HEAT TRANSFER THROUGH GASES CONTAINED BETWEEN TWO VERTICAL CYLINDERS AT DIFFERENT TEMPERATURES

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Heat transfer through gases contained between vertical concentric cylinders was investigated when the radius of the inner cylinder is small compared to the radius of the outer cylinder. Experiments were performed in a modified hot wire type thermal conductivity cell in which end effects could be evaluated in addition to the overall heat transfer between the cylinders. Measurements were made with argon, helium, and neon in the pressure range 0.1-620mm Hg, for the temperature differences between the cylinders of 10,50 and 100^{o}C, and for the length to the outer diameter ratios between 3.3 and 6.7. The results show that below certain Rayleigh numbers and length to diameter ratios heat is transferred from the hot to cold boundary by conduction only. End effects contribute only in the corner regions. The distance to which these end effects penetrate was determined experimentally, and compared to analytical results obtained from a simple heat balance in the corner region. Relationships, based on the experimental data, were also obtained for the average heat transfer both in the corner regions and over the entire length of the cylinders. It is shown that these results may be used to estimate the error introduced in thermal conductivity and thermal accommodation coefficient measurements by neglecting end effects.

INTRODUCTION

A knowledge of heat transfer through gases contained between concentric cylinders at different temperatures is required in many problems of practical interest. It is needed, for instance, in the experimental determination of thermal conductivities and thermal accommodation coefficients of gases. Heat transfer between concentric cylinders has been measured in connection with such experiments*. To determine thermal conductivities or thermal accommodation coefficients not the total heat transfer, but only the heat conducted through the gas must be known. Thus, considerable efforts have been made in these experiments to minimize the effects of natural convection with the result that in most of the available data heat transfer due to convection is negligible. This investigation was undertaken to study heat transfer by both conduction and convection between two vertical concentric cylinders. Radiative transfer will not be considered. In most cases it is not negligible but is independent of the conductive and convective heat transfer and once the experimental conditions have been specified it can be either measured or calculated with reasonable accuracy.

In addition to studying the overall heat transfer between the cylinders, special attention is given to the evaluation of the end effects**. End effects become of particular importance in thermal conductivity and thermal accommodation coefficient measurements when the temperature difference between the inner and outer cylinders is large; a condition that has been encountered recently, in attempts to extend the use of the concentric cylinder geometry to higher temperatures [3 - 5]. This investigation was motivated by the problems arising due to end effects in such experiments. The results will be presented in general form, however, so as to

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* Good summaries of these experiments may be found in references [1] and [2].

** It is noted here that the end effects can be caused predominantly by convection in the gas but are not limited to convection effects only.

be applicable to other problems of similar geometry.

EXPERIMENTAL

Here we shall be concerned with the following problem: a gas is contained between two vertical concentric cylinders of radii a, and b, and length S. The radius of the outer cylinder, b, is very large compared to a ($b/a \gg 1$) and the length S is large compared to b($S/b \gg 1$). The temperatures of the inner and outer cylinders are T_a and T_b respectively. The ratio $(T_a - T_b)/T_a$ is not necessarily small compared to one and thus the properties of the gas cannot be assumed to be constant. The two horizontal surfaces which bound the gas layer are impervious to heat and mass flow.

The aim was to construct an apparatus which approximates well the above conditions and also allow the variation of the significant parameters over a wide range. The experiments were performed in a modified hot wire type apparatus [6] in which the gas, the pressure, the temperature difference between the cylinders, and the lengths of the cylinders could be varied independently. The radii a and b were fixed.

The test section, shown schematically in Fig. 1, consists of a thin tungsten filament supported axially in a vertical Pyrex tube. Three sections of the filament, differing in length, were isolated by potential leads similar in concept to those described in refs. [7-9]. The three sections were included in order to evaluate the influences of wire lengths on the end effects. The effective length of the outer cylinder (distance S in Figures 1,5) about each of these sections can be varied by moving teflon discs inside the tube. These moving disc are mounted on a 3/16 inch diameter stainless steel rod which has a left and right threaded portion, designed so that the distance between the disc can be altered by turing the rod. The threaded rod and also the center filament are supported by two stationary discs installed near each end of the tube and connected by two 1/8 inch diameter supporting rods. The two supporting rods and the threaded rod are placed 120° apart very close to the walls of the Pyrex tube. It is understood that the positions and sizes of these rods might affect the heat transfer. To test this, thermal conductivities of gases were measured with the apparatus. The results of these experiments [6] indicate that in the range of present experimental conditions the rods have negligible effects on the heat transfer.

The data was obtained by the following experimental procedure. At each pressure and at each temperature difference between filament and tube, the position of the movable discs was varied ranging from a distance equal to the length of the filament section (S = H, see Fig. 5) to the largest possible distance between the discs. For each position of the discs (i. e., for each S value) the power input to the filament section under consideration (A-A, B-B, or C-C, Fig. 1) was measured both in the presence of the gas, Q_t, and in vacuum, Q_v ($\sim 1 \times 10^{-7}$ mm Hg). It was found that the Q_v values were independent of the position of the discs, as long as the distance between the discs was larger than the distance between the potential leads being used. The total power input to the filament may be expressed as

$$Q_t = Q_k + Q_e + Q_r + Q_f \qquad (1)$$

Q_k is the heat conducted by the gas from the filament, Q_e is the heat transferred from the filament to the gas due to end effects, Q_r is the heat loss from the filament by radiation and Q_f is the heat conducted along the filament to the supports. From eq. (1) the sum $Q_H = Q_k + Q_e$ is

$$Q_H = Q_t - Q_v \qquad (2)$$

In evaluating Q_H from eq. (2) it is implicitly assumed that Q_v is the sum of the radiation and support losses ($Q_v = Q_r + Q_f$) and that it is independent of pressure. Data was taken after steady state conditions were reached.

The temperature of the outer cylinder was maintained constant by immersing it in an oil bath. The temperature of this bath was held at 34.50 ± 0.02°C during the tests. In the experiments that follow the inner surface of the test tube was presumed to have been at this same temperature, denoted as T_b. The average temperature of the filament, T_a, was determined by measuring its resistance [10]. Recognizing that the temperature distribution along the filament is not entirely uniform, the term filament temperature, as used here, refers to the average temperature corresponding to the measured resistance of the appropriate filament section. The temperatures of the teflon discs were not recorded.

