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SYSTEM STUDY CONCERNING AN ANTENNA SUITABLE FOR A SPINNING-GONIOMETER DIRECTION-FINDING SYSTEM

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ABSTRACT

Three antenna arrays suitable for a spinning-goniometer direction-finding system have been analyzed, and their relative advantages and disadvantages with respect to a standard, four-element Adcock system have been studied. An eight-element configuration using two signal goniometers was found to be superior to any of those studied. A double Adcock array consisting of two such eight-element configurations is proposed. It is shown that the bearing error of the double Adcock is on the order of a tenth of a degree, and the sensitivity factor exceeds that of the standard, four-element Adcock by an order of magnitude.

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1. INTRODUCTION

In this study an attempt was made to determine an "optimum" antenna configuration for a spinning-goniometer radio direction-finding system. This system was to be optimized with respect to four quantities: (1) error, (2) sensitivity, (3) bandwidth, and (4) cost. The objective was an antenna configuration which had better sensitivity than the standard, four-element Adcock with a bandwidth of 2 to 30 Mc and with bearing errors for all frequencies of less than 2 degrees.

The various configurations were studied, first with respect to their inherent bearing errors, and secondly with respect to their relative signal sensitivities.

2. BASIS FOR COMPARISON

The first step taken was the investigation of various ways of connecting antenna elements together; that is, would advantages be gained by connecting adjacent elements in series, parallel, or some other type of connection? As a basis for all comparison the four-element Adcock antenna was studied first.

The plan view of this system is shown in Fig. 1. Let O be the phase center of the array, with a wave impinging in the direction ıown. Then the voltages induced in each element are:

$$e_N = A\, e^{j(\omega t + \varphi_N)}$$

$$e_S = A\, e^{j(\omega t - \varphi_S)}$$

$$e_E = A\, e^{j(\omega t + \varphi_E)}$$

$$e_W = A\, e^{j(\omega t - \varphi_W)} \qquad (1)$$

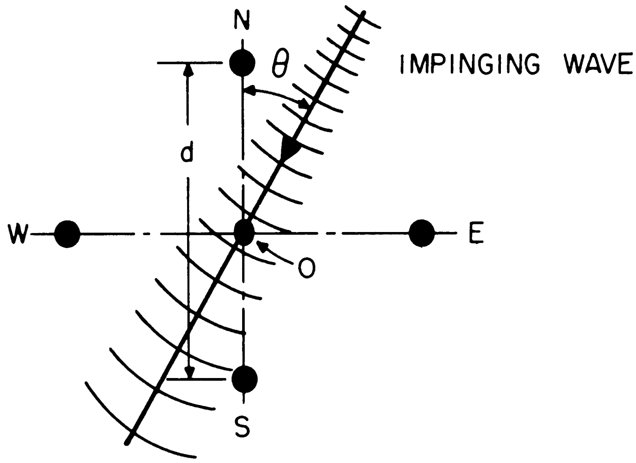


Fig. 1. Plan view of Adcock antenna.

where:

$$\varphi_N = \varphi_S = \frac{\pi d}{\lambda} \cos\theta$$

$$\varphi_E = \varphi_W = \frac{\pi d}{\lambda} \sin\theta \qquad (2)$$

In the goniometer one forms

$$e_N - e_S = +2j\, A\, e^{j\omega t} \sin\varphi_N \qquad (3)$$

$$e_E - e_W = +2j\, A\, e^{j\omega t} \sin\varphi_E \qquad (4)$$

and the indicated angle of arrival, φ, assuming no errors are introduced by the goniometer, is found from:

$$\tan \varphi = \left| \frac{e_E - e_W}{e_N - e_S} \right| = \frac{\sin \varphi_E}{\sin \varphi_N} = \frac{\sin \left[\frac{\pi d}{\lambda} \sin \theta \right]}{\sin \left[\frac{\pi d}{\lambda} \cos \theta \right]} \tag{5}$$

If $\frac{\pi d}{\lambda}$ is small:

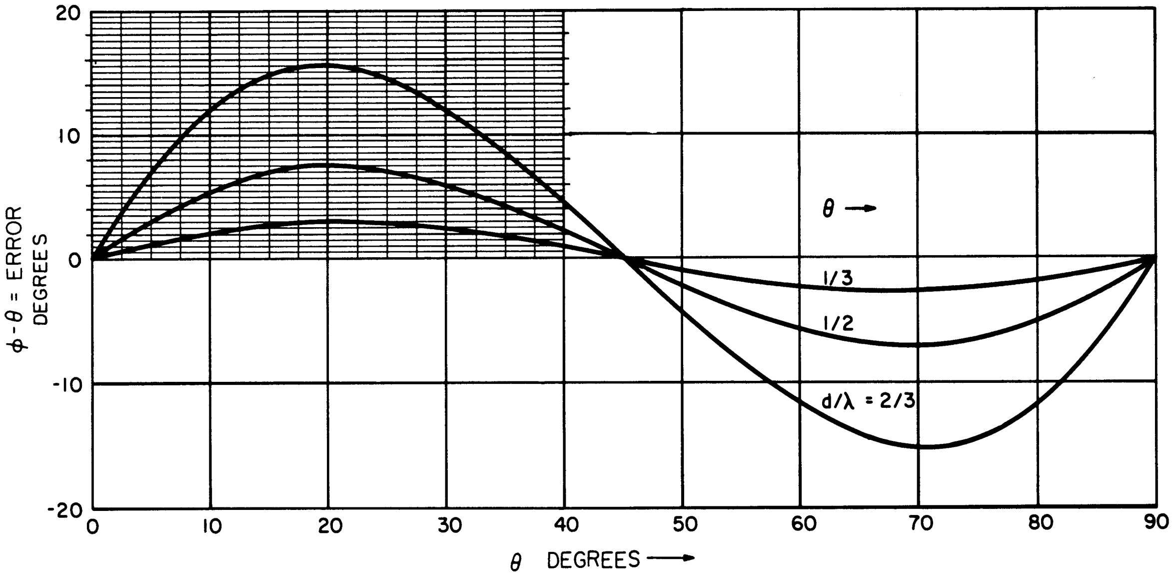
$$\tan \varphi \cong \frac{\frac{\pi d}{\lambda} \sin \theta}{\frac{\pi d}{\lambda} \cos \theta} = \tan \theta, \text{ and } \varphi \cong \theta = \text{the true bearing.}$$

However, in order to obtain the greatest sensitivity at the lowest frequency received, d should be as large as possible (within practical limits). The array diameter, d, will be limited by the greatest error which can be tolerated at the highest frequency to be received. It is of interest, then, to calculate the error for various spacings, d, and to plot $(\varphi - \theta)$ vs. θ for various relative spacings, d/λ. This is shown in Fig. 2.

3. CONFIGURATION NUMBER 1

The first configuration investigated is shown in Fig. 3. Let O be the phase reference point and θ be the angle of the incoming plane wave measured clockwise from a center line passing through antennas 1, 5, 6, and 2.

The idea here is to add the voltages of antennas 1 and 5, and those of 6 and 2, and subtract (6 + 2) from (1 + 5) to form the north-south antenna-pair voltage. In like manner, the east-west antenna-pair voltage is formed.



$$\theta = \text{true bearing (in degrees)}$$

$$\varphi = \tan^{-1} \frac{\sin\left[\frac{\pi d}{\lambda} \sin\theta\right]}{\sin\left[\frac{\pi d}{\lambda} \cos\theta\right]} = \text{indicated bearing}$$

Fig. 2. Octantal error curves for Adcock antenna.

Mathematically:

$$\begin{aligned} e_1 &= A\,e^{j(\omega t + \varphi_1)} & e_3 &= A\,e^{j(\omega t + \varphi_3)} \\ e_2 &= A\,e^{j(\omega t - \varphi_2)} & e_4 &= A\,e^{j(\omega t - \varphi_4)} \\ e_5 &= A\,e^{j(\omega t + \varphi_5)} & e_7 &= A\,e^{j(\omega t + \varphi_7)} \\ e_6 &= A\,e^{j(\omega t - \varphi_6)} & e_8 &= A\,e^{j(\omega t - \varphi_8)} \end{aligned} \qquad (6)$$

where:

$$\varphi_1 = \varphi_2 = \frac{\pi d_1}{\lambda} \cos\theta \qquad \varphi_3 = \varphi_4 = \frac{\pi d_1}{\lambda} \sin\theta$$

$$\varphi_5 = \varphi_6 = \frac{\pi d_2}{\lambda} \cos\theta \qquad \varphi_7 = \varphi_8 = \frac{\pi d_2}{\lambda} \sin\theta \qquad (7)$$

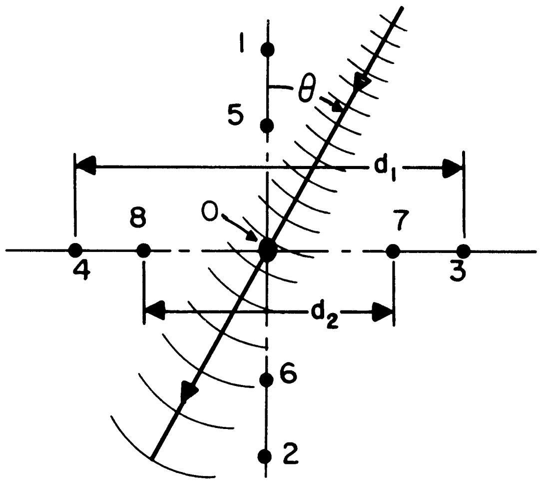


Fig. 3. Plan view of Configuration No. 1.

