

# CAVE NOTES

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## CAVE CONDUIT ENLARGEMENT BY NATURAL CONVECTION

by Rane L. Curl\*

When limestone goes into solution in water, or in water containing dissolved carbon dioxide, there is an increase in solution density. The resulting density difference between the solution near a conduit's walls and in the bulk flow can induce a circulation of the water—downward at the dissolving surface and upward within the bulk. As relatively fresh solvent is brought to the ceiling or upper walls by this density-induced flow, we may expect a tendency toward formation of features such as upward conduit enlargement or ceiling "pots" or tubes.

In real situations, there is also a forced flow through the conduit which causes limestone dissolution without a similar preference for the location and orientation of the surface. Which mode of dissolution is dominant may be expected to depend on the flow rate, the properties of the water and solutes, and the concentration difference between the solution at the walls and in the bulk flow. An estimate is offered here for the relation between these variables, which divides the density current, or natural convection regime, from that of forced convection dissolution. This estimate is based on information concerning laminar flow in horizontal tubes.

Eubank and Proctor (1951) have correlated data for laminar flow heat transfer in horizontal tubes in which transfer by natural as well as forced convection is important. Their relation may be applied to mass transfer (dissolution) because of the analogy between the two processes (Bird, Stewart, Lightfoot, 1960: p. 642-647). No similar correlation is available for turbulent flow. The transverse velocities associated with turbulence probably diminish the effects of natural convection while increasing forced convection transfer. Nevertheless, natural convection effects undoubtedly extend into the turbulent regime, especially with larger density differences.

The Eubank-Proctor relation for the mass transfer coefficient is

$$\frac{kd}{D} = 1.75 \left[ \frac{\pi d^2 U}{4LD} + 0.04 \left( \frac{d^4 \rho g \alpha}{\mu LD} \right)^{0.75} \right]^{1/3} \quad (1)$$

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where the variables are:

- k: mean mass transfer coefficient over distance L (cm /sec)  
 d: conduit diameter (cm)  
 D: solute (calcium bicarbonate) diffusivity (cm<sup>2</sup>/sec)  
 L: conduit length (cm)  
 g: acceleration of gravity (981 cm/sec<sup>2</sup>)  
 $\mu$ : solution viscosity (gm/cm sec)  
 $\rho$ : solution density (gm/cm<sup>3</sup>)  
 $\alpha$ : fractional density difference between wall and bulk solutions.

The first term on the right in Equation (1) is contributed by the forced convection mechanism, and the second, by that of natural convection (note that gravity is regarded as a variable). Therefore, we may estimate the relation between the variables which divides the forced from the natural convection regimes on the basis of the relative magnitudes of the two groups. That is, natural convection becomes important when

$$0.04 \left( \frac{d^4 \rho g \alpha}{\mu DL} \right)^{0.75} > \frac{\pi d^2 U}{4LD}. \quad (2)$$

This may be rearranged to

$$d > \frac{v^{2/5}}{g^{3/5}} \left( \frac{\pi}{0.16} \right)^{4/5} \left( \frac{d}{L} Sc \right)^{1/5} \frac{U^{4/5}}{\alpha^{3/5}} \quad (3)$$

where  $v = \mu/\rho$ , the kinematic viscosity, and  $Sc = \mu/\rho D$ , the Schmidt number. The variable L has been made to appear on the right-hand side in the ratio  $d/L$ , which accounts for the greater average influence of natural convection in long tubes due to the greater development of the denser region near the wall. It certainly enters into the situation for which the correlation was developed, but irregularities in cave conduits tend to cause some mixing in the flow. Moreover,  $d/L$  appears to the 1/5 power, so that its effect is small. Assuming that the effective development distance for natural convection is but one conduit diameter, we take  $d/L = 1.0$ . This gives a conservative estimate of the lower limit (in  $d$ ) to the natural convection regime. At 10°C,  $v = 0.0131$  cm<sup>2</sup>/sec (Hunsaker and Rightmire, 1947: p. 449) and  $D = 0.66 \times 10^{-5}$  cm<sup>2</sup>/sec (Reid and Sherwood, 1958). With these values, Equation (3) becomes

