MANUFACTURING SYSTEM VARIATION REDUCTION THROUGH FEED-FORWARD CONTROL CONSIDERING MODEL UNCERTAINTIES

by

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A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
(Industrial and Operations Engineering)
in The University of Michigan
2009

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DEDICATION

To My Parents

ACKNOWLEDGEMENTS

I would like to sincerely thank my advisor and Co-chair, Professor Jianjun Shi, for sharing his vast knowledge and vision throughout the formation of this dissertation, as well as his guidance beyond mere research problems. My earnest gratitude has far exceeded what words can express. I would also like to thank my Co-chair Professor Jionghua (Judy) Jin for her continuous support and encouragement, and all the discussion of my research and projects.

I would like to thank my other committee members, Professors S. Jack Hu and Professor Gary Herrin, for their careful review and constructive suggestions for this dissertation.

My gratitude also goes to General Motors Corporation, for their support of this work. I would like to thank especially Dr. Charles Wampler, for all his willingness and dedication in the discussion of the research, as well as his great efforts and supports in the experiments. I would also like to thank all the members in the GM CRL lab, and the staff I cooperated with in Lansing Grand River and Lansing Delta Township assembly plants. The experiments would never have been accomplished without their contributions.

I also want to thank all the students in Professor Shi's research lab for their warm friendship and constructive comments. In particular, I would like to thank Dr. Luis Eduardo Izquierdo; from whom I learnt and benefited greatly from his hard-working attitude and mild personality. I would also like to thank Dr. Jian Liu for his invaluable input and discussions, which helped me improve my research in many ways.

Last, but not least, I would like to express my hearty gratitude to my parents, for their love and unconditional support that sustained me through this critical stage of life. It is to them that this dissertation is dedicated.

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ABSTRACT

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Today's manufacturing industry is facing greater challenges than ever. To meet the higher and stricter challenges and demands, advanced manufacturing paradigms such as flexible manufacturing and reconfigurable manufacturing are widely used by manufacturers to perform complex manufacturing operations. Complex manufacturing is characterized by a diverse product mix, various sources of disturbances, a large number of operations and stations, and the inevitable complex interactions among stations, and between processes and products. This dissertation deals with modeling and process control to enhance product quality produced in complex manufacturing processes, including multistage manufacturing processes. The successful deployment of these techniques will lead to new levels of quality and robustness in manufacturing.

Fundamental research has been conducted on active control of multistage manufacturing systems. This includes three topics related to control and modeling, which are:

- *Development of feed-forward controllers for manufacturing processes*: Feed-forward controllers allow deviation compensation on a part-by-part basis using programmable tools. The control actions take into consideration not only process mathematical models and in-line measurements, but also the modeling and measurement uncertainties. Simulation results show that the proposed control approach is effective in variation reduction, both for a data-driven model and for an engineering-driven model.
- *Stream of Variation (SoV) Modeling with consideration of model uncertainties*: To model the variation propagation and model changes in Multistage Manufacturing Processes (MMPs) for control purposes, it is necessary for the model to capture the impact of model uncertainties that are due to the errors of incoming parts or errors arising from other process variations. This development of a modeling method considering model uncertainties enables the development of the above-mentioned control strategy.
- *Model and controllability validation in real multistage manufacturing processes*: As the theoretical basis for model-based predictive controls and many other applications in multistage manufacturing, the SoV model is validated in real manufacturing processes. At the same time, the controllability in MMPs also needs to be validated in real processes. The results of experiments provide a solid theoretical basis in the SoV theory and its applications including active control.

CHAPTER 1

INTRODUCTION

1.1 Motivation

Today's manufacturing industry faces greater challenges than ever due to the increasing levels of competition led by the emergence of new technologies, more demanding customers, stricter regulations, and globalization (Koren, 2003). To meet these challenges and demands, advanced manufacturing paradigms such as flexible manufacturing and reconfigurable manufacturing are widely used by manufacturers to perform complex manufacturing operations. Complex manufacturing is characterized by a diverse product mix, various sources of disturbances, a large number of operations and stations, and the inevitable complex interactions among stations, and between processes and products.

Quality and productivity are the key issues in cost reduction and manufacturing process performance improvement, and quality assurance is the more important one of these. This is because all performance measures are related to the variations in key product characteristics (KPCs). Thus variation reduction has been a primary means for product quality assurance in manufacturing. Traditionally, variation control was accomplished through the methodologies of robust design and Statistical Process Control (SPC). Robust design methodology attempts to tune the parameters in a manufacturing system so that the process and products are insensitive to variations. However, it does not completely eliminate the sources of variation, nor can it utilize the abundant on-line information that is provided by today's advanced sensing systems. SPC methodologies have been successfully applied in out-of-control condition detection and root cause identification, but these methods do not provide systematic means for automatic compensation and variation reduction. In the last decade, however, the idea of variation reduction through active control has been discussed in the literature (Svensson, 1985; Wu *et al.*, 1994). In this approach, active control systems together with in-line sensing systems provide the capability of improving final product quality in a part-by-part dimensional control basis in manufacturing. The focus of this thesis will be the development and validation of system-level active control that takes modeling uncertainties into consideration.

Real-time automatic control systems have long been employed in manufacturing industries including the semiconductor and chemical industries. In assembly processes, it was originally introduced to improve manufacturing responsiveness to the variety of product mix, but it can also serve as an automatic dimensional controller. One example of such a tool is the FANUC C-Flex robot that serves as a fixture to hold parts in automobile assembly lines, as shown in Figure 1-1. This category of reconfigurable tools is also known collectively as Programmable Tooling (PT). Another enabler of real-time control is a sensing system such as the Optical Coordinate Measuring Machine (OCMM) in the automotive industry, which provides in-line measurements that can be used as control input signals.

Figure 1-1 A C-Flex unit (Fanuc, 2007)

Automatic control cannot be implemented without another critical necessity, which is the mathematical model of the process. In order to derive the process models, two approaches, i.e. the data-driven approach and engineering-driven modeling, were developed in the literature. In processes where it is difficult to obtain models directly from process design knowledge or parameter settings, controllers based on data-driven models have been proposed. Examples include the methodology of Design of

Experiment (DOE) -based Automatic Process Control (APC) (Jin and Ding, 2004). In the DOE-based APC method, the system models are estimated from designed experiments, and the strengths from both SPC and active control are combined to achieve active process compensation. At the same time, in processes where engineering knowledge is available, control strategies can then be developed based on engineering-driven models. Specifically, in MMPs, Stream of Variation modeling methodology has emerged to derive system models from design blueprints (Jin and Shi, 1999; Shi, 2006). This method has been widely applied in diagnosis, process design, and active control in MMPs. The tooling adjustment or compensation based on the SoV Model was developed to achieve an effective improvement in final product quality (Djurdjanovic and Zhu, 2005; Djurdjanovic and Ni, 2006; Izquierdo et al., 2007).

However, the mathematical basis for control, both in the data-driven model and in the engineering-driven model, has embedded and inevitable modeling errors. This is because uncertainties can enter the system not only as noise from the sensors and from disturbances in the system, but also as variations in the model itself. These model uncertainties can be statistical model estimation errors in DOE modeling, or part fabrication and other process-induced uncertainties in SoV modeling. This thesis will develop an automatic controller that takes these uncertainties into consideration.

1.2 Dissertation Research Overview

1.2.1 Research Problems

In the control of complex manufacturing processes, several fundamental problems and challenges need to be addressed.

(i) **Control strategy under model uncertainties**: Mathematical modeling of manufacturing processes is one critical enabler to achieve active control for variation reduction. The two approaches of process model development (data-driven modeling and engineering-driven modeling) both inherently have model uncertainties. For data-driven modeling, the uncertainties come from the process variations as well as measurement noises, because the data used to derive models are sampled from the true underlying process; and thus the statistical model is always estimated with errors. For processes for which models can be built from engineering knowledge, the true process will randomly deviate from product and process design due to uncertainties in fabrication. As a result of these modeling uncertainties, the performance of a controller may degrade during manufacturing processes, if the controller was designed only on the basis of these designated models. The development of control strategies for manufacturing processes that take into consideration these modeling and measurement uncertainties will significantly improve control performance and robustness in process variation reduction.

(ii) **Variation propagation modeling for Multistage Manufacturing Processes:** To accomplish variation reduction, it is necessary to understand the propagation of the variation in MMPs by mathematically describing the propagation at the system level. Stream of Variation modeling methodology for MMPs has been proposed and theoretically thoroughly studied, but it has not yet taken into consideration the model's uncertainties that are due to the errors of incoming parts or errors arising from variances in locating positions. It is desirable to model the variation propagation and model changes in MMPs for control purposes. This model will catch the impact of process uncertainties in modeling and enable the development of the above-mentioned control strategy.

(iii) **Model and controllability validation in real-life MMPs**: As the basis for model-based predictive controls, as well as many other applications in multistage manufacturing, the SoV model has not yet been validated in real manufacturing processes, nor has the controllability in MMPs. Successful validation will demonstrate a solid theoretical basis in the SoV theory and its applications, including the active control in MMPs.

1.2.2 Research Objectives

The objective of this research is to improve control performance for in-process active compensation that takes into consideration modeling and observation uncertainties.

The effective application of these subjects will significantly improve product quality, and reduce production and maintenance costs. In this thesis, the knowledge of process variations and their propagation will be used for modeling and active control in manufacturing process, as illustrated in Figure 1-2. The four blocks whose names are in bold font indicate the areas on which this dissertation focuses and to which it provides major contributions.

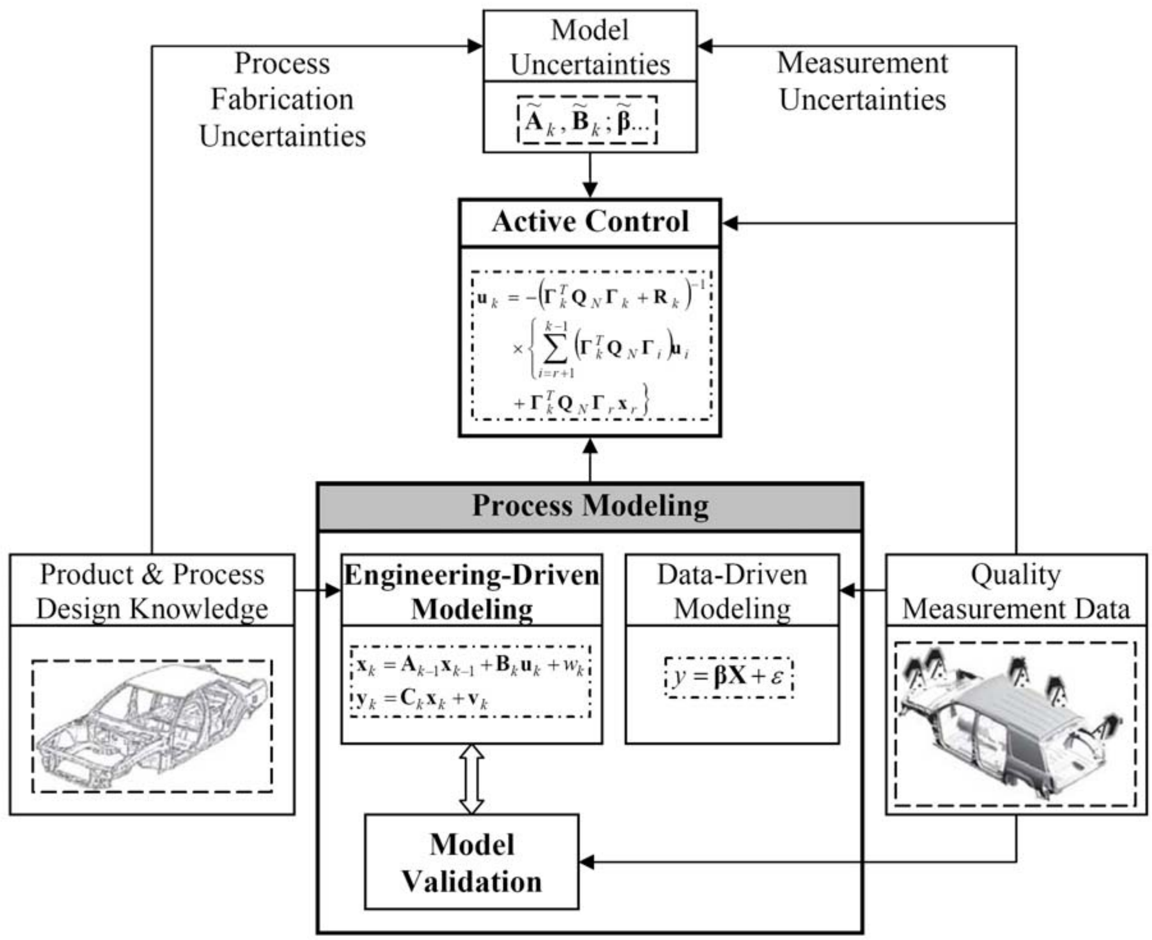

Figure 1-2 Diagram of thesis research scheme

The more specific research tasks for achieving the proposed objective are:

1. To develop a feed-forward control strategy that takes into consideration modeling and control uncertainties based on a regression model that is estimated from observation of the process and product variables through designed experiments;

2. To develop a modeling method for multistage manufacturing processes that takes into consideration modeling uncertainty. Such a model will include errors inherited from part fabrication errors as well as errors accumulated from previous assembly dimensional errors;

3. To develop a feed-forward control strategy that takes into consideration the modeling and control uncertainties for multistage manufacturing processes;

4. To design and conduct simulation experiments for both control strategies; and

5. To conduct an experiment that validates the SoV model and controllability validation in a real-life production environment.

1.3 Related Work

Corresponding to the research objectives defined in the previous section, a review of existing research will be conducted in this section. This review covers topics of modeling uncertainties, active control based on experimental design and its application in MMPs, and variation propagation modeling for MMPs.

1.3.1 Modeling Uncertainties

The problem of model uncertainty has drawn much attention in the control community. In control systems, models of the system to be controlled always have inherent errors due to imperfect data, lack of process knowledge and system dynamics, and complexity. The research dealing with the above-mentioned dynamics and disturbances has formed the area of robust control, with a variety of methodologies having been developed. Among them, Model Reference Adaptive Control (MRAC) (Åström, 1996) takes into consideration the system dynamic by designing the controller with parameters that can be updated according to system output. H_2 or H_∞ control (Basar and Bernhard, 1995; Kwakernaak, 2002) seeks to minimize the maximum power or energy gain of the system so as to stabilize it. Fuzzy Control (Tanaka and Sugeno, 1992) has the ability to control the system without requiring complex mathematical modeling. However, MMPs need to control a particular discrete subassembly throughout the process in finite stages, and because of the different nature of manufacturing systems, those well-developed robust control techniques cannot be directly applied in the context of MMPs.

1.3.2 Active Control Based on Experiment Design

In complex manufacturing processes, there are many process variables that interact in a complicated manner. In general, these variables can be classified into control (or controllable) factors, **x** (variables that can be easily manipulated), and noise (or uncontrollable) factors, **n** (variables that vary randomly and are difficult to manipulate in real time).

Taguchi's robust parameter design (RPD) is considered a cost-effective tool for reducing process variability, and it aims to set the values of controllable factors to eliminate the effect of noise factors on response (Taguchi, 1986). This is done by exploiting the control-by-noise interactions, and setting the controllable factors to "optimal" levels so that the response is not sensitive to the variations in noise factors within certain ranges. Experimental design is then employed to obtain the relationship among the dependent and independent variables.

RPD is essentially an off-line technique for determining the control factors' settings at the design stage in order to maximize the robustness of process performance to the disturbances of noise. In this effort, only the distributions of noise factors are assumed to be known.

Some research efforts have been made that use online observations of noise factors for process variation reduction. One approach is explicitly introducing online measurable noise factors into a designed experiment, as described in (Pledger, 1996). This research showed that the additional information gained from online observations can improve the selection of values for the controllable factors. The robust parameter design methodologies in the presence of feed-forward or feedback control systems and measurable noise factors were proposed later (Joseph, 2003; Tirthankar and Wu, 2006). These approaches, however, do not implement the off-line control variables under automatic control scheme.

Recently, the methodology named DOE-based Automatic Control (DOE-based APC) emerged in literature, as an automatic control strategy based on regression models obtained from DOE (Jin and Ding, 2004). In this research, noise factors were classified into "measurable noise factors" and "immeasurable noise factors". This initial approach assumed that all controllable factors are adjustable on-line, which limits the applicability of the methodology in the case where some control factors cannot be changed in real-time. Controllable factors can be further classified into on-line controllable and off-line controllable factors, with the latter denoting factors that are difficult to adjust online but can be set off-line at the design stage (Shi *et al.*, 2005).

1.3.3 Variation Propagation Modeling

In order to conduct variation reduction across the stages, it is important to mathematically understand the variation flow and accumulation in MMPs. The part and variation flow is shown in Figure 1-3, where $\mathbf{x}_k$ represents the state of part quality at stage k, $k = 1, 2, \ldots, N$. $\mathbf{y}_k$ is the measurement vector of KPCs at stage k. $\mathbf{u}_k$ is the system input vector, containing process faults in fixtures, welding gun or machine tools, as well as the control action if station k is equipped with control actuators. $\mathbf{w}_k$ and $\mathbf{v}_k$ represent the un-modeled process error and measurement error at stage k, respectively.

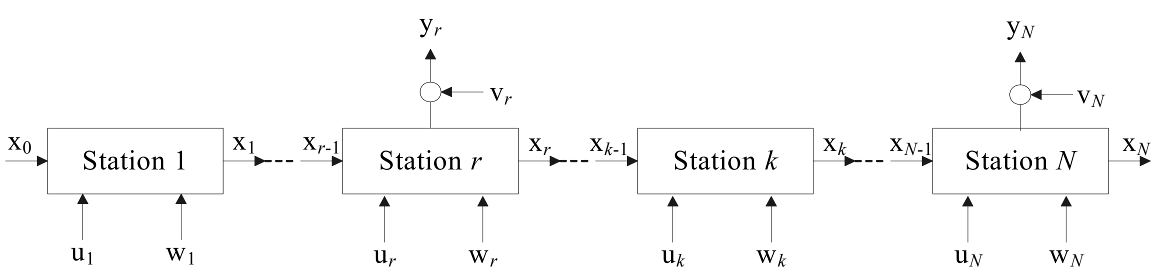

Figure 1-3 Stream of variation in an MMP

Since product and process design are available for MMPs, it is possible to develop engineering-driven modeling methods. But due to the complex inter-stage correlation, modeling for MMP is relatively new. The essence of engineering modeling is to mathematically represent the knowledge in terms of relationships between potential process faults and quality of KPCs. The state space model concept in automatic control theories was first applied in discrete-manufacturing process modeling to describe the variation propagation in a 2-D automotive body assembly process (Jin and Shi, 1999). A "datum flow chain" (DFC) concept was proposed to identify and define the kinematic constraints and mates in the assembly process (Mantripragada and Whitney, 1998). The state space model concept was then adopted for modeling multistage machining processes (Huang *et al.*, 2003). These variation propagation modeling techniques provided the basis for the process control as well as for other applications in MMPs, and the modeling technique proposed in this dissertation research will provide the ability for further reduction of variations under modeling uncertainties including the impacts of initial part variations.

1.3.4 Active Control in MMPs

Application of in-process control in MMPs is complicated, in the sense that variation is propagated and accumulated throughout the production line. Therefore, a successful control strategy for MMPs should be a strategy with system-level optimization, rather than stage-level optimization, as the control objective.

In-process control for MMPs has undergone intensive study, and promising results have been reported in the literature, which will be reviewed in the subsequent paragraphs. There are two basic mechanisms in control theory, namely a feedback control and a feed-forward control. Between them, the feedback control mechanism, by using information from downstream stages to determine the control actions at the current stage, is only effective when the control objective is to reduce the shift in mean values. Only the feed-forward control scheme fits the research objective of the process variation reduction in MMPs.

Many research efforts focusing on a feed-forward control in manufacturing processes have been reported in the literature. Most of them are on stage-level variation reduction. Feed-forward control with a sensor system has been employed in various assembly processes (Svensson, 1985; Wu *et al.*, 1994), and has been adopted by automobile manufacturers like Nissan (Sekine *et al.*, 1991). However, stage-level active control does not factor in the propagation and cross-stage relationship of variation from previous stages. Thus it is effective only at the last stage of an MMP, or it is effective if the compensated KPC is located on parts that will not be affected by downstream operations. Otherwise, the stage-level optimal compensation may not be optimal at the system level and thus unable to deliver the best final product at the end of the MMP.

For system-level active control, an optimal control scheme was proposed in mechanical assembly using state transition models (Mantripragada and Whitney, 1999). This approach treats control as a stochastic discrete-time linear optimal regulator problem, and obtains a deterministic controller, considering parts as the only source of variation in the process. A similar optimal control problem was analyzed, with application in semiconductor manufacturing (Fenner *et al.*, 2005). They used Dynamic Programming (DP) as the optimization tool, taking the control magnitudes in each direction on each single part, or controllable environmental variables at each manufacturing stage, as the decision variable. However, in an MMP such as an automobile assembly, which usually involves large numbers of stages and assemblies, the possible control actions form a solution space of extremely high dimensions. This is because each subassembly introduces six degrees of freedom as decision variables, and even if the slip plane is only considered as a 2-D case, it will introduce three degrees of freedom. Thus the curse of dimensionality for DP will limit the application of the above-mentioned analytical global-optimal controller in MMP control. In this case of a high dimensional solution space, a simpler sub-optimal controller with adequate performance will be an ideal alternative to the one that attempts to solve global optima. Under this simplified objective, a controller that adjusts the position of fixtures and the tool path to improve the final product quality was proposed for multistage machining process (Djurdjanovic and Zhu, 2005). The realization of a feed-forward control using a Programmable Tooling (PT) in a multistage assembly process was also analyzed, with the target of the minimization being deviation, rather than the variation of the final product (Djurdjanovic and Ni, 2006). A feed-forward controller that aims at reducing final part KPC variation, and which takes into account controllability and measurement noises was developed later (Izquierdo *et al.*, 2007). These studies investigated control strategies that consider the controllability and capability of the actuator respectively, but without taking into account model uncertainty. The research in this dissertation will provide a control strategy that works under modeling errors.

1.4 Dissertation Outline

This dissertation is presented in a multiple-manuscript format. Each of Chapters 2, 3, and 4 is written in the format of an individual research paper, which consists of an introduction, the main body sections, conclusions, and references. The chapters are as follows.

Chapter 2 describes the design of a feed-forward controller based on models that are obtained from designed experiments, where the true process variable relationships can be captured only by statistical modeling. The results of a case study indicate that this approach can efficiently improve controller performance.

Chapter 3 is devoted to the SoV model validation, which was previously verified using only simulation and commercial software. A real-life experiment also demonstrates the controllability in multistage manufacturing using actuators together with an in-line sensing system.

Chapter 4 explores the problem of control with uncertainties in the SoV model and discrete manufacturing processes.

Finally, Chapter 5 summarizes the conclusions and contributions of the dissertation. Several topics for future research are also proposed.

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CHAPTER 2

DESIGN OF DOE-BASED AUTOMATIC PROCESS CONTROLLER WITH CONSIDERATION OF MODEL AND OBSERVATION UNCERTAINTIES

Abstract

Robust parameter design (RPD) has been widely used as a cost-effective tool in quality control to reduce variability, in which the controllable factors are set to minimize the variability of response variables due to noise factors, assuming their distributions are known. It is essentially an off-line tool without considering that some noise factors can be measured on-line. Recently, the concept of DOE-based automatic process control has been proposed for on-line process control based on regression models obtained from DOE and with consideration of the on-line measurement of noise factors. The existing literature investigates the DOE-based APC with assumption that both regression models and the on-line noise measurement are precisely known, which limits the applicability of the technique. This paper develops the DOE-based APC scheme that considers both the observation and the modeling uncertainties. The controller is implemented under two APC strategies, i.e. cautious control strategy and certainty equivalence control strategy. The comparison among on-line APC and off-line robust design approaches demonstrates that automatic controller with consideration of both uncertainties can achieve better process performance than conventional off-line design, and is more stable than normal DOE-based APC controllers. The proposed approach is illustrated using an industrial process.

