THE UNIVERSITY OF MICHIGAN

INDUSTRY PROGRAM OF THE COLLEGE OF ENGINEERING

A STUDY OF THE RADIATION CONTROLLED ARC WITH EMPHASIS ON ARC DISCHARGE WIND TUNNELS

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A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy in The
University of Michigan
Department of Electrical Engineering
1964

June, 1964

IP-674

PREFACE

Throughout this dissertation I have attempted to create an understanding of the basic scientific principles of the electric arc that drives an arc discharge hypervelocity wind tunnel. The large energy required for the discharge has precluded significant previous investigation. Therefore, I have approached the problem from fundamentals. The data that was used and the assumptions that have been made have been clearly stated.

The data, particularly the photographs of the arc at 100 atmospheres and 50,000 kilowatts will probably be the most valuable contribution that I have made. But the photographs and electrical data could not have been made without the power. There are, therefore, many to whom I owe much. I am sincerely grateful to the following people who contributed so much:

Dean S.S. Attwood, Associate Dean G.V. Edmonson, of the College of Engineering, Prof. W.G. Dow, Chairman, Electrical Engineering, Prof. W.C. Nelson, Chairman, Aeronautical and Astronautical Engineering, and Prof. W. Kerr, Chairman, Nuclear Engineering, for assuming responsibility for the installation of power supply and hypervelocity facility;

P.O. Hays for his cooperation in operating the power supply;

The United States Air Force for supporting the work on the extended electrode surface chamber (Contract No. AF 40(600)-934);

Dr. J.T. Wilson and the Institute of Science and Technology for supporting the arc photography experiments;

H.C. Early, member of my doctoral committee, for guidance, particularly in the early stages;

Prof. J.A. Nicholls, Prof. W. Kerr, Prof. G. Hok and Prof. E.A. Martin, members of my committee, for their constructive criticism of this dissertation.

Prof. W.G. Dow, Chairman of my committee, for his guidance, criticism, help, patience, and hope.

Ann Arbor, Michigan
April, 1964

William N. Lawrence

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CHAPTER I

INTRODUCTION

The conduction of electricity by a gaseous medium has been one of the phenomenon of nature that has intrigued men practically since the beginning of time. Electrical conduction by gases is an extremely complex phenomenon which depends upon a multitude of factors of which only a few can be controlled directly. The laboratory experimentalist can control the current or voltage, but generally not both; the gas composition, the pressure of the gas, the electrode material and the initial geometric configuration of the electrodes. In addition, the experimentalist may cause forces to be applied to the media of the arc column through such methods as the application of a magnetic field, a directed gas jet, or the effective change of the gravitational constant by applying certain accelerations to the vessel which contains the arc discharge. The laws of physics, however, control the method by which the forces are distributed throughout the volume.

Such factors as the mean free path, the collision cross-section, the ionization potential, the atomic structure the thermal conductivity of the gas, and the work function and subsequent geometric configuration of the electrodes after there has been some erosion, the interaction between the contamination from the electrodes and the gas media and the velocity distributions of the various charged and uncharged particles are among the factors which cannot be directly controlled. Quite commonly there is an interaction between all of these various parameters so that the analysis of any one given situation in the broad realm of gaseous conduction, is wrought with numerous difficulties.

The arc discharge had been subjected to very little basic research compared to the amount of effort that went into attaining empirical data prior to say 1950. There was continuing interest in the electric arc as part of the commercial development of switch gear and welding equipment. There has also been some work done in the development of light sources[14] in which reasonably large percentage of the effort has been spent in determining the basic nature of the discharge. At pressures in considerable excess of one atmosphere and electric currents of thousands of amperes there was little experimental work, and very little analytical work, done on the electric arc.

Subsequent to World War II, at least three major technological fields have demanded the development of the basic understanding of the physical processes involved in the high energy, high power electric discharge. First, the possibility of controlling the hydrogen bomb for the production of electric power has led to a broad research program for the study of hydrogen plasma[53] at extremely high temperatures. Second, with a parallel motive, there has been an intensive effort in the field of magneto-hydrodynamics where the ultimate goal is to take high temperature gases and use the internal energy of the gas to directly generate electricity. Third, the demand for high speed military aircraft and the very closely related space vehicles, has required the development of wind tunnels capable of producing very high velocities.

Since the velocity of the gas in the wind tunnel test section is directly related to the stagnation temperature of the gas prior to expansion, it is necessary to preheat the gas in order to attain hypersonic velocities within wind tunnel test sections. The directly heated wind tunnel uses air which has been heated by blowing it across either a pre-heated pebble bed or electrically heated filaments. In order to develop even higher temperatures it is necessary to heat the air with an electric arc. Two basic types of wind tunnels have developed in this line. A continuous running device is normally referred to as an "arc air heater" or plasma jet driven wind tunnel. The other type is heated by an impulse of electrical energy in a high pressure arc chamber. This is the arc discharge or so-called "Hot Shot"[33] wind tunnel.

The simulation of hypersonic flight in an arc discharge wind tunnel requires a high pressure, high temperature, stationary gas which is expanded through a nozzle to the test section of the wind tunnel. The stationary gas pressures are of the order of 10,000 to 100,000 psi at temperatures which are generally in the order of 3000°K to 5000°K. These conditions can only be reached in a closed chamber on an interim or pulse basis.

The power supply for a hot-shot wind tunnel is a very expensive piece of equipment for any university to install because it not only requires expensive apparatus, but it also requires a small building in which to house it. In view of the fact that the supply could also be used to study high power electric arcs and also that the equipment could be used in association with controlled thermonuclear fusion experimental equipment, the University of Michigan undertook a program for the development and installation of the required high power pulse equipment.

The power supply, which will be described in detail later, stores energy in an inductance coil. The coil can be discharged rapidly to generate power pulses up to 6,000,000 KW. The power supply and its associated equipment became operational in 1962. The equipment was used throughout 1962 and the first part of 1963 to develop the present interpretation of the behavior of the electric arc in a closed chamber in very dense gas.

The work that is presented here is the outgrowth of an investigation of the behavior of the arc within a special arc chamber[18] which had been designed for possible use with a hypersonic arc discharge wind tunnel. The original investigation was intended to show the feasibility of utilizing an arc chamber in which the entire interior wall surface of the chamber could be used as an electrode. This was referred to as an "extended electrode surface" arc chamber and was intended to reduce the metallic contamination in the gas. Typical characteristics for the arc discharge were as follows:

Current: 100,000 amperes, initial

Arc voltage: 2000 volts

Time of discharge: 6 milliseconds

Total Energy: 600,000 joules

Initial Pressure: 25 atmospheres or 400 psi

Final Pressure: 600 atmospheres or 10,000 psi

Throughout this text then, the term "high pressure", unless otherwise modified, shall be used to refer to a pressure in the range of 10 to 1000 atmospheres, and the term "high current" shall refer to currents of the order of 50,000 to 100,000 amperes. The background for this dissertation, then, is based on the development of equipment for a specific experimental project. It is the purpose of this dissertation to present some of the experimental results that were attained and to attempt to correlate the experimental results with a theoretical aspect of an electric arc as it existed within this chamber. This dissertation covers three main research tasks which may be briefly summarized as follows:

1) High power pulse equipment was installed to produce electric arcs in the range of 100,000 KW to 400,000 KW in gas at pressures up to 12,000 psi or 800 atmospheres. The voltage, current and gas pressure as functions of time were determined. The erosion marks on the interior of the chamber were used to infer the probable behavior of the arc.

2) The equipment was modified in order to take high speed (14,000 frames per sec) photographs of the arc and thus determine the size and shape of the arc and also the velocity with which it moved.

3) The data that was collected was analyzed in terms of heat transfer theory. The predominant heat transfer mechanism in the early stages of the discharge was found to be radiation with convection and conduction playing relatively minor roles. The theory, as presented here, has been derived specifically for this case.

The special experimental technique that was developed for photographing the arc is a combination of old methods which could be combined only because of the extremely high intensity of the radiation from the arc. The chamber was transformed into a pin-hole camera which put an image of the interior arc onto an exterior ground glass plate. The glass plate image was photographed with a high speed camera.

There has been some photographic work of high pressure electric discharges but there has not been equipment available previously to undertake a study such as this one. It is possible to find information on electric arcs at lower current[44], lower pressures,[29] or significantly shorter discharge times,[40] but there has been no endeavor elsewhere to produce and scientifically study the arc that was photographed here. The photographs that appear in Figures 23, 24 and 25 are, therefore, unique.

The size and shape of the arc renders it possible to analyze this arc in terms of heat transfer mechanisms which constitutes the third part of this dissertation. It should be emphasized early in this work that the mechanism that controls the behavior of this arc is radiation, in contrast to the situation normally encountered in the arc discharge, where either conduction or convection heat losses can be used to describe the behavior of the arc. The derivation of the radiation characteristics of the arc which in turn is based on previous work that has appeared in the literature. However, the exact derivation as used here has not previously appeared explicitly in the literature.

The background theory for the analysis of the electric arc was formulated simultaneously by Suits[56,57,58,59] in the United States of America at the General Electric Co. and by a group which included Steenbeck,[54,55] von Engle,[15] Kesseling,[27,28] and Foitzik[16] among others in Germany at the Siemens-Konzern. Suits work was primarily concerned with arcs where the heat transfer mechanism was free convection and he showed that the arc could be treated as a solid bar under these conditions.

The German group was more interested in the arc where conduction was the primary heat transfer mechanism. Arcs were studied in a rotating chamber to eliminate convection effects. Recent efforts by Maeker,[38,39] Schirmer[50,51] and Goldenberg[22] have yielded excellent agreement between theory and experiment for the conduction controlled arc. Perhaps the most interesting aspect was the evolution of Steenbeck's "Minimum Principle"[55] which postulates the principle that the temperature of the arc shall be adjusted so that the voltage gradient along the arc shall be a minimum.

Dow[11] extended the work of Suits to the case of forced convection and combined the minimum principle into the theory in order to determine the temperature. Thus the temperature relationship which eluded Suits was incorporated into the theory for convectively controlled arcs. The work of Suits and Dow is summarized in Appendix A. The theory for the radiatively controlled arc as developed in Chapter VI follows along the lines that Steenbeck used to describe the conductively controlled arc and that Dow used to describe the forced convection case. Steenbeck did derive a theory which covers any general heat transfer mechanism,[55] but he used such general terms that it is not applicable here. It was also limited in that it could not be used to evaluate the individual parameters, such as arc diameter, because it lacked the concepts of gas dynamics that are associated with the mobility and the Saha Equation.

The minimum principle was stated by Steenbeck as a postulate. He had much experimental evidence but no theoretical argument on which to base it. This author believes that the minimum principle is correct insofar as it can be directly derived from the theory of irreversible processes because the arc temperature-voltage gradient relationship must be based on the laws of thermodynamics. For this reason, a rather extensive discussion of the minimum principle in terms of the stationary states of thermodynamic systems is included.

Part I

EXPERIMENTAL ASPECTS OF THE ARC AND ARC CHAMBER FOR A HYPERVELOCITY WIND TUNNEL

CHAPTER II

EXPERIMENTAL EQUIPMENT

DESCRIPTION OF POWER SUPPLY

The University of Michigan inductive energy power[51] supply stores energy over an extended period of time and then dissipates the energy very rapidly into a load with a very high power pulse. The operation of the system is shown schematically in Figure 1. Electrical energy is taken from a power line over approximately 20 minutes by a 150 horsepower motor. The motor, which is connected to a magnetic clutch, drives a large fly wheel up to a maximum rotational speed of 10,000 RPM. The fly wheel is connected to an Allis-Chalmers Unipolar generator which is capable of transferring the kinetic energy from the fly wheel to a large inductance coil over a period of about 3 seconds. When the current in the coil reaches a maximum, a switch across a load is opened, causing the current to be diverted into the load. The energy stored in the coil is dissipated in a period ranging from 5 to 20 milliseconds, depending on the load characteristics. The resistance between the coil and the load serves to prevent current from passing through the load prior to the opening of the switch.

The motor-generator-coil system is shown in Figure 2. The coil is shown in the background, the generator is on

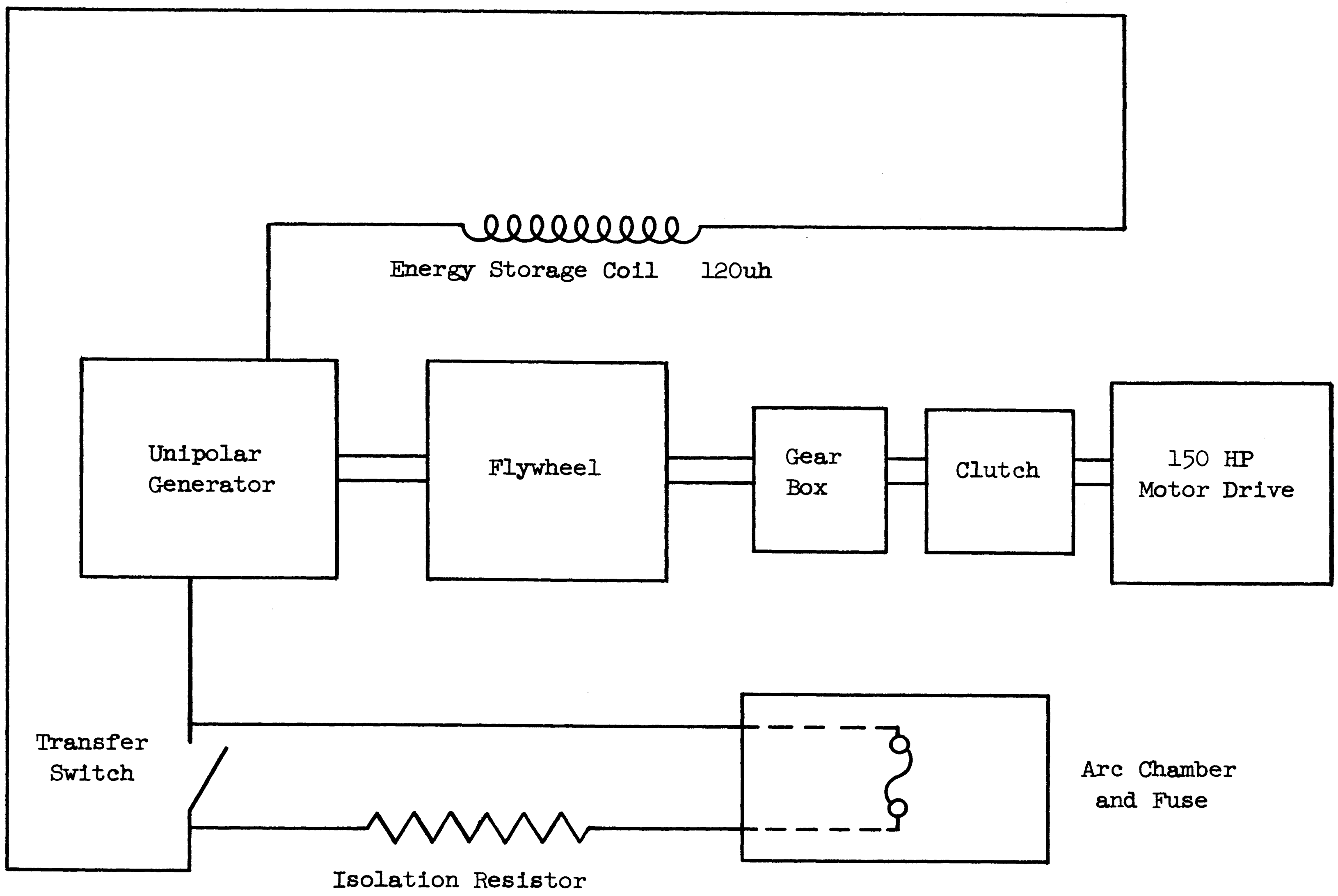


Figure 1. Schematic drawing of the power supply.

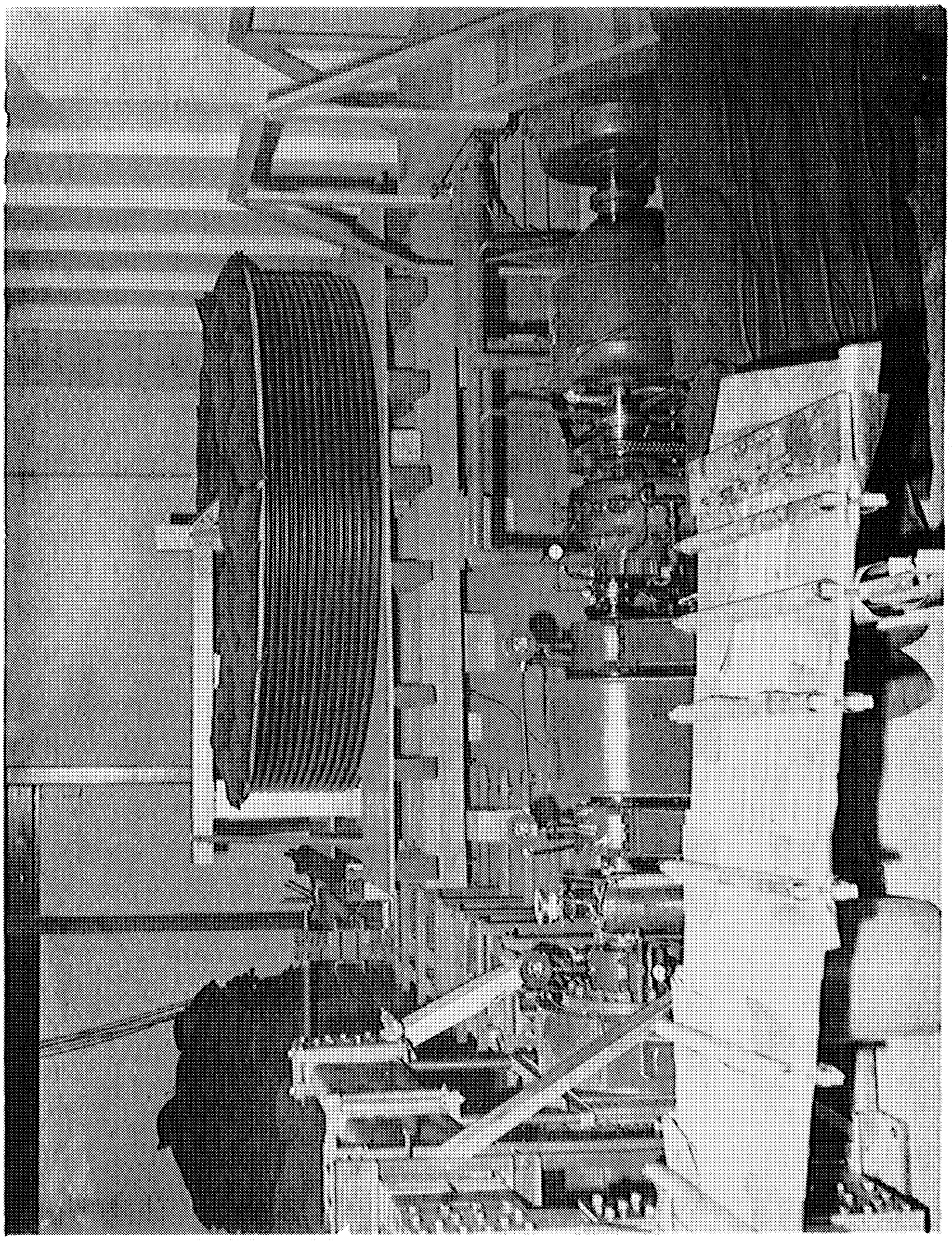


Figure 2. The inductive energy power supply

the left in the foreground. From left to right, the items of equipment are the generator, the flywheel, the gearbox, the magnetic clutch and, at the extreme right, the 150 horsepower electric motor. The flat sheet resistor, which in appearance resembles a metal table, is seen in the extreme foreground.

The generator and flywheel are shown in Figure 3. The Allis-Chalmers Unipolar generator is rated at 7200 RPM, 30 volts and 60,000 amperes. For pulse operation the rating of the generator is considerably higher than the nameplate ratings. For pulse operation the maximum speed of 10,000 RPM will produce a maximum terminal voltage of 45 volts. The absolutely maximum current that could be drawn from this generator has been estimated to be as high as 2,000,000 amperes. The maximum useful current from the generator, that is, the current that could be used for pulse operation without danger of overheating or mechanical failure is approximately 500,000 amperes. The generator is currently installed in a circuit which will produce approximately 300,000 amperes. The generator differs from normal direct current generators in that it uses liquid metal brushes instead of carbon brushes to make electric contact with the spinning rotor. The liquid metal is a eutetic mixture of sodium and potassium, NaK, which is liquid at normal temperatures since the melting point is approximately 60°F.

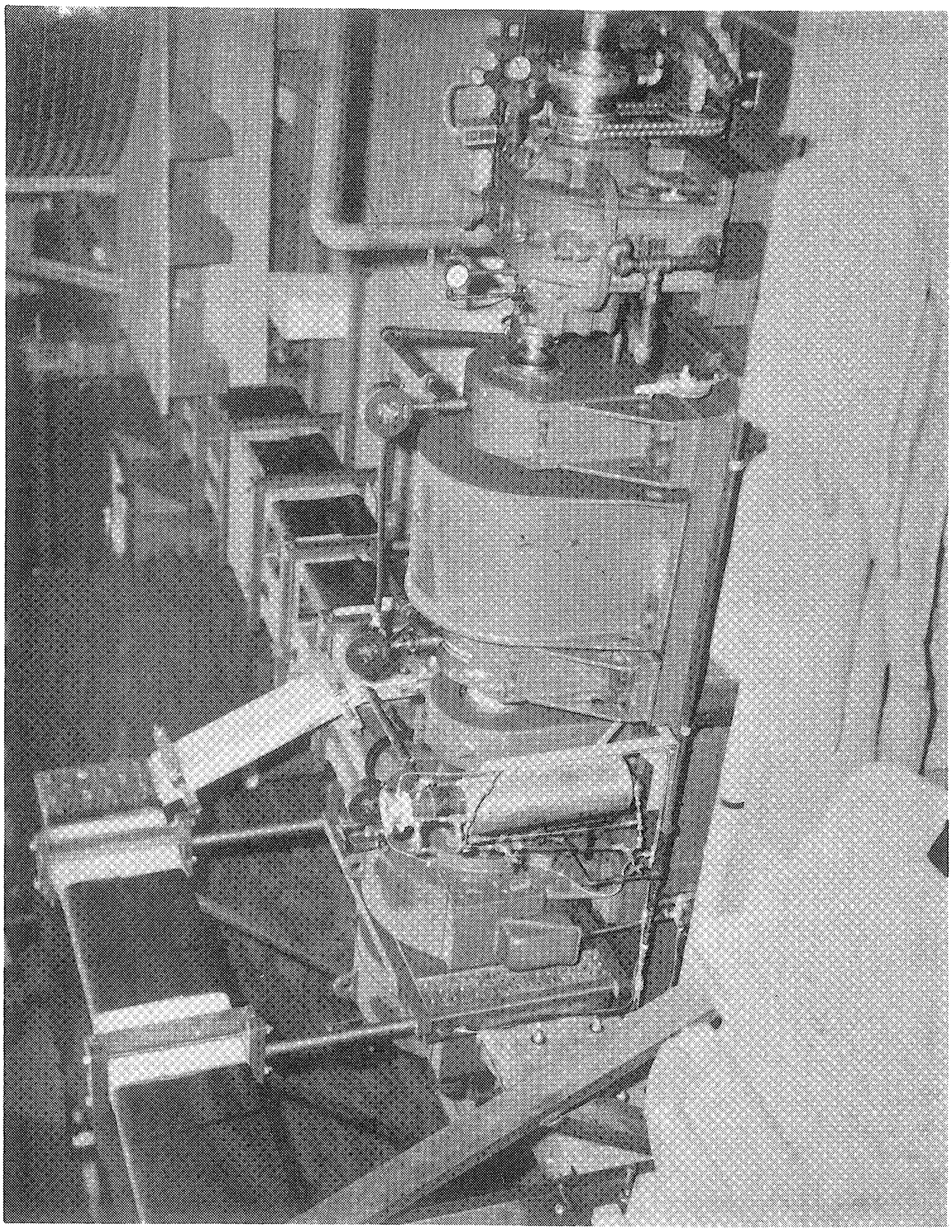


Figure 3. The unipolar generator and twenty megajoule flywheel

The coil has an inductance of 120 microhenries and a DC resistance of 47 micro-ohms. The generator is electrically analogous to a capacitor so that this circuit behaves as a simple RLC circuit to a first approximation. The resistance in the circuit is nearly the resistance for critical damping. The peak current is, therefore, a function of the circuit resistance. The circuit has a total resistance of 100 micro-ohms, including the resistance in the connecting bus bars and switch assembly. This is somewhat more than the 70 micro-ohms in the circuit that is used for the hypersonic wind tunnel. The slightly increased resistance in this circuit reduces the maximum current attainable from the generator to approximately 275,000 amperes. Inasmuch as the maximum current that was used experimentally was only 100,000 amps, this reduction was not a technical limitation during these experiments.

The switch that is required to transfer the current into the arc chamber is a critical part of this system. The development of switch gear was an important part of the development of stored energy as reported by Early and Walker.[17,60] From the beginning of the development work on inductive energy storage, the switch has always been required only to transfer the current into a second device, some form of fusable link, which actually performed the switching. The fuse is replaced after each experiment.

The transfer switch is considerably different from most existing switchgear when considered from a theoretical standpoint. A rather detailed discussion of the theoretical aspects of the transfer switch was given in a report by Dow, Lawrence, and Rozian.[9]. Briefly summarized here, the switch may be contrasted to a standard circuit breaker which would be capable of handling the currents involved in this system, in the following manner:

> In a normal circuit breaker there is a "race" between the voltage recovery and the dielectric recovery in the gap created by the opening switch. In the case of the transfer switch the race can be controlled by the operator to allow sufficient time for the gap to recover its dielectric strength sufficiently to withstand the voltage required by the arc in the arc chamber. The arc required less than 4000 volts during the sequence of experiments when a fusable link was used inside of the arc chamber. When an external fuse was used a voltage of 8000 volts was attained. From what is now known of high current arcs in the type of chambers that are presently used, it is reasonable to assume that approximately 5000 volts is the maximum that will be developed by this system due to the characteristics of the load. This voltage is small compared to the voltages on power transmission lines.

Next, there is a contrast in the dissipation requirement between the circuit breaker and the transfer switch. In both systems the energy dissipated by the arc is related to the lead inductance in the circuit. In the hot-shot system the loop inductance is held to a minimum. In a typical transmission line this loop may be extremely large. It is because of the size of this inductance in the transmission system that such devices as current limiting fuses and circuit breakers can stand severe faults because the inductance itself acts as a limit to the current. In this system the current is controlled by the operator, not by a fault.

Finally, the transfer switch is designed to operate under specific conditions, designed to create a certain effect in the arc chamber. The operator, then, can depend upon high currents whereas in a power system the circuit breakers must be able to operate under any condition from no load to maximum rating. The obvious consequence here is that magnetic forces can be used to assist in the operation of the switch.

DEVELOPMENT OF THE TRANSFER SWITCH

Experience from previous work with inductive energy storage coils played an important role in the design of the fast acting current transfer switch and a fuse system. The largest current previously used was 5000 amperes which was so much less than the 100,000 to 300,000 amperes that would be used in the new system, that a new design in switching apparatus had to be developed.

The transfer switch had to meet three basic requirements with a fourth design consideration:

1) The switch had to be small to be fast acting and yet give sufficiently low contact resistance so that it would not melt during the charging cycle;
2) It had to be able to develop enough voltage to transfer the current into a parallel load;
3) It had to develop substantial dielectric strength within some reasonable time after the current was transferred into the fuse element in order to withstand the voltage that appears across the switch when the fuse melts.
4) It should be designed so that in the case of a failure, with subsequent large energy dissipation in the switch, the necessary repairs can be made rapidly and inexpensively.

Several types of fast acting transfer switches were considered, each with a minimum mass in order to attain the fastest possible action. The maximum surface area for minimum mass in the switch contact arm is attained in a knife switch which utilizes the surface area on both sides of the blade to reduce the contact resistance and current density. The Detroit Edison Company donated, from their warehouse supply, 24 knife switches to this program. Each switch was rated at 2400 amperes and had been used on 600 volt D.C. street railway service in the City of Detroit. The transfer switch used herein is a modification of one of these switches.