$$d > 0.141 \frac{U^{4/5}}{\alpha^{3/5}} \quad (4)$$

with  $d$  in cm and  $U$  in cm/sec. Equation (4) is plotted in Figure 1. The fractional density difference  $\alpha$  is shown in terms of parts per million, or  $10^6 \alpha$ . For conditions lying above a particular  $\alpha$  line, natural convection becomes dominant. Also shown in Figure 1 are the laminar-turbulent transition boundary (for flow in a smooth round tube) and lines giving the total volumetric flow in gallons per minute (U.S.), for any combination of  $d$  and  $U$ . The exclusion of the turbulent regime is, as indicated above, due to lack of information, rather than an assumption that natural convection is absent.

The dependence of  $\alpha$  on solution composition differences can be estimated from information on the partial molal volumes of the solutes. For very dilute solutions the density depends on composition according to

$$\alpha = \left[ \frac{M}{\rho} - \bar{V}^0 \right] \Delta C \quad (5)$$

where  $M$  and  $\bar{V}^0$  are the molecular weight and partial molal volumes, respectively, of the solute at infinite dilution, and  $\Delta C$  is the solute concentration difference between the surface and bulk solutions expressed in gram moles/cm<sup>3</sup> (Harned and Owen, 1950: p. 250). In the solution of limestone in dilute carbonic acid, the latter combines with the former to produce soluble calcium bicarbonate, but simultaneous diffusion of all species may produce compositions other than that indicated by the simple reaction  $\text{CaCO}_3 + \text{CO}_2 + \text{H}_2\text{O} = \text{Ca}^{+2} + 2\text{HCO}_3^{-1}$ . However, we can at least estimate the net partial molal volume associated with this process by considering it as the volume effect associated with the addition of one mole of calcium bicarbonate and the simultaneous subtraction of one mole each of water and carbon dioxide. The values for partial molal volumes (at 25 °C) that will be used are given in Table I. These give  $\bar{V}^0 = -22.7 \text{ cm}^3/\text{gm mol}$  for the assumed process, while the net  $M$  is only the molecular weight of  $\text{CaCO}_3$ , 100 gm/gm mol. Consequently,  $\alpha = 123\Delta C$  or, in terms of ppm  $\text{CaCO}_3$  difference,

$$\alpha = 1.23 \times 10^{-6} \Delta(\text{ppm CaCO}_3) \quad (6)$$

Note that a 1.0 ppm concentration difference causes a 1.23 ppm density difference, due essentially to solvent contraction upon dissolving limestone.

Figure 1 shows that, with sufficiently slow water circulation through limestone conduits, natural convection may be the primary flow mode for limestone removal. This is possible with even extremely small composition differences. The common upward enlargement of anastomoses above bedding planes could be explained as a solution feature dominated by natural convection dissolution. While small flow velocities are required for natural convection to be important, such small flows certainly occur. Whether they last for periods long enough to control anastomosis development remains a question.

