THE UNIVERSITY OF MICHIGAN

INDUSTRY PROGRAM OF THE COLLEGE OF ENGINEERING

THE EVALUATION OF NUCLEAR REACTOR PARAMETERS FROM MEASUREMENTS OF NEUTRON STATISTICS

By

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A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy in the
University of Michigan
1958

March, 1958

IP-270

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ACKNOWLEDGMENTS

Valuable guidance was given the writer by the members of his Doctoral Committee under the chairmanship of Professor S. W. Churchill. R. B. Mesler was a member of the Committee while an Assistant Professor at the University.

The writer is particularly grateful to the Argonne National Laboratory for the opportunity of joining the Reactor Engineering Division as a Resident Student Associate during this investigation. The unsurpassed facilities of the Experimental Physics Section under the direction of W. C. Redman made this work possible. All the staff members and technicians of this section freely gave of their aid and advice. The members of the ZPR-V staff, W. Y. Kato group leader, and comprised of C. E. Cohn, G. J. Fischer, and B. J. Toppel made really substantial contributions of time and technical criticism. F. H. Martens made ZPR-IV' available for measurements and assisted in the interpretation of results. Margaret Butler, Eleanor Funkey, and Mildred Schlapkohl programmed and carried out the machine computations.

P. Herman and R. White designed and built some of the early equipment and the rest of the writer's co-workers in the Phoenix Memorial Laboratory were always ready with a helping hand.

Ernestine Wark did her usual fine typing job on the first draft, and this final manuscript was published by the Industry Program of the College of Engineering.

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INTRODUCTION

THE EVALUATION OF NUCLEAR REACTOR PARAMETERS FROM MEASUREMENTS OF NEUTRON STATISTICS

This investigation was undertaken to provide insight into the statistical behavior of the neutrons in neutron multiplying systems such as nuclear reactors and to increase the amount of experimental data available on this phenomenon. The primary aim was to develop a statistical method for evaluation of nuclear reactor parameters: in this particular case the mean prompt neutron lifetime and the absolute power level. A further aim was to demonstrate that the neutron statistics may be measured, at least in the case of light water moderated systems, using conventional neutron counting apparatus. The statistical behavior of the neutron population in a neutron multiplying medium, although fundamental in nature, has not received much attention to date.

The behavior of the neutron population of a neutron multiplying medium is commonly described by differential equations. In reality, however, neutrons are discrete particles taking part in discrete processes and their behavior is not subject to complete description by the continuous representation of differential equations. That this is true may be confirmed by examining the trace of a sensitive neutron detector sampling the population of a multiplying system, such as a nuclear reactor. The trace will have a mean which, because of the extremely large number of neutrons in practical systems, appears continuous and exhibits a behavior predictable by differential equations. However, large fluctuations about the mean of the trace will also be apparent and close examination will reveal that the fluctuations are larger than would be expected if they were random in orgin. These fluctuations, often referred to as "pile noise", are a manifestation of the discrete and statistical nature of the neutron processes.

An appreciation of the orgin of the neutron population fluctuations requires an inspection of the system on the microscopic or single particle level. On this scale each individual neutron interaction is describable only in terms of the laws of probability. The quantitative parameters of these laws are the interaction probabilities known as "cross sections". The nature of the individual neutron events which requires probabilistic description is the basic cause of the neutron fluctuations.

The absorption of a neutron by a fissionable isotope can, with a certain probability, cause nuclear fission which releases a number of additional neutrons. The fluctuations in the number of these neutrons released by fission are a prime contributor to the overall neutron population statistics and an example of the probability distributions encountered on the microscopic level which are replaced with an average parameter on the macroscopic scale and in the continuous model. In illustration, the mean number of neutrons released by the fission of U-235 is 2.47 but emission probabilities for the release of from 0 to 7 neutrons have been measured (6).

Consider a multiplying medium, such as described above, driven by a random source of neutrons. A source neutron can be immediately lost to the system by non-productive absorption or by leakage across the system boundaries. Alternately, it can initiate a chain of neutron production. The length of such chains is not fixed but varies widely since each neutron interaction is subject to the laws of chance. This distribution of neutron histories and chain lengths results in greater-than-random fluctuations for the combination of random source plus neutron multiplying medium. In addition, random-in-time source fluctuations become correlated-in-time chain fluctuations whose time scale is dictated by the time constants of the neutron multiplying system. The neutron chains are composed of connected links of discrete neutron events. The times between the connected or correlated events and hence the periods of the fluctuations are determined by the prompt neutron lifetime and the delayed neutron half-lives.

The neutron statistics of multiplying systems is important in practical applications and is a fruitful field for fundamental research. Pile noise imposes a limit on the precision of neutron measurements such as those taken in connection with pile oscillators (11, 15). The fluctuations should influence the design of reactor control instrumentation such as period channels and servo controls. Since the phenomenon is intrinsic to the system its study yields information on fundamental parameters such as system multiplication, mean neutron lifetime, effective fraction of delayed neutrons, dispersion in the number of neutrons emitted in fission, and detector efficiency. It may be possible to extend the theoretical and experimental results of studies of neutron multiplying systems to multiplying systems such as microbe populations which are more difficult to study.

Neutron fluctuations may be measured and studied using several techniques. The time distribution of counts is measured in the so-called "Rossi-alpha" experiment (3,27) to yield values of the prompt neutron period and mean prompt neutron lifetime. Moore (25) proposes to evaluate the reactor transfer function from measurements of the auto correlation function of pile noise at steady power. The work to be described here deals with the study of a particular statistic, the variance to mean ratio of counts, as a function of counting time.

The variance to mean ratio was investigated by Feynman, de Hoffmann, and Serber (13) as a means of evaluating the dispersion in the number of neutrons emitted in fission. This technique is extended here as a method of evaluating mean prompt neutron lifetime and absolute reactor power. The parameters to be measured, neutron lifetime and power level, are difficult to measure by ordinary methods because they are integral properties of the system. Conventional lifetime measurement techniques typically entail complete real and adjoint flux maps (28,29), pile oscillators (18), or a uniform poisoning technique (31). Power calibrations made at low power generally utilize a complete flux map (30). In contrast, the proposed neutron statistics method would exploit the theoretical prediction that the neutron population fluctuations are themselves an integral property of the system. Therefore, if the validity of the method is established, measurements of fluctuations at a single point may be utilized to yield integral properties of the system. Also, the statistics measurement technique employed is conventional and requires only simple counting equipment available to most laboratories.

The theoretical statistics of a neutron counting experiment on a multiplying medium is developed from several points of view.

The theory is derived from essentially physical arguments. Alternately, the same results are obtained from a more abstract model. Although one of the developments includes the effects of the delayed neutrons the results are reduced to a working equation which describes only the prompt neutron induced fluctuations. This equation is adequate to describe the fluctuations measured under selected conditions.

The neutron statistics of two reactors, ZPR-IV' and ZPR-V, built and operated by the Argonne National Laboratory were measured by a counting type experiment.

Commercially available counting equipment was employed. A boron trifluoride counter in the reactor detected the neutrons. Its signal was amplified by a pre-amplifier and a linear amplifier. The pulse-amplitude-discriminator output of the linear amplifier drove a scaler whose counting or gate time was precisely controlled by a crystal oscillator time-base-generator. The output of the scaler was recorded by dictation or by a digital recorder.

The measurement technique consisted of counting the number of neutrons detected during a fixed counting or gate time. This was repeated for many gates and the number of counts obtained at the fixed gate time was analyzed for the ratio of variance to mean. Data on the behavior of the variance to mean ratio as a function of gate time was obtained by repeating the whole at gate times ranging from .001 to 1.000 seconds. The effect of gate time was studied at reactivities varying from -1.720×10^{-2} to -2.135×10^{-4}. The experimental data are correlated in terms of the theory in order to evaluate the parameters of the theory and hence of the reactors studied. The mean prompt neutron lifetimes and absolute power levels of the reactors are calculated from the measured parameters and are compared with independent estimates and measurements of these quantities. The applicability and limitations of this statistical method of evaluating nuclear reactor parameters are discussed both for the light water moderated systems investigated and for systems with longer and shorter prompt neutron lifetimes.

THEORY

The theoretical statistics of a neutron multiplying system as measured by a counting type experiment will be developed in this section. Three separate derivations will be presented based on arguments ranging from those essentially physical in nature to abstract mathematical ones. This will demonstrate the relationship between the various theories in the literature and will emphasize the generality of the results.

The reference experiment consists of counting the number of neutrons registered by a suitable neutron detector over a fixed counting or gate time. A large number of gates are taken and the distribution of counts obtained at the fixed gate time is analyzed for the ratio of variance to mean. This experiment is then repeated at various gate times and the behavior of the variance to mean ratio is examined as a function of gate time.

The derivations to follow will be based on a single neutron velocity model and will neglect any effects due to system size. The theory will treat only those fluctuations attributable to the prompt neutrons but arguments will be advanced for the validity of such a theory under certain optimum measuring conditions.

Physical Derivation

Average Behavior

The derivation based on essentially physical arguments is due to de Hoffmann (5).

Consider the steady state operation of a neutron multiplying medium being driven by a random source emitting S neutrons per second. The average neutron behavior may be represented by parameters averaged over the various probability distributions involved and is usually characterized by the system prompt reproduction factor, K'. The prompt reproduction factor is defined here as the ratio of the number of prompt neutrons (those neutrons emitted instantaneously in fission) in one generation to the number of neutrons of all origins in the previous generation. The prompt reproduction factor K' is related to the total reproduction factor K by the relation $K' = K(1-\beta)$ where β is the effective fraction of delayed neutrons. If S source neutrons are introduced into the system SK' prompt neutrons will appear in the first generation, SK'^2 in the second generation, SK'^3 in the third, etc. The total number of neutrons produced by and including the original S is then:

$$S(1 + K' + K'^2 + K'^3 + \text{etc.}) \qquad (1)$$

If the system is subcritical, $K' < 1$, this series converges to:

$$\frac{S}{1-K'} = F \qquad (2)$$

This is the mean neutron production rate when a source of strength S neutrons per second is multiplied by the system. The production rate excluding the extraneous source neutrons is:

$$\frac{S}{1-K'} - S = \frac{SK'}{1-K'} = FK' \qquad (3)$$

The mean kinetic behavior of the system may be described in terms of the reproduction factor and the mean prompt neutron lifetime to loss by absorption or leakage, τ. The increase in the neutron population, N, is (K'-1)N per generation. Since the neutrons have a lifetime or generation time of τ:

$$\frac{dN}{dt} = (K'-1)\frac{N}{\tau} \tag{4}$$

where t = time. If N_o is the number of neutrons present at time zero, integration yields:

$$N = N_o e^{-at} \tag{5}$$

where $a = \frac{1-K'}{\tau}$.