It can be shown that:

$$(e_1 + e_5) - (e_2 + e_6) = +4jAe^{j\omega t} \sin \frac{\varphi_1 + \varphi_5}{2} \cos \frac{\varphi_1 - \varphi_5}{2} \qquad (8)$$

and

$$(e_3 + e_7) - (e_4 + e_8) = +4jAe^{j\omega t} \sin \frac{\varphi_3 + \varphi_7}{7} \cos \frac{\varphi_3 - \varphi_7}{2} \qquad (9)$$

The indicated bearing, φ, is:

$$\tan \varphi = \frac{\sin \frac{\varphi_3 + \varphi_7}{2} \cos \frac{\varphi_3 - \varphi_7}{2}}{\sin \frac{\varphi_1 + \varphi_5}{2} \cos \frac{\varphi_1 - \varphi_5}{2}} = \frac{\sin\left[\frac{\pi(d_1 + d_2)}{2\lambda} \sin \theta\right] \cos \left[\frac{\pi(d_1 - d_2)}{2\lambda} \sin \theta\right]}{\sin\left[\frac{\pi(d_1 + d_2)}{2\lambda} \cos \theta\right] \cos \left[\frac{\pi(d_1 - d_2)}{2\lambda} \cos \theta\right]}$$

Letting now $d_2 = kd_1$, where $0 \leqq k \leqq 1$:

$$\tan \varphi = \frac{\sin \left[(1 + k) \frac{\pi d_1}{2\lambda} \sin \theta\right] \cos \left[(1 - k) \frac{\pi d_1}{2\lambda} \sin \theta\right]}{\sin \left[(1 + k) \frac{\pi d_1}{2\lambda} \cos \theta\right] \cos \left[(1 - k) \frac{\pi d_1}{2\lambda} \cos \theta\right]} \qquad (10)$$

A few observations can now be made. Consider first antennas 1, 2, 5, and 6. The voltages of antennas 1 and 5 can be added together through the use of transformers, as can those of antennas 6 and 2. The connection is shown in Fig. 4.

The subtraction of $(e_2 + e_6)$ from $(e_1 + e_5)$ is obtained when the proper connections are made to the goniometer. Actually the above operation [the subtraction of $(e_2 + e_6)$ from $(e_1 + e_5)$] can be thought of in a different way. e_2 and e_6 can be subtracted from e_1 and e_5, respectively, and the resulting voltages can then be added. This operation can be obtained using two goniometers (see Fig. 5). If the transformer scheme were used for implementing Configuration No. 1, eight transformers and one goniometer would be required; whereas only two goniometers would be required for implementing it if the goniometer connection were used. A polyphase goniometer of proper design could also be used.

If Eqs. 3 and 4 are compared with (8) and (9), it is seen that the output voltage of the system is doubled. The above analysis does not, however, take into account mutual effects between antennas, but it does appear that something can be gained by using eight antennas instead of four, as far as sensitivity is concerned.

If k is set equal to 1 or 0 in Eq. 10, the antenna system degenerates into the ordinary four-element Adcock system. If d_1/λ is set equal to 2/3, error curves are as plotted in Fig. 6 for k = 1/2, k = 1/3, and k = 2/3. By comparing these curves with those of Fig. 2, it is seen that for a spacing of $d_1/\lambda = 2/3$, the error has been reduced by approximately 5°. Clearly, we appear to be moving in the right direction.

If we examine Eq. 10, we see that it can be divided into two multiplicative terms:

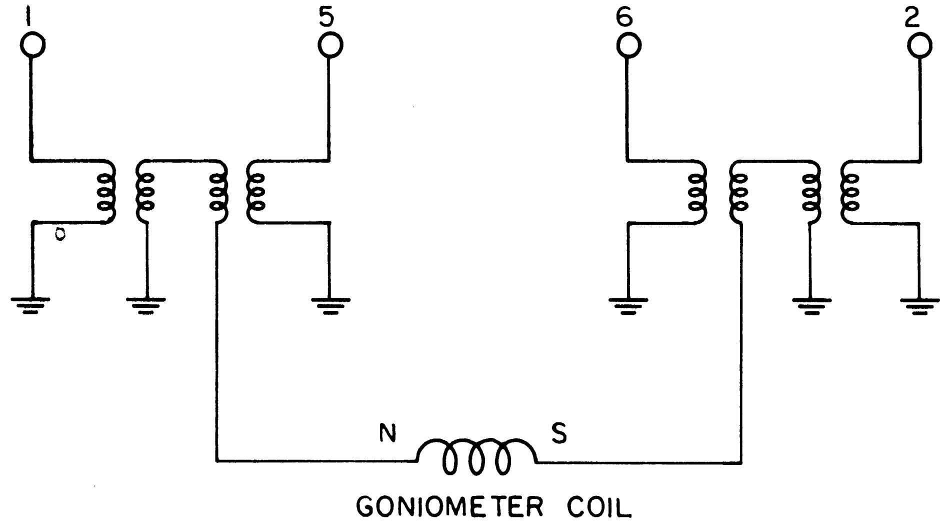


Fig. 4. Transformer connection for Configuration No. 1.

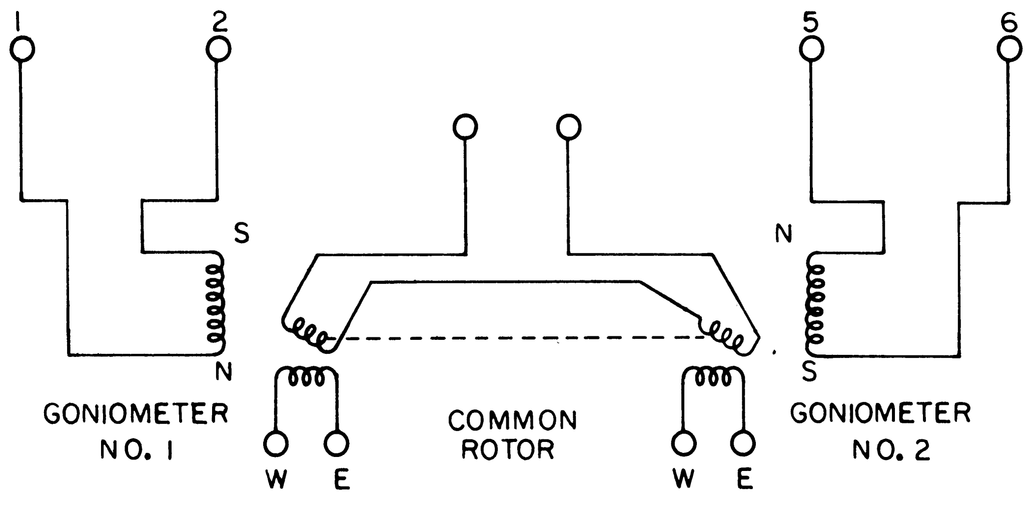
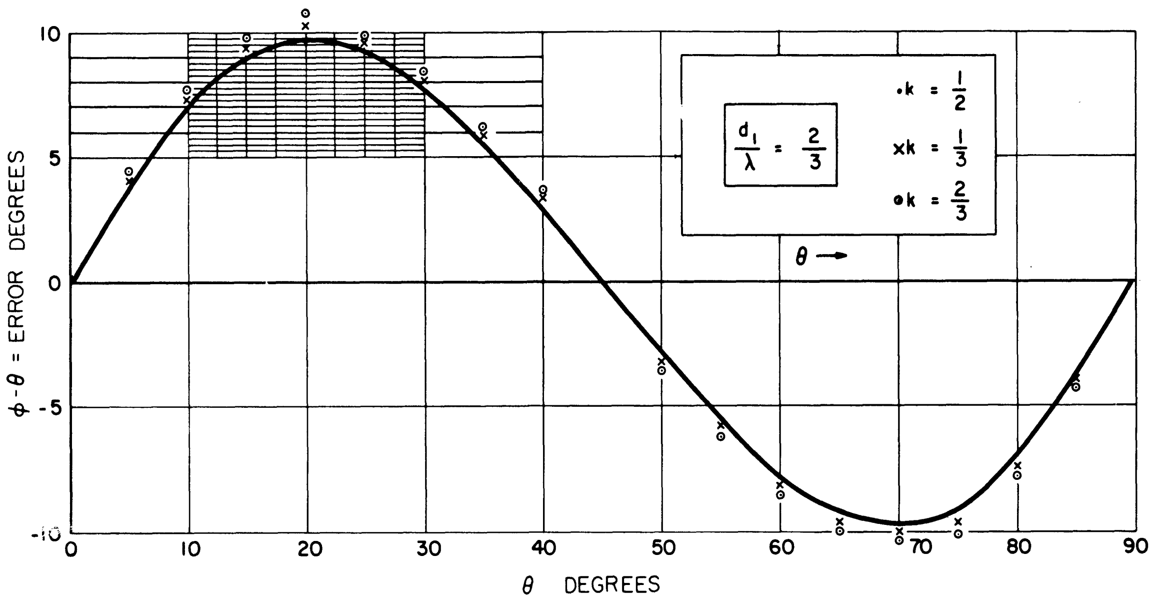


Fig. 5. Two-goniometer connection. The rotors are tied together and phased such that their induced voltages are additive.



