THE UNIVERSITY OF MICHIGAN

INDUSTRY PROGRAM OF THE COLLEGE OF ENGINEERING

AN EMPIRICAL APPROACH TO WORKING STRESS DIAGRAMS

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INTRODUCTION

It is often found that design analysis using conventional failure theories and working stress diagrams does not adequately nor reliably evaluate the fatigue strengths of machine components. Apparently, the factors involved and the conditions occuring during the lives of these components are too complex to be approximated by simple energy and/or stress criteria.

This lack of correlation is not surprising in light of the following observations on fatigue failures.

Fatigue, from a macroscopic viewpoint, is a three stage process. The crack initiation stage appears to play the predominant role in determining the fatigue strength in some cases, while in others, the crack propagation stage appears predominant: in the former cases, the relative duration of these two stages seems to be a variable which influences not only the fatigue strength, but the endurance limit as well. Simple energy and/or stress criteria cannot deal effectively with this situation.

Energy and stress criteria can often be exceeded in localized areas, particularly at the root of notches, and not produce failure. Factors such as stress history, strain-hardening characteristics, stress redistribution, residual stresses, etc., are equally as pertinent as failure criteria in this situation.

Some materials exhibit a "knee" in their S-N curves while others do not. The mechanism of fatigue may be different in these two cases: if this is so, the same "law of failure" probably should not apply to both. Indeed, there is some indication that several mechanisms of fatigue exist and that different mechanisms occur at different stress levels.

A portion of the nominal stress-strain relationship can be altered by stressing relatively soft steel cyclically below the yield strength (of the virgin material). During such cycling, sub-microscopic anelastic behavior occurs, the cumulative result of which is a lower yield strength and a non-linear stress-strain relationship in the vicinity of the fatigue strength. (Other materials are affected to different degrees and in different manners.) Thus, the estimation of the true elastic constants at any given point in the material is difficult and assumptions of homogeneity, isotropy, independence of stress history, etc., lack credulity.

The fatigue strength of steel can be altered by cyclic stressing below the yield strength --- understressing and overstressing.

The observations point out difficulties in applying general energy and/or stress criteria to all fatigue applications. The most accurate and reliable approach to fatigue design is the empirical approach of compiling data for each material and mode of loading.

DEFINITION OF FAILURE

The term "fatigue" will be used in this paper for the mode of failure resulting from fluctuating loadings which induce <u>no</u> noticeable plastic deformations.

Thus, failure above the yield strength in Figure 1 cannot strictly be called fatigue because plastic deformations do occur. In order to distinguish this mode of failure from others, it will be termed "yield-fatigue failure". It is the transition mode which lies between static failure and fatigue failure and is closely related to the plastic strains induced(1)[3]-- implying that it results from a different fatigue mechanism than exists at the endurance limit.

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[3] Numbers in parenthesis refer to similarly numbered references in bibliography at end of paper.