Statistical Behavior

Measurement of the neutrons necessitates the introduction of a neutron detector. A random detector will have an average response to the neutrons and will show fluctuations about this average. The mean counting rate of a detector with random response is proportional to the mean rate of neutron loss from the system which at steady state is equal to the mean rate of production. The constant of proportionality, E, is the detector efficiency, defined as the mean number of counts in the detector per neutron lost from the system. If F is the average steady state neutron production (loss) rate then the mean counting rate is:

$$\frac{\overline{dc}}{dt} = F\,E \tag{6}$$

The mean number of counts accumulated over a counting or gate time T is:

$$\bar{c} = F\,E\,T \tag{7}$$

Attention will now be turned to the neutron chains which are the source of the greater-than-random fluctuations in multiplying systems. In particular, the expected number of counts occurring in pairs, one count at t_1 and one count at t_2 with $t_1 < t_2$ will be studied. The origin of such pairs of counts is twofold. The extraneous source is continuously initiating neutron chains in the assembly and there are therefore a certain number of pairs to be expected comprised of one count from each of two different randomly initiated chains. Such are termed "accidental" pairs. In a multiplying system an additional number of chain related or "coupled" pairs are also to be expected. A coupled pair arise from the same chain and may be traced back to a common fission. The total number of expected pairs will be expressed as the sum of the accidental and coupled pairs.

If the mean neutron production (loss) rate is F the expected number of accidental pairs is just the probability of a count in dt_1 at t_1, F E dt_1 multiplied by the probability of a count in dt_2 at t_2, F E dt_2. The two individual probabilities are multiplied to give $F^2 E^2 dt_1 dt_2$ since the counts are independent, originating as they do from two different randomly initiated chains.

Coupled pairs are traceable to a common fission which occurred at time t, $-\infty < t \leq t_1$. The expected number of coupled pairs is composed of the following independent factors.

1. The mean neutron production rate from fission is FK' from (3). Since a mean number of neutrons, ν, are produced per fission, the probability of the common fission occurring in dt about t is:

$$\frac{FK'}{\nu} dt$$

2. This fission releases ν neutrons with probability:

P_ν

3. The probable number of neutrons present after a time $t_1 - t$ is given by (5). The loss rate at any time is given by the number present divided by their mean life. Therefore the probability of a count in dt_1 about t_1 due to the release of ν fission neutrons at t becomes:

$$E \, \nu \, e^{-a(t_1 - t)} \frac{dt_1}{\tau}$$

4. The probability of a count in dt_2 about t_2 given that ν neutrons were born at t and one has been lost through detection at t_1 is written:

$$E \, (\nu - 1) \, e^{-a(t_2 - t)} \frac{dt_2}{\tau}$$

The expected number of coupled pairs is obtained by integrating the product of the above factors over the range of t and then taking the sum over the distribution of neutrons emitted per fission.

$$\sum_{\nu} \int_{-\infty}^{t_1} \frac{FE^2K'P_\nu}{\tau^2\bar{\nu}} \, \nu(\nu-1) \, e^{-a(t_1+t_2)} dt_1 dt_2 \, e^{2at} dt \qquad (8)$$

Integration gives:

$$\frac{FE^2K'}{2a\tau^2\bar{\nu}} \, e^{-a(t_2 - t_1)} \, dt_1 dt_2 \sum_{\nu} P_\nu \, \nu(\nu - 1) \qquad (9)$$

The sum over the discrete ν distribution yields the factor $(\bar{\bar{\nu}} - \bar{\nu})$. The total expected number of pairs is the sum of those accidental and coupled in origin.

$$F^2E^2dt_1dt_2 + \frac{FE^2K' \,(\overline{\nu^2}-\overline{\nu})}{2a\tau^2\overline{\nu}} \; e^{-a(t_2-t_1)} \, dt_1dt_2 \qquad (10)$$

If the neutrons are counted over a counting or gate time T the expected number of pairs registered in that interval is obtained by integration.

$$\int_{t_1=o}^{t_1=t_2} \int_{t_2=o}^{t_2=T} F^2E^2dt_1dt_2 + \frac{FE^2K' \,(\overline{\nu^2}-\overline{\nu})}{2a\tau^2\overline{\nu}} \; e^{-a(t_2-t_1)} \, dt_1dt_2 \qquad (11)$$

This gives:

$$\frac{F^2E^2T^2}{2} + \frac{FE^2K' \,(\overline{\nu^2}-\overline{\nu})}{2a^2\tau^2\overline{\nu}} \left[1- \frac{(1-e^{-aT})}{aT}\right] T \qquad (12)$$

Equation (7) identifies FET as $\overline{c}$, the mean (M_c) of c. The number of combinations of c things taken two at a time is $\frac{c(c-1)}{2}$ so in a count of c the expected number of pairs of counts is $\overline{\frac{c(c-1)}{2}}$ which is an identity with (12). Making the substitutions:

$$\frac{\overline{c^2}-\overline{c}}{2} = \frac{\overline{c}^2}{2} + \frac{\overline{c}\; EK'(\overline{\nu^2}-\overline{\nu})}{2a^2\tau^2\overline{\nu}} \left[1 - \frac{(1-e^{-aT})}{aT}\right] \qquad (13)$$

Multiplying both sides of (13) by $2/\overline{c}$, substituting $\frac{1-K'}{\tau}$ for "a" from (5), and recognizing $\overline{c^2}-\overline{c}^2$ as the variance (V_c) of c yields:

$$\frac{V_c^{\,T}}{M_c} = 1 + \frac{EK'}{(1-K')^2} \left[\frac{\overline{\nu^2}-\overline{\nu}}{\overline{\nu}}\right] \left[1 - \left(\frac{1-e^{-aT}}{aT}\right)\right] \qquad (14)$$

In dealing with statistical theory and measurements one must discriminate between population mean, M_i, and population variance, V_i, and measured sample mean, indicated by a superbar, and sample variance, s^2. Mean and variance are defined functionally by Equations (53) and (54) respectively. Population statistics are obtained from these equations in the limit as n approaches infinity and sample statistics are obtained when n is finite. All theoretical statistics derived here are population statistics and this is the reason for the use of population notation in the final writing of Equation (14) although it was convenient to use the more familiar bar notation in the preceding development. The use of population notation for theory will be adhered to in the following except for the continued use of the superbars to indicate the population means of ν and ν^2.

The above relation indicates the results to be expected upon the performance of the reference experiment. The reason for studying the ratio of variance to mean is that this ratio equals one when a random distribution is sampled with a random response detector. When a multiplying system is driven with a random source the variance to mean ratio can be greater than one indicating the effect of multiplication. Note that when the multiplication is reduced to zero, $K' \rightarrow 0$, that the response approaches that expected for the random source alone. Also, as the detector efficiency is reduced the response becomes random. At low efficiency the bursts or chains of neutrons are not sampled in number and in the limit only one neutron from each chain is detected and this is equivalent to counting the random source neutron which initiated the chain.

Although the above equation considers explicity only the effects due to prompt neutrons it has a useful range of validity. The total fluctuation is comprised of contributions from the prompt and delayed neutrons. The period of each fluctuation is dictated by its inherent time constant; the mean prompt neutron lifetime of the prompt neutrons and the halflives of the delayed neutrons. Since in many reactors there is an appreciable difference between the prompt neutron period and the shortest delay period it is possible to take measurements at gate times long compared to the prompt period yet short compared to the shortest delay period. At such optimum gate times the full prompt neutron fluctuation is measured but the delays make a negligible contribution since they effectively act like a random background source of constant strength. Under these conditions all the parameters of Equation (14) are those of the prompt neutrons.

Probability Generating Functions

The method of probability generating functions(12) facilitates the development of the theory of chain processes. Certain useful properties will now be displayed in anticipation of their application in the following sections.

A probability generating function (p.g.f.) can be defined by:

$$B(y) = b_0 + b_1 y^1 + b_2 y^2 + \ldots\ldots b_m y^m \ldots\ldots \qquad (15)$$

where the coefficient of y^m is the probability of the result m occurring. The variable y is a dummy variable much like an integration variable. The B distribution is described completely by the individual probabilities b_m. B(y) has the obvious property B(1) = 1.

Moreover, the moments of the B distribution are generated by the derivatives of its p.g.f. If a superprime denotes a derivative with respect to y:

$$M_b = \sum_m m b_m = B'(1) = \text{mean} \qquad (16)$$

and

$$M_b^2 = \sum_m m^2 b_m = \sum_m m(m-1)\ b_m + \sum_m m b_m$$

$$M_b^2 = B''(1) + B'(1) \qquad (17)$$

Since $V_b = M_b^2 - (M_b)^2 = \text{variance}$

$$V_b = B''(1) + B'(1) - B'^2(1) \qquad (18)$$

A certain physical situation appears time and again in the description of a multiplying process. It is represented mathematically by the sum:

$$S_M = X_1 + X_2 + X_3 + \ldots\ldots X_M \tag{19}$$

where the X_m are mutually independent random variables with a common distribution and therefore a common p.g.f., H(y), and where the number of terms, M, is also a random variable which is independent of the X_m, and has a p.g.f. G(y). The value of the sum is thus dependent on the number of terms and the value of each individual term. The probability that the sum has a certain value j is given by:

$$P(S_M=j) = \sum_m P(M=m)\ P(X_1 + \ldots\ldots + X_m=j) \tag{20}$$

The probabilities for all j are generated by the p.g.f. which by a theorem of probability generating functions is a compounding of H and G of the form:

$$S(y) = G\left[H(y)\right] \tag{21}$$

Here the G operates on the H.