θ = true bearing (in degrees)

$$\tan \varphi = \frac{\sin\left[(1 + k)\ \frac{\pi d_1}{2\lambda} \sin \theta\right]}{\sin\left[(1 + k)\ \frac{\pi d_1}{2\lambda} \cos \theta\right]} \cdot \frac{\left[\cos(1 - k)\ \frac{\pi d_1}{2\lambda} \sin \theta\right]}{\left[\cos(1 - k)\ \frac{\pi d_1}{2\lambda} \cos \theta\right]}$$

φ = indicated bearing

$$k = \frac{d_2}{d_1}$$

Fig. 6. Octantal error curves.

$$\tan \varphi = \left\{\frac{\sin\left[(1 + k)\ \frac{\pi d_1}{2\lambda} \sin \theta\right]}{\sin\left[(1 + k)\ \frac{\pi d_1}{2\lambda} \cos \theta\right]}\right\} \cdot \left\{\frac{\cos\left[(1 - k)\ \frac{\pi d_1}{2\lambda} \sin \theta\right]}{\cos\left[(1 - k)\ \frac{\pi d_1}{2\lambda} \cos \theta\right]}\right\} \qquad (11)$$

Intuitively, this fact suggests that it may be possible to place the antennas in such a way that one of the multiplicative terms would correct for the octantal error. This is indeed the case, as will now be shown.

We shall use the two-goniometer method for taking the sums and differences of the proper antenna voltages. It appears from the above that if the inner four antennas are rotated by 45^o in a counter-clockwise direction about the phase reference point, and the rotor of the goniometer associated with these antennas is rotated such that its null position for a signal arriving from North is the same as for the goniometer associated with the North-South, East-West Adcock array, then error cancellation will occur. Actually the inner array diameter should be increased until all antennas lie on a circle with diameter d. This antenna array will now be analyzed and will be called Configuration No. 2 (see Fig. 7).

4. CONFIGURATION NUMBER 2

The induced antenna voltages are:

$$\begin{aligned} e_1 &= A_a e^{j(\omega t + \varphi_1)} & e_5 &= A_a e^{j(\omega t + \varphi_5)} \\ e_2 &= A_a e^{j(\omega t - \varphi_2)} & e_6 &= A_a e^{j(\omega t - \varphi_6)} \\ e_3 &= A_a e^{j(\omega t + \varphi_3)} & e_7 &= A_a e^{j(\omega t + \varphi_7)} \\ e_4 &= A_a e^{j(\omega t - \varphi_4)} & e_8 &= A_a e^{j(\omega t - \varphi_8)} \end{aligned} \qquad (12)$$

where:

$$\begin{aligned} \varphi_1 &= \varphi_2 = \frac{\pi d}{\lambda} \cos \theta & \varphi_5 &= \varphi_6 = \frac{\pi d}{\lambda} \cos \left(\theta - \frac{\pi}{4}\right) \\ \varphi_3 &= \varphi_4 = \frac{\pi d}{\lambda} \sin \theta & \varphi_7 &= \varphi_8 = \frac{\pi d}{\lambda} \sin \left(\theta - \frac{\pi}{4}\right) \end{aligned} \qquad (13)$$

Since the goniometer rotor positions and connections are important, the conventions used in the analysis are shown in Fig. 8.

The input voltages to the goniometers are:

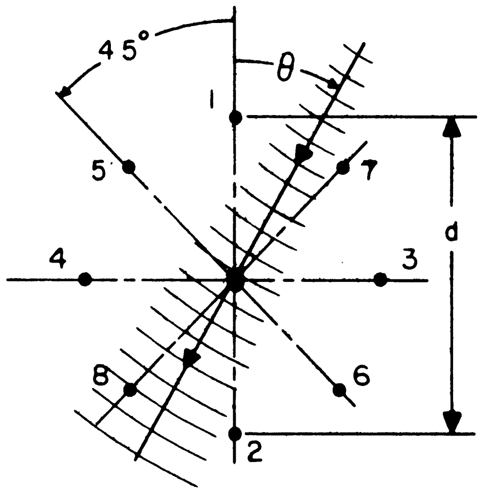
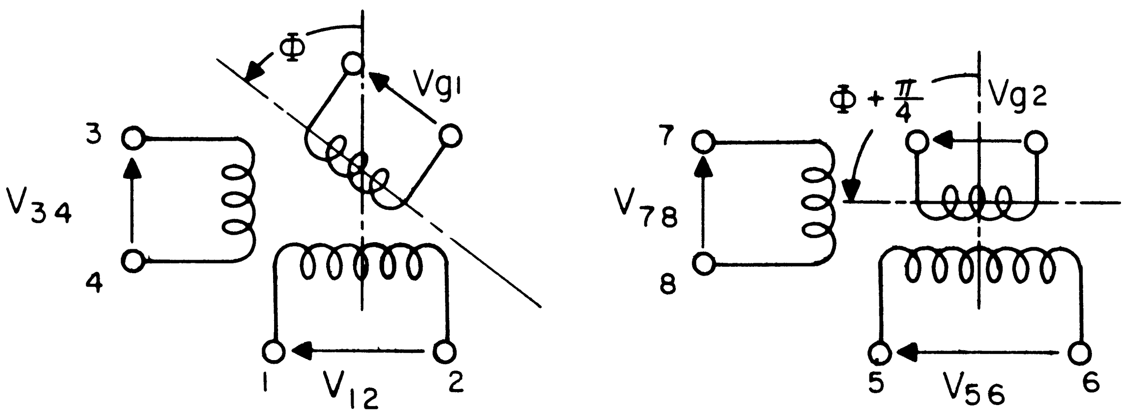


Fig. 7. Plan view of Configuration No. 2.



Goniometer No. 1. Goniometer No. 2.

$\Phi = \omega_m t$ where ω_m = rotation rate of goniometers.

Fig. 8. Goniometer connections and rotor positions used for Configuration No. 2.

$$V_{12} = Re[e_1 - e_2] = -2\ A_a \sin \varphi_1 \sin \omega t$$

$$V_{34} = Re[e_3 - e_4] = -2\ A_a \sin \varphi_3 \sin \omega t$$

$$V_{56} = Re[e_5 - e_6] = -2\ A_a \sin \varphi_5 \sin \omega t$$

$$V_{78} = Re[e_7 - e_8] = -2\ A_a \sin \varphi_7 \sin \omega t \qquad (14)$$

The complex output voltages of the goniometers are:

$$V_{g1} = A_{g1} \left\{ -jRe[e_1 - e_2]\ e^{j\Phi} + Re[e_3 - e_4]\ e^{j\Phi} \right\}$$

$$V_{g2} = A_{g2} \left\{ -jRe[e_5 - e_6]\ e^{j(\Phi + \frac{\pi}{4})} + Re[e_7 - e_8]\ e^{j(\Phi + \frac{\pi}{4})} \right\} \qquad (15)$$

where:

$A_{g1} = A_{g2} = A_g$ are constants depending upon the goniometers used.

Let V_T = output voltage fed to the receiver = $Re \left\{ V_{g1} + V_{g2} \right\}$. It can be shown that:

$$Re\ V_{g1} = -2\ A_g A_a \sin \omega t [\sin \varphi_1 \sin\Phi + \sin \varphi_3 \cos \Phi]$$

$$Re\ V_{g2} = -2\ A_g A_a \sin \omega t [\sin \varphi_5 \sin (\Phi + \tfrac{\pi}{4}) + \sin \varphi_7 \cos (\Phi + \tfrac{\pi}{4})] \qquad (16)$$

Therefore

$$V_T = -2\ A_g A_a \sin \omega t \left\{ \left[\sin \varphi_1 + \frac{1}{\sqrt{2}} \sin \varphi_5 - \frac{1}{\sqrt{2}} \sin \varphi_7 \right] \sin \Phi + \left[\frac{1}{\sqrt{2}} \sin \varphi_5 + \sin \varphi_3 + \frac{1}{\sqrt{2}} \sin \varphi_7 \right] \cos \Phi \right\} \qquad (17)$$

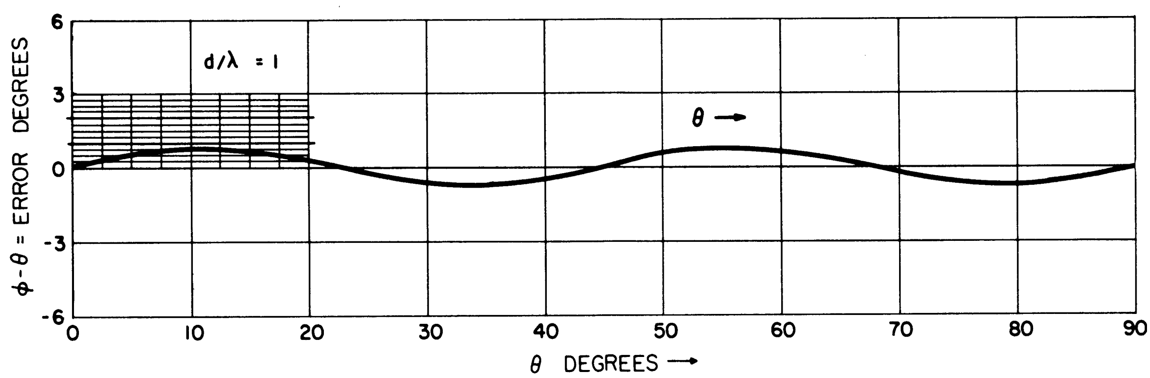
The indicated angle of arrival can now be determined from

$$\tan \varphi = \frac{\frac{1}{\sqrt{2}} \sin \varphi_5 + \frac{1}{\sqrt{2}} \sin \varphi_7 + \sin \varphi_3}{\frac{1}{\sqrt{2}} \sin \varphi_5 - \frac{1}{\sqrt{2}} \sin \varphi_7 + \sin \varphi_1}$$

or

$$\tan \varphi = \frac{\sin\left[\frac{\pi d}{\lambda} \cos(\theta - \frac{\pi}{4})\right] + \sin\left[\frac{\pi d}{\lambda} \sin(\theta - \frac{\pi}{4})\right] + \sqrt{2} \sin\left[\frac{\pi d}{\lambda} \sin \theta\right]}{\sin\left[\frac{\pi d}{\lambda} \cos(\theta - \frac{\pi}{4})\right] - \sin\left[\frac{\pi d}{\lambda} \sin(\theta - \frac{\pi}{4})\right] + \sqrt{2} \sin\left[\frac{\pi d}{\lambda} \cos \theta\right]} \tag{18}$$

If we let now $d/\lambda = 1$, the bearing error, $\varphi - \theta$, can be calculated; this is shown in Fig. 9. It is clearly seen that a large decrease in error is achieved using this scheme, and it appears that a considerable improvement has been achieved.