The test tube was connected to the vacuum pumps and the gas supply tanks through a glass vacuum system. Pressures were measured with a McLeod gauge, a U-tube mercury manometer and an ionization gauge. The electrical measurements were made with a high precision d. c. Millivolt Standard, together with an optical galvanometer and suitable standard resistors. Test gases of higest quality ("Airco" in Pyrex) were used throughout the experiments, but no other efforts were made to obtain clean surfaces.

The experimental data reported in the following were obtained with argon, neon, and helium for temperature differences of 10, 50 and 100°C and in the pressure range 0.1 - 620mm Hg.

PENETRATION DEPTH

Under certain conditions, in the center part of the cylinders the heat transfer will be by conduction only; i.e., at a distance from the ends the heat transfer through the gas is described by the Fourier equation [11]. First, the conditions will be determined under which such a "conduction regime" exists.

The experiments of Eckert and Carlson [11] on natural convection in air between two vertical plates show that local heat transfer conditions are different in directly opposite corners (two lower, or two upper corners) and are similar in diagonially opposite ones. The two different types of corners are denoted as starting and departure corners. The distance from the end where the ends effects become negligible and the conduction regime begins is referred to as the penetration depth Z_p. Although the penetration depths in the starting and departure corners may be somewhat different [11], here no distinction will be made between them. In order to find the penetration depth we apply a heat balance in the corner region [11, 12].

$$Q_c = Q_s + Q_d = \int_0^{Z_p} q_s(Z)\,dZ - \int_0^{Z_p} q_d(Z)\,dZ \tag{3}$$

Q_c is the enthalpy carried by convection through a horizontal plane at Z_p, and q_s and q_d are the heat fluxes per unit length in the starting and departure corners (Fig. 2). Equation (3) can be applied only when the end effects do not penetrate beyond the center plane of the cylinders ($Z_p < H/2$). It will be assumed now a) that q_s varies as $Z^{-\frac{1}{4}}$ [13] b) that at $Z = Z_p$, q_s is equal to the conductive heat transfer per unit length, q_k, and c) that q_d is independent of the position and is equal to q_k. With these assumptions eq. (3) may be integrated to yield

$$Z_p = \frac{3Q_c}{q_k} \tag{4}$$

The parameters Q_c and q_k are given by the equations

$$Q_c = \left[\int_a^b 2\pi r \rho w(r)\,dr \int_{T_a}^{T} C_p\,dT \right]_{Z=Z_p} \tag{5a}$$

$$q_k = \left[2\pi a\, k \frac{dT}{dr} \right]_{\substack{r=a \\ z=z_p}} \tag{5b}$$

w(r) is the axial velocity and all other symbols are defined in the nomenclature. Equation (5a) is based on the assumption that in the conduction regime ($Z = Z_p$) the flow is laminar and fully developed. In order to evaluate Q_c and q_k, the temperature and velocity distributions must be known at $Z = Z_p$. In the conduction regime heat is conducted in the radial direction only and thus, at $Z = Z_p$, the conservation equations for mass, momentum and energy are

$$\frac{1}{r}\frac{d}{dr} r \rho' w = 0 \tag{6a}$$

$$\frac{1}{r}\frac{d}{dr}\left(r \mu \frac{dw}{dr}\right) = g(\rho' - \bar{\rho}) \tag{6b}$$

$$\frac{1}{r}\frac{d}{dr}\left(r k \frac{dT}{dr}\right) = 0 \tag{6c}$$

where

$$\rho' = \rho(1 + C) \tag{6d}$$

C is a small correction to the density. It was introduced so that the continuity and momentum equations could be satisfied simultaneously without knowledge of the detailed flow field in the corners [3]. In the calculations that follow, at any point C was always less than 0.1 and generally was about 0.01. The average density has been defined as

$$\bar{\rho} = \frac{2}{b^2 - a^2} \int_a^b \rho\, r\, dr \tag{7}$$

In eq. (6b) the approximation has been made that density differences due to temperature differences are only of importance in producing differences in the buoyancy force. The equation of state for gases then has the form [12].

$$\rho(r)T(r) = \bar{\rho}\,\bar{T} = \text{const} \tag{8}$$

The boundary conditions corresponding to eqs. (6a - 6c) are

$$\begin{aligned} r &= a \qquad w = 0 \qquad T = T_a \\ r &= b \qquad w = 0 \qquad T = T_b \end{aligned} \tag{9}$$

The temperature difference between the cylinders may be large ($T_b/T_a \gg 1$) and hence k and μ cannot be taken to be constants. Equations (6-9) were integrated numerically using a high speed digital computer. Solutions for eqs. (4-9) were obtained for a wide range of conditions, namely for three monatomic gases (A, Ne, He) and four diatomic gases (CO_2, O_2, N_2, and H_2), four temperature differences (ΔT = 10, 50, 100, 500° C), four pressures (400, 600, 760 mm Hg) and five radius ratios (b/a = 10, 25, 50, 100, 250, using a = 0.5, 0.05, 0.005, and 0.0005 cm). The gas properties used in calculations were taken from refs. [14, 15]. The computed numerical results were transformed into the following dimensionless groups:

penetration depth $\quad Z_p^* = \dfrac{Z_p}{2b}$

axial velocity $\quad W^* = \left.\dfrac{w \rho c_p b}{k}\right|_{\substack{T=\bar{T} \\ Z \geq Z_p}}$ (10)

Rayleigh number $\quad Ra = Gr\,Pr = \dfrac{g\,\rho^2 \Delta T (2b)^3}{\mu^2\,\bar{T}} \left.\dfrac{c_p \mu}{k}\right|_{T=\bar{T}}$

It was found that for the entire range of parameters employed in the solutions both w* and Z_p^* can be correlated extremely well with the Rayleigh number defined above. Calculated axial velocity profiles are shown in Fig. 3 for various Rayleigh numbers. These profiles change very little for different radius ratios as long as the radius ratio, b/a, is larger than about 10.

