WHAT NONVERBAL INTERACTIONS WITH DIAGRAMS TEACHERS PERCEIVE AS MEANINGFUL ELEMENTS OF STUDENTS' MATHEMATICAL WORK

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The work of teaching includes processing input from many students and in many modalities: their speech, but also their facial expressions, gestures, and actions. We investigate the rationality invested by teachers in perceiving important nonverbal interactions of students with diagrams. Through examining how geometry teachers re-enacted and then discussed a conversation between two students about a proof we show examples of the kind of nonverbal interactions that are meaningful for teachers. This examination helps extend our knowledge of the work of teaching and guide the work of developing media for representing classroom episodes.

STUDYING THE RATIONALITY OF TEACHING

The research reported here is inscribed in an effort to understand the rationality of teachers of high school geometry. Building on Bourdieu's (1998) work on practical reason and on Goodwin's (1994) notion of "professional vision," we hypothesize that teachers have (often tacit) professional resources (or dispositions) that enable them to perceive and appreciate events and things in their practice. The need for those professional resources is justified on a theoretical perspective on the complex work of teaching according to which the teacher is responsible to manage (symbolic) transactions in the classroom (Herbst, 2006). The teacher has to manage exchanges between, on the one hand, the work done by the class in moment-to-moment interactions among people and things and, on the other hand, the objects of knowledge that are at stake in such interactions (and that the class is expected to lay claim on). The rationality alluded is adapted to handle the demands of particular instructional situations, thus specific to the curricular obligations of subject matter.

From an observer's perspective, the moment-to-moment interactions between people and ideas that a teacher would need to account for are potentially realized in multiple modes of communication. Students communicate mathematically through language and inscriptions; but other modalities, including facial expressions, gesture, body positioning, tool use, and kinesthetics can be used for communication as well (e.g., Arzarello et al., 2005; Jurow, 2004). Yet, it is one thing to note that students might make use of those various modalities to interact with content, and quite another thing to claim that interaction in all those modalities is perceived and accounted for by the teacher as mathematical work, pursuant of the knowledge at stake. This report is focused on that latter issue with particular reference to nonverbal interactions students may have with geometric diagrams in the situation of installing a theorem. What nonverbal interactions with diagrams do teachers of high school geometry perceive as meaningful elements of students' mathematical work? We study the question by stimulated, backward reconstructions of that perception: We involve teachers of geometry in pondering a classroom scenario in which students had produced a problematic diagram, and we examine how they reconstruct possible ways in which students might have produced the diagram. The teachers' conversation data helps us identify actions that students might have done on (or with) diagrams that from the teachers' perspective might matter in the value of the work done.

TEACHERS' PERCEPTION OF INTERACTIONS WITH DIAGRAMS

We contribute to two questions about the rationality of teaching. First a theoretical question predicated on the notion that the work of the teacher requires processing a huge number of overt behaviors as possible signs of mathematical work: Inasmuch as teachers do find signs of students' mathematical work in their interaction with diagrams, what kinds of mathematical work do they see in there? Secondly, a technical question that informs the further development of a methodology for the study of teachers' rationality. We have been creating and using representations of teaching in the form of animations, slide shows, and comic books with cartoon characters that represent teacher and students in classroom scenarios (Herbst & Chazan, 2006; Miyakawa & Herbst, 2007). These scenarios are used as prompts for commentary by and conversations among teachers. The discussions those scenarios help precipitate are the main data source in our search for the rationality of teaching. A consequential decision in the creative aspect of such work is what to consider in the creation of the cartoon characters with which the scenarios are represented. Partly informed by our understanding of the hypothesis of "the uncanny valley" in robotics, according to which human viewers empathize more with less humanlike characters than with characters that pretend to be humanlike but fail to act like humans do (MacGillivray, 2007; Mori, 1970), the cartoon character sets we have created embody considerable simplification in the way the human body (and individual differences) are modeled (see Figures 1a, 1b). To the extent that some of the information that a teacher relies on when managing classroom exchanges comes through behavioral modalities, a basic technical problem in the development of these character sets is to find out what are the sets of meanings that may need to be represented. Our design of character sets to represent classroom interaction would benefit from improved knowledge of the granularity of teachers' perception of those students' behaviors that count (for the teacher) as mathematical work. Chen & Herbst (2007) have shown how argumentation in geometry can involve students in the use of specialized iconic gestures and finger patterns to sign parallelism or incidence when making an argument about angle measures. After that work we have begun developing the character set ThExpians P (Figure 1b) that among other things has a richer meaning potential (i.e., a better arm-and-hand model) than the set ThExpians B (Figure 1a) that we have to date used in many of our animations. The question posed in this paper looks into teachers' perception of that kind of behaviors to inform the development of future animations and conversations among teachers about those.

