $t_H \succ_j t_M$ and $t_M \succ_i t_L$. Then, there is an implicit way toevaluate the relative strength of the above changes of the jurors information. First, one may compare (s_H, t_M) to (s_M, t_H) and then (s_M, t_L) to (s_L, t_M). In the example,

- If the third juror has the realization z_1, then the first inequality shows that having t_3 instead of t_2 is stronger news for conviction then having s_2 instead of s_3 is for acquittal.
- If the third juror has the signal realization z_2, then the second inequality shows that having t_3 instead of t_1 is weaker news for conviction then having s_1 instead of s_3 is for acquittal.
- And finally, with z_3, having t_2 instead of t_1 is stronger news for conviction then having s_1 instead of s_2 is for acquittal.

However, a problem occurs since the change from t_1 to t_2 dominates the change from s_2 to s_1 and the change from t_2 to t_3 dominates the change from s_3 to s_2, however the change from t_1 to t_3 is dominated by the change from s_3 to s_1.

If efficient voting exists, then the sum of votes for all signals profiles such that the efficient decision is convict is bigger than the sum of votes for all to the signal profiles such the efficient decision is acquit. By this requirement, any private information structure induces a system of inequalities that has to be solvable. The no flip-flop condition ensures that any two-element subset of the inequality system is consistent in the sense that it has solution. However, this condition does not guarantee a solution for the entire system. Example 4 presents an information structure such that, although, any two inequalities are solvable, there is system of three inequalities that is inconsistent. Hence, the whole system has no solution.

As the number of jurors and the possible signal values increases, the system becomes more and more difficult. Fortunately, there is an alternative way to represent the problem and an easily understandable condition is available which is equivalent to the set of inequality conditions.

There is a widely discussed question in utility theory, namely what are the properties of a preference relation that allows for an additive separable utility representation. This problem mathematically is very similar to the question whether there are voting profiles that represents the information content of the signals well, i.e. the sum of the votes are higher whenever the signal profile is better news for conviction.

However, an earlier remark suggested that efficient information aggregation is not necessary for efficient voting. The jurors have to get a binary decision right, so as long as the sum of the votes are positive whenever the efficient verdict is convict the voting is efficient.

Hence, I define a binary relation on the signal profiles that is implied by the efficient decision rule: A signal profile is ‘bigger’ then another whenever the efficient decision is convict for the first and acquit for the second profile. For $s, s' \in S$

$$s \succ_N s' \Longleftrightarrow \Omega^e(s) = C \text{ and } \Omega^e(s') = A. \tag{3.6}$$

Notice that this relation is not complete.

Then, I ask what characteristics of this binary relation ensure that there exist voting functions d^i that represent the binary relation in the sense that the sum of the votes is positive if and only if the efficient decision is convict, or formally, so that there exist functions $d^i : S^i \to \mathbb{R}$ such that:

$$\Omega^e(s) = C \Longleftrightarrow \sum_{\mathscr{N}} d^i(s^i) \geq 0. \tag{3.7}$$

Definition 3.5. *A function* $\phi : S \to \mathbb{R}$ is an additive separable representation of a binary relation *if there exist functions* $\phi^i : S^i \to \mathbb{R}$ *such that*

$$s \succ s' \Longleftrightarrow \phi(s) = \sum_{\mathscr{N}} \phi^i(s^i) > \sum_{\mathscr{N}} \phi^i(s'^i) = \phi(s').$$

The first result shows that the existence of the voting profile that is characterized by Equation (3.7) is equivalent to the existence of an additive separable representation of a binary relation on the signal space.

Lemma 3.2. *An efficient voting profile in a voting game (∞, α) exists if and only if there is an additive separable representation of the binary relation defined by Equation (3.6).*

Proof. A voting strategy itself is an additive separable representation since $s \succ_N s' \iff \Omega^e(s) = C$ and $\Omega^e(s') = A \iff \sum_{\mathscr{N}} d^i(s^i) \geq 0 > \sum_{\mathscr{N}} d^i(s'^i)$.

If there exist functions $\{\phi^i\}_{i\in\mathscr{N}}$ representing a binary relation then the functions $\{d^i | d^i = a\phi^i + b^i, a > 0\}_{i\in\mathscr{N}}$ represent the binary relation as well. $s \succ s'$ if and only if $\sum_{\mathscr{N}} \phi^i(s^i) > \sum_{\mathscr{N}} \phi^i(s'^i)$. Notice that $\sum_{\mathscr{N}} \phi^i(s^i) > \sum_{\mathscr{N}} \phi^i(s'^i) \iff \sum_{\mathscr{N}} a\phi^i(s^i) > \sum_{\mathscr{N}} a\phi^i(s'^i) \iff \sum_{\mathscr{N}} a\phi^i(s^i) + \sum_{\mathscr{N}} b^i > \sum_{\mathscr{N}} a\phi^i(s'^i) + \sum_{\mathscr{N}} b^i \iff \sum_{\mathscr{N}} d^i(s^i) > \sum_{\mathscr{N}} d^i(s'^i)$.

Hence, one can transform the functions ϕ^i representing $\succ_N$ into valid, efficient voting strategies. There are two conditions to satisfy: (i) valid voting functions map into $[-\alpha, 1-\alpha]$ and (ii) $\sum_{\mathscr{N}} d^i(s^i) \geq 0$ if and only if $\Omega^e(s) = C$.

By construction, there exists $\overline{\phi}$ with the property that for all s such that $\Omega^e(s) = C$, $\phi(s) \geq \overline{\phi}$ and for all s' such that $\Omega^e(s) = A$, $\phi(s') < \overline{\phi}$. Then setting $b^i = \frac{\overline{\phi}}{N}$ gives that $\Omega^e(s) = C$ if and only if $\sum_{\mathscr{N}} d^i(s^i) > 0$. Finally, a small enough a can make sure that the voting functions d^i map into $[-\alpha, 1-\alpha]$. $\square$

Examples 3 and 4 indicated that we may need to consider combinations of signal profiles comparisons and to sum votes across these combinations. Therefore, I start with introducing formal sums of signal profiles. Since the sum of the signal profiles has no meaning in the signal space, it is more convenient to think in terms of the following vector representation.

Define $K^i = |S^i|$ and $K = \sum_{\mathscr{N}} |S^i|$. Then every signal realization can be written in the form of a vector of zeros and ones of length K, and hence there is a set $X \subset \{0,1\}^K$ such that each elements of X represents and element of S and all elements of S is represented in X. Consider the order $\succ_N$ on S and denote the inherited order on X by $\succ_X$.

Given the vectors in X, I define the set $Y \subset \mathbb{Z}^K$ as the additive span of X. A vector y is element of Y if and only if it is the finite sum of elements of X, i.e. $y = \sum_{m\leq M} x_m$ for any $x_m \in X$ and $M \in \mathbb{N}$. The relation $\succ_X$ can be extended to Y in the following way: $y = \sum x_m$ is greater than $y' = \sum x'_m$, i.e. $y \succ_Y y'$ if for all m, $x_m \succ_X x'_m$.

Next, I state the main theorem. The proof is adopted from Krantz et al. (1971) Theorem 9.1

Theorem 3.2. *Efficient voting is possible if and only if the binary relation $\succ_Y$ is irreflexive.*[13]

In the proof I show that the existence of a voting profile is equivalent to the existence of a solution of a system of linear inequalities (Step 1) and that the reflexivity of the relation $\succ_Y$ is equivalent to the existence of a solution of an other system of linear inequalities (Step

[13] The theorem in Krantz et al. (1971) originally allows for indifference between elements of the set. Here, I do not discuss this case, although the proof easily goes through.

3). I refer to a duality theorem stated in Krantz et al. (1971) Theorem 2.7 to demonstrate that the two systems are dual pairs and hence exactly one of the system has a solution (Step 2).

Proof. By Lemma 3.2 the existence of an efficient voting profile is equivalent to the existence of an additive separable representation $\phi^i(s^i)$ of $\succ_N$.

STEP 1: The set $\{\Omega^{-1}(C), \Omega^{-1}(A)\}$ is a partition of S. Denote by $\{X^C, X^A\}$ the respective partition of X. Then for any $x^c \in X^C$ and $x^a \in X^A$, $x^c \succ_X x^a$. Denote by$K^A = |X^A|$ and by $K^C = |X^C|$.

Any functions $\phi^i(s^i)$ imply a K-dimensional vector δ such that the first K^1 entries of δ equal to the K^1 values of $\phi^1(s^1)$, then the next K^2 entries of δ equal to the K^2 values of $\phi^2(s^2)$, and so on. At the same time, any K-dimensional vector imply a family of ϕ^i functions.

Thus, $\phi^i(s^i)$ is a representation of $\succ_N$ if and only if there is a δ such that $x^c\delta > x^a\delta$ for all $x^c \in X^C$ and $x^a \in X^A$.

Define $K^C K^A$ vectors $r_n = x^c - x^a \in \{0,1\}^K$ with $x' \in X^C$ and $x \in X^A$. Then, collect all these vectors into an integer matrix R of size $(K^C K^A) \times K$ such that r_n is the nth row of the matrix. Then there is a representation of $\succ_N$ if and only if $R\delta \gg 0$.[14]

Thus, I have shown so far that an efficient voting profile exists if and only if there exists a vector $\delta \in \mathbb{R}^K$ such that $R\delta \gg 0$.