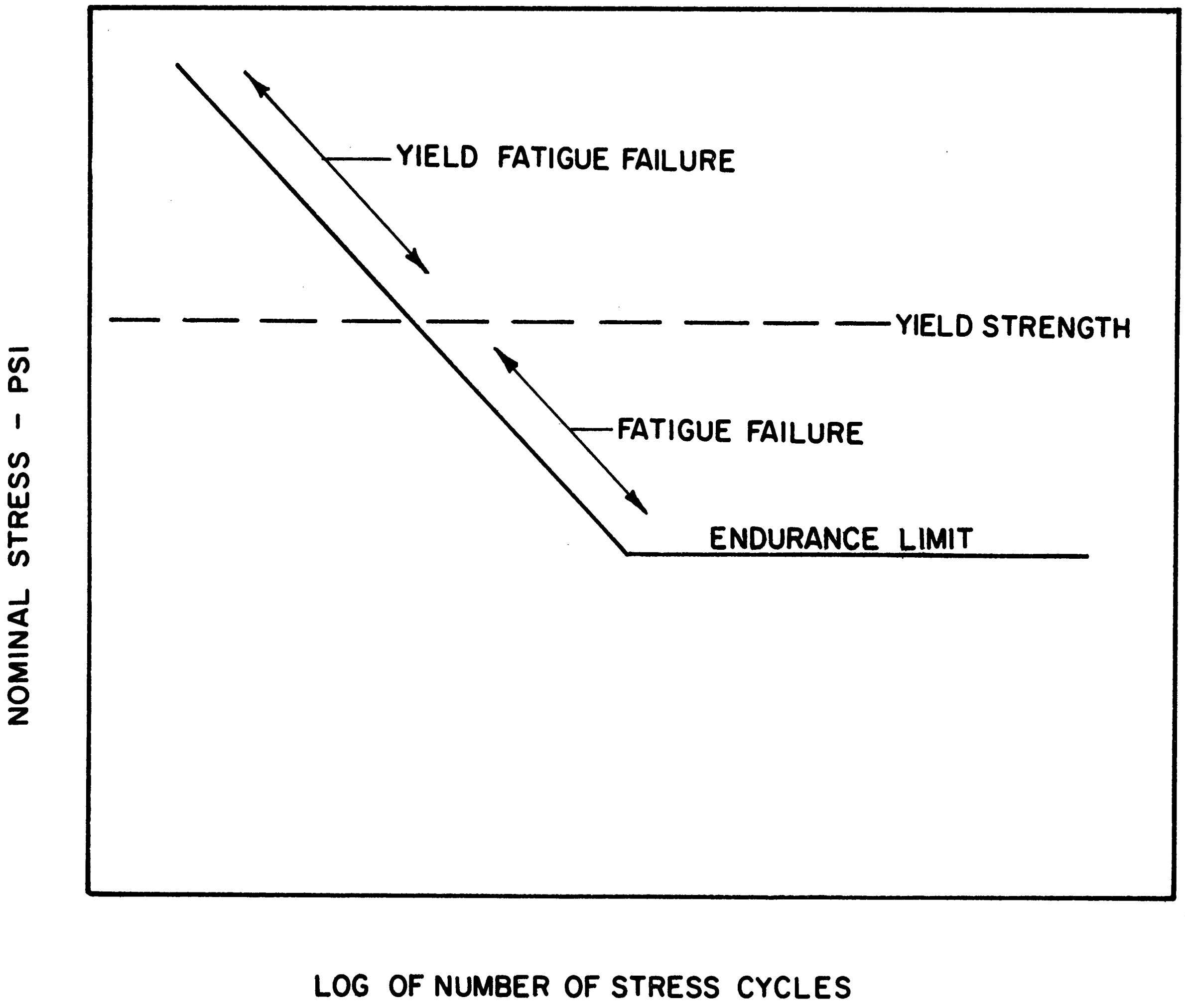


Figure 1. Typical S-N Curve for Axial Fatigue Loading.

It is very important to make this distinction for loadings which have a steady state or mean component as shown in Table 1.

Permanent deformations can occur for fluctuating loadings with relatively high mean components even though the yield strength is not exceeded. This is termed "creep-fatigue failure" in this paper. Normally, fatigue tests are too short in duration to fully observe this mode of failure and often the creep effect, when observed, is attributed to a relaxation effect. (The expression "creep effect" is used here because it seems more appropriate than"relaxation effect" --- neither expression is precisely correct.) Failure resulting from small permanent deformations is essentially a matter of definition and this mode of failure is generally repressed by rigidity requirements.

SCOPE

Mean stress fatigue data are compiled for axial, bending, and torsional loading of unnotched steel specimens. The compilation is confined to ultimate tensile strengths between 60 and 175 KSI. Although steels with tensile strengths above 175 KSI are very important in fatigue applications, there is not enough mean stress data published in order to develop accurate working stress diagrams nor enough to allow confident extrapolation of diagrams for softer steels.

The mean stress data for aluminum does not exhibit a consistent trend and no reasonable diagram can be developed if all data are weighed equally.

WORKING STRESS DIAGRAMS

The best way to construct an allowable working stress diagram is to: (a) draw the limiting deformation line (it can be visualized as roughly the static yield strength line) and, then (b) draw the fatigue failure locus through only that data which displays less permanent deformation prior to failure.

The results of the compilation of mean stress data are given in Figures 2-5. The data of Figure 2 are believed more reliable than that of Figure 3.

While working stress diagrams are not developed for combined modes of loading, it can be seen that the effect of mean stress is relatively small for stresses not exceeding the yield strength. The portions of these diagrams for mean compressive stresses are not shown --- horizontal lines are recommended. Although there can be little doubt that mean compressive stresses increase the fatigue strength, it is unlikely that these increases will often be realized in practice due to the less than perfect alignment of components.