Research on students' learning has addressed the role of nonverbal behaviors and gesture in particular. Our intent, however, is to model the problem that the teacher runs into when teaching a class. It is thus reasonable to elicit teachers' perception of students' behaviors from their reconstruction of students' artifacts (diagrams in this case) and plausible that this perception will be of coarser grain than that of researchers on students' thinking. Our second, technical question is aimed (on the long run) at developing a cartoon model that contains the physical resources that a teacher would need were he or she to represent the behaviors that have value in the instructional situations they manage.

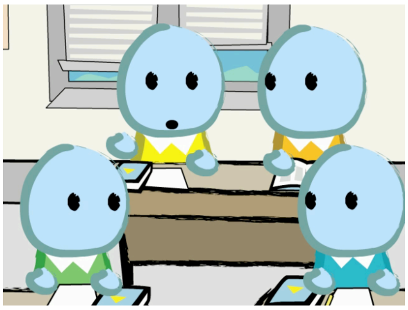

Figure 1a. ThExpians B

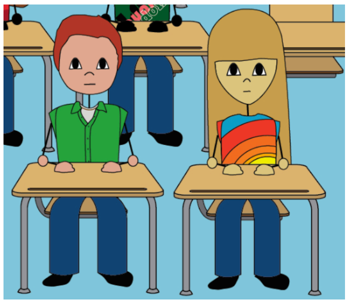

Figure 1b. ThExpians P

To narrow down this investigation we focus on the instructional situation of "installing a theorem" in the geometry class (Herbst & Nachlieli, 2007). "Installing a theorem" is the period between two states of public knowledge (as perceived by the teacher): the state of not yet being accountable for knowing a particular theorem, and the state of already being accountable for knowing that theorem (Herbst & Miyakawa, accepted). To research that instructional situation means for us to construct a model that can describe what is expected to happen when a class installs a theorem. As regards the role of proof in installing a theorem, we want to model when a proof is needed, who can do it, what does it need to have in order to be acceptable. The data presented here responds to a scenario in which a teacher had stated a particular theorem —that congruent chords in a circle are equidistant to the center — and that he had commissioned groups of students to work proving it. The scenario was designed to immerse teachers in the problem of appraising a set of students' responses.

METHODOLOGY

We present these stories in monthly study groups where experienced teachers of high school geometry gather to watch and discuss animated representations of possible teaching episodes. The particular session analyzed here was the third meeting of the year. Nine teachers were in attendance, along with a moderator and two researchers. The agenda was to begin by reading aloud together a script for a story that we told the participants was in the pipeline for development as an animation. Study group participants were assigned individual roles and asked to perform the script as their characters. Before distributing scripts to participants, the moderator put the story in context by reading a prior moment in the episode. In this prior moment the teacher reviews the definition of distance from a point to a segment and writes on the board a new theorem ("In a circle two congruent chords are equidistant from the center"). The teacher then asks students to work in pairs to prove the theorem.

Following the reading of this framing scene, our study group participants were divided into four groups, each of which was given a different script containing the dialogue of a pair of students working together on the problem. Each script contained one or more diagrams showing what the students drew, but did not contain any behavioral descriptions, nor did it include any step-by-step description of the process by which the students produced the diagram. The diagrams were drawn using conventional notation for geometric objects and their markings, but neither the script nor the diagrams contained signs to signal students' gestures. Each group of participants saw only one of the four scripts. Participants had approximately 20 minutes to read and discuss the scripts in their possession, after which they had to perform the script for the rest of the study group. Participants stood at an easel pad and, as part of their performance, drew the diagrams that accompanied the scripts.