The calculated penetration depths are shown in Fig. 4. The results of the calculations are represented by a solid line and can be well approximated for $b/a > 10$ by the relation

$$Z_p^* \cong \frac{Ra}{4400} \qquad (Ra \geq 4000) \tag{11}$$

It will be shown presently that eq. (11) is applicable only for Ra > 4400.

The above expression is expected to be a reasonable approximation for Z_p^* only as long as the conductive and convective heat transfers are of the same order of magnitude in the corner region. When the convective heat transfer is small compared to the heat conduction then dimensional considerations [3] suggest that the penetration depth becomes constant, its value being of the order of the difference between the radii (b-a).

Penetration depths were measured in argon, helium and neon with the apparatus described in the previous section. The penetration depth was determined by measuring the heat transfer from a certain section of length H of the filament (AA, BB, or CC, Fig. 1) while varying the actual length S, of the cylinders (Figs. 1,5). A typical result indicating the variation of the heat transfer with S is shown in Fig. 5. In principle, the penetration depth is reached when there is no further change in the measured heat transfer with a change in the distance S. Here, however, the penetration depth was taken to correspond to the conditions, that for a one cm change in S the heat transfer changes less than 0.1% compared to its maximum value.

The experimentally determined penetration depths are shown in Fig. 4. In certain cases the condition given above could not be reached and the penetration depth was obtained by extrapolation of the data. These test points are indicated by a short line attached to them. In Fig. 4, penetration depths are also shown deduced from Gregory and Marshall's experiments in O_2, N_2, and CO_2 [16, 17]. Gregory and Marshall used two test tubes of different lengths in gaseous connection to measure thermal conductivities. The penetration depths in their experiments can be evaluated from their carefully detailed data.

For Rayleigh numbers greater than about 4400 the data agrees fairly well with the analytical result (eq. 11), provided that $Z_p < H/2$. At lower Rayleigh numbers the penetration depth becomes a constant and equal to the diameter of the outer tube

$$Z_p^* = 1 \qquad (Ra < 4400) \tag{12}$$

The results in Fig. 4 also indicate that, as expected, the penetration depth is independent of the cylinder length as long as the end effects do not extend beyond the center plane of the cylinders.

In the experiments the pressure was reduced to such low levels that rarefaction effects became important. The highest Knudsen number (based on the diameter of the filament) in the experiments was about 10. At this value nearly free molecule conditions exist in the gas [18]. It is interesting to note, however, that even at such a high degree of rarefaction the penetration depths remain equal to 2b. It is noted here also that in the present experiments the Knudsen numbers corresponding to Ra = 4400 were about 0.01. At these Knudsen numbers the temperature jump effects are negligible, eqs. (6,9) are valid, and consequently eq. (11) may be used in estimating the penetration depth.

The conditions can now be established under which a conduction regime exists when end effects do not penetrate beyond the center plane of the cylinders, i.e. when $Z_p < 2H$. Using eqs. (11,12) the limiting conditions for the conduction regime were determined and are shown in Fig. 6. In this figure the Rayleigh number is based on the radius b so that the results for the concentric cylinder geometry can be compared with the results obtained by Eckert and Carlson [11] and by Batchelor [12] for vertical parallel plates. As can be seen the Rayleigh numbers limiting the conduction regime as given by the present results are similar to the values presented by Eckert and Carlson, and are less than the values given by Batchelor. However, the slope of the line bounding the conduction regime is identical to the one given by Batchelor.

Heat Transfer in the Conduction Regime

The conductive and convective heat transfer between the two cylinders will be now evaluated based on heat transfer measurements made in argon, neon, and helium. In these measurements the moveable discs were positioned at either one of the potential leads (AA, BB, or CC) resulting in length to diameter ratios H/D = 3.3, 5, 6.7

Corner Region Without distinguishing between the starting and departure corners an average Nusselt number is defined in the corner region

$$\overline{Nu}_c = \frac{\overline{h}_c D}{k} \qquad (13)$$

D is the diameter of the outer tube and $\bar{h}_c$ is the overall heat transfer coefficient in the corner region given by the equation ($Z_p < H/2$)

$$Q_H = \bar{h}_c \pi D \Delta T \, 2 z_p + \frac{2\pi k \Delta T}{\log b/a} (H - 2 z_p) \qquad (14)$$

Equations (13) and (14) may be rearranged to yield

$$\overline{Nu}_c = \frac{Q_H - Q_k}{2\pi k z_p \Delta T} + \frac{2}{\log b/a} \qquad (25 < Ra < 2 \times 10^4) \qquad (15)$$

For the present data, eq. (15) can be applied only in the Rayleigh number range indicated. At $Ra < 25$ rarefaction effects become significant, k is not a constant and the heat conducted from the filament, Q_k, cannot be calculated from the Fourier equation. At $Ra > 2 \times 10^4$ the condition $Z_p < H/2$ is not valid anymore.

Corner Nusselt numbers calculated from eq. (14) are shown in Fig. 7. In calculating $\overline{Nu}_c$, average values for the thermal conductivity were used corresponding to the temperatures $(T_a + T_b)/2$, and Z_p was computed from eqs. (11) or (12). For Q_H the measured heat transfer values were substituted. The results indicate that in the appropriate range of Rayleigh numbers $\overline{Nu}_c$ is nearly a constant, at 0.355.* In the present experiments log (b/a) = 5.86, and $\overline{Nu}_c$ can be expressed as

$$\overline{Nu}_c \cong 0.013 + \frac{2}{\log b/a} \qquad (25 < Ra < 2 \times 10^4) \qquad (16)$$

*Note that the average Nusselt number remains constant at least up to $Ra \cong 1 \times 10^5$ (Fig. 7) even though $Z_p > H/2$

<u>Average heat transfer</u>. An average heat transfer coefficient for the heat transfer between the inner and outer cylinders including the corner regions may be defined by the equation

$$Q_H = \pi D \bar{h} \Delta T H \tag{17}$$

Using eqs. (14,17) one obtains for the average Nusselt number

$$\overline{Nu} = \frac{\bar{h} D}{k} = \frac{2}{\log b/a} + \frac{D}{H}\left(2\overline{Nu}_c - \frac{4}{\log b/a}\right)\frac{Z_p}{D} \tag{18}$$

Substituting eqs. (11,12 and 16) into eq. (18) we get

$$\overline{Nu} \cong \frac{2}{\log b/a} + 0.026\frac{D}{H} \qquad (25 < Ra < 4400) \tag{19}$$

and

$$\overline{Nu} \cong \frac{2}{\log b/a} + 5.9\times 10^{-6}\frac{D}{H} Ra \qquad (4400 < Ra < 2\times 10^4) \tag{20}$$

The above results (eq. 19,20) may be used to estimate the errors arising in thermal conductivity measurements due to the neglecting of end effects. For example, in typical hot wire type thermal conductivity cells H/D = 20 and b/a = 200. Then, for Ra = 8800, by including the end effects the Nusselt number is 0.379, while neglecting end effects it is 0.377. Thus, in this case end effects may introduce about a one percent error into the measurements.