DESIGN OF NOTCHED COMPONENTS

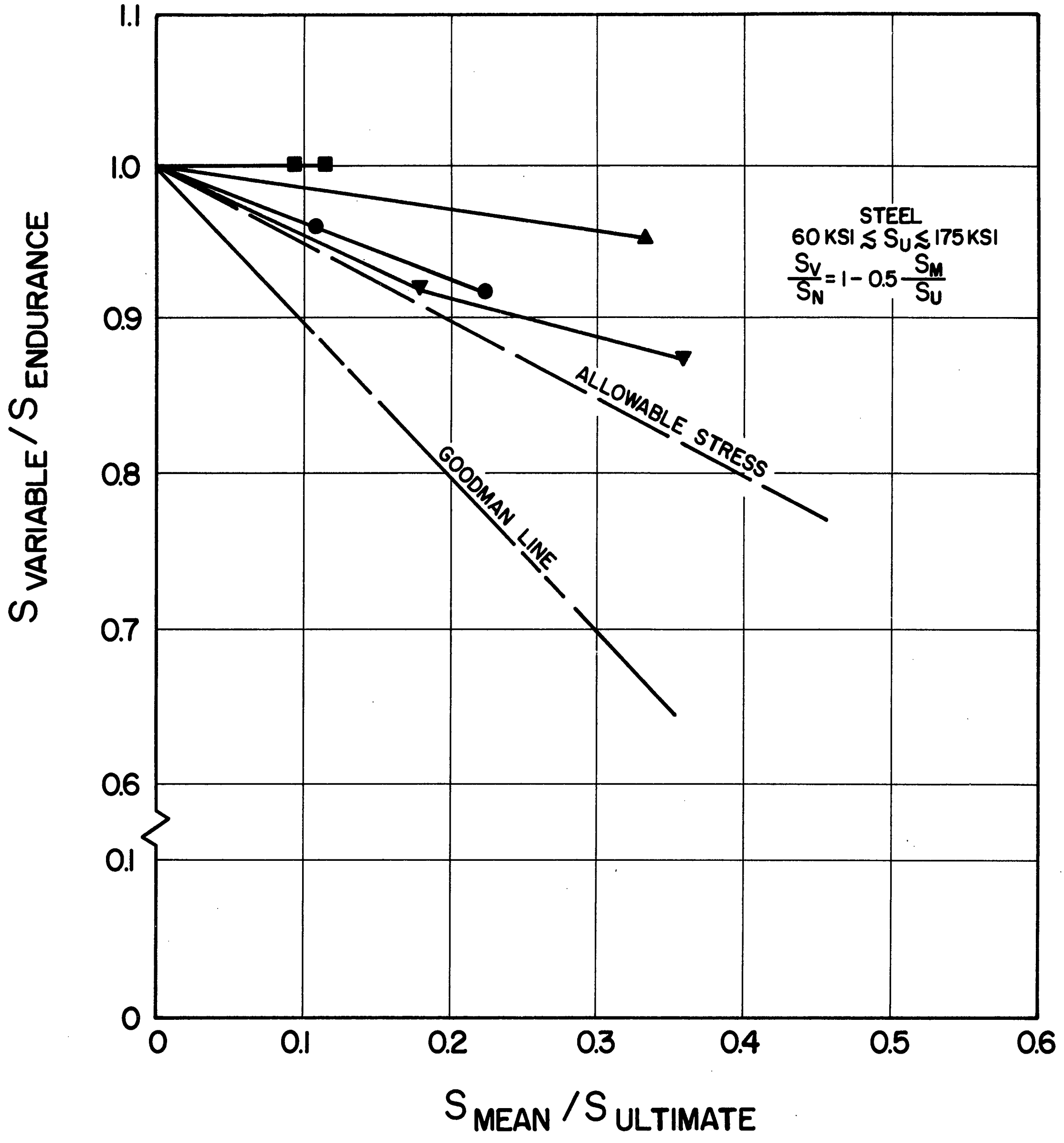
ELASTIC RANGE

There are two common working stress diagram approaches to design of notched components:

(1) Apply the fatigue strength reduction factor to both components of the loading.

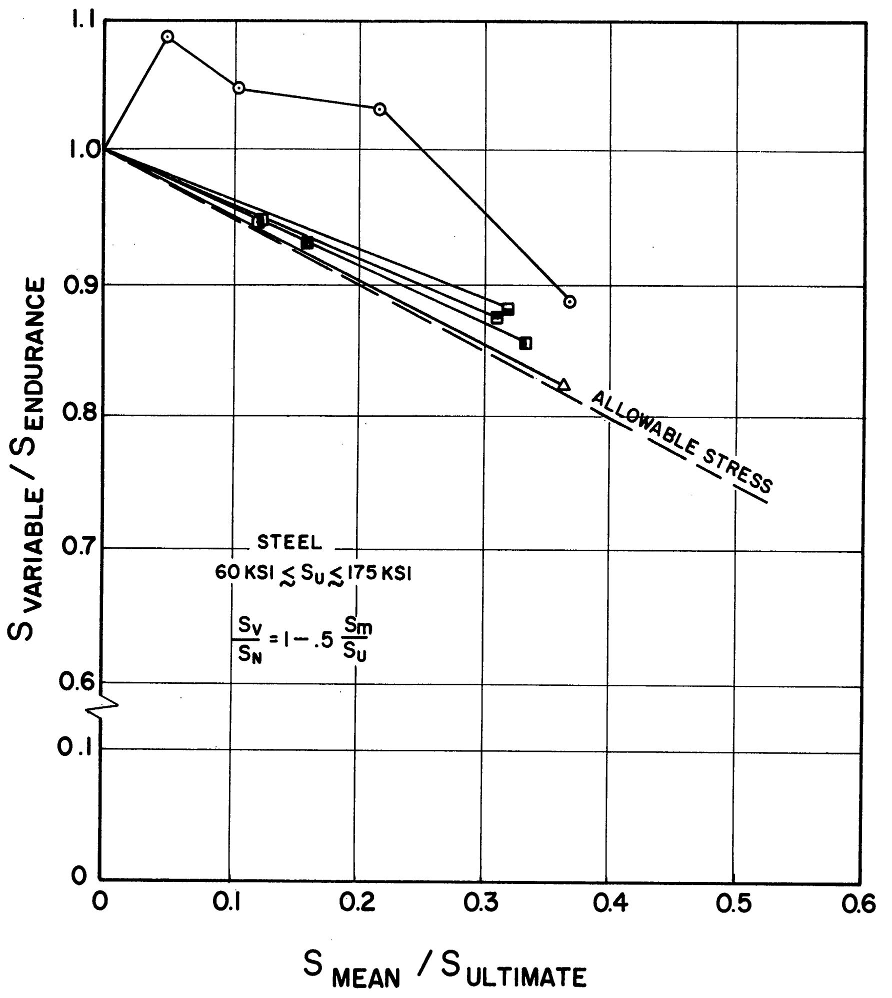
(2) Apply the fatigue strength reduction factor to only the variable component of the loading.