STEP 2: The Theorem 2.7 in Krantz et al. (1971, pp. 62) states that the system $R\delta \gg 0$ has a solution if and only if the system $R^T\rho = 0$ such that $\rho > 0$ has no solution.[15]

Moreover, immediately after the theorem they argue that if the elements of the matrix R are rational numbers then the system $R\delta \gg 0$ has a rational solution if and only if the system $R^T\rho = 0$ such that $\rho > 0$ has no rational solution. Also, if ρ is a solution of the second system, than for any $a > 0$, $a\rho$ is a solution as well. Therefore, the theorem implies that if $R\delta \gg 0$ has no solution then the system $R^T\rho = 0$ such that $\rho > 0$ has an integer solution.

STEP 3: Now, I show that the existence of a $K^A K^C$-dimensional integer vector, $\rho > 0$ with $R^T\rho = 0$ is equivalent to $\succ_Y$ being reflexive.

$R^T\rho = 0$ is equivalent to saying that $\sum_{X^A}\sum_{X^C}\rho^{x^a,x^c}(x^c - x^a) = 0$, where x^c are the elements of X^C, x^a the elements of X^A and $\rho^{x^a,x^c} \in \mathbb{N}$ the $K^A K^C$ dimensions of ρ. Recall, that ρ is weakly positive and integer vector, hence each entries of it is a natural number.

[14] $v \gg 0$ denotes that every component of the vector v is strictly positive.
[15] $v > 0$ denotes that every component of v is weakly positive and that $v \neq 0$.

Thus, if there exists $\rho > 0$ with $R^T\rho = 0$ then

$$\sum_{X^A}\sum_{X^C}\rho^{x^a,x^c}x^c = \sum_{X^A}\sum_{X^C}\rho^{x^a,x^c}x^a.$$

Since any ρ^{x^a,x^c} is a non-negative integer there are $x_1^c, x_2^c, \ldots, x_M^c \in X^C$ and $x_1^a, x_2^a, \ldots, x_M^a \in X^A$, not necessarily distinct vectors, such that $\sum_M x_m^c = \sum_M x_m^a \in Y$ which is the definition of $\succ_Y$ being reflexive.

On the other hand, if $\succ_Y$ is reflexive then there are $x_1^c, x_2^c, \ldots, x_M^c \in X^C$ and $x_1^a, x_2^a, \ldots, x_M^a \in X^A$, not necessarily distinct vectors, such that $\sum_M x_m^c = \sum_M x_m^a \in Y$. Then a vector $\rho \in \mathbb{N}^{K^A K^C}$ can be generated in the following way. For all $n \in K^A K^C$, the nth entry $\rho_n \equiv \{\#m | r_n = x_m^c - x_m^a\}$. Then $R^T\rho = 0$.

To summarize, I have shown that an efficient voting profile exists $\Longleftrightarrow$ there exists an additive representation $\phi^i(s^i)$ of $\succ_N$ $\Longleftrightarrow$ there exists a K-dimensional vector δ with $R\delta \gg 0$ $\Longleftrightarrow$ there exists no $K^A K^C$-dimensional vector $\rho > 0$ such that $R^T\rho = 0$ $\Longleftrightarrow$ the relation $\succ_Y$ is irreflexive. $\square$

Notice that while the elements of the set X represent signal profiles, the elements of the set Y represents collections of signal profiles. Recall the information structure in Example 3, there, $(s_1,t_1) \succ_{1,2} (s_2,t_1)$ and $(s_2,t_2) \succ_{1,2} (s_1,t_2)$. Hence, the representations of the collection $\{(s_1,t_1),(s_2,t_2)\}$ and the collection $\{(s_1,t_2),(s_2,t_1)\}$ in Y are related according to $\succ_Y$. Moreover, it is easy to see that they are equivalent in Y, so $\succ_Y$ is irreflexive.

Similarly, in Example 4 $(s_2,t_3,z_1) \succ_{1,2,3} (s_3,t_2,z_1)$, $(s_3,t_1,z_2) \succ_{1,2,3} (s_1,t_3,z_2)$ and $(s_1,t_2,z_3) \succ_{1,2,3} (s_2,t_1,z_3)$. Therefore the collection $\{(s_2,t_3,z_1),(s_3,t_1,z_2),$ $(s_1,t_2,z_3)\}$ is related to the collection of $\{(s_3,t_2,z_1),(s_1,t_3,z_2),(s_2,t_1,z_3)\}$. Moreover, they are equivalent in Y.

Hence, an element y in the set Y such that $y \succ_Y y$ suggest that there are multiple ways - explicit or implicit - of comparing two signal sub-profiles and the implied relationship between them is ambiguous. Hence, it is impossible to assign efficient votes to those sub-profiles.

3.5 Conclusion

In this paper, I have studied the joint decision problem of a committee of privately informed individuals. I argued that allowing multiple votes for the members, improves the quality of the joint decision made by the committee. I also showed that for conditionally independent signals, full efficiency can be reached if there are sufficient number of votes available. I discussed that with correlated private information full efficiency may not be possible for any number of votes. Moreover, I provided conditions to ensure that full efficiency exists. To summarize, allowing multiple votes makes a voting mechanism better and is a remedy for a certain type of inefficiencies in the joint decision problem. However, the efficient information aggregation with correlated private signal would require a different class of decision mechanisms. In both cases, individuals want to express their private information but due to some institutional constraint are unable to do so.

One can think about an additional obstacle to aggregate private knowledge in a committee setting. As it is shown by Li et al. (2001) if individual members have conflicting interests, they may not want to communicate their private signals even if that would be possible.

Chapter 4

Costly Voting, Polarization and Turnout

4.1 Introduction

When the margin in elections decreases, does the turnout increase? Blais (2000, pp. 58 - 62) surveys a variety of studies from different countries and concludes that “the verdict is crystal clear [...]: closeness has been found to increase turnout in 27 of the 32 different studies that have tested the relationship, in many different settings and with diverse methodologies. There are strong reasons to believe that [...] more people vote when the election is close” (p. 60).

To understand the connection between margin and turnout theoretically, one might turn to an equilibrium model of rational voting with voting costs. It seems relatively obvious that individual incentives for participation increase as margin decreases because a decreasing margin should increase the probability of any individual vote being pivotal, and rational voting decisions are exclusively based on the comparison between the benefits from voting if the vote is pivotal and the costs of voting. Indeed, Blais (2000), in the second sentence quoted above, writes that the effect found in the studies that he cites is “as predicted by rational choice theory” (p. 60).

However, a change in margin is presumably the result of a change in the distribution of voters’ preferences over the candidates or decisions voted upon. While for any voter with *given* preferences the incentive to vote increases as the margin decreases, a prediction of the effect of margin on turnout must combine this observation with an analysis of the effect on turnout of the change in the distribution of voters’ preferences that caused margin to decrease in the first place.

In the language of game theory the question thus becomes whether equilibrium turnout increases if the distribution of voters’ preferences changes in a way that causes margins to decrease. In other words: if we observe equilibrium voting decisions by populations of voters with different preference distributions will we find that there is a negative relation between margin and turnout? As noted by Krasa and Polborn (2008, footnote 13) this question is, in fact, an open question in the game theoretic literature on costly voting.

In an equilibrium model of rational voting with a finite population of voters the question that I have posed is difficult to address because of the intricacies of analyzing the comparative statics of the probability with which any vote is pivotal. I propose in this paper a simplified model in which the mapping that relates turnout and margin into voters' perception of their probability of being pivotal is exogenous. I construct this mapping so that it shares some properties with the exact relationship between turnout, margin, and probability of being pivotal in a model with a finite but large population of voters. Thus, the model might either be interpreted as a model of boundedly rational voters who misperceive in an exogenously specified way the probability of being pivotal, or as an approximation of a model of fully rational voters in the case that the number of voters tends to infinity.[1]

Within this model I provide a sufficient condition that guarantees that a decrease in margin leads to higher turnout. The condition is that the change in the distribution of voters' preferences that causes margin to decrease must not involve a decrease in the *polarization* of voters. The notion of "polarization" is defined formally in the paper. Informally, polarization decreases if voters move from the extreme ends of the preference distribution to the center of the preference distribution. The condition that polarization must not decrease is a very restrictive condition. It is, of course, only sufficient. I do not prove that it is necessary, but I do show by means of an example that, if the condition is violated, the result need not be true. This may explain why the literature, such as the study by Blais (2000) cited above, finds exceptions from the rule that decreases in margin lead to higher turnout.

Related Literature. The theory of rational voter dates back to the seminal work of Downs (1957). According to his views, rational voters vote when the expected benefit of changing the election result outweighs the strictly positive costs of voting. Downs puzzling observation was that, since the probability of being pivotal is minuscule in large election, rational voter theory implies zero turnout. Among the many attempts to resolve this paradox of voting, Ledyard (1984) and Palfrey and Rosenthal (1983) proposed a game theoretic revision of the election models. They pushed the argument one step further observing that if voter turnout is expected to be zero then the pivotal probability is actually high. Therefore, complete absenteeism is not a Nash Equilibrium of the voting game. They emphasize positive voter turnout in equilibrium, however decreasing turnout rate as the

[1]Note that I do not provide an exact limit result that would support this latter interpretation.

size of the electorate increases.