Each of the requirements was tested prior to the final design of the switch. The test instrument was the transformer coil in the Plasma Engineering Laboratory that was built in 1957.[60] This coil has a primary winding of 117 turns with a two turn secondary, which yields after losses a current ratio of approximately 50:1. The primary is operated at up to 5000 amperes and 40,000 volts during the transient. The secondary rating is 250,000 amperes at 800 volts.

The effect of joule heating in the switch is easily calculated and, in the absence of the coil, calculations would have been considered satisfactory for design. However, it was a relatively simple task to connect 480 volts A.C. directly across the primary of the coil, giving a primary current of 500 amperes and a secondary current of 25,000 amperes. The knife switch, which normally has three blades, each three inches wide, was fitted with a single blade two inches wide. The blade carried 25,000 amperes rms for 2.2 seconds, which corresponded to a normal charging cycle at 150,000 amperes peak current. There were no adverse or unexpected events.

The voltage developed by the transfer switch was measured on the secondary of the coil at 100,000 amperes. The switch opened into a load of 160 micro-ohms and 0.5 microhenry in approximately one millisecond developing about 60 volts. This switch opened due to the self-magnetic force field alone in nine milliseconds, there being no externally applied opening force.

A switch was specially constructed at this time to perform the next two series of tests. It utilized the fingers from one of the heavy switches, but used a smaller blade since the current was relatively low. The switch was actuated by an air cylinder which was preloaded and trip-released by a toggle brace. An air blast was directed towards the fingers to clear the arc away from the copper contacts and increase the voltage on the air column. The switch was tested first at very low current levels and subsequently at 5000 amperes.

The transfer switch was intentionally allowed to arc over in order to determine the magnitude of the voltage in the switch, and to gain insight into the amount of damage that could be expected if the switch should arc over when used with the large coil of the inductive storage power supply. For comparison, Figure 4 shows oscilloscope traces for a normal switching operation. The current switched rapidly into a fuse which held for about three milliseconds. The voltage trace shown in Figure 5 was recorded during the switch arc. It was determined that the switch would develop nearly 1000 volts and since most of the voltage appeared across the arc column, relatively little damage was done to the swtich fingers and blade.

A series of switch tests were made toward determining the rate of recovery of the dielectric strength once the current goes to zero in the swit ch. The dielectric voltage recovery tests were made on the primary of the coil at currents of 5000 amps. The current was transferred from the switch to a fuse which after a short time would break and apply the high voltage across the switch. The fuses which were used could develop at least 50,000 volts, but a voltage of 15,000 to 20,000 volts was considered adequate. A ball gap was used to limit the voltage and dissipate the energy after breakdown. The size of the fuse controlled the time lag between zero current and high voltage on the switch. An upper limit of one millisecond, and probably less, was established as the time for the switch to recover sufficient electric strength to withstand 15,000 volts after transferring 4500 amperes. Several attempts were made to shorten the hold time of the fuse, but smaller fuses have a higher resistance and the switching would not be complete

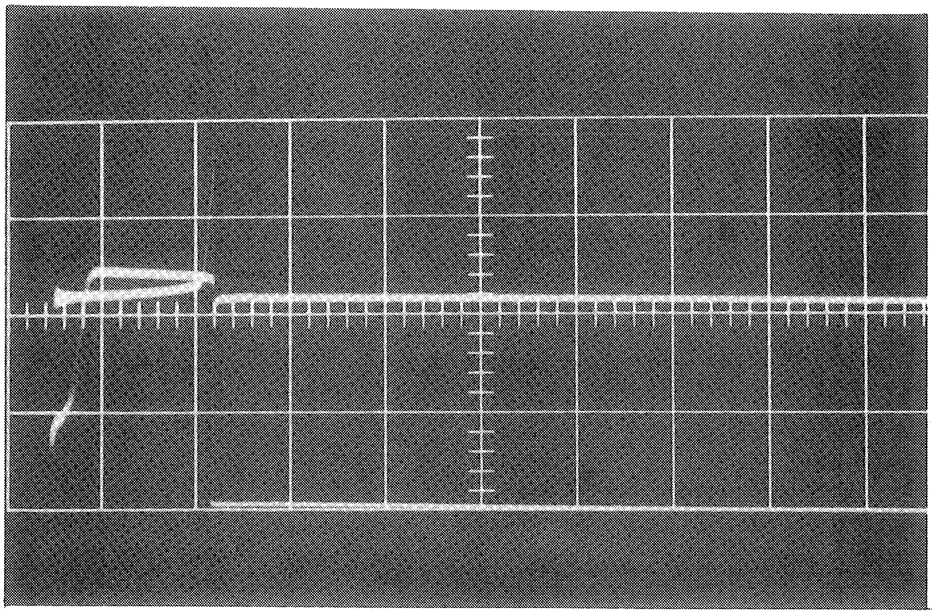


Figure 4. Fuse current (lower trace, 2000 amperes per division) and fuse voltage (upper trace, 10 KV per division). A ball gap was used as the load. The time scale is 2 ms per division

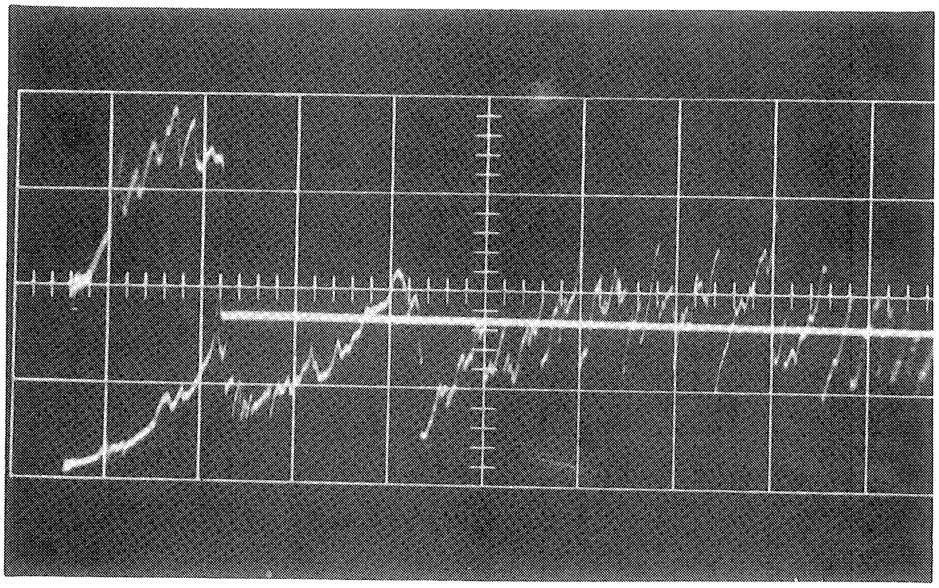


Figure 5. The experimental switch arc-over

by the time the fuse blew up, resulting in continuing arc in the switch blades without a current zero. Knowledge obtained during this series of tests led to confidence that a knife switch with a suitable actuating mechanism could reasonably be expected to operate at high currents on the large coil power supply.

The switch which was installed is shown in Figure 6. This system uses an air cylinder both to accelerate and decelerate the switch blade. The air cylinder is pressurized prior to use but is prevented from pulling the switch open by a toggle brace. The switch opens when the toggle is tripped by an electrically detonated squib (or blasting cap). An air blast is directed at the point of last contact between the blades and fingers.

The entire switch assembly is shown in Figure 7. The switch is shown in the open position. The isolation resistor plates extend to the right from the switch. The chamber normally connects directly to the ends of these plates. The air for the blast is stored (at 100 psi) in the 30 gallon air tank seen at the left. The blast is started 0.2 seconds prior to the switch actuation. The air flow is limited by the seven orifices in the air-blast "nozzle." These orifices have a total cross sectional area of one-half square inch. The pressure in the tank drops to 60 psi in one second during this flow.

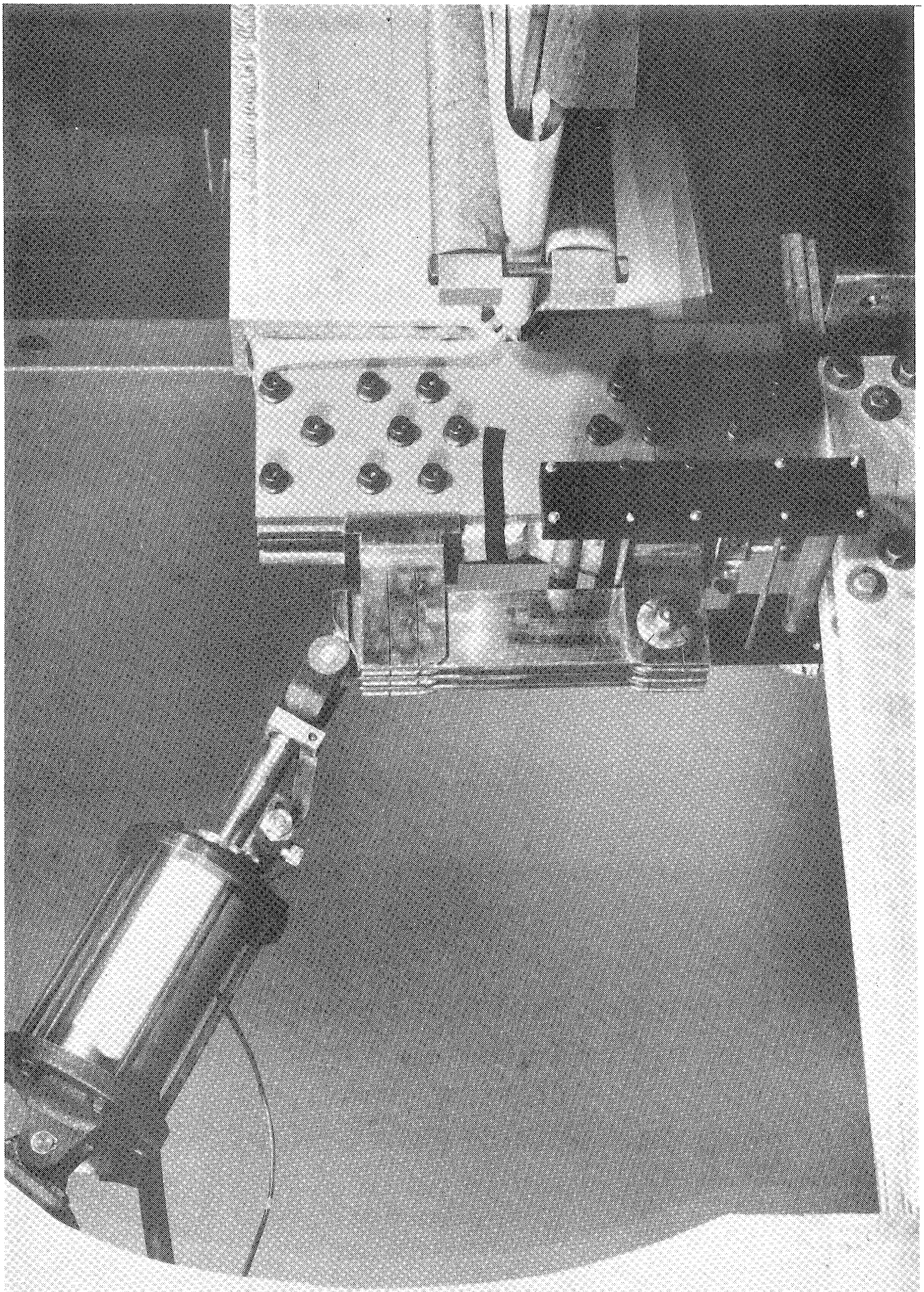


Figure 6. The transfer switch.

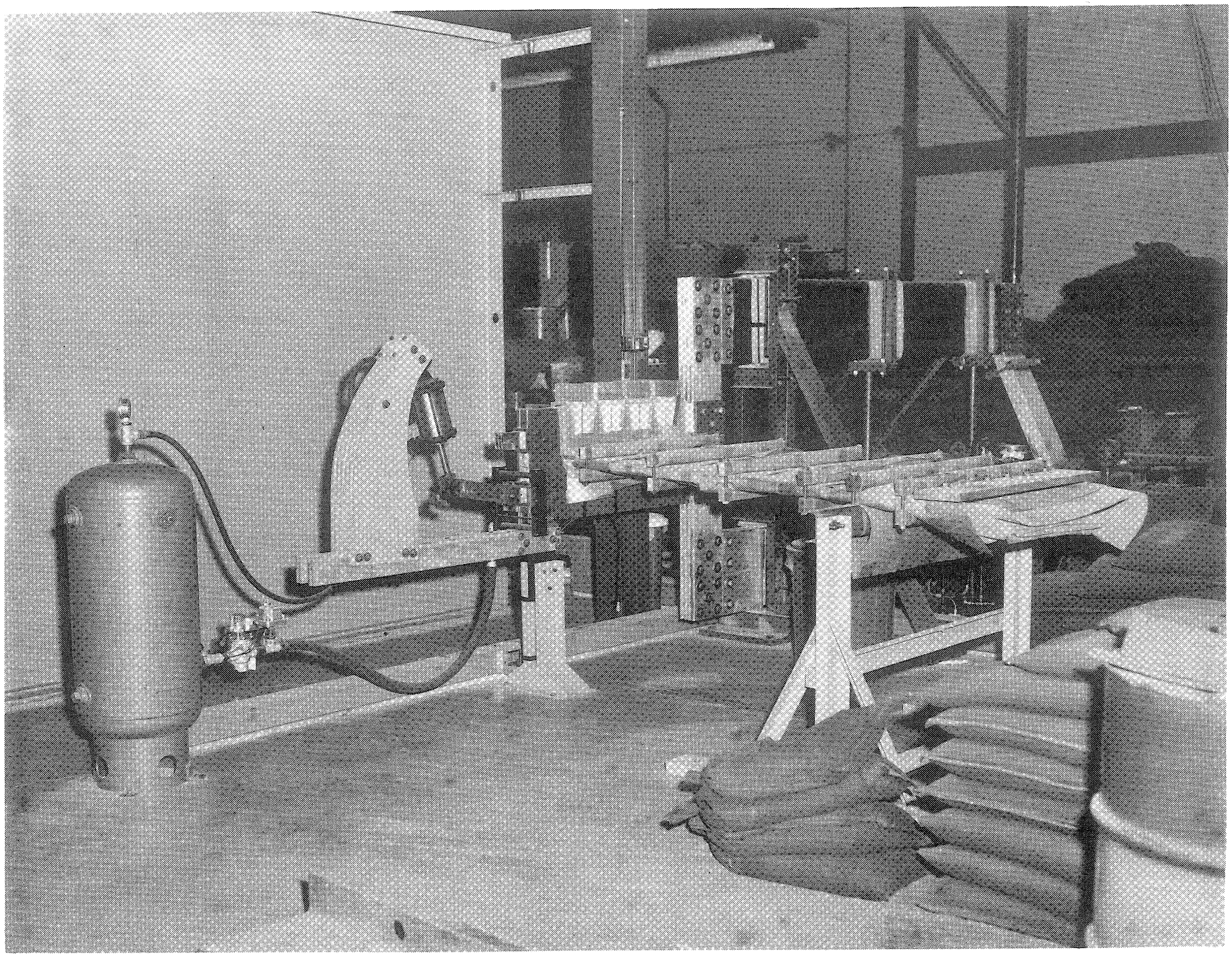


Figure 7. The transfer switch assembly

The initial tests on the switch were made at 30,000 amperes. The current was transferred into a load of 250 micro-ohms on the first test. The load was increased on subsequent tests to 1500 micro-ohms. The maximum voltage was attained when the switch transferred 38,000 amperes into 1500 micro-ohms with a voltage of 56 volts. The inductance associated with the switching losses was only 0.1 to 0.2 microhenries, and was associated almost entirely with the switch blade. The inductive voltages were, therefore, quite small.

This switch was used successfully at currents up to 100,000 amperes in the arc chamber test operations which followed. Data shows that the switch has regularly cleared in 1.0 to 1.5 milliseconds.

One aspect of this switch mechanism is not considered to be completely satisfactory. When the switch opens, it swings to the farthest point, being stopped by the air pressure in the cylinder. It then swings back and recloses about 0.08 seconds after it originally opened and begins to recharge the coil. It then reopens, but this time much more slowly. The second opening probably causes about half of the total damage to the switch blades, but otherwise it is not important. Inasmuch as only one set of blades has been used for the entire program it has not been worth the effort to modify the switch. Eventually, the switch will be modified so that it will not reclose. It might be noted at this point that consideration previously had been given to designing a switch system which would utilize a reclosing operation in the main switch, to limit the duration of the arc. However, the electrical characteristics of the two operations were not sufficiently similar to make it practical.

USE OF FUSES

As applied to arc chamber technology, there are two designations of fuses, the internal fuse which is inside the arc chamber and part of the chamber design, and the external fuse which is outside the chamber and may be designed independently of the chamber. Both perform the same task of allowing the switch to clear. The internal fuse breaks, and simultaneously initiates the arc and causes the high voltage to appear in the same physical location. The external fuse outside of the chamber causes the high voltage but does not directly initiate the arc within the chamber. Gas breakdown within the chamber can be most conveniently accomplished in dense gas by using a small wire inside the chamber which will explode. The initiating wire would normally have only about 2% of the cross sectional area of the internal fuse.

The techniques of using an external fuse to switch a coil were initially developed some time ago in order to switch the current from the primary to the secondary of the transformer coil. Fuses were used for currents in the range of 1000 to 5000 amperes and were required to develop up to 50,000 volts. The fuse cartridge was an oil filled cylindrical tube of glass reinforced plastic. The fuse wire was 15 inches long with a bore not exceeding one fourth inch. A piece of copper wire varying in size from #22 to #18 depending on the current, and about 18" long. The oil fuse developed the high voltage in a very short time. A fuse that held for two milliseconds would develop 40,000 volts in 0.2 to 0.3 milliseconds.

EXTERNAL FUSE DESIGN

During the initial design of the 6 megajoule energy-storage power supply it was assumed that an external fuse could be used to accomplish the switching, although the exact form of the fuse was to be designed later. Data that was then available[2] together with a short investigation of fuse characteristics indicated the following statements were empirically true for fast-acting fuses:

1) The length of time the fuse will hold is inversely proportional to the square of the current density;
2) The length of time required to be destroyed (the blow-up time) is proportional to the time the fuse carried the current;

3) The shape of the voltage trace from the time the voltage starts to rise is approximately exponential;

4) The energy that is dissipated in the fuse is proportional to the maximum voltage, to the initial current, and to the time that the fuse takes to blow.

The first statement leads to, and is supported through exploding wire technology,[1] by the expression:

$$\int J^2 dt = \text{Const.} \tag{1}$$

Since the current is generally constant in time in this work, as will be shown later in this chapter;

$$J^2 t_h = 3.5 \times 10^{10} \ \frac{\text{amp.}^2\text{-sec.}}{\text{in}^4} \tag{2}$$

where J = current density, amperes/meters^2

t_h = time the fuse holds

This expression has been found to be correct within 10% throughout all of the fuse work that has been done.

The second statement can be expressed as:

$$t_h = K_1 t_b \tag{3}$$

where

t_b = time from the start of the voltage rise to the maximum voltage.

This statement is a first approximation, with the value of K_1 varying from 10 for a two to four millisecond fuse to 15 for a much slower 20 or 30 millisecond fuse. The number will only be valid, however, as long as the blow-up time is short so that confinement of the arc can be maintained.

The third statement expressed mathematically is

$$v = V_o \exp\left(\frac{t - t_h}{\tau}\right) \qquad (4)$$

$$\text{for} \quad t_h \leq t \leq t_h + t_b$$

where

v = the time-varying voltage across the fuse

V_o and τ are constants of the operation which will be explained below.

This last equation describes the rise in voltage from the time that the fuse wire reaches the melting point until the fuse reaches its maximum value, at which point there is some form of electrical breakdown, either within the fuse itself, in the case of a failure, or within the arc chamber if the arc is properly struck. The constant, V, may be interpreted as the v oltage drop across the fuse wire when the wire has just attained its melting temperature, but prior to any change of phase. Because of the heating in the wire, this voltage drop is considerably higher than the voltage drop across the fuse initially, but is still low compared to the voltage that the fuse will generate. Typically, this voltage will not exceed 100 volts. The time constant, τ , that describes the rate of rise of the voltage, has no simple physical meaning. The physical process is extremely complex. Once that any part of the fuse changes phase to the liquid state, it immediately flows because of pinch forces, which will then cause the formation of a short arc column between the liquid droplets that are created by the pinching action of the current. With each instant additional sections of the wire break up forming droplets and short arcs. The number of such arcs probably increase exponentially and each short arc probably has approximately a constant voltage across it. Therefore is characteristic of the rate of the random change from solid to ionized vapor of the individual parts along the fuse wire.

The voltage rises to many times the initial voltage, typically from 50 to 300 times the voltage at the beginning of the fuse action, and this infers that an interval of from four to six times τ is required to develop the voltage. There exists, then, one approximate empirical relationship that

$$t_b = K_2 \tau \tag{5}$$

where K_2 has been found experimentally to have a value of about five for the type of fuse that was employed in the present project.

Finally, the last statement may be written as

$$W_f = K_3 V_{max} I_o t_b \qquad (6)$$

$$W_f = K_4 V_{max} I_o t_h \qquad (7)$$

where I_o is the initial current in the fuse, and W_f is the total energy dissipated in the fuse. There is the ten to one ratio between t_h and t_b by equ. (5). The value of the constant K_4 is approximately .05 for the fast acting oil filled fuses that have been used, subject to a variation of ± 20%.

There is a theoretical basis for equ. (2) which is based on the fact that the electrical joule heating of the metal is equal to the thermal heating. For a unit volume, this may be expressed as

$$(J^2/\sigma)\,dt = c\,m\,dT \qquad (8)$$

where

σ = conductivity, mhos per meter.

c = kilogram calories per kilogram, °K

m = mass, kilograms

T = temperature, °K

Since the specific heat and conductivity are functions of temperature only, this simple differential equation may be written in the form

$$J^2 dt = \sigma c m \, dT \tag{9}$$

Both sides may be integrated easily so that

$$\int_0^{t_1} J^2 dt = \int_{T_0}^{T_f} \sigma c m \, dT \tag{10}$$

If the final temperature, T_f, is the melting point the time, t_1, becomes the hold time, t_h, for the fuse wire.

The temperature integral can be evaluated by either numerical methods or by approximating the resistivity with an analytical function. Either method yields a value, which has been checked experimentally, of

$$J^2 t_h = 3.5 \times 10^{10} \tag{2}$$

There is also a theoretical basis that relates the two equations

$$V = V_0 \exp\left(\frac{t - t_h}{\tau}\right) \tag{11}$$

and

$$W_f = K_4 V_{max} I_0 t_h \tag{12}$$

The energy dissipated in the fuse is dissipated in two steps. Some energy is dissipated in the form of joule heating, raising the fuse element to the melting point temperature. Additional energy is dissipated in the melting of material and the arc formation as the fuse blows up. Expressed mathematically this can be stated as

$$W_f = W_{fm} + W_{fe} \qquad (13)$$

where

W_{fm} = energy required to bring the fuse to the melting temperature,

W_{fe} = energy dissipated in the fuse during the blow-up time.

The term, W_{fe}, includes the effects of whatever changes of phase, from solid to vapor to ionized vapor takes place. Fortunately the first term in this equation, the energy lost in melting, can be calculated quite accurately utilizing the elementary equation

$$W_{fm} = \int_{T=20^\circ}^{T=1060^\circ} cm \, dT \qquad (14)$$

where c is specific heat, m is mass of copper heated, ΔT is temperature rise. The temperature rise is the change in temperature from room temperature to the melting point of copper which is approximately 1060°C. The specific heat is tabulated in HANDBOOK OF PHYSICS AND CHEMISTRY and while not constant over this entire range, does not vary by more than about 5% having a mean value of .095

in cgs units. The mass in the fuse length can either be determined by calculation from the product of density and volume or by direct measurement. For analytical purposes the mass may be written in the following form

$$m = \rho l A \tag{15}$$

where

ρ = density, kilograms per meter^3

l = length

A = sectional area.

It was noted earlier that the square of the current density times the time that a fuse takes to reach the melting point is approximately a constant which may be written as

$$J^2 t_h = \frac{I_o^2 t_h}{A^2} = 3.5 \times 10^{10} \tag{16}$$

This may be rearranged to the form

$$A = K_5 I_o \sqrt{t_h} \tag{17}$$

where K_5 is a new constant.

The length of the fuse wire will normally be held to a minimum so that the transfer switch may be operated with as little back voltage as possible. For any given voltage requirement there is a minimum fuse length that must be used in order to obtain the required voltage to transfer the current into the load when the fuse blows up. This may be stated as an equation using still another constant, K_6:

$$l = K_6 V_{max} \tag{18}$$

On substitution of eqns. (18) and (17) together with (15) back into (14), one comes to the conclusion that the total energy dissipated in melting can be expressed as

$$W_{fm} = K_7 V_{max} I_o \sqrt{t_h} \tag{19}$$

where K_7 has an obvious meaning.

The form is interesting in that this says that the energy that goes into melting is a function of the required fuse voltage, the current that it has to switch, and the square root of the time that the fuse has to hold. It will be shown below that only a small error will be involved in assuming that the melting energy will vary linearly with the time as equ. (4) indicated.

The energy that goes into the fuse destruction can be written as the time integral of the product of voltage and current in the fuse which can be written as

$$W_{fe} = \int_{t_h}^{t_h+t_b} V I_o \, dt \tag{20}$$

or

$$W_{fe} = V_o I_o \int_{t_h}^{t_h+t_b} exp\left(\frac{t-t_h}{\tau}\right) dt \tag{21}$$

This requires an elementary integration which takes the form

$$W_{fe} = V_o I_o \left[exp\left(t_b/\tau\right)\right] \tau \qquad (22)$$

if the value of the lower limit is neglected because it is negligibly small. By use of the definition of the maximum voltage this is given the form

$$W_{fe} = I_o V_{max} \tau \qquad (23)$$

It was noted under the discussion of fuses that the time to blow up is approximately 5 time constants yielding a final form for the energy in the explosion of

$$W_{fe} = 0.2 \; I_o V_{max} t_b = 0.02 \; I_o V_{max} t_h \qquad (24)$$

The combination of these two sources of energy dissipation takes the final form of

$$W_f = 0.02 \; I_o V_{max} t_h \left(1 + \frac{K_8}{\sqrt{t_h}}\right) \qquad (25)$$

The value of $\frac{K_8}{\sqrt{t_h}}$ for a 2 ms, 40,000 volt 5000 ampere fuse calculates out to 1.6. The value would be decreased for a lower voltage or longer time, but the variation in the total fuse energy is not great with the final result that equ. (7) yields adequate accuracy if K_4 is given a value of .05.

APPLICATION OF FUSE EQUATIONS

These equations can be used first to gain an insight into the feasibility of using fuses in the operational system, and, second, can be used quantitatively in the design of a fuse. The first conclusion to be drawn, specifically from equ. (7) is that for any system the fuse time must be held to a minimum and, therefore, the transfer switch must open, clear, and develop dielectric strength in the shortest possible time. The transfer switch on the transformer coil was designed to operate in two milliseconds at 5000 amperes and the transfer switch on the large coil used in the present project has operated in four milliseconds at 100,000 amperes in order to comply with this criterion.

It should be noted, from equ. (7) that the energy dissipated in the fuse is proportional to the current. The stored energy in the coil is proportional to the square of the current. The fraction of the energy lost varies inversely as the current. Thus, if an external fuse can be constructed which will use one third of the energy at 50,000 amperes, it should be possible to build a fuse which uses only one sixth of the energy at 100,000 amperes, and only one eighteenth, or approximately 5% of the energy at 300,000 amperes, providing that the time and voltage conditions remain unchanged.

The necessity for fast acting switches is further indicated by equ. (3) which shows that it is not possible to build fuses which will hold for a long time but blow in a short time, unless something can be done which will significantly alter the shape and correspondingly the rate of voltage rise of the fuse for the voltage as given in equ. (3). More will be said about the possibility of attaining this later.

If equ. (7) indicates a fast switch requirement, and inertial effects place a limit on the switch motion, then a compromise must be made. As examples, for 5000 amperes, 40,000 volts, a hold time of 2 ms, the calculated energy lost in the fuse is 20 kilojoules. The transformer coil stores 200 kilojoules at 5000 amperes so that the fuse energy is 10%. A fuse which holds for 5 ms would consume perhaps 25% of the energy. It would then be feasible to consider using a 5 ms fuse, but a shorter fuse time would be highly desirable.