2.1 Introduction

In complex manufacturing processes, there are many process variables that interact in a complicated manner. In general, these variables can be classified into control (or controllable) factors, $\mathbf{x}$ (variables that can be easily manipulated), and noise (or uncontrollable) factors, $\mathbf{n}$ (variables that vary randomly and are difficult to manipulate in real time). Let y denote a quality response of interest, then the relationship between $\mathbf{x}$, $\mathbf{n}$ and y can be generally expressed as

$$y = f(\mathbf{x}, \mathbf{n}) \tag{1}$$

Taguchi's robust parameter design is considered a cost-effective tool for reducing process variability, which aims to set the values of controllable factors to eliminate the effect of noise factors on response (Taguchi, 1986). This is done by exploiting the control-by-noise interactions and setting the controllable factors to "optimal" levels so that the response is insensitive to the variations in noise factors within certain ranges. Experimental design is then employed to obtain the relationship of $f(\cdot)$ in (1).

RPD is essentially an off-line technique for determining the control factors' settings factors at design stage to maximize the robustness of process performance to the disturbances of noise. In this effort, only the distributions of noise factors are assumed to be known. While with the advancement of sensing technology, some noise factors can be measured or estimated through in-process sensing during production. Examples of such factors include temperature, humidity, etc. It is reasonable to anticipate the process performance to be further improved if the process control factors are adjusted according to the on-line sensing information of noise factors, rather than based only on the assumptions of their distributions.

Some research efforts have been made to utilize online observations of noise factors for process variation reduction. One approach is explicitly introducing online measurable noise factors into a designed experiment (Pledger, 1996). This work shows that the additional information gained from online observations can enhance the choice of values for the controllable factors. A robust parameter design methodology in the presence of feed-forward or feedback control systems and measurable noise factors was proposed later (Joseph, 2003; Tirthankar and Wu, 2006). However they do not implement the off-line control variables under automatic control scheme.

The study of automatic control strategy based on regression models obtained from DOE has emerged, where noise factors are classified into "measurable noise factors" and "immeasurable noise factors" (Jin and Ding, 2004). They investigated two types of control strategies: cautious control and certainty equivalence control, and compare them with robust parameter design. Their approach assumes all controllable factors are on-line adjustable, which limits the applicability of the methodology in the case where some control factors can not be changed in real-time. The controllable factors were further classified into on-line controllable and off-line controllable ones, with the latter denoting factors that are difficult to be adjusted online but can be set off-line at the design stage (Shi *et al.*, 2005). A corresponding control strategy is proposed in the paper.

The aforementioned DOE-based APC approaches are developed based on the precise regression models with no assumptions or considerations of modeling error. However, in practice, the model parameters are often estimated from experimental data, and these estimates will be affected by the design and unknown random effects in experiments. Thus, it is unavoidable that all regression models used in the APC design have inherent modeling errors. Other than modeling uncertainties, the precision of on-line sensing of noise factors is constrained by the sensing capabilities. Therefore, it is important to investigate the impacts of those modeling and sensing errors on the performance of the DOE-based APC control strategy, as well as further develop effective ways to improve the robustness of the control strategy to both uncertainties.

This paper develops a generic APC method based on experimental design and modeling, which considers modeling and sensing errors simultaneously. The paper is organized as follows. In Section 2.2, a new process modeling and an on-line control strategy is proposed with consideration of model uncertainties. Section 2.3 illustrates the proposed strategies using an injection molding process and carries out comparisons to existing approaches. Finally, the paper is summarized in Section 2.4.

2.2 Online Control Algorithm

2.2.1 General Model and Assumptions

A generic response model with single response y can be expressed in terms of $\mathbf{X}$, $\mathbf{U}$, $\mathbf{e}$, and $\mathbf{n}$ as:

$$y = f(\mathbf{X}, \mathbf{U}, \mathbf{e}, \mathbf{n}) + \varepsilon, \tag{2}$$

where the controllable factors are classified into off-line setting factors, **X**, and on-line controllable factors, **U**, while noise factors are classified into measurable noise, **e**, and immeasurable noise, **n**, and ε is the regression residual error.

Regression model $f(\mathbf{X}, \mathbf{U}, \mathbf{e}, \mathbf{n})$ is generally non-linear for inputs but still linear for model parameters. A model of interest should include main effects of all factors together with control-by-noise interactions. Thus, model (2) becomes:

$$y = \beta_0 + \boldsymbol{\beta}_1^T \mathbf{X} + \boldsymbol{\beta}_2^T \mathbf{U} + \boldsymbol{\beta}_3^T \mathbf{e} + \boldsymbol{\beta}_4^T \mathbf{n} + \mathbf{X}^T \mathbf{B}_1 \mathbf{e} + \mathbf{U}^T \mathbf{B}_2 \mathbf{e} + \mathbf{X}^T \mathbf{B}_3 \mathbf{n} + \mathbf{U}^T \mathbf{B}_4 \mathbf{n} + \varepsilon, \tag{3}$$

where $\mathbf{X} \in \mathrm{R}^{m \times 1}$, $\mathbf{U} \in \mathrm{R}^{n \times 1}$, $\mathbf{e} \in \mathrm{R}^{p \times 1}$, $\mathbf{n} \in \mathrm{R}^{q \times 1}$, and other vectors and matrices are of appropriate dimensions. This model has the typical form of regression models obtained from a designed experiment. The details of modeling strategy and procedures can be found in (Wu and Hamada, 2000). The higher order interactions among controllable variables and noise variables are not included in (3), since the consideration of these interactions will only add to the complexity of solving the problem, but will not on the optimization procedure.

Define $\boldsymbol{\beta}$ as $\boldsymbol{\beta} \equiv \left[\beta_0,\ \boldsymbol{\beta}_1^T, \cdots,\ \boldsymbol{\beta}_4^T,\ \mathrm{vec}\left(\mathbf{B}_1\right)^T, \cdots, \mathrm{vec}\left(\mathbf{B}_4\right)^T\right]^T$, representing the set of all model parameters. Here vec(**B**) is the stack up of column vectors of a matrix **B**.

To develop the process control methodology, the following assumptions have been made:

A1. The manufacturing process is time-invariant for the period of time when the same control law is applied. This would be appropriate for many real-life manufacturing processes with stable and in-control productions. The model and control law can be adjusted in case the process setting has changed. The parameters are estimated from designed experiments, denoted by $\hat{\boldsymbol{\beta}} = \boldsymbol{\beta} + \tilde{\boldsymbol{\beta}}$, where $\boldsymbol{\beta}$ is the underlying true model parameter, $\hat{\boldsymbol{\beta}}$ is its estimate from experiments, and $\tilde{\boldsymbol{\beta}}$ is the estimation error. The estimation uncertainty is represented by $Cov(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta}) = \Sigma_{\tilde{\boldsymbol{\beta}}}$. This modeling uncertainty is of the same definition as the parameter estimation error. The estimation error is assumed to be normally distributed.

A2. The noise terms **e**, **n** and ε are independent of each other, with $E(\mathbf{e}) = \mathbf{0}$, $Cov(\mathbf{e}) = \Sigma_{\mathbf{e}}$, $E(\mathbf{n}) = \mathbf{0}$, $Cov(\mathbf{n}) = \Sigma_{\mathbf{n}}$, and $E(\varepsilon) = 0$, $Var(\varepsilon) = \sigma_{\varepsilon}^2$. The model residual errors, represented as ε's, are independently and identically distributed.

A3. The online noise observer can provide an unbiased estimation of measurable noise factor **e**, denoted by $\hat{\mathbf{e}} = \mathbf{e} + \tilde{\mathbf{e}}$, where **e** is the true value of the observable noises, $\hat{\mathbf{e}}$ is its observation, and $\tilde{\mathbf{e}}$ is the observation error. The observation of noise variable is unbiased, i.e. $E\left[\hat{\mathbf{e}} - \mathbf{e} \mid \hat{\mathbf{e}}\right] = 0$, and $Cov(\hat{\mathbf{e}} - \mathbf{e} \mid \hat{\mathbf{e}}) = \Sigma_{\tilde{\mathbf{e}}}$ represents the observation uncertainty.

The modeling uncertainty, $\Sigma_{\tilde{\boldsymbol{\beta}}}$, in assumption A1 can be obtained from experimental design together with the model coefficients (Wu and Hamada, 2000). The observation uncertainty, $\Sigma_{\tilde{\mathbf{e}}}$, in assumption A3 can be estimated from the specifications of observer, or gauge repeatability and reproducibility study.

2.2.2 Optimal Control Strategy

The objective process control is usually to keep the deviation of response y from the target value as small as possible, which is called a "nominal-the-best problem". A quadratic loss function $L(y,t) = k(y-t)^2$ should be chosen as the performance measure, where t is the target value and k is a monetary coefficient. Since there is only one quality response characteristic y considered in this paper, k can be assumed to be 1 without loss of generality. The optimal control algorithm is developed to minimize $L(y,t)$.

Thus, when the process parameter $\hat{\boldsymbol{\beta}}$ is estimated and measurement $\hat{\mathbf{e}}$ is obtained, the conditional control objective function can be expressed as

$$
\begin{aligned}
& J_{APC}\left(\mathbf{X},\mathbf{U}\mid\hat{\mathbf{e}},\hat{\boldsymbol{\beta}}\right) \\
& = E_{\mathbf{e},\mathbf{n},\tilde{\boldsymbol{\beta}},\varepsilon}\left[L(y,t)\middle|\hat{\mathbf{e}},\hat{\boldsymbol{\beta}}\right] = E_{\mathbf{e},\mathbf{n},\tilde{\boldsymbol{\beta}},\varepsilon}\left[\left(y\middle|\hat{\mathbf{e}},\hat{\boldsymbol{\beta}}\right)-t\right]^2 \\
& = \left(E_{\mathbf{e},\mathbf{n},\tilde{\boldsymbol{\beta}},\varepsilon}\left[y\middle|\hat{\mathbf{e}},\hat{\boldsymbol{\beta}}\right]-t\right)^2 + Var_{\mathbf{e},\mathbf{n},\tilde{\boldsymbol{\beta}},\varepsilon}\left(y\middle|\hat{\mathbf{e}},\hat{\boldsymbol{\beta}}\right).
\end{aligned}
\tag{4}
$$

In this equation, if $\mathbf{a}$ is a random vector and b is a random variable as a function of $\mathbf{a}$, $E_{\mathbf{a}}[b]$ represents the expectation of b taken over the distribution of $\mathbf{a}$. Similarly, $Var_{\mathbf{a}}(b)$ represents the variance of b taken over the distribution of $\mathbf{a}$. For regression model (3), it can be shown that (see Appendix 1 for details):

$$
\begin{aligned}
& J_{APC}(\mathbf{X},\mathbf{U}\middle|\hat{\mathbf{e}},\hat{\boldsymbol{\beta}}) \\
& = \sigma_\varepsilon^2 + \left(\hat{\beta}_0 - t + \hat{\boldsymbol{\beta}}_1^T\mathbf{X} + \hat{\boldsymbol{\beta}}_2^T\mathbf{U} + \hat{\boldsymbol{\beta}}_3^T\hat{\mathbf{e}} + \mathbf{X}^T\hat{\mathbf{B}}_1\hat{\mathbf{e}} + \mathbf{U}^T\hat{\mathbf{B}}_2\hat{\mathbf{e}}\right)^2 \\
& \quad + \left(\hat{\boldsymbol{\beta}}_3 + \hat{\mathbf{B}}_1^T\mathbf{X} + \hat{\mathbf{B}}_2^T\mathbf{U}\right)^T \Sigma_{\tilde{\mathbf{e}}} \left(\hat{\boldsymbol{\beta}}_3 + \hat{\mathbf{B}}_1^T\mathbf{X} + \hat{\mathbf{B}}_2^T\mathbf{U}\right) \\
& \quad + \left(\hat{\boldsymbol{\beta}}_4 + \hat{\mathbf{B}}_3^T\mathbf{X} + \hat{\mathbf{B}}_4^T\mathbf{U}\right)^T \Sigma_{\mathbf{n}} \left(\hat{\boldsymbol{\beta}}_4 + \hat{\mathbf{B}}_3^T\mathbf{X} + \hat{\mathbf{B}}_4^T\mathbf{U}\right) \\
& \quad + \sigma_{\tilde{\beta}_0}^2 + \mathbf{X}^T \Sigma_{\tilde{\boldsymbol{\beta}}_1} \mathbf{X} + \mathbf{U}^T \Sigma_{\tilde{\boldsymbol{\beta}}_2} \mathbf{U} + \hat{\mathbf{e}}^T \Sigma_{\tilde{\boldsymbol{\beta}}_3} \hat{\mathbf{e}} + E_{\mathbf{n}}\left[\mathbf{n}^T \Sigma_{\tilde{\boldsymbol{\beta}}_4} \mathbf{n}\right] + Var_{\tilde{\boldsymbol{\beta}}}\left(\mathbf{X}^T\mathbf{B}_1\mathbf{e}\right) \\
& \quad + Var_{\tilde{\boldsymbol{\beta}}}\left(\mathbf{U}^T\mathbf{B}_2\mathbf{e}\right) + E_{\mathbf{n}}\left[Var_{\tilde{\boldsymbol{\beta}}}\left(\mathbf{X}^T\mathbf{B}_3\mathbf{n}\right)\right] + E_{\mathbf{n}}\left[Var_{\tilde{\boldsymbol{\beta}}}\left(\mathbf{U}^T\mathbf{B}_4\mathbf{n}\right)\right].
\end{aligned}
\tag{5}
$$

The optimal controller should minimize $J_{APC}\left(\mathbf{X},\mathbf{U}\middle|\hat{\boldsymbol{\beta}}\right) = E_{\hat{e}}\left[J_{APC}(\mathbf{X},\mathbf{U}\middle|\hat{\mathbf{e}},\hat{\boldsymbol{\beta}})\right]$, i.e., taking into account of all possible observations of noise factors. If constraints on the controllable factors are considered, this optimization problem can be written as

$$
(\mathbf{X}^*,\mathbf{U}^*) = \arg \min_{\|\mathbf{X}\|_\infty \le \mathbf{1}, \|\mathbf{U}\|_\infty \le \mathbf{1}} J_{APC}\left(\mathbf{X},\mathbf{U}\middle|\hat{\boldsymbol{\beta}}\right),
\tag{6}
$$

where $\|\cdot\|_\infty$ is the maximum absolute row sum norm of the corresponding matrix, and **1** indicates the experimental region and constraints for the controllable variables.

Since $\mathbf{X}$ is the off-line setting factor that cannot be adjusted during production, the following **two-step approach** is proposed to obtain an optimal solution to the optimization problem.

(i) Solve the optimal control law of $\mathbf{U}^*$. The restricted solution is given as:

$$\mathbf{U}^* = \arg\min_{\|\mathbf{U}\|_\infty \le \mathbf{1}} J_{APC}\left(\mathbf{X},\mathbf{U} \mid \mathbf{X},\hat{\mathbf{e}},\hat{\boldsymbol{\beta}}\right). \tag{7}$$

A closed-form solution of $\mathbf{U}^* = h(\mathbf{X},\hat{\mathbf{e}},\hat{\boldsymbol{\beta}},\Sigma_{\tilde{\mathbf{e}}},\Sigma_{\mathbf{n}}\,\Sigma_{\tilde{\boldsymbol{\beta}}})$ can be obtained from (5) and (7) by solving the equation of $\dfrac{\partial J_{APC}\left(\mathbf{X},\mathbf{U} \mid \mathbf{X},\hat{\mathbf{e}},\hat{\boldsymbol{\beta}}\right)}{\partial \mathbf{U}} = 0$. However this closed-form solution is a function of $\mathbf{X}$, and is not guaranteed to be within the constrained region. A numerical search for optimum should be employed under this situation using optimization methods such as DIRECT (Jones *et al.*, 1993; Björkman and Holmström, 1999), which will be illustrated in details in case study.

(ii) Minimize the quadratic loss at $\mathbf{U} = \mathbf{U}^*$ over the distribution of $\hat{\mathbf{e}}$. Thus the constrained optimal setting of $\mathbf{X}^*$ can be obtained as:

$$\mathbf{X}^* = \arg\min_{\|\mathbf{X}\|_\infty \le \mathbf{1}} E_{\hat{\mathbf{e}}}\left\{J_{APC}\left(\mathbf{X},\mathbf{U}^* \mid \hat{\mathbf{e}},\hat{\boldsymbol{\beta}}\right)\right\}. \tag{8}$$

It should be noted that, the second step requires a direct plug-in of the closed-form solution of $\mathbf{U}^*$ obtained in the first step. Since it cannot be foreseen whether this solution falls into the constrain region or not before numerically solving it, this step might not lead to a globally optimal $\mathbf{X}^*$, however the sub-optimal performance is guaranteed as comparing to Robust Design. This is because the sub-optimal is still the best solution in the feasible region, which includes the solution from Robust Design. The efficiency will be shown in the case study.

Then the control strategy can be implemented as:

1. Off-line set $\mathbf{X} = \mathbf{X}^*$ at the process setup stage;

2. On-line adjust $\mathbf{U}$ to $\mathbf{U}^*$ according to observations during production.

If there are no measurement errors, i.e. $\Sigma_{\tilde{\mathbf{e}}}$ is zero, the corresponding control law becomes certainty equivalence control, which will be discussed in the following case study.

2.3 An Injection Molding Process

2.3.1 Injection Molding Process Description

Injection molding is widely used in the manufacturing of fabricated plastic products. The process is very complex due to the high degree of interaction of material, machine and process variables (Smud *et al.*, 1991).

In this experiment, percentage of shrinkage of the molded parts is determined as the quality response. The problem is considered as a nominal-the-best as discussed in section 2.2.2.

A designed experiment on this injection molding process was reported (Engel, 1992). The experiment consists of seven controllable factors and three noise factors as listed in Table 2-1. The lowercase letters are used to denote the process variables in each set of factors.

Table 2-1 Factors in the injection molding experiment

Controllable Factors	
Off-Line (**X**)	*On-Line* (**U**)
X_1: Mold temperature X_2: Cavity thickness X_3: Gate size	U_1: Cycle time U_2: Holding pressure U_3: Injection speed U_4: Holding time
Noise Factors	
Measurable (**e**)	*Immeasurable* (**n**)
e_1: Moisture content e_2: Ambient temperature	N_1: Percentage reground

For classification of the noise factors, percentage reground is considered immeasurable, since it is very costly to be measured during the process run. Ambient temperature can be easily measured; moisture content can be estimated through the measurements of ambient humidity and the amount of time the material is exposed to the air (Pledger, 1996). Thus these two variables are considered measurable noise factors. The experiment design response values are shown in Table 2-2, as presented in literature (Steinberg and Bursztyn, 1994).

Table 2-2 Design and responses for the injection molding experiment

	Controllable Factors							Percent Shrinkage for Noise Factors n_1, e_1, e_2			
Cell	u_1	x_1	x_2	u_2	u_3	u_4	x_3	(-1,-1,-1)	(-1,1,1)	(1,-1,1)	(1,1,-1)
1	-1	-1	-1	-1	-1	-1	-1	2.2	2.1	2.3	2.3
2	-1	-1	-1	+1	+1	+1	+1	2.5	0.3	2.7	0.3
3	-1	+1	+1	-1	-1	+1	+1	0.5	3.1	0.4	2.8
4	-1	+1	+1	+1	+1	-1	-1	2.0	1.9	1.8	2.0
5	+1	-1	+1	-1	+1	-1	+1	3.0	3.1	3.0	3.0
6	+1	-1	+1	+1	-1	+1	-1	2.1	4.2	1.0	3.1
7	+1	+1	-1	-1	+1	+1	-1	4.0	1.9	4.6	2.2
8	+1	+1	-1	+1	-1	-1	+1	2.0	1.9	1.9	1.8

By fitting a straight line in the half-normal probability plot of all main effects and two-factor interactions (as shown in Figure 2-1), any effect or interaction whose corresponding point falls off the line is considered significant.

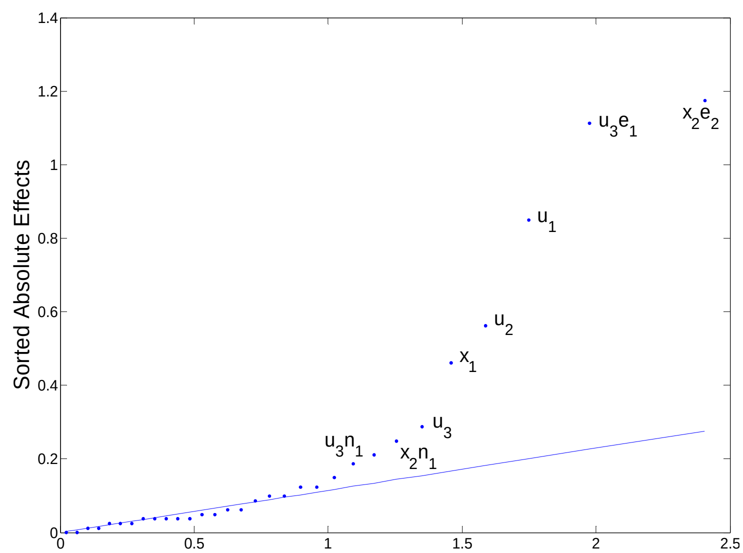

Figure 2-1 Half-normal probability plot of main effects and interactions

Table 2-3 Effect estimates

Main Effects				2-way Interactions			
Effect	Estimate	Effect	Estimate	Effect	Estimate	Effect	Estimate
X_1	-0.150	u_2	-0.563	$X_2 e_1$	1.175	$x_2 n_1$	-0.250
X_2	0.125	u_3	0.288	$U_3 e_1$	-1.113	$u_2 n_1$	-0.188
X_3	-0.463	e_1	0	$X_1 n_1$	0.125	$u_3 n_1$	0.213
U_1	0.850	n_1	-0.100				

The estimated effects of all significant terms and some non-significant main effects (x_1, x_2, e_1 and n_1) are listed in Table 2-3.

From the above results, the following model can be obtained

$$y = \beta_0 + \boldsymbol{\beta}_1^T \mathbf{X} + \boldsymbol{\beta}_2^T \mathbf{U} + \beta_3 e_1 + \beta_4 n_1 + \mathbf{X}^T \mathbf{B}_1 e_1 + \mathbf{U}^T \mathbf{B}_2 e_1 + \mathbf{X}^T \mathbf{B}_3 n_1 + \mathbf{U}^T \mathbf{B}_4 n_1 + \varepsilon, \qquad (9)$$

where $\mathbf{X} = [x_1, x_2, x_3]^T$ and $\mathbf{U} = [u_1, u_2, u_3, u_4]^T$. Estimated model parameters are $\beta_0 = 2.25$, $\boldsymbol{\beta}_1^T = [-0.075 \quad 0.063 \quad -0.232]$, $\boldsymbol{\beta}_2^T = [0.425 \quad -0.282 \quad 0.144]$, $\beta_3 = 0$, $\beta_4 = -0.05$, $\mathbf{B}_1^T = [0 \quad 0.588 \quad 0]$, $\mathbf{B}_2^T = [0 \quad 0 \quad -0.557]$, $\mathbf{B}_3^T = [0.063 \quad -0.125 \quad 0]$, and $\mathbf{B}_4^T = [0 \quad -0.094 \quad 0.106]$. Other controllable factors not appearing in the model can be set according to economic advantages during the production. The uncertainty of estimated model parameters $\Sigma_{\tilde{\boldsymbol{\beta}}}$ is $4.084 \times 10^{-4} \cdot \mathbf{I}^{21\times21}$, as can be obtained as a standard output of statistical regression process.

2.3.2 Implemented Process Control Strategy

This section first develops the robust parameter design and the optimal control scheme for the injection molding. They will be implemented under both the cautious control strategy and certainty equivalence control strategy in the next section.

1) Robust Parameter Design: According to the transmitted variance model approach (Wu and Hamada, 2000), the variance of $\hat{y}$ is:

$$\begin{aligned} Var(\hat{y}) &= (-0.05 + 0.0625x_1 - 0.125x_2 - 0.0938u_2 + 0.1063u_3)^2 \sigma_{n_1}^2 \\ &\quad + (0.5875x_2 - 0.5563u_3)^2 \sigma_{e_1}^2. \end{aligned} \qquad (10)$$

To minimize the transmitted variance $Var(\hat{y})$ over the experimental region, the robust settings can be obtained as $(x_1, x_2, u_2, u_3) = (0.4664, 0, -0.2222, 0)$ by solving a quadratic problem for continuous x_i's and u_i's. Factors u_1 and x_3 are significant effects in the response model but not in the transmitted variance model, so they are set to values between -1 and +1 to bring the expected response close to target value, which means:

$$\mathbf{X}^* = \begin{bmatrix} 0.4664 & 0 & x_3^* \end{bmatrix}^T,\ \mathbf{U}^* = \begin{bmatrix} u_1^* & -0.2222 & 0 \end{bmatrix}^T \tag{11}$$

where u_1 and x_3 are given in Table 2-4, based on different target value (percent shrinkage) t.