Finally, it is noted here that at Ra 1×10^5 steady state conditions cannot be reached and significant fluctuations in heat transfer are observed. It has yet to be determined whether these fluctuations are due to turbulence or to instabilities such as observed by Elder [19] and by Vest [20] between parallel plates using fluids with high Prandtl numbers.

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NONEMCLATURE

a	radius of inner cylinder (cm)
b	radius of outer cylinder (cm)
c_p	specific heat (watts/gm-^oC)
D	diameter of outer cylinder (cm)
g	gravitational acceleration (cm/sec^2)
h	heat transfer coefficient (watts/cm-^oC)
H	distance between potential leads (cm)
k	thermal conductivity (watts/cm-^oC/cm)
q	heat flow per unit length (watts/cm)
Q	heat flow (watts)
r	radial coordinate (cm)
S	actual length of cylinders (cm)
T	temperature (^oC)
ΔT	temperature difference = $T_a - T_b$ (^oC)
w	axial velocity (cm/sec)
Z	distance from corner in axial direction (cm)
Z_p	penetration depth (cm)
ρ	density (gm/cm^3)
ρ'	corrected density = $\rho(1+C)$ (gm/cm^3)
λ	mean free path (cm)
μ	viscosity (gm/cm-sec)
C	correction to density
Nu	Nusselt number = hk/D

Pr	Prandtl number = $\left.\frac{c_p \mu}{k}\right\|_{T=\bar{T}}$
Ra	Rayleigh number = Gr Pr = $\frac{g \rho^2 \Delta T (2b)^3}{\mu^2 \bar{T}} \left.\frac{c_p \mu}{k}\right\|_{T=\bar{T}}$
w*	axial velocity = $\left.\frac{w \rho b c_p}{k}\right\|_{T=\bar{T},\ z \geq z_p}$
Z^*_p	penetration depth = $Z_p/2b$

<u>Indices</u>

a	evaluated at a
b	evaluated at b
c	convective
d	departure corner
e	due to end effects
f	along the filament
H	based on height, H, when S=H
k	conductive
r	radiation
s	starting corner
t	total
v	in vacuum
-	indicates average value

FIGURE CAPTIONS

Fig. 1 Schematic of test section

Fig. 2 Sketch of lower end of tube indicating the corner region.

Fig. 3 Calculated axial velociy distributions ($a = 0.005$ cm)

Fig. 4 Penetration depth

Fig. 5 Data, showing the variation of heat transfer from length H of the filament due to changes in the distance S.

Fig. 6 Limits of the conduction regime. (Rayleigh number is based on radius b)