These methods give results which are identical within the accuracy of most determinations of fatigue strengths because the mean stress has such a small effect on fatigue in the elastic range. Method 1 is recommended for



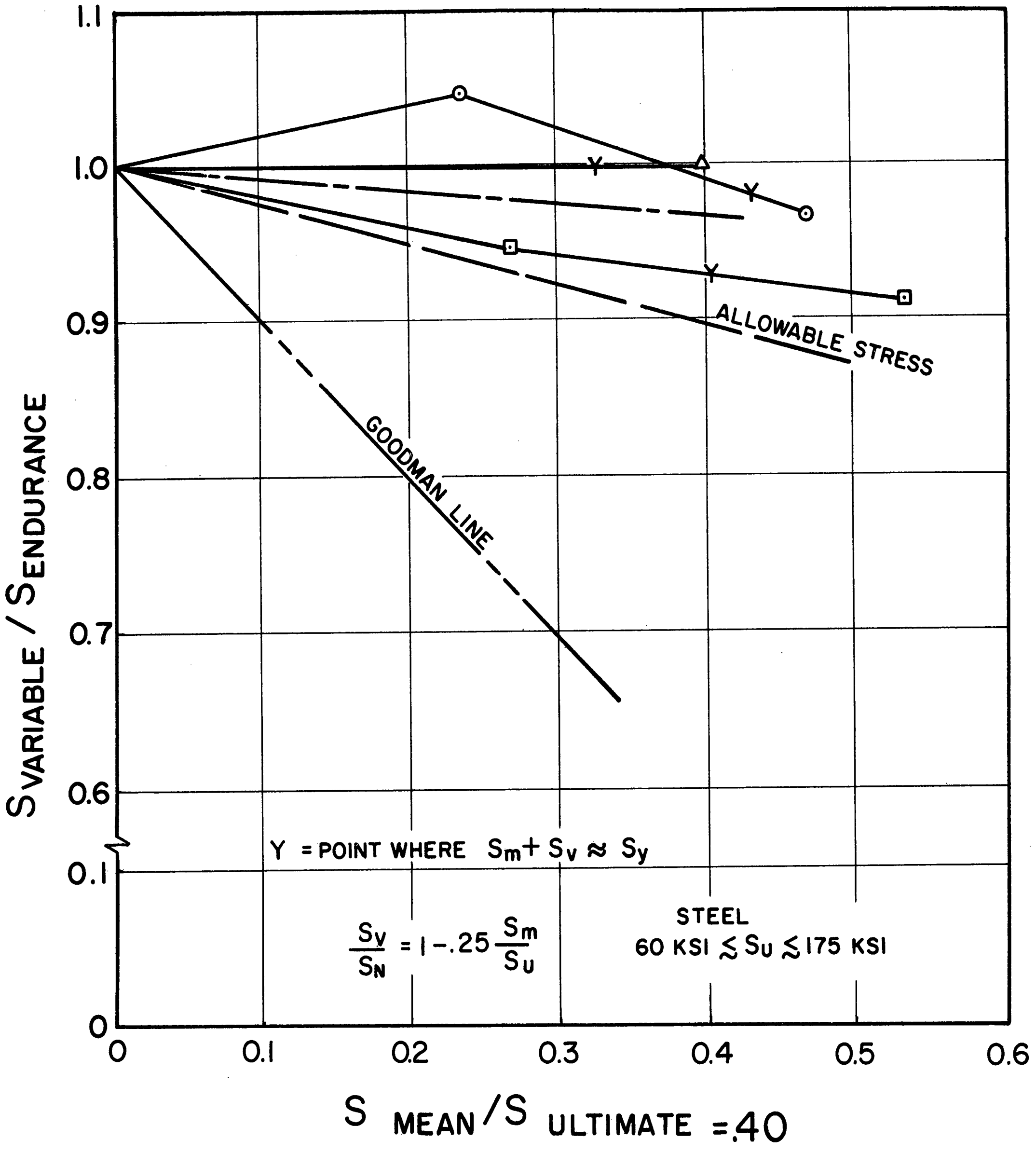
REFERENCE	S_U (KSI)	SYMBOL	YEAR	INVESTIGATOR	STEEL
(3)	59	●	1939	GOUGH & WOOD	.12C
(4)	120 (EST.)	▲	1954	FINDLEY	SAE 4340 Q & T - D
(5)	125	▼	1956	O'CONNOR & MORRISON	Ni-Cr-Mo Q & T
(6)	119	■	1955	FITCHIE	Ni-Cr Q & T

Figure 2. Working Stress Diagram Showing Selected Mean Stress Fatigue Data for Axial Loading in the Nominal Elastic Range.