The data we examine here is taken from a conversation that began 57 minutes into the study group session. Prior to this conversation, a first group of teacher participants performed their assigned script, and the group as whole discussed that script for some twenty minutes. After that discussion, teacher participants Edwin and Tina presented a proof proposed by students Tau and Epsilon[1]. Tau and Epsilon's proof was based on a diagram that depicted the special case of two congruent chords drawn parallel to each other. We do not reproduce here the complete scenario read by Edwin and Tara, since the discussion we report focuses on the students' decisions to outfit the diagram before doing the proof. In the script, Tau says to Epsilon, "These are the chords and distances" and the diagram of Figure 2 is shown. In reply Tau says "We just complete the triangles" and the diagram of Figure 3 is shown. Tau and Epsilon proceed to prove that the distances from the center to the chords are congruent by showing that each of those segments is a leg of a pair of right triangles that are congruent by virtue of having its hypotenuses congruent and two pairs of congruent angles, one of which pairs is congruent by virtue of being vertical angles.

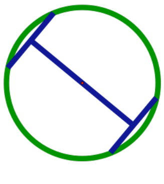

Figure 2.

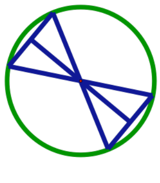

Figure 3.

We look into discourse and gestures from teachers for the set of meanings that they would look for in students' behaviors. That is, each element of interaction with a

[1] The fictional students in the ThEMaT stories are named after Greek letters.

diagram noticed by teachers in the study group (noticed by way of the teachers' words or gestures) identifies one meaning or one referent for which a sign must exist in the classroom (a behavior by students). We note how teachers mark those behaviors in conversation in order to claim that those behaviors are taken as signs of mathematical work (we note what participants did or say, how they did it or say it, to convey the sense that such behaviors had to be attended to by a teacher in class).

FINDINGS

Study group participants marked relevant interactions that Tau and Epsilon had with diagrams as Edwin and Tina re-enacted the classroom episode. Teachers used their own gestures to talk about the students' thinking they could attest to when imagining students act. In the study group discussion, teachers realized that students in the script made an assumption about the diagram, and that this assumption was key in their proposed proof of theorem. They used gestures to point to key actions in students' making of the diagram and students' assumptions about that situation. That was aimed to figuring out whether the solution putatively proposed by a pair of students was valuable at all. Three interactions with diagrams were singled out.

The "H" (one long segment)

As soon as participants started discussing the episode, Mara noted that students had made an unwarranted assumption when they drew the diagram: in her words, the theorem "never said anything about those lines being parallel," and "all of a sudden they're drawing these segments through the center of the circle." Tina described what students did as making "a little H", and accompanied this description with a gesture: with her right index finger she traced a vertical stroke, then another vertical stroke to the right of the former, then a horizontal one connecting strokes, thus making an H in the air. She described the students' intent as "they just start off like that way.... Just make it easy because they want to go through the center." If they had not made that assumption — if, as Tina said, "I just drew two congruent chords" — then, as Mara put it, "If that's all I have done, then I don't know if I am going through the center any more." As Mara said this, Edwin lifted both his arms up, then brought them down with palms open and swept his right palm from the right across to the imaginary line drawn by the left palm, indicating that the segment transversal to the two vertical strokes of the H is one long segment crossing from one chord to the other.

In these exchanges teachers seemed to agree that the figure drawn was special, but disagreed in relation to what the drawing meant. For Mara, the drawing was a response to students' incorrectly assuming that the chords had to be parallel, whereas Tina seemed to present a scenario in which parallelism had just resulted from the students' desire to ease the drawing—which she expected some of her students would do. Mara's retort and Edwin's gesture indicated that the students' decision was still mathematically problematic—a straight segment from a chord's midpoint to the other chord's midpoint might or might not pass through the center of the circle.

Two distinct segments of distance between the center and the chord

Mara pointed to what the correct construction ought to be with her gestures: two hands together at the “center” that moved horizontally in opposite directions and stopped after traversing the same distance--thus indicating that the diagram needed two distinct segments originating at the center and extending outward to the chords (see Figure 4a, 4b). With this gesture, Mara showed that it was possible for the two distance segments to form a straight line, but that students were not entitled to replace the two segments by a single long one, straight from one chord to the other. Glen made this explicit, saying “It may not be a line. Maybe two separate,” while Mara gestured with both hands pointing up towards each other at a slant to form an up-side-down V, see Figure 5), indicating that the chords might not be parallel.