On the 6 megajoule coil, a test was made to prove the feasibility of an external fuse as a switching device. The actual conditions in the test were 8,000 volts, 42,000 amperes, and 4 ms hold time which calculated out to 67,000 joules into the fuse out of an original stored energy of 105,000. An integration of the voltage trace indicated 25,000 amperes and 40,000 joules remained in the circuit and was dissipated in the arc chamber after the fuse went out, which was excellent correlation.

From these two examples it becomes apparent that for the various inductive systems that are presently in existence the switching must be accomplished in a very few milliseconds in order to use an external fuse. An external fuse system should not even be considered for use in conjunction with a switch that requires in excess of 10 milliseconds to clear.

FUSE FILLERS

Oil filled fuses have been used extensively to accomplish the switching of the transformer type of coil. However, it was deemed worthwhile to investigate alternate fuse fillers. Fuses have been used for some time and have reached a high degree of reliability. The most common fillers in standard fuses are quartz, or sand of fairly fine size, and boric acid. Boric acid is normally considered to be an effective filler because the arc inside of the fuse will break down the crystal structure and release the water of hydration. Quartz, or sand, apparently merely acts as an extended surface heat sink which causes a relatively high voltage. Extensive work on both types of fuse fillers has been reported.[2]

A series of tests were made with the transformer coil using a fuse that was filled with boric acid. Both wire and copper tape were used as fuse elements. The rate of voltage rise in these tests was considerably less than the rate of rise using an oil filled fuse.

Another series of tests was made to investigate the possibility of using a sand or quartz filled fuse. A commercial 2500 volt, 100 amp fuse was procured and was tested for this application. The fuse contained about 30 very fine, estimated no. 26 silver wires in parallel. The wires were embedded in a fine clean white sand. The fuse was tested at 3500 amperes and was found to hold for approximately 20 milliseconds with the rate of rise being quite rapid, about 3 milliseconds, to a voltage of about 10 kv. The fuse was dismantled, rebuilt, using 10 no. 26 copper wires, which gives a total cross section that is slightly larger than one no. 18 wire. The fuse was then inserted in the circuit and tested. The voltage rise on this test was quite similar to that obtained with the original fuse wires. At this point it appeared that it might be feasible to utilize a sand filled fuse instead of the oil filled fuse even though the rate of voltage rise was not quite as fast as with an oil filled fuse.

Further experimentation with the sand filled fuses indicated that the peak voltages that were attainable for this type of fuse were considerably less than the voltage attainable with an oil filled fuse for the same length.

Typically, a sand filled fuse about 2 feet long would develop from 8 to 10 kilovolts. A similar oil filled fuse could be expected to generate nearly 100 kilovolts. The length of the fuse would need to be quite long, and the number of parallel wires seemed to be excessive. In order to obtain the required voltage it was finally concluded that it would be most advisable to utilize the oil filled fuse for initial tests. However, if the blast flame from the oil filled fuse should pose a particular hazard it will be possible in the future to perform tests directly on the large coil and determine the feasibility of using a sand filled fuse.

STAGED FUSES

The oil filled fuse has a large capacity to recover its dielectric strength as soon as the arc is extinguished within the fuse. According to equ. (7) above the energy dissipated in the fuse is directly proportional to the maximum voltage and the time the fuse has to hold. If either factor can be significantly reduced, the energy dissipated in that fuse will be correspondingly reduced. The feasibility of using a system of fuses and ball gaps was investigated. The transfer switch was used to transfer the current into a fairly large fuse, that is, one that had a long hold time. When this fuse blew it generated a voltage only sufficient to break down a low voltage ball gap. The current then transferred into a fuse with a much shorter hold time. The first fuse would recover its dielectric strength while the second fuse carried the current. When the second fuse blew it would cause the required high voltage. As the hold time was only that necessary for the oil filled fuse to recover its dielectric strength, not the time required for the transfer switch to recover its dielectric strength, the total energy dissipated in the two fuses could be made less than the energy required to accomplish the switching using one fuse The voltage trace from an experiment utilizing two fuses is shown in Figure 8. The first fuse was a boric acid filled fuse and the second was an oil filled fuse. The voltage generated by the first fuse is probably not in excess of 2 kilovolts and it recovered its dielectric strength in about 4½ milliseconds to sufficient extent to withstand a voltage of approximately 12 kilovolts.

A similar experiment was performed utilizing two oil filled fuses at 5000 amperes with the second fuse holding only for approximately 0.8 millisecond. Even though the first fuse only generated approximately 2 kv, it withstood an estimated 15 kilovolts 0.8 of a millisecond later. Thus, it was shown that there is an alternative method to accomplish the switching at high currents and high voltages that does not consume as much energy as indicated in

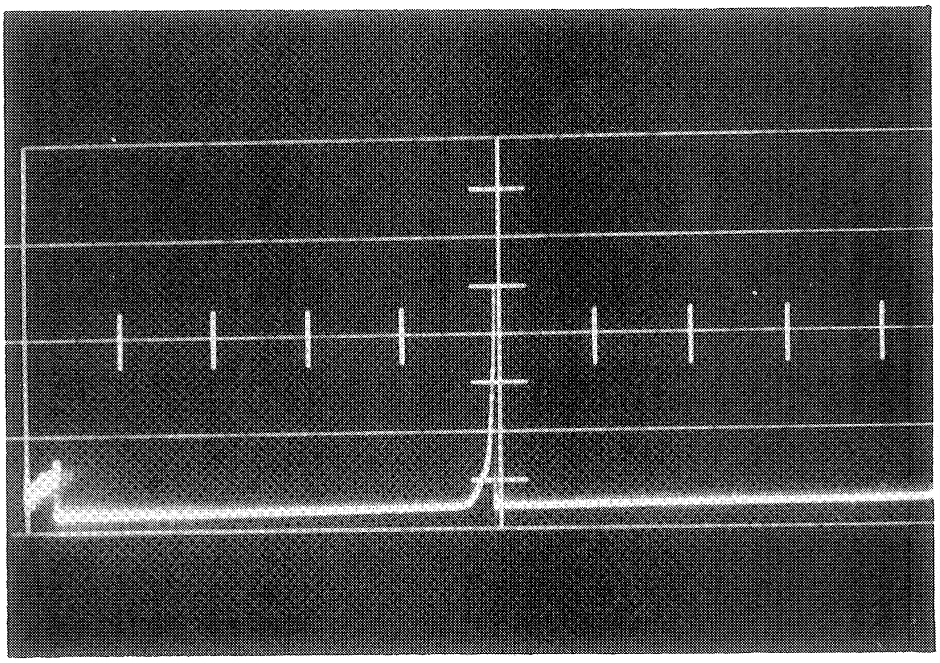


Figure 8. Voltage trace for staged fuses at 3000 amperes. The first fuse filled with boric acid withstood the voltage from the second fuse which was oil filled. The sensitivities are 5000 volts/div. and 1 ms/div.

the equations above if a system of fuses is used instead of one single fuse.

ARC CHAMBER

The arc investigations have been made in an arc chamber[18] that was designed to reduce the contamination of the gas by utilizing extended electrode surfaces. The chamber is shown in Figures 9 and 10, in assembled and disassembled conditions. The chamber is composed of two copper canisters which are held in place by two larger steel canisters which are in turn held in place by an external bolting mechanism. The upper and lower halves of the chambers are held together by two hydraulic pullers, each rated at 160,000 lbs of force. The current is brought into the chamber on two parallel copper plates, each ½ inch thick and 12 inches wide. The use of the sheet conductors minimizes the inductance between the arc chamber and the transfer switch and thus reduces the load on the switch.

The basic concept of this design was that the self-magnetic field of the high current arc would be sufficient to drive the arc rapidly over all of the internal surfaces of the chamber so that arc spots would not burn craters into the electrodes, providing that the aspect ratio, that is, the ratio of length to diameter, were correct. The aspect ratio is approximately 2½ to 1 for this chamber

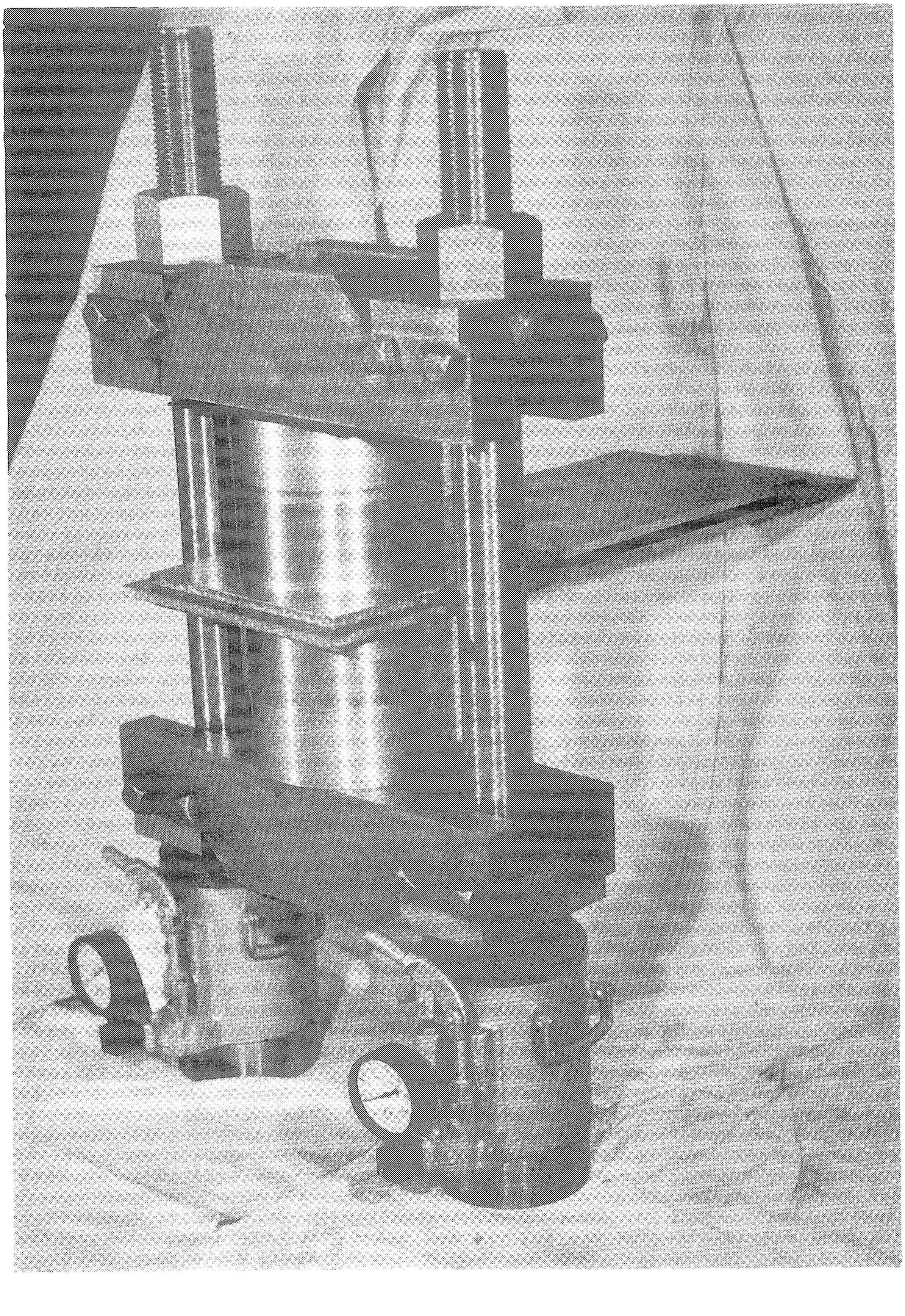


Figure 9. The extended-electrode surface arc chamber

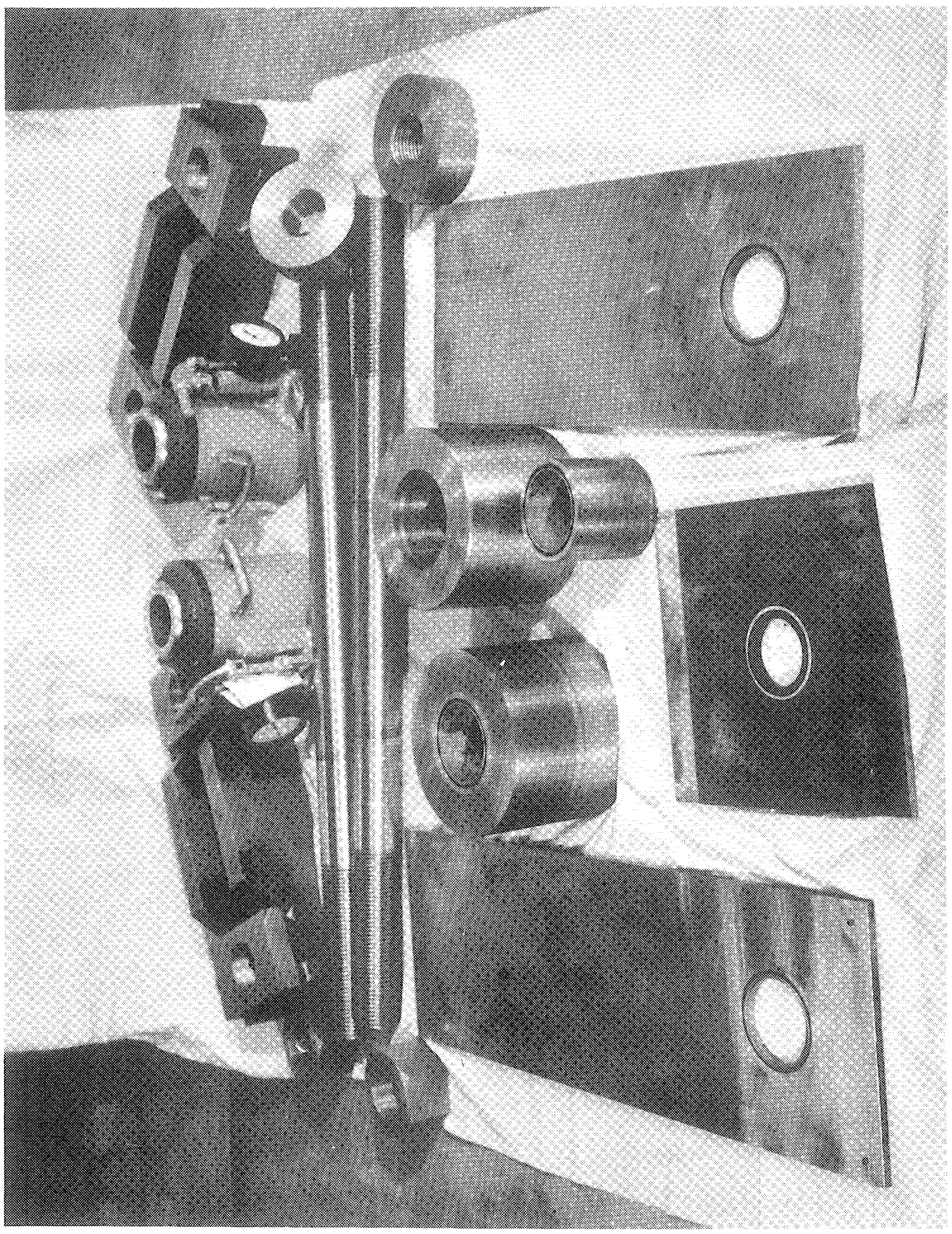


Figure 10. The component parts of the arc chamber

The values used in the design of the arc chamber were based primarily on a maximum allowable pressure. The thermodynamic conditions used in calculations were,(24)

Temperature:	4000°K
Initial pressure:	1470 psi
Density ratio:	100
Final pressure:	22,500 psi
Internal energy:	120,000 joules/mole
Specific volume:	220 cc/mole.

While the University of Michigan energy storage coil will store 6,000,000 joules, the chamber was designed to hold 1,000,000 joules. This determines the size of the chamber.

Number of moles:	8.3
Volume:	110 cubic inches or 1,830 cc.
Diameter of cavity:	3.80 inches
Length of cavity	9.5 inches
Ratio, length/diameter	2.5

The walls of the copper liner are approximately ½ inch thick. The liners are cut from standard copper bar stock.

Nominal O.D.	5 inches
Finished O.D.	4.970 inches
Length stock	10 inches
Finished length, each	4-3/4 inches.

The chamber walls were cut from 4340 steel, a nickel, chromium, molybdenum alloy which was heat treated to the maximum strength which would leave the material machinable. No heat treatment was used after machining.

Hardness:	Rockwell C30
Nominal tensile:	130,000 psi
Nominal yield:	120,000 psi

For the design pressure the required minimum allowable yield strength of the metal based on each of the following conditions is,[4]

Inner surface at yield, yield strength is	56,000
Outer surface at yield, yield strength is	33,000
Bursting, minimum yield strength is	30,000
To withstand tensile forces, yield strength is	42,500
Maximum allowable stress based on maximum strain theory,[52] yield strength is	50,000
Safety factor to equipment	2.0
Safety factor to personnel	4.0

The insulator is NEMA grade G-10, which is made of 182 glass cloth and epoxy resin. It meets military specification MIL-P-8013. The copper plates are hard rolled.

Minimum dry tensile, plastic	43,000
Nominal yield, copper	40,000

Minimum yield points for copper and plastic plates, allowable

Inner surface yield, minimum yield is	48,000
Outer surface yield, minimum yield is	23,000

Bursting (estimated), minimum yield is 20,000

Safety factor, equipment: 2.0

The chamber mount is constructed from the following materials:

Pullers, Simplex 803B, 80 ton each

Studs, 1112 steel, 3 inches diameter, 4 threads per inch

Nuts, 1015 mild steel, 3 in.-4

Cross bars, 2" x 6", 1018 cold drawn steel

Loading blocks, 1015 hot rolled steel

The chamber was originally fitted with a pressure transducer and a gas inlet port so that the chamber may be pre-charged to any specific density. Experiments have been made with densities that range from 10 to 100 times normal atmospheric density.

The flat plate leads connect onto the transfer switch through the assembly which was shown in Figures 2 and 6. This assembly consists of six pieces of flat $\frac{1}{2}$ x 4" aluminum bus bar in parallel leading to the chamber and six pieces of $\frac{1}{2}$ x 4" stainless steel in parallel for the return line. The stainless steel is the resistor shown in Figure 1 which prevents the charging current from preheating the fuse.

CHAPTER III

GROSS CHARACTERISTICS OF ARC WITHIN THE ARC CHAMBER

The experimental program to determine the behavior of the arc in the split chamber was carried on at the same time that the experimental equipment was given its initial tests so that experimentation started at a relatively low energy level. The first experiment was made at 30,000 amperes in the coil which stored approximately 54,000 joules of energy. The chamber was pressurized to 400 psi, approximately 25 atmospheres, with pure nitrogen. The arc chamber was fitted with an internal fuse that was cut from copper plate 1/8 inch thick and had a cross sectional area of .024 sq. in. The fuse was designed to hold for a period of approximately 20 milliseconds. The fuse was bolted into the two halves of the arc chamber with bolts which were sealed to prevent leakage of the gas.

Following the discharge the chamber was disassembled and the upper half of the chamber, the anode was photographed immediately upon disassembly. The amount of contamination within the chamber was very slight, consisting mostly of a very fine brown dust believed to be primarily condensed copper vapor as shown in Figure 11. The fuse was almost totally destroyed. Figure 12 indicates that the fuse was only slightly damaged by the magnetic forces and that most

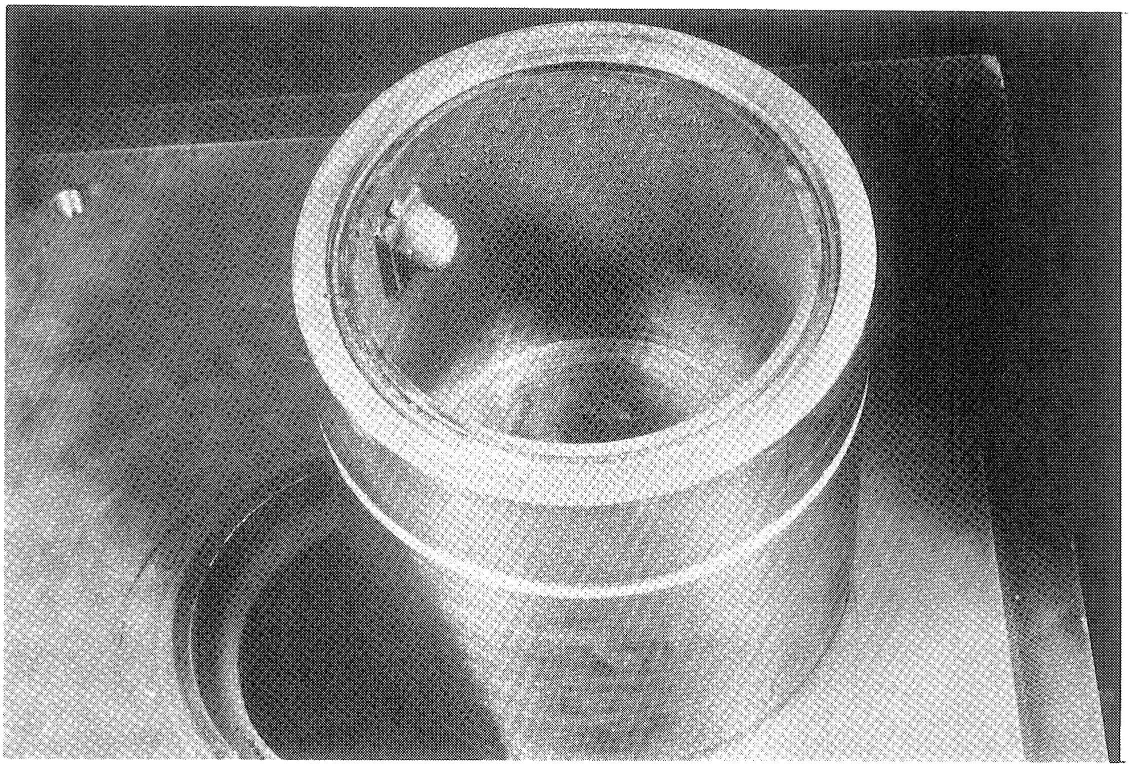


Figure 11. The anode, the upper half of the chamber, after the first experiment at 30,000 amperes and 54,000 joules

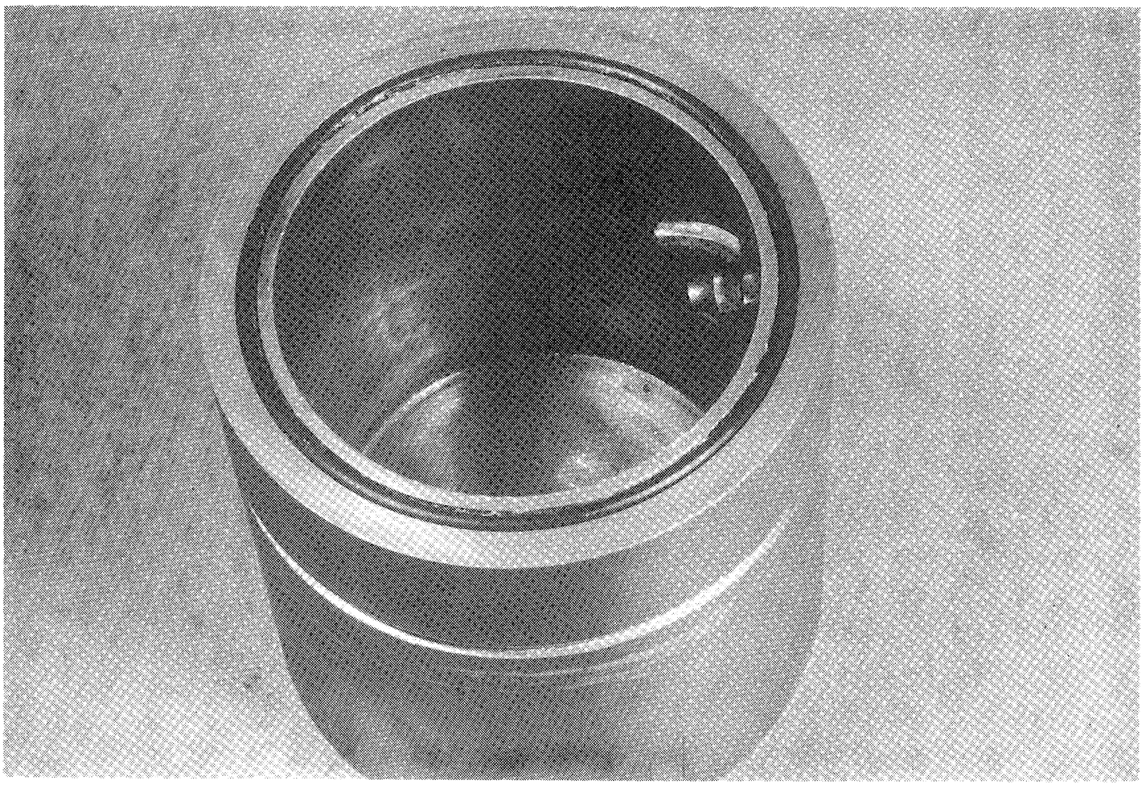


Figure 12. The cathode after cleaning with soap and water.

of the fuse destruction was caused by heating effects. This was anticipated because calculations indicated that the magnetic forces should become effective only if the current was above about 40,000 amperes.

After cleaning the chamber halves it was found that except in the immediate vicinity of the fuse attachment there was very little damage to the arc chamber. There were surface marks that were very shallow and easily removed with emery paper. On the anode in particular there were a number of very fine marks not over a tenth of a millimeter in width. These ran in the general pattern to indicate that the arc moved away from the insulation, perhaps with multiple anode spots.

A second test was made at approximately the same current level and the effects on the arc chamber are shown in Figure 13. The arc apparently blew directly across the chamber from the initial starting point and attached to the opposite wall. It moved, again leaving the extremely fine marks that are shown in the Figure. The contamination was relatively small. The fuse had been modified slightly for this second experiment. The cross sectional area at the break points was the same, but it had been designed with three notches. The purpose of the notches was to insure that only short sections of the fuse would be melted. Long sections would be thrown out of the arc region by the

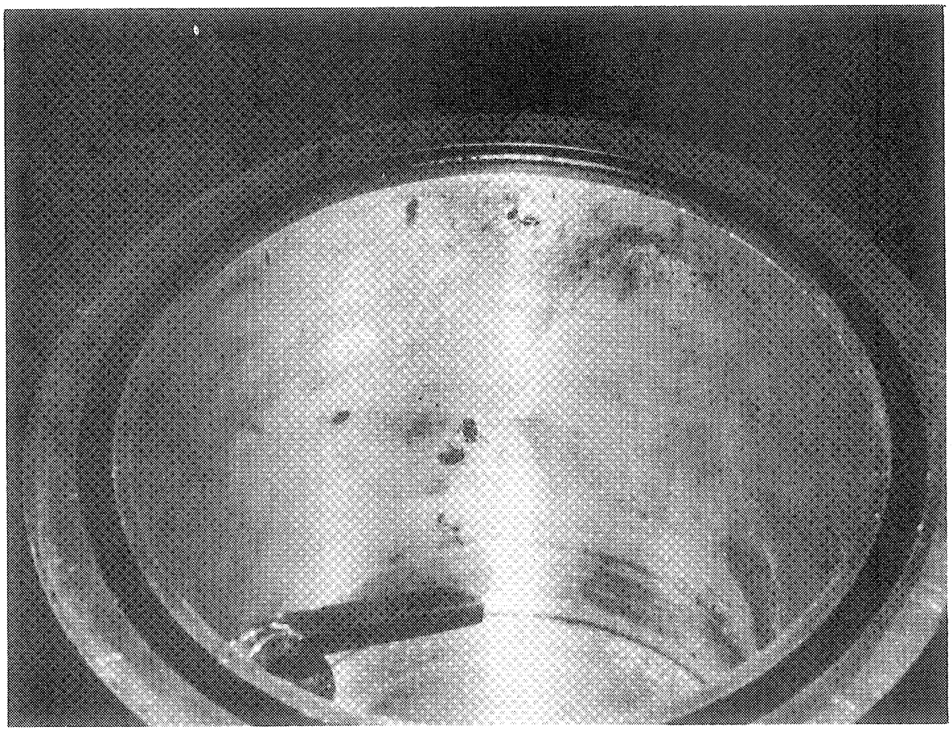


Figure 13. Fine traces on the anode after the second test at 31,000 amperes.