The moments of S may be generated from derivatives of S(y) by the above methods they are:

$$M_S = M_G M_H \tag{22}$$

and

$$V_S = V_H M_G + V_G (M_H)^2 \tag{23}$$

Physical-Abstract Derivation

This second derivation is based on the work of Courant and Wallace(4). The starting point will be the expression they develop for V_N the variance of the neutron population. The variance V_n of the number of neutrons lost over a given time interval will be deduced from V_N. The statistics of the response of a random detector will then be investigated using the methods of probability generating functions.

The results of Courant and Wallace which are of interest here will now be listed. The expressions are general in that they include the delayed neutrons but these will be reduced to working equations which consider only prompt neutrons.

The variance of the neutron population as measured by a detector with resolving time Tr over a measuring time Tm is given as:

$$V_{N\,Tm}^{\;Tr} = V_N^{Tr} - V_N^{Tm} \tag{24}$$

The resolving time of the Courant and Wallace theory is identified with the gate time T of the experiment under study here. The measuring time becomes the time required to take j counts or gates and equals jT + (j-1) T_d where T_d is the display time between gates. V_N^{t} is determined from the relation:

$$V_N^{t} = \sum_s \frac{2\,k_s}{(\gamma_s t)^2}\,(\gamma_s t - 1 + e^{-\gamma_s t}) \left[V_N + \sum_i \frac{\lambda i\, V_{Ni}}{(\lambda i - \gamma_s)} \right] \tag{25}$$

V_N^{Tr} and V_N^{Tm} are evaluated by substituting Tr and Tm for t in the above.

The absence of a superscript on the V's indicates population variances as would be measured with a detector of zero resolving time over infinite measuring time. Subscript Ni denotes the mixed variance of the neutrons and the i^{th} delay precursors. The γ's are the periods of the various components of the fluctuation and are determined from the inhour equation.

$$\gamma = \frac{(1-K')}{\tau} - \frac{K}{\tau} \sum_i \frac{\lambda i\, \beta_i}{(\lambda i - \gamma)} \tag{26}$$

The coefficients k are the coefficients of the various periods and are evaluated from:

$$k_s = \frac{\gamma_s \tau}{(1-K) + K \sum_i \dfrac{\beta i}{\left(1 - \dfrac{\lambda i}{\gamma_s}\right)^2}} \tag{27}$$

Finally, the neutron population variance including delayed neutrons is:

$$V_N = M_N \left[1 + \frac{K}{2(1-K)} \frac{\left[1 - K + (\lambda i\tau)^*\right] \left[\frac{\overline{\nu^2} - \overline{\nu}}{\overline{\nu}}\right]}{\left[1 - K + K \sum_i \beta i + (\lambda i \tau)^*\right]} \right] \tag{28}$$

The rate of neutron loss from the system is proportional to the instantaneous number (population) present. The number lost over time T is obtained from:

$$n(T) = \int_0^T \frac{N(t)}{\tau} dt = \overline{N} \frac{T}{\tau} \tag{29}$$

In terms of population mean values:

$$M_n = M_N \frac{T}{\tau} \tag{30}$$

V_n is related to V_N through the relation:

$$V_n = V_N \left(\frac{\partial n}{\partial N}\right)^2 = V_N \left(\frac{T}{\tau}\right)^2 \tag{31}$$

The general equations will now be reduced to the case of no delayed neutrons. If the delays are neglected each V_{Ni} is zero. Also, there is only one period, the prompt period, which from (26) equals $\frac{1-K'}{\tau} = a$. The coefficient k of the single period is unity. Equation (25) then simplifies to:

$$V_N^t = \frac{2}{at} \left[1 - \left(\frac{1 - e^{-at}}{at}\right) \right] V_N \tag{32}$$

If the measuring time, Tm, is taken long compared to the prompt neutron period V_N^{Tm} approaches zero and (24) may be written:

$$V_{N_{Tm}}^{Tr} = V_N^{Tr} \tag{33}$$

Noting that Tr is equivalent to the gate time T:

$$V_{N_{Tm}}^{Tr} = V_N^T = \frac{2}{(1-K')} \left[1 - \frac{(1-e^{-aT})}{aT} \right] \frac{\tau}{T} V_N \tag{34}$$

Substituting for V_N from (28) after setting $\beta i = \lambda i = o$, $K = K'$, and neglecting the 1 compared to the $\frac{1}{1-K'}$ term:

$$V_N^{\ T} = \frac{K'}{(1-K')^2} \left[\frac{\overline{\nu^2}-\overline{\nu}}{\overline{\nu}}\right] \left[1 - \frac{(1-e^{-aT})}{aT}\right] \frac{\tau}{T} M_N \qquad (35)$$

The variance of the number of neutrons lost during T derives from (35) and may be simplified by noting that $M_n = M_N \frac{T}{\tau}$.

$$V_n^{\ T} = V_N\left(\frac{T}{\tau}\right)^2 = \frac{K'}{(1-K')^2} \left[\frac{\overline{\nu^2}-\overline{\nu}}{\overline{\nu}}\right] \left[1 - \frac{(1-e^{-aT})}{aT}\right] M_n \qquad (36)$$

During the gate time T the neutron detector is presented with a number n of neutron loss events. The detector either counts each event with probability E or does not count the event with probability (1-E). The situation is identical to that discussed above Equation (20). Here the role of the G distribution is played by n and that of the H distribution by the detector. The count distribution is compounded from the neutron loss (n) and the detector (D) distributions.

The previous results of (22) and (23) yield:

$$V_c = V_D M_n + V_n (M_D)^2 \qquad (37)$$

and:

$$M_c = M_D M_n \qquad (38)$$

The p.g.f. of the D distribution may be written down immediately.

$$D(y) = (1-E) + Ey \qquad (39)$$

Its moments are:

$$M_D = D'(1) = E \qquad (40)$$

$$V_D = D''(1) + D'(1) - D'^2(1) = M_D - (M_D)^2 \qquad (41)$$

Substituting into (37):

$$\frac{V_c}{M_D M_n} = \frac{V_c}{M_c} = 1 + M_D\left[\frac{V_n}{M_n} - 1\right] \tag{42}$$

This equation may be expressed in terms of the previously developed moments of the n and D distributions. Substituting and neglecting the 1 in the brackets gives the familiar Equation (14):

$$\frac{V_c^T}{M_c} = 1 + \frac{EK'}{(1-K')^2}\left[\frac{\overline{\nu^2}-\overline{\nu}}{\overline{\nu}}\right]\left[1 - \frac{(1-e^{-aT})}{aT}\right] \tag{14}$$

Interesting Reductions

Useful information can be gained by investigating the reduced forms of Equation (14) at short and long gate times.

As the gate time is reduced the counting experiment approaches an instantaneous sampling of the neutron population. Specifically, in the limit of counting time approaching the mean neutron lifetime the statistics become those of the neutron population itself. The exponential in the gate time brackets may be expanded and high order terms neglected.

$$\begin{aligned}\lim_{T\to\tau}\frac{V_c}{M_c} = \frac{V_{N*}}{M_N} &= \lim_{T\to\tau}\left[1 + \frac{EK'}{(1-K')^2}\left[\frac{\overline{\nu^2}-\overline{\nu}}{\overline{\nu}}\right]\frac{aT}{2}\right] \\ &= \lim_{T\to\tau}\left[1 + \frac{EK'}{(1-K')^2}\left[\frac{\overline{\nu^2}-\overline{\nu}}{\overline{\nu}}\right]\frac{(1-K')T}{2\ \ \tau}\right] \\ &= 1 + \frac{EK'}{2(1-K')}\left[\frac{\overline{\nu^2}-\overline{\nu}}{\overline{\nu}}\right]\end{aligned} \tag{43}$$

The * denotes the distribution obtained when the neutron population statistic $\frac{V_N}{M_N}$ is sampled with a detector of efficiency E. The magnitude of the fluctuation is now reduced by a factor 1/1-K'. This is due to the fact that the instantaneous sampling, because it does not measure over time, does not see the fluctuations in chain length. The counting experiment measures chain behavior comprised of many generations while the instantaneous experiment measures the statistics of the individual generations. The above expression may, of course, be obtained directly from (28).

At the other extreme is the reduction of Equation (14) obtained at long measuring time when the full influence of chain behavior is felt. In this instance time should be measured in units of the fluctuation period 1/a:

$$\lim_{aT \to \infty} \frac{V_c}{M_c} = 1 + \frac{EK'}{(1-K')^2} \left[\frac{\overline{\nu^2} - \overline{\nu}}{\overline{\nu}} \right] \qquad (44)$$

The identity of the variable sampled in such a long counting experiment will be established in the next section.

Abstract Derivation

It is not necessary to carry a derivation through within the framework of a neutron multiplying system. A general model may be used to arrive at general functional equations which then may be particularized to a given multiplying system. Such an approach emphasizes the generality of results and the fact that nuclear reactors are specific examples of a larger class of multiplying systems which includes such members as human and microbe populations and cosmic ray cascades(16,26,32). A derivation does not have to consider the process as occurring continuous in time either. A discrete-in-time generation by generation view may be taken.

This is, in a sense, a more basic viewpoint since by definition a multiplying process consists of a connected chain of discrete multiplying stages. The treatment of Hawkins and Ulam(17) is an example of the discrete-generation functional approach. Their results will be used to determine the statistics of the sum of the number of members of a chain.

Under the general assumptions that all individuals are alike, that each acts independently of the other, and that each is governed by probabilities common to all, Hawkins and Ulam generate the following functional relation for the p.g.f. U of the sum of the members of a chain initiated by a single individual:

$$U = x\,F\left[U(x)\right] \tag{45}$$

Here F is the single generation p.g.f. for the members of a second generation starting from a single member in the first. It is not necessary to solve this equation for U since its moments may be generated by differentiation.