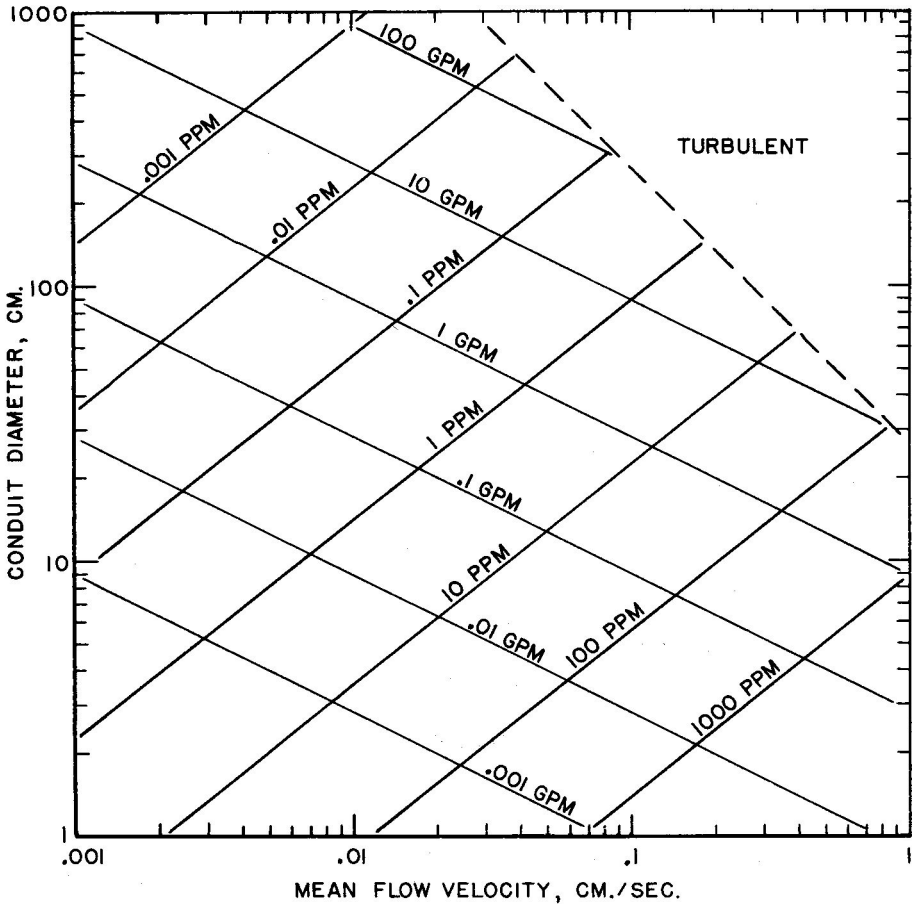
The transition from natural to forced convection is smooth. If we call the "transition region" that in which one of the terms in Equation (2) is no more than twice the other, the transition region for constant  $U$  lies between 0.57d and 1.74d, while the transition flow rates for constant  $d$  lie between 0.5U and 2U, where  $d$  and  $U$  are the "boundary" values from Figure 1.

This analysis gives no indication of the degree of morphological modification to be expected from natural convection since only the relative importance of the two regions are compared. A circular conduit with only forced convection dissolution would remain circular, but if natural convection is also present the flow is complex. The problem of finding local dissolution rates around the perimeter of a soluble conduit with significant natural convection has not been solved. We would expect the development of vertical asymmetry but the changing shape would also modify the flow, giving an even more intractable problem.

Model experiments for cave development performed with water flowing between, say, salt blocks should be scaled in all significant phenomena. When scaling the modelled process, we can maintain forced convection similarity by keeping the model Reynolds' number and Schmidt number the same as in the prototype. To maintain similarity for natural convection we must also keep the Grashof number  $d^3\rho^2g\alpha/\mu^2$ , the same. Thus for a model 1/10 natural scale we require flow velocities 10 times as great for Reynolds'

TABLE I

$\text{Ca}^{+2} + 2\text{HCO}_3^{-1}$ .....	29.3 cm <sup>3</sup> /gm mol (Couture, et al., 1957, p. 207; and Owen, et al., 1942, p. 461).
$\text{CO}_2$ .....	44.0 cm <sup>3</sup> /gm mol (Lauder, 1959, p. 40)
$\text{H}_2\text{O}$ .....	18.0 cm <sup>3</sup> /gm mol



**Figure 1.** Natural and forced convection dissolution regimes. Above a line showing the solution density difference (in parts per million, PPM) natural convection predominates over forced convection. The volumetric flow (GPM, U.S.) is also shown as a function of conduit size and flow velocity.

number similarity and  $\alpha$  1000 times as great for Grashof number similarity. For a density difference of 10 ppm in nature, a 10,000 ppm density difference is required in the model. This is equivalent to dissolving salt with water already about 99% saturated. Using water alone, the effect of natural convection is grossly exaggerated. The salt solution undersaturation in any particular case must be adjusted to equivalence to the prototype density difference—an unknown which introduces another degree of freedom into model experiments.

Natural convection flows may arise from temperature differences as well as varying compositions. Wall temperatures may lag behind varying water temperatures, although in this case the direction of the effect will reverse as stream temperatures rise and fall. A more stable situation may arise where two streams of different temperature join. This is a possible alterna-

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tive to the "Mischungskorrosion" hypothesis for certain ceiling and wall tubes, or avens, if the upper stream is colder than the main stream. At present, however, this possibility remains conjectural.

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