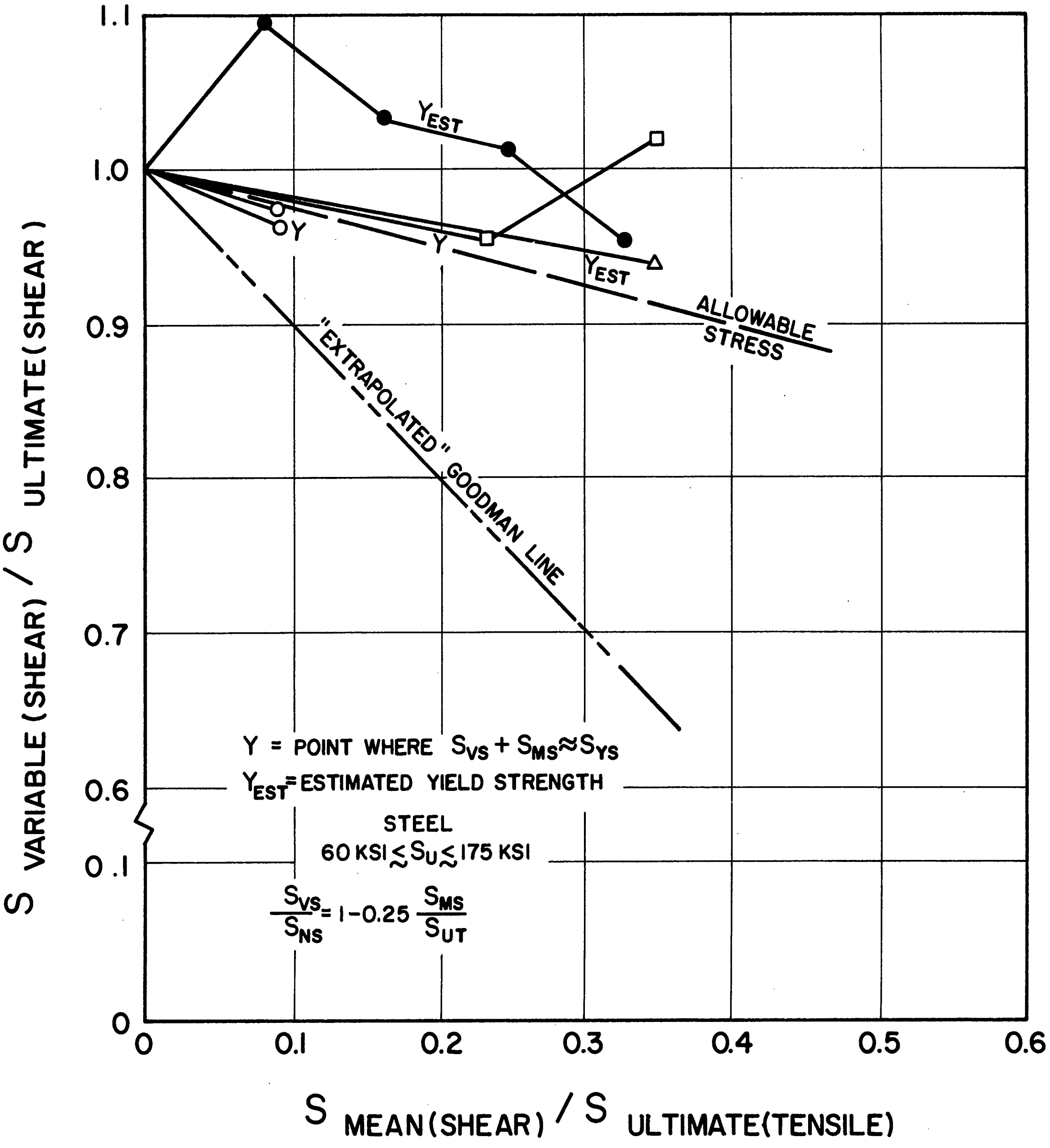
REFERENCE	S_U (KSI)	SYMBOL	YEAR	INVESTIGATOR	STEEL
(7)	117	⊙	1951	GROVER, BISHOP AND JACKSON	SAE 4130 NORM
(8)	159	△	1953	TRAPP & SCHWARTZ	SAE 4340 Q & T
(9)	115	⬒	1941	HEMPEL & LUCE	CR - MO Q & T
"	136	⬓			CR - MO Q & T
"	150	◧			NI - CR - MO Q & T
"	61	◨			ST 34. 11 (08C)
"	89	■			ST 50. 11 (LOW ALLOY)
"	114	⊡			ST 70. 11 (LOW ALLOY)

Figure 3. Working Stress Diagram Showing Other Mean Stress Fatigue Data for Axial Loading in the Nominal Elastic Range.