Building on the model of Ledyard (1984) the works of Börgers (2004) and Krasa and Polborn (2008) pose normative questions, whether the aggregate welfare can be improved as additional incentives are present for voting (mandatory voting or voting subsidies). Subsidies increase participation, since they implicitly make the preference distribution more polarized. Krasa and Polborn (2008) shows that whether this increased participation is welfare enhancing depends on the relative preference towards the two alternatives in the population as well as on the number of voters.

My setup is the closest to Campbell (1999). He models a private value election with heterogeneous voters and homogeneous voting costs. His key observation is that with positive voting costs voters with weak preference abstain, hence if minority voters have more intense preferences than majority voters, it might happen that the minority wins the election, since supporters of the majority might abstain in much higher number. Campbell's paper therefore emphasize the role of voters with intense preference and thus points to the importance of polarization in the turnout.

In Section 4.2, I describe the model and discuss my main assumptions. Section 4.3 characterizes the equilibrium, proves existence and gives necessary and sufficient conditions for uniqueness. In Section 4.4, I turn to the comparative statics of the equilibrium turnout and margin and prove that they are negatively related when the polarization of the underlying preference distributions coincides. Section 4.5 concludes.

4.2 A Model of Elections

Next, I describe the model of an electorate where voting is costly. The main deviation from the models often used in the literature is that I allow voters who misperceive the probabilities with which the different election outcomes happen, moreover, they misperceive their effect on those probabilities. In this section, after describing the model, I discuss the assumptions about those subjective probabilities and illustrate them with an example.

Alternatives. I describe an election in which one of two parties, $LEFT$ or $RIGHT$, is chosen.

Players. A continuum electorate[2] participate in a binary election. A voter's preferences are described by a parameter z_t distributed independently across voters on $[-1,1]$ according to

[2]Recall the two possible interpretations of the setup discussed in the introduction. Here, I present the model as an approximation of an election with a large number of voters and I state the results accordingly. Though, the results can be restated in terms of finite and boundedly rational voters. In that case, the implica-

a continuously differentiable cumulative distribution function F that is common knowledge among the voters.

Actions. The voters can either vote *L*eft or *R*ight or not vote, *A*bstain. Let a_t the action of player t, hence $a_t \in \{L, A, R\}$. A pure strategy of player t, $v_t(z)$, is a measurable function assigning actions to player t's preference type. A symmetric strategy profile is denoted by v, without an index referring to the player.

Outcome / Election Margin. The winner of the election is the party, $LEFT$ or $RIGHT$, that receives the majority of the votes. Denote by $\mu(v)$ the measure of right voters and by $\lambda(v)$ the measure of left voters when all voters follow the symmetric strategy v. I define the election margin $M(v) \equiv \mu(v) - \lambda(v)$ as the difference between the support of the right and the left party. The winner of the election is the $RIGHT$ party whenever the margin is positive and the $LEFT$ party whenever it is negative.

Payoffs. I characterize the voters' preferences in the following way: A voter with a preference parameter z has payoff $u_t(z, LEFT) = -z$ in case of a $LEFT$ victory and $u_t(z, RIGHT) = z$ in case of a $RIGHT$ victor. One can see that for types z below 0, $LEFT$ is the preferred alternative and for types z above 0, $RIGHT$ is the preferred alternative. The cost of voting is $c > 0$ for all voters.

Before I proceed, let me discuss that an alternative model of costly voting. The work of Börgers (2004), Krasa and Polborn (2008) and others assumes binary preferences, a voter either prefers the $LEFT$ or the $RIGHT$ with the same intensity, however the cost of voting is diverse.

Independently of the framework, a voter's payoff is determined by the election outcome - which is an uncertain event - and her own action, whether she votes or not. If there is a utility function on the space of the consequences: $(LEFT, vote), (LEFT, abstain)$ and $(RIGHT, vote), (RIGHT, abstain)$ that gives an expected utility representation of the voter's preferences then any affine transformation of that utility function also represents the same preferences. The following tables show the utility for a voter with $z < 0$ in the preference heterogeneity model and a for a left voter in the cost heterogeneity model.

u	$LEFT$	$RIGHT$
vote	$-z-c$	$z-c$
abstain	$-z$	z

v	$LEFT$	$RIGHT$
vote	$1-\kappa$	$-\kappa$
abstain	1	0

tions are about the *expected* margin and the *expected* turnout. Note, however, that considering empirical implications the two interpretations differ since the expected values of the election margin and election turnout are not observable.

Setting $\alpha = -\frac{1}{2z} > 0$ and $\beta = \frac{1}{2}$, it is transparent that the utility in the cost heterogeneity model is an affine transformation of the utility in the preference heterogeneity model via $\alpha u + \beta = v$. A similar transformation is possible whenever $z > 0$ and the voter has a right preference, respectively. Therefore the two models lead to the same voting behavior which is the focus of this paper.[3]

Subjective Probability and the Influence. I intend to model voters who misperceive their effect on the election outcome. A voter's subjective probability of a right victory is

$$Pr(RIGHT, M, T, a) \equiv H(M, T, a)$$

and it depends on the predicted margin, $M \in [-1, 1]$, the predicted turnout, $T \in [0, 1]$, and her own action, $a \in \{L, A, R\}$. Similarly, a voter' subjective probability of a left victory is

$$Pr(LEFT, M, T, a) \equiv 1 - H(M, T, a).$$

I assume the following about the subjective probability:

1. $H(M,T,R) > H(M,T,A)$ and $(1 - H(M,T,L)) > (1 - H(M,T,A))$, whenever $M \in (-1, 1)$. Intuitively, a vote cast for a party strictly increases the winning chances of the party.
2. $H(M,T,R) - H(M,T,A) = (1 - H(-M,T,L)) - (1 - H(-M,T,A))$. Holding the expected margin and the turnout fixed, the effect of a vote cast for either party is independent of the identity of the party, given the lead for that party held constant. Notice that the election margin is by definition the lead of the right party, and hence $-M$ is the lead for the left party.
3. $H(M,T,R) - H(M,T,A) = (1 - H(M,T,L)) - (1 - H(M,T,A))$. Holding the expected margin and the turnout fixed, the effect of a vote cast for either party is independent of the identity of the party.
4. The probability $H(M,T,a)$ is continuous and differentiable in M and T, and

$$\begin{aligned}
&\frac{\partial}{\partial M}[H(M,T,R) - H(M,T,A)] > 0 \qquad \text{if} \quad M < 0 \\
&\frac{\partial}{\partial M}[H(M,T,R) - H(M,T,A)] = 0 \qquad \text{if} \quad M = 0 \\
&\frac{\partial}{\partial M}[H(M,T,R) - H(M,T,A)] < 0 \qquad \text{if} \quad M > 0 \\
&\frac{\partial}{\partial T}[H(M,T,R) - H(M,T,A)] \leq 0.
\end{aligned}$$

The effect of a vote is the highest when the election is a tie and it decreases as the

[3]However, note that the welfare differs in these models. In my model, the welfare is fully described by the turnout and the election outcome, while in the alternative model, it matters who voted.

absolute value of the election margin increases. Moreover, the voter's influence decreases in the average turnout.

A left voter believes that for given margin M, her action increases the winning probability of her favored party by

$$I_R(M,T) \quad = \quad H(M,T,R) - H(M,T,A).$$

Similarly, for a right voter the impact is:

$$I_L(M,T) \quad = \quad (1 - H(M,T,L)) - (1 - H(M,T,A)).$$

The above assumptions imply the following properties for a voter's impact:

1. $I_R(M,T) > 0$ and $I_L(M,T) > 0$ whenever $M \in (-1,1)$.
2. $I_R(M,T) = I_L(-M,T)$
3. $I_R(M,T) = I_L(M,T)$
4. Both $I_R(M,T)$ and $I_L(M,T)$ are continuous and differentiable in M, and they are symmetric and single peaked in zero. Moreover, the influence is weakly decreasing in T.

Hence, I can define the *influence* function: $I(M,T) \equiv I_R(M,T) = I_L(M,T)$ such that it is strictly positive on $(-1,1)$, it is symmetric: $I(M,T) = I(-M,T)$, is single peaked at zero-margin and it is continuous and differentiable in M and it is continuous, differentiable and weakly decreasing in T. Moreover, by construction $I(M,T) \in [0,1]$.

Next, I compare my assumptions regarding the influence function I to the exact relationship between margin, turnout, and the probability of a voter being pivotal. First note that in my model, if all voters have the same probability of voting for either of the two candidates, and the same probability of abstaining, these probabilities are unambiguously pinned down by M and T. Now suppose that there were a finite population of voters of size N. We can then calculate the probabilities $\Pi^L_{M,T}, \Pi^R_{M,T}$ with which a voter who casts a vote for the left or the right party is pivotal. The influence function may be seen as the voter's perception of this functions $\Pi^L_{M,T}$ and $\Pi^R_{M,T}$.

With exact probabilities, intuitively, as the expected election margin decreases, having equal (almost equal) number of left and right voters is more likely. However, the pivotal probability of a right voter is the average of the probability of the realized margin among all the other voters is either 0 or -1 which is the highest when the left party has a slight lead.

The effect of the average turnout is two-fold. Intuitively, if the turnout increases, then the expected number of voters who participate in the election increases, hence it is less likely that one is pivotal. On the other hand, if the election margin is relatively high, there is an additional effect, namely, increasing the turnout while holding the margin constant makes the group of actual voters more balanced and hence, increases the chance of being pivotal. Moreover, as the number of the voters grows large, the effect of the average turnout is diminishing compared to the effect of the margin, except in the neighborhood of zero margin, where is has a negative effect on the pivotal probability. To summarize, the effect of the turnout is likely to be smaller than the effect of the expected margin and where it matters it has a negative effect on the pivotal probability.