θ = true bearing

φ = indicated bearing

$$\tan \varphi = \frac{\sin\left[\frac{\pi d}{\lambda} \cos(\theta - \frac{\pi}{4})\right] + \sin\left[\frac{\pi d}{\lambda} \sin(\theta - \frac{\pi}{4})\right] + \sqrt{2} \sin\left[\frac{\pi d}{\lambda} \sin \theta\right]}{\sin\left[\frac{\pi d}{\lambda} \cos(\theta - \frac{\pi}{4})\right] - \sin\left[\frac{\pi d}{\lambda} \sin(\theta - \frac{\pi}{4})\right] + \sqrt{2} \sin\left[\frac{\pi d}{\lambda} \cos \theta\right]}$$

Fig. 9. Error curve.

5. SENSITIVITY CONSIDERATIONS

One must, however, also consider the sensitivity of the various configurations. A figure of merit of the sensitivity is given by a quantity which we shall call "sensitivity factor." This is a measure of the magnitude of the magnetic field set up inside the goniometer by a signal impinging upon the antenna array. In general it is a function of the bearing of the incoming wave and the antenna array being used. As before, the four-element Adcock antenna was used as a basis for comparison.

The output voltage of the signal goniometer of the standard Adcock antenna is given by:

$$V_T = -[2\ A_a\ A_g \sin \varphi_N \sin \Phi + 2\ A_a\ A_g \sin \varphi_E \cos \Phi] \sin \omega t \qquad (19)$$

where Φ is the instantaneous goniometer position as defined in Fig. 8. The sensitivity factor can be immediately determined from this expression and is defined as the square root of the sum of the squares of the coefficients of the $\sin \Phi$ and $\cos \Phi$ terms; hence,

Sensitivity Factor =

$$\text{S.F.} = 2\ A_a\ A_g \sqrt{\sin^2 \left[\frac{\pi d}{\lambda} \cos \theta\right] + \sin^2 \left[\frac{\pi d}{\lambda} \sin \theta\right]} \qquad (20)$$

This equation was plotted as a function of θ in Fig. 10 for values of $d/\lambda = 1/3$, $d/\lambda = 1/2$, and $d/\lambda = 2/3$. The quantity $2\ A_a\ A_g$ was set equal to 1 to normalize the result.

It can clearly be seen from this that one must consider the sensitivity of the array as a function of the angle of the incoming electromagnetic wave. For the standard Adcock antenna configuration

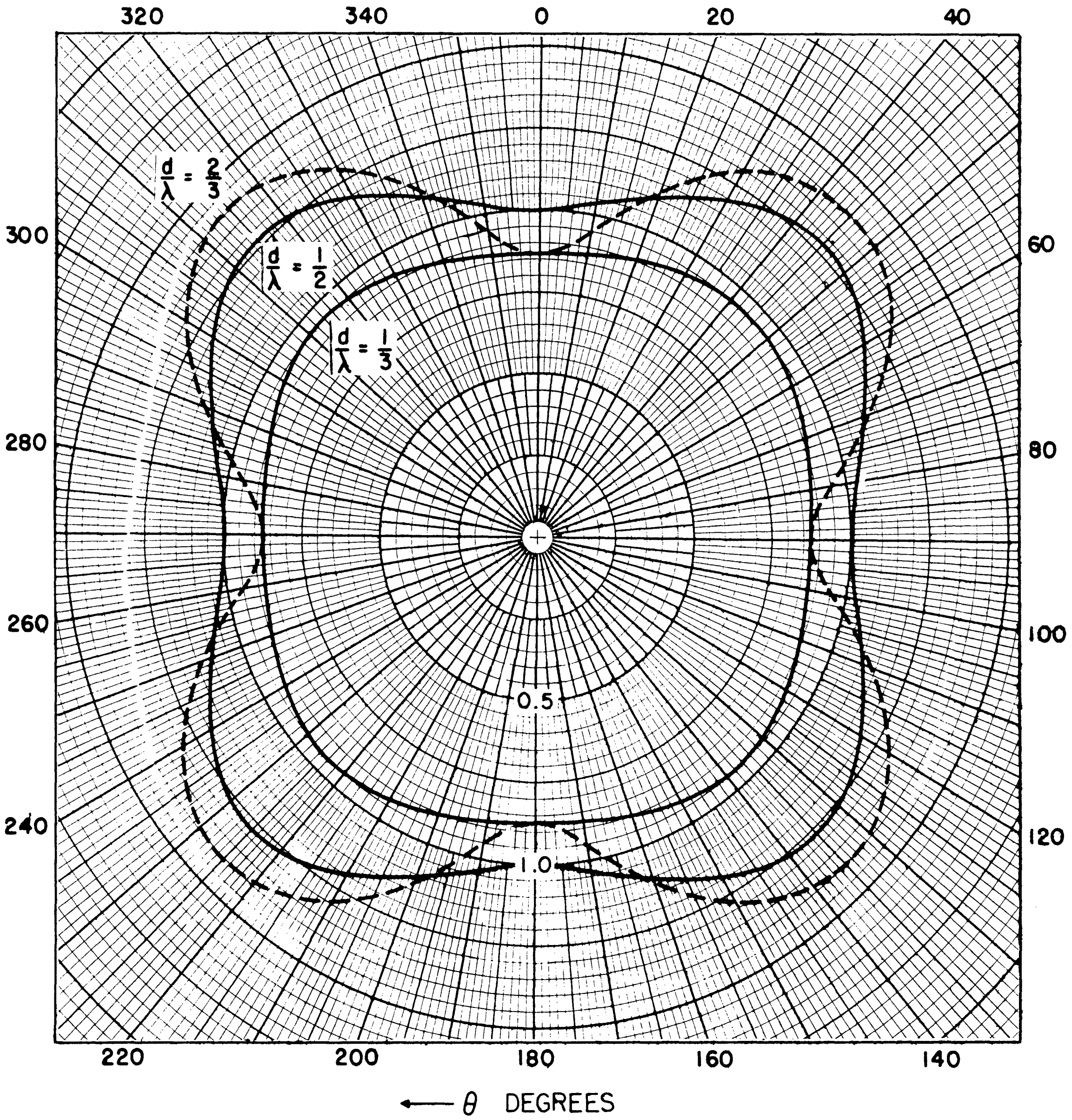


Fig. 10. Sensitivity factor; Adcock antenna.

the greatest overall sensitivity occurs when $d/\lambda = 1/2$. When $d/\lambda > 1/2$, it is true that the sensitivity is increased for some directions, but in others it is decreased below that at $d/\lambda = 1/2$.

The output voltage of the goniometer for Configuration No. 1 is:

$$V_T = -4 A_a A_g \left\{ \sin\left[(1 + k) \frac{\pi d_1}{2\lambda} \sin \theta\right] \cos \left[(1 - k) \frac{\pi d_1}{2\lambda} \sin \theta\right] \cos \Phi + \sin\left[(1 + k) \frac{\pi d_1}{2\lambda} \cos \theta\right] \cos \left[(1 - k) \frac{\pi d_1}{2\lambda} \cos \theta\right] \sin \Phi \right\} \sin \omega t \tag{21}$$

From this the sensitivity factor is given by:

$$S.F. = 4 A_a A_g \sqrt{A_1^2 + B_1^2} \tag{22}$$

where:

$$A_1 = \sin\left[(1 + k) \frac{\pi d_1}{2\lambda} \sin \theta\right] \cos \left[(1 - k) \frac{\pi d_1}{2\lambda} \sin \theta\right]$$

and

$$B_1 = \sin\left[(1 + k) \frac{\pi d_1}{2\lambda} \cos \theta\right] \cos \left[(1 - k) \frac{\pi d_1}{2\lambda} \cos \theta\right] .$$

For $k = 1/2$ and $d_1/\lambda = 2/3$, Fig. 11 shows the variation of sensitivity factor as a function of arrival angle, θ; again, $2 A_a A_g$ was set equal to 1 for comparison purposes. Here we see the same type of behavior as in the case of the standard Adcock. The sensitivity factor is approximately twice that given by Eq. 20. It must be kept in mind, however, that the mutual interaction of the antennas was not taken into account.