REFERENCE	S_u (KSI)	SYMBOL	YEAR	INVESTIGATOR	STEEL
(10)	145	⊡	1951	GOUGH & CLENSHAW	Ni-C_R Mo Q&T
(11)	122	△	1951	HANLEY	SAE 4340 Q&T-D
(12)	172	⊙	1953	FINLEY, MERGEN & ROSENBERG	SAE 4340 Q & T
(4)	~70 (EST)	—·—	1954	FINDLEY	SAE 1020

Figure 4. Working Stress Diagram Showing Mean Stress Fatigue Data for Reversed Bending Loading in the Nominal Elastic Range.



REFERENCE	S_U (KSI)	SYMBOL	YEAR	INVESTIGATOR	STEEL
(10)	145	△	1951	GOUGH & CLENSHAW	Ni - Cr - Mo Q & T
(13)	125	○	1956	CHODOROWSKI	Ni - Cr - Mo Q & T
(11)	122	●	1951	HANLEY	SAE 4340 Q & T - D
(12)	172	□	1953	FINDLEY, MERGEN & ROSENBERG	SAE 4340 Q & T
NOTE: S_{MS}/S_{US} IS MORE CONSERVATIVE WITH RESPECT TO ALLOWABLE STRESS LINE					

Figure 5. Working Stress Diagram Showing Mean Stress Fatigue Data for Torsional Loading in the Nominal Elastic Range.

design and is a few percent more conservative than method 2. Extrapolation of either method into the plastic range is not completely justified by existing data.

PLASTIC RANGE

Plastic strains induced by loadings which have high mean components do not appear to reduce seriously the strength of notched steel specimens provided that the bulk of the specimen's critical cross-section is elastic. (Mean stress data for unnotched specimens, subjected to either bending or torsion, follow this same general trend.) The dashed line of Figure 6 reflects the trend of the data.

It seems that more parameters are needed in order to correlate accurately the data in this range. In addition, there remains the question of where, on Figure 6, the dashed line should be terminated --- it is in this area that creep-fatigue failure occurs and more study is needed to develop design methods.

ENDURANCE LIMITS

Since working stress diagrams are often developed using only static strengths, a more accurate method of estimating the endurance limit is desirable.

It has long been known that certain microstructures are superior to others in fatigue applications. A rough, but very effective, index to this effect of microstructures is the Sy/Su ratio (yield strength/ultimate strength). Figure 7 shows approximate "B-20" lines as determined by a compilation of recent data.