In the appendix, I calculate the exact pivotal probabilities and I point to the effects mentioned above. I show analytically how the margin changes the pivotal probabilities for a a significant subset of the possible margin and turnout values. I refer to numerical computations in characterizing the effect of the turnout.

The most important misperception by the voters is that they overestimate their effect on the election outcome.[4] Furthermore, they disregard the slight asymmetry in the margin that appears in the exact probabilites and also simplify the effect of the turnout greatly. My assumptions on the exogenous form of the influence function result a tractable, symmetric form of equilibria in my model and allow comparative statics results as the preferences of the electorate changes.

Example. This example illustrates the construction above for the case that the effect of the turnout is zero. I provide a subjective probability of a right victory given the different actions of the voters and then show that influence function that these probabilities imply. Intuitively, the higher the lead of the right party, the higher the probability that *RIGHT* wins. Consider the following family of valid density functions on $[-1,1]$

$$\gamma_\alpha(x) = \frac{1}{2} - \alpha x.$$

Then the distribution Γ describes the winning probability of the right party as its lead increases.

$$\Gamma_\alpha(M) = \int_{-1}^{M} \left(\frac{1}{2} - \alpha x\right) \mathrm{d}x = \frac{1+\alpha}{2} + \frac{M}{2} - \alpha\frac{M^2}{2}.$$

Then $H(M,A) = \Gamma_0(M)$, $H(M,R) = \Gamma_\alpha(M)$ and $H(M,L) = \Gamma_{-\alpha}(M)$ are valid subjective probabilities for any $\alpha \in (0,1/2]$. The influence function I that is implied by this

[4] Klor and Winter (2008) report an experiment in which voters overestimate the probability that their vote is pivotal. Note, however, that the experiment is more closely related to committee decision than to large elections. Also the experiment results find overestimated impact for voters who belongs to the majority whereas in my model all voters are subject to misperception.

specification of H is:

$$I(M) = \frac{\alpha}{2} - \frac{\alpha}{2}M^2.$$

Intuitively, as α increases, the voter's perceived impact on the outcome of the election increases.

Subjective Expected Payoff. The following is the expected payoff of a voter choosing action a such that $a \in \{L, A, R\}$ with a preference parameter z if the other voters follow strategy profile v:

$$\pi(a,z;v) = (1 - H(M(v),T(v),a))(-z) + H(M(v),T(v),a))z - C(a).$$

The first part of this expression describes the probability that the outcome is $LEFT$ multiplied by the payoff in that case. The second part is the expected payoff in case of a $RIGHT$ victory. Finally, $C(a)$ denotes the cost related to the action followed, it is equal to c for the actions L and R and equal to zero for the action A.

Turnout. The turnout is defined as the overall measure of the players who vote, i.e. choose action L or R: $T(v) \equiv \lambda(v) + \mu(v)$.

Equilibrium Concept. I consider pure strategy Nash equilibria of the game defined above such that the strategies are symmetric across voters.

4.3 Equilibrium of the Voting Game

I show that best response strategies of voter t in the voting game are in symmetrical cutoff strategies with two critical types $\underline{z}_t \in [-1,0)$ and $\overline{z}_t \in (0,1]$ that satisfy $\underline{z}_t = \overline{z}_t$. The type $\underline{z}_t$ separates the range of voters who optimally vote for the left party from the ones who do not vote. Type $\overline{z}_t$ separates the range of non-voters from right voters. For $z \in (0,1]$, I denote by $\mathbf{z}_t$ the symmetric cut-off strategy of voter t with cutoff values $\underline{z}_t = -z$ and $\overline{z}_t = z$: vote Left for $z_t \leq \underline{z}_t$, Abstain for all $z_t \in (\underline{z}_t, \overline{z}_t)$, and vote Right for all $z_t \geq \overline{z}_t$.

Since any best response is always in cutoff strategies, an equilibrium must have the same feature. Voters with strong preference for one of the alternatives vote for this alternative and voters with weak preference between the alternatives abstain in equilibrium.

Lemma 4.1 (Monotone Best Response of a Voter)**.** *Given any strategy profile v followed by the other voters, the best response for voter t is a symmetric cut-off strategy $\mathbf{z}_t$, defined* *by*

$$\underline{z}_t = \frac{-c}{2I(M(v),T(v))}$$
$$\bar{z}_t = \frac{c}{2I(M(v),T(v))}.$$

Proof. First, for any voter with $z<0$ voting *R*ight is strictly dominated strategy and for any voter with $z>0$ voting *L*eft is strictly dominated strategy. The payoff differences between actions A and L and R and A are the following:

$$\pi(A,z;v)-\pi(L,z;v) = 2zI(M(v),T(v))+c$$
$$\pi(R,z;v)-\pi(A,z;v) = 2zI(M(v),T(v))-c.$$

A voter weakly prefers to vote *L*eft if the first difference is non-positive and she weakly prefers to vote *R*ight if the second difference is non-negative. Therefore, all types $z < -\frac{c}{2I(M(v),T(v))}$ vote left and all types $z > \frac{c}{2I(M(v),T(v))}$ vote right. □

From the above equations, one can also see that $\underline{z}_t < \bar{z}_t$ as the influence is always positive. Hence, there is a strictly positive measure of types who abstain. In particular we have the following corollary:

Corollary 4.1. *Voters who are indifferent between the two alternatives, i.e. with preference parameter* 0 *abstain.*

Example. In this example, I calculate the equilibrium for an electorate such that the voters preferences are uniformly distributed, i.e. $f(z)=1/2$ for all $z\in[-1,1]$. Consider the influence function derived in Section 4.2, $I(M)=\alpha/2-(\alpha/2)M^2$ Hence, x represents an equilibrium if $2(\alpha/2-\alpha/2M(x)^2)x=c$. Notice that the margin is necessarily zero in this electorate, since for all z, $F(-z)=1-F(z)$. Therefore for general α and c, the cutoff types in equilibrium as $\underline{x}=-(c/\alpha)$ and $\bar{x}=c/\alpha$. Finally, the turnout here is $1-c/\alpha$.

Next, I discuss existence and uniqueness of equilibrium. Every symmetric strategy profile with symmetric cutoff types is characterized by a single number $z\in(0,1]$ defining the right cutoff type in every voter's cutoff strategy $\mathbf{z}_t$. I use the notation $\mathbf{z}$ for the corresponding symmetric strategy profile v.

The condition for this symmetric strategy profile being an equilibrium is that the expected benefit from voting for the cutoff types $\underline{z}$ and $\bar{z}$ is equal to the participation costs

c:[5]

$$\begin{aligned} -2\underline{z}I(M(\mathbf{z}),T(\mathbf{z})) &= c \qquad (4.1) \\ 2\overline{z}I(M(\mathbf{z}),T(\mathbf{z})) &= c. \end{aligned}$$

One can see that these equations are closely related, namely, whenever there is a $z \in (0,1]$ that satisfies the second one such that $\mathbf{z} = (-z,z)$ then $-z$ satisfies the first one. Therefore, I investigate only one of the above indifference conditions. Moreover, with a slight abuse of notation, I write M and T as functions of z rather than $\mathbf{z}$.

Finally, I define a partial order on the set of cutoff strategies.

Definition 4.1. *A cutoff strategy* $\mathbf{x}$ *is* more extreme *than* $\mathbf{y}$ *if*

$$\underline{x} < \underline{y} < \overline{y} < \overline{x}.$$

Theorem 4.1 (Existence)**.** *Define* $\overline{c} = 2I(0,0)$. *If* $c \geq \overline{c}$ *then the unique equilibrium is such that no one votes.*

If $c < \overline{c}$ *then there exists Nash Equilibrium in monotone strategies,* $\mathbf{x}$*, with non-zero turnout. In all equilibria, the critical types* $\underline{x}$ *and* $\overline{x}$ *are symmetric about 0. Moreover, if* $\mathbf{x}$ *and* $\mathbf{y}$ *are both equilibrium strategies then the critical values are ordered and the equilibrium with more extreme cutoffs has lower turnout and higher margin.*

Proof. First, by the previous argument in any equilibrium $\underline{z} + \overline{z} = 0$. If $c > \overline{c}$ then $2zI(M(z),T(z)) \leq \overline{c} \leq c$ for all z and the only equilibrium is that no voter participates.

If $c < \overline{c}$ the function $2zI(M(z),T(z))$ is continuous in z and takes on the value $2I(0,0) = \overline{c} > c$ at $z = 1$ and the value $0 < c$ at $z = 0$. By the intermediate value theorem there must be $z \in (0,1]$ with $2zI(M(z),T(z)) = c$. □

The next Theorem assumes that the influence function does not depend on the average turnout and states a sufficient and necessary condition such that the equilibrium is unique for *all* influence function and cost parameters.[6] Uniqueness requires that the preference distribution reflects a uniform population bias for either party.

[5]Note the form of the marginal benefit of voting here. First, there is no need to tie break here, since the margin is a continuous variable, second, the multiplier 2 comes from the fact that the benefit from changing the election outcome for a voter with type z is actually $2z$, the difference between the utilities assigned to the two alternatives.