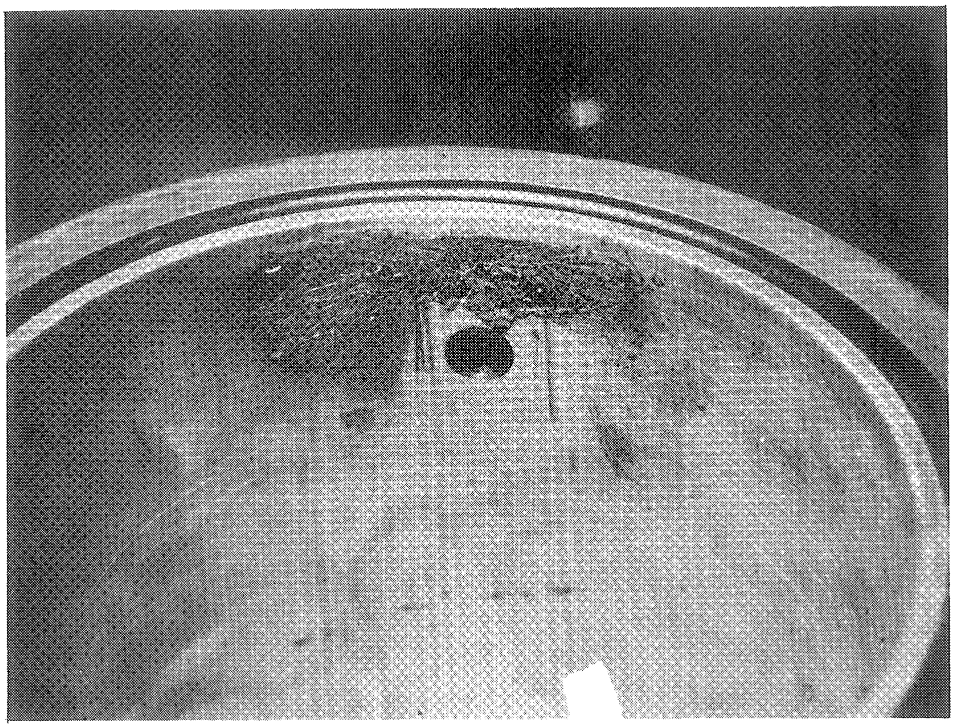


Figure 14. The cathode after 45,000-ampere experiment

magnetic forces without adding significantly to the contamination in the gas.

For the next experiment, the current was increased to 45,000 amperes. At this current level the fuse showed a tendency to break rather than melt because of the increase in magnetic forces. From this point on two sections of fuse were invariably found within the chamber indicating that the three notches all broke at approximately the same time. Oscilloscope traces obtained at a later date indicated that three notches would break within approximately one half of a millisecond of each other. The arc, however, appears to have anchored to the cathode rather than moving directly up the cathode wall. The damage caused by this is shown in Figure 14.

The experimental program continued on up to the level of 100,000 amperes without any significant delays. At this level a series of tests were conducted at various pressures in order to determine the effect of gas density upon the erosion and on the voltage and current characteristics. At currents in the range of 50,000 to 100,000 amperes the very fine arc tracks that were seen initially were no longer observed. Those tracks that were left in the copper indicated that the surface underwent considerable melting. All arc tracks were relatively difficult to remove, requiring the use of a small hand grinder in order to remove the tracks sufficiently to proceed to the next test. The overall erosion, however, was not particularly severe. In Figures 15

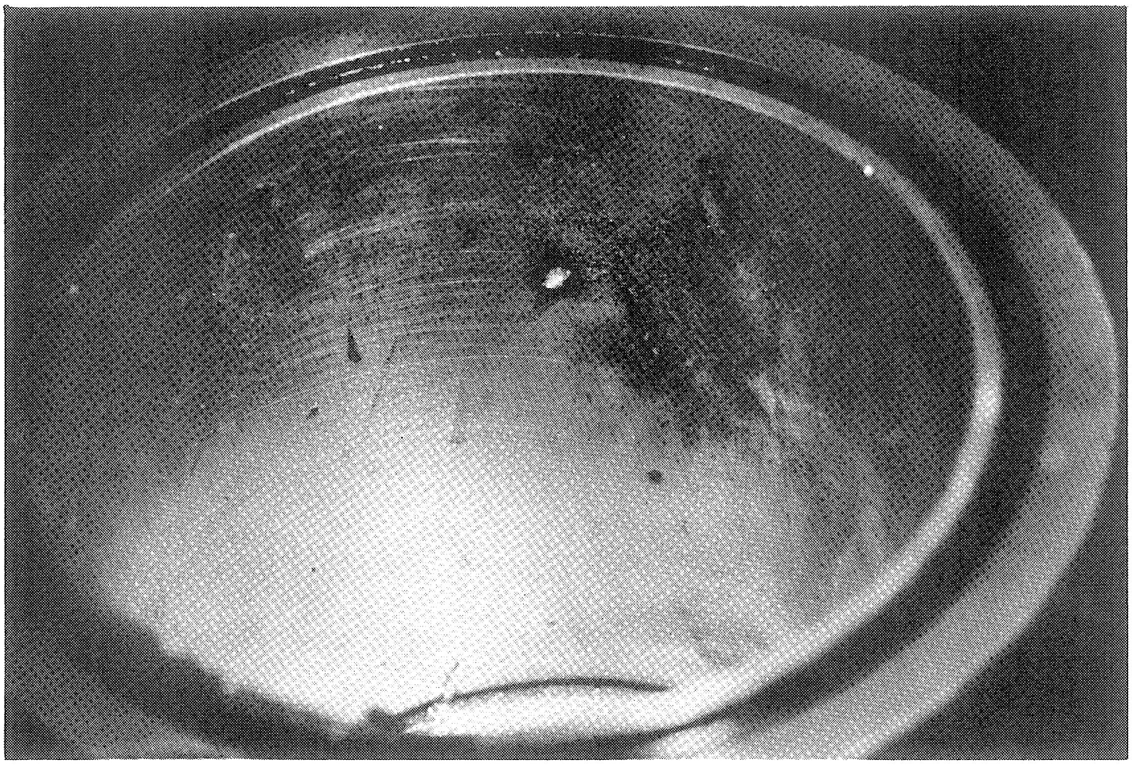


Figure 15. The cathode marks at I_o = 100,000 amperes, P_o = 1500 psi. The anode for this test is shown below.

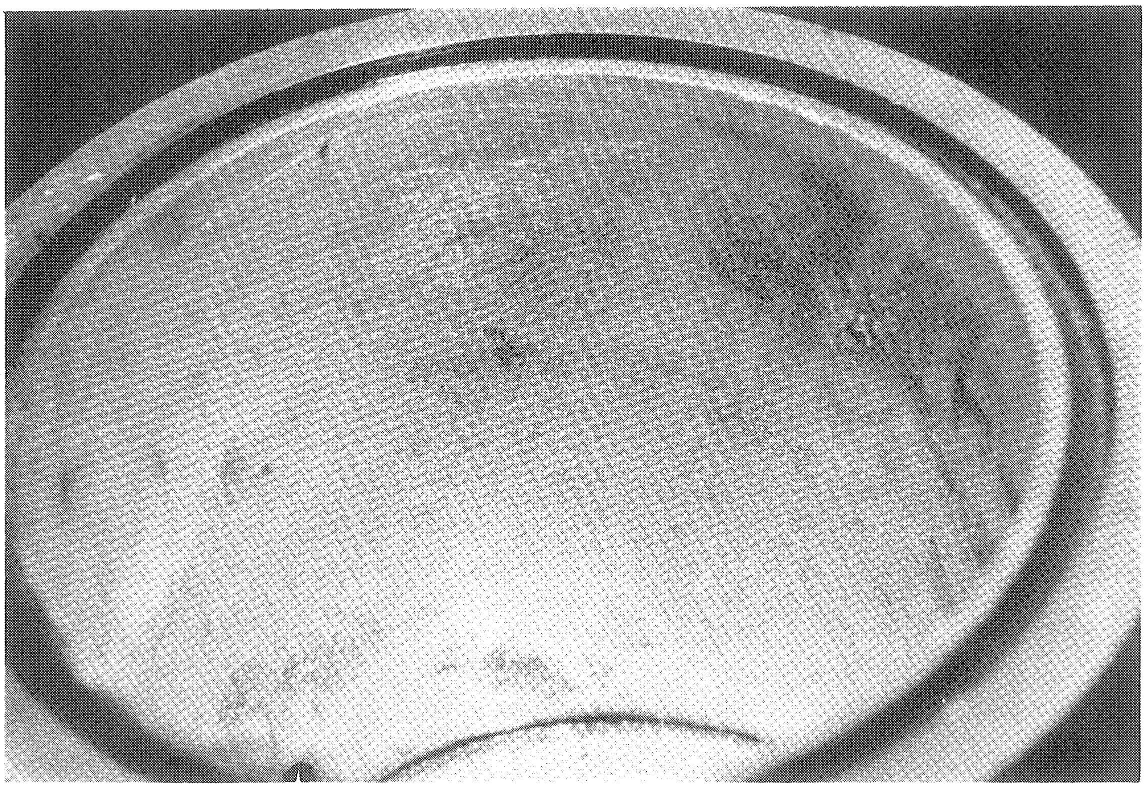


Figure 16. The anode marks.

and 16 the cathode and anode are shown after an experiment at 100,000 amperes with initial gas density of 100 times normal atmospheric density. The cathode spot appears to anchor in one location and then move, probably discontinuously to another position, never moving very far up the side wall of the arc chamber. The anode appears to move in a more continuous fashion. The initial pattern, that is the pattern nearest the insulator or end of the copper liner, shows a broad pattern of surface erosion. The anode spots, however, appear to have moved up almost to the end wall. Tracks have been found on the end wall of the anode.

Even at the higher current levels the cathode spot still anchors to the fuse termination, as shown in Figures 17 and 18. The effects of the cathode anchoring are greatly affected by the pressure or density of the gas. In Figure 17 the photograph was taken after a test at 100,000 amperes with an initial pressure of 1500 psi. Figure 18 was taken after an identical discharge except that the initial pressure had been 150 psi. There is a difference by a factor of ten in pressure between the two experiments, but all other variables were held constant. Note in the upper picture that the spot is most severly eroded close t o the nut that holds the remains of the fuse in place, but that the region of severe erosion probably is not more than one and one-half inches in diameter. Part of the apparently eroded area,

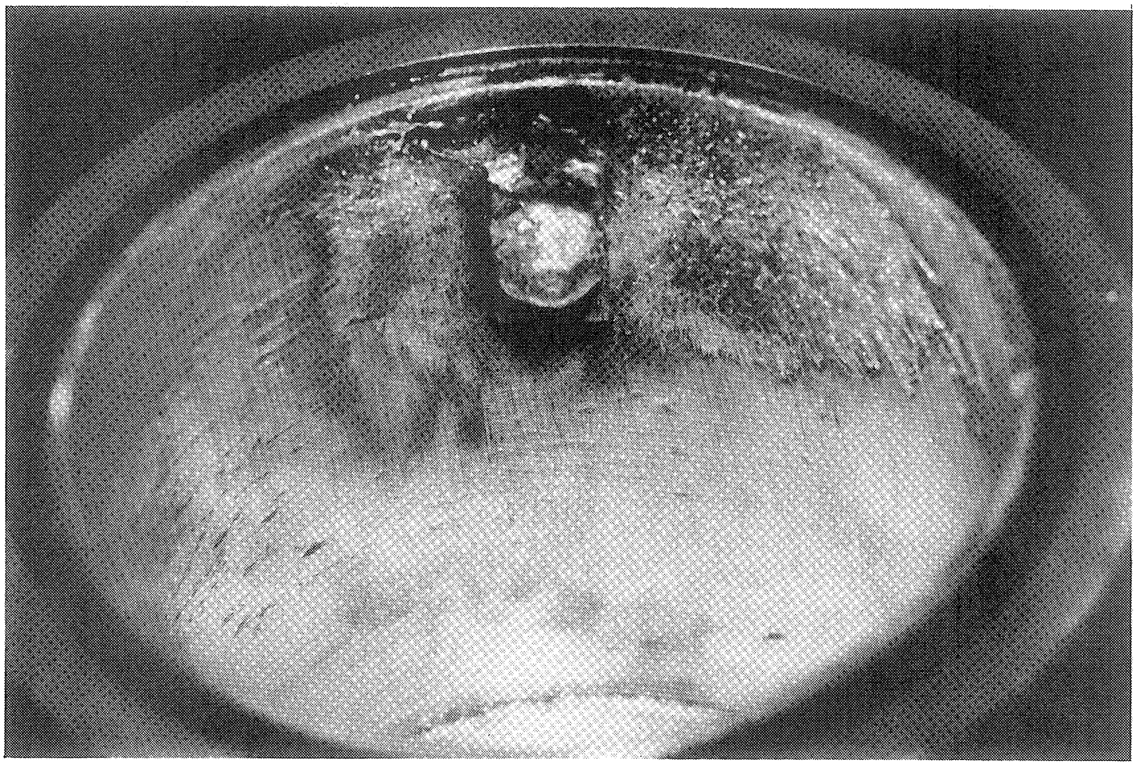


Figure 17. The cathode spot at the fuse terminal at I_o = 100,000 amperes, P_o = 1500 psi.

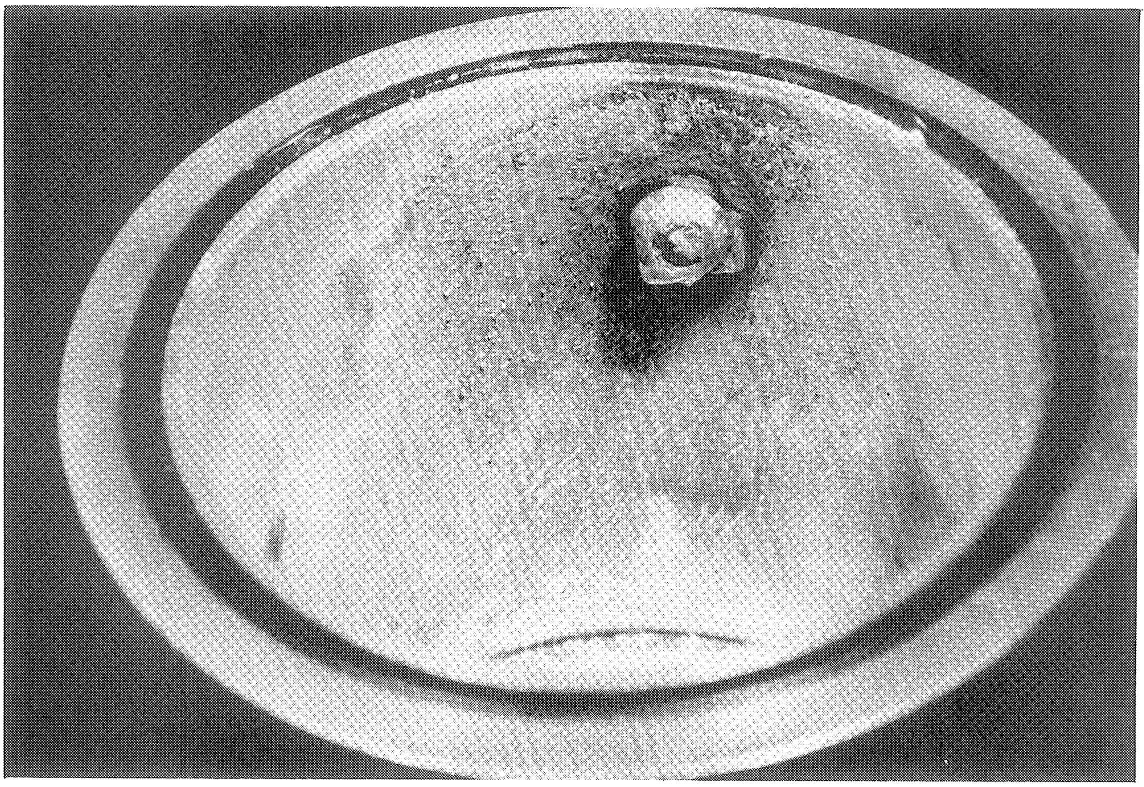
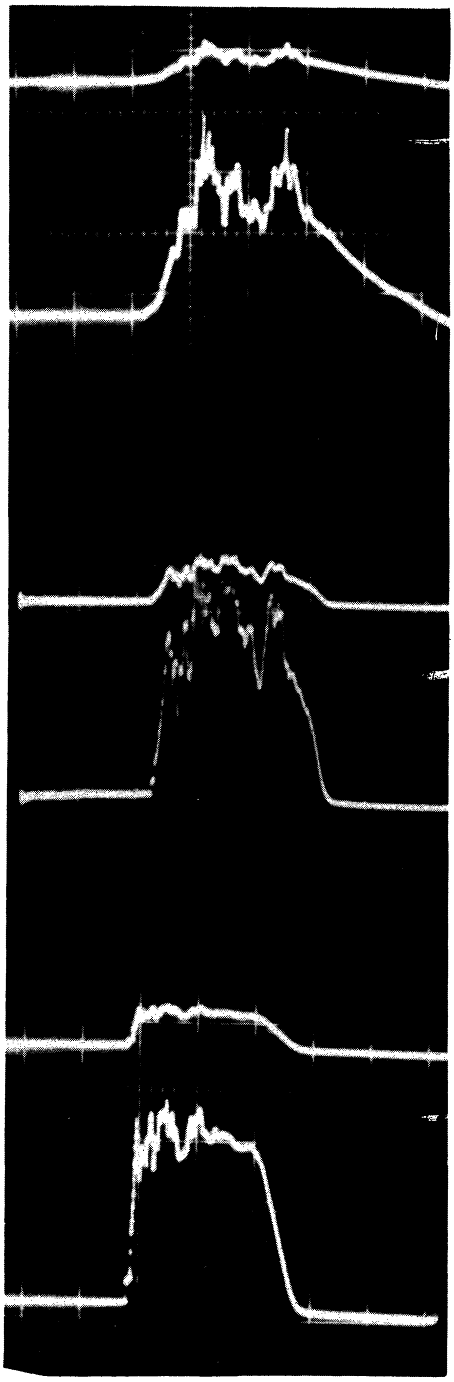


Figure 18. The cathode spot for I_o = 100,000 amperes, P_o = 150 psi.

that near the outer edges, is due to spraying of the melted metal rather than actual erosion. In Figure 18 the nut was almost completely melted away and the damaged area is perhaps twice the diameter or four times the area.

The density of the gas within the arc chamber plays an important part in the determination of the voltage characteristic of the arc during the discharge, in addition to affecting the amount and character of erosion on the surface of the copper liner. In Figure 19 there are three sets of traces that were recorded on a Tektronix 551 dual beam oscilloscope. The first set of traces was recorded during a test where the initial pressure was 150 psi. The next set of traces was recorded on a test where the initial pressure was 400 psi and the third set of traces was recorded during a test where the initial pressure was 1500 psi. The voltage fluctuations are much more severe in the lower density discharge. The rate of rise to the first voltage peak is also much slower in the low density discharge. In the very high density case, the voltage appears to be almost a square function. This contradicts the original concept involved in the design in the arc chamber, where it was assumed that the small column associated with the high density discharge would behave much more erratically than the larger diameter and more diffuse column of a low density discharge.



P_o = 150 psi

P_o = 400 psi

P_o = 1500 psi

Figure 19. The voltage across the arc chamber for three initial densities at I_o = 100,000 amperes. Each voltage is recorded with two traces, the upper at 3500 volts per division and the lower at 700 volts per division.

The average voltage during each of these discharges has been calculated for the duration between the time when the voltage reaches it's first maximum and the time when the voltage begins to trail off, that is, gradually drop towards zero. A plot of these average voltages versus the corresponding initial pressure are given on Figure 20. The graph indicates that the initial density in the gas has a small effect on the average voltage during the discharge.

The data indicates that the average arc voltage increases as the current level during the discharge is increased. Figure 21 shows the average voltage during the central portion of the discharge plotted as a function of initial current in the discharge for three different initial pressures. The slopes of the curves for 150 and 400 psi initial pressure are positive, indicating that the voltage is increased when the current is increased. This was partially anticipated in the design of the chamber when it assumed that the magnetic forces would tend to drive the arc through the air with sufficient velocity to increase the arc voltage. The results obtained here tend to verify this assumption.

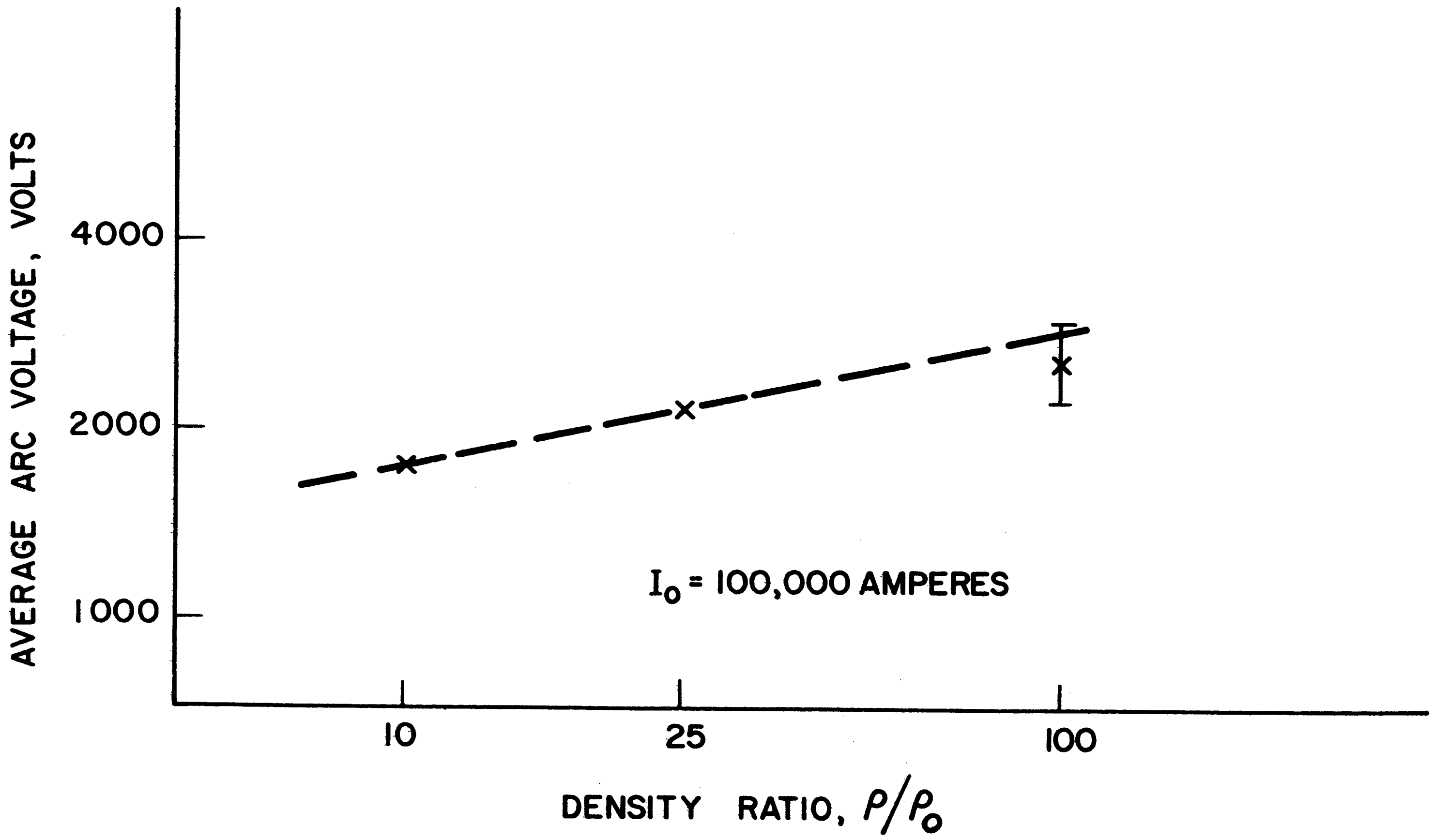


Figure 20. Voltage dependence on initial chamber gas density.

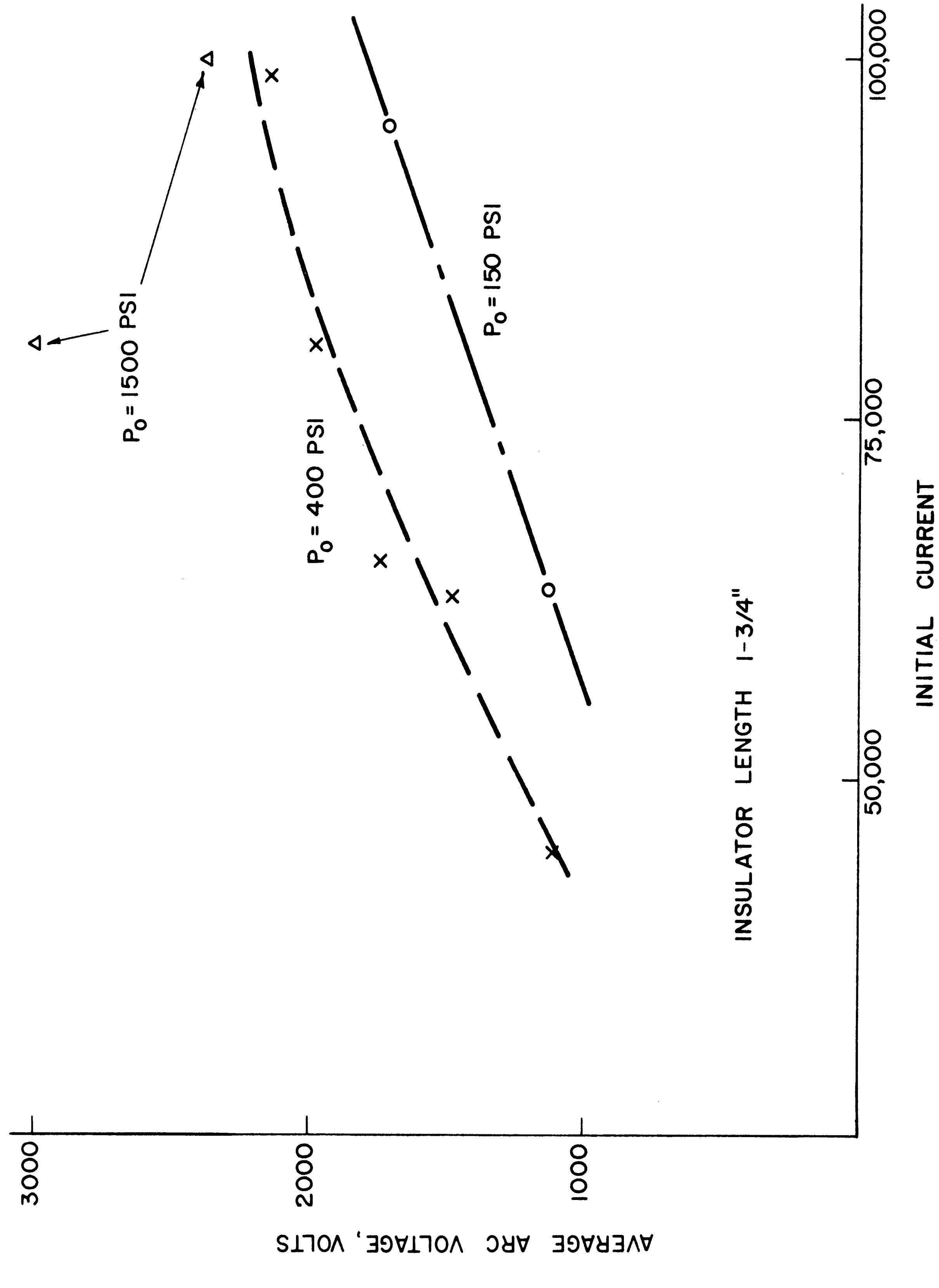


Figure 21. Arc chamber voltage.

CHAPTER IV

ARC CHAMBER MODIFICATIONS FOR PHOTOGRAPHIC INVESTIGATIONS

The initial investigations had clearly shown that a magnetically self-driven arc existed inside of the split arc chamber during the discharge. Furthermore, it was reasonably well established indirectly from the voltage traces that the arc diameter was a function of gas density and that the velocity with which the arc moved was a function of arc diameter or gas density. However, the only direct indication of arc behavior was determined from observation of the tracks left on the anode and cathode as the arc swept across the extended electrode surface. Information was being taken from the anode and cathode spots which is of value in the study of contamination, but it is the motion of the arc column that is really of interest from a thermodynamic standpoint.