The variance to mean ratio is given by (16) and (18) as:

$$\frac{V_U}{M_U} = \frac{U'' + U' - U'^2}{U'}$$

evaluated at x = 1. Differentiating (45) and substituting:

$$\frac{V_U}{M_U} = 1 + \frac{F''}{(1-F')^2} + \frac{2F'-1}{(1-F')} \tag{46}$$

This is the expression for a general multiplying system with the assumed properties. The result may be specialized to a neutron multiplying system by specifying the form of the single generation p.g.f., F. If K', the reproduction factor, is the mean number of neutrons produced in the second generation per neutron in the first generation then $\frac{K'}{\bar{\nu}}$ is the probability of fission. The probability of non-productive loss is then $1-\frac{K'}{\bar{\nu}}$. Furthermore, if fission does occur an additional zero, one, two, three, etc., neutrons are produced with probability v_0, v_1, v_2, v_3, etc. The number of neutrons produced on fission is described completely by the p.g.f.:

$$V(x) = v_0 + v_1 x + v_2 x^2 + v_3 x^3 + \ldots\ldots \qquad (47)$$

The p.g.f., F, may then be written:

$$F(x) = 1 - \frac{K'}{\overline{\nu}} + \frac{K'}{\overline{\nu}} v_0 + \frac{K'}{\overline{\nu}} v_1 x + \frac{K'}{\overline{\nu}} v_2 x^2 + \frac{K'}{\overline{\nu}} v_3 x^3 + \text{etc.} \qquad (48)$$

More compactly:

$$F(x) = 1 - \frac{K'}{\overline{\nu}} + \frac{K'}{\overline{\nu}} V(x) \qquad (49)$$

The moments of F(x) are:

$$F'(1) = \frac{K'}{\overline{\nu}} V'(1) = \frac{K'}{\overline{\nu}} \overline{\nu} = K' \qquad (50)$$

and

$$F''(1) = \frac{K'}{\overline{\nu}} V''(1) = \frac{K'}{\overline{\nu}} (\overline{\nu^2} - \overline{\nu}) \qquad (51)$$

Substituting these into (46) yields:

$$\frac{V_U}{M_U} = 1 + \frac{K'}{(1-K')^2} \left[\frac{\overline{\nu^2} - \overline{\nu}}{\overline{\nu}} \right] + \frac{2K'-1}{(1-K')} \qquad (52)$$

which to the first order is identical to (44) except for a detector efficiency which may be introduced by compounding this distribution with that of a random detector. The above result explicitly identifies the reduction of the basic equation at long gate times, (44), as the statistics of the sum of the neutrons in a chain. This is not unexpected, of course, since a counting process is a summation.

The general functional approach may also be used to derive the statistics of the neutron population as given by (43).

Neutron Energy Spectrum and System Size

The theory which was derived here was developed on the basis of a single group model. Each particle was assumed identical to every other particle in the sense that the probabilities of various events were assumed common to all. In actual physical systems there is a spectrum of neutron energies and particles are present in all geometric locations within the system. Such real particles of differing energy and location are subject to different probabilities. An exact general theory would account for the continuum of particle energy and position.

R. P. Feynman(13) has investigated the general model but as yet these calculations are unpublished. He concluded that the results of the continuous energy model are identical with those of the one group model. His calculations on the geometric effect for the particular case of the Los Alamos water boiler indicated that for this small symmetric machine this effect amounted to only about 1 per cent. Accepting these results, the one group theory may be applied to systems with a continuous spectrum of neutron energies but caution should be exercised when using it to describe large, spatially heterogeneous systems. Hopefully, this situation will soon be rectified by the release of Feynman's work.

Hawkins and Ulam(17) and Everett and Ulam(8,9,10) have generalized the discrete, functional theory to include a finite number of different classes of particles. The particles could be classified by energy or spatial distribution or both.

MEASURING SYSTEM

One of the advantages of the method of reactor parameter measurement under study is the conventionality of the measuring system. Furthermore, of the methods of measuring neutron statistics this one employs the most conventional techniques. Essentially what is required is a means of detecting and counting neutrons. Most laboratories should be able to put together a suitable system from available equipment. If not, commercially available components may be combined to build up an apparatus such as is to be described.

Apparatus

The counting system utilized in these experiments is diagramed in Figure 1. A BF_3 counter placed in the core of the reactor detected the neutrons. Its output was amplified at the reactor by the pre-amplifier and carried to the linear amplifier in the control room through existing coaxial cables. The pulses were further amplified in the linear amplifier whose amplitude discriminator output drove the gate-scaler combination. The counts registered on the scaler were either read out by dictation or printed on paper tape by a digital recorder. The invididual components and manufacturers are listed in Table I.

Some of the considerations which governed the choice and use of the more critical components will now be discussed.

Detector

The efficiency with which neutrons are detected and the energy of the detected neutrons is a function of the choice of detector. In the predominantly thermal spectrums investigated it was desired to measure thermal neutrons. The success of the experiment hinged on

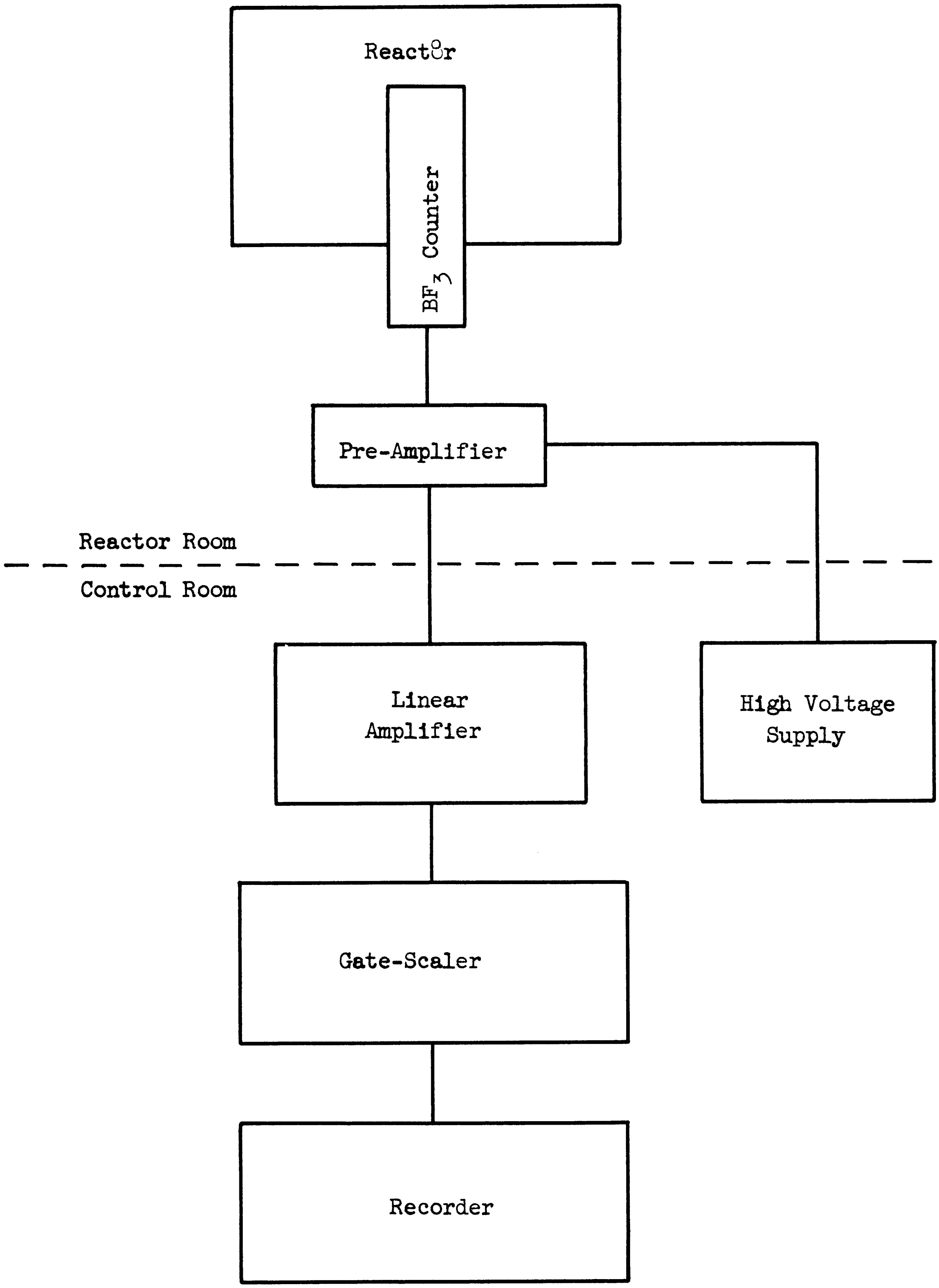

Figure 1. Measuring System.

TABLE I

Equipment List

Component	Type	Manufacturer
Detector	BF_3 counter, 40 cm Hg pressure, 1" diameter by 17" long, 12" active volume, Serial G 10380	N. Wood Counter Laboratory, 1525 East 53rd Street, Chicago Illinois
Pre-amplifier	Linear, Model DR9, Serial A 194	B. J. Electronic Borg-Warner Corp. 330 Newport Avenue Santa Ana, Calif.
Amplifier	A1D Linear, Model DA5, Serial 7517	Detectolab 6544 N. Sheridan Rd. Chicago, Illinois
Gate-Scaler	Computing Digital Indicator, Dynac Model DY-2500, gate time .001 to 9.999 seconds, temperature controlled oven for stabilized crystal, Serial 28	Dynac, Inc. 395 Page Mill Road Palo Alto, Calif.
High Voltage Supply	Dual high voltage supply, Serial B-180	Radiation Instrument Development Laboratory 5737 South Halsted Chicago, Illinois
Recorder	Digital Recorder, Model 560A, Serial 100 and 160	Hewlett-Packard Co. 275 Page Mill Road Palo Alto, California

getting a high counting efficiency. The counting efficiency in a given neutron density is dependent on the geometric size of the detector and on the sensitivity with which incident neutrons are counted. BF_3 counters are more sensitive than other common thermal neutron counters such as fission chambers and boron-coated counters except for scintillation counters which were unavailable. Sensitivity increases with gas pressure but so does operating voltage and attendent problems. The 40 cm Hg pressure used was a compromise between sensitivity and operating voltage.