Table 2-4 Settings of factor u_1 and x_3 in terms of percent shrinkage

Percent shrinkage t	Factor u_1^*	Factor x_3^*
≤1.85	-1	1
(1.85, 2.25]	-1	-1
(2.25, 2.70]	1	1
>2.70	1	-1

2) Optimal Control: The model (9) has one measurable noise factor e_1 and one immeasurable noise factor n_1. Assuming the variance of the immeasurable noise factor n_1 is $\sigma_{n_1}^2$ and the measurement uncertainty of the measurable noise factor e_1 is $\sigma_{\tilde{e}_1}^2$, the objective loss function for this application example becomes:

$$\begin{aligned}
& J_{APC}\left(\mathbf{X},\mathbf{U}\middle|\hat{e}_1,\hat{\boldsymbol{\beta}}\right) \\
& = \sigma_\varepsilon^2 + \left(\hat{\beta}_0 - t + \hat{\boldsymbol{\beta}}_1^T\mathbf{X} + \hat{\boldsymbol{\beta}}_2^T\mathbf{U} + \hat{\beta}_3^T\hat{e}_1 + \mathbf{X}^T\hat{\mathbf{B}}_1\hat{e}_1 + \mathbf{U}^T\hat{\mathbf{B}}_2\hat{e}_1\right)^2 \\
& \quad + \left(\hat{\beta}_3 + \hat{\mathbf{B}}_1^T\mathbf{X} + \hat{\mathbf{B}}_2^T\mathbf{U}\right)^T \Sigma_{\tilde{e}_1} \left(\hat{\beta}_3 + \hat{\mathbf{B}}_1^T\mathbf{X} + \hat{\mathbf{B}}_2^T\mathbf{U}\right) \\
& \quad + \left(\hat{\beta}_4 + \hat{\mathbf{B}}_3^T\mathbf{X} + \hat{\mathbf{B}}_4^T\mathbf{U}\right)^T \Sigma_{n_1} \left(\hat{\beta}_4 + \hat{\mathbf{B}}_3^T\mathbf{X} + \hat{\mathbf{B}}_4^T\mathbf{U}\right) \\
& \quad + \sigma_{\tilde{\beta}_0}^2 + \mathbf{X}^T \Sigma_{\tilde{\boldsymbol{\beta}}_1} \mathbf{X} + \mathbf{U}^T \Sigma_{\tilde{\boldsymbol{\beta}}_2} \mathbf{U} + \hat{e}_1^2\sigma_{\tilde{\beta}_3}^2 + \sigma_{n_1}^2\sigma_{\tilde{\beta}_4}^2 + \hat{e}_1^2\mathbf{X}^T \Sigma_{\tilde{\mathbf{B}}_1} \mathbf{X} \\
& \quad + \hat{e}_1^2\mathbf{U}^T \Sigma_{\tilde{\mathbf{B}}_2} \mathbf{U} + \sigma_{n_1}^2\mathbf{X}^T \Sigma_{\tilde{\mathbf{B}}_3} \mathbf{X} + \sigma_{n_1}^2\mathbf{U}^T \Sigma_{\tilde{\mathbf{B}}_4} \mathbf{U}.
\end{aligned} \tag{12}$$

Thus the closed-form control law of $\mathbf{U}^*$ can be obtained as a function of $\mathbf{X}$ (see Appendix 2 for details):

$$
\begin{aligned}
\mathbf{U}^* = -\Big\{ & \left(\hat{\boldsymbol{\beta}}_2 + \hat{\mathbf{B}}_2 \hat{e}_1\right)\left(\hat{\boldsymbol{\beta}}_2 + \hat{\mathbf{B}}_2 \hat{e}_1\right)^T \\
& + \hat{\mathbf{B}}_2 \sigma_{\tilde{e}_1}^2 \hat{\mathbf{B}}_2^T + \hat{\mathbf{B}}_4 \sigma_{n_1}^2 \hat{\mathbf{B}}_4^T + \Sigma_{\tilde{\boldsymbol{\beta}}_2}^2 + \hat{e}_1^2 \Sigma_{\tilde{\mathbf{B}}_2} + \sigma_{n_1}^2 \Sigma_{\tilde{\mathbf{B}}_4} \Big\}^{-1} \\
& \times \Big\{ \left(\hat{\beta}_0 - t + \hat{\boldsymbol{\beta}}_1^T \mathbf{X} + \hat{\beta}_3^T \hat{e}_1 + \mathbf{X}^T \hat{\mathbf{B}}_1 \hat{e}_1\right)\left(\hat{\boldsymbol{\beta}}_2 + \hat{\mathbf{B}}_2 \hat{e}_1\right) + \hat{\mathbf{B}}_2 \sigma_{\tilde{e}_1}^2 \left(\hat{\boldsymbol{\beta}}_3 + \hat{\mathbf{B}}_1^T \mathbf{X}\right) \\
& + \hat{\mathbf{B}}_4 \sigma_{n_1}^2 \left(\hat{\boldsymbol{\beta}}_4 + \hat{\mathbf{B}}_3^T \mathbf{X}\right) \Big\}.
\end{aligned} \tag{13}
$$

As aforementioned, the optimal off-line setting $\mathbf{X}^*$ can be obtained by first substituting the opt imal $\mathbf{U}^* = \begin{bmatrix} u_1^* & u_2^* & u_3^* \end{bmatrix}^T$ as (13) into the quadratic loss function (12), and then minimizing (12) by integrating over the distribution of e_1.

2.3.3 Case Study

A simulated case study is conducted here to evaluate and compare the performance of (i) robust parameter design, (ii) control law proposed in literature (Shi *et al.*, 2005), and (iii) on-line cautious process control developed in this paper.

The residual term ε in model (9) represents the model accuracy, which is determined by the DOE model structure, factor test levels, effect de-aliasing, sensitivity analysis, and parameter estimation methods. Therefore, reduction of the model residual can only be obtained by improved designed experiments and/or modeling algorithms. A control strategy alone can not reduce the variation contributed by the residual noise in the model. Thus, the following performance evaluation will compare only the response value of the predicted model without considering the regression residual ε, or the residual variance σ_ε^2.

The optimization problems were solved by using DIRECT, an algorithm that is able to search the global minimum of a multivariate function subject to simple bounds on the variables (Jones *et al.*, 1993; Björkman and Holmström, 1999). The optimization was carried out by using Matlab.

If the measurement uncertainty $\Sigma_{\tilde{\mathbf{e}}}$ is considered, the optimal control law is known as "cautious control (CC) law". The basic idea is that magnitudes of the controllable factors adjustments consider both the estimated noise factor levels and the covariance of the estimator errors. When the measurement errors $\tilde{\mathbf{e}}$ is not considered, i.e., $\Sigma_{\tilde{\mathbf{e}}} = \mathbf{0}$, the designed controller is known as "certainty equivalence (CE) control". Furthermore, if a controller does not consider the model parameter estimation error, nor the measurement error, it is a traditional APC controller.

The process performances of robust parameter design and on-line process control are compared, and APC is carried out under cautious control, certainty equivalence control and traditional control.

In this simulation study, both n_1 and e_1 are assumed to follow zero-mean normal distributions with variance of 0.25, i.e. $n_1 \sim \mathrm{N}(0, 0.25)$, $e_1 \sim \mathrm{N}(0, 0.25)$. Thus 95% of the random noises will fall in the region of [-1, 1]. $\hat{e}_1$ is the measurement of e_1 with certain observation errors, and the performance of the control laws are compared under different levels of noise estimation uncertainties. The model parameter uncertainties are set to 4.084×10^{-4}, which is obtained from the deigned experiment data.

Figure 2-2 shows the comparison of the performances of different controllers. The horizontal axis of the figure is the ratio of uncertainty of sensor noise ($\sigma_{\tilde{e}_1}$) to that of measurable noise factor (σ_{e_1}). The vertical axis is the ratio between controller performances and robust design performance ($J_{\mathrm{APC}}/J_{\mathrm{RPD}}$), and thus a value smaller than 1 indicates a preference of employing the corresponding APC strategy at that noise level. The figure shows that robust design is outperformed by both control strategies at a lower noise level, since it ignores the uncertainties of model parameter estimation and observation. However, CE control works well only if there is no (or small) estimation errors. As the uncertainties get larger, ($\sigma_{\tilde{e}_1} / \sigma_{e_1} > 0.24$ in this study), the CE controller deteriorates and performs worse than the RPD. While a CC further considers the observation uncertainty and is 'cautious' about each control action it takes, thus its performance is better than the RPD until a large noise level.

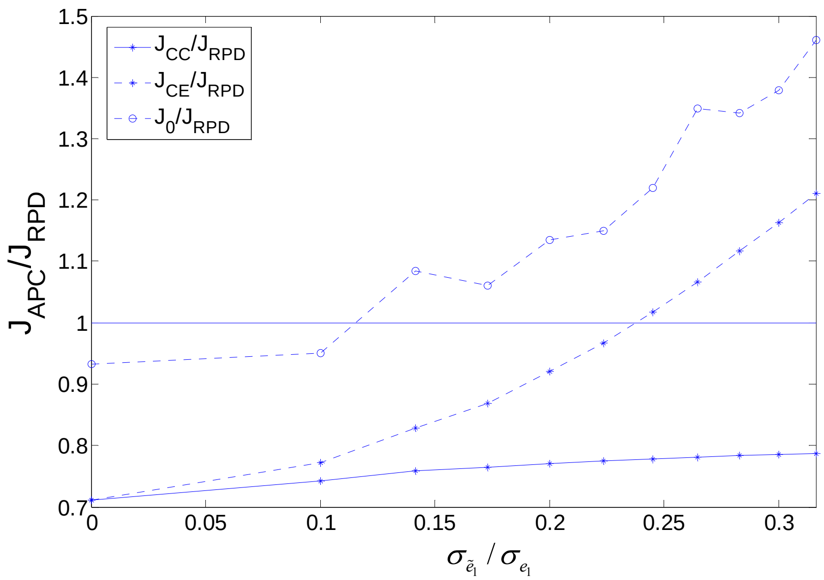

Figure 2-2 Comparison of variability of $\hat{y}$ under RPD and APC Strategies

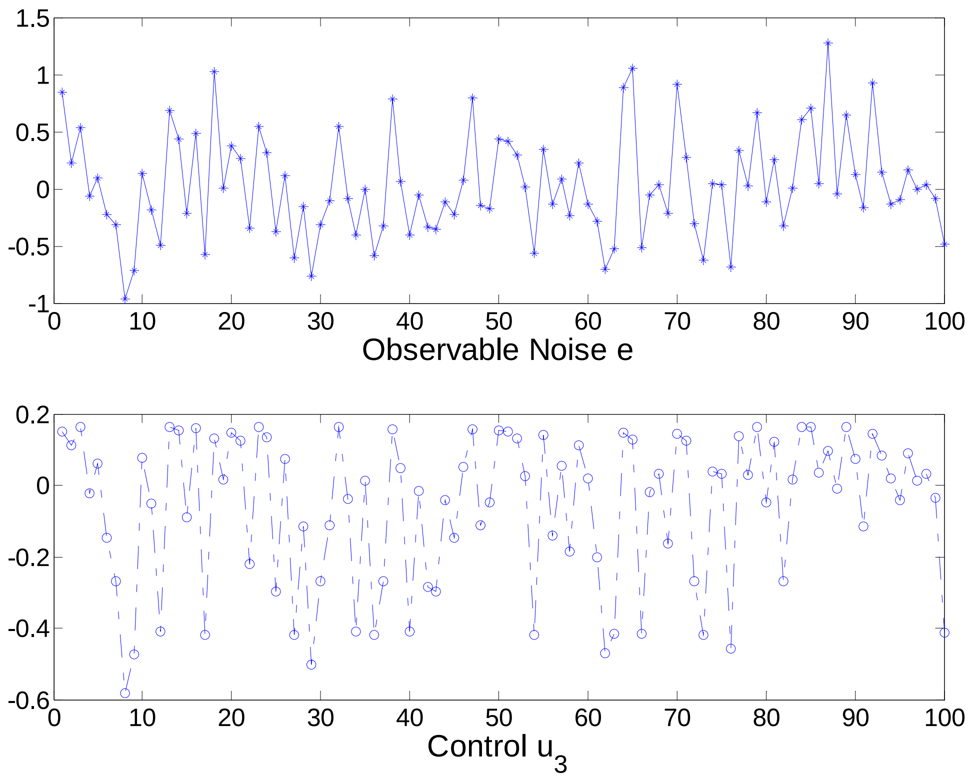

Figure 2-3 Examples of observable noises and control actions

Figure 2-3 shows an example of 100 runs of observable noise e and online controllable factor u_3. The control amount is adjusted according to the changes of observed noise.

The control performances for different target values are also compared, as shown in Figure 2-4. The horizontal axis indicates that the comparison of the performances is carried out under shrinkage percentages ranging from 1% to 3.6%. This range contains the designed experimental range, within which the correctness of the DOE regression model can be ensured. The vertical axis in Figure 2-4 represents the performances (or quadratic loss) under different control strategies. The peaks of the RPD are resulted from the adjustment of u_1 and x_3 according to different target regions.

A closer look of Figure 2-4 indicates that (i) the control performance is improved by considering modeling error; and (ii) the control performance is still acceptable when other strategies are not functioning properly. It is also observed that the controller performance gets worse closer to the edge of target range. This performance deterioration is because the optimal controlled solution is close to being out of the experimental region and needs to be set to the boundary, which greatly decreases the control efficiency.

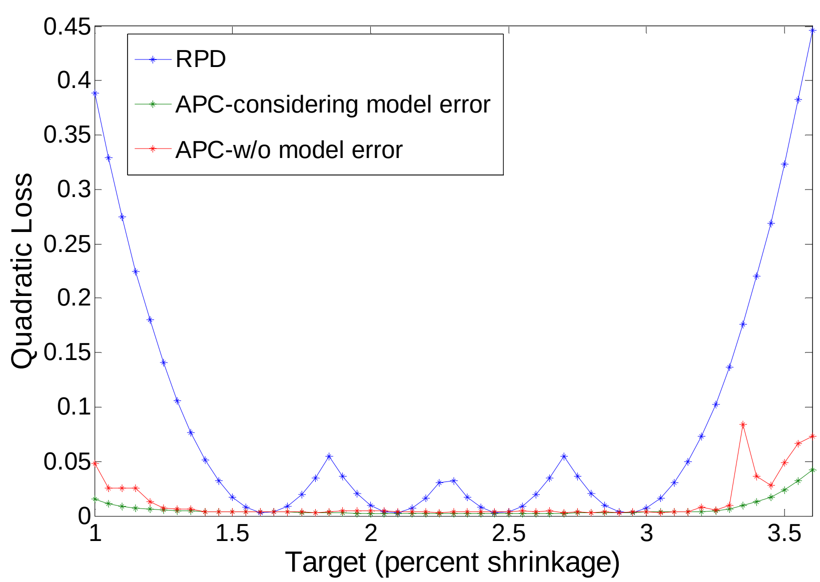

Figure 2-4 Comparison of quadratic losses of the three approaches

Figure 2-5 provides a zoom-in comparison from Figure 2-4. This figure focuses on the two APC control strategies, ranging from 2.0% to 2.6 %, which is in the center of the previously mentioned working range of controllers. This closer look clearly shows that APC with consideration of modeling errors is consistently better than that without considering them.

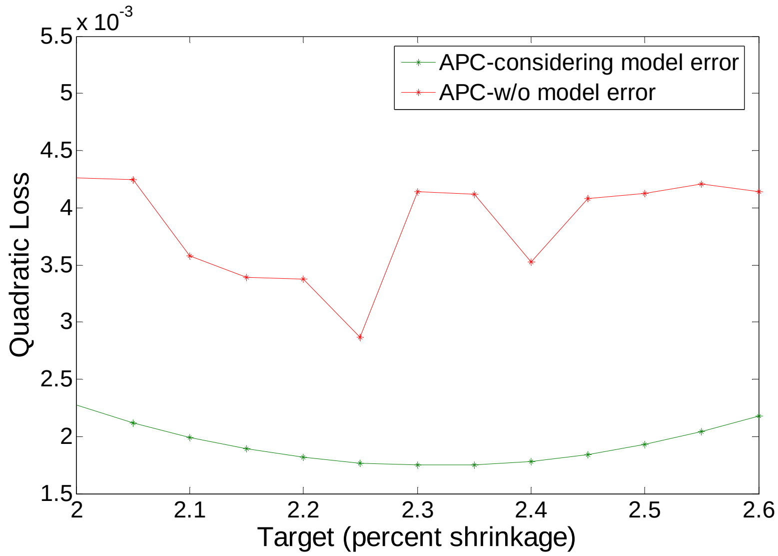

Figure 2-5 Comparison of quadratic losses of two APC

2.4 Conclusion

An automatic process control strategy is developed based on regression models obtained from design of experiments. The control strategy takes into account modeling and observation uncertainties and demonstrated superior performance when the uncertainties are large. The performances of the proposed control strategy and existing control approaches have been compared via a case study on an injection molding process. Generally, the proposed strategy can significantly improve the control performance by considering in-process observation of noise factors and errors in parameter estimation. The comparison study also indicates that the certainty equivalence control provides better performance than robust design when on-line sensing uncertainties are small.

There are some open issues to be addressed in the future research. Some examples of those topics include (a) incasing the model (3) complexity by including high interaction terms among control variables and noise variables; (b) considering model uncertainties due to model structure errors; (c) investigating the sensitivity of the control performance to the model assumptions. More efforts are needed to consider the integrated system modeling and control to achieve better control performance.

Acknowledgments

The financial support from NSF Grant # 0217395 is gratefully acknowledged.

Appendix 1: Proof of (5)

Consider the quadratic loss function $L(y,t)=\left[y-t\right]^2$. The objective function is:

$$\begin{aligned} J_{APC}\left(\mathbf{X},\mathbf{U}\mid\hat{\mathbf{e}},\hat{\boldsymbol{\beta}}\right) &= E_{\mathbf{e},\mathbf{n},\tilde{\boldsymbol{\beta}},\varepsilon}\left[L\left(y,t\right)\middle|\hat{\mathbf{e}},\hat{\boldsymbol{\beta}}\right]=E_{\mathbf{e},\mathbf{n},\tilde{\boldsymbol{\beta}},\varepsilon}\left[\left(y\middle|\hat{\mathbf{e}},\hat{\boldsymbol{\beta}}\right)-t\right]^2 \\ &=\left(E_{\mathbf{e},\mathbf{n},\tilde{\boldsymbol{\beta}},\varepsilon}\left[y\middle|\hat{\mathbf{e}},\hat{\boldsymbol{\beta}}\right]-t\right)^2+Var_{\mathbf{e},\mathbf{n},\tilde{\boldsymbol{\beta}},\varepsilon}\left(y\middle|\hat{\mathbf{e}},\hat{\boldsymbol{\beta}}\right). \end{aligned} \tag{14}$$

Plugging in the response model (3), the two terms in (14) becomes:

$$\begin{aligned} &E_{\mathbf{e},\mathbf{n},\tilde{\boldsymbol{\beta}},\varepsilon}\left[y\middle|\hat{\mathbf{e}},\hat{\boldsymbol{\beta}}\right] \\ &=E_{\mathbf{e},\mathbf{n},\tilde{\boldsymbol{\beta}},\varepsilon}\left[\beta_0+\boldsymbol{\beta}_1^T\mathbf{X}+\boldsymbol{\beta}_2^T\mathbf{U}+\boldsymbol{\beta}_3^T\mathbf{e}+\boldsymbol{\beta}_4^T\mathbf{n}+\mathbf{X}^T\mathbf{B}_1\mathbf{e}+\mathbf{U}^T\mathbf{B}_2\mathbf{e}\ +\mathbf{X}^T\mathbf{B}_3\mathbf{n}+\mathbf{U}^T\mathbf{B}_4\mathbf{n}+\varepsilon\middle|\hat{\mathbf{e}},\hat{\boldsymbol{\beta}}\right] \\ &=\hat{\beta}_0+\hat{\boldsymbol{\beta}}_1^T\mathbf{X}+\hat{\boldsymbol{\beta}}_2^T\mathbf{U}+E_{\mathbf{e}}\left[\boldsymbol{\beta}_3^T\mathbf{e}\mid\hat{\mathbf{e}},\hat{\boldsymbol{\beta}}\right]+E_{\mathbf{e}}\left[\mathbf{X}^T\mathbf{B}_1\mathbf{e}\mid\hat{\mathbf{e}},\hat{\boldsymbol{\beta}}\right]+E_{\mathbf{e}}\left[\mathbf{U}^T\mathbf{B}_2\mathbf{e}\mid\hat{\mathbf{e}},\hat{\boldsymbol{\beta}}\right] \\ &=\hat{\beta}_0+\hat{\boldsymbol{\beta}}_1^T\mathbf{X}+\hat{\boldsymbol{\beta}}_2^T\mathbf{U}\ +\hat{\boldsymbol{\beta}}_3^T\hat{\mathbf{e}}+\mathbf{X}^T\hat{\mathbf{B}}_1\hat{\mathbf{e}}+\mathbf{U}^T\hat{\mathbf{B}}_2\hat{\mathbf{e}}, \end{aligned}$$

$$Var_{\mathbf{e},\mathbf{n},\tilde{\boldsymbol{\beta}},\varepsilon}\left(y\middle|\hat{\mathbf{e}},\hat{\boldsymbol{\beta}}\right)=E_{\mathbf{n}}\left(Var_{\mathbf{e},\tilde{\boldsymbol{\beta}},\varepsilon}\left[y\middle|\hat{\mathbf{e}},\hat{\boldsymbol{\beta}},\mathbf{n}\right]\right)+Var_{\mathbf{n}}\left(E_{\mathbf{e},\tilde{\boldsymbol{\beta}},\varepsilon}\left[y\middle|\hat{\mathbf{e}},\hat{\boldsymbol{\beta}},\mathbf{n}\right]\right). \tag{15}$$