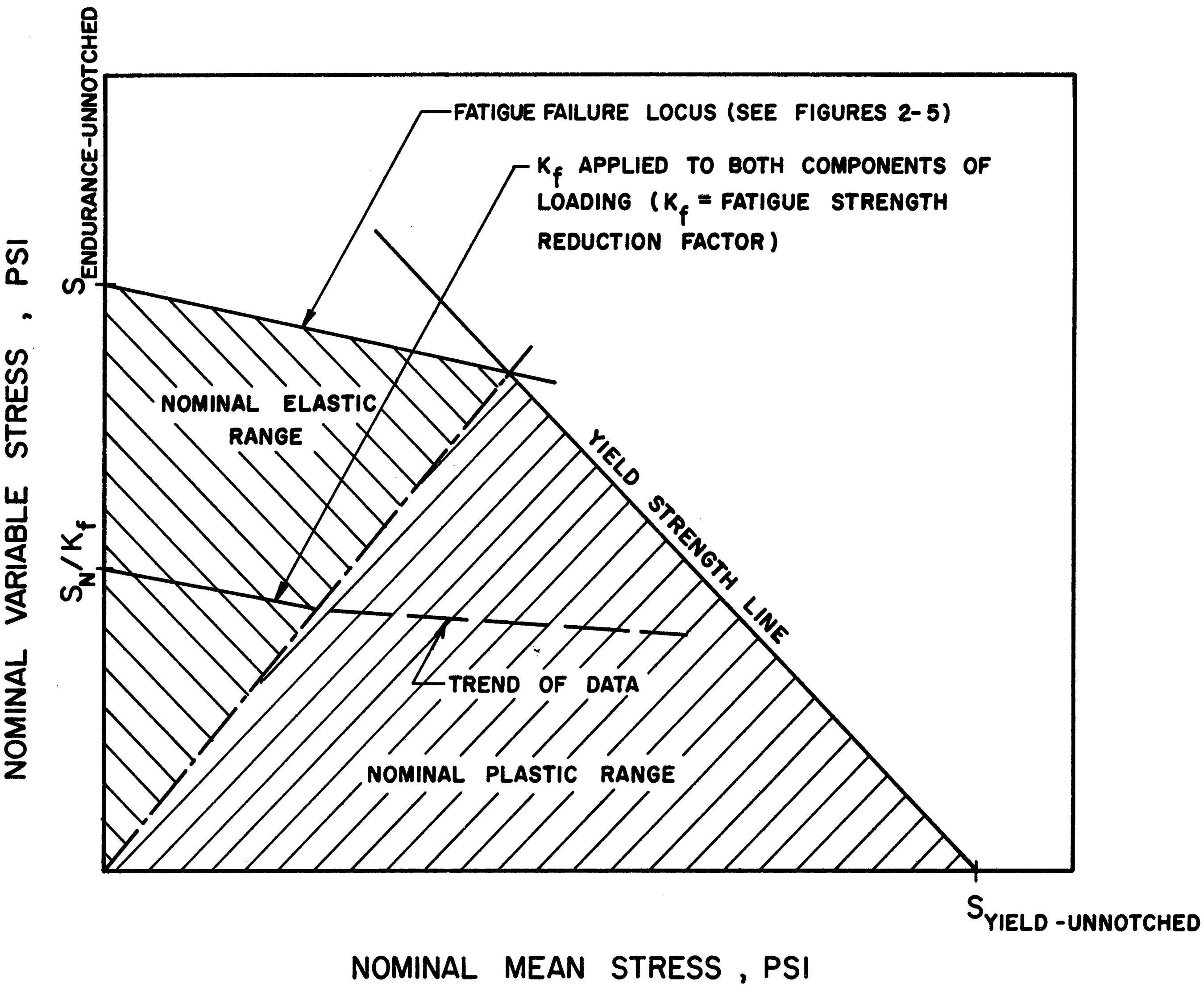


Figure 6. Working Stress Diagram Showing Trend of Mean Stress Fatigue Data in the Nominal Plastic Range.

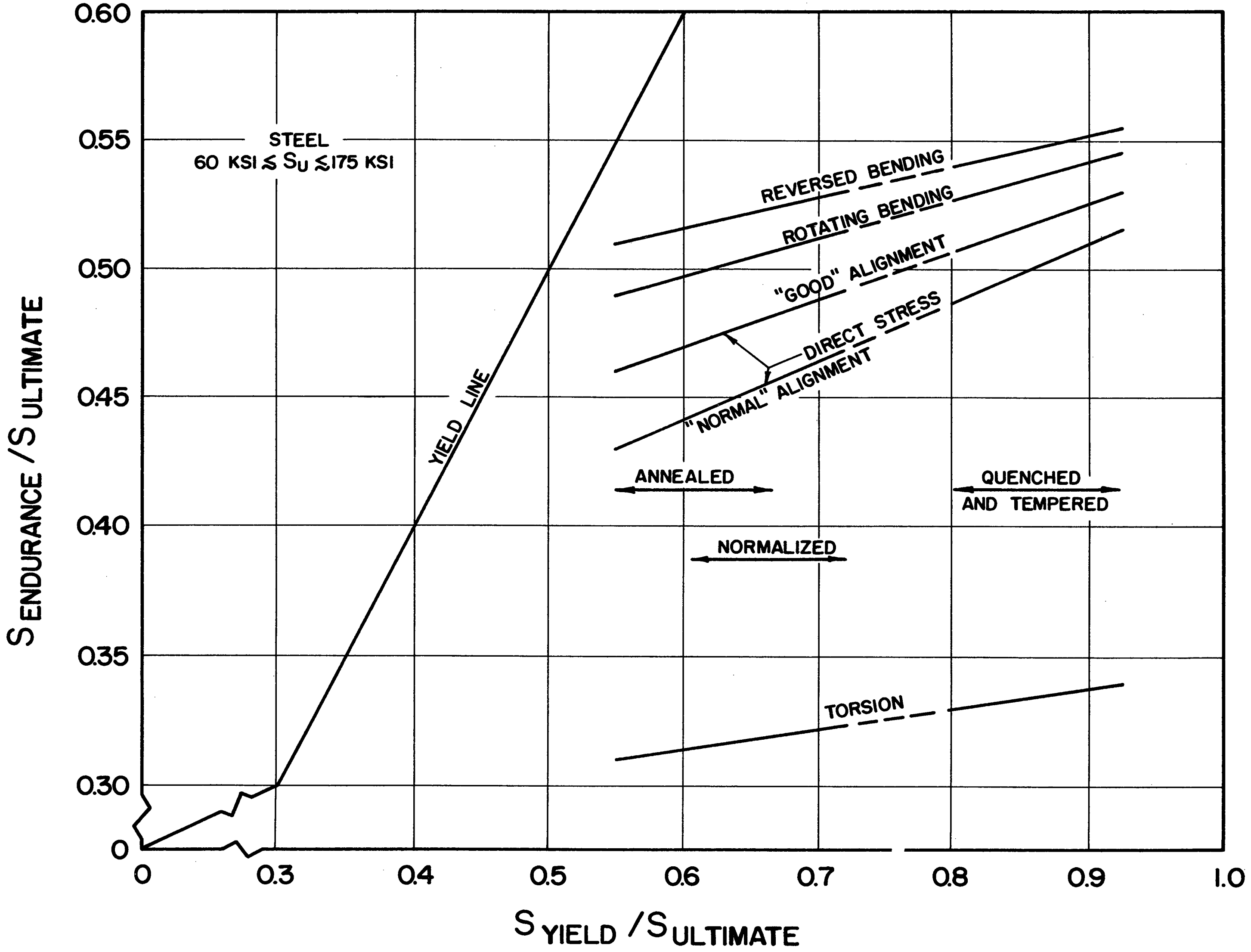


Figure 7. Correlation Between the Static Strength and the Endurance Limit--Approximate "B-20" Lines for Mechanically Polished Fatigue Specimens.

