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A criterion for the fragmentation of bubbly magma based on brittle failure theory

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The fragmentation of bubbly magma is a defining point in a volcanic eruption—before fragmentation the magma flows relatively slowly, during fragmentation the bubbles break up to release compressed gas and, afterwards, the eruption becomes a violent gas flow carrying suspended magma particles. Seemingly benign lava flows or domes can suddenly fragment into deadly pyroclastic flows[1–3]. Several criteria have been proposed to define the point of magma fragmentation or foam stability[4–7]. The criterion of Papale[7] is based on melt relaxation theory and equates magma strain rate with the rate of increase of flow velocity with distance. It ignores, however, the role of bubble pressure in causing fragmentation. Two empirical approaches[4,5] consider the role of high bubble pressure in causing fragmentation but do not address the underlying physics of magma fragmentation. Here I develop a fragmentation criterion for bubbly magma based on brittle failure theory and apply it to the fragmentation of lava domes and flows. On the basis of this theory, a bubbly magma will fragment when the tensile stress at the inner walls of bubbles exceeds the tensile strength of the magma. The fragmentation conditions depend strongly on initial water content, with calculated vesicularity and final water levels coinciding reasonably well with those in observed pumices. This suggests that the proposed criterion captures the essence of the fragmentation process in bubbly magma.

Table 1 Calculated conditions for fragmentation

Given conditions				Fragmentation conditions	
T (°C)	S (bar)	P_{out} (bar)	$H_2O_{t,i}$ (wt%)	Vesicularity (%)	$H_2O_{t,av}$ (wt%)
700	60	1	1.0	57	0.66
700	60	3	1.0	63	0.63
700	50	1	1.0	47	0.74
700	50	1	0.7	77	0.35

$H_2O_{t,av}$ is the average H_2O_t in the melt shell at the time of fragmentation.

Fragmentation can be viewed as the result of brittle failure of many bubbles at roughly the same time. Hartog[8] summarized classic theories of material strength and concluded that (1) the maximum-strain theory has been discredited by experiments; (2) the maximum-stress theory applies well to the failure of brittle materials; and (3) the maximum-shear theory applies well to the beginning of yield in ductile materials. The maximum-stress theory states that brittle failure occurs when the maximum tensile stress (σ_{max}) exceeds the tensile strength of the material (S):

$$\sigma_{max} > S \quad (1)$$

In modern treatment of fracture mechanics, brittle failure occurs when the stress intensity factor exceeds the fracture toughness[9]. The stress intensity factor can be calculated, given the size and shape of any microcrack or visible crack. However, because the distribution, size, and shape of the initial microcracks and weaknesses are not known *a priori* in a magma, it is difficult to apply the modern approach. Nevertheless, the application of modern fracture mechanics arguments to a material containing numerous random small cracks leads to the modified Mohr theory of failure, which is equivalent to the maximum-stress theory if the sum of the greatest and least principal stresses is positive (J. R. Barber, personal communication). Because the maximum-stress theory is relatively easy to apply, it will be used in this work.

A bubbly magma system is complicated by the non-uniform distribution of bubbles of variable sizes. A first-order approximation is to assume that all bubbles are spherical, of the same size, and spaced regularly (Fig. 1a). Because the stress distribution and bubble growth rate in this case is still too complicated, it is further approximated by assuming that each spherical bubble is surrounded by a spherical shell (Fig. 1b), for which an analytical solution of stress distribution can be obtained and a numerical solution of bubble growth is available. Where the pressure at the outside surface of the shell is P_{out} and the pressure in the bubble is P_{in}, the stress at the bubble wall (inner wall of the shell) can be expressed as[10]:

$$\sigma^{rr} = -P_{in} \quad (2)$$

$$\sigma^{tt} = (P_{in} - P_{out})\frac{1+2x}{2(1-x)} - P_{out} \quad (3)$$

Here σ^{rr} is the radial stress (and one of the principal stresses), σ^{tt} is the tangential stress (and two of the principal stresses), and x is vesicularity. Tensile stress is positive and compressive stress is negative. ($P_{in} - P_{out}$) is referred to as ΔP hereafter and can be identified as the dynamic pressure, P_{dyn} (refs 11, 12). That ΔP is non-zero means that stress in the magma is not dissipated by viscous flow. That is, the fragmentation criterion for bubbly magma using the maximum-stress theory also automatically incorporates consideration of the liquid–glass transition.