Configuration No. 2 has the following output voltage:

$$V_T = -2 A_a A_g \left\{ A_2 \cos \Phi + B_2 \sin \Phi \right\} \tag{23}$$

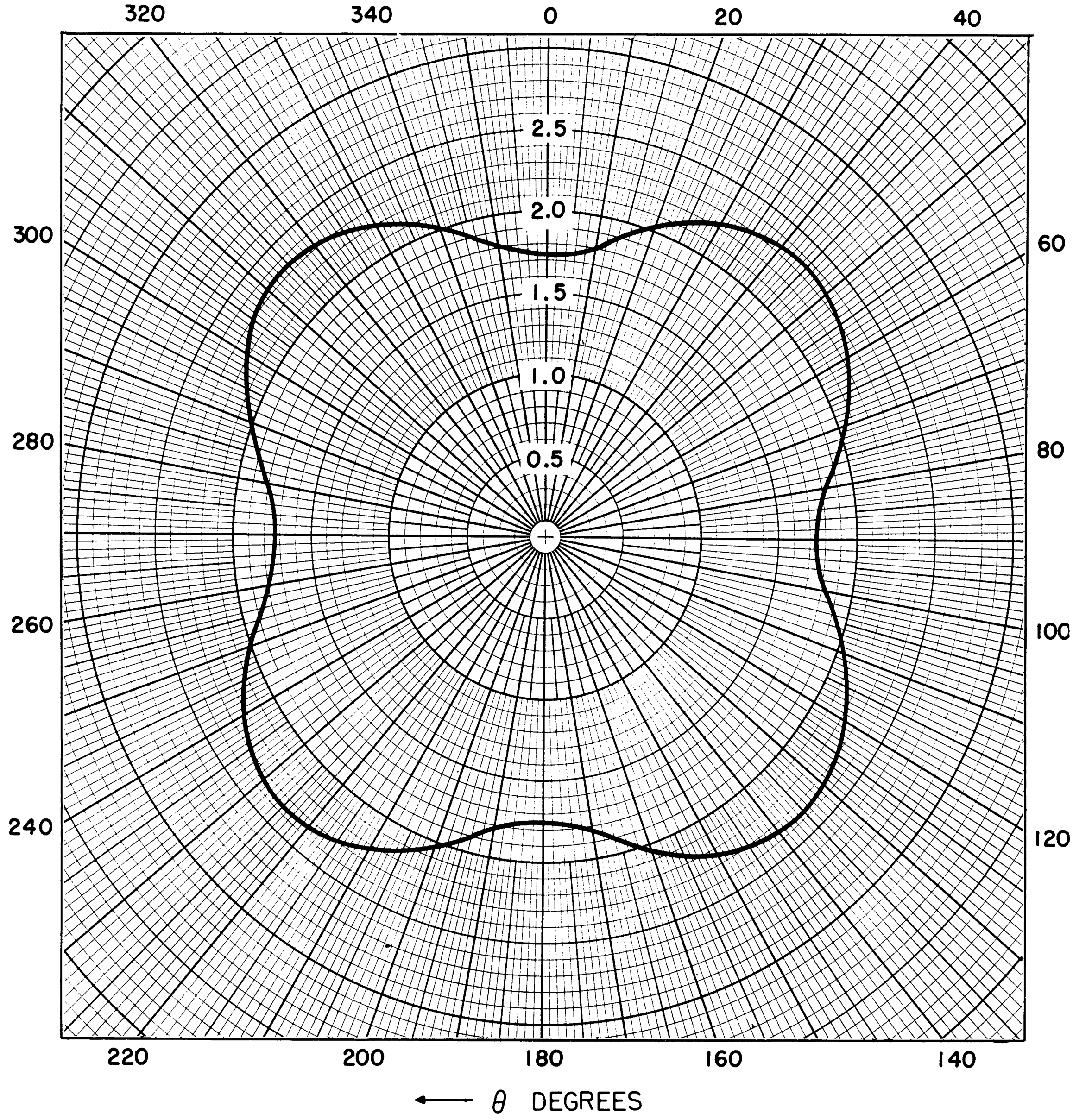


Fig. 11. Sensitivity factor; Configuration No. 1.

where:

$$A_2 = \frac{1}{\sqrt{2}} \sin\left[\frac{\pi d}{\lambda} \cos\left(\theta - \frac{\pi}{4}\right)\right] + \frac{1}{\sqrt{2}} \sin\left[\frac{\pi d}{\lambda} \sin\left(\theta - \frac{\pi}{4}\right)\right] + \sin\left[\frac{\pi d}{\lambda} \sin \theta\right]$$

and

$$B_2 = \frac{1}{\sqrt{2}} \sin\left[\frac{\pi d}{\lambda} \cos\left(\theta - \frac{\pi}{4}\right)\right] - \frac{1}{\sqrt{2}} \sin\left[\frac{\pi d}{\lambda} \sin\left(\theta - \frac{\pi}{4}\right)\right] + \sin\left[\frac{\pi d}{\lambda} \cos \theta\right].$$

The sensitivity factor is obtained from this equation and is given by:

$$S.F. = 2\, A_a A_g \sqrt{A_2^{\,2} + B_2^{\,2}} \qquad (24)$$

where A_2 and B_2 are defined as above. Figure 12 shows a plot of Eq. 24 as a function of $\underline{\theta}$ with $d/\lambda = 1$ and $2\, A_a A_g = 1$. From this figure it is seen that not only is the sensitivity greater than that of the standard Adcock, but it is more uniform as a function of the angle of arrival, θ, of the electromagnetic wave.

In summary, three antenna systems have been studied with respect to sensitivity and inherent errors. Of the three, Configuration No. 2 has the greatest merit and shows the most improvement over the 4-element Adcock. Incidentally, it is interesting to note that this antenna system is currently being used in the U. S. Navy GRD-6 direction-finding system; however, as far as the author knows, no detailed analysis of the system has ever been published. Indeed, it has many features which were not initially obvious when the study was first begun. The remaining part of this report will deal with further details concerning Configuration No. 2.

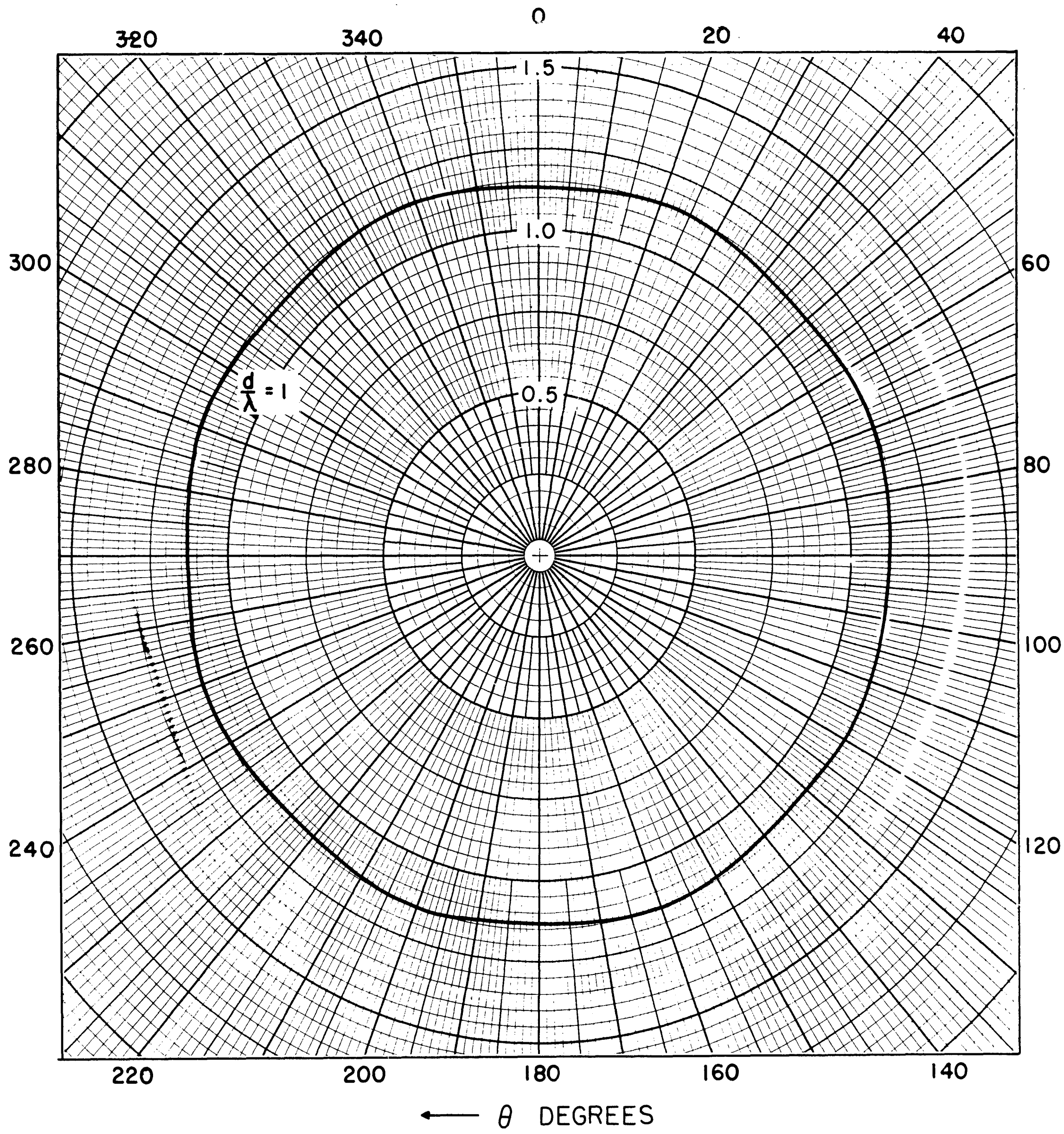


Fig. 12. Sensitivity factor; Configuration No. 2.