Fig. 7 Average Nusselt numbers in the corner region

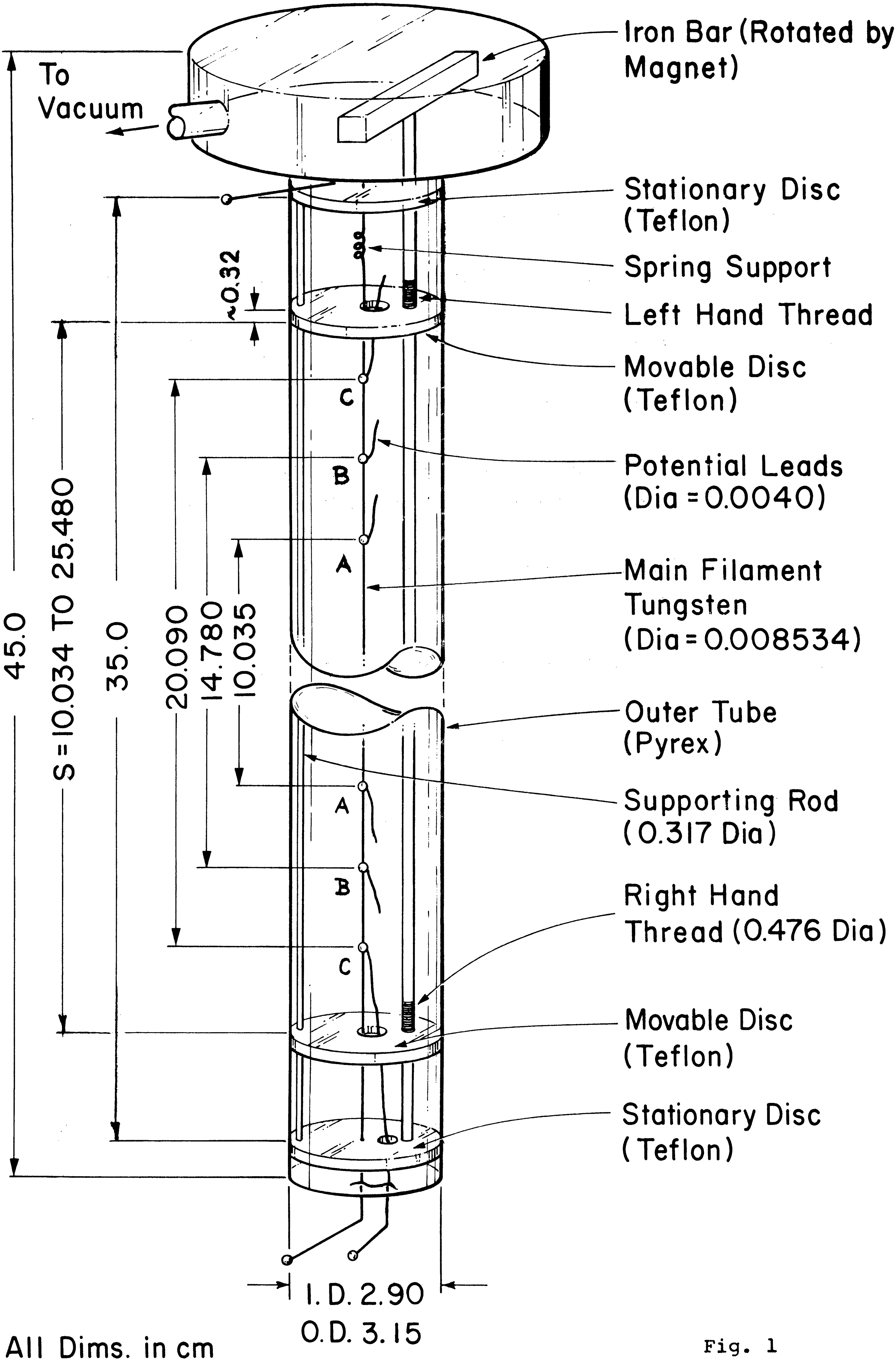

Iron Bar (Rotated by Magnet)
To Vacuum
Stationary Disc (Teflon)
Spring Support
Left Hand Thread
~0.32
Movable Disc (Teflon)
Potential Leads (Dia = 0.0040)
Main Filament Tungsten (Dia = 0.008534)
Outer Tube (Pyrex)
Supporting Rod (0.317 Dia)
Right Hand Thread (0.476 Dia)
Movable Disc (Teflon)
Stationary Disc (Teflon)
45.0
S = 10.034 TO 25.480
35.0
20.090
14.780
10.035
A
B
C
I. D. 2.90
O.D. 3.15
All Dims. in cm