[6]Notice that the failure of this condition does not mean that there are necessary multiple equilibria. If the condition is violated, there may exist influence functions such that the equilibrium is unique, however this will not be true for all influence functions.

Theorem 4.2 (Uniqueness). *If $c < \overline{c}$ then the voting game has a unique equilibrium for all influence functions I and costs c, if and only if it is either $f(-z) \leq f(z)$, for all $z \in (0,1]$, or $f(-z) \geq f(z)$ for all $z \in (0,1]$.*

Proof. The strategy $(-z, z)$ is an equilibrium if and only if it satisfies the indifference conditions, in other words, if the marginal benefit from voting $2zI(M(z))$ is equal to the cost of voting c.
IF: Next notice that $M'(z) = -f(z) + f(-z)$ therefore the condition implies that the derivative of the margin is either weakly increasing or weakly decreasing. Given that $M(1) = 0$, it follows that $|M(z)|$ is weakly decreasing in the cutoff z. Thus, $I(M(z))$ is weakly increasing in z. Finally, the marginal benefit is a product of a weakly and a strictly increasing function, hence it is strictly increasing. Thus, the equilibrium is unique.
ONLY IF: I show that whenever the condition of the theorem fails then there exists an influence function and voting cost such that there are multiple equilibria.

The condition fails if and only if there exists $z', z'' \in [0,1]$ such that $f(-z') > f(z')$ and $f(-z'') < f(z'')$. This is equivalent to having $z', z'' \in [0,1]$ such that $M'(z') > 0$ and $M'(z'') < 0$. So assume that this is the case.

STEP 1: I show that there is a z such that $M(z) > 0$ and $M'(z) > 0$ or a z' such that $M(z) < 0$ and $M'(z) < 0$.

We know that $M(1) = 0$.

If $M(0) > 0$ then it is either true that for all $z \in [0,1]$, $M(z) \geq 0$ or there exists $\hat{z} \in [0,1]$ such that $M(\hat{z}) < 0$.

Consider the case when for all $z \in [0,1]$, $M(z) \geq 0$. By assumption there is a $z' \in [0,1]$ such that $M'(z') > 0$. Then $M(z')$ is either strictly positive itself or $M(z') = 0$. If $M(z') = 0$ by the continuous differentiability of the election margin in the critical type, there exists $z^o > z'$ close to z' such that $M'(z^o) > 0$ and $M(z^o) > 0$.

Now consider the case when a $\hat{z} \in [0,1]$ exists such that $M(\hat{z}) < 0$. Then since $M(0) > 0$ and $M(1) = 0$ by the differentiability of the margin in the critical type there must be a $z^o \in [0,1]$ such that $M'(z^o) < 0$ and $M(z^o) < 0$.

If $M(0) < 0$ a similar proof shows that there exists z^o such that the signs of $M(z^o)$ and of $M'(z^o)$ coincide.

STEP 2: I show that Step 1 implies the claim.

Pick the value z^o such that the the signs of $M(z^o)$ and of $M'(z^o)$ coincide. Without loss of generality assume that $M'(z^o) > 0$ and $M(z^o) > 0$. Then there exists an influence function, that is sensitive enough to the changes in the election margin at the value $M(z^o)$, such that $\frac{\mathrm{d}}{\mathrm{d}z} 2zI(M(z))|_{z=z^o} < 0$. Let $c = 2z^o I(M(z^o))$.

Then there exists a $\tilde{z} < z^o$ such that $2\tilde{z}I(M(\tilde{z})) > 2z^oI(M(z^o)) = c$.

We know that $2zI(M(z)) = 0$ if $z = 0$. Therefore by the Intermediate Value Theorem, there is a $z^* \in [0, \tilde{z}]$ such that $2z^*I(M(z^*)) = c$.

Thus both z^o and z^* are equilibria. $\square$

4.4 Election Margin and Voter Turnout

After having characterized the equilibria of the voting game in the previous section, I now focus on the comparative statics of equilibrium turnout and margin with respect to changes of the preference distribution. I prove the main result of this paper, Theorem 4.3. If polarization is weakly higher under distribution G than under F and some some equilibrium under G has a weakly lower margin than some equilibrium under F then the equilibrium under G must have weakly higher turnout.

Before I turn to the formal discussion and the definition of the polarization of a preference distribution, I would like to take up the example from the introduction and use the model to show how the first intuition on the negative correlation of election margin and voter turnout can be misleading in general.

Example. Recall the example in Section 4.3 calculating the equilibrium for a uniform electorate. Here, I find the equilibrium for a more polarized type distribution than the uniform one and show that it has both higher margin and turnout. Define the following type distribution:

$$g(z) = \begin{cases} 1 & \text{if} \quad z < -0.8 \\ 1/4 & \text{if} \quad z \in [-0.8, 0.8] \\ 2 & \text{if} \quad z > 0.8. \end{cases}$$

Consider the parameter values $\alpha = 0.2$ and assume that the cost of voting is 0.1. Substituting these values into the formula derived in the previous example, we get that in case of a uniform electorate the critical preference intensity is 0.5, the equilibrium margin is zero while the turnout is 0.5. Now, I calculate the related values under the reference distribution defined above.

As before, the equilibrium is characterized by the indifference condition: $2(0.2/2 - 0.2/2M(x)^2)x = 0.1$. Notice that whenever the critical type $x < 0.8$, the election margin is: 0.2(2-1) = 0.2. Therefore, if x is a solution of the equation $0.2(1-0.04)x = 0.1$ and $x < 0.8$ then it represents an equilibrium. Finally, the value $x = 50/96 < 0.8$ satisfies the indifference condition. Hence, there is an equilibrium with election margin 0.2 and turnout $1-(25/96)$, and both of these values are higher than the respective values under the uniform type distribution.

The equilibrium is also unique in this example. Hence, no $z<x$ can satisfy the indifference condition since $M(z)=M(x)=0.2$ and therefore $(\alpha-\alpha M(z)^2)z<(\alpha-\alpha M(x)^2)x=c$. Also, if $z>x$, $M(z)\leq M(x)$ and hence $(\alpha-\alpha M(z)^2)z>(\alpha-\alpha M(x)^2)x=c$.

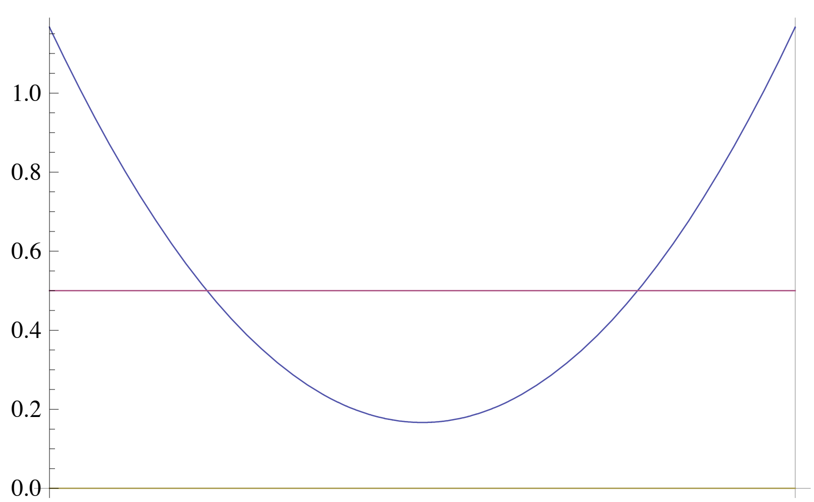
While it is true that a closer election under the uniform distribution is associated with "more types participating in the election", this effect is overcompensated by the fact that there are more extreme voters under the distribution G.

The example above illustrates the need to disentangle the change of voters' preferences shifting from left to right from the change of voters' preferences becoming more extreme (for either one of the possible outcomes).

I now introduce the formal notion of a distribution having the same or a higher polarization than another. This notion needs to capture a sense of a large part of the electorate having strong preferences of L over R (represented by z close to 0) or strong preferences of R over L (represented by z close to 1).

Definition 4.2. *Distribution G is weakly* more polarized *than distribution F if and only if $\forall z\in(0,1]$: $G(-z)+1-G(z)\geq F(-z)+1-F(z)$.*

A distribution is more polarized than an other if there are more voters with strong preference for either outcome under the first distribution than under the second one. Note that this defines solely a partial order on distributions. For arbitrary distributions F, G it can generally be the case that there are both more indifferent voters (with z close to 0) and more extreme voters (with z close to -1 or 1) under distribution F than under G. The next graph shows two possible type distributions with different level of polarization (the type on the horizontal and the density on the vertical axes).



An alternative way to define an order for polarization is based on the average absolute preference of the voters: $\int_{-1}^{1}|z|f(z)dz$ or the average squared preference of the voters $\int_{-1}^{1}|z|^2 f(z)dz$. While the former weights differences in preferences equally across the whole spectrum the latter emphasizes changes of types that are already extreme.

Actually, it is possible to show that the above definition of polarization can be interpreted as the joint of all possible such measures.

Lemma 4.2. *The distribution F has the same polarization as the distribution G if and only if*

$$\int_{-1}^{1}|z|^{\alpha}f(z)dz = \int_{-1}^{1}|z|^{\alpha}g(z)dz \tag{4.2}$$

for all values of $\alpha \in [0,\infty]$.