For (15),

$$\begin{aligned} &Var_{\mathbf{n}}\left\{E_{\mathbf{e},\tilde{\boldsymbol{\beta}},\varepsilon}\left[y\middle|\hat{\mathbf{e}},\hat{\boldsymbol{\beta}},\mathbf{n}\right]\right\} \\ &=Var_{\mathbf{n}}\left\{\hat{\beta}_0+\hat{\boldsymbol{\beta}}_1^T\mathbf{X}+\hat{\boldsymbol{\beta}}_2^T\mathbf{U}+\hat{\boldsymbol{\beta}}_3^T\hat{\mathbf{e}}+\hat{\boldsymbol{\beta}}_4^T\mathbf{n}+\mathbf{X}^T\hat{\mathbf{B}}_1\hat{\mathbf{e}}+\mathbf{U}^T\hat{\mathbf{B}}_2\hat{\mathbf{e}}\ +\mathbf{X}^T\hat{\mathbf{B}}_3\mathbf{n}+\mathbf{U}^T\hat{\mathbf{B}}_4\mathbf{n}\right\} \\ &=\left(\hat{\boldsymbol{\beta}}_4+\hat{\mathbf{B}}_3^T\mathbf{X}+\hat{\mathbf{B}}_4^T\mathbf{U}\right)^T\Sigma_{\mathbf{n}}\left(\hat{\boldsymbol{\beta}}_4+\hat{\mathbf{B}}_3^T\mathbf{X}+\hat{\mathbf{B}}_4^T\mathbf{U}\right), \end{aligned} \tag{16}$$

$$
\begin{aligned}
& E_{\mathbf{n}}\left\{Var_{\mathbf{e},\tilde{\boldsymbol{\beta}},\varepsilon}\left(y\middle|\hat{\mathbf{e}},\hat{\boldsymbol{\beta}},\mathbf{n}\right)\right\} \\
&= E_{\mathbf{n}}\left\{E_{\mathbf{e}}\left[Var_{\tilde{\boldsymbol{\beta}},\varepsilon}\left(y\middle|\hat{\mathbf{e}},\hat{\boldsymbol{\beta}},\mathbf{n},\tilde{\mathbf{e}}\right)\right]+Var_{\mathbf{e}}\left[E_{\tilde{\boldsymbol{\beta}},\varepsilon}\left(y\middle|\hat{\mathbf{e}},\hat{\boldsymbol{\beta}},\mathbf{n},\tilde{\mathbf{e}}\right)\right]\right\} \\
&= E_{\mathbf{n}}\left\{E_{\mathbf{e}}\left[\sigma_{\varepsilon}^2+\Sigma_{\tilde{\beta}_0}+\mathbf{X}^T\Sigma_{\tilde{\boldsymbol{\beta}}_1}\mathbf{X}+\mathbf{U}^T\Sigma_{\tilde{\boldsymbol{\beta}}_2}\mathbf{U}+\mathbf{e}^T\Sigma_{\tilde{\boldsymbol{\beta}}_3}\mathbf{e}+\mathbf{n}^T\Sigma_{\tilde{\boldsymbol{\beta}}_4}\mathbf{n}+Var_{\tilde{\boldsymbol{\beta}}}\left(\mathbf{X}^T\mathbf{B}_1\mathbf{e}\right)\right.\right. \\
&\quad \left. +Var_{\tilde{\boldsymbol{\beta}}}\left(\mathbf{U}^T\mathbf{B}_2\mathbf{e}\right)+Var_{\tilde{\boldsymbol{\beta}}}\left(\mathbf{X}^T\mathbf{B}_3\mathbf{n}\right)+Var_{\tilde{\boldsymbol{\beta}}}\left(\mathbf{U}^T\mathbf{B}_4\mathbf{n}\right)\right] \\
&\quad \left. +Var_{\mathbf{e}}\left[\hat{\beta}_0+\hat{\boldsymbol{\beta}}_1^T\mathbf{X}+\hat{\boldsymbol{\beta}}_2^T\mathbf{U}+\hat{\boldsymbol{\beta}}_3^T\hat{\mathbf{e}}+\hat{\boldsymbol{\beta}}_4^T\mathbf{n}+\mathbf{X}^T\hat{\mathbf{B}}_1\hat{\mathbf{e}}+\mathbf{U}^T\hat{\mathbf{B}}_2\hat{\mathbf{e}}+\mathbf{X}^T\hat{\mathbf{B}}_3\mathbf{n}+\mathbf{U}^T\hat{\mathbf{B}}_4\mathbf{n}\right]\right\} \\
&= \sigma_{\varepsilon}^2+\Sigma_{\tilde{\beta}_0}+\mathbf{X}^T\Sigma_{\tilde{\boldsymbol{\beta}}_1}\mathbf{X}+\mathbf{U}^T\Sigma_{\tilde{\boldsymbol{\beta}}_2}\mathbf{U}+\hat{\mathbf{e}}^T\Sigma_{\tilde{\boldsymbol{\beta}}_3}\hat{\mathbf{e}}+E_{\mathbf{n}}\left[\mathbf{n}^T\Sigma_{\tilde{\boldsymbol{\beta}}_4}\mathbf{n}\right] \\
&\quad +Var_{\tilde{\boldsymbol{\beta}}}\left(\mathbf{X}^T\mathbf{B}_1\mathbf{e}\right)+Var_{\tilde{\boldsymbol{\beta}}}\left(\mathbf{U}^T\mathbf{B}_2\mathbf{e}\right)+E_{\mathbf{n}}\left[Var_{\tilde{\boldsymbol{\beta}}}\left(\mathbf{X}^T\mathbf{B}_3\mathbf{n}\right)\right]+E_{\mathbf{n}}\left[Var_{\tilde{\boldsymbol{\beta}}}\left(\mathbf{U}^T\mathbf{B}_4\mathbf{n}\right)\right] \\
&\quad +\left(\hat{\boldsymbol{\beta}}_3+\hat{\mathbf{B}}_1^T\mathbf{X}+\hat{\mathbf{B}}_2^T\mathbf{U}\right)^T\Sigma_{\tilde{\mathbf{e}}}\left(\hat{\boldsymbol{\beta}}_3+\hat{\mathbf{B}}_1^T\mathbf{X}+\hat{\mathbf{B}}_2^T\mathbf{U}\right).
\end{aligned}
$$

(16) holds because $Cov(\mathbf{e}|\hat{\mathbf{e}}) = Cov(\tilde{\mathbf{e}}|\hat{\mathbf{e}}) + Cov(\hat{\mathbf{e}}|\hat{\mathbf{e}}) = \Sigma_{\tilde{\mathbf{e}}}$.

Thus the expected quadratic loss function is given by:

$$
\begin{aligned}
& J_{APC}\left(\mathbf{X},\mathbf{U}\middle|\hat{\mathbf{e}},\hat{\boldsymbol{\beta}}\right) \\
&= \sigma_{\varepsilon}^2+\left(\hat{\beta}_0-t+\hat{\boldsymbol{\beta}}_1^T\mathbf{X}+\hat{\boldsymbol{\beta}}_2^T\mathbf{U}+\left(\hat{\boldsymbol{\beta}}_3^T+\mathbf{X}^T\hat{\mathbf{B}}_1+\mathbf{U}^T\hat{\mathbf{B}}_2\right)\hat{\mathbf{e}}\right)^2 \\
&\quad +\left(\hat{\boldsymbol{\beta}}_3+\hat{\mathbf{B}}_1^T\mathbf{X}+\hat{\mathbf{B}}_2^T\mathbf{U}\right)^T\Sigma_{\tilde{\mathbf{e}}}\left(\hat{\boldsymbol{\beta}}_3+\hat{\mathbf{B}}_1^T\mathbf{X}+\hat{\mathbf{B}}_2^T\mathbf{U}\right) \\
&\quad +\left(\hat{\boldsymbol{\beta}}_4+\hat{\mathbf{B}}_3^T\mathbf{X}+\hat{\mathbf{B}}_4^T\mathbf{U}\right)^T\Sigma_{\mathbf{n}}\left(\hat{\boldsymbol{\beta}}_4+\hat{\mathbf{B}}_3^T\mathbf{X}+\hat{\mathbf{B}}_4^T\mathbf{U}\right) \\
&\quad +\sigma_{\tilde{\beta}_0}^2+\mathbf{X}^T\Sigma_{\tilde{\boldsymbol{\beta}}_1}\mathbf{X}+\mathbf{U}^T\Sigma_{\tilde{\boldsymbol{\beta}}_2}\mathbf{U}+\hat{\mathbf{e}}^T\Sigma_{\tilde{\boldsymbol{\beta}}_3}\hat{\mathbf{e}}+E_{\mathbf{n}}\left[\mathbf{n}^T\Sigma_{\tilde{\boldsymbol{\beta}}_4}\mathbf{n}\right]+Var_{\tilde{\boldsymbol{\beta}}}\left(\mathbf{X}^T\mathbf{B}_1\mathbf{e}\right) \\
&\quad +Var_{\tilde{\boldsymbol{\beta}}}\left(\mathbf{U}^T\mathbf{B}_2\mathbf{e}\right)+E_{\mathbf{n}}\left[Var_{\tilde{\boldsymbol{\beta}}}\left(\mathbf{X}^T\mathbf{B}_3\mathbf{n}\right)\right]+E_{\mathbf{n}}\left[Var_{\tilde{\boldsymbol{\beta}}}\left(\mathbf{U}^T\mathbf{B}_4\mathbf{n}\right)\right].
\end{aligned}
$$

This proves (5).

Appendix 2: Proof of (13)

From (12), take the first-order partial derivative of $J_{APC}\left(\mathbf{X},\mathbf{U}\middle|\mathbf{X},\hat{e}_1,\hat{\boldsymbol{\beta}}\right)$ on $\mathbf{U}$ and set it zero,

$$
\begin{aligned}
&\frac{\partial J_{APC}\left(\mathbf{X},\mathbf{U}\middle|\mathbf{X},\hat{e}_1,\hat{\boldsymbol{\beta}}\right)}{\partial \mathbf{U}} \\
&= 2\left(\beta_0 - t + \hat{\boldsymbol{\beta}}_1^T\mathbf{X} + \hat{\beta}_3^T\hat{e}_1 + \mathbf{X}^T\hat{\mathbf{B}}_1\hat{e}_1\right)\left(\hat{\boldsymbol{\beta}}_2 + \hat{\mathbf{B}}_2\hat{e}_1\right) + 2\left(\hat{\boldsymbol{\beta}}_2 + \hat{\mathbf{B}}_2\hat{e}_1\right)\left(\hat{\boldsymbol{\beta}}_2 + \hat{\mathbf{B}}_2\hat{e}_1\right)^T\mathbf{U} \\
&\quad + 2\sigma_{n_1}^2\hat{\mathbf{B}}_4\left(\hat{\beta}_4 + \hat{\mathbf{B}}_3^T\mathbf{X}\right) + 2\left(\hat{\mathbf{B}}_4\sigma_{n_1}^2\hat{\mathbf{B}}_4^T\right)\mathbf{U} + 2\sigma_{\tilde{e}_1}^2\hat{\mathbf{B}}_2\left(\hat{\beta}_3 + \hat{\mathbf{B}}_1^T\mathbf{X}\right) + 2\left(\hat{\mathbf{B}}_2\sigma_{\tilde{e}_1}^2\hat{\mathbf{B}}_2^T\right)\mathbf{U} \\
&\quad + 2Var\left(\hat{\boldsymbol{\beta}}_2^T\mathbf{U}\right) + 2\hat{e}_1^2 Var\left(\hat{\mathbf{B}}_2^T\mathbf{U}\right) + 2\sigma_{n_1}^2 Var\left(\hat{\mathbf{B}}_4^T\mathbf{U}\right) \\
&= 0.
\end{aligned}
$$

It follows that if $\left(\hat{\boldsymbol{\beta}}_2 + \hat{\mathbf{B}}_2\hat{\mathbf{e}}\right)\left(\hat{\boldsymbol{\beta}}_2 + \hat{\mathbf{B}}_2\hat{\mathbf{e}}\right)^T + \hat{\mathbf{B}}_2\sigma_{\tilde{e}_1}^2\hat{\mathbf{B}}_2^T + \hat{\mathbf{B}}_4\sigma_{n_1}^2\hat{\mathbf{B}}_4^T + \sum_{\tilde{\boldsymbol{\beta}}_2}^2 + \hat{e}_1^2\sum_{\tilde{\mathbf{B}}_2}$ $+\sigma_{n_1}^2\sum_{\tilde{\mathbf{B}}_4}$ is invertible, then

$$
\begin{aligned}
\mathbf{U}^* = &-\left\{\left(\hat{\boldsymbol{\beta}}_2 + \hat{\mathbf{B}}_2\hat{e}_1\right)\left(\hat{\boldsymbol{\beta}}_2 + \hat{\mathbf{B}}_2\hat{e}_1\right)^T + \hat{\mathbf{B}}_2\sigma_{\tilde{e}_1}^2\hat{\mathbf{B}}_2^T + \hat{\mathbf{B}}_4\sigma_{n_1}^2\hat{\mathbf{B}}_4^T + \sum_{\tilde{\boldsymbol{\beta}}_2}^2 + \hat{e}_1^2\sum_{\tilde{\mathbf{B}}_2} + \sigma_{n_1}^2\sum_{\tilde{\mathbf{B}}_4}\right\}^{-1} \\
&\times\left\{\left(\hat{\beta}_0 - t + \hat{\boldsymbol{\beta}}_1^T\mathbf{X} + \hat{\beta}_3^T\hat{e}_1 + \mathbf{X}^T\hat{\mathbf{B}}_1\hat{e}_1\right)\left(\hat{\boldsymbol{\beta}}_2 + \hat{\mathbf{B}}_2\hat{e}_1\right)\right. \\
&\left. + \hat{\mathbf{B}}_2\sigma_{\tilde{e}_1}^2\left(\hat{\beta}_3 + \hat{\mathbf{B}}_1^T\mathbf{X}\right) + \hat{\mathbf{B}}_4\sigma_{n_1}^2\left(\hat{\beta}_4 + \hat{\mathbf{B}}_3^T\mathbf{X}\right)\right\}.
\end{aligned}
$$

This proves (13).

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CHAPTER 3

FEED-FORWARD PREDICTIVE CONTROL STRATEGY WITH CONSIDERATION OF MODEL UNCERTAINTY FOR MULTISTAGE MANUFACTURING PROCESSES

Abstract

Active control for dimensional variation reduction in multistage manufacturing processes is a challenging issue for quality assurance. It is desirable to implement a system-level control strategy to minimize the end-of-line product variance, which is propagated from upstream manufacturing stages. Research has been conducted to realize such objective, based on the mathematical variation propagation model derived according to the designated parameters from product and process design. However, due to the uncertainties caused by the significant changes of process parameters from their nominal, such designated model will be different from the one of the real process, and will not accurately represent the physics of the process. Thus, the performance of controller may degrade under process uncertainties if it is designed based on the designated model. This paper proposed a feed-forward control strategy for multistage assembly process that explicitly takes into account the uncertainties of model coefficients. The case study demonstrates that the controller derived from the proposed approach outperformed the one without considering the model uncertainty.

3.1 Introduction

Dimensional variation reduction is a critical yet challenging problem in quality assurance of multistage manufacturing processes (MMPs). In MMPs, multiple operation stages are involved in generating designated Key Product Characteristics (KPCs) or functionality of a product. At a certain stage *k*, special causes, such as part fabrication error, fixture error, welding gun error and robot positioning error, will increase the measurements variation of some KPCs to a level that exceeds their specification limits. Compounded with the input quality transmitted from preceding stages, these quality problems will be further propagated to the downstream stages and accumulated to the final product. This variation propagation makes the process monitoring and root cause diagnosis especially challenging.

In order to increase the process stability and reduce the product variation, three types of approaches are widely adopted in practice: i) robust process design, which aims to reduce the impact of variation sources at the design stage; ii) Statistical Process Control (SPC), which detects variation sources from quality measurements on KPCs; and iii) in-process active control, which compensates the error based on in-process measurements. Among them, robust design is usually conducted in the design phase of product realization and will not react to in-process disturbances; SPC focuses mainly on fault detection, rather than process adjustment; in-process control actively corrects deviation of each assembly, and can thus achieve variation reduction on a part-by-part base.

Application of in-process control in MMPs is complicate, in the sense that variation is propagated and accumulated throughout the production line. Therefore, a successful control strategy for MMPs should be the one that compensates error at an intermediate stage, aiming at achieving the overall best quality at the final stage, rather than the best quality at the stage where the control action is taken. In other words, the system-level optimal, not the stage-level optimal, is the control objective.

In-process control for MMPs has received intensive study and promising results have been reported in literatures, which will be reviewed in the following paragraph. There are two basic mechanisms in control theory, namely feedback control and feed-forward control. Feedback control mechanism, by utilizing information from downstream stages to determine the control actions at the current stage, is effective when the control objective is to reduce the shift in mean values. This is because that in discrete party manufacturing, the observation made on a particular product at a specific downstream stage is not correlated with the quality of another product at the current stage. If a mean shift presents in the process, all parts will be affected in a deterministic way until a corrective control action is taken, and thus feedback control will be effective in correcting mean shifts. However, when the variance of process increases, the random KPC deviations will be different from a downstream product to the one at the current stage. Under this situation, the control actions derived based on the downstream observation may not effectively correct the random deviation at the current stage. Thus, only feed-forward control scheme fits the research objective of this paper, which is the process variation reduction in MMP.

Many research efforts have been reported in the literature on feed-forward control in manufacturing processes. Most of them are focused on stage-level variation reduction. For example, feed-forward control with sensor system has been employed in various assembly processes (Svensson, 1985; Wu and Hamada, 2000), and also adopted by automobile manufacturers such as Nissan (Sekine *et al.*, 1991). However, stage-level active control does not consider the propagation and cross-stage relationship of variation from previous stages. Thus it is effective at the last stage of an MMP, or if the compensated KPC is located on parts that will not be affected by downstream operations. Otherwise, the stage-level optimal compensation may not be optimal at system-level and deliver the best final product at the end of the MMP.

For system-level active control, an optimal control scheme was proposed in mechanical assembly using state transition models (Mantripragada and Whitney, 1999). This approach treats control as stochastic discrete-time linear optimal regulator problem, and obtains a deterministic controller, considering parts as the only source of variation in the process. A similar optimal control problem is analyzed, with the application in semiconductor manufacturing (Fenner *et al.*, 2005). The authors used Dynamic Programming (DP) as the optimization tool, taking the control magnitudes in each direction on each single part, or controllable environmental variables at each manufacturing stage, as the decision variable. However, a MMP, such as an automobile assembly line, usually involves large numbers of stages and assemblies, with each subassembly introducing three degrees of freedom as the decision variables, even only consider the slip plane as a 2-D case. Thus the possible control actions for a MMP will form a solution space with extremely high dimensions, and the curse of dimensionality for DP will limit the application of the above mentioned analytical global-optimal controller in MMP control. In such case of a high dimensional solution space, a simpler sub-optimal controller with adequate performance will be an ideal alternative to the one targeting to solve global optima. Under this simplified objective, controller that adjusts the position of fixtures and tool path to improve the final product quality was proposed for multistage machining process (Djurdjanovic and Zhu, 2005). The realization of feed-forward control using PT in multistage assembly process is also analyzed, with the minimization target being deviations of KPCs, rather than variation of the final product (Djurdjanovic and Ni, 2006). A feed-forward controller that aims at reducing final part KPC variation, considering controllability and measurement noises is developed later (Izquierdo *et al.*, 2007). These studies investigated control strategies, considering controllability and actuator capabilities respectively, without taking into account the model uncertainty.

In real assembly processes, uncertainties can enter the system not only as noises of the sensors, disturbances in system, but also as variation of the system model itself. The uncertainty of model is generated as a result of part fabrication process, such as part initial errors from stamping, errors accumulated from previous stages, fixture, welding gun and robot error, and other process uncertainties, which are inevitable in real-life production. The system models used in previous works are obtained from the first principle, where the parameters come from the designed blue print. However, the model coefficients will randomly deviate from their designated values because of the former mentioned model uncertainties. A control strategy derived from the nominal model without considering model uncertainty may lead to even worse control performances than ordinary production, especially under large noise environment.

The problem of model uncertainty has drawn much attention in control community. In control systems, due to the imperfect data, lack of process knowledge and system dynamics and/or complexity, models of the system to be controlled always have inherent errors. The research dealing with the above mentioned unknown dynamics and disturbances has formed the area of robust control, with a variety of methodologies developed. Among them, Model Reference Active Control (MRAC) catches the system dynamic by designing the controller with parameters that can be updated according to system output (Åström, 1996); H_2 or H_∞ control (Basar and Bernhard, 1995; Kwakernaak, 2002) seeks to minimize the maximum power or energy gain of the system so as to stabilize it; Fuzzy Control (Tanaka and Sugeno, 1992) has the ability to control without requirement for complex mathematical modeling. However, due to the different nature of manufacturing systems, those well-developed robust control techniques cannot be directly applied in the context of MMP.

This paper proposes a Stream of Variation model based predictive controller for MMPs variation reduction, as illustrated in Figure 3-1. The enablers of online feed-forward control in MMP include actuators, such as Programmable Tooling (PT), real-time sensing technologies, as well as a mathematical variation propagation model, as shown bold border blocks in Figure 3-1. PT allows the high precision automatic adjustment of locators and clamps that are used to mount the parts, and thus it provides the capability of variation reduction on a part-by-part adjustment base. Accurate real-time dimensional measurement sensors, such as OCMM system, can obtain real-time quality information for in-line controller to decide control action. The last but not least critical element in control of MMP, is a mathematical model of the process that links the final product quality with variation sources in upstream stages, and precisely predicts the final quality. This paper will focus on the SoV model with model uncertainties and the derived predictive controller.

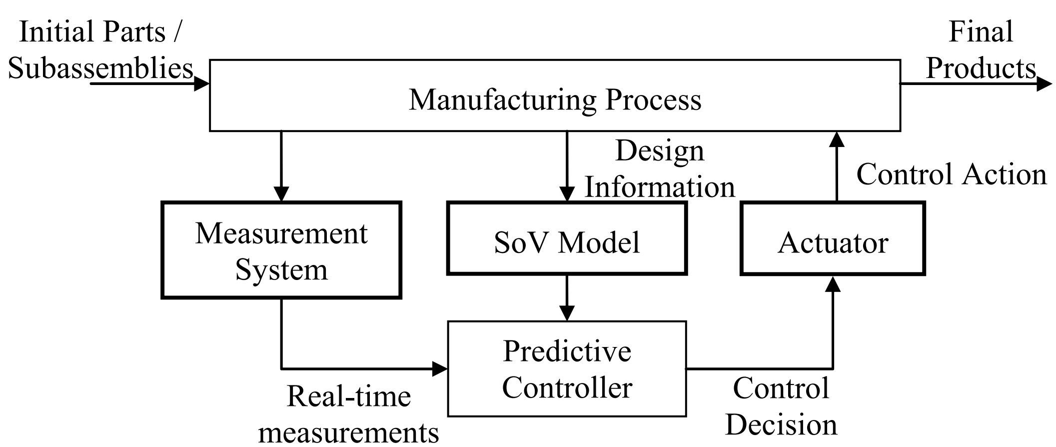


Figure 3-1 Diagram of control scheme in MMP

The rest of this paper is organized as follows: section 3.2 introduces the SoV model and extends the model representation to explicitly represent potential model uncertainty; section 3.3 derives predictive control strategy based on this model considering uncertainties; section 3.4 demonstrates the effectiveness of the proposed method with a real life production example. The method will be summarized in section 3.5.

3.2 Stream of Variation Model and Model Uncertainty

This section introduces the uncertainty of SoV model, and extends the model representation to consider model uncertainty generated from geometric errors, such as part fabrication problems, or errors accumulated from previous stages.

3.2.1 Representation of Part Deviations

State space model was introduced to mathematically represent the dimensional variation propagation in MMP (Jin and Shi, 1999; Shi, 2006). Two coordinate systems are necessary to properly represent part deviation in such a process, namely global (or body) coordinate system and part coordinate system, with the former one remain unchanged for all stages, and the latter one attached onto each part or assembly. Each part is characterized by its deviation from nominal position, which can be represented by x, y, and z as translation coordinate variables, and α, β, and ϕ as corresponding rotation coordinate variables. In most assembly circumstances, parts are joined on slip planes, where the deviations can be simplified into a 2-D case, since the 3rd direction is constrained by part connecting surface (Ding *et al.*, 2000). This simplified deviation is represented by 3 coordinate variables, x, y and β, as shown in Figure 3-2. The subscripts (i, k) of coordinate variables in Figure 3-2 indicate part i on stage k, for i=1,…, n_p, k=1,…, N, where n_p is the total number of parts, and N is the total number of stages in MMPs. For a subassembly that consists of more than one part, the subscript becomes (s, k) with s denoting the index of subassembly.

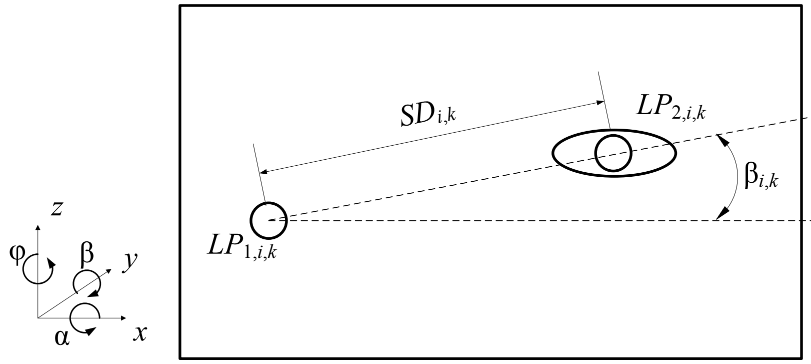


Figure 3-2 Representation of part deviation

Two reference points are necessary to represent each part coordinate in 2-D space, denoted as $LP_{1,i,k}$ and $LP_{2,i,k}$. The position of a part at stage k can then be represented by the position of $LP_{1,i,k}$, i.e., $\left(LP_{1,i,k(\mathrm{x})}, LP_{1,i,k(\mathrm{z})}\right)$ in global coordinate system, together with the part orientation represented by the angle between the line connecting $LP_{1,i,k}$ and $LP_{2,i,k}$ and the x-axis of global coordinate system, i.e., $\beta_{i,k}$ in Figure 3-2. In 2-D rigid body assembling, the 4-way hole and 2-way slot pair that are used to mount the subassembly, are usually chosen as the reference points for the subassembly for simplicity, and are denoted as $LS_{1,s,k}$ and $LS_{2,s,k}$.