Fig. 1

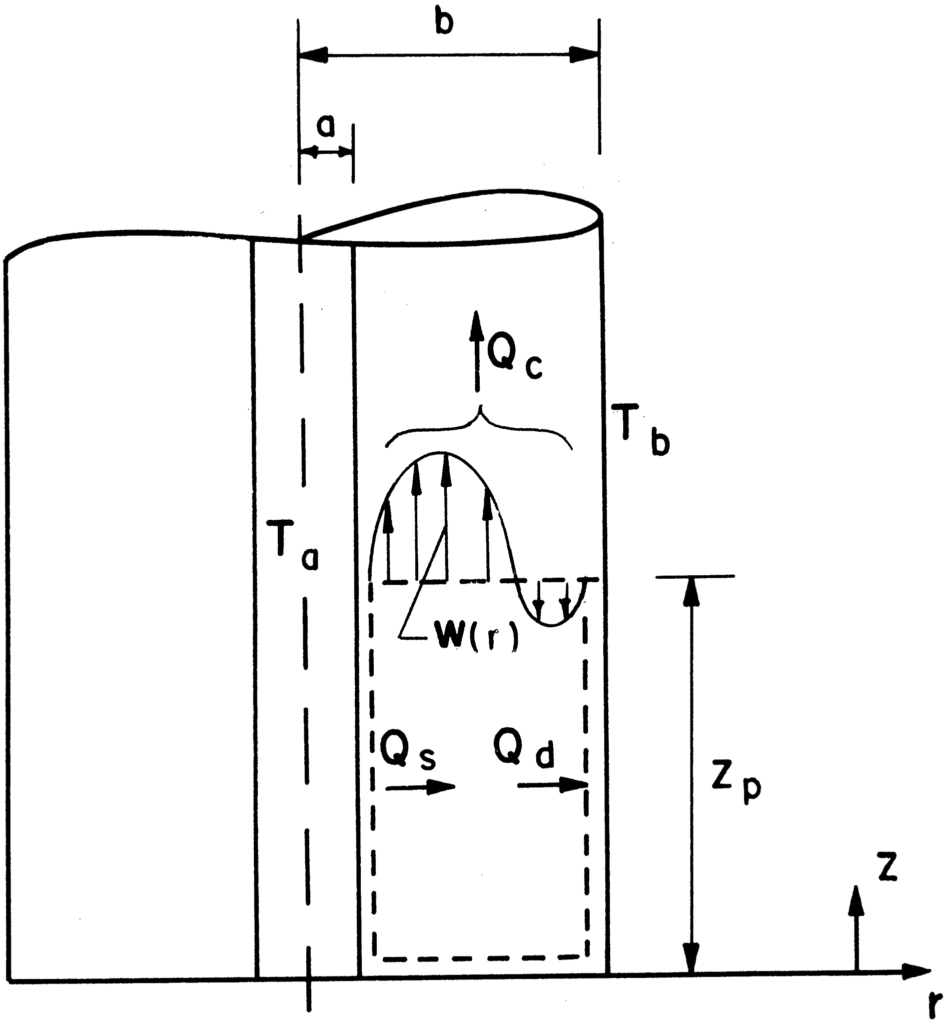
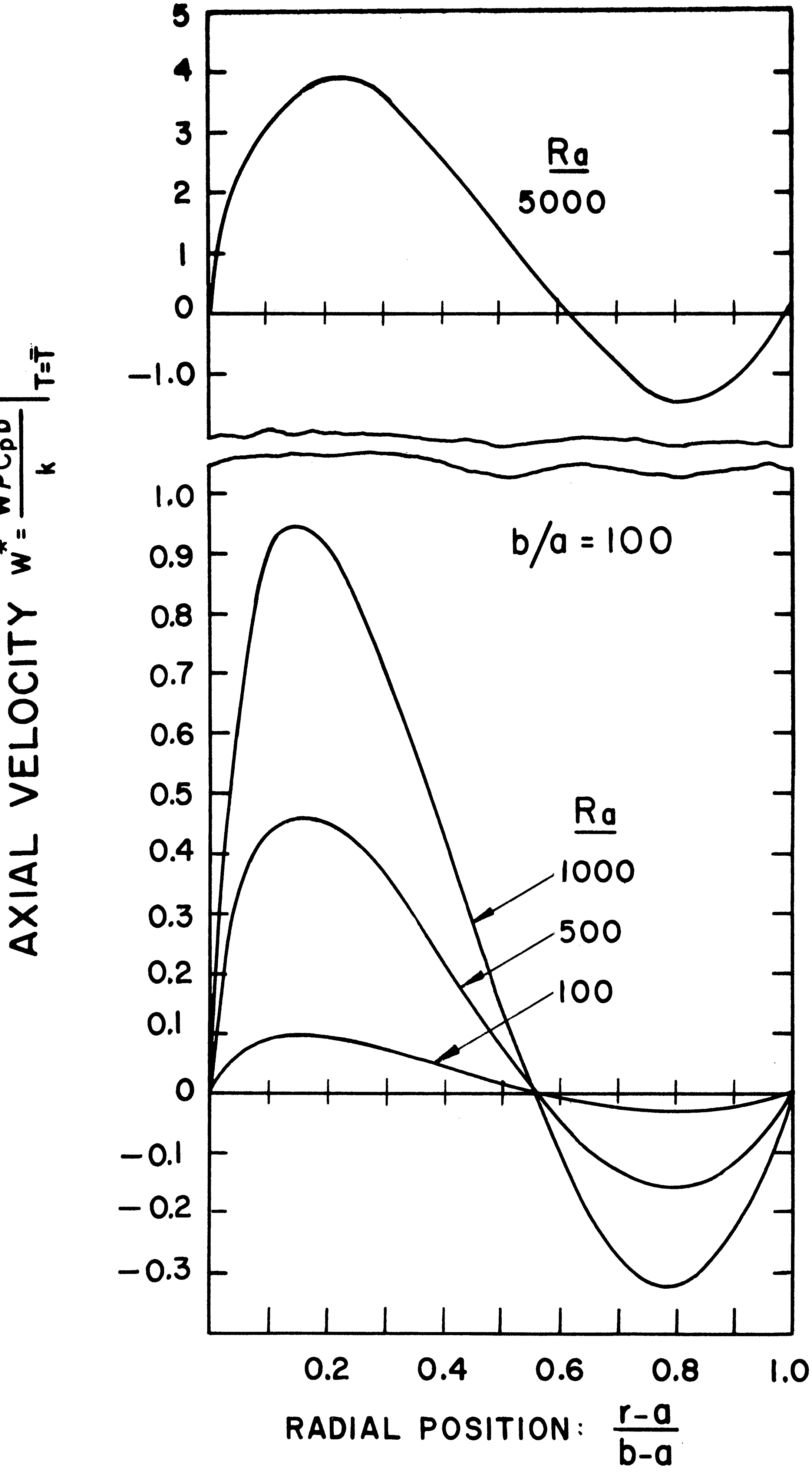
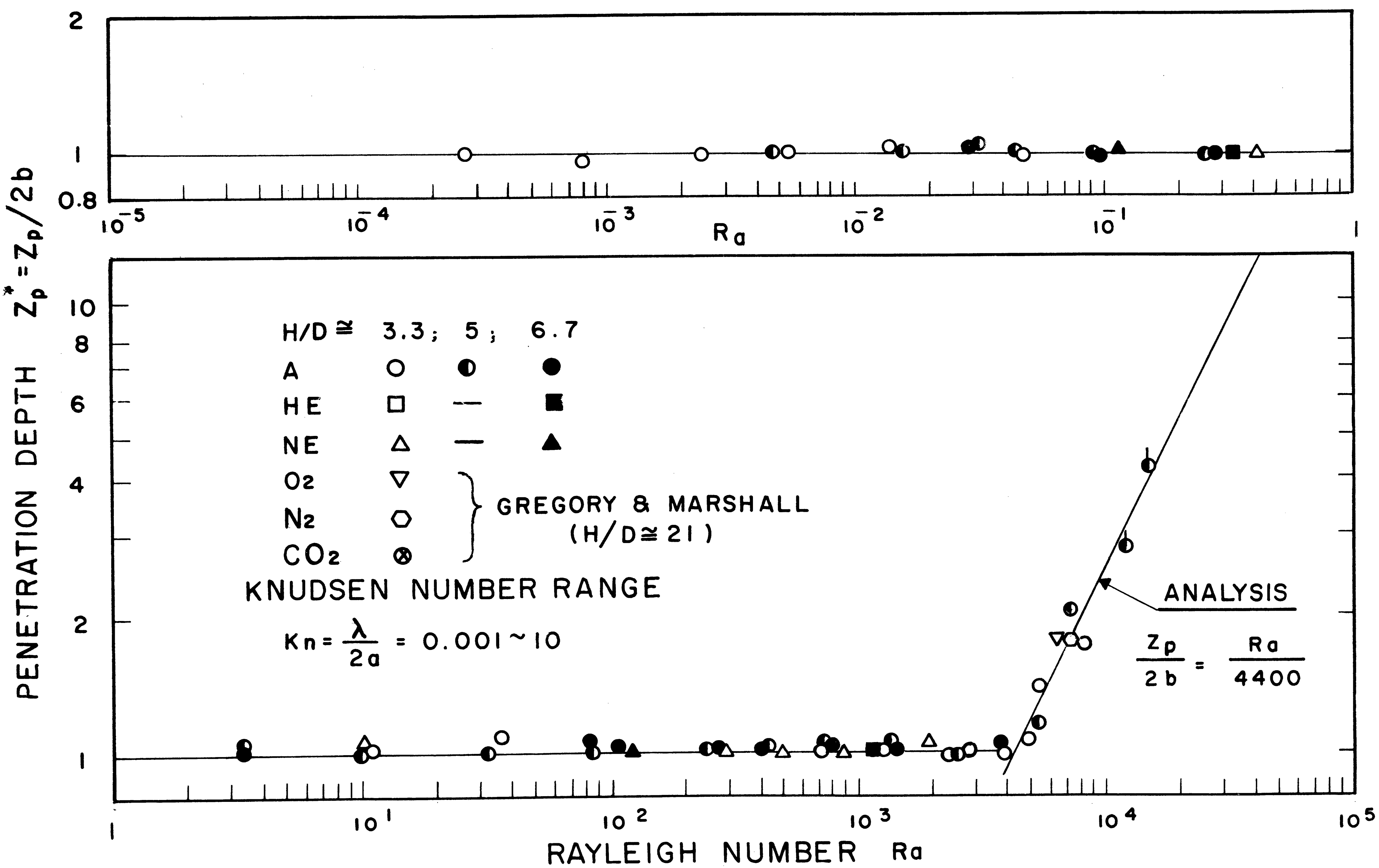