6. THE DOUBLE-ADCOCK ANTENNA SYSTEM

The frequency band will be split into two parts: (1) from 2 to 8 Mc, and (2) from 8 to 30 Mc. The antenna configuration being considered for coverage of the entire frequency range consists of two rings of eight antennas each. Figure 13 shows a plan view of the system envisioned.

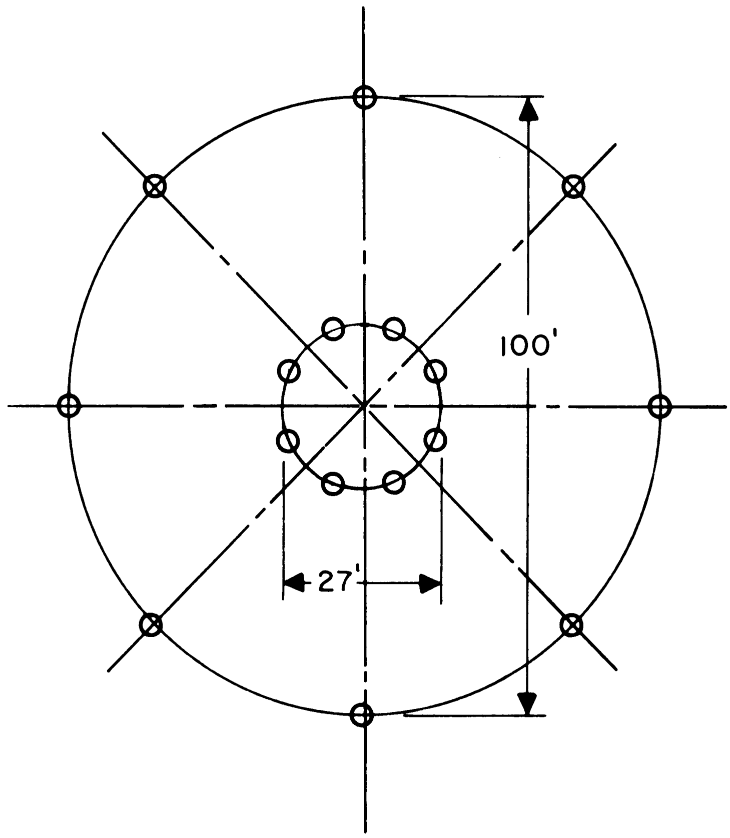


Fig. 13. Plan view of double-Adcock Antenna System.

The inner ring is to be used for the higher frequency range (8 - 30 Mc); the outer, for the lower frequency range (2 - 8 Mc). Let the diameter of the outside ring be 100 feet and that of the inside ring 27 feet. As usual, the spacing is a compromise between the greatest bearing errors at the higher end of the frequency band and the minimum sensitivity of the array which occurs at the lower end. It will now be of interest to determine the relative increase of sensitivity of this double Adcock configuration with respect to a four-element Adcock which has a maximum error of only one degree in the band being considered.

Through the use of Eq. 5 it can be shown that a spacing of $d/\lambda = 0.204$ gives approximately a one-degree maximum error for the four-element Adcock. Hence, for the low band at 8 Mc: $d_{lo} = (0.204)(37.5) = 7.65$ meters; and for the high band at 30 Mc: $d_{hi} = (0.204)(10.0) = 2.04$ meters. Obviously, these values for the diameter of the high and low bands are unrealistic. It was decided, therefore, to compare the double-Adcock array shown in Fig. 13 with that of the AN/TRD-4A direction-finding set. For this array $d_{lo} = 33'$ and $d_{hi} = 18.83'$. For these spacings $\left(\frac{d_{hi}}{\lambda}\right) = 0.574$ for 30 Mc and $\left(\frac{d_{lo}}{\lambda}\right) = 0.268$ for 8 Mc. From the literature[1] one can obtain the values of maximum error for the above antenna spacings. They are: (1) for the low band at 8 Mc, $\text{Error}_{max} = (\varphi - \theta)_{max} \cong 1.9^\circ$; and (2) for the high band at 30 Mc, $\text{Error}_{max} = (\varphi - \theta)_{max} \cong 10^\circ$.

We shall now calculate the maximum errors which occur in the proposed double-Adcock array. From Fig. 9 it is seen that the maximum error for this array type occurs at approximately 11.25° and at odd, integral multiples thereof. With $\theta = 11.25^\circ$, $d_{lo} = 100'$, $d_{hi} = 27'$, and through the use of Eq. 18, it can be shown that for the high band (8 - 30 Mc) at 30 Mc: $\text{Error}_{max} = (\varphi - \theta)_{max} \cong 0.12^\circ$; and for the low band

[1] P. G. Redgment, W. Struszynski, and G. J. Phillips, "An Analysis of the Performance of Multi-Aerial Adcock Direction-Finding Systems," J. Inst. Elec. Engrs. (London), Vol. 94, Part III A, No. 15, 1947, pp. 751-761.

(2 - 8 Mc) at 8 Mc: $Error_{max} = (\varphi - \theta)_{max} \cong 0.11^{o}$. Clearly the results here are far superior to these achieved by the reference AN/TRD-4A antennas.

A comparison must now be made of the relative sensitivities of the AN/TRD-4A antennas and the double Adcock antennas. This was accomplished by calculating the sensitivity factors at 2 and 8 Mc for the low band and at 8 and 30 Mc for the high band of both antenna systems. The results are shown in Figs. 14, 15, 16, and 17. From these curves it is easy to compare the two systems. Table I summarizes the results with $\theta = 0$, since at this angle of arrival the sensitivity factor is minimum. This table gives the ratios of the sensitivity factor of the double Adcock to that of the TRD-4A antenna for the limiting frequencies of the two ranges of coverage. In all cases $2\ A_a A_g$ was set equal to 1 in Eqs. 20 and 24.

	Frequency	Double Adcock	TRD-4A	Ratio
Low Band	2 Mc	1.213	0.209	5.8:1
	8 Mc	1.930	0.746	2.6:1
High Band	8 Mc	1.298	0.462	2.8:1
	30 Mc	1.896	0.973	1.9:1

Table I. Sensitivity factor comparison.

Although Table I shows considerable improvement in sensitivity with the double Adcock system, this is not the whole story. One might well ask what value of d/λ would give the greatest minimum sensitivity. For the standard Adcock this occurs at $d/\lambda = 0.5$, which can be seen in Fig. 10; however, the bearing error is too great for the system to be of any use

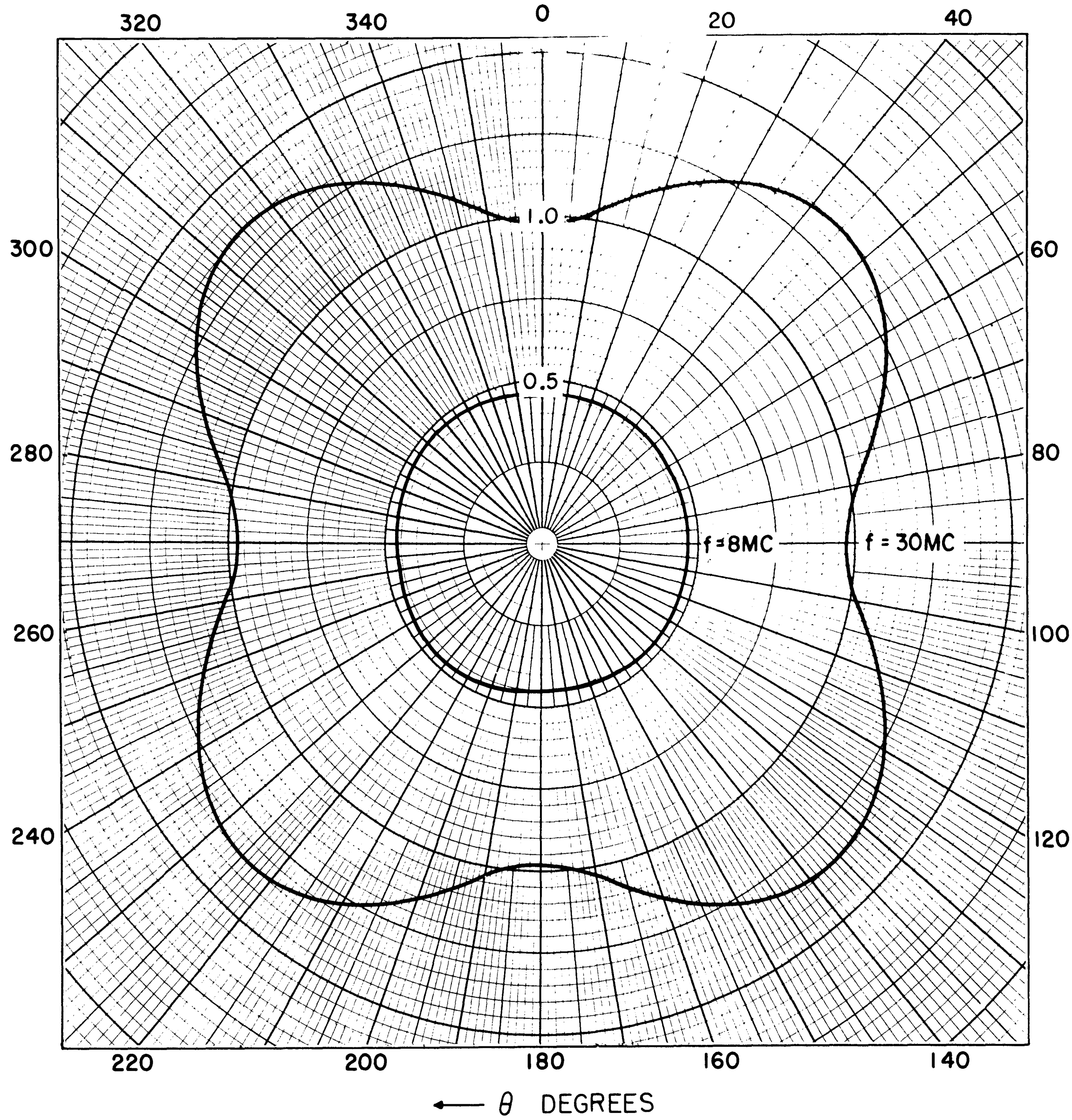


Fig. 14. Sensitivity factor; standard AN/TRD-4 Adcock; high band.