Several probing techniques were considered in order to determine the position and motion of the arc and the diameter of the column. There were two most promising techniques. The high magnetic field of the arc column should induce a voltage into a pick-up loop located inside the arc chamber.[29] The transient voltage generated by the arc moving past the loop would indicate the arc behavior. The alternative was to attempt to locate a window in the chamber and try to photograph or observe visually the size, shape and motion of the arc. The former technique would have been simpler, but would have been subject to extreme difficulties when it came time to analyze the output of the magnetic probe. The photographic system was certainly the most desirable, but was at the same time the most difficult to install. There was also question as to whether a reasonable photograph of the arc column could be obtained through the dense, hot gas between the arc column and the observation window. Calculation of the expected intensity of the light from the arc column indicated that a system could be used which would incorporate a very high f-number f or the overall photographic system, perhaps somewhere in the range of f100, and still obtain an image on the film of a Fastex camera which exposes 14,000 frames per second. The possibility of using this very high f-number gave some hope that a reasonable depth of field and thus an adequate picture of the arc inside the chamber could be obtained.

The photographic system that was used was what is best described as a two stage optical system. The optics are shown schematically in Figure 22. The combined requirements of extreme depth of field and high f-number permitted the consideration of a pin-hole camera which in theory has an infinite depth of field. The image of the arc was projected on a ground glass plate. This image then was flat

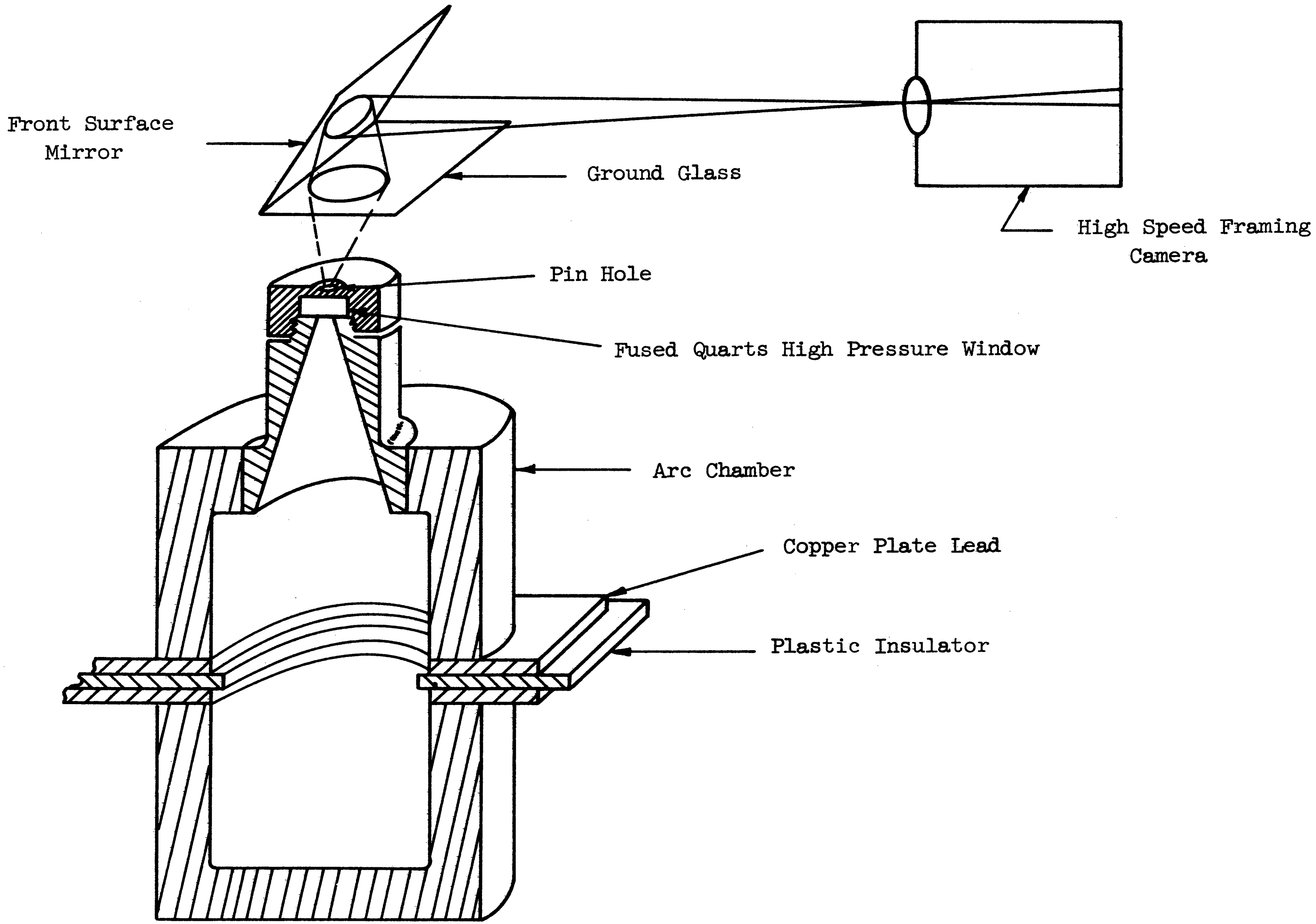


Figure 22. Optical modification of the arc chamber.

and represented a picture of the arc coming from a volume with virtually an infinite depth of field. The high speed camera was focused on the ground glass plate. The camera photographs an image which has zero depth of field and therefore could be operated with a large lens aperature. In the final series of photographs it was found that the Fastex camera could be operated in the range from f4 to f8 and that at these aperatures there was sufficient light to expose the film.

The window is a piece of fused quartz, 1 inch thick and 2 inches in diameter. The actual orifice to the chamber is ½" in diameter. The pinhole was drilled in a small piece of brass shim stock and has a diameter of .042 inches. The ratio of the pinhole diameter to the distance to the ground glass projection plate is the reciprocal of the f-number of the pinhole optical system. The magnification of the system is the ratio of the distance to the arc from the pinhole to the distance of the pinhole to the ground glass. The total magnification of the system is defined by the size of the source and image. The inside of the chamber is approximately 3-3/4 inches in diameter, and was imaged on one frame of a normal 8mm exposure which is the height of 4 millimeters. The f-number, then, will be effected by the location of the ground glass. As the distance from the pin-hole to the ground glass is increased, the effective f-number of this part of the system is correspondingly increased, but to maintain the same overall magnification requires that the Fastex camera be removed to a distance farther away but its f-number is not a function of its position. In operation, the ground glass was located approximately 3 inches from the pinhole and the Fastex camera with a 3 inch telephoto lens was located approximately $2\frac{1}{2}$ ft. away from the image on the ground glass.

The window was designed to withstand a maximum pressure of 10,000 psi. Prior to any electrical tests the chamber was filled with oil and pumped up to a pressure of 13,000 psi, as measured with a dead-weight tester. This was, in fact, the highest pressure to which the arc chamber has yet been subjected.

Subsequently a series of tests were made at various current levels and various initial densities starting with a current of 30,000 amperes and finishing at a maximum of 70,000 amperes. The optical system worked quite well. The initial parts of the discharge were recorded in all cases with quite satisfactory results.

Some of the individual images have been reproduced in Figures 23, 24 and 25. In Figure 23, the initial pressure was 1200 psi, while in Figure 24, the initial pressure was 150 psi. In both of these tests the initial current was approximately 65,000 amperes. In Figure 25, the initial pressure was

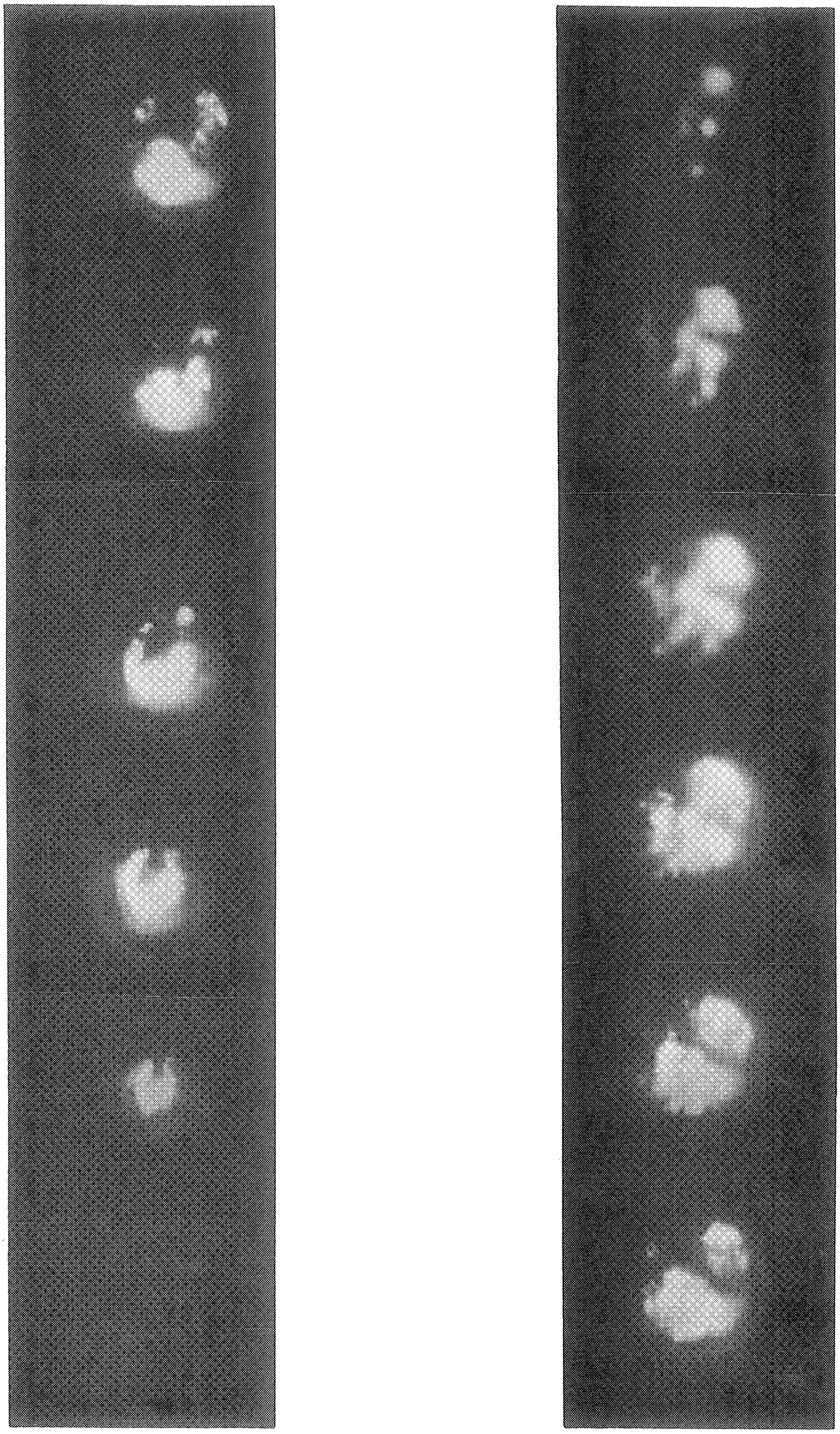


Figure 23. Photographs of the arc at P_o = 1200 psi, I_o = 66,000 amperes

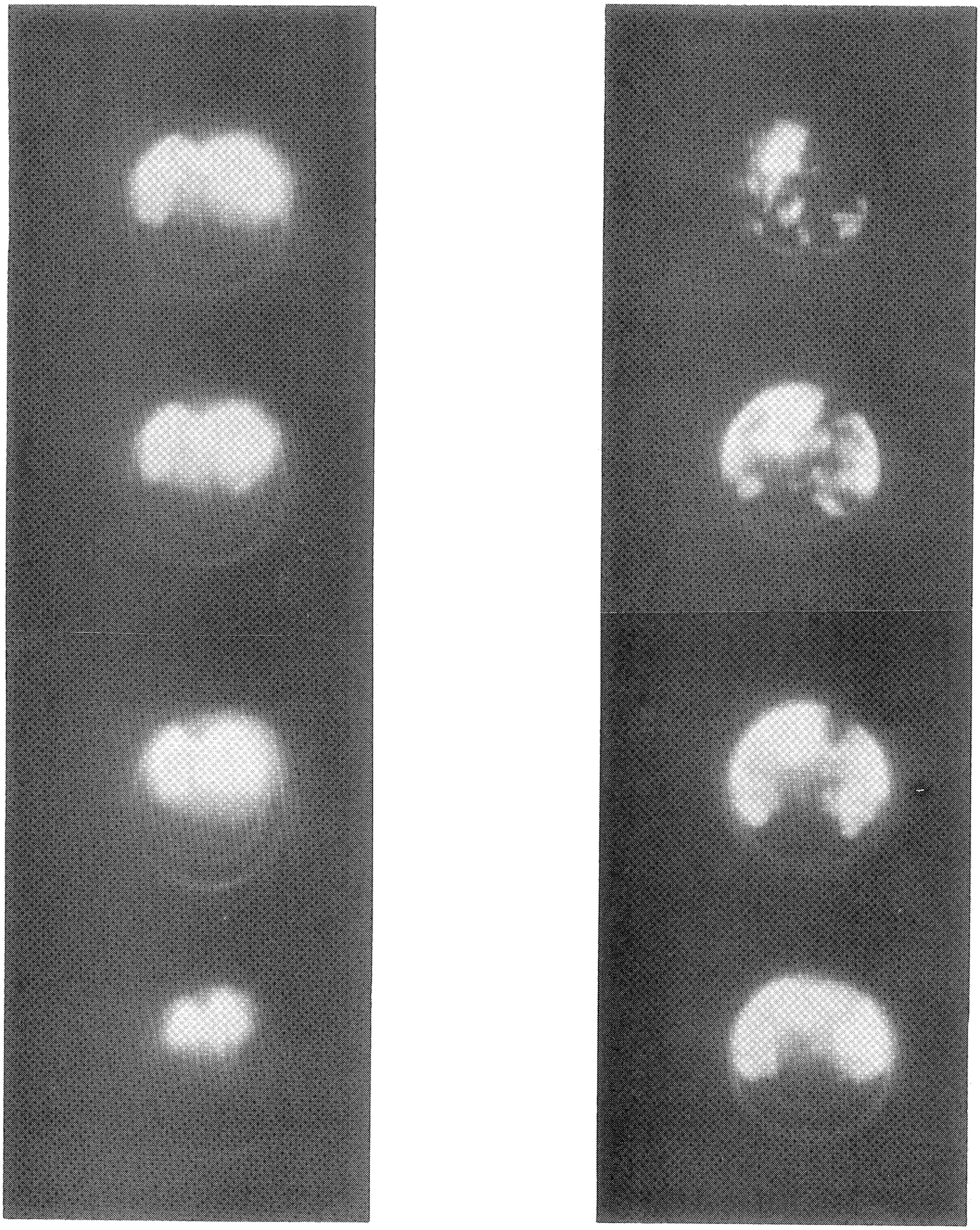


Figure 24. Photographs of the arc at $P_o = 150$ psi, $I_o = 64{,}000$ amperes

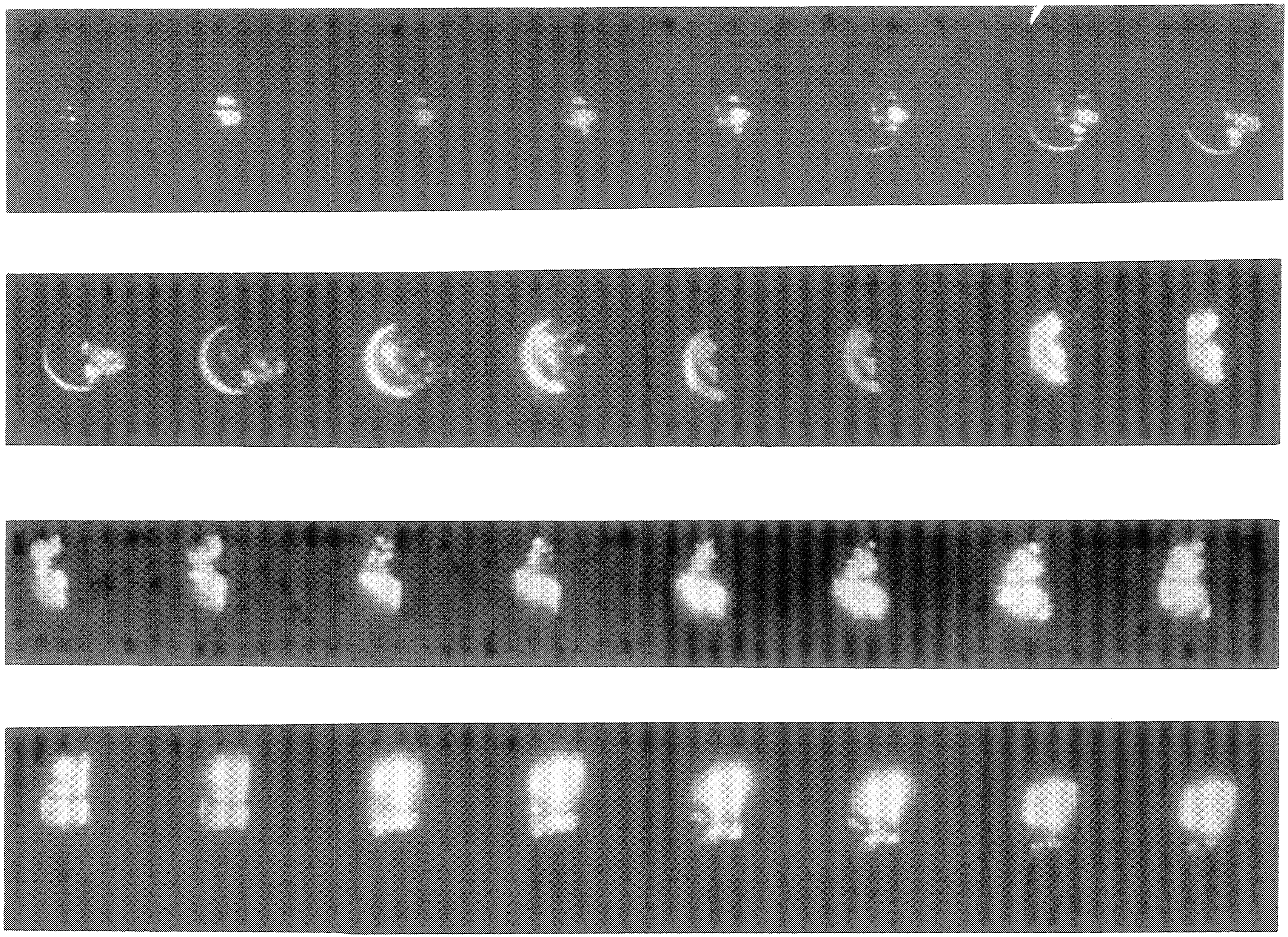


Figure 25. Photographs of the arc at p_o = 400 psi, I_o = 45,000 amperes

400 psi, but the initial current was only approximately 45,000 amperes. The pictures were obtained for approximately the first 3 milliseconds of the discharge. It was characteristic of the series of photographs, the best of which have been reproduced here, that the duration during which the image could be photographed was considerably shorter at higher current levels.

For a specific case which is to be used in the analysis, the frames in Figure 23 were carefully measured. The current was 66,000 amperes and the initial pressure was 1200 psi in nitrogen. The fourth frame has been redrawn in Figure 26 to indicate the approximate position, shape and size of the arc, relative to the arc chamber. If one assumes that the visable cross section of this arc is approximately the conducting cross section of the arc so that the luminous part of the column can be considered the actual arc column diameter, then the diameter of the arc is just under three centimeters. The arc length, at this particular time, is a little bit more difficult to estimate, but careful scaling of the shape of the arc on the previous frames and on subsequent frames indicates that the arc length must be between 10 to 16 centimeters with a length of 13 centimeters being the best estimate of the true length.

The speed of the camera was approximately 14,000 frames per second at the time this picture was taken and this picture

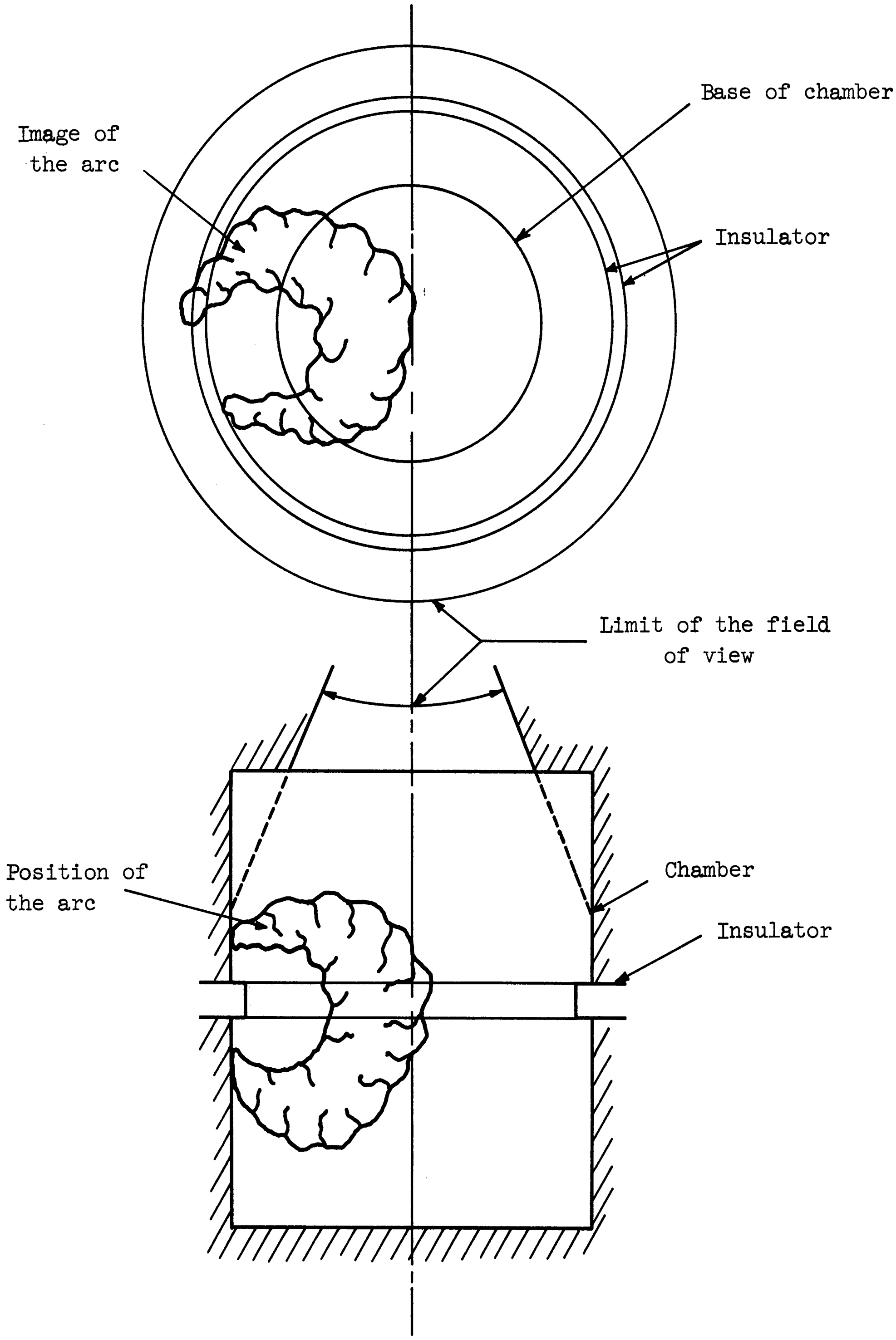


Figure 26. Sketch of the arc within the chamber.

is the fourth frame. Therefore, the time from the instant that the voltage started to rise until the arc had assumed this position was approximately 0.2 milliseconds. If the remaining frames are treated in a similar manner it is possible to determine the size of the arc, the arc column diameter and length, the volume which it occupies, and the velocity with which it is moving.

One of the interesting aspects of this series of pictues is that the arc moves as an entire column. It does not leave behind a luminescent slowly decaying plasma column, nor, on the other hand, does the column appear to slice through the air with a very sharp edge as one might anticipate observing, if the arc column were perhaps a millimeter in diameter. The column apparently moves fairly uniformly, maintaining approximately a circular cross section and as soon as the trailing part of the plasma column cools sufficiently to cease conducting it apparently ceases radiating as well. The concept that the visual diameter are approximately equal in size has been developed quite fully by Elenbaas(14) for the high pressure Mercury arc. The arc in this chamber appears to behave as a unit moving almost as a spoke through the very dense gas. In all cases there was sufficient contamination generated to obliterate the image on the ground glass after the first several frames. The contamination appears to come from the fusable link.

In all cases there was sufficient contamination generated to obliterate the image on the ground glass. The individual images which have been reproduced in Figures 23 and 24 show the arc for the first $\frac{1}{2}$ millisecond. In all cases the total duration of the arc discharge was some six to eight milliseconds. As these images appear reproduced here individually it is difficult to determine what caused the arc to "fade out".

However, when the original films are run through a movie projector, the cause of the failure to get an image after the first millisecond becomes clear. In each case the arc initiation is clearly observable as the fusable link breaks. Starting at the chamber wall where the fuse breaks, what can only be described as a black cloud can be seen to roll up the wall of the chamber towards the window and this cloud eventually rolls across the top of the arc obliterating the image. It appears almost certainly that this cloud is condensed copper vapor from the copper that is eroded from the base of the arc. The vapor is forced away from the cathode spot quite violently by the forces on the cathode spot. When the vapor strikes or mixes with the cool surrounding air, it condenses into microscopic copper dust particles. These then continue to blow across the window, obscuring vision. The chamber has always contained a fairly large volume of extremely fine dust after each shot.

The source of this dust, then would appear to be the copper that is evaporated from the cathode and anode spots.

Some consideration was given to the possibility that the black cloud could be a propagating shock wave with sufficient turbulence behind it to destroy the image of the arc column. However, assuming that this shock wave is propagating into air at 300°K, its velocity must be in excess of 1000 ft/sec, and yet at this velocity of sound wave, it would take no more than 1/3 millisecond to propagate sufficiently to cut off the view of the arc. In some cases the image of the arc can be seen for 10 times this duration, and in all cases for at least twice this duration. It seems safe to conclude, then, that the limitation of the optical system as designed is in the contamination generated within the arc chamber and not an inherent limitation of the gas dynamics of the hot air - cold air interface.

The photographic experimentation produced two significant results. First, the velocity of the arc traveling through the air could be established quite accurately by determining the time in terms of the number of frames on the film that it took the arc to move from one side of the arc chamber to the other. In some cases it was possible to determine where the arc moved and how fast it moved after the first contact with an opposite wall. This can be determined most readily from the series of pictures shown in Figure 23 where observation of the moving film has indicated that the arc travelled across the arc chamber in a period of approximately 0.4 milliseconds where the distance is approximately 3-3/4 inches. The velocity, then, comes out to be approximately 800 ft/sec which is very close to the velocity of sound in air of a temperature of 300°K.

In the work of Early and Walker[61] the arc was found to move at a velocity of 145 ft/sec when the current was 4.9 amperes and in applied external field of 6000 gauss was perpendicular to the current. The velocity of this arc is about three times as great as the velocity measured by Early, but when one considers that the current involved is of the order of 1000 times as great and the magnetic fields associated with the high current arc must be even larger than the magnetic fields that they used, this is a surprisingly small increase in the velocity of the arc. In none of the films that were observed was there an indication that any arc velocity was significantly greater.

One is very tempted to suggest that the velocity of this arc might be increased as the temperature of the gas goes up with the addition of energy inside the closed chamber. However, the general theory of arc motion due to a magnetic field, is not clear. It should prove most interesting in the future to build another arc chamber designed explicitly for photographing the arc under its self-magnetic and perhaps externally applied magnetic fields. It would then be possible to establish the relationships between the current, magnetic field, and velocity as a function of gas density and temperature on an experimental basis. Once the experimental effects were available it should be possible to develop a theory that would adequately explain the motion of the arc.

The diameter of the arc is one of the more important parameters that determines the arc characteristics as will be shown in a later section. The photographs give a reasonable indication of the approximate size of the diameter when one considers that the arc chamber has an inside diameter of 3-3/4 inches and the arc is small compared to this dimension. At the other extreme the arc is very definitely not a filimentary, microscopic or hairlike arc as might be anticipated at the high pressures. It is reasonable to assume that at the range of 50,000 amperes and pressures of the order of 100 atmospheres the arc diameter is of the order of one inch, judging by the large group of pictures that have been developed here.