When in the reactors the counter was placed in a dry aluminum tube 1-1/16" ID with 1/16" walls. The counter was wrapped in plastic to limit electrical contact with the tube.

Amplifier

The linear amplifier amplified the output of the pre-amplifier to a level at which the associated pulse amplitude discriminator could be used to discriminate against non-neutron pulses. Discriminator plateaus are more convenient to measure than voltage plateaus and use of a discriminator has the added advantage of providing information on the pulse size distribution in the amplifier and hence on possible amplifier overloading.

Special care was taken to insure that the amplifier operated as stably and with as good pulse resolution as possible. Settings of voltage and gain were made such that only a few of the amplified pulses were greater than 100 volts in amplitude. Overloading of the amplifier with large pulses causes periods of insensitivity after the overloading pulses. Operation of the amplifier on the one-microsecond delay-line pulse shaping circuit gave good pulse resolution to begin with. The stability of operation of the amplitude discriminator was improved by operating with the dc restorer diode V_{12} in the circuit. This circuit component helps to keep pulse height measurement independent of pulse rate.

Noise suppression is a problem in systems involving high amplification such as the one used here. Noise would obviously be detrimental in an apparatus whose purpose is the measurement of statistics. The design of the commercial amplifier incorporates features to limit the noise level but additional precautions must be taken when using and interconnecting the equipment. The line voltage was regulated and the complex was grounded at only one point. The single ground was at the amplifier. Efforts made to prevent other incidental grounds included wrapping the counter in plastic and spraying the pre-amplifier chassis with Krylon. Special attention was devoted to making good connections to assure continuity of the one available ground loop. Proper shielding is also important in preventing the introduction of noise into a system. The amplifier was enclosed in a metal cabinet in addition to the shielding already provided on the chassis. The background noise level under these conditions and at the settings used for taking data was 3 volts as measured by the amplitude discriminator. Measurements were generally made at a discriminator setting of 30 to 40 volts. However, noise of even this amplitude could be injected by flicking switches such as the safety rod control. Such noise could not be eliminated and it was lived with by taking care to see that the offending switches were not manipulated during data taking.

Gate-Scaler

The heart of the counting setup was the gate-scaler. In the particular equipment used the gate and scaler were combined in the single chassis of the DY-2500. The operation of the gate-scaler may best be understood by reference to the block diagram of Figure 2. The Digital Display Counter counts and displays the number of pulses supplied to its input. Pulses from an external source must pass through the Signal Gate to reach the Display Counter. When the Signal Gate is open external input pulses are transmitted unimpeded to the Display Counter. When closed, further input is blocked and the Counter displays the accumulated count. The action of the Signal Gate is controlled by the Preset Counter. The Preset Counter is similar to the Display

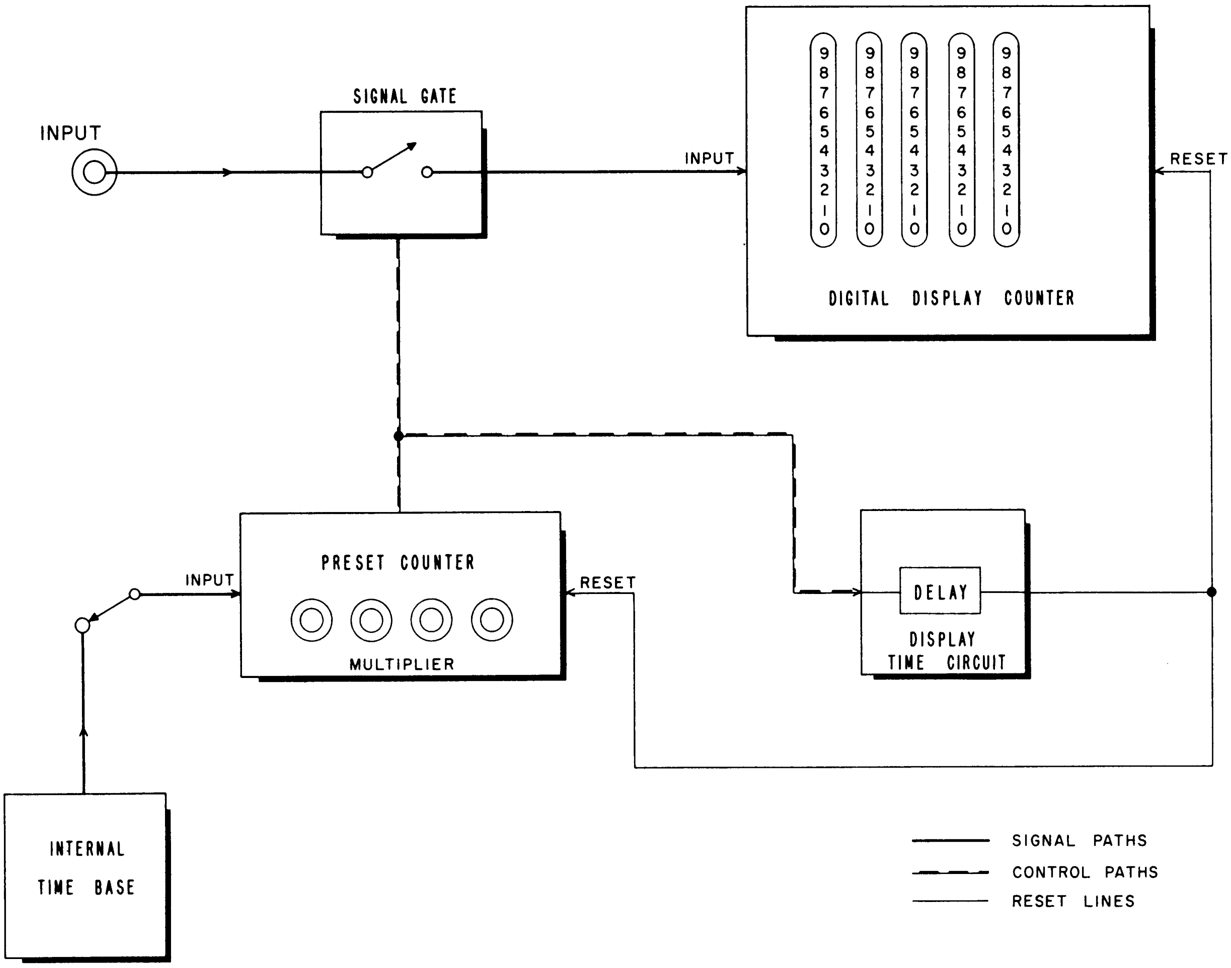

FIGURE 2 GATE – SCALER

Counter only with a comparator circuit added. The comparator is set with the Multiplier dials. The Preset Counter holds the Signal Gate open while counting the Internal Time Base until coincidence is registered between the accumulated count and the count preset in the comparator circuit with the Multiplier dials. At this time the Signal Gate is closed and the circuit is maintained in this status for a certain display time by the Display Time Circuit. Then the Display Counter and Preset Counter are cleared and a new gate cycle initiated.

The precision and accuracy of the gate is directly attributable to the corresponding properties of the Internal Time Base. In the DY-2500 the Internal Time Base is a 100 kc crystal oscillator. The crystal is housed in a temperature controlled oven to improve its stability. The basic crystal frequency is attenuated through a 100 to 1 frequency divider to give gate times variable from .001 to 9.999 seconds in millisecond increments. The same instrument is available with gate times from .0001 to .9999 seconds.

In addition to accuracy, a crystal oscillator is capable of supplying the required precision and stability. The mean counting rates during the measurements were in the range of 1000 to 10,000 counts per second. At these counting rates and at the shortest gate time of .001 second only 1 to 10 counts were accumulated in a gate. Any fluctuation or lack of precision in the gate which would throw in or lose one or more pulses could cause large percentage-wise deviations in the counts at short gate times. It was imperative that the gate be sufficiently precise so that any count fluctuations introduced by its lack of precision did not mask the count fluctuations caused by the physical phenomena under investigation. Calculations indicate that for the reactors studied the crystal frequency had to have a standard deviation of less than 3 parts per million to permit the taking of satisfactory measurements. This specification was met by the temperature stabilized crystal used which had a stability of 5 parts per million per week.

Because the DY-2500 was designed for industrial frequency measurements it was necessary to make a minor modification to provide reliable service in the present application. This involved the installation of a new amplitude discriminator designed specifically to be triggered by the output of the amplitude discriminator of the A1D amplifier. The new discriminator was a plug in type unit like the one provided with the DY-2500 and it was only necessary to exchange units. A circuit diagram of the Argonne installed discriminator is given in Figure 3.

The new discriminator was considerably faster than the one provided with the DY-2500 and it operated reliably on the A1D discriminator output which the DY-2500 discriminator did not do. The input to the discriminator was the 15-volt negative, 0.2 μs rise time, output pulse of the A1D discriminator. Its output was a 60-volt negative, 0.1 μs rise time, 2 μs decay time, pulse which was sufficient to reliably operate the first stage of the display counters.

Commensurate with the use of the special discriminator with its high (10v) trigger level it was necessary to replace the 1000 ohm resistor in the plate circuit of the oscillator diode with a 11K resistor. This increased the amplitude of the oscillator pulses to a level which would trigger the new discriminator when the check circuit was activated.