Under ideal production condition, parts are positioned in their designed location, with neither fixture variation nor part variation. However in reality, locating tools and/or part fabrications are not perfect, the part will deviate from its nominal position. Instead of using the absolute global coordinate, the actual position of an assembly in space is described by its deviations from the nominal positions, i.e., $\mathbf{x}_{i,k} \equiv \begin{bmatrix} \delta x_{i,k} & \delta y_{i,k} & \delta z_{i,k} & \delta\alpha_{i,k} & \delta\beta_{i,k} & \delta\phi_{i,k} \end{bmatrix}^T$. It denotes the random deviations associated with each of the six degrees of freedom (d.o.f's) of part i at stage k, where the notation δ in front of a coordinate variable represents a small deviation. $\mathbf{x}_{i,k}$ for a 2-D assembly can be simplified as $\begin{bmatrix} \delta x_{i,k} & \delta z_{i,k} & \delta\beta_{i,k} \end{bmatrix}^T$, since the part will only have two d.o.f.'s of translation and one d.o.f. of rotation under the constraint. For all the n_p parts emerged in the assembly process, the deviation of the whole assembly on stage k can be expressed as $\mathbf{x}_k \equiv \begin{bmatrix} \mathbf{x}_{1,k}^T & \cdots & \mathbf{x}_{n_p,k}^T \end{bmatrix}^T$, which is usually referred to as the state of the part.

If part i has not yet appeared in stage k, the corresponding $\mathbf{x}_{i,k} = \mathbf{0}$. The deviation state notation, $\mathbf{q}_{s,k}$, is used for a multi-part subassembly s at stage k, to differentiate from $\mathbf{x}_{i,k}$ which is used for individual parts.

In MMPs with control system, parts or subassemblies are held by the locators of actuators, such as Programmable Tooling (PT), at the locating holes/slots, i.e., at $LS_{1,s,k}$ and $LS_{2,s,k}$. The actuator control action vector, denoted by $\mathbf{u}_k$ at stage k, contains the movements of each part represented by its locator pair deviation. For example, for the locator pair that supports subassembly s on stage k, its control action will be described as $\mathbf{u}_{s,k} = \left[\delta u_{LS_{1,s,k}}(x) \quad \delta u_{LS_{1,s,k}}(z) \quad \delta u_{LS_{2,s,k}}(x) \quad \delta u_{LS_{2,s,k}}(z) \right]^T$, where each element denotes the adjustment in each direction respectively. For all the n_k locator pairs used on stage k, the control action vector is $\mathbf{u}_k \equiv \left[\mathbf{u}_{1,k}^T \quad \cdots \quad \mathbf{u}_{n_k,k}^T \right]^T$.

The general SoV modeling procedures requires part design knowledge, in particular, the designed geometric layout, eg., the CAD model and/or assembly process plan. The models derived in literature are assumed to remain constant through the production process. However in practice, as stated earlier, the inherent uncertainties in all process will introduce variation into the state transition matrices of SoV model and will further impact on decisions made based on the model, including control actions. The following sections of this paper will present the SoV model considering these model uncertainties first. New notations will be engaged to differentiate the designated and real process models. An original matrix represents the one derived from the part/process design geometry; matrix with superscript $\frown$ is the one with true process parameters, and matrix with superscript $\sim$ is the deviation of the true matrix from its designed value, e.g., $\widehat{\mathbf{R}} = \mathbf{R} + \tilde{\mathbf{R}}$. With this notation system, the two necessary corollaries for model derivation (Shi, 2006) can be extended as Lemma 1.

***Lemma* 1** If subassembly s at stage k is mounted at points $LS_{1,s,k}$ and $LS_{2,s,k}$, then its deviation state due to small deviations at the locating points is

$$\mathbf{q}_{s,k} = \widehat{\mathbf{R}}_3^{s,k} \begin{bmatrix} \delta LS_{1,s,k}(x) \\ \delta LS_{1,s,k}(z) \\ \delta LS_{2,s,k}(x) \\ \delta LS_{2,s,k}(z) \end{bmatrix} = \widehat{\mathbf{R}}_3^{s,k} \cdot \mathbf{u}_{s,k}, \tag{1}$$

where

$$\widehat{\mathbf{R}}_3^{s,k} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \frac{\sin\widehat{\beta}_{s,k}}{\widehat{SD}_{s,k}} & -\frac{\cos\widehat{\beta}_{s,k}}{\widehat{SD}_{s,k}} & -\frac{\sin\widehat{\beta}_{s,k}}{\widehat{SD}_{s,k}} & \frac{\cos\widehat{\beta}_{s,k}}{\widehat{SD}_{s,k}} \end{bmatrix} = \mathbf{R}_3^{s,k} + \tilde{\mathbf{R}}_3^{s,k}, \tag{2}$$

and $SD_{s,k}$ is the distance between $LS1_{,s,k}$ and $LS2_{,s,k}$.

The next lemma characterizes the re-orientation operation, which induces deviation on a part when a subassembly is transferred from one stage to the next, even if the current locating points are free of error.

***Lemma* 2** Suppose that subassembly s is mounted at $LS1_{,s,k}$ and $LS2_{,s,k}$ at stage k and also assume that these two locating points are free of error at the current stage k. The deviation state of subassembly s on stage k when moving from stage k-1 to stage k can be expressed as a linear combination of deviations accumulated in its locating points $LS1_{,s,k}$ and $LS2_{,s,k}$ at the previous stage k-1.

$$\mathbf{q}_{s,k} = \widehat{\mathbf{R}}_4^{k-1,k} \begin{bmatrix} \delta LS_{1,s,k-1}(x) \\ \delta LS_{1,s,k-1}(z) \\ \delta LS_{2,s,k-1}(x) \\ \delta LS_{2,s,k-1}(z) \end{bmatrix}, \tag{3}$$

where

$$\widehat{\mathbf{R}}_4^{k-1,k} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ -\frac{\sin\widehat{\beta}_{s,k}}{\widehat{SD}_{s,k}} & \frac{\cos\widehat{\beta}_{s,k}}{\widehat{SD}_{s,k}} & \frac{\sin\widehat{\beta}_{s,k}}{\widehat{SD}_{s,k}} & -\frac{\cos\widehat{\beta}_{s,k}}{\widehat{SD}_{s,k}} \end{bmatrix} = \mathbf{R}_4^{k-1,k} + \tilde{\mathbf{R}}_4^{k-1,k}. \tag{4}$$

The above two lemmas are used to derive the state space model.

3.2.2 SoV Model with Part Induced Uncertainty

Figure 3-3 illustrates the stream of variation in a multistage manufacturing process. Parts from supplier enter the production at Stage 1 with initial fabrication errors, $\mathbf{x}_0$. In Stage 1, the part control action $\mathbf{u}_1$ is applied through PT first, while other un-modeled process errors, such as process background disturbances, higher order terms due to linearization, $\mathbf{w}_1$, also add to the variation of the parts. The designed operation at Stage 1 then takes place, and the state of the subassembly changes to $\mathbf{x}_1$. The subassembly is then transferred to the next stage, and the variations propagate and accumulate similarly as more parts/subassemblies are joined together, until the finished assembly exits the production line at the final Stage *N*. The KPC will be monitored at the final Stage *N* as well as some intermediate stages such as Stage *k*. The measurement $\mathbf{y}_k$ is obtained with sensor errors $\mathbf{v}_k$.

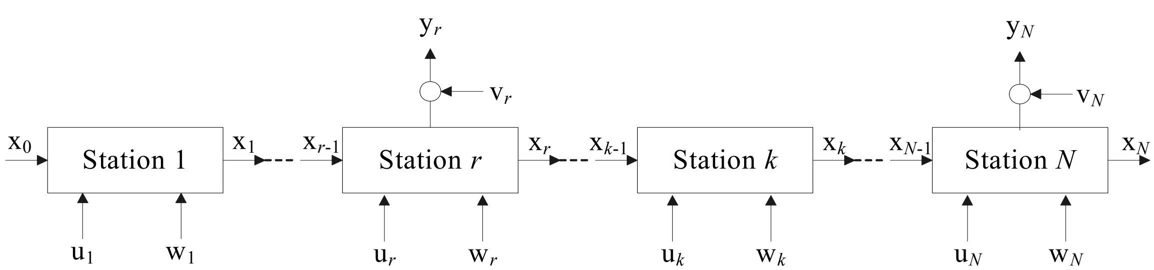


Figure 3-3 Multistage manufacturing process

Based on the two lemmas presented in the previous section, temporarily without considering model uncertainties, a state space model can be build to mathematically represent the variation propagation in MMPs, as defined in (5),

$$\begin{cases} \mathbf{x}_k = \mathbf{A}_{k-1}\mathbf{x}_{k-1} + \mathbf{B}_k\mathbf{u}_k + \mathbf{w}_k \\ \mathbf{y}_k = \mathbf{C}_k\mathbf{x}_k + \mathbf{v}_k \end{cases}, \ k = 1,\ldots,N. \tag{5}$$

The first equation is called the state equation, where matrix $\mathbf{A}_k \in \mathbb{R}^{n\times n}$ is the reorientation matrix, which describes the error transferred from previous stage through part reorientation. Matrix $\mathbf{B}_k \in \mathbb{R}^{n\times p}$ describes the impact of fixture deviations on the state of the system. The second equation is called the observation equation, where matrix $\mathbf{C}_k \in \mathbb{R}^{m\times n}$ represents how the part deviation will be transformed to the deviation of KPC. This flow chart also shows un-modeled disturbance, $\mathbf{w}_k \in \mathbb{R}^n$, and the measurement noises, $\mathbf{v}_k \in \mathbb{R}^m$. The modeling details and procedures can be found in literature (Ding *et al.*, 2000; Shi, 2006). $\mathbf{y}_N$ is the deviation of final product KPCs, whose variance/covariance, $\Sigma_{\mathbf{y}_N}$, is the index that a control strategy should aim to minimize. Let the state transition matrix $\mathbf{\Phi}_{k,i} = \mathbf{A}_{k-1}\mathbf{A}_{k-2}\ldots\mathbf{A}_i,\ k > i$ describe the deviation transited between stage *i* and *k*, and $\mathbf{\Phi}_{i,i} = \mathbf{I}_{n\times n}$, which is an identity matrix, then (5) can be written in an input-output format (Ding *et al.*, 2000),

$$\mathbf{y}_N = \sum_{i=1}^{N}\mathbf{\Gamma}_i\mathbf{u}_i + \mathbf{\Gamma}_0\mathbf{x}_0 + \sum_{i=1}^{N}\mathbf{\Psi}_i\mathbf{w}_i + \mathbf{v}_N, \tag{6}$$

where $\mathbf{\Gamma}_i = \mathbf{C}_N\mathbf{\Phi}_{N,i}\mathbf{B}_i$, $\mathbf{\Gamma}_0 = \mathbf{C}_N\mathbf{\Phi}_{N,0}$, and $\mathbf{\Psi}_i = \mathbf{C}_N\mathbf{\Phi}_{N,i}$.

Assuming measurements taken at stage *r*, i.e., $\mathbf{y}_r$ is known, $\mathbf{x}_r$ can be obtained through the inverse of observation equation in (5), as $\hat{\mathbf{x}}_r = \mathbf{C}^{\dagger}\mathbf{y}_r$. The superscript $\dagger$ represents generalized inverse, since **C** is typically not a square matrix due to the measurement redundancy. The output-input format then becomes,

$$\mathbf{y}_N = \sum_{i=r+1}^{N}\mathbf{\Gamma}_i\mathbf{u}_i + \mathbf{\Psi}_r\hat{\mathbf{x}}_r + \sum_{i=r+1}^{N}\mathbf{\Psi}_i\mathbf{w}_i + \mathbf{v}_N. \tag{7}$$

Thus the sensors are not mandatory to be installed in the same stage as the actuators, which releases the requirements proposed in previous works (Izquierdo *et al.*, 2007).

However, the deviations of subassembly from the designated dimensions, will not only appear as part errors in state vector $\mathbf{x}_k$ as considered in literature, but will also change the state transition matrices which is derived from the design geometry. The model uncertainty is thus introduced into the production system. With the same notation mechanism as in (5), the system can be described as

$$\begin{cases} \mathbf{x}_k = \widehat{\mathbf{A}}_{k-1}\mathbf{x}_{k-1} + \widehat{\mathbf{B}}_k\mathbf{u}_k + \mathbf{w}_k \\ \mathbf{y}_k = \mathbf{C}_k\mathbf{x}_k + \mathbf{v}_k, \end{cases}, \ k = 1,\ldots,N, \tag{8}$$

where $\widehat{\mathbf{A}}_k$ denotes the true state transition matrix, whose coefficients randomly deviates from that of the designate $\mathbf{A}_k$. The deviation matrix is denoted by $\tilde{\mathbf{A}}_k = \widehat{\mathbf{A}}_k - \mathbf{A}_k$. The sets of notations $\mathbf{B}_k$ and $\mathbf{\Gamma}_k$ are defined in a similar way. In the following derivation, $\mathbf{C}_k$ is considered constant, which means the locating points are monitored directly and precisely. By using model (8) instead of (5), the model uncertainty will be explicitly considered in control algorithm development.

3.3 Predictive Control Strategy

In MMP with control capability, when the subassemblies are mounted onto the fixtures, the sensors take measurements of the part deviations as input to the predictive controller. The controller then decides the corrective action based on these measurements as well as control actions taken in previous stages, which can be read from the Programmable Logic Controller (PLC). The control action is carried out by PT, before the subassemblies are joined together, and moved to next stage. This controller is called model predictive controller because it is designed based on the variation of KPCs at the final stage predicted by process model.

This section proposes a predictive control strategy with consideration of model uncertainty. It is first formulated as an optimization problem, and then a general optimal control strategy is derived.

3.3.1 Model Predictive Control Index

Model predictive control (MPC) refers to computer control algorithms that utilize an explicit process model to predict the future response of a process (Maciejowski, 2002). The controller employed can be any widely used control algorithms, such as Linear-Quadratic-Gaussian (LQG), and common model forms such as Input–output (IO), first-principles (FP), in linear and non-linear systems (Qin and Badgwell, 2003). MPC has been widely used in industries such as chemicals industry (Lee and Cooley, 1997), mining, and on automotive actuators (Cairano *et al.*, 2007). However, MPC requires a mathematical model that can predict the final product quality with respect to the process adjustments. Due to the lack of analytical process models, the application of MPC in MMP was limited in literature, and the presence of SoV modeling method has provided the possibility.

In MPC for MMP, the objective of predictive control is to improve quality through minimizing the variances of final product quality $\mathbf{y}_N$. With this purpose, the control action should also be punished based on its magnitude, since large control adjustment might result in unstable system response. The more important reason for punishing large control action is that it may result in the part geometry falling out of linearity approximation range, which is one assumption of SoV model. Taking the evaluations of quality and control penalty both into account, the optimization index at stage *k* can be written as

$$J_k = E\left[\hat{\mathbf{y}}_{N/k}^T \mathbf{Q}_N \hat{\mathbf{y}}_{N/k} + \mathbf{u}_k^T \mathbf{R}_k \mathbf{u}_k\right], \tag{9}$$

where $\hat{\mathbf{y}}_{N/k}$ denotes the final product quality of stage *N* that is predicted at stage *k*, and penalty coefficients $\mathbf{Q}_N \in \mathbb{R}^{m\times m}$ and $\mathbf{R}_k \in \mathbb{R}^{n\times n}$ are the weight matrices. Similar to the common requirements in control theory, $\mathbf{Q}_N$ is assumed to be semi-positive definite, and $\mathbf{R}_k$ is positive definite to make sure the inevitability of the analytic solution.

Dynamic Programming (DP) is the desirable tool to solve a globally optimal solution for the objective function (9) in MMP, however, since each subassembly introduces 6 d.o.f's into the system, the curse of dimensionality of DP will result in the search in an extremely high dimensional solution space, which is not applicable. Approximation is necessary to analytically derive the control action for part currently at stage *k*. This is because that when part is mounted in stage *k*, all downstream control actions are not yet decided, and will be dependant on the decision at current stage. Considering this dependency, one possible approximated control strategy can be the one that assumes no control actions will take place in later stages, i.e., $\mathbf{u}_{k+i} = 0,\ 1 < i < N-k$.

This approximation provides a sub-optimal controller whose efficiency can be demonstrated in later sections, and the controller serves as an ideal alternative for a global optimal one. This assumption can be formulized as a constraint to the objective function presented in (9), and lead to

$$\begin{aligned} J_k^* = \min_{\mathbf{u}_k} J_k = \min_{\mathbf{u}_k} E\left[\hat{\mathbf{y}}_{N/k}^T \mathbf{Q}_N \hat{\mathbf{y}}_{N/k} + \mathbf{u}_k^T \mathbf{R}_k \mathbf{u}_k\right] \\ s.t.\ \mathbf{u}_{k+i} = 0,\ 1 < i < N-k. \end{aligned} \quad (10)$$

Thus the optimization index (10) considers the variation impact on final product as well as the constraints on large control actions.

3.3.2 Control Law Derivation

To optimize the control objective, the following assumptions are necessary:

A1. As stated previously, $\mathbf{u}_{k+i} = 0,\ i = 1, \ldots N-k$, for stage k;

A2. The uncertainty matrix is assumed to be zero-mean, and the covariance between two uncertainty matrices is assumed to be known, e.g., $E\left[\tilde{\mathbf{B}}_k\right] = \mathbf{0}$, $E\left[\tilde{\mathbf{B}}_k^T \tilde{\mathbf{B}}_k\right] = \Sigma_{\tilde{\mathbf{B}}_k}$, etc. The latter covariance can be obtained either from geometric relationship derivation, or from Monte Carlo simulation of the process errors. The expectation of higher order interactions are assumed to be ignorable, i.e., **0**;

A3. The expectations of un-modeled system errors and measurement noises are **0**, i.e. $E\left[\mathbf{w}_i\right] = \mathbf{0}$ and $E\left[\mathbf{v}_i\right] = \mathbf{0}$, and their variances are know.

Under the above three assumptions, the optimal $\mathbf{u}_k$ that minimizes the index $J_k = E\left[\hat{\mathbf{y}}_{N/k}^T \mathbf{Q}_N \hat{\mathbf{y}}_{N/k} + \mathbf{u}_k^T \mathbf{R}_k \mathbf{u}_k\right]$, can be obtained by solving $\frac{dJ_k}{d\mathbf{u}_k} = 0$. The solution to this optimization problem is

$$\begin{aligned} \mathbf{u}_k = &-\left(\boldsymbol{\Gamma}_k^T \mathbf{Q}_N \boldsymbol{\Gamma}_k + E\left[\boldsymbol{\Gamma}_k^T \mathbf{Q}_N \tilde{\boldsymbol{\Gamma}}_k\right] + E\left[\tilde{\boldsymbol{\Gamma}}_k^T \mathbf{Q}_N \boldsymbol{\Gamma}_k\right] + E\left[\tilde{\boldsymbol{\Gamma}}_k^T \mathbf{Q}_N \tilde{\boldsymbol{\Gamma}}_k\right] + \mathbf{R}_k\right)^{-1} \\ &\times\left\{\sum_{i=r+1}^{k-1}\left(\boldsymbol{\Gamma}_k^T \mathbf{Q}_N \boldsymbol{\Gamma}_i + E\left[\tilde{\boldsymbol{\Gamma}}_k^T \mathbf{Q}_N \boldsymbol{\Gamma}_i\right] + E\left[\boldsymbol{\Gamma}_k^T \mathbf{Q}_N \tilde{\boldsymbol{\Gamma}}_i\right] + E\left[\tilde{\boldsymbol{\Gamma}}_k^T \mathbf{Q}_N \tilde{\boldsymbol{\Gamma}}_i\right]\right)\mathbf{u}_i \right. \\ &\left. +\left(\boldsymbol{\Gamma}_k^T \mathbf{Q}_N \boldsymbol{\Psi}_r + E\left[\tilde{\boldsymbol{\Gamma}}_k^T \mathbf{Q}_N \boldsymbol{\Psi}_r\right] + E\left[\boldsymbol{\Gamma}_k^T \mathbf{Q}_N \tilde{\boldsymbol{\Psi}}_r\right] + E\left[\tilde{\boldsymbol{\Gamma}}_k^T \mathbf{Q}_N \tilde{\boldsymbol{\Psi}}_r\right]\right)\hat{\mathbf{x}}_r\right\}. \end{aligned} \quad (11)$$

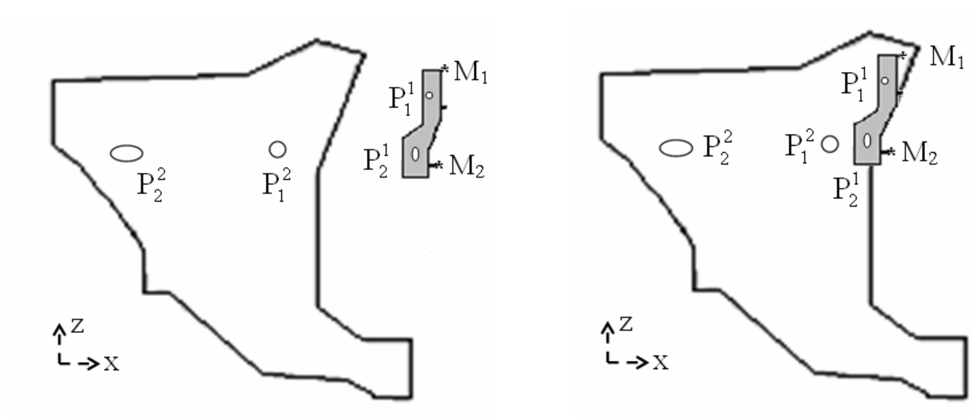
The detailed derivation procedure is presented in Appendix 1. The minimum is guaranteed by the positive-definite second order derivative. If model uncertainty is not considered, then model uncertainty terms can be ignored, and equation (11) will converge to $\mathbf{u}_k = -\left(\mathbf{\Gamma}_k^T \mathbf{Q}_N \mathbf{\Gamma}_k + \mathbf{R}_k\right)^{-1} \times \left\{\sum_{i=r+1}^{k-1} \left(\mathbf{\Gamma}_k^T \mathbf{Q}_N \mathbf{\Gamma}_i\right)\mathbf{u}_i + \mathbf{\Gamma}_k^T \mathbf{Q}_N \mathbf{\Psi}_r \hat{\mathbf{x}}_r\right\}$, i.e., the result derived using traditional optimal control, without considering model uncertainty.

3.4 Case Study

In this section, a real production case is studied to illustrate the effectiveness of the control strategy developed above.

3.4.1 Product and Process Description

Figure 3-4 shows an assembly process that joins two parts, a hinge pillar inner panel and a bracket.



(a) Contour of the parts before welding (b) Parts after welding

Figure 3-4 Hinge pillar inner panel and bracket

Figure 3-4 (a) shows the individual parts before welding. In this contour plot, the part on the left-hand-side is the hinge pillar, which is denoted as Panel (2) in Table 3-1. The smaller part on the right-hand-side of Figure 3-4 (b) is the bracket, denoted as Bracket (1) in Table 3-1. Figure 3-4 (b) shows the relative position of the two parts after the welding operation, with the bracket contour highlighted. The annotation P_i^j in these plots represents the locating hole/slot of each part. The superscript j indicates the index of the part (either 1 or 2), and subscript i indicates whether it is a 4-way hole (1) or a 2-way slot (2). Both of the two measurement points (MLPs) are located on bracket (M_1 and M_2), and are marked using asterisks.

Table 3-1 shows the designated global coordinates of each locating pin/holes in a tabular form. Y coordinates are not needed under 2-D rigid part assumption, and is not presented in the table. The global coordinates of the two measurement points are shown in Table 3-2.

Table 3-1 Coordinates of fixture locators (PLPs) (Unit: mm)

Part Name	4-way Pin (1)		2-way Pin (2)	
	x	Z	X	Z
Bracket (1)	2250	955	2250	905
Panel (2)	2200	900	2000	900

Table 3-2 Coordinates of measurement points (MLPs) (Unit: mm)

MLP	M_1	M_2
(x, z)	(2284.54, 991.73)	(2280, 850)

The corresponding SoV model can be derived as shown in Appendix 2.

3.4.2 Control Performance

The controller can be obtained by plugging process variables into equation (11), where $\mathbf{\Gamma}_k = \mathbf{C}_N \mathbf{\Phi}_{N,k} \mathbf{B}_k$, and $\mathbf{R}_k$ and $\mathbf{Q}_k$ follow the common definition in control area, i.e. $\mathbf{Q}_k = \lambda \mathbf{I}$ and $\mathbf{R}_k = \mathbf{I} - \mathbf{Q}_k$, where $\mathbf{I}$ is an identity matrix of appropriate dimension. $\hat{\mathbf{x}}_r$ is the observed system state in intermediate measurement stage r. The analytic solution is solvable, but this process is tedious and thus the detailed format will be omitted here.