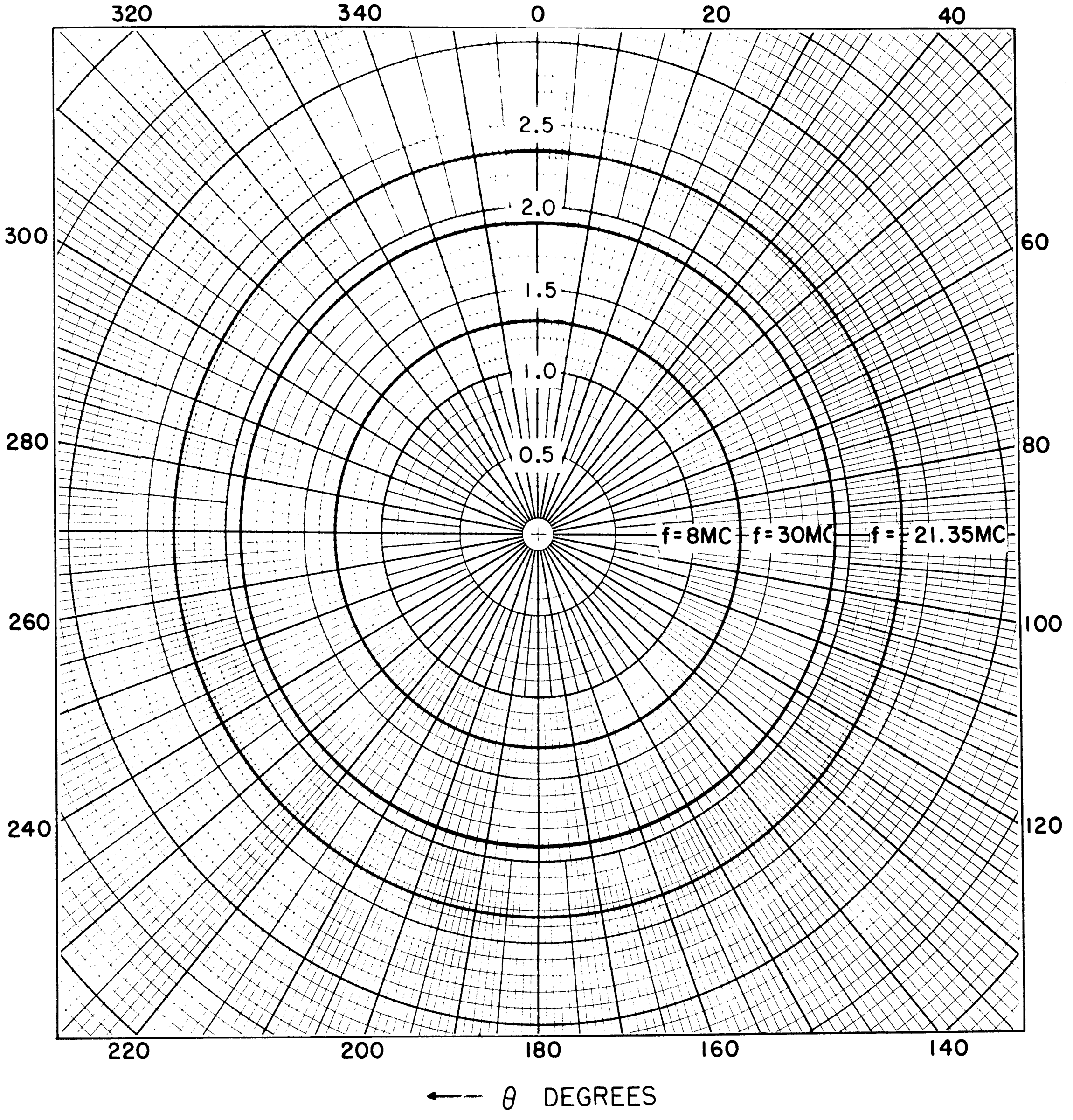


Fig. 15. Sensitivity factor; double Adcock; high band.

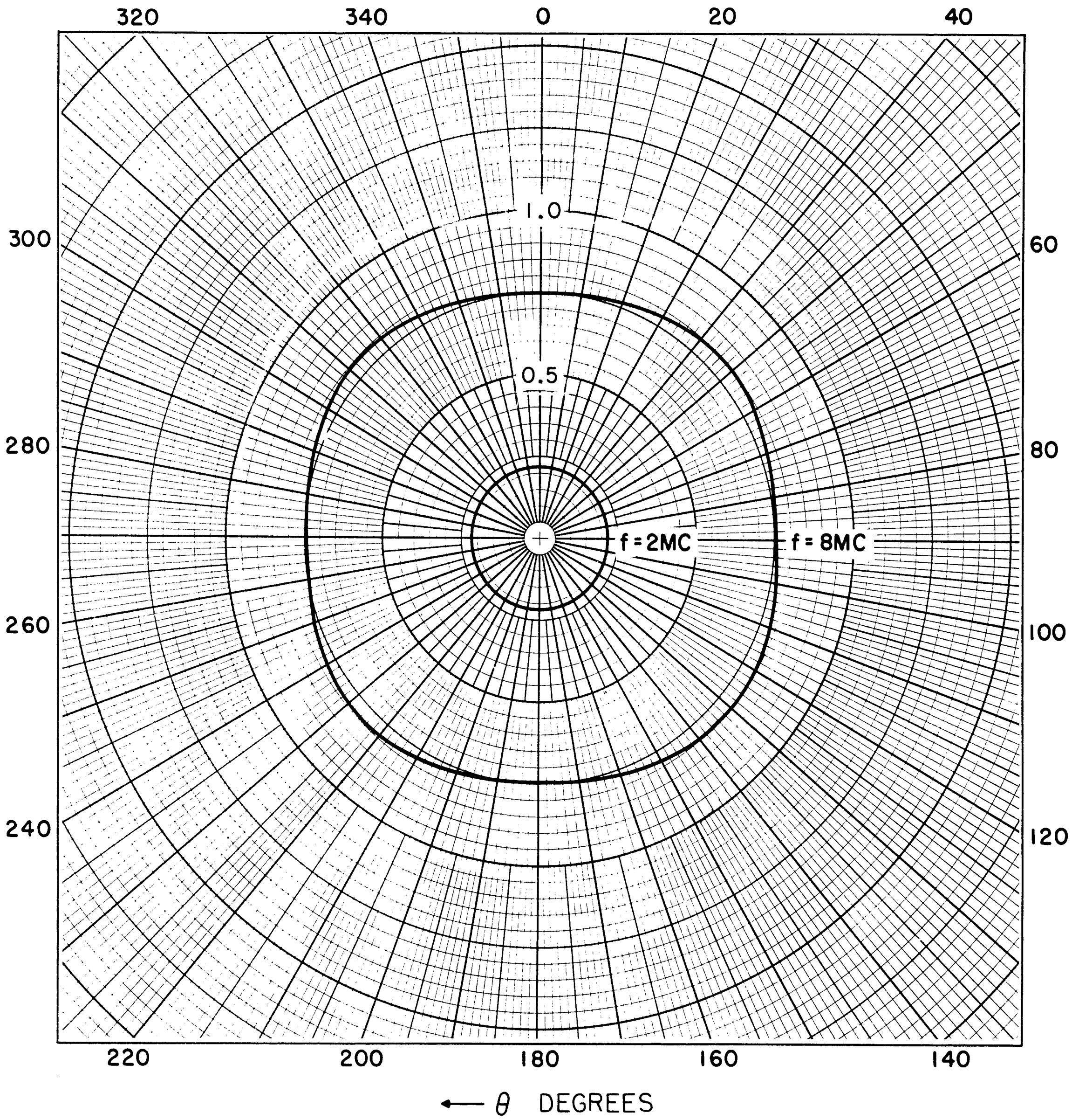


Fig. 16. Sensitivity factor; standard AN/TRD-4 Adcock; low band.