Recorder

The output of the scaler must somehow be recorded. This may be done with more or less economy depending on the keeness of sight and the en-

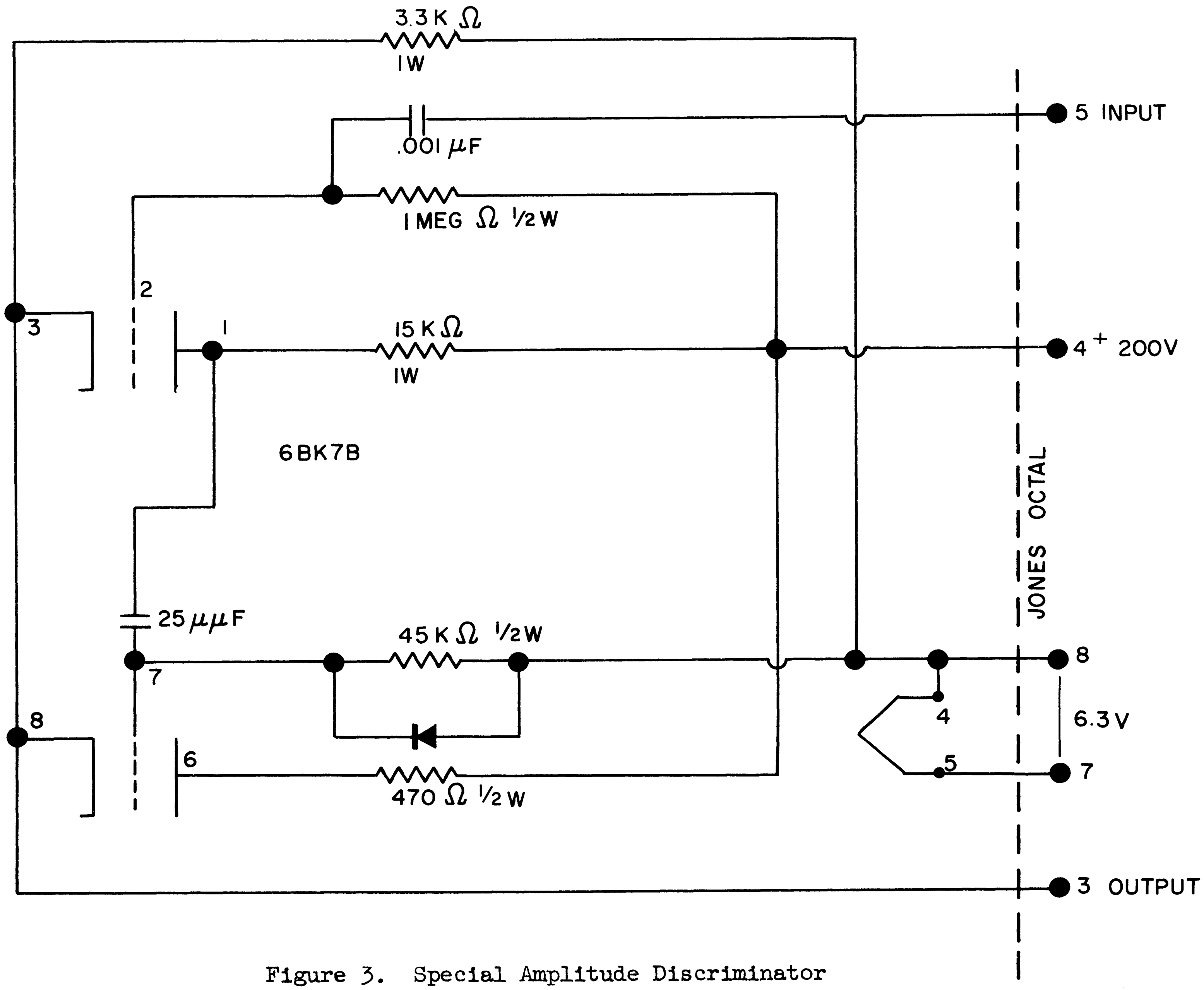

Figure 3. Special Amplitude Discriminator

durance of the experimenter. Some of the data were taken by dictating the numbers as they were displayed and later transcribing them. This scheme proved satisfactory but exhausting for counts consisting of 100 gates at each gate time. The economy derived from this approach depends on the relative price of human data takers and a recorder. The personalized technique has several important drawbacks. The validity of a series of gates can be destroyed by one gross misreading of the scaler. Since the variance of the counts is quite large and the counts are displayed quite rapidly such errors are inevitable. Furthermore, a human recorder requires the display time to be longer than the minimum display time possible, thus lengthening the measuring time and increasing the importance of reactor drift and delayed neutron effects.

The recorder-printer used to record the majority of the data was designed specifically to record the staircase output of Hewlett-Packard counters. It is very fast being capable of printing up to five lines a second yet it operates as a slave to the counter to which it is connected. The recorder simultaneously prints on adding machine tape and provides an analog output proportional to the number displayed by the counter. Use of this recorder permitted operation of the scaler at minimum display time which decreased the measuring time. It also was possible to take more data than would have been feasible with human readout.

The ultimate in data recording would be to transcribe the data directly on the input medium for the data analysis operation. Punching directly on cards or tape would be an easily taken step in this direction. It would again eliminate hand operations susceptible to human error. A feasible but advanced system would employ a computer to take and analyze the data directly and give the final results all in one operation.

Calibration, Checkout, and Settings

Calibration

It was necessary to make certain measurements to characterize the counting system. Calibration consisted of measurements of system dead time, scaler display time, and BF_3 counter plateaus.

Dead time as used here is defined as that time period following the registration of a pulse by the scaler during which a second neutron event in the detector will not be registered by the scaler. For this measuring system which employed a proportional counter and a wide band amplifier the dead time of the system was that of the first binary of the scaler. This dead time was measured, as described in Appendix I, utilizing a double pulse generator and by observing the output of the first binary of the scaler. The dead time so determined was 5.1 microseconds.

The display time of the scaler is that time after the signal gate is closed and before reset of the display and preset counters during which the accumulated count is displayed on the display counter and printed by the recorder. The minimum display time dictated the interval between gates and thus the length of the measuring time. All counts taken using the recorder were made at minimum display time. To measure this quantity, the display time was set at minimum, the gate time was set at .001 seconds, and the number of gates per minute was counted. Ninty-six gates per minute were observed. Since a series started and ended with a gate and since the total gate time (.096 sec.) was negligible this meant there were 95 displays per minute which gave a minimum display time of 0.63 sec. The minimum time between gates was dictated by this display time even when the scaler was connected to the recorder.

When taking data by dictation it was necessary to increase the display time beyond the minimum. The average display time under these circumstances was 1.5 seconds.

It was desirable to use a counter with a good plateau so that in the event of slight counter voltage or discriminator level fluctuation a minimum deviation would have been introduced into the count. Unfortunately, this specification and the specification that few pulses be allowed to overload the amplifier were found difficult to meet simultaneously. This was due to the wide spread in neutron pulse heights of the counters tested. In order to raise all the neutron pulses above a reasonable discriminator setting it was necessary to increase the counter voltage so high that many of the large pulses exceeded the maximum desirable amplifier pulse amplitude of 100 volts. A number of plateaus of the counter appear in Appendix II. The plateau characteristics of the counter, while not outstanding, were deemed satisfactory.

Checkout

The proper functioning of the entire system was conveniently established by counting a random source. A random or Poisson distribution should have a variance to mean ratio of one. A large number of counts of a Ra-Be source were taken during the course of the measurements. These are tabulated in Appendix II and summarized in Figure 4. The data are separated in two sets: those based on 100 gates and those based on 525 gates. Some of the 100 gate data were taken by dictation and some consist of the first one or two groups of 100 gates of a full 525 gate series taken with the recorder. It was standard checkout procedure to hand calculate the first two 100 gate groups of 525 gate series taken as part of the daily checkout. This was done before data taking began to verify proper operation of the equipment. The full 525 gates, including the first 200, were later reduced on the computer. It

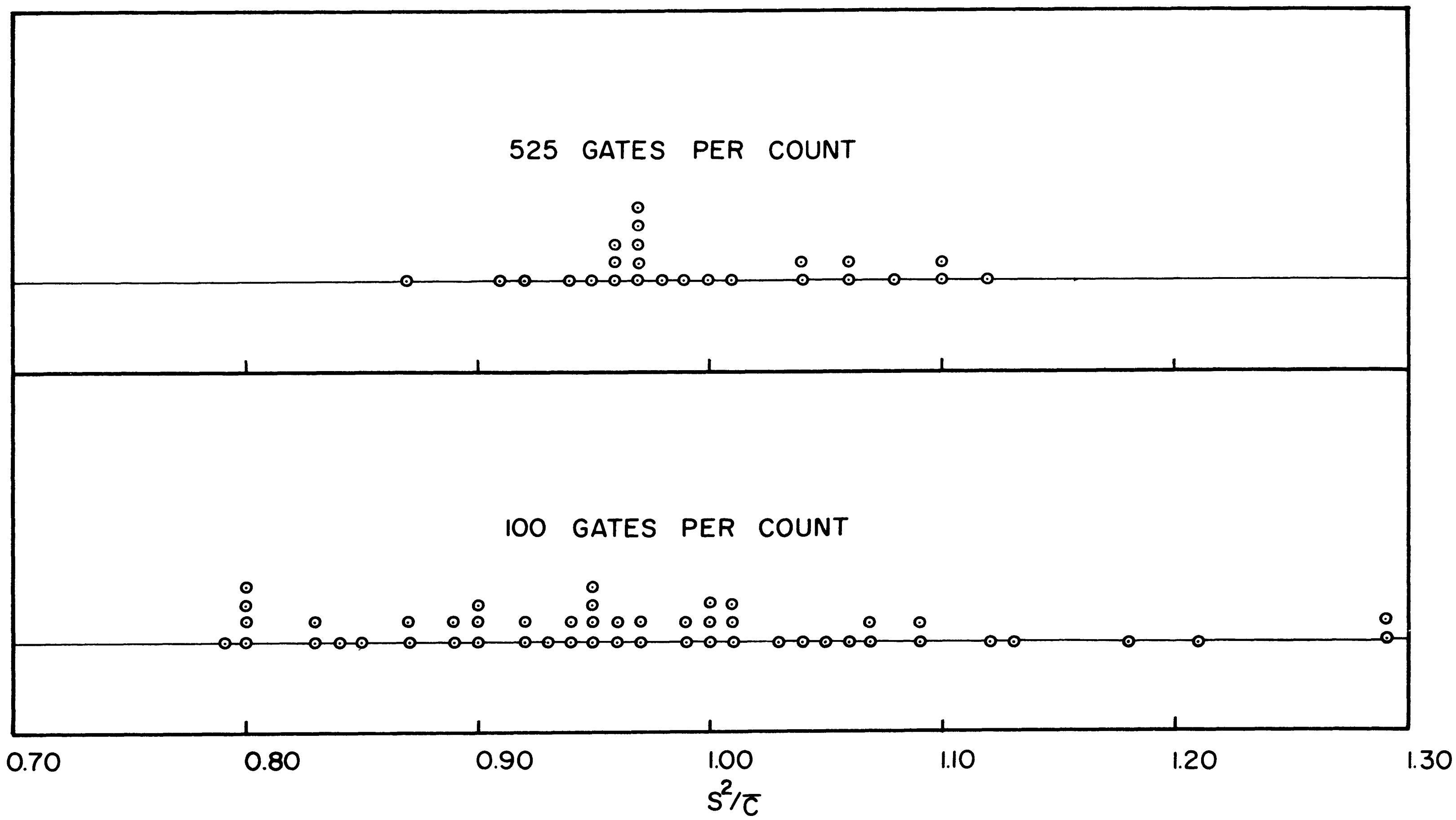

Figure 4. Random Scource Statistics.