Fig. 2



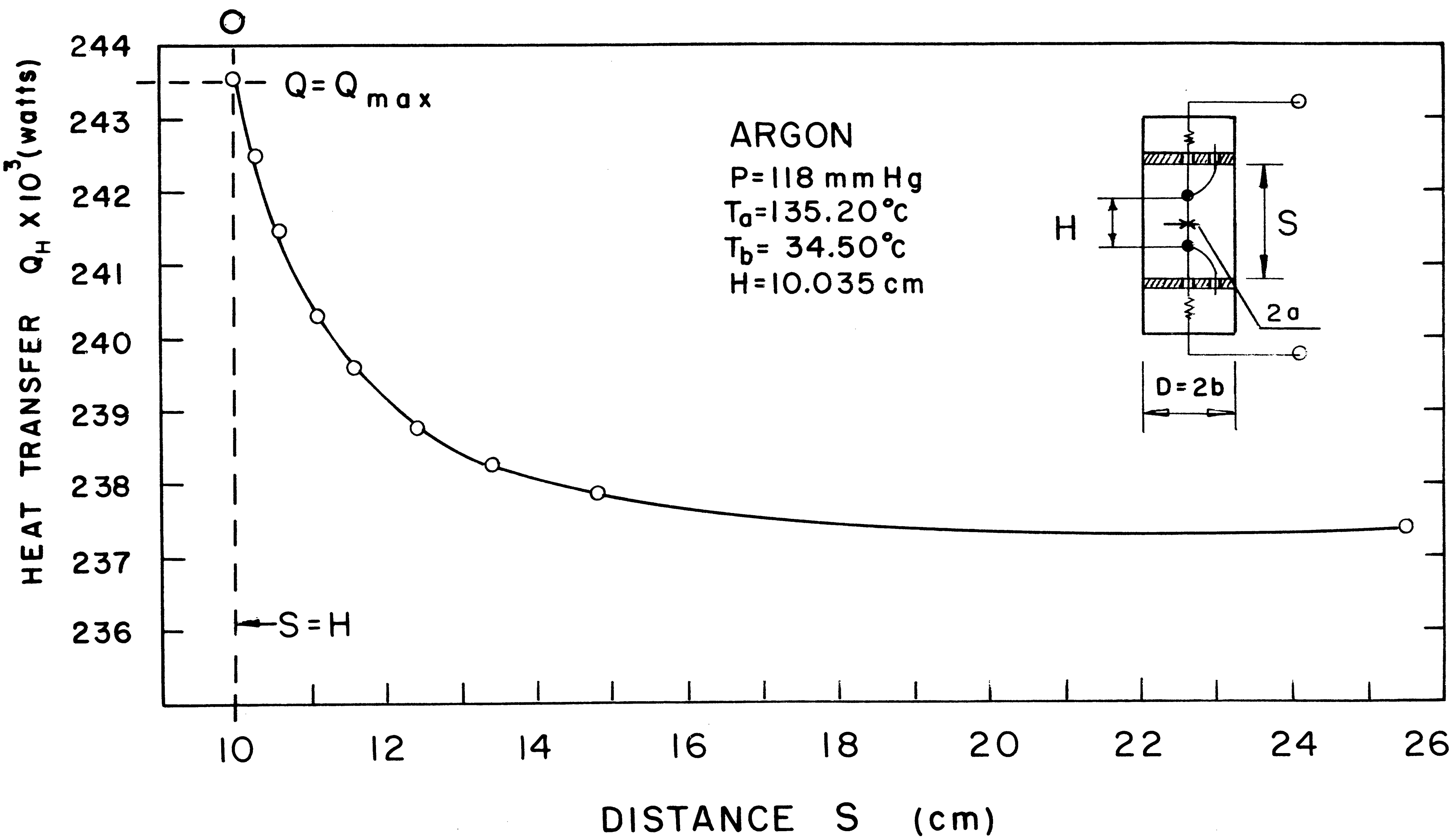
AXIAL VELOCITY $w^* = \frac{w\rho C_p b}{k}\Big|_{T=\bar{T}}$
Ra
5000
b/a = 100
Ra
1000
500
100
RADIAL POSITION: $\frac{r-a}{b-a}$

Fig. 3



PENETRATION DEPTH $Z_p^* = Z_p/2b$
2
1
0.8
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
Ra
10
8
6
4
2
1
H/D ≅ 3.3; 5; 6.7
A
HE
NE
O2
N2
CO2
GREGORY & MARSHALL
(H/D ≅ 21)
KNUDSEN NUMBER RANGE
$Kn = \frac{\lambda}{2a} = 0.001 \sim 10$
ANALYSIS
$\frac{Z_p}{2b} = \frac{Ra}{4400}$
1
10^1
10^2
10^3
10^4
10^5
RAYLEIGH NUMBER Ra

Fig. 4



HEAT TRANSFER Q_H X 10^3 (watts)
244
243
242
241
240
239
238
237
236
O
Q=Q max
S=H
ARGON
P=118 mm Hg
Ta=135.20°c
Tb= 34.50°c
H=10.035 cm
H
S
2a
D=2b
10
12
14
16
18
20
22
24
26
DISTANCE S (cm)