PART II

ANALYSIS OF THE EXPERIMENTAL RESULTS AND EXTENSION OF THE THEORY TO THE RADIATION CONTROLLED ARC

CHAPTER V

AN ENERGY BALANCE

The first five frames of the photographs in Figure 23 can be used to determine an energy balance for the switch period of the discharge. The equation for the energy balance can be considered in terms of a power balance first to evaluate the important factors. The power equation is

$$VI = \frac{dU}{dt} + \frac{dW_k}{dt} + P_{rad} + P_{conv} + P_{cond} \qquad (26)$$

where

V = arc voltage

I = arc current, amperes

U = internal energy of the plasma column, joules

W_k = work done by the column, joules

P_{rad} = total power radiated by the column, watts

P_{conv} = total power lost by convection, watts

P_{cond} = total power lost by conduction, watts

The components can be estimated in terms of the fourth frame of Figure 23 where the volume of the arc is 100 cm^3, the area is 65 cm^2, the lapsed time since initiation is .21 ms, the voltage is 850 volts, the current is 63000 amperes and the pressure is about 100 atmospheres. The input power is:

$$VI = 54\ Mw \tag{27}$$

The time rate of change of internal energy can be estimated as

$$\frac{dU}{dt} = \frac{U}{v}\frac{dv}{dt} \tag{28}$$

As will be shown later, the energy density in the arc is about 70 joules/cm^3 and the volume change from zero to 65 cm^3 in .21 ms about 3×10^5 cm^3/sec. The product of these numbers yields

$$\frac{dU}{dt} = 21\ Mw \tag{29}$$

This is a little bit lower than the actual value but indicates that this factor is significant.

The power radiated can be estimated from the Steffan-Boltzman black body radiation law.

$$P_{rad} = \sigma_B T^4 \tag{30}$$

where

$\sigma_B = 5.67 \times 10^{-8}$ watts/m^2/$(^oK)^4$,

the Stefan-Boltzman constant.

If the temperature is taken as 12,000^{o}K and the area as 65 cm^2,

$$P_{rad} = 12\ Mw \tag{31}$$

which is a significant heat transfer rate. A slight difference in temperature will cause a much greater variation in this factor, of course.

The work done by the expanding column can be estimated from

$$\frac{dW_k}{dt} \cong p\frac{dv}{dt} \tag{32}$$

The pressure is about 100 atmospheres and the rate of change of volume is 3×10^5 cm^3/sec. Since one atmosphere-cm^3 is near enough .1 joules,

$$\frac{dW_k}{dt} = 3\ Mw. \tag{33}$$

This term is rather small compared to the radiation losses and will be neglected as an effect on the arc but it should be pointed out that it is probably a significant factor in the process of distributing the energy throughout the chamber which leads to thermal equilibrium when the arc extinguishes.

The loss due to convection cannot be easily estimated. The loss depends on particle transfer which is not readily estimated. The effect can be calculated by the method in Appendix A. This is done in Chapter VI and depends on the thermal conductivity of the gas. If the thermal conductivity is taken as .02 watts/$cm^2/^oK$ the power loss is estimated to be

$$P_{conv} \cong 0.15\ Mw \qquad (34)$$

Finally, the conduction loss can be estimated in terms of a reasonable maximum temperature gradient. For an arc temperature of 12,000^{o}K and an ambient temperature 1.0 cm from the edge of the arc, the gradient is 12,000^{o}K/cm. Then,

$$P \cong 0.02\ Mw \qquad (35)$$

The conduction and convection losses are completely negligible.

The following process will be used to determine the energy balance. First the diameter and the approximate arc length will be determined from each picture. The diameter can be scaled in terms of the size of the image in relationship to the diameter of the arc chamber which is 3.8 inches. The length of the arc is somewhat more difficult to determine accurately because the camera tends to look along the length of the arc. However, reasonable estimates of the minimum and maximum arc length can be determined. From this data then the volume occupied by the arc and its surface can be calculated.

The framing rate was 14,000 frames per second so there is 0.7 milliseconds between each frame. The electrical data that was recorded has been used to determine the voltage and pressure as functions of time. The voltage rises linearly to a peak value of 2,400 volts in 0.6 milliseconds so that the rate of voltage rise is 4×10^6 volts/sec. The pressure was known to be 1,200 psi at the initiation and rises at an approximate rate of 2,600 psi per millisecond. The initial current was 66,000 amperes and this decreases slightly during the period in which these exposures were taken. An average value of 63,000 amperes will be assumed for the value of the current during the first 0.3 millisecond of this discharge.

The electrical input to the arc column in terms of total power can be readily determined from the experimental data. If the voltage is assumed to rise at a rate of 4×10^6 volts per second and the current is constant at 63,000 amperes, the total electrical input can be expressed simply as

$$P = (63{,}000)(4 \times 10^6 t) \tag{36}$$

The integral of this equation is the total energy that has been dissipated in the arc chamber.

The real gas tables[25] can be used to obtain a reasonable approximation of the total energy that is stored in the plasma. While it might seem surprising at first, the energy that is stored in the plasma is almost independent of the temperature of the gas column. The internal energy of the gas is much more a function of the pressure. The value of the internal energy as a function of both pressure and temperature was calculated, and it was established from the tables that the pressure is the significant variable if the arc column is assumed to be anywhere in the range of 10,000°K to 15,000°K.

The relationship between the internal energy and the pressure can be shown in terms of the perfect gas law for a monatomic gas by the simple relations that,

$$U = \frac{3}{2} n k T \tag{37}$$

and,

$$p v = n k T \tag{38}$$

The ratio of these two equations will eliminate the number of particles and the gas temperature with the result that the internal energy stored in the plasma per unit volume is directly proportional to the pressure of the gas, or

$$\frac{U}{v} = \frac{3}{2} p \tag{39}$$

The actual dependence of the energy density is constant with temperature. Thus it is relatively easy to calculate the value of the energy stored within the plasma column without requiring detailed knowledge of the temperature or temperature distribution within the arc column, if the dimensions have been reasonably well determined.

For the balance that is made here it is assumed that radiation is the only mechanism which removes a significant amount of energy from the column. It is further assumed at this point that the temperature is 12,000°K. The energy balance consists of measuring the total energy that has been dissipated by the arc and balancing this against the energy that has gone into creating the plasma plus that which has been radiated. All other effects are neglected. The results of the balance are listed in Table 1.

The results of the energy balance have been drawn on the graph, Figure 27. There is a slight discrepancy associated with the calculation for the second frame at the instant 70 microseconds after the initiation of the arc. Otherwise, the correlation is extremely good, indicating that approximately 3/4 of the energy is used to raise the internal energy of the gas and that about 1/4 of the energy is radiated. Furthermore, the assumed temperature of 12,000°K seems to be a very realistic value for the arc temperature. As will be shown in the next chapter, there is a good theoretical reason for choosing the value of 12,000°K.

TABLE 1

ENERGY BALANCE

Exposure	Diam (cm)	Min Length	Max Length	Est. Length	Volume	t(ms)	V
1	Just initiating					0	0
2	1.9	6.5	7.5	7.0	21	.07	280
3	2.2	8.0	12.0	10.0	38	.14	560
4	2.5	10.0	16.0	13.0	65	.21	840
5	3.0	14.0	20.0	16.0	110	.28	1120

Exposure	Pressure psi	Pressure at.	Energy density	Energy Stored Kjoules	Elec. Input Kjoules	Radiation Power Watts	Energy Radiated Kjoules
1							
2	1200	82	50	1.0	.65	5.0	0.2
3	1450	97	60	2.3	2.6	8.4	0.6
4	1700	115	70	4.5	5.9	12	1.4
5	1950	131	72	8.0	10.4	18	2.4

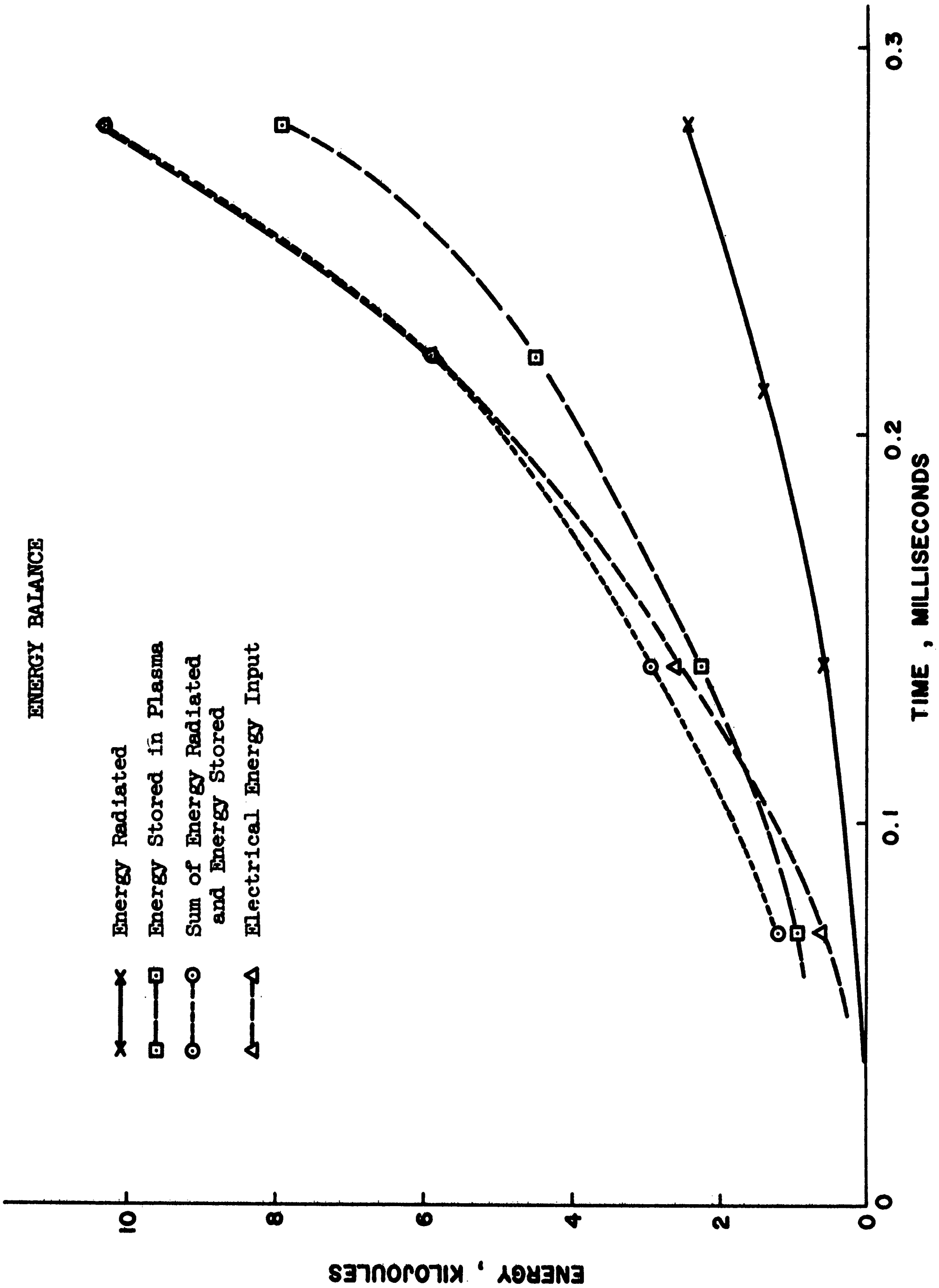


Figure 27. The energy balance.

CHAPTER VI

THE STEADY STATE RADIATION-CONTROLLED ARC

It will be shown that the characteristics of the electric arc inside of the pressure chamber are adequately described in terms of a steady state arc that is controlled by radiation mechanisms of heat transfer. The arc is subject to extreme changes during its ten millisecond lifetime because the current starts at a maximum and decreases almost linearly to zero while the pressure starts at a moderate value, typically 25 atms, and rises to a final value, depending on the total energy, in range of 300 to 800 atms. (4,500 to 12,000 psi). In order to make a comparison between experiment and theory, data as used in the energy balance will be used for comparison. Parameters such as current and pressure will be considered as constants which have been measured. Corrections for the effects due to energy storagy in the plasma will be considered prior to comparison.

The physical behavior of the arc in the steady state is controlled by the conductivity of the gas and the method by which the energy is removed from the column. These are functions primarily of temperature, but are affected by pressure, current, sometimes by magnetic field, and by the composition of the gas. It is possible to write a complete set of equations which describe the arc behavior in terms of these parameters. For simplicity here, the radiative loss will be considered in terms of an arc which radiates as a perfect black body.

The arc column is described in terms of the current or current density, voltage gradient, diameter, and the temperature. The temperature will be assumed to be constant across the arc diameter, which is usually nearly true providing the diameter is taken as the electrical conduction diameter. This last statement is reasonable because the electrical conductivity depends upon an exponential term in the Saha equation so that a decrease of a few per cent in temperature will decrease the conductivity by a large amount.

The concept of arc diameter is somewhat more nebulous. It will be defined as the diameter within which current flows but will be measured in terms of the part of the arc that radiates in the visible spectrum. Since the diameter is the least accurately determined parameter this should introduce no significant additional error.

The equations can be written starting with the heat transfer equations in terms of the Stefan-Boltzman equation for the idealized case of unit emissivity,

$$P_{rad} = \sigma_B T^4 \qquad (40)$$

For a cylindrical arc column the power input per unit length is here assumed to be equal to the radiation power per unit of length so that

$$EI = \pi D \sigma_B T^4 \tag{41}$$

where

E = voltage gradient, volts/m

D = arc diameter

The radiation towards the column from the surrounding medium is neglected.

The current conduction equation is

$$J = \sigma E \tag{42}$$

where σ = electrical conductivity, mhos/meter

or for the total crossection if the electrical conductivity is assumed to have a constant value across the arc diameter

$$I = \frac{\pi}{4} D^2 \sigma E \tag{43}$$

The conductivity is a function of temperature. It can be calculated in terms of the electron density and electron mobility, which are both functions of temperature. The more important of the two is the electron density, n_e. The electron density can be calculated from the Saha thermal ionization equation which is given in its general form for first ionization[40] by

$$\frac{n_i n_e}{n_o} = 2 \frac{B_1(T)}{B_2(T)} \left(\frac{2\pi m_e kT}{h^2} \right) e^{-\frac{q_e V_i}{kT}} \tag{44}$$

where

n_e = electron density, per meter^3

n_i = positive ion density, per meter3

n_o = neutral particle density, per meter^3

q_e = charge of electron, coulombs

m_e = mass of electron, kilograms

k = Boltzman constant

V_i = ionization potential of the gas

h = Planck's constant

$B_o(T)$ = partition function for the atom

$B_1(T)$ = partition function for the ion

The partition functions, $B_o(T)$ and $B_1(T)$, are constant up to temperatures near 50,000°K. The fraction of ionization for this arc is small and neutral and the perfect gas low holds reasonably well provided the number of atoms, not molecules, is used since the gas is almost completely dissociated. Thus the simplified version of the Saha equation in MKS units

$$\frac{n_e^2}{n_o} = 2.42 \times 10^{21} T^{\frac{3}{2}} e^{-\frac{q_e V_i}{kT}} \tag{45}$$

This equation also appears in the literature[4] in the logarithmic form

$$\log \frac{n_e^2}{n_o} = -\frac{5040}{T} + \frac{3}{2} \log T + 21.38 \tag{46}$$

The value of neutral particle density is relatively easy to estimate but quite difficult to evaluate exactly because it is implicitly a function of the pressure as well as temperature and ionization potential. The exact solution can be obtained by using the real gas tables in a computer solution, but a physical picture is the object of this discussion and a simplifying assumption will be made in order to obtain a closed solution.

If the pressure and temperature are known, the real gas tables[25] can be used to determine the condition of air, which is very similar to nitrogen. The state of the gas can be determined in the following manner. The tables are tabulated in terms of pressure, p, temperature, T, mass density ratio ρ/ρ_0 and compressibility factor, Z. They also contain information on the internal energy, enthalpy, and entropy. The density, ρ_0 , is the density of air at $273^{\circ}K$ and one atmosphere. The compressibility factor is defined by the equation, for one mole of gas,

$$Z = \frac{p v}{R T} \tag{47}$$

where

p = pressure in atmospheres

R = universal gas constant

The equation of state for a real gas is in one form,

$$\frac{p}{p_0} = Z\left(\frac{\rho}{\rho_0}\right)\left(\frac{T}{T_0}\right) \tag{48}$$

If Z has a value of unity, this is the perfect gas low.

The form of the equ. (48) is particularly informative because Z represents the deviation from the perfect gas low for a given mass density. Another way to say this is that once the number of moles or kilograms is established in a closed volume, the non-linear (or real gas) effects in the pressure temperature relationship are defined in terms of Z. For application in this problem, Z will include the effects of dissociation and ionization.

An example taken from the real gas tables yields the following conditions

$$\begin{aligned} p &= 19.74 \text{ atmos.} \\ T &= 15{,}000^{o}K \\ {}_{ko} &= .158 \\ Z &= 2.27 \end{aligned} \qquad (49)$$

The value of Z indicates that each original diatomic particle is dissociated and that about 13% of these particles are ionized. This is a good indication of the condition of the gas at the range of pressure and temperature that are of interest here. Even if the pressure is increased to 600 atmospheres at 15,000^{o}K the factor Z will still be greater than 2.0. At 12,000^{o}K and 100 atms Z is 1.95, still indicating almost total dissociation and 2% ionization. Thus, the approximations are made that:

a) the neutral gas particles are totally dissociated atoms.

b) the number of electrons (and ions) is small compared to the neutral gas particles, or the partial pressure of the neutrals is equivalent to the total pressure

The neutral particle density then is given by the perfect gas law,

$$n_o = \mathcal{L} p \frac{273}{T} \tag{50}$$

where

$\mathcal{L}$ = Loschmidt number,
2.687 x 10^{25} atoms/meter^3 (51)

The neutral particle density can be used in the Saha equation to yield an expression for the electron density.

$$n_e^2 = \mathcal{L} p \left(\frac{273}{T}\right)\left(\frac{2\pi m_e k}{h^2}\right) T^{\frac{3}{2}} e^{-\frac{q_e V_i}{kT}} \tag{52}$$

or

$$n_e = 4.2 \times 10^{24}\, T^{\frac{1}{4}} p^{\frac{1}{2}} e^{-\frac{q_e V_i}{2kT}} \tag{53}$$

The electron mobility is a function of the mean free path of an electron in the gas and the random velocities of electron. The mobility is of the form[10]

$$g_e = \text{const.} \times \frac{q_e l_e}{m_e \bar{C}_e} \tag{54}$$

where

g_e = mobility of electron meters^2/volt sec (55)

ℓ_e = electron mean free path, meters

$\bar{C}_e$ = average velocity of the electrons that cross a plane in one direction

The constant of proportionality h as a value near unity. It depends on assumptions. The value of $8/3\pi$ was computed by Compton[6] and is used hereafter. If the electrons have a Maxwellian distribution of velocities and, correspondingly, a temperature, then

$$\bar{C}_e = \sqrt{\frac{8kT}{\pi m_e}} \tag{56}$$

The mean free path of the electrons is more difficult to define because it is more properly described in terms of the collision probability which is a function of electron energy. The lack of information a bout the collision probability in a high gas density situation places a severe limit on the accuracy of the mean free path. Since the value used for the electron mean free path in air wether measured in terms of Van der Waals effects, viscosity effects, or low density direct electrical measurements agree within roughly thirty per cent[31], the standard of comparison mean free path will be used here and a temperature, or energy variation will be neglected. The standard of comparison mean free path is defined as $4\sqrt{2}$ times gas particle mean free path in solid elastic spheres, and varies according to the perfect gas low relationships. The relationship is expressed as,

$$l_e \cong l_n \cong l_{no} \frac{1}{p} \cdot \frac{T}{273} \tag{57}$$

where

l_n = standard of c omparison electron mean free path
(58)
l_{no} = standard of comparison electron mean free path and 1 atmos. and 273°K.

The term, l_{no} has a value of 4 x 10^{-4} meters a 1 mm Hg or 5.3 x 10^{-7} meters at one atmosphere.

It is now possible to rewrite the mobility equation. It takes the form,

$$g_e = \frac{8}{3\pi} \left(\frac{q_e}{m_e}\right)\left(\frac{l_{no} T}{p \times 273}\right)\sqrt{\frac{\pi m_e}{8 k T}} \tag{59}$$

The electrical equations can now be combined so that

$$\frac{J}{E} = \sigma = n_e q_e g_e \tag{60}$$

The substitution of equs. (53) and (59) permit the elimination of the electron density and mobility

$$\frac{I}{E} = \pi D^2 q_e \mathcal{L}^{\frac{1}{2}} \left(\frac{2\pi m_e k}{h^2}\right)^{3/2} T^{\frac{1}{4}} p^{\frac{1}{2}} e^{-\frac{q_e V_i}{2kT}} \left(\frac{8}{3\pi}\right)\left(\frac{q_e}{m_e}\right)\left(\frac{l_{no} T}{p \times 273}\right) \tag{61}$$

This equation can be written in a simpler form by evaluating the constants

$$\frac{I}{E} = 9.5 \times 10^4 D^2 T^{\frac{3}{4}} p^{-\frac{1}{2}} e^{-\frac{q_e V_i}{2kT}} \tag{62}$$

The heat transfer equation was

$$EI = \pi D \sigma_B T^4 \tag{63}$$

The current and pressure are measured or controlled parameters and all the other terms are constants of mature. Thus there are three variables, E, T, and D which remain. The final relationship is the Steenbeck Minimum Principle[54,55] which is expressed as

$$\frac{\partial E}{\partial T} = 0 \tag{64}$$

The expression will be discussed at length later. For now, it means only that it is assumed that the temperature will be adjusted to create the minimum possible voltage gradient.

The diameter, D, may be eliminated by substitution which yields an explicit solution between the voltage gradient and temperature. The diameter is given by

$$D = \frac{EI}{\pi \sigma_B T^4} \tag{65}$$

and when this is substituted in the conduction equation the result is

$$\frac{I}{E} = 9.5 \times 10^4 \left(\frac{E^2 I^2}{\pi^2 \sigma_B^2 T^8} \right) T^{\frac{3}{4}} p^{-\frac{1}{2}} e^{-\frac{q_e V_i}{2kT}} \tag{66}$$

This can be solved for the gradient so that

$$E^3 = 3.3 \times 10^{-19} \frac{T^{\frac{20}{4}}}{I} p^{\frac{1}{2}} e^{+\frac{q_e V_i}{2kT}} \tag{67}$$

or

$$E = 6.5 \times 10^{-7} \frac{T^{\frac{29}{12}}}{I^{\frac{1}{3}}} \, p^{\frac{1}{6}} \, e^{+\frac{q_e V_i}{6kT}} \tag{68}$$

The minimum of the voltage gradient is determined by taking the derivative of the function of temperature. This is readily accomplished with the result that the minimum occurs when

$$T = \frac{q_e V_i}{6k\left(\frac{29}{12}\right)} \tag{69}$$

The value of ionization potential, V_i, for nitrogen is 14.48 volts and using a perfect black body radiation law, the temperature of the arc is 11,500°K according to these calculations.

This value of temperature can be used to evaluate the other parameters. For a specific example, values cited perviously will be used here. The initial current was 66,000 amperes and initial pressure in nitrogen was 1200 psi. The time was taken as 0.21 milliseconds after the arc initiation. At this instant the current was calculated to be 63,000 amperes and the pressure had risen to 1700 psi or 115 atmospheres. These values predict that the voltage gradient should be 2400 volts/meter. The measured arc length was between 10 cm and 16 cm with the value of 13 cm being the best value which predicts an arc voltage of 310 volts.

The power requirement is 1.5 megawatts/cm which compared well with an experiment value of 1.0 megawatts per cm. Finally, the arc diameter can be calculated for the same 63,000 ampere case. From equ. (65) the diameter is calculated to be 4.05 cm. This is too large by a fraction of only 30%.

The theory that is outlined here can be used to calculate the gradient, the temperature, and arc diamter with reasonable accuracy. The errors involved appear to be negligible for temperature, a factor of about 30% in the voltage gradient and 40% in the diameter. While the accuracy leaves something to be desired, it does indicate that the basic concepts are probably correct and that it is the idealized laws and models that lead to errors. After all, the arc that is being subjected to this analysis was designed to heat air for a wind tunnel, not to be a perfectly controlled experimental arc. There is no spectroscopic information to indicate the temperature or the temperature distribution across the arc nor was the fact that the experimental arc was required to store energy in the plasma given more than first order consideration. In the next section, the possible effects of some of the correction factors will be considered.

CHAPTER VII

ADDITIONS TO THE RADIATION THEORY

NON-RADIATIVE MECHANISMS

The electric arc characteristics for lower current and pressure, that is, the more commonly encountered arcs, are controlled by the convective or conductive heat transfer mechanism. The work of Foitzik[16] with a rotating chamber indicates the extremes that are necessary to prevent convective effects. Elenbaas[14] illustrates also that the purely conductive case produces a very wide arc column. As demonstrated in Appendix A, Suits has shown that natural convection and Dow has shown that forced convection can account for the heat transfer in arcs that operate at about one atmosphere and at currents up to some 10,000 amperes. There is also work reported[13,40] for the solution of the Elenbaas-Heller equation in the form

$$\sigma E^2 - S(T) + div(\kappa \; grad \; T) = 0 \qquad (70)$$

where the term S(T) accounts for small radiative losses from the arc column.

It is the purpose of this section to show that the convective or conductive heat transfer mechanisms cannot account for any appreciable fraction of the energy that is dissipated in t he 60,000 ampere high pressure arc. The work that has been done experimentally on electric arcs in which conduction is the primary mechanism has used apparatus that is expressly designed to eliminate confection loss. Natural convection losses are caused by gravitational forces and, thus, convection can be eliminated by containing the arc with walls or by containing the arc in a rotating chamber so that the time average gravitational force is eliminated. Experiments with a rotating chamber have generally been limited to currents of the order of 10 amperes. Thus, the general mass of experimental evidence indicates that the convective mechanism is the dominant process in high current, unconfined electric arcs.

The convective losses are increased if the gas flow is increased around the arc, or for a given geometry, the forced convection heat transfer is greater than the natural convection heat transfer, and even though there are small geometric changes in an arc under forced convection, it is well established that the arc voltages must be higher for a wind blown arc than a stationary arc. Indeed, this is the process that is utilized in circuit breakers to increase the switch voltage.

There are two methods of producing forced convection in an electric arc. Either an air blast may be directed along or across the arc or magnetic forces on the arc may be used to move the arc through stationary gas. The self-magnetic forces of the arc are adequate to move the arc very rapidly when the current is as high as it was in these experiments. It is postulated that the magnetic forces are sufficient to cause convective heat transfer but it can be shown that the convective losses cannot account for any reasonable fraction of the power that was dissipated in the chamber.

Consider a case of convective heat transfer where the arc column moves through still gas due to magnetic forces. If the gas temperature is taken as 12,000°K and the pressure as 1 15 atmospheres the real gas tables indicate that the density is 1.3 times the density at 273°K and one atmosphere or 1.65 gr. per liter. The velocity of the arc was measured to be 800 ft/sec or 24,000 cm/sec. With these measured parameters, the heat transfer due to forced convection can be estimated through the use of the theory in Appendix A.

The heat transfer under forced convection is a function of the Reynolds Number. For the velocity and density above, the mass flow is 40 gr/sec/cm^2. For an arc diameter of 3cm and a viscosity of 5 x 10^{-3}(8,31,34) at this temperature, the Reynolds Number has a value of about 25,000. The corresponding value of the Nusselt Number is about 150 where the Nusselt Number, N_{Nu}, is defined as

$$N_{Nu} = \frac{h_m D}{K_f} \tag{71}$$

where

h_m = mean value of the heat transfer coefficient

D = diameter

κ_f = mean film value of the thermal conductivity

The Nusselt number is a dimensionless quantity.