is not known what caused the concentration of the data below a value of $s^2/\bar{c} = 1$. Many explanations could have been offered, on the other hand for an increased variance and $s^2/\bar{c}$ values greater than one. The deviation is probably an indication of the statistics of the statistic $s^2/\bar{c}$. The drawback in using this method to test the system is that one wild $s^2/\bar{c}$ ratio is not enough to absolutely establish the malfunction of the apparatus. However, it is a warning sign that testing should be continued until proper functioning is assured. Improper operation of the equipment was detected by this method several times. Variance to mean ratios measured in these instances ranged from .52 to 2.9.

Prior to taking data a standarized checkout procedure was followed. It was usually not necessary to warm up the equipment since it was left on continously. If the recorder was in use it was the output of the recorder which was examined during checkout. The steps were as follows:

1. A visual check of the condition of the system was made and cable connections were tightened.

2. The background noise level of the amplifier was checked with the counter voltage off and with the amplifier settings the same as during data taking. This was done by noting at what discriminator level dis-discharge occurred. This was generally in the region 3-5 volts.

3. The DY-2500 was tested at various settings of the gate with the built-in check circuit.

4. The amplifier-scaler-recorder combination was checked by driving the amplifier with a 60 cycle Hg pulser.

5. At least two 100 gate counts of a Ra-Be source were taken and the variance to mean ratio calculated.

6. Whenever the counter voltage was turned on its operating value was

determined on the basis of a discriminator plateau. The voltage was set so that not more than 1 per cent of the counts at a setting of 40 or not more than 40 counts per second were registered with the discriminator set at 100. The operating discriminator level was chosen on the basis of these plateaus. Typical operating plateaus are given in Appendix II.

Operating Settings

Although not critical, the following settings were used almost without exception when taking data:

Counter voltage:	1850v
Amplifier coarse gain:	64
Amplifier fine gain:	1
Amplifier bandwidth:	Delay line
Discriminator level:	40v

EXPERIMENTS

Measurements were made on two zero power reactors (ZPR) built and operated by the Argonne National Laboratory; the Internal Exponential Experiment (ZPR-V) and the Experimental Source Reactor (ZPR-IV'). Brief descriptions of each reactor follow.

ZPR-V

The internal exponential experiment (20) is a coupled slow-fast system consisting of a central fast core surrounded by a light water moderated thermal annulus. The fast and slow sections were coupled through a natural uranium isolation blanket one inch in thickness. The configuration of the reactor is shown in Figure 5.

The fast core is comprised of enriched and natural uranium plates which along with aluminum plates are contained in iron fuel cans 3" x 3" by 24" long. The uranium was present in a U-238 to U-235 atomic ratio of 5 to 1. The volume composition of the fast core was 35 percent uranium, 10 percent iron, 15 percent aluminum and 40 percent void. The fast core is reflected on top and bottom by an iron-uranium reflector one foot thick.

A light water filled tank 5 feet in diameter surrounds the central fast core. Immediately against the fast core are the natural uranium isolation blankets, then comes the thermal fuel baskets, and finally the outer water section which acts as a water reflector. The enriched thermal fuel is clad with aluminum to form .060 in. thick fuel plates each loaded with 16.2 grams of U-235. These thermal plates are contained in baskets which hold up to 70 plates apiece and which are located two baskets to a side of the fast core. The spacing between plates is such that the metal to water ratio is 0.4 to 1. The thermal annulus is further reflected and shielded by a 10 foot diameter shield tank.

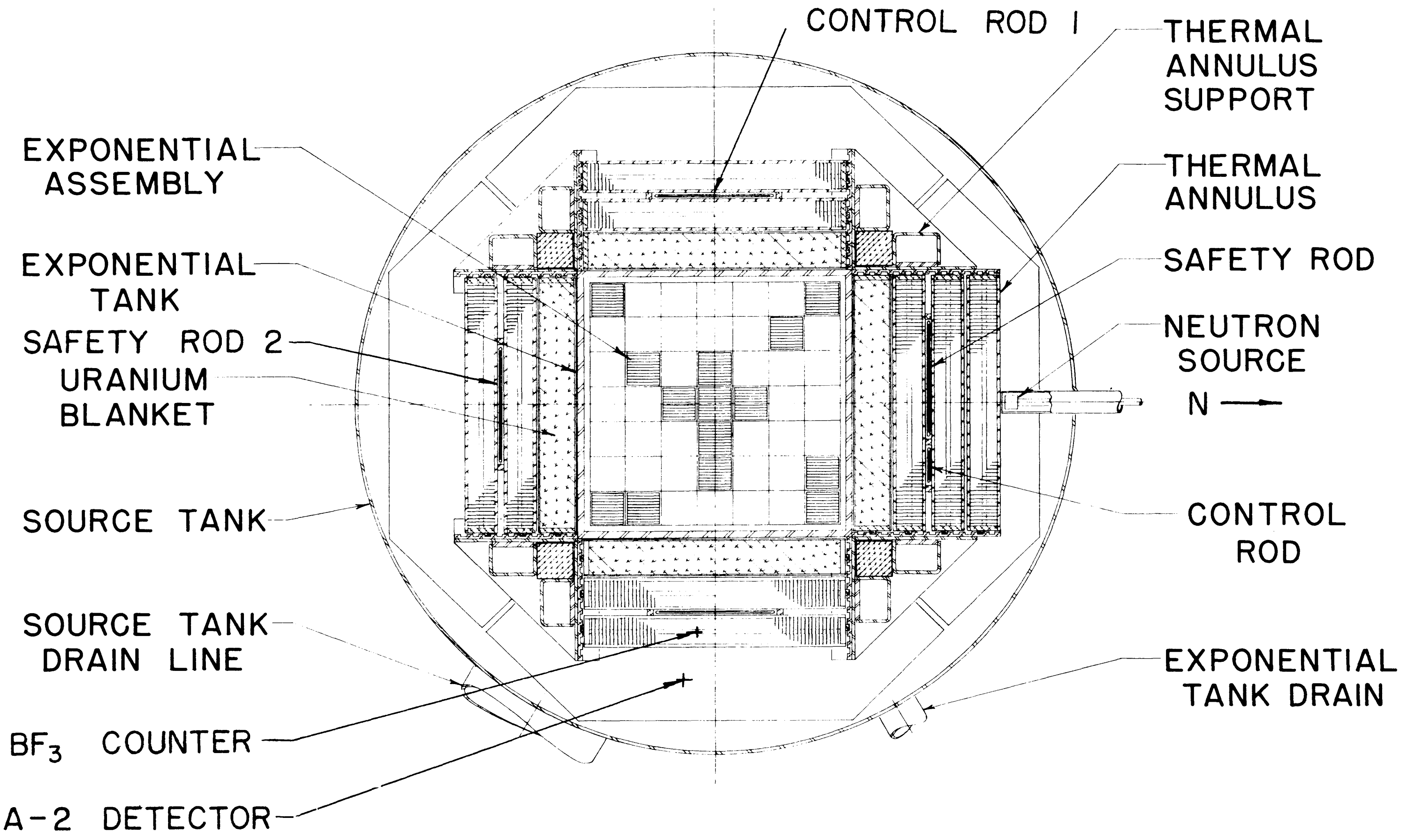

FIGURE 5 ZPR-V REACTOR
PLAN VIEW

0 1 2 3
SCALE IN FEET

Safety rod No.2 and control rod No.1 located between thermal fuel baskets as shown in Figure 5 were used to adjust reactivity in these experiments. These blade rods were 9-5/8 in. wide and consisted of .060 in. of cadmium sandwiched between 1/8 in. sheets of stainless steel.

An Sb-Be source provided neutrons for startup and subcritical operation. It was comprised of a beryllium cylinder fixed in the thermal annulus into which an activated antimony ball was shot using an air cylinder. The Sb-Be source was augmented by a large residual spontaneous fission source. There was some question about the contribution to the residual source due to the (γ,n) reaction on the beryllium of reactor produced gamma rays. To answer this, a removable Be block was installed in place of the fixed Be cylinder for some of the experiments.

The BF_3 detector was placed in the second thermal basket on the east side of the core as indicated. Channel A-2 of the ZPR-V instrumentation was used to monitor the neutron level. The detector for this channel, a gamma compensated ionization chamber, was located as shown and fed a multirange micro-ammeter whose output was recorded.

ZPR-IV'

The Experimental Source Reactor (7) is pictured in Figure 6. It consists essentially of a Borax type-fuel-element light-water-moderated core with a high leakage face to supply neutrons to experiments.