Fig. 5

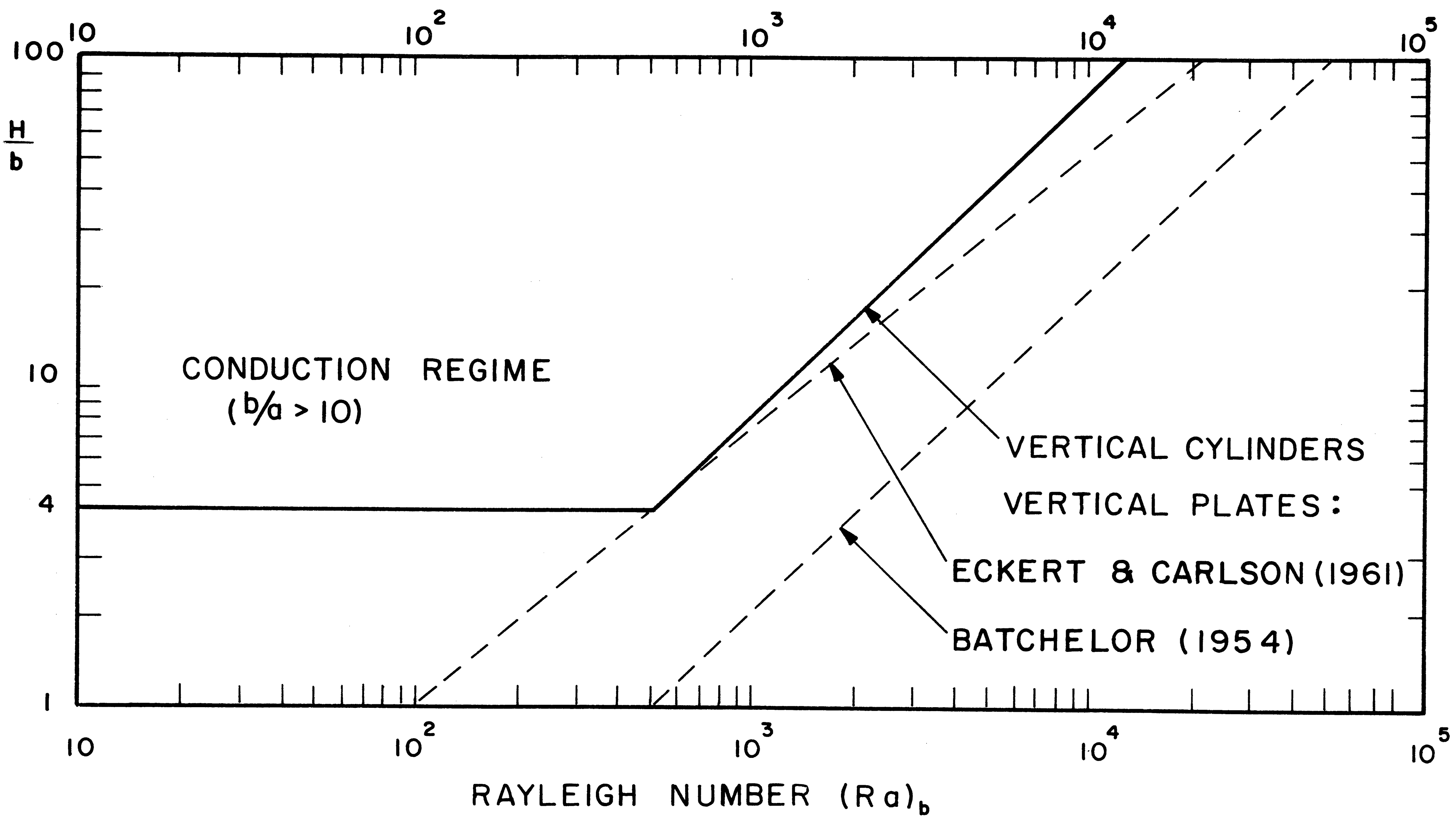

100
10
4
1
H/b
10
10^2
10^3
10^4
10^5
CONDUCTION REGIME
(b/a > 10)
VERTICAL CYLINDERS
VERTICAL PLATES:
ECKERT & CARLSON (1961)
BATCHELOR (1954)
RAYLEIGH NUMBER (Ra)b

Fig. 6

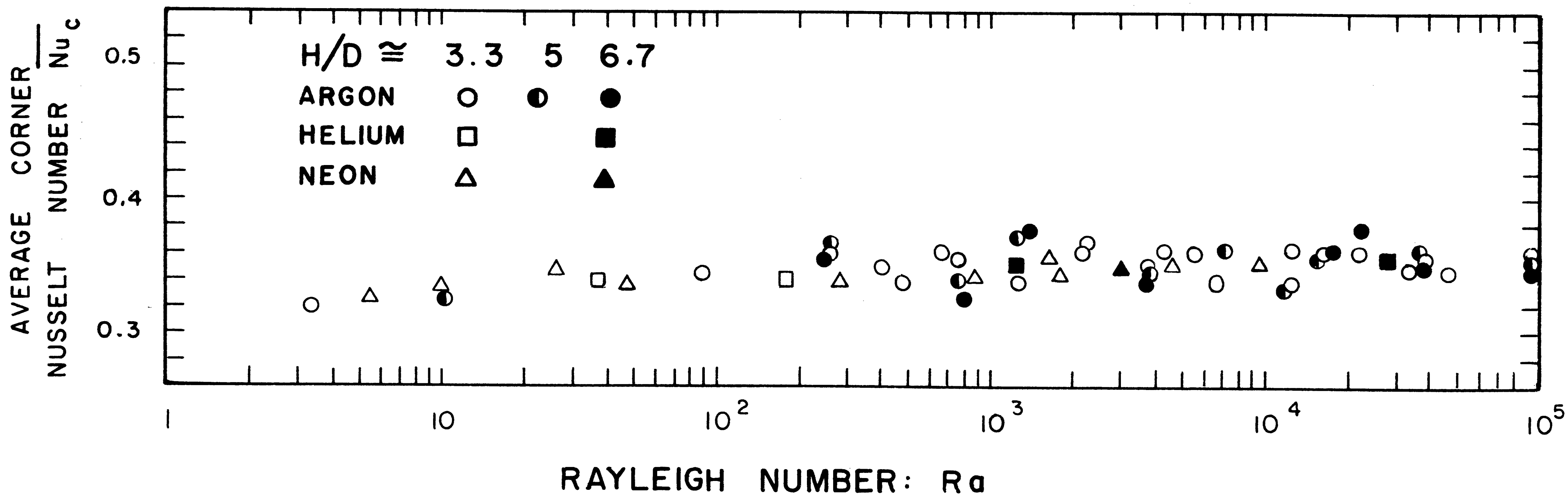

AVERAGE CORNER NUSSELT NUMBER $\overline{Nu_c}$
0.5
0.4
0.3
H/D ≅ 3.3 5 6.7
ARGON
HELIUM
NEON
1
10
10^2
10^3
10^4
10^5
RAYLEIGH NUMBER: Ra

Fig. 7