When $P_{in} > P_{out} > 0$, then σ^{rr} is compressive and σ^{tt} can be tensile. The fracture criterion under compressive stresses is more complicated and is not considered here. When $\sigma^{tt} + \sigma^{rr}$ is positive, the maximum-stress theory applies. The maximum tensile stress is the tangential stress at the bubble wall (σ^{tt} above). When the maximum tensile stress (σ^{tt}) exceeds the tensile strength of the melt, the shells surrounding bubbles fail. That is, brittle failure occurs when the following holds:

$$\frac{1+2x}{2(1-x)}\Delta P - P_{out} > S \quad (4)$$

Increasing P_{in} or increasing x (that is, decreasing the shell thickness) increases the tensile stress, and hence increases the likelihood of brittle failure. Because the crack volume is small and because P_{in} is exerted by the gas, the development of cracks does not significantly reduce ΔP and hence does not relieve the stress. Therefore, the cracks will grow until the bubble wall is broken (and the stress is thus relieved). This is in contrast to cracking induced by thermal stress (cooling or heating). In the latter case, the development of cracks relieves the stress and hence crack growth may stop without completely breaking the material. In a bubbly magma, there is a range of bubble sizes and different bubbles are under different stress conditions. Bubbles with a narrow size distribution and under similar stress conditions break roughly simultaneously, leading to fragmentation.

When equation (4) is compared with the earlier empirical fragmentation criterion[4,5], $x\Delta P \geqslant S$, the difference is large. Although the earlier empirical approach does not apply quantitatively, it does capture the role of bubbles in fragmentation, unlike other work[7].

Equation (1) above is general, and equation (4) represents a first-order approximation of the complicated real magma systems. In real volcanic systems, bubbles are of different sizes and distributed randomly (instead of uniformly); shells surrounding bubbles are not spherical, but consist of films separating two bubbles and plateau borders[6] (Fig. 1); and the films have variable thicknesses. Because stress tends to concentrate at corners or thin films, all these complexities help to concentrate the stress and make the system fragment more easily. Furthermore, only part of the outside surface of a bubble is exposed to P_{out} and the rest is embedded in the magma body (Fig. 1a). Future work using equation (1) and incorporating these complexities (numerical solution of stress distribution) is necessary to produce more realistic fragmentation models.

The strength of magma has been investigated experimentally. Webb and Dingwell[13] determined the tensile strength of fibre basaltic to rhyolitic melt to be 2500–5000 bar. Decrepitation experiments[14] confirmed that dry glasses with CO_2 or Xe vesicles have very high strength, but H_2O-bearing glasses have much lower strengths of 15–55 bar (the strengths are calculated from equation (4) using data[14] with the reported vesicularity; this is a correction to previous work[14] using a zero vesicularity). Experiments have been carried out to investigate the fragmentation of the dacitic melt[15] of Mount St Helens. For a 36% vesicularity of the melt, of which open vesicularity accounts for 29% and closed vesicularity accounts for only 7%, fragmentation has been shown to occur (Alidibirov *et al.*, personal communication) at $\Delta P \geqslant 30$ bar at 900°C and $\geqslant 90$ bar at 20 °C. Hence, using equation (4), $S \approx 40$ bar at 900 °C and $S \approx 120$ bar at 20 °C. By simple interpolation, $S \approx 60$ bar at 700 °C. However, because most bubbles in the experiments are interconnected, the gas in the bubbles escapes easily and may not exert much stress on bubble walls. Furthermore, closed bubbles were probably not pressurized during the fragmentation experiments. Hence the obtained strength is a maximum. That is, actual strength is likely to be less than 60 bar at 700 °C.

Once the tensile strength of the magma is known, the fragmentation point can be determined from equation (4) by calculating P_{dyn} as a bubble grows in a shell of magma (Fig. 1) using a program[12]. The accuracy of bubble growth calculation has been confirmed experimentally[16] using newly assessed H_2O diffusivity and solubility[17], provided a small correction is made to the viscosity model[18] (a factor of 1.4–3.6, well within the stated uncertainty).