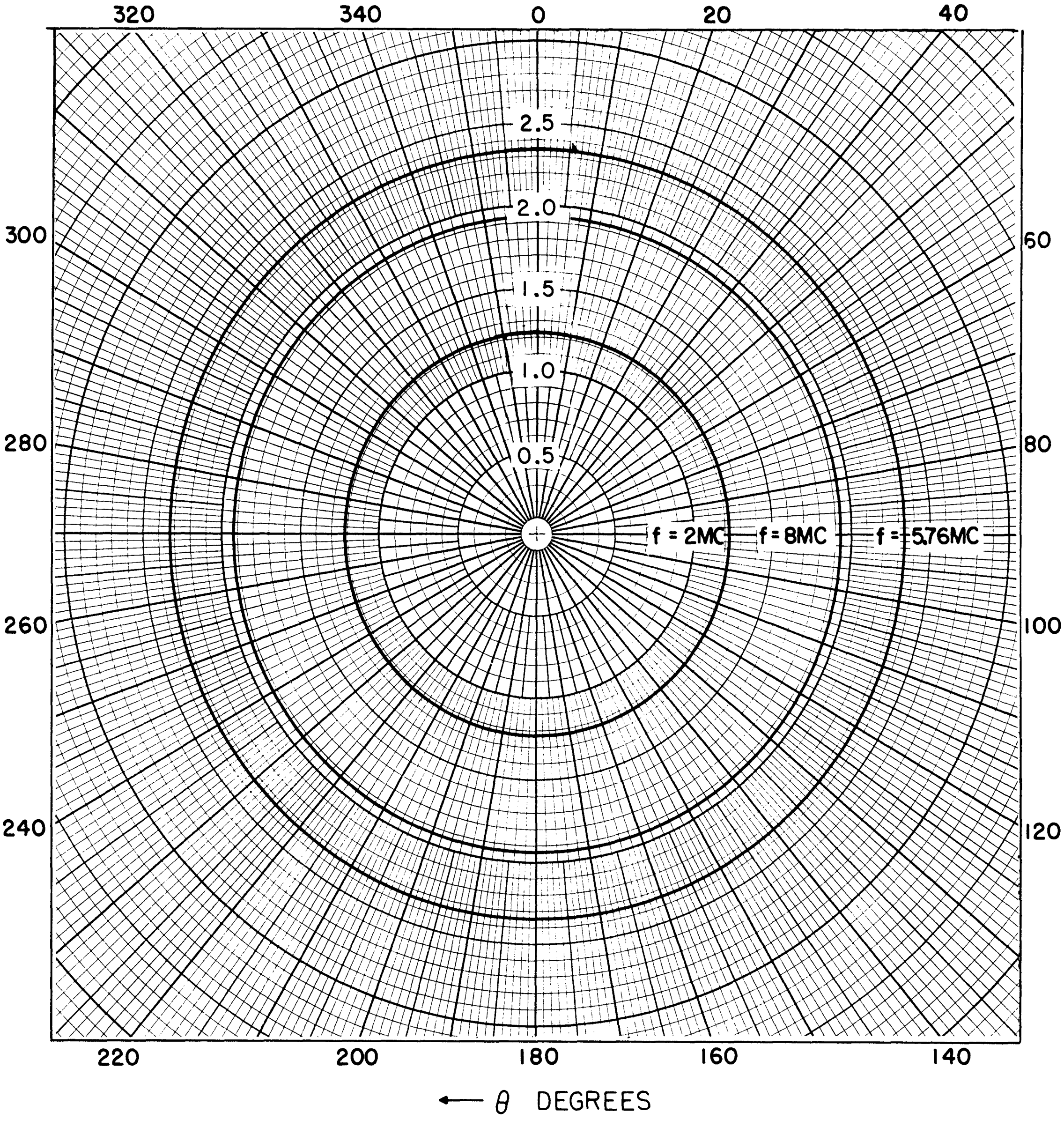


Fig. 17. Sensitivity factor; double Adcock; low band.

to us. For the eight-element antenna, on the other hand, a spacing of $d/\lambda = 0.586$ gives the greatest minimum sensitivity (see Appendix). This is useful in our case because this spacing occurs within the frequency ranges of interest. Figures 15 and 17 show plots of the sensitivity factors at the frequencies where the greatest minimum sensitivity occurs. For this type of antenna, then, the sensitivity increases from the lower frequency limit of the band covered, reaches a maximum at approximately the mid-frequency, and then decreases again when the highest frequency limit is reached. In contrast to this the sensitivity of the four-element Adcock increases monotonically from that at its lowest operating frequency to its greatest sensitivity at the high end of the band covered. The effect of sensitivity peaking near midband in each frequency range is another strong point in favor of the double-Adcock configuration.

A few more points must be brought out with regard to the antenna configuration proposed. In all of the above, mutual coupling between antennas was not taken into account. In reality, of course, there will be mutual coupling taking place. For this reason it is imperative that symmetry of the array be maintained. Unless the antennas are equally placed on the circumference of a circle, bearing errors caused by mutual antenna coupling between elements will occur. Another problem which must be solved appears when the inner array is being used. Since the outer system will probably approach its inherent resonance at some frequency in the high band, something must be done to reduce the radiation coupling between the outer and inner antenna rings. It is believed that some sort of passive filter could be placed at the bases of the antennas in the outer ring which would absorb energy at their inherent resonance points and which would not affect them in their own region of operation. Evidence to the fact that this can be done occurs in a report of a similar double-Adcock system manufactured by the Telefunken Co. in West Germany. It is true that such a technique will cause a shielding effect on the inner antenna ring, but one never gets something for nothing. All these effects will have to be evaluated experimentally to determine the actual, overall quality of the proposed system.

7. SUMMARY AND CONCLUSIONS

Three antenna systems have been analyzed, and their relative advantages and disadvantages have been studied with respect to a standard, four-element Adcock system. An eight-element configuration using two signal goniometers has been found superior to any of those studied. It is proposed that two such eight-element configurations be erected in two concentric rings with diameters of 27 and 100 feet, respectively. The inner ring will be used for the higher frequency band (8-30 Mc); and the outer ring will be used for the lower frequency band (2-8 Mc). At the highest frequency in each band the bearing error is on the order of a tenth of a degree, and the sensitivity factor exceeds that of the standard four-element Adcock by an order of magnitude. Practical problems remain which will have to be worked out before the system configuration becomes wholly adequate, but it is believed that they can be solved.

APPENDIX

Determination of the Value of d/λ Which Gives the Greatest Minimum Sensitivity for Configuration No. 2

From Eq. 24:

$$S.F. = 2 A_a A_g \sqrt{A^2 + B^2} \tag{25}$$

where:

$$A = \frac{1}{\sqrt{2}} \sin\left[\frac{\pi d}{\lambda} \cos(\theta - \frac{\pi}{4})\right] + \frac{1}{\sqrt{2}} \sin\left[\frac{\pi d}{\lambda} \sin(\theta - \frac{\pi}{4})\right] + \sin\left[\frac{\pi d}{\lambda} \sin \theta\right] \tag{26}$$

$$B = \frac{1}{\sqrt{2}} \sin\left[\frac{\pi d}{\lambda} \cos(\theta - \frac{\pi}{4})\right] - \frac{1}{\sqrt{2}} \sin\left[\frac{\pi d}{\lambda} \sin(\theta - \frac{\pi}{4})\right] + \sin\left[\frac{\pi d}{\lambda} \cos \theta\right] \tag{27}$$

The minimum sensitivity factor occurs when $\theta = 0$.

Therefore, (26) and (27) become:

$$A = 0 \text{ and } B = \frac{2}{\sqrt{2}} \sin\left[\frac{\pi d}{\sqrt{2}\lambda}\right] + \sin \frac{\pi d}{\lambda} \tag{28}$$

Consequently, the sensitivity factor becomes

$$S.F. = 2 A_g A_a \left[\sqrt{2} \sin\left(\frac{a}{\sqrt{2}}\right) + \sin a\right], \tag{29}$$

where:

$$a = \frac{\pi d}{\lambda}.$$

Taking the derivative of (29) with respect to <u>a</u> and setting the result equal to zero gives:

$$\frac{d(S.F.)}{da} = 2 A_g A_a \left[\cos \frac{a}{\sqrt{2}} + \cos a\right] = 0 \tag{30}$$

Solving now for values of <u>a</u> which satisfy (30) results in:

$$\cos \frac{a}{\sqrt{2}} + \cos a = 0$$

$$2 \cos \frac{1}{2}\left(a + \frac{a}{\sqrt{2}} \right) \cos \frac{1}{2}\left(a - \frac{a}{\sqrt{2}} \right) = 0$$

$$\cos \frac{a}{2}\left(1 + \frac{1}{\sqrt{2}}\right) = 0 \quad \text{when} \quad \frac{a}{2}\left(1 + \frac{1}{\sqrt{2}}\right) = \frac{\pi}{2} \pm n\,\pi \text{ where } n = 0,1,2, \ldots$$

and

$$\cos \frac{a}{2}\left(1 - \frac{1}{\sqrt{2}}\right) = 0 \quad \text{when} \quad \frac{a}{2}\left(1 - \frac{1}{\sqrt{2}}\right) = \frac{\pi}{2} \pm n\,\pi \text{ where } n = 0,1,2, \ldots$$

Therefore

$$a = \frac{\pi(1 \pm 2\,n)}{1 + \frac{1}{\sqrt{2}}} \quad \text{where } n = 0,\ 1,\ 2,\ \ldots \tag{31}$$

and

$$a = \frac{\pi(1 \pm 2\,n)}{1 - \frac{1}{\sqrt{2}}} \quad \text{where } n = 0,\ 1,\ 2,\ \ldots \tag{32}$$

From (31) it can be shown that a maximum occurs when $n = 0$; this results in $d/\lambda = \dfrac{1}{1 + \frac{1}{\sqrt{2}}} = 0.5858$. All other maxima obtained from (31) and (32) give values of d/λ which are greater than one and, hence, are unuseable because the bearing errors of the antenna system would be intolerable.