Assume the standard deviations of the fabrication error of all parts are equally 1.5mm. Also assume the measurement noise level, i.e., each element of $\sigma_{\mathbf{w}_i}$, is 0.05mm, and that un-modeled noise level $\sigma_{\mathbf{v}_2}$ is as small as 0.0005mm, which is smaller than usual un-modeled error assumptions, because considering modeling error would significant reduce that uncertainty.

The simulation results of 100 car samples are shown in Table 3-3. The first number in each cell of the table is the mean of performances J_2 under each situation, and the second number in apprentice is the corresponding standard deviation. From this table, the application of optimal control strategy not only improves the mean by 37.23% comparing to a controller without considering modeling error, but also achieves a reduction in variations of all KPCs.

Table 3-3 Simulated mean and std for J_2 under different control (Units: mm)

	M1.x	M1.z	M2.x	M2.z
No control	0.18 (4.97)	-0.01 (4.59)	-0.08 (2.20)	0.00 (4.42)
Control without modeling error	-0.03 (0.18)	0.02 (0.18)	0.01 (0.10)	0.03 (0.16)
Control with modeling error	-0.01 (0.13)	0.02 (0.14)	0.02 (0.09)	0.02 (0.13)

Figure 3-5 shows the mean of controller performances J_2 over different noise level. The horizontal axis is the level of part fabrication uncertainty in terms of standard deviation, and the vertical axis is the final part quality measured by J_2. J_k^* and J_2 represent the performance of controller that considering and without considering modeling error respectively.

The proposed controller outperforms the one without considering modeling uncertainty, especially under conditions when part fabrication processes have high variations. This indicates that the proposed controller is more robust under modeling and observation uncertainties within certain ranges. This result also shows the possibility of releasing the tolerances allocated to suppliers when assembly process is incorporated with this control strategy. The above simulated case study shows the efficiency of the proposed controller in MMP.

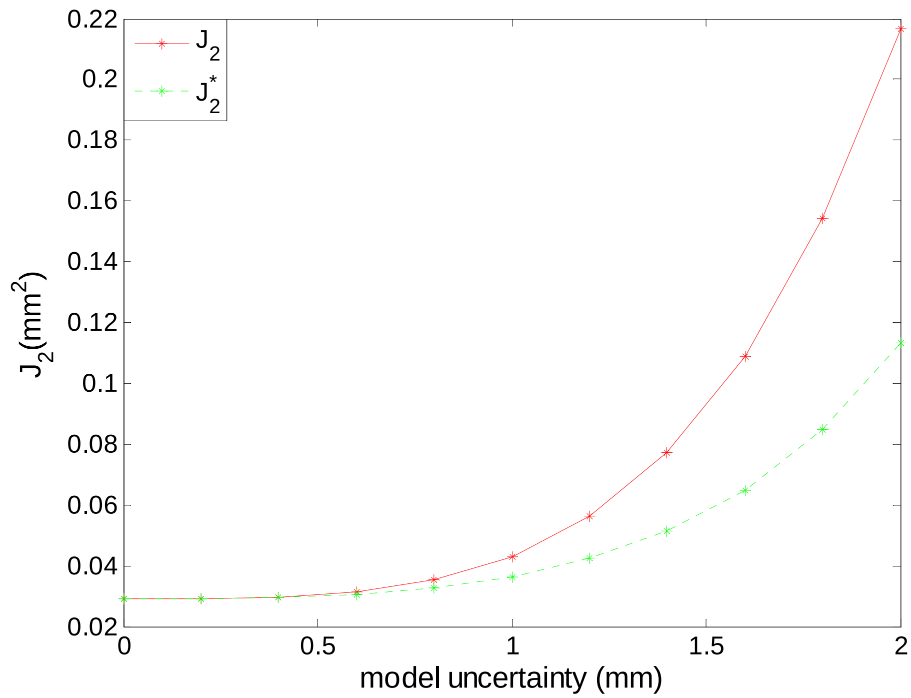


Figure 3-5 Control performances of different controllers

3.5 Conclusion

In MMP, random part fabrication and process induced errors will introduce uncertainty to the variation propagation models that are derived from nominal product/process design data. A control methodology without considering the model uncertainty will result in a degradation of the controller performance in practice. A controller that is robust to the noises would be preferable under such cases of model uncertainties. This paper first derives a mathematical MMP variation propagation model with model uncertainties considered. Based on the proposed model, a predictive control strategy is derived, with consideration of model uncertainty. The effectiveness of the proposed control strategy has been demonstrated through a case study of a simulated assembly processes.

The proposed approach also has limitations when number of stages of a MMP is large. A complete SoV model can be derived from process design blue print, however the analytic solution can become too complicated due to the high order interaction terms of variations of model parameters. Numeric approximations should be introduced under such conditions, which can be the future work in extending the application of the proposed methodology.

Acknowledgements

This research was supported in part by the General Motors Collaborative Research Lab in Advance Vehicle Manufacturing at The University of Michigan.

Appendix 1: Derivation of (11)

Denoted the prediction of $\mathbf{y}_N$ at stage k as $\hat{\mathbf{y}}_{N/k}$, which can be expended in similar way to (7) as,

$$\hat{\mathbf{y}}_{N|k} = \sum_{i=r+1}^{N} \widehat{\mathbf{\Gamma}}_i \mathbf{u}_i + \widehat{\mathbf{\Psi}}_r \hat{\mathbf{x}}_r , \tag{12}$$

where $\widehat{\mathbf{\Gamma}}_i = \mathbf{C}_N \widehat{\mathbf{A}}_{N-1} \ldots \widehat{\mathbf{A}}_i \widehat{\mathbf{B}}_i = \mathbf{\Gamma}_i + \tilde{\mathbf{\Gamma}}_i$, $\widehat{\mathbf{\Psi}}_i = \mathbf{C}_N \widehat{\mathbf{A}}_{N-1} \ldots \widehat{\mathbf{A}}_i = \mathbf{\Psi}_i + \tilde{\mathbf{\Psi}}_i$, and Stage r is the intermediate measurement stage. It can be easily observed that, $\tilde{\mathbf{\Gamma}}_i$ is a function of all $\mathbf{A}_j$'s, $\tilde{\mathbf{A}}_j$'s, $\mathbf{B}_j$'s and $\tilde{\mathbf{B}}_j$, $j = i, \ldots, N-1$. Similarly, $\widehat{\mathbf{\Psi}}_i = \mathbf{C}_N \widehat{\mathbf{A}}_{N-1} \ldots \widehat{\mathbf{A}}_i = \mathbf{\Psi}_i + \tilde{\mathbf{\Psi}}_i$.

Equation (12) can then be written as:

$$\begin{aligned} \mathbf{y}_N &= \sum_{i=r+1}^{N} \left(\mathbf{\Gamma}_i + \tilde{\mathbf{\Gamma}}_i \right) \mathbf{u}_i + \left(\mathbf{\Psi}_r + \tilde{\mathbf{\Psi}}_r \right) \hat{\mathbf{x}}_r + \sum_{i=r+1}^{N} \left(\mathbf{\Psi}_i + \tilde{\mathbf{\Psi}}_i \right) \mathbf{w}_i + \mathbf{v}_N \\ &= \left(\mathbf{\Gamma}_k + \tilde{\mathbf{\Gamma}}_k \right) \mathbf{u}_k + \sum_{i=r+1}^{k-1} \left(\mathbf{\Gamma}_i + \tilde{\mathbf{\Gamma}}_i \right) \mathbf{u}_i + \left(\mathbf{\Psi}_r + \tilde{\mathbf{\Psi}}_r \right) \hat{\mathbf{x}}_r + \sum_{i=r+1}^{N} \left(\mathbf{\Psi}_i + \tilde{\mathbf{\Psi}}_i \right) \mathbf{w}_i + \mathbf{v}_N \end{aligned}$$

Plug the above equation into the expression of J_k,

$$
\begin{aligned}
J_k &= E\left[\mathbf{y}_N^T\mathbf{Q}_N\mathbf{y}_N + \mathbf{u}_k^T\mathbf{R}_k\mathbf{u}_k\right] \\
&= E\Big[\mathbf{u}_k^T\mathbf{R}_k\mathbf{u}_k + \mathbf{u}_k^T\left(\boldsymbol{\Gamma}_k+\tilde{\boldsymbol{\Gamma}}_k\right)^T\mathbf{Q}_N\left(\boldsymbol{\Gamma}_k+\tilde{\boldsymbol{\Gamma}}_k\right)\mathbf{u}_k + 2\mathbf{u}_k^T\left(\boldsymbol{\Gamma}_k+\tilde{\boldsymbol{\Gamma}}_k\right)^T\mathbf{Q}_N \\
&\quad \left(\sum_{i=r+1}^{k-1}\left(\boldsymbol{\Gamma}_i+\tilde{\boldsymbol{\Gamma}}_i\right)\mathbf{u}_i + \left(\boldsymbol{\Psi}_r+\tilde{\boldsymbol{\Psi}}_r\right)\hat{\mathbf{x}}_r + \sum_{i=r+1}^{N}\left(\boldsymbol{\Psi}_i+\tilde{\boldsymbol{\Psi}}_i\right)\mathbf{w}_i + \mathbf{v}_N\right) \\
&\quad +\left(\sum_{i=r+1}^{k-1}\left(\boldsymbol{\Gamma}_i+\tilde{\boldsymbol{\Gamma}}_i\right)\mathbf{u}_i + \left(\boldsymbol{\Psi}_r+\tilde{\boldsymbol{\Psi}}_r\right)\hat{\mathbf{x}}_r + \sum_{i=r+1}^{N}\left(\boldsymbol{\Psi}_i+\tilde{\boldsymbol{\Psi}}_i\right)\mathbf{w}_i + \mathbf{v}_N\right)^T \\
&\quad \times\mathbf{Q}_N\left(\sum_{i=r+1}^{k-1}\left(\boldsymbol{\Gamma}_i+\tilde{\boldsymbol{\Gamma}}_i\right)\mathbf{u}_i + \left(\boldsymbol{\Psi}_r+\tilde{\boldsymbol{\Psi}}_r\right)\hat{\mathbf{x}}_r + \sum_{i=r+1}^{N}\left(\boldsymbol{\Psi}_i+\tilde{\boldsymbol{\Psi}}_i\right)\mathbf{w}_i + \mathbf{v}_N\right)\Big]
\end{aligned}
$$

The derivation with respect to $\mathbf{u}_k$ is:

$$
\begin{aligned}
\frac{dJ_k}{d\mathbf{u}_k} &= E\Big[2\mathbf{R}_k\mathbf{u}_k + 2\left(\boldsymbol{\Gamma}_k+\tilde{\boldsymbol{\Gamma}}_k\right)^T\mathbf{Q}_N\left(\boldsymbol{\Gamma}_k+\tilde{\boldsymbol{\Gamma}}_k\right)\mathbf{u}_k + 2\left(\boldsymbol{\Gamma}_k+\tilde{\boldsymbol{\Gamma}}_k\right)^T \\
&\quad \times\mathbf{Q}_N\left(\sum_{i=r+1}^{k-1}\left(\boldsymbol{\Gamma}_i+\tilde{\boldsymbol{\Gamma}}_i\right)\mathbf{u}_i + \left(\boldsymbol{\Psi}_r+\tilde{\boldsymbol{\Psi}}_r\right)\hat{\mathbf{x}}_r + \sum_{i=r+1}^{N}\left(\boldsymbol{\Psi}_i+\tilde{\boldsymbol{\Psi}}_i\right)\mathbf{w}_i + \mathbf{v}_N\right)\Big] \\
&= 2E\Big[\left(\mathbf{R}_k + \left(\boldsymbol{\Gamma}_k+\tilde{\boldsymbol{\Gamma}}_k\right)^T\mathbf{Q}_N\left(\boldsymbol{\Gamma}_k+\tilde{\boldsymbol{\Gamma}}_k\right)\right)\mathbf{u}_k \\
&\quad +\left(\boldsymbol{\Gamma}_k+\tilde{\boldsymbol{\Gamma}}_k\right)^T\mathbf{Q}_N\left(\sum_{i=r+1}^{k-1}\left(\boldsymbol{\Gamma}_i+\tilde{\boldsymbol{\Gamma}}_i\right)\mathbf{u}_i\right) + \left(\boldsymbol{\Gamma}_k+\tilde{\boldsymbol{\Gamma}}_k\right)^T\mathbf{Q}_N\left(\boldsymbol{\Psi}_r+\tilde{\boldsymbol{\Psi}}_r\right)\hat{\mathbf{x}}_r\Big]
\end{aligned}
$$

Each of the above 3 terms can be calculated as:

$$
\begin{aligned}
&E\left[\left(\left(\boldsymbol{\Gamma}_k+\tilde{\boldsymbol{\Gamma}}_k\right)^T\mathbf{Q}_N\left(\boldsymbol{\Gamma}_k+\tilde{\boldsymbol{\Gamma}}_k\right)+\mathbf{R}_k\right)\mathbf{u}_k\right] \\
&= E\left[\boldsymbol{\Gamma}_k^T\mathbf{Q}\boldsymbol{\Gamma}_k + \boldsymbol{\Gamma}_k^T\mathbf{Q}\tilde{\boldsymbol{\Gamma}}_k + \tilde{\boldsymbol{\Gamma}}_k^T\mathbf{Q}\boldsymbol{\Gamma}_k + \tilde{\boldsymbol{\Gamma}}_k^T\mathbf{Q}\tilde{\boldsymbol{\Gamma}}_k + \mathbf{R}_k\right]\mathbf{u}_k \\
&= \left(\boldsymbol{\Gamma}_k^T\mathbf{Q}\boldsymbol{\Gamma}_k + E\left[\boldsymbol{\Gamma}_k^T\mathbf{Q}\tilde{\boldsymbol{\Gamma}}_k\right] + E\left[\tilde{\boldsymbol{\Gamma}}_k^T\mathbf{Q}\boldsymbol{\Gamma}_k\right] + E\left[\tilde{\boldsymbol{\Gamma}}_k^T\mathbf{Q}\tilde{\boldsymbol{\Gamma}}_k\right] + \mathbf{R}_k\right)\mathbf{u}_k
\end{aligned}
$$

$$
\begin{aligned}
&E\left[\left(\boldsymbol{\Gamma}_k+\tilde{\boldsymbol{\Gamma}}_k\right)^T\mathbf{Q}_N\left(\sum_{i=r+1}^{k-1}\left(\boldsymbol{\Gamma}_i+\tilde{\boldsymbol{\Gamma}}_i\right)\mathbf{u}_i\right)\right] \\
&= \sum_{i=r+1}^{k-1}E\left[\boldsymbol{\Gamma}_k^T\mathbf{Q}_N\boldsymbol{\Gamma}_i + \boldsymbol{\Gamma}_k^T\mathbf{Q}_N\tilde{\boldsymbol{\Gamma}}_i + \tilde{\boldsymbol{\Gamma}}_k^T\mathbf{Q}_N\boldsymbol{\Gamma}_i + \tilde{\boldsymbol{\Gamma}}_k^T\mathbf{Q}_N\tilde{\boldsymbol{\Gamma}}_i\right]\mathbf{u}_i \\
&= \sum_{i=r+1}^{k-1}\left(\boldsymbol{\Gamma}_k^T\mathbf{Q}_N\boldsymbol{\Gamma}_i + E\left[\boldsymbol{\Gamma}_k^T\mathbf{Q}_N\tilde{\boldsymbol{\Gamma}}_i\right] + E\left[\tilde{\boldsymbol{\Gamma}}_k^T\mathbf{Q}_N\boldsymbol{\Gamma}_i\right] + E\left[\tilde{\boldsymbol{\Gamma}}_k^T\mathbf{Q}_N\tilde{\boldsymbol{\Gamma}}_i\right]\right)\mathbf{u}_i
\end{aligned}
$$

$$
\begin{aligned}
&E\left[\left(\boldsymbol{\Gamma}_k+\tilde{\boldsymbol{\Gamma}}_k\right)^T\mathbf{Q}_N\left(\boldsymbol{\Psi}_r+\tilde{\boldsymbol{\Psi}}_r\right)\hat{\mathbf{x}}_r\right] \\
&= E\left[\boldsymbol{\Gamma}_k^T\mathbf{Q}_N\boldsymbol{\Psi}_r + \boldsymbol{\Gamma}_k^T\mathbf{Q}_N\tilde{\boldsymbol{\Psi}}_r + \tilde{\boldsymbol{\Gamma}}_k^T\mathbf{Q}_N\boldsymbol{\Psi}_r + \tilde{\boldsymbol{\Gamma}}_k^T\mathbf{Q}_N\tilde{\boldsymbol{\Psi}}_r\right]\hat{\mathbf{x}}_r \\
&= \left(\boldsymbol{\Gamma}_k^T\mathbf{Q}_N\boldsymbol{\Psi}_r + E\left[\boldsymbol{\Gamma}_k^T\mathbf{Q}_N\tilde{\boldsymbol{\Psi}}_r\right] + E\left[\tilde{\boldsymbol{\Gamma}}_k^T\mathbf{Q}_N\boldsymbol{\Psi}_r\right] + E\left[\tilde{\boldsymbol{\Gamma}}_k^T\mathbf{Q}_N\tilde{\boldsymbol{\Psi}}_r\right]\right)\hat{\mathbf{x}}_r
\end{aligned}
$$

Set the derivative to **0**, thus the sub-optimal $\mathbf{u}_k$ can be obtained as:

$$
\begin{aligned}
\mathbf{u}_k = &-\left(\mathbf{\Gamma}_k^T\mathbf{Q}_N\mathbf{\Gamma}_k + E\left[\mathbf{\Gamma}_k^T\mathbf{Q}_N\tilde{\mathbf{\Gamma}}_k\right] + E\left[\tilde{\mathbf{\Gamma}}_k^T\mathbf{Q}_N\mathbf{\Gamma}_k\right] + E\left[\tilde{\mathbf{\Gamma}}_k^T\mathbf{Q}_N\tilde{\mathbf{\Gamma}}_k\right] + \mathbf{R}_k\right)^{-1} \\
&\times\left(\sum_{i=r+1}^{k-1}\left(\mathbf{\Gamma}_k^T\mathbf{Q}_N\mathbf{\Gamma}_i + E\left[\tilde{\mathbf{\Gamma}}_k^T\mathbf{Q}_N\mathbf{\Gamma}_i\right] + E\left[\mathbf{\Gamma}_k^T\mathbf{Q}_N\tilde{\mathbf{\Gamma}}_i\right] + E\left[\tilde{\mathbf{\Gamma}}_k^T\mathbf{Q}_N\tilde{\mathbf{\Gamma}}_i\right]\right)\mathbf{u}_i\right. \\
&\left.+\left(\mathbf{\Gamma}_k^T\mathbf{Q}_N\mathbf{\Psi}_r + E\left[\tilde{\mathbf{\Gamma}}_k^T\mathbf{Q}_N\mathbf{\Psi}_r\right] + E\left[\mathbf{\Gamma}_k^T\mathbf{Q}_N\tilde{\mathbf{\Psi}}_r\right] + E\left[\tilde{\mathbf{\Gamma}}_k^T\mathbf{Q}_N\tilde{\mathbf{\Psi}}_r\right]\right)\hat{\mathbf{x}}_r\right)
\end{aligned}
$$

The above expression takes a common format of an optimal control law.

The second derivate of J_k with respect to $\mathbf{u}_k$ is $E\left[\left(\mathbf{\Gamma}_k + \tilde{\mathbf{\Gamma}}_k\right)^T \mathbf{Q}\left(\mathbf{\Gamma}_k + \tilde{\mathbf{\Gamma}}_k\right)\right] + \mathbf{R}_k$, which is positive definite, which grantees that it is the optimal minimal point.

Appendix 2: Corresponding state transition matrices for case study

$$
\mathbf{A}_1 = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 55 \\ 0 & 1 & 0 & 0 & -1 & -50 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},
$$

$$
\mathbf{B}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.0200 & 0.0004 & 0.0200 & -0.0004 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0050 & 0 & -0.0050 \end{bmatrix},
$$

$$
\mathbf{B}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0050 & 0 & -0.0050 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0050 & 0 & -0.0050 \end{bmatrix}, \text{ and}
$$

$$\mathbf{C}_2 = \begin{bmatrix} 1 & 0 & -91.37 & 0 & 0 & 0 \\ 0 & 1 & 84.54 & 0 & 0 & 0 \\ 1 & 0 & 50 & 0 & 0 & 0 \\ 0 & 1 & 80 & 0 & 0 & 0 \end{bmatrix}.$$

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CHAPTER 4

EXPERIMENTAL VALIDATION OF A STREAM OF VARIATION MODEL AND PROCESS CONTROLLABILITY IN A PRODUCTION ENVIRONMENT

Abstract

Stream of Variation models, which capture variation propagation in multistage manufacturing processes, have been thoroughly studied in various applications, such as process modeling, fault identification, resource allocation. This paper presents experiments that test the validity of a Stream of Variation model of an automotive body assembly process, as well as testing the controllability of online part-by-part feed-forward control. The experiments were carried out in an automotive assembly plant that was equipped with Programmable Tooling and an in-line measurement system. The results support the validity of the SoV model and indicate the feasibility of a part-by-part compensation scheme in a real-life manufacturing environment.

4.1 Introduction

In a multistage manufacturing process (MMP), products are manufactured through multiple operations or stages. The product quality is typically reflected by the variations of Key Product Characteristics (KPCs). During production, the KPCs of a subassembly will deviate from nominal position due to part variations and process variations, such as fixture error and welding gun error, at each stage. These variations will be carried to the next stage and further interact with the assembly process, and thus these variations can be propagated to the downstream stages and accumulate to the final product. If the final accumulation is large enough, the production process will have quality problems.

The propagation of variation in a MMP raises a challenge for feed-forward control. This is because when doing optimization for a MMP at each intermediate stage, the target to be optimized should be the product quality at the end of production line, rather than at the current stage. Minimizing the variation of subassembly at the current stage alone may not lead to the best final product quality. To achieve effective online feed-forward control in a MMP, three components are necessary. These enablers include a model that captures the variation flow, i.e., the Stream of Variation model, real-time sensing technologies to measure the variation, and Programmable Tooling (PT) to perform control actions to suppress the variation.

SoV modeling has been proposed to describe the propagation of variation in a MMP, through exploring the relationship of variation sources and geometric information of each operating station based on design information, especially product and process geometry (Jin and Shi, 1999). It has then been utilized as the mathematical basis in various applications such as process modeling, design evaluation, diagnosis, tolerance synthesis, active control, and other areas (Shi, 2006).

Theoretical studies of SoV models have been thoroughly investigated, however, validation of the model has been carried out through numerical simulation and calibration with commercial software (Ding *et al.*, 2000), or through experimental validation in a research lab (Zhou *et al.*, 2004). To our knowledge, no accounts of validation experiments in a real production environment have appeared in the literature, nor has any controllability study for the application of active dimensional control been published. To fill this gap, this paper will present experiments testing the validity of a SoV model and studying the controllability of the process. Thus the first task of this paper is the validation of a SoV model in a real-life manufacturing environment.

The second task of this paper is to test the feasibility of active dimensional control in a MMP. Programmable Tooling (PT) was initially installed for production flexibility in the process studied. It mounts the subassembly/part onto locators and clamps that can be adjusted precisely according to computer commands to produce several different products on the same equipment. This has the beneficial side effect that it also provides the capability of making adjustments in real-time, thus, in principle, enabling variation reduction by making part-by-part adjustments to counteract measured deviations.

In-process active control for final product variation reduction is a newly emerging application area of SoV models, and simulations that show promising results have been reported in the literature. In MMP, the possible control actions form a high dimensional solution space because of the large numbers of stations and assemblies involved. To avoid unnecessary complication, a simpler sub-optimal controller with good-enough performance can be used as an alternative to the global optimal one obtained through Dynamic Programming. In the literature, a controller for improving final product quality was first proposed for a multistage machining process (Djurdjanovic and Zhu, 2005), and then for a multistage assembly process (Djurdjanovic and Ni, 2006), taking the path of deviation minimization, rather than variation reduction. This strategy brings the mean of KPC deviations as close as possible to 0, but will not necessarily reduce the variation of these deviations. A feed-forward controller that aims to minimize final part KPC variation was developed later (Izquierdo *et al.*, 2007), and a controller with model uncertainty is also proposed to consider the part fabrication and process induced errors (Zhong *et al.*, 2008).