The fragmentation criterion developed above can be used to address volcanological problems. A long-standing puzzle for volcanologists is the sudden fragmentation of seemingly benign lava domes and flows into dangerous pyroclastic flows[1–3]. To investigate the conditions for such fragmentation, I calculate bubble pressure and bubble wall stress using the method described above,

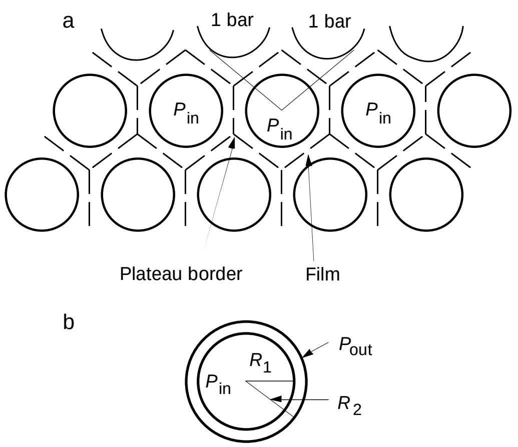

Figure 1 Simplification of real bubbly magma. **a,** By ignoring the variable bubble sizes, and non-uniform distribution, regularly spaced spherical bubbles of the same size are obtained. The upper surface is the broken (fragmented) surface. Even in this idealized case, each bubble (even if it is completely inside the magma) is not surrounded by a perfectly spherical shell. Instead, each bubble is surrounded by a melt shell with a spherical inner surface and polyhedral outer surface. **b,** Further approximation leads to spherical bubbles each surrounded by a spherical shell.

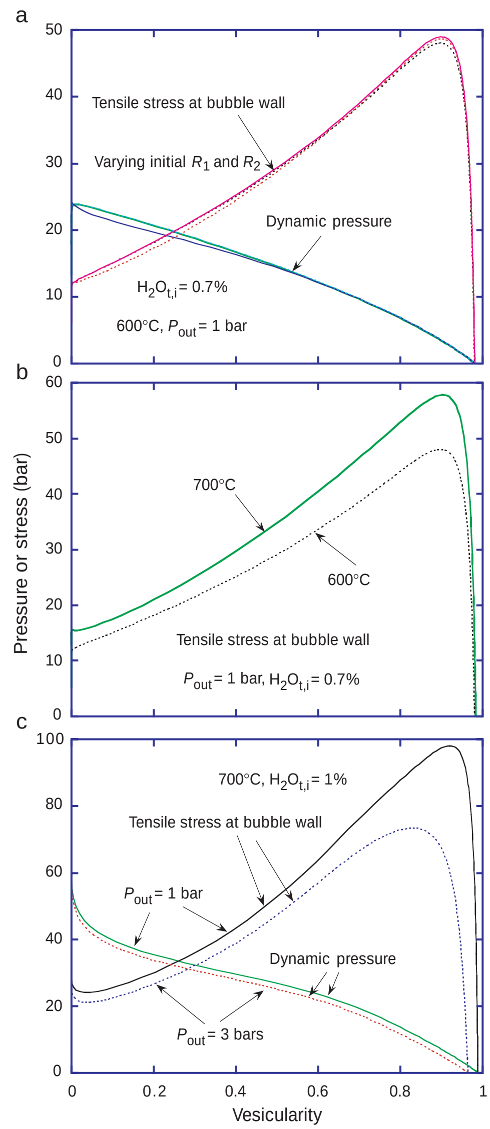

Figure 2 Calculated tangential stress (σ^{tt}) in bubble walls as a function of vesicularity. **a,b,c,** Different experimental conditions. The program of Proussevitch and Sahagian[12] is used for calculation with H_2O diffusivity and solubility from Zhang[17]. The viscosity is that of Hess and Dingwell[18] divided by two. The calculated stress may exceed the tensile strength because fragmentation is not incorporated into the calculation. The fragmentation point is obtained by comparing the calculated stress in the figure with the tensile strength. In **a,** the initial bubble radius R_1 varies from 1 to 5 μm and initial bubble+shell thickness R_2 varies from 50 to 200 μm.

without considering fractures. The calculated stress is then compared with tensile strength to determine the fragmentation point. Figure 2a shows that the calculated stresses are almost independent of the choice of the initial bubble radius (R_0) or initial melt volume surrounding each bubble (related to number density of bubbles). Hence the effect of the initial bubble size or number density is ignored. Figure 2b shows that the σ^{tt} versus x relation depends on the temperature. In a slowly growing dome when temperature decreases with time from 700 to 600 °C, it is expected that the σ^{tt} versus x relation would lie between the two trends in Fig. 2b. Figure 2c shows that σ^{tt} decreases as Pout increases. A comparison of Fig. 2b and c shows that the calculated stress increases with initial total H_2O ($H_2O_{t,i}$).