These lines should not be used for "as quenched" structures. As a further refinement to their use: spherodized structures generally have slightly higher endurance limits than shown, whereas tempered structures with "easily resolved" carbides have slightly lower endurance limits than shown.

Data on size effect are anomalous and, in general, shows this effect to be very small in relation to the uncertainties involved in estimating fatigue strength reduction factors, surface factors, stresses, etc. However, for very small and very large components, metallurgical and processing effects should be considered.

DISCUSSION OF COMPILATION

Several general observations on fatigue resulting from this study are listed below.

The results given in Figures 2-5 show that as the maximum stress approaches the yield strength, the fatigue strength (the allowable variable component of stress) decreases. The same relationship holds for the fully reversed stressing of specimens as shown in Figure 7.

The level of the maximum stress with respect to the yield strength determines the level of the anelastic behavior occurring --- the higher the maximum stress for a given variable stress, the more pronounced the effect of the anelastic behavior in decreasing the fatigue strength. The level of the variable stress determines the intensity of the anelastic behavior: its effect is greater than that of the mean stress. Thus, a significant parameter for all types of loadings (by extrapolation) is the relative difference between the maximum stress and the yield strength.

If this difference is large, increasing the mean stress will have little or no effect on the fatigue strength. (Yield strength, in the above discussion, is used as a reference value. Proportional limit, elastic limit, etc., can be used.)

The type of deformation is relatively important in estimating fatigue strengths and endurance limits. Mohr's circle analysis methods do not predict the greater the greater strengths for bending than for axial loading. The restraining influence of neighboring grains in allowing only those microscopic deformations which are compatible with the macroscopic deformations is probably responsible for this increase. This factor should be considered when analyzing complex states of stress.

When a reasonable comparison of data is possible, reversed bending fatigue strengths are consistently higher than those for rotating bending. This anomaly can apparently be explained by the difference in the crack propogation stages. Rotating bending specimens not only have more area susceptible to fatigue cracks but apparently do have more propogating cracks at the time of rupture. The difference of strengths is much less noticeable when the Sy/Su ratio is high since the number of potential sites is substantially decreased. It has been observed that, in rotating bending tests, it is much more difficult to initiate a single crack in mild steel (low Sy/Su) than in a chrome-nickel steel (high Sy/Su).(2)

Hence, the use of knowledge of the fatigue fracture appearance can help refine the estimation of the fatigue strength. Consider, for example, the fatigue strengths, in rotating bending, of hollow versus solid specimens --- hollow specimens are consistently weaker. The macroscopic deformations involved are virtually identical during the crack initiation stages: the difference in the crack propogation stages evidently accounts for the difference of strengths as well as for the "knee" occuring at a lesser number of cycles for the hollow specimens.

The difference of fatigue strengths carries over to the endurance limits in both cases above. Although the correlation factor does not appear to be unity, a definite relationship exists between the endurance limit and the fatigue strength. (The relationship between the yield-fatigue strength and either the fatigue strength or the endurance limit is much less certain and depends, at least in part, on the mode of loading.)

The endurance limits of various tests in bending and torsion are often compared as meaningful --- usually in relationship to a failure theory. The overall ratios, as read from Figure 7, are:

Torsion/Reversed Bending -------------------------.61
Torsion/Rotating Bending -------------------------.63
Torsion/Direct Stress----------------------------.65
Direct Stress/Reversed Bending--------------------.93
Direct Stress/Rotating Bending--------------------.95

No comparison above has special significance since the data are from many various sources and several factors cannot be controlled.

The latter two ratios above reflect the tendency of recent data to show less loading effect than found in older tests. In fact, there is some indication that, when a reasonable comparison of data is possible, the loading effect is insignificant for high Sy/Su ratios.

COMMENTS

Many "traditional" references are not included in this compilation because these tests are not at the level of more recent tests: data from these references should not be considered as reliable as recent data. In addition, several references do not include sufficient static data in order to plot accurate curves. However, in all cases examined, the trends follow the allowable stress lines in a conservative manner as nearly as can be ascertained.

The "good" and "normal" direct stress "B-20" endurance limits were determined by compiling data from the periods 1945-1960 and 1915-1935, respectively.

TABLE I

CYCLIC DEFORMATIONS - DIRECT STRESS(5)

Magnitude of Stress of Cycle Tons/Sq. In. (= 2240 Lb./sq.in.)			Life-Number of Cycles X 10^{-6}	Reduction of Area-Percent
Maximum	Mean	Variable		
50.1	20	30.1	0.02	70.3
48.5	20	28.5	0.055	68.4
48.1	20	28.1	0.164	0.5
47.8	20	27.8	1.012	0.1
47.5	20	27.5	1.210	0.2
47.2	20	27.2	10.000 (Unbroken)	Zero

Material: Ni-Cr-Mo Alloy Steel-Quenched and Tempered

Static Properties: Tons/Sq. In.	Ultimate	Lower Yield	Upper Yield
	55.9	48.7	50.4
	±1.5	±1.5	±1.4

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