The power balance can be written for the forced convection effect as

$$\left(\frac{dQ}{dt}\right)_{conv} = \pi D h_m T \tag{72}$$

when the temperature of the ambient is small compared to the arc temperature. The term $(dQ/dt)_{conv}$ represents the heat loss. This can be written

$$\left(\frac{dQ}{dt}\right)_{conv} = \pi T \kappa_f N_{Nu} \tag{73}$$

The value of the thermal conductivity can be estimated from either McAdams[34] by extrapolation, or from an estimation of the viscosity, Prandtl number and specific heat at 12,000°K. Either yields a value of .020 watts/cm/°K. The power loss becomes

$$\left(\frac{dQ}{dt}\right)_{conv} = \pi \times 12{,}000 \times \frac{1}{50} \times 150 \tag{74}$$

$$\cong 150{,}000 \text{ watts per cm of arc length}$$

This value for convection loss is based on a value of thermal conductivity at 12,000°K rather than an averaged value. The resultant loss even so is quite small compared to the measured value of 1,000,000 watts per cm, and the value used here for thermal conductivity is twenty times as large as the thermal conductivity which has been estimated for the mercury vapor discharge[14].

Thus, it would seem that the convective losses from the column are relatively small and can be neglected in the analysis of the arc. This should not infer, however, that the magnetic forces are unimportant in the o peration of the hypersonic tunnel chamber. The turbulence created by the magnetic field is instrumental in distributing the energy after the gas is heated.

MODIFICATION OF THE RADIATION EQUATION

The results obtained through the use of the black body radiation equation, conductivity and minimum principle are sufficiently accurate to indicate that the physical principles are basically correct for the arc that was analyzed. The total power input was measured quite accurately and the power per unit length is known within an error of $\pm$ 20%. The calculated diameter was approximately equal to the measured diameter when based on the calculated temperature. The calculated voltage gradient seems to be slightly higher than would be indicated by the measured power. Overall, this appears to be a good example of an arc discharge that is completely described in terms of radiation losses.

The statement was made above that convection is usually found to be the controlling heat transfer mechanism in unconfined arcs. Some attention should be paid to the "near black body" case where the arc column may not be optically thick and yet the radiative losses may still be large compared to convection losses. The author estimates based on past experience and based on Peters[46] work that this regime probably occurs in air at pressures of 10 atmospheres, currents of 10,000 amperes as an example.

The theory as developed here shows that the temperature is given by,

$$T = \frac{q_e V_i}{6k(r)} \tag{75}$$

where the value of r = 29/12 corresponds to a perfect black body. The ionization potential decreases in a very dense gas[40] but should remain a constant of 14.48 volts for nitrogen. Thus, the temperature will have a higher value when the exponent, r, is lower.

It should be clarified here that the exponent, r, was dependent upon two assumptions. The first was that the radiation power varied exactly as the fourth power of the temperature. The second was that the radiation came from the surface of the arc, not from points throughout the volume of the arc. If the radiation originates uniformly throughout the volume of the arc, then equ. (65) should read

$$D = \frac{EI}{\pi P_r(T)} \tag{76}$$

where $P_r(T)$ = power radiated per unit volume of arc

It is interesting to note that if equ. (76) is substituted into the conduction equation, equ. (62), the result is

$$E^2 = 3.3 \times 10^{-6} P_r(T) T^{\frac{3}{4}} p^{\frac{1}{2}} e^{+\frac{q_e V_i}{2kT}} \tag{77}$$

The arc voltage is to a first approximation independent of the current. In the experiments where the pressure is a function of the total energy (or integrated power) that has been dissipated, as in the arc chamber experiments, the pressure increases with current. The change in pressure is approximately proportional to the joule heating of the gas or as the square of the current. Thus. equ. (77) should indicate implicitely that the voltage increases with increasing current. In arc terminology, then, this arc should show a positive volt-ampere characteristic. This situation has been found experimentally and reported without explanation previously(9), even for the case where the black body situation presumably occurs.

With this much insight into the problem, it seems worthwhile to pay closer attention to the "near black body" radiation condition. The first step is to consider the effect of gas density on the type of radiation that eminated from a plasma. At low density the radiation is found to have a line spectrum which originates from essentially isolated atoms. The atoms may have been excited by either condition in very low density plasma or by collision processes. As the density is increased the spectral lines tend to be broadened and a low intensity continuum will appear, arising from electron transitions from free to a bound state. If there are temperature gradients, there may be re-absorption and the intensity of the central part of the line will be reduced. As the gas density is increased further the lines become broader and the background continuum becomes stronger until finally in the limit the radiation distribution obeys Planck's law.

The intensity of the lines within a line spectrum are related to the temperature of the gas particles.[8] The actual determination of gas temperature from line intensity is quite difficult because of extraneous effects such as re-absorption, etc. In the black body continuum case, the temperature is related to radiation terms of the Planck relationship. Except for first order corrections, such as the determination of temperature from the Doppler line broading effect, in between these two extremes according to Loeb[31] the theory is not adequate to relate the gas temperature to the shape of the radiation distribution curves.

An experimental verification of the transition from a line spectrum to near-black-body was carried out by Peters.[46] He used a pressure chamber which confined a water stabilized[35,36] arc at pressures up to 1000 atms. Currents were between 100 amps and 200 amps. The hydrogen lines were well defined at 80 atms. At 300 atms, the intensity of the continuum between vestigal lines attained 20% of the black body intensity for the corresponding temperature. At 1000 atms the radiation was within 20% of the corresponding black body distribution for a temperature of 12,000°K.

Therefore, on the assumption that the radiation characteristic deviates from a black body, the radiation power can be expressed as

$$P_{rad} = \left(\sigma_B T^4\right) F(T) \tag{78}$$

where the function, F(T), is a measure of the deviation from black body conditions. It must have a value less than unity. The existence of this function also infers that the arc is, to some small extent, transparent, or that the light that leaves the surface may have been generated at some finite distance below the surface which constitutes a volume effect. However, since this is nearly the black body condition, the volume effects will be neglected and the function, F(T) will be treated as a surface modification.

The modified equation for the diameter becomes

$$D = EI/\sigma_B T^4 F(T) \tag{79}$$

and substitution directly into the conductivity equation gives

$$\frac{I}{E} = 9.5\times10^4\left(\frac{EI}{\pi\sigma_B T^4 F(T)}\right)^2 T^{\frac{3}{4}} p^{-\frac{1}{2}} e^{+\frac{q_e V_i}{2kT}} \tag{80}$$

the minimum principle yields a temperature for the arc in terms of a temperature derivative of the function, F(T). At this point only the maximum value of F(T) is known. Nothing is known about its functional dependence on the temperature, and so the derivative is equally uncertain.

As a first approximation to F(T), let it be of the form

$$F(T) = A_s T^s \tag{81}$$

This permits the temperature to be calculated in the form

$$T = \frac{q_e V_i}{6k\left(\frac{29+8s}{12}\right)} \tag{82}$$

From thermodynamic considerations, s, must be negative which will yield a higher temperature.

If the conduction equation is rewritten as

$$ED^2 = \left(\frac{I p^{\frac{1}{2}}}{9.5\times10^4}\right) T^{\frac{3}{4}} e^{+\frac{q_e V_i}{2kT}} \tag{83}$$

this equation can be used to evaluate the temperature. This results in a situation where the experimental arc gradient and diameter should predict a value of temperature and that temperature should depend only on the relationship between the radiation power and temperature which also can be measured experimentally. There is no data available at the present time to check this hypothesis.

CHAPTER VIII

INTRODUCTION TO THE THERMODYNAMICS OF IRREVERSIBLE PROCESSES*

The concept of true thermal equilibrium is one of the most valuable tools of physics. The attainment of perfect thermal equilibrium is a rarity. To be more specific, many processes, including chemical reactions, biological life processes, heat transfer in stellar atmospheres and electric current flow through an ionized gas are of great interest but cannot be described by equilibrium thermodynamics. In between the extremes are a number of cases of engineering importance that can be approximated by the concept of local thermal equilibrium. The electric arc is a case of a borderline condition where local thermal equilibrium is assumed in order to calculate the electron density, but which requires the use of the laws of irreversible processes to determine the gross characteristics. This Chapter is devoted to a brief introduction to the concept of entropy production, which will in turn be used to develop the "minimum principle" of Steenbeck.

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* The theory that is presented here is taken from the works of Prigogine with some modifications by de Groot. The material has been organized by the author for inclusion at this point for two reasons. First, a rudimentary knowledge of the thermodynamics of irreversible processes is manditory prior to reading Chapter IX. Second, this theory is not the common knowledge among electrical engineers that it is among thermodynamicists and this dissertation is primarily for electrical engineers.

The field of non-equilibrium thermodynamics is the result of a successful attempt by L. Onsager[43] to show that individual particles in a statistical distribution obey a principle of "time reversal invariance". The principle follows from the fact that the equations of motion are symmetric with respect to time, which is another way of saying that the particles will re-trace their trajectories if all velocities are reversed. There is then a reciprocal relationship associated with a distribution function.

The second major accomplishment was due to I. Prigogine[47] sixteen years later when he applied the Onsager reciprocal relations to nonequilibrium chemical reactions and successfully demonstrated that entropy production could be a controlling factor in the rate of chemical reaction.

The major portions of the theory were developed in a remarkably short time, probably less than ten years. In 1955 Prigogine published his first book[48] on the subject, with the second edition appearing in 1961 without major revision. Major contributions were made by S.R. de Groot and P. Mazur particularly during the early 1950's and they culminated their work by collaborating to write a book[23] which was published in 1961. References to virtually all of the work in non-equilibrium thermodynamics can be found in these two texts.

The analysis outlined here will follow the text of Prigogine because it presents the fundamentals clearly and in a readily understandable fashion. The text by de Groot and Mazur is without a doubt a much more rigorous and complete presentation of many of the derivations in terms of statistical mechanical methods, but for these very reasons is beyond the scope of this introduction.

Entropy is a function of state of a gas in thermal equilibrium in a closed system, according to the principles of classical thermodynamics. It is defined in terms of the differential equation,

$$dS = \frac{dQ}{T} \tag{84}$$

where

dQ = incremental heat added to a closed system by any reversible processes, kilogram calories

dS = incremental entropy, kilogram calories per degree Kelvin

The change in entropy of an entire system can be written in two parts;

$$dS = d_i S + d_e S \tag{85}$$

where

d_iS = incremental change in entropy due to the internal behavior of the gas, and

d_eS = incremental change in entropy due to interaction of the gas with the surrounding part of the system.

There are many apparently different statements of the second law of thermodynamics but perhaps the simplest formulation for a system is,

$$dS \geq \frac{dQ}{T} \tag{86}$$

The equality holds for reversible processes and the inequality for any irreversible process. This can be written as

$$d_i S + d_e S \geq \frac{dQ}{T} \tag{87}$$

and since T is the temperature of the gas

$$d_e S = \frac{dQ}{T} \tag{88}$$

so that

$$d_i S \geq 0 \tag{89}$$

for an insulated or adiabatic system. Examples of internal entropy production may be found in chemical reactions, mixing of gases, electric joule heating of a media, etc.

Conceptually, the internal entropy change may be considered to be taking place within any given volume of the gas and the total internal entropy change is then the sum of parts. This may be carried one step further to an infinitesimal volume within which there is an entropy source. The entropy production of the entire volume is

$$\frac{d_i S}{dt} = \int_{\mathcal{V}} \sigma_S \, d\mathcal{V} \tag{90}$$

where

σ_S = entropy production rate density, kilogram calories per meter^3 per $^{\circ}$Kelvin per second.

$\mathcal{V}$ = volume, meter^3

The term, σ_S , is a function of position and exists for every point throughout the volume. The significance of this is that it is now no longer necessary to consider the entropy as a function of state for a closed system. It becomes a function that may be used whether the system is either open or closed, because the entropy production is considered in terms of localized point phenomena.

The concept that there can exist continuity of mass, energy and/or momentum at each point throughout a volume is quite common and the continuity equations are written in a form, for example, as for mass,

$$\frac{\partial \rho}{\partial t} + \operatorname{div} \rho u = \text{source strenth} \tag{91}$$

where

u = velocity, meters per second

If one has conservation of mass, the source strength is zero, It is necessary to develop a similar equation for the continuity of entropy. Within any volume, the entropy is a function of the state of the gas which may be expressed as

$$S = \int_{v} s_{v}\, dv \tag{92}$$

where

s_v = entropy density, on intensive property.

For a fixed volume

$$\frac{dS}{dt} = \frac{\partial}{\partial t}\int s_{v}\, dv \tag{93}$$

The entropy flow can be expressed as a vector which has the same direction as the energy flow, and if it is integrated over the surface of the volume the total change of entropy can be expressed as

$$\frac{d_e S}{dt} = \int -\Phi \cdot dA \tag{94}$$

where

Φ = entropy flow

The entropy change can be equated to the sum of the two causes of change so that

$$\frac{dS}{dt} = \frac{d_e S}{dt} + \frac{d_i S}{dt} \tag{95}$$

If Gauss' theorem is applied to the entropy flow,

$$\int \frac{\partial s_v}{\partial t}\, dv = \int -\operatorname{div} \Phi\, dv + \int \sigma_s\, dv \tag{96}$$

or

$$\int \left(\frac{\partial s_v}{\partial t} + \operatorname{div} \Phi - \sigma_s \right) dv = 0 \tag{97}$$

Since this must hold for any arbitrary volume, the entropy continuity equation can be expressed as

$$\frac{\partial s_v}{\partial t} + \operatorname{div} \Phi = \sigma_s \tag{98}$$

By definition of entropy production

$$\begin{aligned} &\sigma_s = 0 \text{ for a reversible process} \\ &\sigma_s > 0 \text{ for an irreversible process} \end{aligned} \tag{99}$$

The calculation of the entropy production follows the technique used to calculate the entropy of a finite system. As an example, consider the entropy that is produced by the conduction of heat through a volume. The entropy change in the volume is zero in a stationary state because it is not subject to change with time. Then

$$\sigma_s = \operatorname{div} \Phi \tag{100}$$

or

$$\int \sigma_s \, dv = \int \Phi \cdot dA \tag{101}$$

If this is considered in one dimension, the entropy products can be readily calculated. If dQ/dt is the rate of heat flow through a volume with insulated sides so that the heat enters one end with a temperature T_1 and leaves the other end with a temperature T_2. The entropy flow into the volume per unit area is

$$\left[\Phi \cdot A\right]_{x_1} = -\frac{dQ}{dt} \cdot \frac{1}{T_1} \qquad (102)$$

and the entropy flow out is

$$\left[\Phi \cdot A\right]_{x_2} = +\frac{dQ}{dt} \cdot \frac{1}{T_2}$$

If the length of the volume is incremental then

$$v = A\,dx \qquad (103)$$

and

$$T_2 = T_1 + \frac{\partial T}{\partial x}dx \qquad (104)$$

The entropy production then becomes

$$\sigma_s = \frac{d(Q/A)}{dt} \cdot \frac{(T_1 - T_1 - \frac{\partial T}{\partial x}dx)}{T_1\,T_2} \qquad (105)$$

or

$$\sigma_s = -\frac{P_x}{T^2}\frac{\partial T}{\partial x} \qquad (106)$$

where

$$P_x = \frac{d(Q/A)}{dt} = heat\ flow \qquad (107)$$

in the x-direction per unit area.

This result for one dimension can be shown to be a specific result of a general vector form,

$$\sigma_s = -\frac{\vec{P}}{T^2} \cdot grad\,T = \vec{P} \cdot grad\left(\frac{1}{T}\right) \qquad (108)$$

where $\vec{P}$ = heat f low vector, kilogram calories per meter^2-sec

THE PHENOMENOLOGICAL EQUATIONS

The principle of entropy production sources can, of course, be extended to other irreversible processes such as diffusion or chemical reaction. Commonly more than one process is involved, e.g., chemical reactions are accompanied by thermal actions, or thermal diffusion can accompany molecular diffusion through a porous plug. If the factors that cause the entropy production are handled carefully, very interesting results can be obtained.

In the previous section it was shown that

$$\frac{d_i S}{dt} = \int_v \vec{P} \cdot \operatorname{grad}\left(\frac{1}{T}\right) \tag{109}$$

It is quite general that

$$\vec{P} = \kappa \operatorname{grad} T \tag{110}$$

(To be very general, the thermal conductivity, κ, should be expressed as a tensor but that is beyond the scope of this text). The temperature gradient controls the flow of heat. In a general sense the temperature gradient may be considered as a "force" which causes a "flow", in this case the heat flow. Similar forces may be found in the pressure gradients across a porous plug, and electric potential gradients along a resistive element which cause mass and current flow, respectively. The entropy produced by the latter systems is a function of both force and flow as was the case for heat flow.

The entropy production can be expressed in terms of the product of a force and its corresponding flow. For the heat transfer case, the generalized heat flow, J_q , is defined by the equation

$$J_q \triangleq \vec{P} \tag{111}$$

and the corresponding force, X_q, is defined by

$$X_q \triangleq grad\left(\frac{1}{T}\right) \tag{112}$$

The entropy production is the product of these,

$$J_q X_q = \frac{d_i S}{dt} \tag{113}$$

The conductivity relation which may be expressed as

$$J_q = L_q X_q \tag{114}$$

where the coefficient is related to the thermal conductivity by the relation

$$K = \frac{L_q}{T^2} \tag{115}$$

When there are two irreversible processes occuring in the same system, the total entropy production is the sum of the entropy production from the individual processes. The flow of the individual processes is subject to interference from the other irreversible process. As long as the flow relations are linear, the equations are called the phenomenological relations and are written for a two process system

$$J_1 = L_{11} X_1 + L_{12} X_2 \tag{116}$$

$$J_2 = L_{21} X_2 + L_{22} X_2 \tag{117}$$

where the J stands for a flow, X a force and L the coefficient. The subscripts refer to the first and second processes.

The interference coefficients, L_{12} and L_{21} , relate the effect of one force on the other flow, i.e., in a thermocouple, the application of a thermal gradient creates a current flow, or an electric current affects the heat transfer (The Thompson heat).

Onsager[43], investigating the thermocouple effects, proved for linear equations from a microscopic approach that

$$L_{12} = L_{21} \tag{118}$$

which is known as the Onsager reciprocal relation.

STATIONARY STATES

A stationary state of an irreversible process is defined as a condition in which the state variables are constant in time. In a case where there are two phases (separated by a porous plug) which may interchange mass and energy, and with each phase being maintained at constant temperature, the stationary state is characterized by zero mass flow but finite energy flow (zero energy flow would indicate equilibrium). For this case the entropy production is given by

$$\frac{d_iS}{dt} = J_q X_q + J_m X_m \tag{119}$$

when the subscript , m, is used to denote mass flow and the phenomenological equations are

$$J_q = L_{qq} X_q + L_{qm} X_m \tag{120}$$

$$J_m = L_{mq} X_q + L_{mm} X_m = 0 \tag{121}$$

and the latter i s zero only for the stationary state. These may be combined so that

$$\frac{d_iS}{dt} = L_{qq} X_q^2 + (L_{qm} + L_{mq}) X_q X_m + L_{mm} X_m^2 \tag{122}$$

Since the internal entropy is always positive, this expression is a definite positive quadratic. If the partial derivative is taken with respect to X_m,

$$\begin{aligned} \frac{\partial}{\partial X_m}\left(\frac{d_iS}{dt}\right) &= (L_{qm} + L_{mq}) X_q + 2 L_{mm} X_m \\ &= 2 L_{mq} X_q + 2 L_{mm} X_m \\ &= 2 J_m \end{aligned} \tag{123}$$

Thus, for the stationary state when the Onsager relation holds and the mass flow is zero,

$$\frac{\partial}{\partial X_m}\left(\frac{d_i S}{d t}\right) = 0 \qquad (124)$$

The stationary state then corresponds exactly to a state of minimum entropy production.

This argument can be extended to a general case of an independent force. If j of these corresponding flows are zero for the stationary state so that

$$J_i = 0 \qquad (i = 1, 2, 3 \cdots j) \qquad (125)$$

there will be j extremum conditions

$$\frac{\partial}{\partial X_i}\left(\frac{d_i S}{dt}\right) = 0 \qquad (i = 1, 2, 3, \cdots j) \qquad (126)$$

Inasmuch as the entropy production is positive, the extremum conditions are minima.

Thus it is shown that a stationary state is associated with a minimum entropy production.[21] But it must be cautioned that the result has been attained only through the use of extensive assumptions such as linear phenomenological laws, the validity of the Onsager reciprocal relations[42], the constancy of the phenomenological coefficients, and local thermal equilibrium or approximate Maxwellian velocity distributions. In addition, Glansdorf[20] and Mazur[41] have been able to show that there are cases where minimum entropy production is not related to a stationary state.

The application of the theory of irreversible processes to the arc is not a simple task, nor have the proofs to date been rigorous, but local thermal equilibrium has been assumed in electric arcs at pressures of an atmosphere or higher with proven experimental success. The results of the assumption of minimum entropy production do lead to the Steenbeck principle which within experimental accuracy does describe the characteristic behavior of the electric arc.

CHAPTER IX

DAS MINIMUMPRINZIP

Throughout this text a basic principle has been used to determine analytically the voltage gradient, current, arc diameter and temperature interrelationships. In each case, the gradient is written as a function of temperature and the derivative of the gradient with respect to temperature has been set equal to zero. This principle was developed during the 1930's primarily by a group of German Engineer-Scientists associated with the Siemens-Werke, a large manufacturing organization in the electrical industry.

By 1940, the group had proposed a theory of the minimum arc gradient and made experiments to prove the correctness of the theory. Foitzik[16] constructed a cylindrical arc chamber with the electrodes placed so that the arc would be horizontal and along the axis. The chamber was rotated about the axis so that the gravitational forces and, thus, the convective force, would average out to zero. After measuring the radiation losses, which were negligible, the total heat loss was ascribed to conduction through the plasma sheath and surrounding air.

Steenbeck[54] analyzed the data of Foitzik for electric arcs in nitrogen and carbon dioxide and obtained excellent agreement between theory and measurement. Steenbeck used the format of Kesselring[27,28] for the heat loss due to conduction

$$EI = \frac{2\pi}{\ln R/r} \int_{T_o}^{T_a} \kappa(T)\, dT \qquad (127)$$

where

$\kappa(T)$ = thermal conductivity as a function of temperature

To,Ta = temperature of wall and arc respectively

R,r = radii of the wall and arc respectively

Kesseling in return took the basic idea from a summary report by Steenbeck[55] in 1932 which was probably the initiation of this series of investigations. Steenbeck here attributes his concept of "Das Mininimumprinzip" to Compton and Morse.[7] It is at this point, however, that a very fine distinction has to be recognized. Comption and Morse analyzed the cathode fall phenomena of a glow discharge. Their technique was to write an equation for the voltage drop from plasma potential to the cathode. The equation was based on computing the energy that an electron will attain during one mean free path and then statistically correlating the energy of the individual electrons to attain an expression for the total energy. The calculus of variation is applied to the expression, according to Hamilton's principle, and an extremum, in this case a minimum, is attained. The assumptions as directly made by Compton and Morse are quoted as follows:

"Consider the condition of a gas when it carries the current, i. There is an infinitive variety of ways in which the potential might be distributed so as to permit this flow of current but of all these ways there is one distribution which is distinguished by the fact that it is the most favorable for the passage of this particular current so the potential drop, v, required to produce the current is less with this particular distribution than with any other. This most favorable distribution is the actual distribution since it permits the passage of the current with the minimum dissipation of energy. We shall, therefore, employ the principle that the actual potential distribution is that which is most favorable to the passage of current subject to the limitations imposed by Potsson's equation."

Steenbeck took this minimum principle, which was postulated as the basis of particle dynamics in a non-equilibrium case and postulated "Das Minimumprinzip" for the case of a continuum of material. Particle dynamics are not used in the derivation other than in the derivation of the Saha equation which expresses the statistically averaged result for an ionized gas in thermal equilibrium, at least locally. This is a very large extrapolation of physical principle and should be questioned. Indeed, it was, Steenbeck, von Engle,(15) Foitzek, Kesselring, and others in the group, spent nearly ten years proving, developing, and testing that it was true at least ot the limits of measurable accuracy.

It was not until 1956 that a reasonable explanation of the physical principle appeared in the literature when Peters(44) applied the principles of irreversible thermodynamics to show that the stationary electric arc corresponds to a minimum entropy production process. The principles are first used to develop the Elenbaas-Heller equation which described the heat transfer without either convection or radiation:

$$\mathrm{div}\,(K\ \mathrm{grad}\ T) + J \cdot E = 0 \tag{128}$$

This part was basically the work of Mazur[23,37] but Peters extended this work and assuming minimum entropy developed the minimum principle and demonstrated it for an idealized "Kanal model" or conducting channel model.

Peter's ideal channel has a constant temperature, T_a, from the center to the radius, r. There is a temperature gradient but no conductivity from the radius, r, to a radius R at which the temperature is the ambient temperature, T_R.

The entropy production source strength is

$$\sigma_S = J_q \cdot \mathrm{grad}\,\frac{1}{T} + \frac{J \cdot E}{T} \tag{129}$$

$$= J_q \frac{d}{dr}\left(\frac{1}{T}\right) + \frac{\sigma E^2}{T}$$

If this expression integrated over all space, the total entropy production is:

$$\frac{d_i S}{dt} = 2\pi \int_0^R \sigma_S\, r\, dr \tag{130}$$

Since

$$EI = (\sigma E^2)(\pi r^2) = 2\pi r J_q$$

$$\frac{d_i S}{dt} = 2\pi \int_0^r \frac{\sigma E^2}{T_a} r\,dr + 2\pi \int_r^R J_q \frac{d}{dr}\left(\frac{1}{T}\right) r\,dr \tag{131}$$

$$= IE/T_a + IE \int_{T_a}^{T_R} d\left(\frac{1}{T}\right)$$

$$= IE/T_a + IE\left(\frac{1}{T_R} - \frac{1}{T_a}\right)$$

$$= IE/T_R$$

This is a stationary state and therefore

$$\delta\left(\frac{d_i S}{dt}\right) = \delta\left(\frac{IE}{T_R}\right) = 0 \tag{132}$$

However, the total current I and the ambient temperature, T_R, are constants so that

$$\delta E = 0 \tag{133}$$

and since the voltage gradient is a function of the temperature it must follow that

$$\frac{\partial E}{\partial T} = 0 \tag{134}$$

This is precisely "Das Minimumprinzip" and holds for any general case where the arc is stationary and the ambient temperature is constant.

In closing this section it is worth considering Peters' remarks on these results as translated directly from the original:

"DISCUSSION OF RESULTS

"In the previous section we have shown the minimum Principle of Entropy production, applied to the electric arc, leads to the Steenbeck Principle of minimum arc voltage as well in the general case as in the simplified presentation of the channel model.

"In summation, we now may look at the numerous fractional results which have been outlined by the application of this principle to the electric arc as experimental proof of validity of the entropy principle, as they can be used better for judging other thermal non-equilibrium system. The arc is precisely a thermodynamic system in which the most effective irreversible processes are specially well adapted to measurement. The irreversible current of charge transport can be read directly from an ammeter and the appropriate "Force" on a voltmeter. Further, spectroscopy makes a determination of the "thermal force" also the temperature gradient and the temperature itself cause no principle difficulty any longer.

"We must point out a conclusion from a peculiar difficulty which has been arisen from the calculation of the (Elenbaas-Heller) equation by variations. There, the coefficients, α_{ik}, which are themselves, in general, functions of this potential, were not varied during the variation derivation of the entropy production from the thermodynamic potentials. Without this disregard (neglect) however, one finds the difficulty to which, among others, Glansdorf has paid attention. The cause for this seeming discrepancy is not apparently in the entropy theorem itself but is to be found in nearness - character of the linear statement of the thermodynamical-phenomenological theory of irreversible processes...

"We can conclude that the entropy principle is valid in a much greater area than the thermodynamic continuum theory and, hence, that the completely exact solution of the variation problems will be accomplished only either by the gas kinetic method of Enskog and Chapman or through introduction of future parts in the continued development of the generalized currents and the thermodynamic functions such as entropy, chemical potential, etc."

The idealization of the channel model yields the expected results, even if the derivation is not rigorous. Peters states, and it seems to this author most porbable that he is correct, that even if the mathematical derivation is weak, the underlying physical principal of minimum entropy production is most assuredly the controlling factor in the determination of the voltage gradient of the electric arc.

CHAPTER X

SUMMARY

Several aspects of a potentially large experimental program have been presented and discussed. Each result in itself seems to suggest that further investigation or application might prove to be of great value.

The arc within the split chamber with extended electrode surfaces was shown to behave as predicted. The arc column moves very rapidly and the arc spots move, although motion of the arc spots, particularly in the early stages, was not as great as was anticipated at the time of the chamber design. The arc spots show greater motion at higher density. It has not been established quantitatively how much contamination there is in the gas, but it does seem safe to assume that the erosion has been somewhat reduced due to motion of the arc. This type of chamber could be considered for application in an arc discharge wind tunnel, particularly if the initial density were to be somewhat higher than the density used in these experiments.