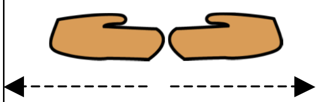		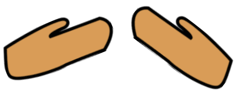	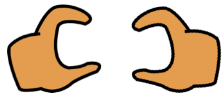
Figure 4a	Figure 4b	Figure 5	Figure 6

If two chords have to be congruent, students might draw them parallel

Like Tina earlier, Edwin recognized that drawing the two chords parallel was a plausible move by students.

> I am sure most kids would do that. If you told them to draw two chords with the same size, I bet a lot of them just draw them across from each other [Edwin extends both arms and places two open palms in front of each other and perpendicular to the floor on both sides of his body, as if each of them was a chord] (102605-338)

And later he explained why students might do that:

> It would be easier to see them being the same. [Edwin’s index and thumb in each hand span invisible congruent segments, where before he had placed his palms, to show the equal length of the two chords; then his two hands move toward each other to meet at the “center”, then move toward their previous location] (102605-341)

Tina continued to unravel how the long segment came to be after students had drawn the chords parallel to ensure they are congruent.

> And then when they joined it to the center point [holding up the script and pointing to the provided diagram, she points from one chord toward the center, then pauses, then from the center to the other chord], they probably realize they were straight across [points from the second chord straight through the center toward the first chord] and now it could [points to the worksheet, indicating other segment connecting the end-points of the chords], you know. (102605-343)

CONCLUSION

The fragments of the conversation shown above suggest that teachers attributed meaningfulness (as mathematical work) to the following actions by students.

The drawing of the congruent chords in the circle:

- Drawing both chords vertically (to ease control that they are congruent)
- Drawing the second chord parallel to the first chord
- Showing that two segments are congruent using a common measuring element (in the teacher's case the index-thumb configuration; Figure 6).

The drawing of the distances:

- Drawing one long segment from the midpoint of one chord to the midpoint of the other chord passing through the center of the circle
- Drawing one long segment from the midpoint of one chord to the midpoint of the other chord not necessarily through the center of the circle
- Drawing segments perpendicular to each chord and to the center of the circle
- Drawing segments perpendicular to each chord and from the circle's center

The main discussion was the extent to which the choices the students had made in creating the diagram really compromised the work they had done. Clearly, to the extent that Tau and Epsilon had considered the "distances" to be just one long segment, which was apparent in their claim that two angles were equal because they were vertical angles (see Fig. 3), their proof was not as general as it could have been. The teachers in the study group were able to notice a fundamental flaw from the start, when Mara and Edwin agreed that "major assumptions" were being made. However they were compelled to look closer at how the diagram could have possibly been generated—taking responsibility to account for what had been done since, as Tina and Edwin noted, their own students might have done the same if given the responsibility to draw the diagram. Their unpacking of what students did highlighted actions taken in producing the diagram, where teachers used different gestures to mark different drawing actions. It appears, from their discussions, that teachers would allocate some value to the decision to draw the two chords parallel inasmuch as doing so would evidence a desire to control the congruence of the given chords. It also appears that teachers would allocate more value to the drawing of the distances as two different strokes that turned out to be aligned than to the drawing of one long stroke passing through the center of the circle. These are all, we note, actions that students could have done without any verbal accompaniment (indeed even the teachers' gestures were far more eloquent than their verbal representations of those actions). In general this illustrates that teachers consider it important to attend to students' drawing actions when they are setting out to prove a theorem, rather than just to the nature of the achieved diagram.

This analysis is far from final in suggesting what characteristics a cartoon model of a student should have in order to be usable to represent diagram production. It does, however suggest that these characters should be able to display the human posture at the beginning, during, and at the end of drawing a straight stroke. To be clear, a character set that could only display postures “before drawing” and “after drawing” might be usable to trigger a conversation, just as the script of Tau and Epsilon’s story was—the analysis above shows that the conversation might endeavor to interpret different possibilities of what happened in between. But if a conversation had to rely on a specific interpretation of how a diagram was produced in a story, the character set should be endowed with finer arm and hand attributes to display the sequence of actions in which the diagram was produced.

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