Proof. ONLY IF: The distributions F and G have the same polarization if and only if the associated pdfs satisfy $f(-z)+f(z) = g(-z)+g(z)$ for all $z \in (0,1]$. Thus,

$$\int_{-1}^{1}|z|^{\alpha}f(z)\mathrm{d}z = \int_{0}^{1}|z|^{\alpha}(f(-z)+f(z))\mathrm{d}z = \\ \int_{0}^{1}|z|^{\alpha}(g(-z)+g(z))\mathrm{d}z = \int_{0}^{1}|z|^{\alpha}g(z)\mathrm{d}z.$$

IF: Set $\psi(z) \equiv f(-z)+f(z)-g(-z)-g(z)$. Then the condition in (4.2) implies that for all $n \in \mathbb{N}$, $\int_0^1 z^n\psi(z)\mathrm{d}z = 0$. Also, $\psi(z) = 0$ for all z is equivalent to the definition of polarization. Hence, to show the if part of the statement, I have to argue that $\int_0^1 z^n\psi(z)\mathrm{d}z = 0$ for all n implies $\psi(z) = 0$. Notice that if $\int_0^1 z^n\psi(z)\mathrm{d}z = 0$ then for any $p(z)$ polynomial of z, it is true that $\int_0^1 p(z)\psi(z)\mathrm{d}z = 0$. Therefore,

$$\int_0^1 \psi(z)^2\mathrm{d}z = \int_0^1 \psi(z)(\psi(z)-p(z))\mathrm{d}z \le \max_z|\psi(z)-p(z)|\int_0^1|\psi(z)|\mathrm{d}z.$$

But by the Weierstrass approximation theorem[7] we can uniformly approximate any continuous function $\psi(z)$ by a polynomial $p(z)$. Thus the RHS can be made arbitrarily small and thus the LHS must be equal to 0, hence $\psi(z) = 0$. □

Among preference distributions with the same polarization, the equilibrium turnout is already determined by the critical voters. While, the critical voters are determined by the election margin. Therefore the intuition that the turnout is higher when the margin is lower is definitely true for changes in preference distributions that leave the polarization unchanged, or that increase the polarization. In this later case, one has to be careful about

[7] See Theorem 1.1 in Rivlin (1981, pp. 1). The theorem states that given a function $f(x)$ that is continuous on $[a,b]$ and $\varepsilon > 0$, there exists a polynomial $p(x)$ such that $|f(x)-p(x)| < \varepsilon$ for all $x \in [a,b]$.

the effect of the turnout on the influence, however, a higher turnout decreases the influence even further and, hence, decreases the preference intensity of the critical types.

This relation between the election margin and the turnout is formalized in the next Theorem.

Theorem 4.3. *If the distribution G is weakly less (more) polarized than the distribution F and the margin is weakly higher (lower) in an equilibrium y under G than in an equilibrium x under F, i.e.* $|M_G(y)| \geq |M_F(x)|$ $(|M_G(y)| \leq |M_F(x)|)$ *then the equilibrium y has a weakly lower (higher) turnout than the equilibrium x, i.e.* $T_G(y) \leq T_F(x)$ $(T_G(y) \geq T_F(x))$.[8]

Proof. I prove only one of the statements in the Theorem. I show that if the equilibrium margin weakly increases while the type distribution becomes weakly less polarized then the turnout weakly decreases. Assume, on the contrary, that the turnout corresponding to the strategy y and the preference distribution G is strictly higher than the turnout corresponding to the strategy x and the preference distribution F. Since the distribution G is weakly less polarized than the distribution F, it is necessary that $y < x$. Thus, the equilibrium conditions require that $I(M_G(y), T_G(y)) > I(M_F(x), T_F(x))$ which implies that $|M_G(y)| < |M_F(x)|$ and that is a contradiction. □

Next, I discuss what kind of shifts in the preference distribution can affect the election margin to rise or fall. I consider a shift in the type distribution such that it represents a new electorate that prefers the right alternative even more. I characterize the change in the equilibrium margin for shifts of this kind. For this result, I assume that the turnout has no effect on the influence function.

Lemma 4.3. *Consider a preference distribution* F_0 *on* $[-1,1]$ *such that for all* $z \in [0,1]$ *the margin* $M(z,0) \equiv 1 - F_0(z) - F_0(-z) = 0$. *Then consider a family of distribution functions* F_a *such that for all z,* $F_a(z)$ *is continuously differentiable and strictly decreasing in a.*[9]

Let $x \in (0,1)$ *be an equilibrium under the preference distribution* F_α. *If*

$$\frac{\partial}{\partial z} 2zI(M(z,a))|_{(x,\alpha)} \neq 0$$

and ε *is small, then there exists a unique continuous equilibrium selection* $\xi : [\alpha - \varepsilon, \alpha + \varepsilon] \to (0,1)$ *such that* $\xi(\alpha) = x$. *Moreover, the election margin* $M_{F_a}(\xi(a))$ *increases in a whenever* $\frac{\partial}{\partial z} 2zI(M(z,a))|_{(x,\alpha)} > 0$ *and it decreases in a whenever* $\frac{\partial}{\partial z} 2zI(M(z,a))|_{(x,\alpha)} < 0$.

[8] Notice, that if $F = G$ this theorem implies that whenever the game has multiple equilibria they are such that a higher turnout coincides with lower margin.

[9] Note that if $\alpha' > \alpha$ then the distribution $F_{\alpha'}$ dominates the distribution F_α in the sense of first order stochastic dominance.

Additionally, if x is the equilibrium with the highest or the lowest level of participation among the equilibria under F_α, then the equilibrium margin necessarily increases in a.

Proof. Denote by $\psi(z,a) \equiv 2zI(M(z,a))$. Then $z \in (0,1)$ is an equilibrium under $F_a(z)$ if $\psi(z,a) = c$, hence $\psi(x,\alpha) = c$.

By assumption, $\psi(z,a)$ is continuously differentiable in both z and a. Therefore, the Implicit Function Theorem implies that, if

$$\frac{\partial}{\partial z}\psi(z,a)|_{(x,\alpha)} = \frac{\partial}{\partial z}2zI(M(z,a))|_{(x,\alpha)} \neq 0$$

then there is a unique function $\xi(a)$ on an open neighborhood A of α, mapping to $(0,1)$ such that $\psi(\xi(a),a) = c$ for all $a \in A$.

Also, $\xi(\alpha) = x$.

Moreover,

$$\left.\frac{\mathrm{d}\xi}{\mathrm{d}a}\right|_{a=\alpha} = -\left.\frac{\partial\psi/\partial a}{\partial\psi/\partial z}\right|_{(x,\alpha)}.$$

Using the definition of ψ, the derivative with respect to a

$$\frac{\partial}{\partial a}\psi(z,a) = 2zI'(M(z,a))\frac{\partial}{\partial a}M(z,a).$$

The above expression is strictly negative since $I' < 0$ and

$$\frac{\partial}{\partial a}M(z,a) = -\frac{\partial F_a(z)}{\partial a} - \frac{\partial F_a(-z)}{\partial a} > 0$$

by assumption.

Therefore the sign of $\frac{\mathrm{d}\xi}{\mathrm{d}a}$ at $a = \alpha$ is determined by the sign of

$$\frac{\partial}{\partial z}\psi(z,a) = \frac{\partial}{\partial z}2zI(M(z,a))|_{(x,\alpha)}.$$

Hence, marginal voter gets more extreme in a whenever

$$\frac{\partial}{\partial z}2zI(M(z,a))|_{(x,\alpha)} > 0$$

and less extreme whenever

$$\frac{\partial}{\partial z}2zI(M(z,a))|_{(x,\alpha)} < 0.$$

To finish the argument, notice that the lemma is stated for influence functions with no turnout effect. Thus the indifference conditions imply that the absolute value of the equilibrium margin is higher whenever the marginal voter is more extreme and it is lower whenever the marginal voter is less extreme.

Finally, notice that if x is the equilibrium with the highest level of participation then the sign of $\frac{\partial}{\partial z} 2zI(M(z,a))|_{(x,\alpha)}$ is necessarily positive. Assume on the contrary that it is negative, then there is an $x' < x$ such that $2x'I(M(x',a)) > c$. Since $2zI(M(z,a)) = 0$ for $z = 0$, by Intermediate Value Theorem there is an $x^* < x' < x$ with $2x^*I(M(x^*,a)) = c$. Thus x is not the equilibrium with the highest turnout. The proof for the equilibrium with the lowest level of participation is analogous, considering that $2zI(M(z,a)) > c$ for $z = 1$. □

By Theorem 4.3, we can conclude that if the shift in the preference distribution from F_α to $F_{\alpha'}$ at the same time decreases polarization, then the turnout decreases in the equilibria with the highest and the lowest participation.

Above, I discussed shifts in preferences such that voters' support one alternative even more. Before I end this section, I take a quick look at a complementary change in preferences that only affects polarization but does not represent a shift in preferences to one side or the other. For this result, I again assume that the turnout has no effect on the influence.