The switching, including both the transfer switch and fuse system, was proved capable of transferring current into a load that required 8000 volts. The fuse technique is adequate for single tests but should be replaced by a different type of high voltage D.C. switch for a series of tests in rapid sequence. The technique of connecting a capacitor across the switch was investigated by Walker.[60] It is an acceptable method for a system which stores a large amount of energy in a coil and then discharges the energy "one capacitor-full" at a time into the load. It becomes impractical from a cost standpoint if the capacitor must be able to absorb any reasonable amount (say 10% in a one megajoule coil or 1% in a one hundred megajoule coil) of the total stored energy.

It is well established that air blown D.C. switches will only generate some 1500 volts. Therefore, the possibility of using other arc media should be investigated. It has been shown here that air at 10,000 psi final pressure only requires some 2000 volts so that high pressure air is not the answer. From the fuse experiments, it would seem that high pressure oil might suffice. At the other extreme, it may be possible to build a high vacuum switch that could be operated repetitively.

The analysis of the arc in terms of radiation as contrasted to large amount of work presently being done on the conduction controlled arc[13,19,22,35,38,39] should emphasize the fact that technology has reached the point where arc can be produced at power levels considerably in excess of discharges 10^8 watts and that a thorough investigation of such arcs in various gases at different pressures, and perhaps with an applied magnetic field, is in order. The investigation, including spectroscopic measurements should be made in a chamber that has been designed for scientific investigations rather than a chamber designed explicitly for wind tunnel operation.

The photographs of the arc within the chamber and the subsequent energy balance has shown that the heating process is quite efficient in the early stages of the discharge. The integrated effect of convection, unfortunately, has not lent itself to a rigorous analysis because contamination obliterated the pictures. An investigation of convection effects should prove to be of value in arc discharge tunnel operation.

The utilization of t he Steenbeck Minimum Principle has led to satisfactory results. Since the author has found only one reference in English to this principle, and that was in survey article,[13] the author hopes that the accent placed on the principle herein will cause it and the underlying concepts of irreversibile thermodynamics to be more widely used.

The arc at 100 atmospheres and 12,000^{o}K has been shown herein to be optically thick. Any arc discharge wind tunnel that expects to operate in this regime will need very careful design. The energy balance has shown that the transfer of energy into the cold gas is quite efficient. The chamber needs to be only long enough to allow the gas to reach the desired temperature across the chamber diameter. The diameter of the chamber only needs to be somewhat larger than the throat diameter of the wind tunnel. The chamber diameter probably could be made smaller than the 4.0 cm diameter calculated in Chapter VI. It would operate with simultaneous current and air flow. The author has referred to this device as "pulsed plasma-jet" which hopefully could be developed to operate for 30 ms at 100 megawatts with existing equipment.

Thus, this past effort should lay the ground work for several scientific and engineering investigations. The equipment is available at the University of Michigan and it should be utilized in the near future to extend the work that has been presented here.

APPENDIX A

Part I

THE SUITS AND PORITSKY EQUATIONS IN MKS UNITS

During the 1930's, C.G. Suits[56,57,58,59] and his associates at the General Electric Research Laboratory did extensive work on the electric arc and the parameters that affected the behavior of the arc. Extensive measurements of total voltage, voltage gradient along the arc column, current, and diameter of an electric arc with the arc media as controlled parameters led to a coherent mathematical relation that showed that the heat transfer mechanism was a controlling factor in the characteristics of the arc. In the range of experimentation where the current was of the order of 2 to 20 amperes and the pressure was between 1 and 1000 atms for various gases, the heat transfer from an arc in free convection was described satisfactorily by the equations for similar free convection around a cylindrical rod.

The heat transfer from a cylinder to the surrounding fluid is described by a dimensionless equation and has been found to depend on three dimensionless numbers. These three numbers are:

$$\text{Nusselt number: } \frac{hD}{K} = \frac{\text{Temperature gradient at the surface}}{\text{Reference temp. gradient}}.$$

Grashoff number:

$$\frac{D^3 \beta_o \rho^2 g \Delta T}{\mu^2} = \frac{\text{(Buoyant force)(inertia force)}}{\text{(viscous force)}^2};$$

Prandtl number:

$$\frac{C_p \mu}{K} = \frac{\text{Molecular diffusivity of momentum}}{\text{" \quad " \quad of heat}}.$$

The individual terms are defined alphabetically as:

C_p = specific heat at constant pressure, kilogram calories per degree K-mole;

D = diameter of the arc, meter units;

g = acceleration of gravity, meters per second^2;

h = heat transfer coefficient, kilogram calories per sec - degree K - meter^2;

K = thermal conductivity, kilogram calories per meter^2-degree K - second;

ΔT = temperature difference, degrees K;

β_o = coefficient of temperature expansion, reciprocal degrees K;

μ = coefficient of viscosity, newton-seconds per meter^2, or kilograms per meter second;

ρ = density, kilograms per meter^3.

Data on heat transfer by natural convection around a horizontal cylinder has been compiled over an extremely wide range by W.H. McAdams.[34] Data has been plotted over a range of 10^{13} for the product of the Grashoff and Prandtl number. The results can be expressed as

$$\frac{hD}{K} = \text{const.} \times \left[\frac{D^3 \beta_o \rho^2 g \Delta T}{\mu^2}\right]^{\alpha} \left[\frac{C_p \mu}{K}\right]^{\alpha} \quad \text{(A-1)}$$

where the power, α , has a value varying from .04 to .25, with the value .20 being an acceptable value for the electric arc condition where the Grashoff number has a value of the order of unity.

The Prandtl number, $C_p\mu/K$, is a constant for each gas according to classical kinetic theory. For a diatomic gas it has a value of .73 and for a monatomic gas a value of .67. If the perfect gas law is assumed, this Prandtl number becomes part of the constant in equ. (A-1) and need not be considered further. This is the accepted practice, for example, in heat exchanger calculations. A discussion of the Prandtl number will be given in Appendix B. For now it will be considered to be constant.

The terms in the Grashof number are subject to simplification under the perfect gas law assumption that was made above. First

$$\beta_o = \frac{1}{v} \cdot \frac{\partial v}{\partial t}\bigg|_p = \frac{1}{T} \qquad \text{(A-2)}$$

and

$$\rho = \frac{m}{v} = \frac{nM}{v} = \frac{Mp}{RT} \qquad \text{(A-3)}$$

where

m = total mass

n = number of moles

M = Molecular weight of the gas

p = pressure

Substitution gives:

$$\frac{hD}{K} = const.\times \left[\frac{D^3 M^2 p^2 g \Delta T}{T \mu^2 R^2 T^2}\right]^{\alpha} \qquad \text{(A-4)}$$

This equation describes the natural convection heat loss around a solid cylinder. It requires the major assumption that a gaseous plasma column is equivalent to a solid bar. This is the assumption that Suits made and then proved in the laboratory that the errors involved could be as low as 7%.

The radial temperature gradient around an arc is very much larger than is normally encountered in heat transfer for a rod. The terms β_o and ρ vary inverse linearly with temperature, and c_p, μ, and K are known to vary with temperature. Thus, these terms are evaluated at the mean temperature of the film

$$T_f = \frac{T - T_a}{2} \qquad \text{(A-5)}$$

where

T_f = film mean temperature

and T_a = ambient temperature

The subscript, f, will be used to denote that the parameter is evaluated at the film temperature, such as K_f, the thermal conductivity at the mean film temperature. Through this extrapolation, an approximate evaluation of the terms hD/K and $D^3 \beta_o \rho^2 / \mu^2$ can be made.

If it is assumed that no other heat-loss mechanisms have any effect the convective heat loss is equal to the electric power input, or,

$$\pi D h \Delta T = \frac{E I}{4.18 \times 10^3} \tag{A-6}$$

where

E = voltage gradient, volts/meter

I = arc current, amperes

Substituting for hD gives

$$E I = const \times \left[\frac{D^3 M^2 p^2 g \Delta T}{T^2 R^2 \mu^2 T} \right]^{\alpha} \tag{A-7}$$

This then expresses the heat loss from the arc column where the mechanism of heat transfer is free convection.

ELECTRICAL CONDITIONS

The physical electronics within the arc can be described as consisting of two basic processes. The first process is the creation of electrons and ions due to thermal ionization. The second is the motion of the electrons to form a current. Ion current is negligible. The conduction current can be expressed as:

$$J = n_e q_e u_e \tag{A-8}$$

where

J = the current density, amperes/meter^2

n_e = electron density, per meter^3,

u_e = the average electron velocity, meters per second.

The electron velocity can be eliminated by the mobility concept:

$$u_e = g_e \bar{E} \qquad \text{(A-9)}$$

where

g_e = mobility, meter^2 per volt-sec.

On taking the total current as the product of the current density and arc cross sectional area, the current equation becomes

$$I = \frac{\pi D^2}{4} n_e q_e g_e E \qquad \text{(A-10)}$$

The electron density, n_e, can be determined by applying the Saha thermal ionization equation. The Saha equation, after Martin(40) takes the form

$$\frac{n_e n_1}{n_o} = \frac{2B_1(T)}{B_o(T)} \left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\frac{q_e V_i}{kT}} \qquad \text{(A-11)}$$

where

n_1, n_o = particle density of the higher and lower states,

$B_1(T)$, $B_o(T)$ = partition functions of the higher and lower states,

k = Boltzman constant

h = Planck's constant

V_i = ionization potential of the higher state

The equation in this form is applicable for any two states, not just the first ionization state, provided the total electron density is used.

For low-temperature gaseous conduction only the first ioniztion state is of interest. For this case it is customary to assume that the perfect gas law holds and that there is electrical neutrality. The partition functions are infinite divergent series, but, following the procedure of Urey and Fermi[3] the first terms of the series are all that are used on the basis that the remainder of the terms arise from quantum states that the electron cannot realistically be expected to attain. Under these conditions, the first term above the ground state predominates and at temperatures below 50,000°K the factor $2\,B_1(T)/B_0(T)$ is a slowly varying function of temperature and has a value very near to unity, i.e., .94 for hydrogen and 1.3 for oxygen at 30,000°K. For this analysis it is reasonable, then, to assume a value of unity. The equation takes the form

$$\frac{n_e^2}{n_0} = \left(\frac{2\pi m_e k}{h^2}\right)^{\frac{3}{2}} T^{\frac{3}{2}} e^{-\frac{q_e V_i}{kT}} \tag{A-12}$$

The constant q_e/k has a value of 11,600. The constant $(2\pi m_e k/h^2)^{\frac{3}{2}}$ has a value of 2.42×10^{21} per meter^3 - (degree Kelvin)$^{3/2}$. The equation can be written in a logarithmic form

$$\log \frac{n_e^2}{n_0} = -\frac{5040}{T} + 1.5 \log T + 21.38 \tag{A-13}$$

At constant temperature and with low ionization, the gas density, n_o, is proportioned to the pressure, p, and therefore, the electron density should vary as the square root of the pressure. Experimentally however, it was found that an increase in pressure of the system caused the temperature of the arc to increase slightly and, correspondinly, the electron density increased at a rate greater than the one-half power of the temperature. Experimentally, it was found by Suits that

$$n_e = const \times p^{\beta} \tag{A-14}$$

where the constant, β , which had an ideal value of 0.5, had a value 1.44 for nitrogen

The mobility, g_e, can be shown to be of the form

$$g_e = const \times \frac{q_e l_e}{m_e \bar{c}_e} \tag{A-15}$$

where the constant coefficient has a value near unity. Suits eliminated this concept by assuming that the mobility is nearly a constant. Since the mean free path varies inversely as the pressure at constant temperature, and the temperature effects are small compared to the temperature effects found in the Saha equation, the current conduction equation takes the form that, nearly enough is

$$I = const\ D^2 E p^{\beta - 1} \tag{A-16}$$

Thus, two equations in I,E,p, and D have been developed.

Holding the temperature constant, except that its effect on electron density shall appear only implicitly through the exponent, β , of the pressure, the logarithmic differential equations can be written from equation (A-7).

$$\frac{dI}{I} + \frac{dE}{E} = 3\alpha \frac{dD}{D} + 2\alpha \frac{dp}{p} \qquad \text{(A-17)}$$

and from equ. (A-16)

$$\frac{dI}{I} = 2\frac{dD}{D} + \frac{dE}{E} + (\beta - 1)\frac{dp}{p} \qquad \text{(A-18)}$$

For p = a constant, eliminating D gives

$$2\frac{dI}{I} + 2\frac{dE}{E} - 3\alpha\frac{dI}{I} = -3\alpha\frac{dE}{E} \qquad \text{(A-19)}$$

or

$$(2+3\alpha)\frac{dE}{E} = -(2-3\alpha)\frac{dI}{I} \qquad \text{(A-20)}$$

or

$$E = const \times I^{-\left(\frac{2-3\alpha}{2+3\alpha}\right)} \qquad \text{(A-21)}$$

For a value of α = 0.2,

$$E = const \times I^{-0.54} \qquad \text{(A-22)}$$

which is in excellent agreement with the laboratory measurements.

For constant current, the diameter of the arc can be predicted as a function of pressure. For $dI = 0$, with the elimination of E,

$$(3\alpha + 2)\frac{dD}{D} + (2\alpha + \beta - 1)\frac{dp}{p} = 0 \qquad \text{(A-23)}$$

$$D = \text{const.} \times p^{-\frac{2\alpha+\beta-1}{3\alpha+2}} \qquad \text{(A-24)}$$

For $\beta = 1.44$ and $\alpha = 0.2$, the diameter should vary according to

$$D = \text{const} \times p^{-.32} \qquad \text{(A-25)}$$

It was assumed above that the mean free path to a first approximation varied inversely as the pressure. To a first approximation then

$$D = \text{const} \times \lambda_e^{+.32} \qquad \text{(A-26)}$$

or the diameter of an arc should vary roughly as the cube root of the mean free path of the electron which is grossly different from the characteristics of low-pressure discharges.

The third case of interest is the determination of the relation between the potential gradient and pressure. Elimination of the diameter yields, holding the current constant,

$$\left(\frac{1}{3\alpha} + \frac{1}{2}\right)\frac{dE}{E} = \left(\frac{2}{3} - \frac{\beta-1}{2}\right)\frac{dp}{p} \tag{A-27}$$

$$E = const \times p^{+\left[\frac{4-3(\beta-1)}{3\alpha+2}\right]\alpha} \tag{A-28}$$

For $\alpha = 0.2$, $\beta = 1.44$, the relation is approximately

$$E = const \times p^{\frac{1}{5}} \tag{A-29}$$

The experimental relation found by this process is quantitatively correct, and in the case of the current-voltage gradient relationship, the quantitative agreement is good. The results are sufficient to justify the concept of heat transfer from a cylinder as being equivalent to the heat transfer from the arc.

The great limitation is the lack of information of temperature. Suits expresses this very well in his comment that "It would be better in general, if the theory included the arc temperature, T, as an explicit variable, but some new relationships not known at present will be required for that purpose."

Part 2

THE ARC IN FORCED CONVECTION

The work of Suits and his colleagues led to the conclusion that an electric arc would be treated as an equivalent cylinder from which the energy is removed by natural convection. The theory, however, lacked an explicit temperature relationship. Dow[11], in his work with high-current electric arcs, modified the Suits theory for the case of forced convection, which requires the substitution of the Reynolds number for the Grashof number, (a step that was indicated by Suits). Dow added a temperature dependence so that the temperature could be considered as part of the theory. His contribution to the analytical procedure was to postulate that the temperature of the arc shall be of such a nature that the arc potential gradient shall be a minimum. The results were well within experimental error.

HEAT TRANSFER

The heat transfer away from a hot cylinder under forced convection is described by McAdams by the equation

$$\frac{hD}{k} = const\left(\frac{DG}{\mu}\right)^n\left(\frac{c_p\mu}{k}\right)^n \quad \text{(A-30)}$$

where

$G = \rho u$ = mass flow kilograms per meter^2-sec

and the other terms have all been defined previously.

The term GD/μ is the Reybolds number which is commonly found as $\rho uL/\mu$. For moderate velocities, several feet/sec, the Reynolds number has a value of the order of 1000. Turbulence is associated with any higher velocity or Reynolds number. Typically, turbulence will be present with an arc in forced convection. Therefore, the exponent, n, is here calculated on the basis of a Reynolds number between 10^3 and 10^4. From McAdams, the exponent has a value of 0.56. If the Reynolds number is between 10^2 and 10^3, the exponent has value of 0.49 and if it is in the range of 10^4 to 10^5, the exponent has a value of 0.64. The Prandtl number is assumed to be a constnt for gases and will not be included in this discussion further. The heat transfer equation for a range of Reynolds number of 1,000 to 50,000 according to McAdams then takes the form

$$\frac{hD}{K} = B\left(\frac{GD}{\mu}\right)^n \qquad \text{(A-31)}$$

where B = 0.24 and n = 0.6

Two additional conceptually simple conditions are available. First, the electrical power input is equal to heat transmission away from the arc, which can be stated as

$$EI = \pi D h (T - T_a) \times 4.18 \times 10^3 \qquad \text{(A-32)}$$

where the ambient temperature, T_a, will be taken as zero. Second, the temperature will adjust so that the voltage gradient shall be a minimum. This can be stated that, if

$$E = E(T, I) \qquad \text{(A-33)}$$

then

$$\left(\frac{\partial E}{\partial T}\right)_I = 0 \qquad \text{(A-34)}$$

The heat transfer equation can now be written as

$$EI = B\left(\frac{DG}{\mu}\right)^n (4.18 \times 10^3)\pi T K \qquad \text{(A-35)}$$

The factors D, μ, K, and I are all affected by the temperature. If the minimum gradient criteria is to be used, then each term must be expressed as a function of the temperature, or it must be elimimated by substitution from other equations. It is possible to eliminate the diameter and the partial differentiation will be made at constant current. Thus, the viscosity and the thermal conductivity remain to be expressed as functions of temperature.

The viscosity can be expressed as

$$\mu = \text{const}\ \frac{m\bar{c}}{\sigma_c^2} \qquad \text{(A-36)}$$

where

σ_c = collision cross-section

$\bar{C}$ = average velocity of particles crossing a plane in one direction

m_a = mass of the atom.

The constant has been evaluated through statistical mechanical procedures but is not of interest at this point. The equation indicates that the viscosity should vary as average particle velocity, $\bar{c}$, which in turn varies as the square root of the temperature, or

$$\mu \sim (T)^{\frac{1}{2}} \tag{A-37}$$

This is not found to be the case for real gases because

$$\sigma_c = \sigma_c(T) \tag{A-38}$$

The effect of the cross section dependence is, in general, to increase the exponent of the temperature.

Thus, information[34] found in the literature up to temperature of 1000°C indicate that the viscosity may be expressed for either air or nitrogen as

$$\mu \sim T^{r} \tag{A-39}$$

where the exponent, r, has value near 3/4. The constants of proportionality are different and the exponentials are slightly different for the two gases, but they will be assumed to be the same for either gas due to the tremendous extrapolation from 1000°C in the range of 5000 to 10,000°K or higher.

The thermal conductivity of air to 550°F and nitrogen to 750°F is given by McAdams. The values are almost identical for the two gases. The temperature dependence is such that

$$\kappa \sim T^{q} \tag{A-40}$$

where the exponent q has a value almost exactly unity.

Over this range of temperatures the Prandtl number, $c_p\mu/\kappa$ is a constant. Since the exponents for the viscosity and thermal conductivity are different, one should expect to find a very slight temperature dependence for the heat capacity. This has been, in fact, found to be the case for most diatomic molecules.

One final comment should be made concerning the viscosity and thermal conductivity. Both factors show a dependence on pressure. However, the effect is largest in the neighborhood of the critical point where interatomic forces come into play. Since the electric arcs under consideration do not operate near the critical point, and moreover, even if they did, the conductivity and viscosity would show similar dependence on the pressure, the pressure effects will be completely ignored, and the exponents will be assumed valid for any pressure range.

The heat transfer equation can be written now as a function of temperature

$$EI = B_1 D^n G^n T^{(1-nr+q)} \tag{A-41}$$

where B_1 is a new constant, the current and mass flow are determined by the experimental controls. The equations relating E, I, D, T, and p , the pressure which is controlled, will now be developed.

ELECTRICAL EQUATIONS

In Part 1 of the Appendix the Saha thermal ionization equation and the electrical conductivity were used along with the perfect gas law. They will be used here subject to identical assumptions. The Saha equation is:

$$\frac{n_e^2}{n_o} = 2.42 \times 10^{21} T^{\frac{3}{2}} e^{-\frac{q_e V_1}{kT}} \tag{A-42}$$

On using the perfect gas law, $p = n_o kT$ where k has a value of 1.38 x 10^{-23} for the pressure in Newtons per meter^2 or 1.45 x 10^{-28} for the pressure in atmospheres, the Saha equation becomes

$$n_e^2 = 1.67 \times 10^{49} p T^{\frac{1}{2}} e^{-\frac{q_e V_1}{kT}} \tag{A-43}$$

or

$$n_e = 0.4 \times 10^{25} p^{\frac{1}{2}} T^{\frac{1}{4}} e^{-\frac{q_e V_1}{2kT}} \tag{A-44}$$

where the pressure is in atmospheres.

The conduction equation is

$$I = \frac{\pi D^2}{4} n_e g_e q_e E \qquad \text{(A-45)}$$

The mobility, g_e, depends primarily on the random velocity in the plasma and on the mean free path. The equation is

$$g = \frac{8}{3\pi} \frac{q_e \, l_e}{m_e \, \bar{c}_e} \qquad \text{(A-46)}$$

and

$$\bar{c}_e = \frac{2}{\sqrt{\pi}} \sqrt{\frac{2 q_e}{m_e}} \sqrt{\frac{T}{11{,}600}} \qquad \text{(A-47)}$$

The true mean free path, l_e , is energy dependent and difficult to evaluate. The mean free path, of a particle moving through elastic spheres can be assumed to be equal to the true mean free path:

$$l_e \cong l_n = l_{no} \frac{1}{p} \cdot \frac{T}{273} \qquad \text{(A-48)}$$

where p = pressure in atmospheres

l_{no} = electron m.f.p. at N.T.P., meters, for an electron moving among solid elastic spheres. For nitrogen, l_{no} has a value of 4 x 10^{-4} meters. The mobility becomes

$$g_e = 4.6 \times 10^{-2} \frac{\sqrt{T}}{p} \qquad \text{(A-49)}$$

Combining the Saha Equation and mobility into two conduction equations will yield for the current,

$$I = const \times D^2 p^{-\frac{1}{2}} E T^{\frac{3}{4}} e^{-\frac{q_e V_i}{2kT}} \qquad \text{(A-50)}$$

At this point, Dow has three basic equations, the current equation, directly above, the heat transfer equation, and the minimum gradient condition which are expressed in terms of three variables, the voltage gradient, the temperature and the arc diameter. The other parameters, the current pressure, gas, etc., are experimentally controlled.

Combining the current equation with the heat transfer equation eliminates the diameter and yields the fundamental arc equation for forced flow:

$$E^{(1+\frac{2}{n})} = B_2 G^n p^{-\frac{1}{2}} I^{(1-\frac{2}{n})} T^{(\frac{2b}{n}-\frac{3}{4})} e^{+\frac{q_e V_i}{2kT}} \qquad \text{(A-51)}$$

$$b = 1 - nr + q$$

At constant temperature this yields the Suits relationship,

$$E = const \; I^{-\frac{2-n}{2+n}} \qquad \text{(A-52)}$$

For a value n = .6, the exponent has a physically reasonable value of .54.

The next step is to take the partial derivative of the arc equation with respect to temperature to determine the minimum gradient. The differentiation can be accomplished readily by taking the logarithm of both sides of the equation prior to differentiation. The result after setting

$$\frac{\partial E}{\partial T} = 0 \quad \text{(A-53)}$$

is

$$\frac{q_e V_i}{2kT} = \frac{2b}{n} - \frac{3}{4} \quad \text{(A-54)}$$

The minimum gradient occurs when the temperature is such as to make the exponent of the temperature term equal to exponent of the exponential term

The value of $(2b/n) - (3/4)$ may be tabulated as follows

n = .6 (exponent of Reynolds Number)

r = 3/4 (exponent of viscosity)

q = 1 (exponent of thermal conductivity)

b = 1-nr+q = 1.55

The temperature thus should be given for nitrogen, for which V_i = 14.5,

$$T = \frac{11{,}600 \times 14.5}{2 \times 4.4} \quad \text{(A-55)}$$

$$T = 19{,}000^\circ K$$

For air with an ionization potential of 13 volts, the temperature under these conditions would be 17,000°K.

These temperatures seem to be very high. However, Dickerman and Morris[8] report that for an air stabilized plasma jet at one megawatt, i.e., approximately 300 volts and 3000 amperes, they measured, spectroscopically, the temperature at the orifice and found its temperature of 17,000°K. The air conditions were 70 gr/sec, 7.5 x 10^4 cm/sec, through an orifice 4 cm in diameter, and an estimate of viscosity would be 10^{-3} poise. The Reynolds number then is near 20,000 for that device, for which the Reynolds number exponent is very close to the value 0.6. The correlation here, then, is excellent

APPENDIX B

PRANDTL NUMBER

In the work of Suits and Dow, the dependence of the heat transfer on the Prandtl number, $C_p \mu / K$, was in the main neglected. Suits neglected it completely and Dow considered it as a second order effect. It is well established that for gases up to 2000°F, which is bout the limit of practical large-scale heat transfer work, the effect of the Prandtl number is negligible. Up to 2000°F the specific heat, C_p , is very nearly a constant, increasing only slightly with temperature. Under the conditions found in the electric arc, and in particular if there is an endothermoc chemical reaction associated with the arc, the specific heat is dependent on temperature to a large extent as the effects, thermal dissociation and thermal ionization, come into effect.

The fundamental concept of the Prandtl number is that it represents ratio of the transfer of momentum to transfer of heat. From a kinetic theory viewpoint, the Prandtl number is a description of the particles that carry the momentum and the heat, but not a description of either momentum or heat.

Texts on kinetic theory emphasize the accuracy of the equation

$$\kappa = \epsilon C_v \mu \tag{B-1}$$

where

C_v = specific heat at constant volume

ϵ = constant

Jeans[26] prefers to use the notation of the partial of internal energy with respect to temperature instead of specific heat. Both Jeans and Loeb[31] state categorically that the errors in the equation are associated with statistics in the evaluation of ϵ, and not in the fundamental physics of the relationship. The value of ϵ depends upon several variables but an excellent approximation (1% to 2%) is attained if

$$\epsilon = \frac{9\gamma - 5}{4} \tag{B-2}$$

where

$$\gamma = C_p/C_v$$

For a perfect monatomic gas where C_p = 5/2, C_v = 3/2 and γ = 1.66, and ϵ has a value of 2.50 so that the corresponding Prandtl number is

$$\frac{C_p \mu}{\kappa} = \frac{\gamma}{\epsilon} = 0.67 \tag{B-3}$$

The values for the Prandtl number of Argon and Helium are 0.66 and 0.71 respectively at 1 atm. and 212°F.[34]

According to the equations above, the variation of Prandtl number with pressure and temperature should be solely a function of the ratio of specific heats, γ , and can be written as

$$\frac{C_p \mu}{k} = \frac{4\gamma}{9\gamma - 5} \qquad \text{(B-4)}$$

The allowable range of γ for a perfect gas is limited by the degrees of freedom of the gas particle. For a monatomic gas γ = 1.66 and for a polyatomic molecule, γ has a limit of unity. These values then set the limits

$$0.67 \leq \frac{C_p \mu}{k} \leq 1.0 \qquad \text{(B-5)}$$

For a first approximation the effect of the Prandtl number is quite small and the approximation of a constant is thus far justified.

However, experiments were made in real gas at extreme peresure and temperature and it is of interest to evaluate the possible real gas effects. Data has been taken from Hilsenrath and Klein[25] and Little[30] in order to demonstrate the negligibly small effect of the ratio of specific heats, even under extreme conditions.

Curves were drawn for the enthalpy at a pressure of 200 atmospheres, and for the internal energy at a specific volume 14,200 c.c. per mole of cool air which is the volume of 28.97 grams of air at 200 atmospheres and 10,100°K.

The curves showed the same general shape. In addition, calculation of the ratio of specific heats, γ , at various points were found to range from approximately 1.1 to 1.4. The range is so narrow that effect on the Prandtl number can be completely neglected.

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