In this paper, we study the relationship between control actions in an upstream experimental station and measurements downstream at the end-of-line (EOL). The objective is to verify the model predictions as well as the effectiveness of the control action. The paper is organized as follows. Section 4.2 introduces SoV modeling, and Section 4.3 describes the characteristics of the selected station, parts and experimental setup, and presents the derived SoV model for this particular experiment. Section 4.4 analyzes the results and Section 4.5 gives the conclusions.

4.2 Stream of Variation Modeling

This section presents the SoV model, which is used to describe the impact that the corrections have on the final product quality (Jin and Shi, 1999; Shi, 2006).

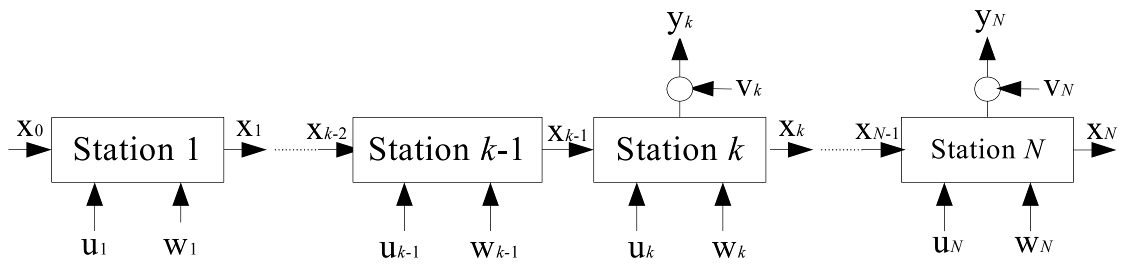


Figure 4-1 Multistage manufacturing process

Figure 4-1 illustrates the stream of variation in a multistage manufacturing process. Parts enter the production line from Station 1 with initial fabrication errors, $\mathbf{x}_0$. In Station 1, the part control action $\mathbf{u}_1$ is first applied through PT, while other un-modeled process errors, $\mathbf{w}_1$, will add to the variation of the parts. The designed operation at Station 1 then takes place and the state of the subassembly changes to $\mathbf{x}_1$. The subassembly is then transferred to the next station, and the variations propagate and accumulate similarly as more parts/subassemblies are joined together, until the finished assembly exits the production line at the final Station *N*. The KPC will be measured at the final Station *N* as well as some intermediate stations such as Station *k*. The measurement $\mathbf{y}_k$ is obtained with sensor errors $\mathbf{v}_k$.

This process can be built to mathematically represent the variation propagation in MMP (Jin and Shi, 1999),

$$\mathbf{x}_k = \mathbf{A}_{k-1} \cdot \mathbf{x}_{k-1} + \mathbf{B}_k \cdot \mathbf{u}_k + \mathbf{w}_k \tag{1}$$

$$\mathbf{y}_k = \mathbf{C}_k \cdot \mathbf{x}_k + \mathbf{v}_k \tag{2}$$

where equation (1) is a state equation, with $\mathbf{x}_k \in \Re^n$ representing the state of the system (part deviations from the nominal) in stage *k*. Variables $\mathbf{u}_k \in \Re^p$ and $\mathbf{w}_k \in \Re^n$ represent the fixture adjustments and the disturbances respectively. To facilitate the application of in real life, the dimension of system matrices are kept unchanged throughout the process, and thus the partition of state and control vectors that corresponding to parts that not yet emerged at station *k* are set to **0**. Matrix $\mathbf{A}_k \in \Re^{n \times n}$ stands for the reorientation matrix, which relates the error transferred between two adjacent stages (*k*-1 and *k*). The effects of fixture deviations on the state of the system are determined by matrix $\mathbf{B}_k \in \Re^{n \times p}$. Equation (2), the observation equation, is used to determine the deviations of the measurement points $\mathbf{y}_k \in \Re^m$, which usually correspond to the KPCs of the product. They are obtained from the state through the observation matrix $\mathbf{C}_k \in \Re^{m \times n}$ adding measurement noise $\mathbf{v}_k \in \Re^m$.

By defining the state transition matrix $\mathbf{\Phi}_{k,i}$ to describe the deviation transmission between stages *i* and *k* (Ding *et al.*, 2000), where $\mathbf{\Phi}_{k,i} \equiv \mathbf{A}_{k-1}\mathbf{A}_{k-2}\cdots\mathbf{A}_{i+1}\mathbf{A}_i$, k>i; $\mathbf{\Phi}_{i,i} \equiv \mathbf{I}$ (**I** is an identity matrix), the observation equation can be written as,

$$\mathbf{y}_N = \mathbf{\Gamma}_0\mathbf{x}_0 + \sum_{k=1}^{N}\mathbf{\Gamma}_k\mathbf{u}_k + \sum_{k=1}^{N}\mathbf{\Psi}_k\mathbf{w}_k + \mathbf{v}_N, \quad (3)$$

where $\mathbf{\Gamma}_k = \mathbf{C}_k\mathbf{\Phi}_{N,k}\mathbf{B}_k$, $\mathbf{\Gamma}_0 = \mathbf{C}_N\mathbf{\Phi}_{N,0}$ and $\mathbf{\Psi}_k = \mathbf{C}_k\mathbf{\Phi}_{N,k}$.

4.3 Experimental Test-bed

4.3.1 Description of the Selected Station and Parts

In this study, an assembly station was selected at an automotive assembly plant, where one bracket is joined to an inner panel. The final station is an existing OCMM (Optical Coordinate Measuring Machine) inspection station at the end of the underbody line (EOL).

The station selected is a secondary assembly line that is connected to the main assembly at an intermediate assembly station, and the production quality, represented by the variation of the KPCs of the final product, is measured at the end of the production line. Figure 4-2 presents a schematic view of the production line. The selected station is marked as Station *s* and circled in bold, and enters the main production line at station *k*. Station *N* is a measurement station at end-of-line.

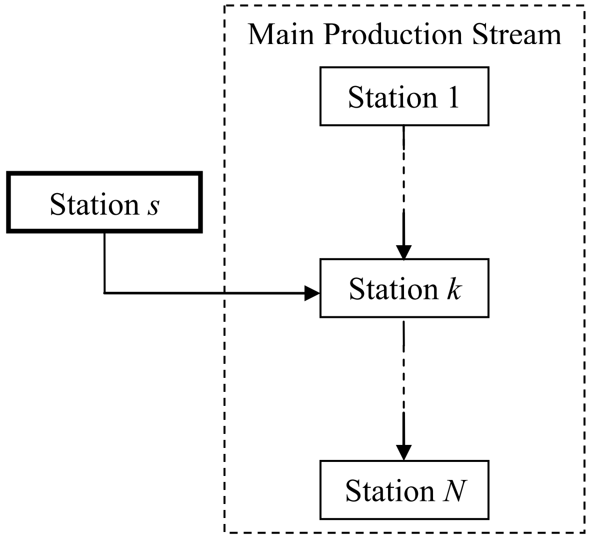
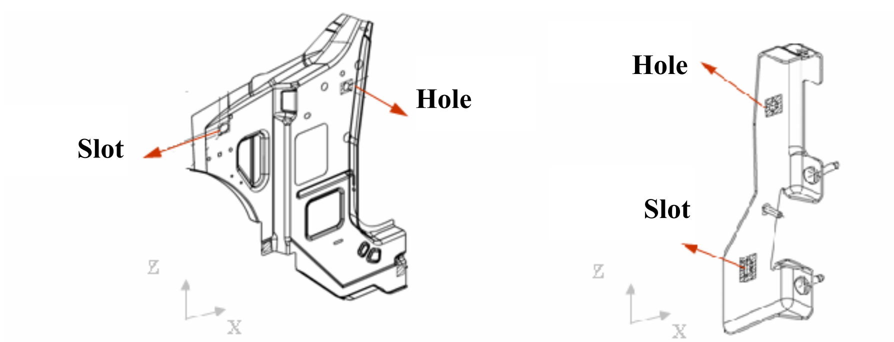


Figure 4-2 Schematic of the assembly flow

Figure 4-3 (a) and (b) present overviews of the parts with locators, and Table 4-1 gives their locations in a global coordinate frame, called the *body coordinate frame*. This frame has its origin at a fixed point in the car body and remains unchanged all through the production. Figure 4-4 shows the position of this subassembly on the car underbody.



(a) Pillar inner panel (b) Bracket

Figure 4-3 View of the parts with locators

Table 4-1 Coordinate of locators (Unit: mm)

Part Name	Hole			Slot		
	X	Y	Z	X	Y	Z
Panel	2236	700.7	1000	1850	734.5	1000
Bracket	2280	700.7	1045	2240	700.7	860

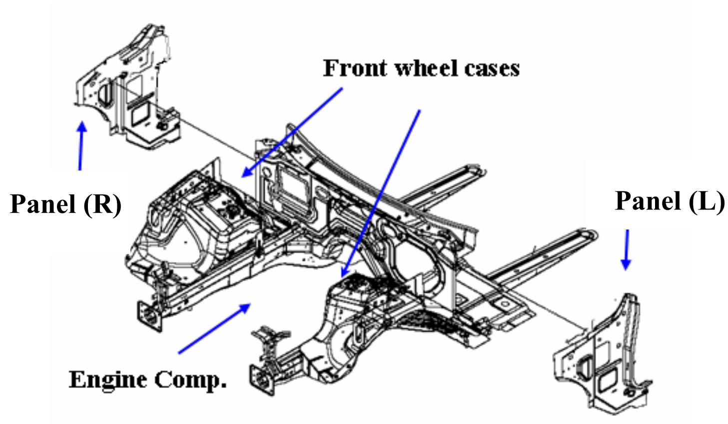


Figure 4-4 Location of the panels in the underbody

In this selected station, the panel is located on fixed fixtures, while the bracket is located by a PT unit, serving as a control actuator. The bracket assembled in the selected station is rigid enough to behave as a rigid body, and the connection between the two parts is slip plane, which means the movement of the bracket is constrained in the flat surface defined by the panel, as illustrated in Figure 4-5 (a). Although SoV models have variations to deal compliant parts (Camelio *et al.*, 2001; Hu and Camelio, 2006) or other types of joints as illustrated in Figure 4-5 (b) and (c) (Liu *et al.*, 2007), the rigid bracket part and its lap joint attachment to the inner panel in the selected station simplifies the model and its validation.

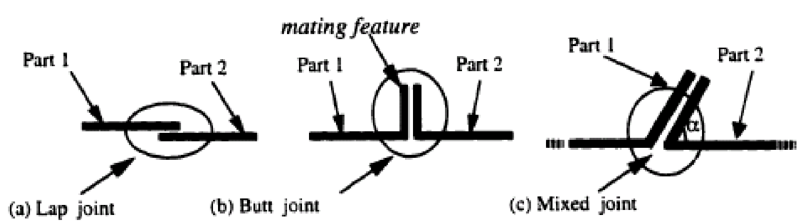


Figure 4-5 Cross sectional view of the joints

The assembly process of interest can be divided in two sequential stations. The first one is the assembly at selected station, where the bracket is joined onto the panel, and the second one is the final measurement process at EOL measurement station.

During the assembly process in selected station, both parts are held at their own locating points by PTs, and its model is presented in Equation (4) (Ding *et al.*, 2000). The initial parts are assumed to be perfect ($\mathbf{x}_0 = \mathbf{0}$), and the fixture variation of the panel and other process disturbances can be considered negligible compared with the shim magnitudes applied on the bracket.

$$\begin{aligned} \mathbf{x}_1 &= \mathbf{A}_0 \cdot \mathbf{x}_0 + \mathbf{B}_1 \cdot \mathbf{u}_1 + \mathbf{w}_1 \\ \mathbf{y}_1 &= \mathbf{C}_1 \cdot \mathbf{x}_1 + \mathbf{v}_1 \end{aligned} \tag{4}$$

In this assembly station with active control, the production procedure is to first load the part, take measurement before welding, then calculate and apply control action, and finally apply welding and move to next station. Since the control action is taken before welding, it will not be effective if welding process introduces too large noise. To study this impact, the part after welding is also measured using the same sensor system. Equation (4) can be further divided into two steps to explain this procedure.

$$\begin{aligned} \mathbf{x}_{11} &= \mathbf{A}_0 \cdot \mathbf{x}_0 + \mathbf{w}_{11} \\ \mathbf{y}_{11} &= \mathbf{C}_1 \cdot \mathbf{x}_{11} + \mathbf{v}_{11} \end{aligned}, \tag{5}$$

and

$$\begin{aligned} \mathbf{x}_1 &= \mathbf{x}_{11} + \mathbf{B}_1 \cdot \mathbf{u}_1 + \mathbf{w}_{12} \\ \mathbf{y}_1 &= \mathbf{C}_1 \cdot \mathbf{x}_1 + \mathbf{v}_{12} \end{aligned}. \tag{6}$$

Equation (5) reflects the positioning of parts, where $\mathbf{x}_{11}$ is the part status before welding and $\mathbf{y}_{11}$ is its observation. Equation (6) captures the variation introduced by control action and welding process, with $\mathbf{u}_1$ representing the control action and $\mathbf{x}_1$ as the part status after welding. $\mathbf{w}_{12}$ includes not only the ordinary process uncertainties, but also the variation introduced by welding process. If this noise term is small, then (5) and (6) can be combined into (4), and the measurement taken before welding is capable to be used as input for control determination.

In the second station, the subassembly is virtually fixed using the locating points of the panel. Equation (7) represents the model of the process in the second station. In this application, part are already joined together and no control action is applied, i.e., $\mathbf{u}_2 = \mathbf{0}$.

$$\begin{aligned} \mathbf{x}_2 &= \mathbf{A}_1 \cdot \mathbf{x}_1 + \mathbf{B}_2 \cdot \mathbf{u}_2 + \mathbf{w}_2 \\ \mathbf{y}_2 &= \mathbf{C}_2 \cdot \mathbf{x}_2 + \mathbf{v}_2 \end{aligned} \tag{7}$$

Here $\mathbf{w}_2$ captures all the process variation introduced by operations between selected station and EOL. The prediction of SoV model at the EOL is $\hat{\mathbf{x}}_2 = \mathbf{A}_1 \cdot \mathbf{x}_1 + \mathbf{B}_2 \cdot \mathbf{u}_2 = \mathbf{A}_1 \cdot (\mathbf{B}_1 \cdot \mathbf{u}_1)$, so if the comparison between $\hat{\mathbf{x}}_2$ and $\mathbf{x}_2$ shows great consistence, this will ensures the correctness of SoV model of the process.

The whole validation experiments will sequentially test, i) the level of sensor noises, $\mathbf{v}_1$ and $\mathbf{v}_2$, to validate if the sensor system is capable for the process; ii) the variation introduced by welding, it can be ignored if the total uncertainty is at the same level of sensor noise; and iii) the model prediction, $\hat{\mathbf{x}}_2$, and measurements at the EOL, $\mathbf{x}_2$. These three steps together validate the SoV modeling and the controllability in the production line.

4.3.2 Measurement Points on Selected Parts

Two OCMM sensors were installed in the selected station for the experiment. These are optical sensors that use a laser beam and a camera to calculate the position of a feature by triangulation. The sensors measure the upper and lower corners of the bracket, and provide four measurements in total, namely, the X and Z coordinates of two corners of the bracket. These measurements will be referred to as X_{Upper}, Z_{Upper}, X_{Lower}, Z_{Lower}, respectively in later sections. Figure 4-6 shows a view of the sensors in the selected station.

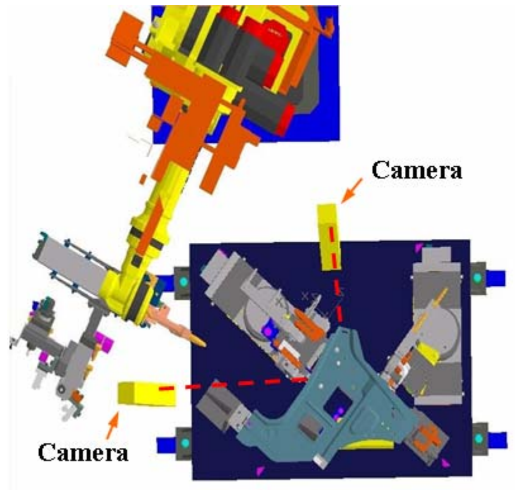
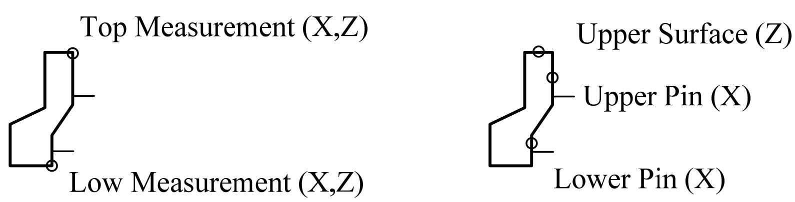


Figure 4-6 Upper view of the station with cameras

Different points are measured at the EOL by OCMM and CMM systems. Figure 4-7 presents the actual location of the measurement points on the part, both in the selected station and at EOL, while only 3 directions are of interest at EOL by OCMM and CMM. Table 4-2 and Table 4-3 then present these measurement locations in body coordinates. For the convenience of comparison, this paper will convert all measurement point deviations to the bracket deviation and rotation. The panel is also measured at EOL, on the locating hole and slot.



(a) In selected station (b) At EOL and CMM

Figure 4-7 Location of measurement points on the bracket

Table 4-2 Location of measurement points on bracket at selected station (Unit: mm)

Selected Station	X	Z
Upper Corner	2315.53	1113.4
Lower Corner	2303.54	791.81

Table 4-3 Location of the measurement points on bracket at the EOL (Unit: mm)

EOL	X	Z
Upper Surface	2280.7	1118.8
Upper Pin	2315.5	1096.0
Lower Pin	2303.5	835.0

Denote the panel as Part 1 and bracket as Part 2, then the state vector $\mathbf{x}_i$ can be defined as $\mathbf{x}_i = \begin{bmatrix} x_{i11} & z_{i11} & \theta_{i1} & x_{i21} & z_{i21} & \theta_{i2} \end{bmatrix}^T$, which is the stack up of the deviations and rotation of both parts. Further denote locating hole as point 1 and slot as point 2, then the control vector in station 1 can be represented as $\mathbf{u}_1 = \begin{bmatrix} x_{11} & z_{11} & x_{12} & z_{12} & x_{21} & z_{21} & x_{22} & z_{22} \end{bmatrix}^T$, where x_{jk} represents the deviation of locating point *k* of part *j* in X direction, and z_{jk} is defined similarly. Finally, the vectors showing deviations of the measurement points at the selected station can be defined as $\mathbf{y}_1 = \begin{bmatrix} x_{\text{upper}} & z_{\text{upper}} & x_{\text{lower}} & z_{\text{lower}} \end{bmatrix}^T$, and the one at the EOL can be defined as $\mathbf{y}_2 = \begin{bmatrix} x_{\text{upper surface}} & z_{\text{upper surface}} & x_{\text{upper pin}} & z_{\text{upper pin}} & x_{\text{lower pin}} & z_{\text{lower pin}} \end{bmatrix}^T$. The corresponding SoV model coefficients can be obtained using location information provided in Table 4-1 and Table 4-2, which is shown in the Appendix.

4.4 Validation of SoV Model

4.4.1 In-line Sensing System Capability Validation

As the prerequisite of the experiments, the sensing systems have to be validated to ensure their capability to the process. The purpose of this experiment is to show that the systems provide reliable and sufficient information of the process state for both statistical and automatic process control.

To validate the in-line sensing system, three tests were performed:

1. Sensor Repeatability and Reproducibility (R&R) study in the selected station;

2. Correlation study between the measurements taken at EOL and in an off-line CMM room, which is the benchmark of product quality;
3. Correlation study between the measurements taken in the selected station and at EOL.

The R&R test provides information about the capability of sensors for a given process (Montgomery, 2005), which is important in system control. R&R is a measure of the capability of sensors to obtain the same measurement reading every time when measuring the same characteristic, which indicates the consistency and stability of the measurement system. This test compares the variation of sensors taking repeated measurements, with the tolerance of the process. If the ratio of the measurement variation and the process tolerance is small (usually less than 0.25 in application), then the sensors and the measurement procedure are considered to be capable for the process.

The correlation analysis between OCMM measurement at EOL and that of the off-line CMM is required. This is because CMM measurement, rather than OCMM measurement, of final product quality is the benchmark used in automotive industry. However, CMM measurement is very time consuming, and is performed just on a few samples of final assemblies, to ensure the general production trend to be in control, and thus the in-line measurement of each product can only be provided by OCMM system. Under this circumstance, if the measurement taken by OCMM is proven to be consistent with CMM, then the EOL OCMM measurements can be used in subsequent analyses for real-time control purpose.

The correlation analysis between the selected station and EOL provides information about the relationship between the intermediate stations and EOL. If the measurements in the selected station are correlated with the ones at EOL, and the measurements at EOL are correlated with CMM, then the measurements in the selected station will be consistent with CMM and can be used for control purposes. A direct correlation study between the selected station and CMM was not performed because of realistic constraints that the measurements taken in the selected station are not measured by CMM.

1. Sensor Gauge R&R test

Gauge R&R test is the study of the repeatability and reproducibility of a measurement system. The repeatability refers to the inherent variation of the gage, and the reproducibility to the variability of the operator (Montgomery, 2005). Since in this application, measurements were performed by automatic systems, only the repeatability is of interest.

The ratio of measurement variation to tolerance, also known as precision to tolerance (P/T), is used as the measure of the process capability. The measurement variation is calculated as six times the standard deviation of the gage and the tolerance as the upper specification limit minus the lower specification limit (2mm). Table 4-4 presents the precision to tolerance ratios of the four measurements.

Table 4-4 Measurement variation to tolerance ratio at selected station

	X_{Low}	Z_{Low}	X_{Top}	Z_{Top}
Ratio	0.11	0.15	0.05	0.04

The ratios of all measurement points are less than 0.25, and thus the measurement system is capable of the process.

2. Correlation between OCMM at EOL and CMM

The correlation analysis between EOL and CMM measurements was performed comparing the deviations of the parts obtained from the measurements in both stations. The OCMM sensors at EOL inspect 100% of the assemblies, while in the CMM room only a few samples are inspected. Since the CMM measurement is an industrially-accepted measurement standard, the objective of this test is to verify the agreement between OCMM and CMM. If the correlation is high, the OCMM measurements at EOL can be considered accurate in subsequent analyses.

Table 4-5 presents the results of the study, where the notation in second column, error, is defined as the CMM measurement minus the OCMM measurement. The linear correlation coefficient (r) of the two systems reveals whether the OCMM measurements follow the same trend as CMM measurements.

Table 4-5 Correlation coefficients between CMM and OCMM measurements

Measurement Point	Error (mm)		r
	mean	σ	
I/P Upper Surface(Z)	1.57	0.09	0.97
I/P Upper Pin (X)	0.66	0.26	0.82
I/P Lower Pin (X)	-0.58	0.23	0.78
Panel 2-Way (Z)	0.30	0.06	0.95
Panel 4-Way (X)	-0.11	0.26	0.72
Panel 4-Way (Z)	0.50	0.07	0.91

The correlation is high for each measurement points; thus, it is reasonable to consider the EOL-OCMM as a reliable measurement system.

3. Correlation between selected station and OCMM at EOL

Table 4-6 presents the deviation correlation of the three degrees of freedom of the bracket between the selected station and EOL, based on the measurements performed during the shim test that is discussed in the next section. The correlations are close to 1, which means measurement in the selected station is similar to what is observed at EOL.

Table 4-6 Correlations between selected station and EOL (shim test)

Rotation	X	Z
0.99	0.99	0.99

4.4.2 Design of Experiment of Shim Test

The validation is carried out using a shim test, which is to purposely adjust the positions of locators held by PT in the selected station, while observing the responses at the EOL. The validity of the SoV model is tested by comparing its predictions of the response to the shim inputs to the true deviations of the bracket observed at EOL.

The number of runs of the shim test was limited by the length of a work shift and by the available manpower. Due to this and the presence of unknown and uncontrollable production factors, a Design of Experiments (DOE) method was utilized to decide the combination of shim commands that would maximize the amount of information obtained (Wu and Hamada, 2000).