Lemma 4.4. *Consider a change in the political landscape from preference distribution F to G so that G has higher polarization while the relative preferences for the left and the right party are the same, i.e. for all $z \in (0,1]$, $f(-z) - f(z) = g(-z) - g(z)$.*[10] *Then, if x is an equilibrium given the preference distribution F then x is an equilibrium given G as well. Moreover, the corresponding equilibrium margin $M_F(x)$ is equal to the equilibrium margin $M_G(x)$ while the corresponding equilibrium turnout is unambiguously higher under G.*

Proof. The condition $f(-z) - f(z) = g(-z) - g(z)$ implies that the election margins $M_F(z)$ and $M_G(z)$ coincide for all symmetric cut-off strategy profiles $(-z,z)$. Therefore, if x is an equilibrium under F, i.e. it satisfies the conditions in (4.1) then it is an equilibrium under G as well. However, the equilibrium turnout is unambiguously higher for the preference distribution G, since that is more polarized. □

[10]Notice that two distributions may represent the same relative preferences for the left and the right party without being the same. For example, any symmetric preference distribution has equally strong relative preferences for the left and the right party.

4.5 Conclusion

I proposed a tractable game-theory model of large elections. My key assumptions were the strictly positive voting cost and strategic voters who misperceive their influence on the election outcome. I have shown that in equilibrium the margin decreases and the turnout increases as the distribution of the voters' preference shifts but the polarization of the preferences remains constant or increases. I also provided an example in which the election margin and turnout exhibited positive relationship. My result sheds light on the importance of polarization of the voters' preference in explaining election turnout and suggests an empirical test of the model where turnout is regressed on closeness controlling for measures of polarization.

4.6 Appendix: Influence in the Finite Model

The negative relation between turnout and margin in Theorem 3 depends crucially on the fact that a voter's perceived probability of being pivotal is increasing in the expected margin. However this fact - that $I(M)$ is decreasing in $|M|$ - is an assumption on the subjective probabilities in my model. In this appendix, I relate this assumption to the properties of the exact pivotal probabilities. Also, I discuss the assumptions about the effect of the turnout on the influence compared to the effect on the exact probabilities.

Consider the election with $N+1$ voters, each with preference types z_t i.i.d. according to the cdf $F(z)$. The preferences over the outcomes and participation are as in Section 2. Just as in Lemma 4.1, the best response strategies are determined by cutoff types $(\underline{z}, \overline{z})$ but the indifference conditions are no longer given by the exogenous influence function but by the actual probability of being pivotal. Equations (4.1) thus turn into

$$\begin{aligned} -2\frac{1}{2}\left(P^0(\mathbf{z}) + P^1(\mathbf{z})\right)\underline{z} &= c \\ 2\frac{1}{2}\left(P^0(\mathbf{z}) + P^{-1}(\mathbf{z})\right)\overline{z} &= c \end{aligned}$$

where $P^m(\mathbf{z})$ is the probability that the realized margin from N voters, each voting according to the cutoff strategy $(\underline{z}, \overline{z})$, is m. Assuming that ties are broken by a fair coin toss, the probability that a potential left voter is pivotal is $\Pi^L_{\mathbf{z}} = \frac{1}{2}(P^0(\mathbf{z}) + P^1(\mathbf{z}))$. Thus, the first equation is the indifference condition for the marginal left voter $\underline{z} < 0$ and the second equation is the equivalent condition for the marginal right voter $\overline{z} > 0$.

Now, I express the pivotal probabilities by the preference distribution and the cutoff types. Then, I change variables and rewrite the expressions by the average margin and turnout. This form allows me to discuss the properties of interest.

Denote by $p(l, r, N-l-r)$ the probability that exactly l voters vote *L*eft and r vote *R*ight under strategy $\mathbf{z}$.

$$p(l, r, N-l-r) = \frac{N!}{l!r!(N-l-r)!} F(\underline{z})^l (1 - F(\overline{z}))^r (F(\overline{z}) - F(\underline{z}))^{N-l-r}$$

Now, I can express the probabilities that the realized margin takes up the values $-1, 0$ and

1.

$$
\begin{aligned}
P^0(\mathbf{z}) &= \sum_{k=0}^{[N/2]} p(k,k,N-2k) \\
P^1(\mathbf{z}) &= \sum_{k=0}^{[(N-1)/2]} p(k,k+1,N-2k-1) \\
P^{-1}(\mathbf{z}) &= \sum_{k=0}^{[(N-1)/2]} p(k+1,k,N-2k-1)
\end{aligned}
$$

Note that the average margin is given by $M = M(\mathbf{z}) = 1 - F(\bar{z}) - F(\underline{z})$ and the average turnout is given by $T = T(\mathbf{z}) = 1 - F(\bar{z}) + F(\underline{z})$. I can thus express the critical probabilities $F(\underline{z}) = \frac{T(\mathbf{z})-M(\mathbf{z})}{2}$ and $1 - F(\bar{z}) = \frac{T(\mathbf{z})+M(\mathbf{z})}{2}$ as functions of the average margin and turnout and substitute these expressions back into the probabilities of being pivotal:

$$
\begin{aligned}
P^0(M,T) &= \sum_{k=0}^{[N/2]} \frac{N!}{k!k!(N-2k)!} \left[\frac{T-M}{2}\right]^k \left[\frac{T+M}{2}\right]^k [1-T]^{N-2k} \\
P^1(M,T) &= \sum_{k=0}^{[(N-1)/2]} \frac{N!}{k!(k+1)!(N-2k-1)!} \left[\frac{T-M}{2}\right]^k \left[\frac{T+M}{2}\right]^{k+1} [1-T]^{N-2k-1} \\
P^{-1}(M,T) &= \sum_{k=0}^{[(N-1)/2]} \frac{N!}{(k+1)!k!(N-2k-1)!} \left[\frac{T-M}{2}\right]^{k+1} \left[\frac{T+M}{2}\right]^{k} [1-T]^{N-2k-1}.
\end{aligned}
$$

One can immediately see that the pivotal probabilities are positive. Also, holding the lead of the preferred party constant, the pivotal probability does not depend on the alternatives, i.e. $P^0(M,T) + P^1(M,T) = P^0(-M,T) + P^{-1}(-M,T)$.

The Effect of the Average Margin on the Pivotal Probabilities. With respect to the effect of the average margin, I assume that (i) the influence function is symmetric at zero margin and (ii) it is decreasing in the absolute value of the margin.

Intuitively, a smaller margin increases the probability that the left and the right votes are equally distributed among the voters and the influence function captures this intuition. However, for the exact probabilities, this property is not true in an equally clean way since the probability of being pivotal is equal to average of the probabilities that the realized margin is 0 and -1, $\frac{1}{2}(P^0(M,T) + P^{-1}(M,T))$.[11] It is transparent that $P^0(M,T)$ is symmetric to zero margin, however, the same is not true for $P^{-1}(M,T)$. The probability $P^{-1}(M,T))$ is actually strictly decreasing at zero margin and thus strictly decreasing as we decrease the margin below 0 by a little. This reflects the intuitive fact that the probability of being

[11] Without loss of generality, I consider a right voter.

pivotal is largest when one's own side is slightly behind, by say half a vote in expectation.

Formally, the pivotal probability of a right voter decreases in the margin whenever the right party has a lead, $M \geq 0$ and it increases in the margin whenever the left party has reasonable lead $M < -\varepsilon$ for $\varepsilon > 0$.[12] Partial differentiation of the pivotal probabilities with respect to M yields:

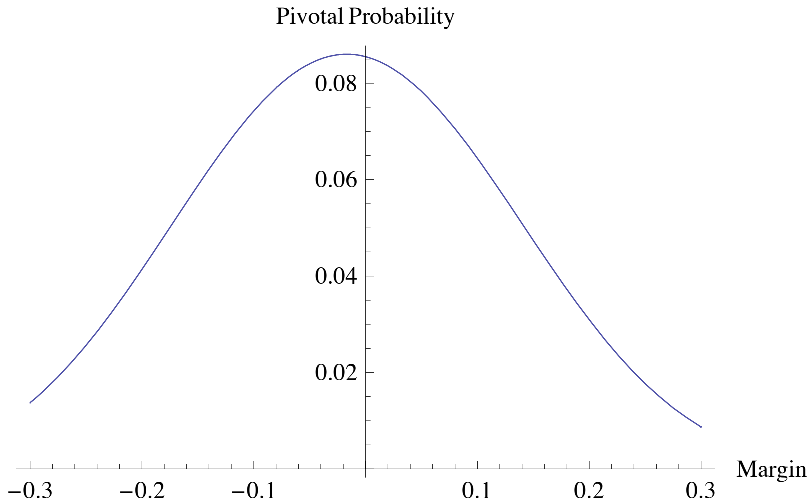
$$
\begin{aligned}
\frac{\partial P^0}{\partial M} &= \sum_{k=0}^{[N/2]} P^0(k)\left[\frac{2k}{T+M} - \frac{2k}{T-M}\right] \\
\frac{\partial P^1}{\partial M} &= \sum_{k=0}^{[(N-1)/2]} P^1(k)\left[\frac{2(k+1)}{T+M} - \frac{2k}{T-M}\right] \\
\frac{\partial P^{-1}}{\partial M} &= \sum_{k=0}^{[(N-1)/2]} P^{-1}(k)\left[\frac{2k}{T+M} - \frac{2(k+1)}{T-M}\right]
\end{aligned}
$$

where $P^0(k)$, $P^1(k)$ and $P^{-1}(k)$ are the kth elements of the respective probabilities. Notice that for $M \geq 0$ both P^0 and P^{-1} are sums of elements that are individually weakly decreasing in the margin. The kth element of $\frac{\partial P^0}{\partial M}$ is the product of the positive probability that exactly k left an k right voters participate and the term $\left(\frac{2k}{T+M} - \frac{2k}{T-M}\right)$ which is necessarily non-positive given $T+M \geq T-M$. Similarly, the kth element of $\frac{\partial P^{-1}}{\partial M}$ is the product of the positive probability that exactly $k+1$ left an k right voters participate and the term $\left(\frac{2k}{T+M} - \frac{2(k+1)}{T-M}\right)$ that is necessarily non-positive.