From Fig. 2, although P_{dyn} decreases with increasing vesicularity, σ^{tt} increases with x to roughly 90% vesicularity. The increase of σ^{tt} despite a decrease in ΔP is owing to the thinning of bubble walls (or decrease in x) so that ΔP must be accommodated in a small distance. Therefore, with gradual bubble growth in a slowly growing lava dome or a slowly advancing lava flow, the vesicularity may reach a critical value so that $\sigma^{tt} > S$, leading to a sudden fragmentation and crumbling of the dome or flow to a pyroclastic flow. The fragmentation conditions, depending strongly on $H_2O_{t,i}$, are listed in Table 1. The calculated vesicularity and final total H_2O level at fragmentation are reasonable values in pumice, suggesting that this criterion captures the essence of fragmentation. Furthermore, as long as $H_2O_{t,i}$ is high and the lava does not cool down rapidly, a dome or lava flow would eventually fragment when the stress reaches 50–60 bar. Hence, the composition $H_2O_{t,i}$ and temperature of a lava dome or flow can be used to forecast whether there will be fragmentation. The predicted vesicularity and final average total H_2O at fragmentation are negatively correlated when other conditions are equal but $H_2O_{t,i}$ is varied (Table 1). This prediction can be used to test my theory.

Factors that may help fragmentation of bubbly magma also include: a trigger to expose fresh interior bubbly lava owing to lava flow or gravitational instability may help fragmentation because of a sudden decrease in P_{out}; or the falling of a piece of lava may send a strong enough shock wave through the piece to break bubble shells, leading to fragmentation.

The application to fragmentation in a volcanic conduit during an eruption is similar in principle, but in practice entails large uncertainties. One such uncertainty is the variation of P_{out} with time, which depends on the dynamics of an eruption, which in turn depends on bubble growth and nucleation rates. In the program[12], the decompression is linear, which is undoubtedly too simple[19,20]. A second uncertainty is the number density of bubbles (that is, the choice of the initial shell thickness). Although it does not affect the stress versus vesicularity relation when P_{out} is constant (Fig. 2a), the number density (and hence new-bubble nucleation) plays an important role in modelling fragmentation in conduit eruptions. For example, for a given ascent rate, a greater number density implies more growth (and hence greater tensile stress) before the magma reaches an exit pressure of 1 bar.

In conclusion, on the basis of the maximum-stress theory for brittle failure, a bubbly magma fragments when the tensile stress at the inner walls of bubbles exceeds the tensile strength of the magma. This fragmentation criterion appears to effectively characterize the fragmentation of slowly growing lava domes and slowly advancing lava flows. Future work should include numerical-stress and bubble-growth calculations around bubbles with non-spherical shells, quantitative understanding of the strength of magma, and interaction between bubble growth/nucleation and conduit eruption dynamics.

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Tube pumices as strain markers of the ductile–brittle transition during magma fragmentation

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Magma fragmentation—the process by which relatively slow-moving magma transforms into a violent gas flow carrying fragments of magma—is the defining feature of explosive volcanism. Yet of all the processes involved in explosively erupting systems, fragmentation is possibly the least understood[1,2]. Several theoretical and laboratory studies on magma degassing[3–7] and fragmentation[8–11] have produced a general picture of the sequence of events leading to the fragmentation of silicic magma[12–14]. But there remains a debate[2] over whether magma fragmentation is a consequence of the textural evolution of magma to a foamed state where disintegration of walls separating bubbles becomes inevitable due to a foam-collapse criterion, or whether magma is fragmented purely by stresses that exceed its tensile strength. Here we show that tube pumice—where extreme bubble elongation is observed—is a well-preserved magmatic 'strain marker' of the stress state immediately before and during fragmentation. Structural elements in the pumice record the evolution of the magma's mechanical response from viscous behaviour (foaming and foam