Since the bracket has 3 degrees of freedom (d.o.f.) in the X-Z plane, the shim test has 3 factors: two translations (ΔX, ΔZ) and one rotation ($\Delta\theta$). For each variable, the experimental range was decided as the maximum displacement of the measurement points to be within 2 mm. Accordingly, the maximum displacements of the hole of the bracket in ΔX and ΔZ directions were equal to 1 mm, and the rotation ($\Delta\theta$) was limited to be within 0.25 degrees. Thus 3 different levels for each variable were defined, which are (-1, 0, 1 mm) for translations, and (-0.25, 0, +0.25 degrees) for rotation. A 3^3_{III} full factorial design will have 27 different runs, but due to time constraints on the realization of the experiment in plant, a factorial design which consists of 9 tests was selected instead, as shown in Table 4-7 (a). This fractional design used confounding of **C**=**AB** (Wu and Hamada, 2000), where **A** corresponds to ΔX, **B** corresponds to ΔZ, and **C** corresponds to $\Delta\theta$, in this experiment respectively. Due to experimental constraints, the DOE was applied in Z and θ (Test 1), and then in X (Test 2) separately.

The sequence of tests was randomized to diminish the effect that sequencing could have in the results. The actual carried out experiment sequence is shown in Table 4-7 (b). Successive replicates were also used for each run level, as shown in the column next to shim.

Table 4-7 Experimental matrices for shim test (Unit: mm)

(a) Randomized 3^{3-1}_{III} fractional factorial experimental design

Run	DOE Factor		
	ΔX	ΔZ	$\Delta\theta$
1	0	0	0
2	-1	-1	0.25
3	-1	1	0
4	1	1	-0.25
5	-1	0	-0.25
6	0	-1	-0.25
7	1	0	0.25
8	1	-1	0
9	0	1	0.25

(b) Matrix of carried out experiment

Run	Test 1				Test 2	
	ΔX	ΔZ	Δθ	Replications	ΔX	Replications
1	0	0	0	5	0	5
2	0	-1	0.25	3	0	3
3	-1	1	0	3	1	3
4	0	1	-0.25	3	-1	3
5	-2	0	-0.25	3	-0.5	2
6	-1	-1	-0.25	3	-0.5	3
7	2	0	0.25	3	0.5	3
8	1	-1	0	3	0.5	3
9	1	1	0.25	3	1	3
10	0	1	0.25	3	1	3
11	-1	0	-0.25	3	0	3
12	0	0	0	5	0	4

The reason for using large shims in the testing is that, in this particular experiment, there are a large number of successive stations between the selected station and EOL, which introduces extra noise to that of the bracket placement and will overwhelm the correlation during normal production. For large shims, these noises are still ignorable compared to control amount, thus the high correlation presented in the shim test result.

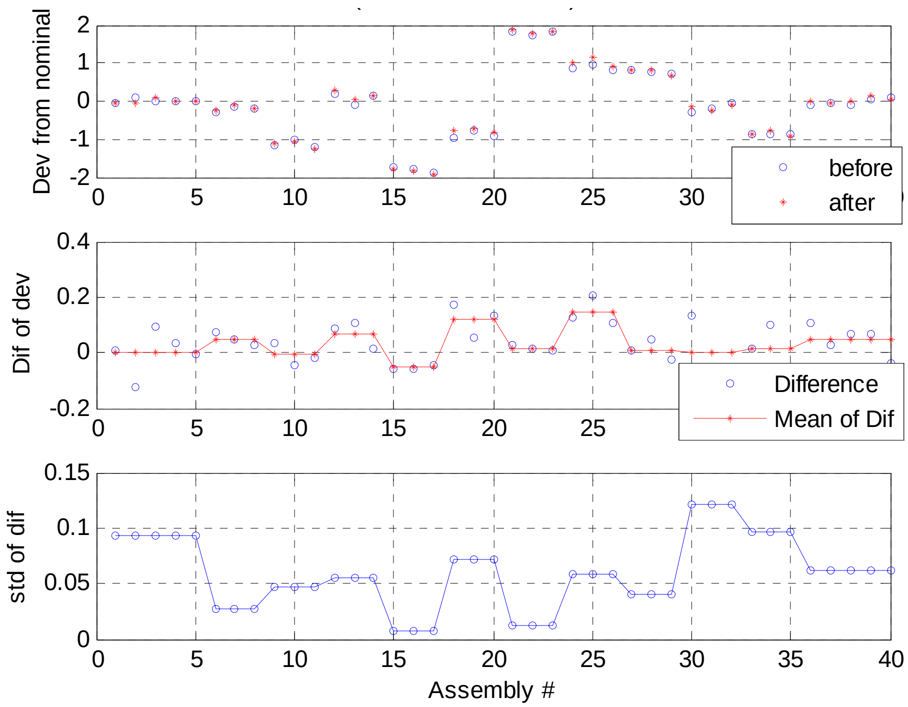
4.4.3 SoV Model and System Controllability Validation

The experimental data are analyzed in two steps. The correlation analysis first compares measurements taken before and after welding in the selected station, which analyses the impact of the welding process. The analysis then focuses on correlation between the measurements in EOL and the predicted deviation using the commanded shims as input to the model, which tests the SoV model's correctness under large deviation, as well as the controllability in selected station.

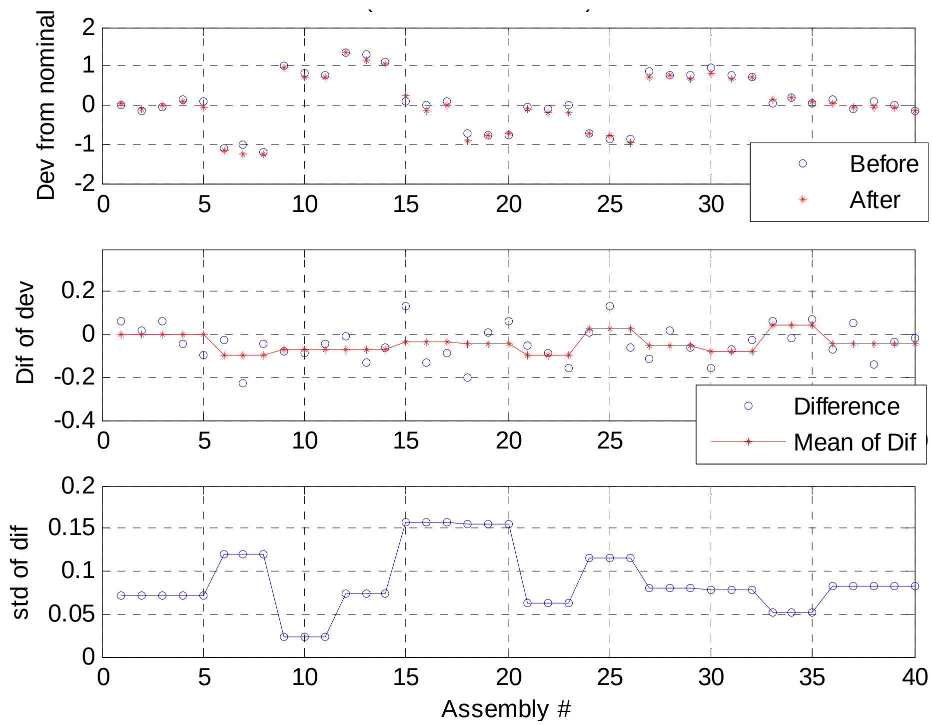
1. Correlation between before and after welding

The purpose of this experiment is to test if the welding process introduces variation into the system. If this variation is significant, then the measurements taken before welding process will not be able to be used to determine the control action, because the position after welding cannot be predicted precisely. On the other hand, if the measurements before and after welding have high correlation, a control strategy can be developed.

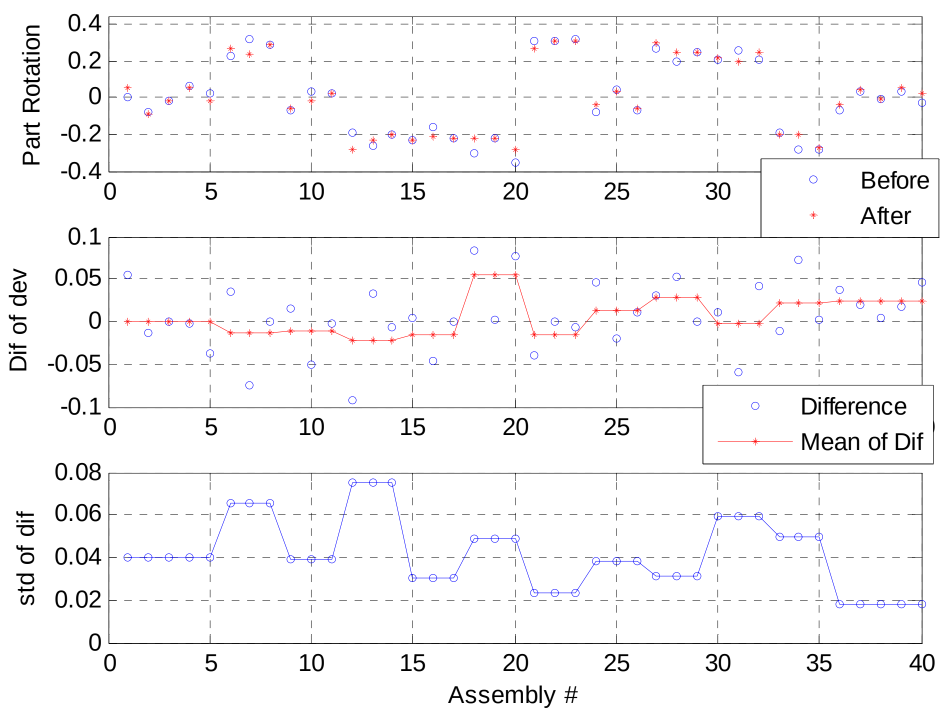
To check this correlation, the measurements taken before welding are compared with those taken just after the bracket is welded to the panel. Figure 4-8 shows the comparison of before and after welding of the 40 parts in Test 1.



(a) Bracket deviations in X direction (Unit: mm)



(b) Bracket deviations in Z direction (Unit: mm)



(c) Bracket rotations (Unit: deg)

Figure 4-8 Comparison of part location before and after welding in the selected station

Table 4-8 presents the correlations between measurements taken before and after welding, which are close to one. This high correlation indicates that welding process does not introduce significant variation into assembly in selected station, and using measurement before welding for part-by-part correction is effective.

Table 4-8 Correlation between before and after welding

	X	Z	θ
Correlation	0.99	0.99	0.98

A *t*-test on the above set of experimental data was carried out, which shows that the difference is not statistically different from zero, which supports the conclusion of welding process does not introduce extra errors in to the production. Thus the welding effect does not need to be modeled, and using the measurements before welding as the basis of feed-forward control is valid.

As a consequence, the suggested sequence in real-life control is, first locate the part, then take measurements of its location, then apply a calculated control action, and finally close clamps and finish the rest of the cycle including welding.

2. Correlation between expected and measured deviation of bracket at the EOL

To validate the SoV model, the inputs (ΔX, ΔZ, $\Delta\theta$) from Table 4-7 (b) are commanded to the programmable tooling at station *s*, and the corresponding part deviation is measured by an OCMM at the end of production line. At the same time, these same inputs are applied to the SoV model to make predictions at the end of line. By comparing the measurement from real production with the predicted from the SoV model, the consistency will indicate whether the SoV model is adequate. This validation process is illustrated as in Figure 4-9.

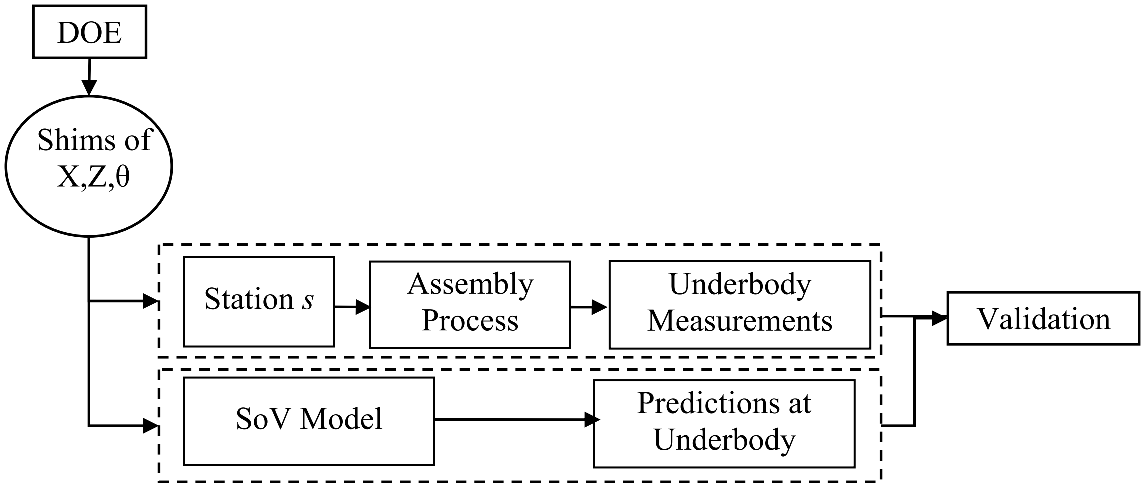
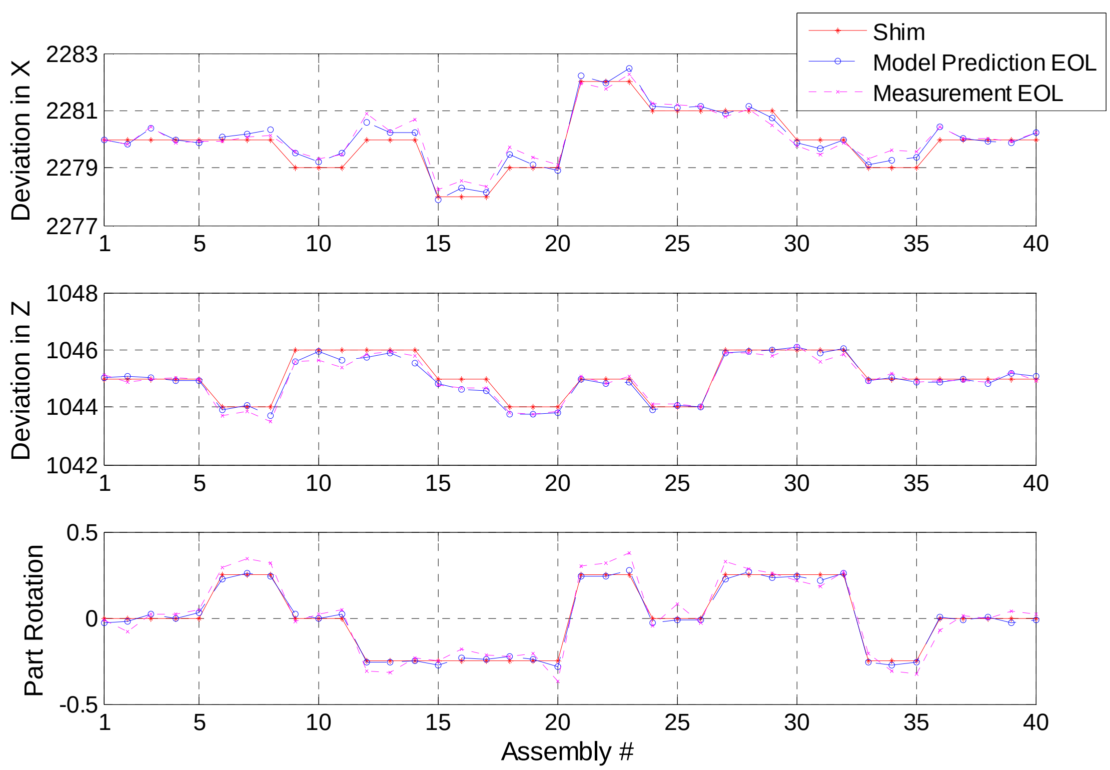
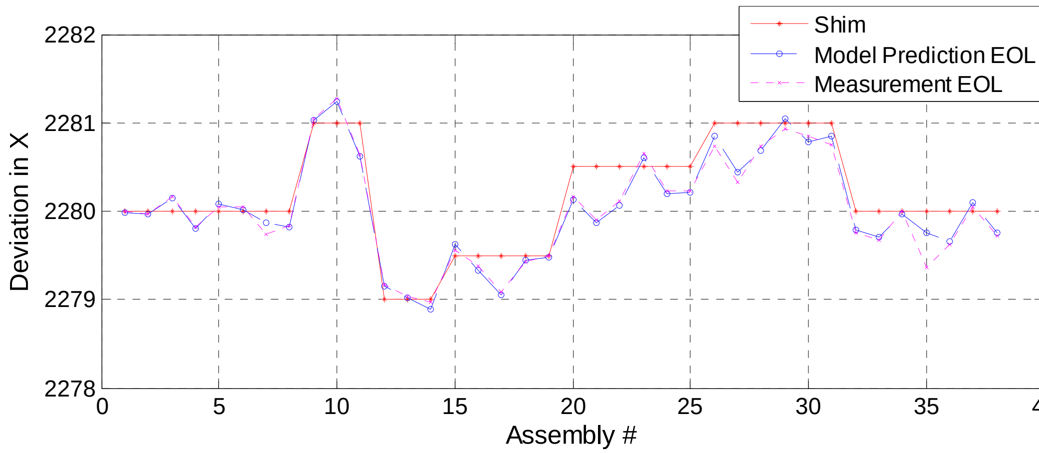


Figure 4-9 Model validation process

Figure 4-10 shows the comparison among shim command, model prediction and the true part deviation and rotation at EOL. The high linearity between measurements in selected station and EOL in test 1 is reflected in Figure 4-10, where all three correlations are close to one.



(a) Comparison for Test 1



(b) Comparison for Test 2

Figure 4-10 Comparison between measurements and model prediction (Unit: mm)

Table 4-9 Correlations between measurements in selected station and EOL for Test 1

d.o.f.	X	Z	Θ
Correlation	0.97	0.99	0.99

The above results show that given the shim command in the selected station, the SoV model closely predicts the final deviation of the bracket at the EOL. The experiment also shows the control action taken in selected station has the expected impact on the final assembly, and thus control can be applied if process information is complete.

4.5 Conclusion

The experiment described in this paper is the first to verify the SoV model in a production environment and to demonstrate the feasibility and controllability using control actions in an intermediate stage on the final product. The results show that, in the selected station, welding has little impact on the part deviation, and the deviations introduced in selected station will appear at the EOL as SoV model predicts. This validates the SoV model in real-life production and shows the controllability of the process based on the SoV modeling technique.

Acknowledgements

The authors would like to acknowledge the assistance of David Powell and Scott Pohl, who work in dimensional control at LGR, and all those who helped install equipment in Station 205 at LGR, especially Al Searles of CCRW. Bob Pryor, of Body Shop Execution, played a key role in facilitating the installation. Finally, the advice and support of Gary Telling of Body PPEC Manufacturing is highly appreciated.

The authors would also like to thank our partners at Perceptron, Inc., especially Wesley Deneau.

Appendix: SOV Model of Selected Station

Following steps of standard SOV model derivation (Shi, 2006), using location information provided in Table 4-1 and Table 4-2, the SoV model of selected station can be obtained as,

$$\mathbf{A}_0 = \mathbf{I}_{6\times 6};$$

$$\mathbf{B}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0026 & 0 & -0.0026 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0011 & 0.0052 & -0.0011 \end{bmatrix};$$

$$\mathbf{C}_1 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 185 \\ 0 & 0 & 0 & 0 & 1 & -40 \end{bmatrix};$$

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 45 & 1 & 0 & 0 \\ 0 & -1 & -44 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix};$$

$$
\mathbf{B}_2 = \begin{bmatrix}
1 & 0 & 0 & 0 & \\
0 & 1 & 0 & 0 & \\
0 & 0.0026 & 0 & -0.0026 & \\
1 & -0.1166 & 0 & 0.1166 & \mathbf{0}_{6\times 4} \\
0 & 1.1140 & 0 & -0.1140 & \\
0 & 0.0026 & 0 & -0.0026 &
\end{bmatrix};
$$

$$
\mathbf{C}_2 = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -386 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & -78.3 \\
0 & 0 & 0 & 0 & 1 & 0.7 \\
0 & 0 & 0 & 1 & 0 & 51 \\
0 & 0 & 0 & 0 & 1 & 35.5 \\
0 & 0 & 0 & 1 & 0 & 210 \\
0 & 0 & 0 & 0 & 1 & 23.5
\end{bmatrix}.
$$

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CHAPTER 5

CONCLUSIONS AND FUTURE WORK

5.1 Conclusions

Process variation reduction through automatic control has been investigated in the literature ever since the advancement of sensing technology and active control systems. In such a system, a controller generates the proper control action based on mathematical models of the process, which comes from either statistical modeling or from product/process design knowledge. Uncertainties in process mathematical models, both data-driven and engineering-driven models, however, may lead to control actions that are responses to noise and that decrease the system performance. This dissertation is the first to explore a control strategy in a multistage manufacturing process that compensates for the final product quality, taking into consideration the existence of modeling uncertainties.

The major achievements of this dissertation can be summarized in four aspects:

1. *Development of a control strategy that takes into consideration modeling uncertainties for data-driven models*

The objective of this study was to develop a control strategy for processes whose mathematical models cannot be derived from engineering design knowledge. For this type of process, statistical models are usually obtained from designed experiments on the system, where model parameters are estimated from experimental data. The parameter estimation inevitably contains uncertainties, which are due to unknown disturbances and randomness in experiments. A controller that generates the control action without considering these uncertainties will underestimate the process variation and may introduce even more noise to the final product. The proposed control strategy derives the control action based not only on DOE models and in-process measurements data, but also on the knowledge of model uncertainty and observation noise. These uncertainties can be obtained from statistical regression procedures, and sensor specifications or gauge R&R, respectively. The application of this controller can significantly improve robustness and dimensional quality, well beyond the improvement offered by an ordinary controller.

2. *Design of a control strategy for quality improvement in multistage manufacturing processes*

The objective of this research was to develop a part-by-part deviation control technique for multistage manufacturing systems, one that takes into consideration the uncertainties of product and processes. Stream of Variation modeling methodology generates the mathematical model from design blueprint data to describe variation propagation in production flow. The SoV model has been widely applied in research on multistage manufacturing systems, including process modeling, diagnosis, and active control. However in production, since part geometry (a) deviates from the nominal because of errors inherited from the part fabrication process, or (b) is accumulated from previous assembly stations, the true process model will deviate from theoretical models accordingly. The proposed controller captures this model variation and these observation uncertainties, and can significantly improve part quality, process robustness, and cost in MMPs.

3. *Validation of the Stream of Variation model in multistage manufacturing processes*

The objective of this work was to validate the correctness and effectiveness of the SoV model. The theoretical applications of the SoV model have been thoroughly studied in literature, but its validation in real manufacturing systems has never been carried out. An experiment has been performed in a selected station where parts perform as a rigid body on a slip plane contact surface. The shim test intentionally adjusts the position of parts in the selected station, and compares the observed responses with the ones predicted by the SoV model. Statistical analysis has validated the model by comparing predictions given by the model with actual product dimensions. This effort fills the gap of the validation of the SoV model in real manufacturing environments.

4. *Validation of automatic control feasibility in real manufacturing environments*

The objective of this research was to validate control feasibility in MMPs. This is a necessary and essential step before the final realization of active control in a real MMP environment. The control feasibility is also shown through a shim experiment. In this experiment, an adjustment in a selected station is applied, and its impact on KPCs is monitored at the end of the production line. The desired control amounts were observed at the end-of-line as expected in theory, and thus the control feasibility is verified in real-life production environment. This validation provides the application basis for future realization of control systems in multistage manufacturing.

5.2 Future Work

To further implement active control in a multistage manufacturing system, there are many more topics that can be explored. Some of the proposed future areas of focus are:

1. *Optimal Sensor Placement*

Sensor distribution plays an important role in automatic deviation control since it determines the system diagnosibility. Studies have been done previously on sensor placement for diagnosis (Ding *et al.*, 2002; Chin *et al.*, 2005) and control (Izquierdo *et al.*, 2007) in multistage assembly processes. However, taking into consideration model embedded uncertainties in realistic processes can change the sensitivity of sensor locations. The problem of sensor placement can be approached from two perspectives: station level and part level. The station level perspective focuses on determining the appropriate stations along the process. Part level perspective focuses on determining the appropriate features of the parts that should be measured in order to improve the estimation of the part deviation.

2. *Robust Fixture/Process Design Considering Modeling Errors*

Robust fixture design has been studied so as to minimize the impact of process variations. When obtaining a process model with uncertainty explicitly expressed, the design can be less conservative, and can utilize additional process knowledge, including tolerance information.

3. *Tolerance Allocation for Controlled Multistage Assembly Systems*

With higher controllability in production, the tolerance allocated to non-key features/parts can be released, as greater levels of control are devoted to key features. This wider tolerance will result in more efficient budgeting and reduction in total cost.

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