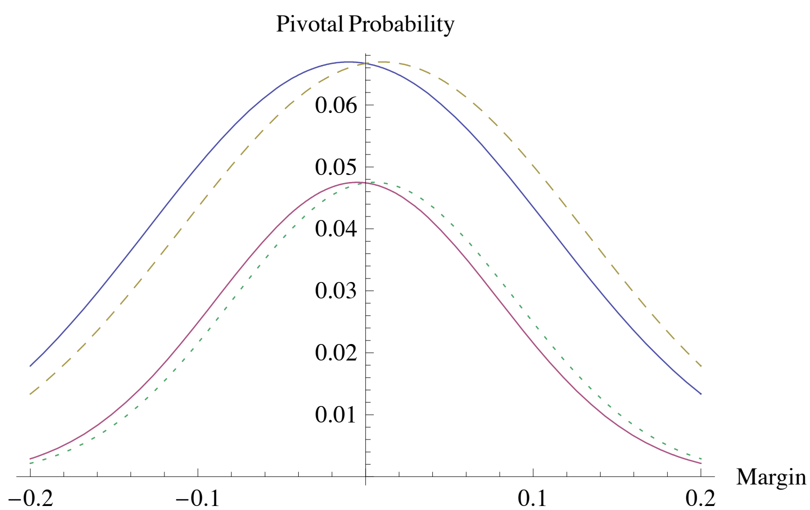
For $M < 0$, the probability P^0 is increasing in M while the properties of P^{-1} are ambiguous. The sign of any element of the derivative again depends on the sign of $\left(\frac{2k}{T+M} - \frac{2(k+1)}{T-M}\right)$ which is positive for all k whenever $\frac{T-M}{T+M} > 2$. So, this condition ensures that the pivotal probability increases in the margin. However, the above condition is only sufficient and the derivative can be positive even for higher values of M (lower absolute values).

The next two graphs illustrate the discussion above. The first graph shows the pivotal probability of a right voter as a function of the average margin if the size of the electorate is $N = 30$ and the expected turnout is $T = 0.7$. Changing these parameter values does not alter the graph qualitatively.

[12] I was not able to derive the value of ε analytically.



The second graph suggests that the pivotal probability becomes symmetric to the margin as the size of the electorate increases. I plot the pivotal probability of a right voter as a function of the expected margin for electorate sizes $N = 50$ and $N = 100$. The curves indicating higher pivotal probabilities gives the values for the population with size $N = 50$. To indicate asymmetry, I included the pivotal probability of a left voter here, marked with dashed and dotted lines.



The Effect of the Average Turnout on the Pivotal Probabilities. I assume that the influence is decreasing in the average turnout.

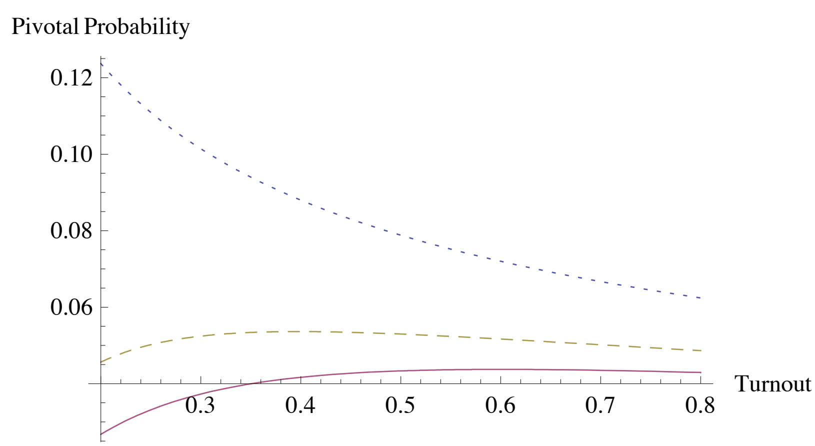
The effect of the turnout on the pivotal probabilities is ambiguous. As the turnout increases, it is more likely that a higher number of people vote, hence the chance of being pivotal decreases. However, keeping the margin constant, increasing turnout levels the expected number of left and right voters and hence increases the chance of being pivotal. These two effects are shown in the partial derivatives.

$$\begin{aligned}
\frac{\partial P^0}{\partial T} &= \sum_{k=0}^{[N/2]} P^0(k)\left[\frac{2k}{T+M}+\frac{2k}{T-M}-\frac{N-2k}{1-T}\right] \\
\frac{\partial P^1}{\partial T} &= \sum_{k=0}^{[(N-1)/2]} P^1(k)\left[\frac{2(k+1)}{T+M}+\frac{2k}{T-M}-\frac{N-2k-1}{1-T}\right] \\
\frac{\partial P^{-1}}{\partial T} &= \sum_{k=0}^{[(N-1)/2]} P^{-1}(k)\left[\frac{2k}{T+M}+\frac{2(k+1)}{T-M}-\frac{N-2k-1}{1-T}\right]
\end{aligned}$$

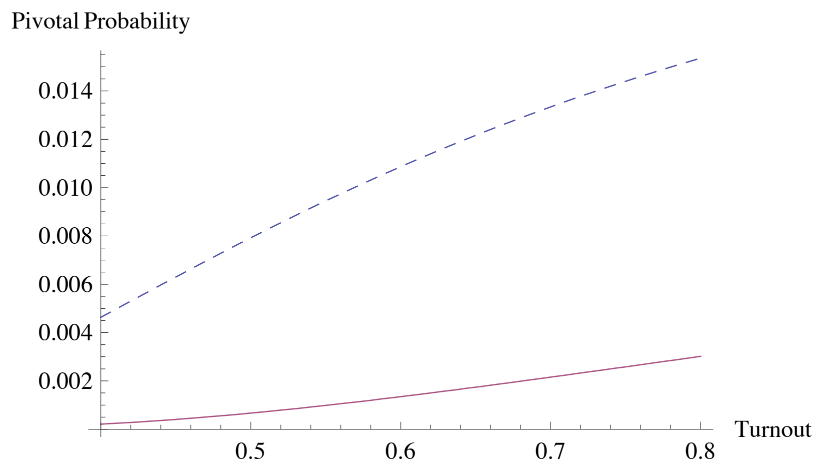
For example, in case of P^0, the first two terms in the expression $\left[\frac{2k}{T+M}+\frac{2k}{T-M}-\frac{N-2k}{1-T}\right]$ refers to the positive effect while the last one reflects the case that increasing turnout, making the expected number of actual voters higher decreases the chance that a voter is pivotal.

Numerical computations show that (i) the positive effect disappears for zero margin but (ii) for high absolute value of margin, the positive effect overtakes as the size of the electorate increases.

The following graph shows the pivotal probabilities as a function of average turnout if the size of the electorate is $N = 50$ and the expected margin is $M = 0$ (dotted line) or $M = 0.1$ (dashed line) or $M = -0.1$ (unbroken line).



To illustrate the positive effect of the turnout on the pivotal probabilities when the margin is relatively high, the next graph plots the pivotal probabilities as a function of the turnout for electorate sizes $N = 50$ (dashed line) and $N = 100$ (unbroken line) and a margin $M = .2$.



Finally, numerical computations also show that the effect of the margin on the pivotal probability is relatively stronger than the effect of the turnout, as the size of the electorate increases, except when, the margin is close to zero.

Chapter 5

Conclusion

My dissertation presented three papers on strategic voting.

In the first paper, which is written jointly with Lones Smith, we proposed a dynamic model of deliberation in a two-member committee. We assumed that the decision making presumes time-cost. We explored the monotone properties of the game and described the set of equilibria. Our main contribution was to show that in a deliberation process with positive time-cost the verdict is almost instantaneous and approximately information-efficient as the period-length vanishes. This result suggests that dynamic models of group decision-making are able to properly aggregate members' private information.

In the second paper, I studied the joint-decision problem of a committee of privately informed individuals whose interests are aligned. I argued that if multiple votes are available for the committee members, then the quality of the joint decision can improve. I also showed that if the committee members have conditionally independent signals, the efficient verdict can be reached if there are a sufficient number of votes available. I discussed that with correlated private information, full efficiency may not be possible for any number of votes. Moreover, I gave a necessary and sufficient condition on the joint distribution of the private signals to ensure that the decision by the committee is efficient. To summarize, allowing multiple votes compared to a single vote improves a voting mechanism and is a remedy for a certain type of inefficiency in the joint-decision problem. However, the efficient decision with correlated private signals would require a different class of mechanisms.

In the final paper, I proposed a tractable game-theory model of large elections. My key assumptions were a strictly positive voting cost and strategic voters who misperceive their influence on the election outcome. I showed that in equilibrium the election margin decreases and the turnout increases as the distribution of the voters' preferences shifts, but the polarization of the preferences remains constant or increases. I also provided an example in which the election margin and turnout exhibited a positive relationship. My result sheds light on the importance of polarization of the voters' preferences in explaining the election turnout and suggests an empirical test of the model where turnout is regressed on closeness, controlling for measures of polarization.

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