The aluminum clad aluminum-uranium alloy fuel is contained in 10 plate Borax type fuel elements loaded to 157 grams of U-235 per element. The core is built up of a five by five array of these elements. The metal to water ratio is 0.4 to 1. The core is reflected on five sides by

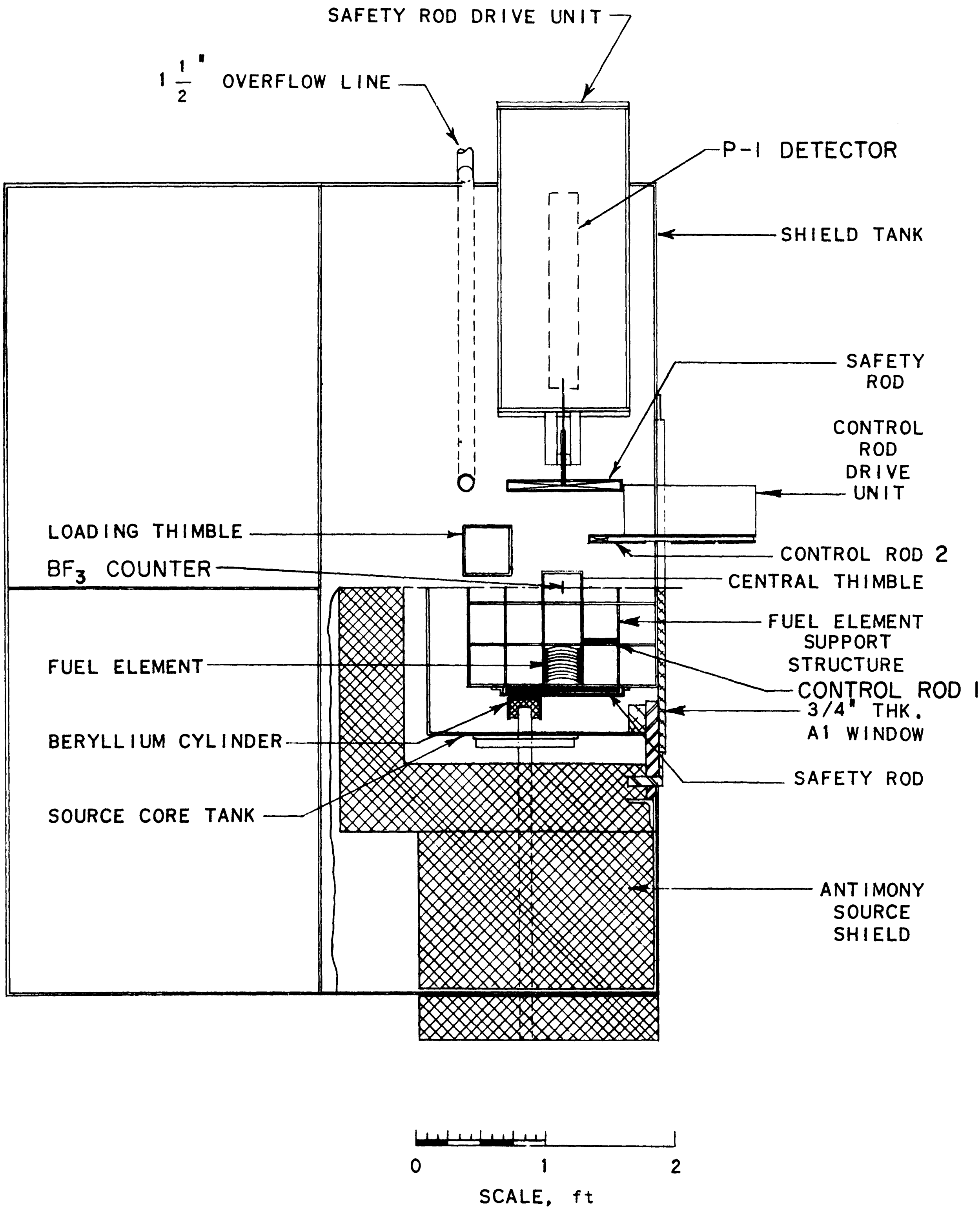

FIGURE 6
ZPR-IV' REACTOR
PLAN VIEW

4 in. of water and surrounded on these sides by a lead and water shield. During these experiments the sixth **side** leakage face was supplying neutrons to a graphite thermal column.

Reactivity was adjusted using the two control rods. These rods are 2-1/2" wide by 28" long and consist of .060" of cadmium clad with aluminum.

The source was Sb-Be as on ZPR-V. Its strength could be adjusted by manipulating the source drive to vary the distance between the Sb and Be. The minimum source strength was much lower than that of ZPR-V, since the large spontaneous fission source of ZPR-V was absent.

The BF_3 counter was located in the central thimble during data taking. The level of the reactor was monitored on ZPR-IV' instrument channel P-1. This channel consisted of a vibrating reed electrometer fed by an uncompensated ionization chamber.

Measurements

Two general classes of measurements, reactivity traverses and gate time traverses, were conducted. In a reactivity traverse data were taken at various reactivities with a fixed gate time. Gate time traverses consisted of counts taken at various gate times at a given reactivity setting. A count, as defined here, consists of the large number of gates made at a particular setting of gate time and reactivity. Traverses are further characterized by the number of gates per count which was either 101 or about 525. Two special runs were made. Series I was designed to evaluate the distribution of $s^2/\bar{c}$ for counts taken at a fixed setting of reactivity and gate time and Series K was a series of source counts at different gate times. The various measurements made are summarized in Table II. Each series was completed in one data taking session.

Table II

Summary of Measurements

Date	Series	Reactor ZPR-	Traverse Type	Gates Per Count	Reactivity* 1-K	Gate Time sec.
8/9	--	V	reactivity	101	various	0.100
8/13	--	V	gate	101	approx. .0038	various
8/15	A	V	gate	101	2.110×10^{-2}	various
8/15	B	V	gate	101	1.922×10^{-2}	various
8/16	C	V	gate	101	1.656×10^{-2}	various
8/16	D	V	gate	101	1.203×10^{-2}	various
8/19	E	V	gate	101	9.641×10^{-3}	various
8/20	F	V	gate	101	6.275×10^{-3}	various
8/20	G	V	gate	101	3.387×10^{-3}	various
8/21	H	V	gate	101	1.474×10^{-3}	various
8/21	I	V	--	101	3.387×10^{-3}	0.100
8/22	J	V	reactivity	101	various	0.100
8/29	K	Ra-Be source	--	525	--	various
8/30	L	V	gate	525	1.794×10^{-3}	various
8/30	N	V	gate	525	6.902×10^{-3}	various
8/30	O	V	gate	525	1.232×10^{-2}	various
9/4	P	IV'	gate	525	1.720×10^{-2}	various
9/5	Q	IV'	gate	525	1.305×10^{-2}	various
9/5	QQ	IV'	gate	525	1.305×10^{-2}	various
9/6	R	IV'	gate	525	9.068×10^{-3}	various
9/6	S	IV'	gate	525	4.838×10^{-3}	various
9/9	T	IV'	gate	525	2.114×10^{-3}	various
9/9	U	IV'	gate	525	3.699×10^{-4}	various
9/10	V	IV'	gate	525	2.135×10^{-4}	various
9/12	W	V	gate	525	1.419×10^{-3}	various
9/13	X	V	gate	525	9.125×10^{-3}	various
9/13	Y	V	gate	525	3.156×10^{-3}	various

*β = .00700

The initial group of traverses taken on ZPR-V at 101 gates per count were exploratory in nature. They confirmed the behavior predicted by theory as gate time and reactivity were varied. They also delineated the range of reactivity over which variance to mean ratios significantly greater than one could be measured and the range of gate time over which the gate time effect saturated. More important, these preliminary runs gave an indication of the scatter to be expected in the $s^2/\bar{c}$ values. The scatter was such at 101 gates per count that it was deemed desirable to increase the number of gates per count. The data of counts consisting of 101 gates were taken by dictation and later transcribed. Fortunately, the recorder became available at the time it was decided to increase the number of gates.

Data collected on and after 8/29/57 were taken with the recorder. The number of gates per count analyzed from the recorder tapes varied due to an error from about 520 to 535. In each case, however, the exact number, n, of gates analyzed was used in forming the variance and mean, The end result of the nonuniformity in gate number is then a negligibly slight difference in the confidence intervals for the individual $s^2/\bar{c}$ values. These counts will be designated as 525 gates counts.

The first several 525 gate count series were taken on ZPR-V. Activity was then moved to ZPR-IV' and the measurements on this machine completed before returning to ZPR-V. In the mean time the shield tank of ZPR-V was painted which necessitated removing and replacing the fuel and the removable beryllium source block was installed. The position of the BF_3 counter was slightly different for the two groups of ZPR-V series. It was more nearly in the center of the fuel basket for the second group than for the first. In each reactor, because of the neutron absorption of the BF_3 counter, it was necessary to add fuel plates to make the reactor critical again after the counter was inserted.

The range of reactivity over which counts could be taken was limited by the residual source strengths of the reactors and by the efficiencies of the counter. It was desirable to make measurements under conditions which would yield a maximum $s^2/\bar{c}$ value significantly greater than the random background value of 1. As reactivity is decreased, the $s^2/\bar{c}$ ratios decrease as predicted by Equation (14). The exact reactivity value at which the maximum ratio becomes smaller than a certain value, say 3 or 4, depends on the detector efficiency E. A higher detector efficiency was obtained in ZPR-IV' than in ZPR-V thus it was possible to take data at (1-K) = 1.72 x 10^{-2} ($s^2/\bar{c}$ max = 4.78) on IV' while on V data was taken down to only (1-K) = 1.23 x 10^{-2} ($s^2/\bar{c}$ max = 3.04).

The other extreme in reactivity, the approach to critical, is limited by the dead time of the apparatus and the residual source strength of the reactor. An increasingly large number of counts are lost in the dead time of a counting system as the counting rate increases. It is good practice to operate at counting rates at which dead time corrections are negligible or small. In view of the 5.1 μs dead time of the apparatus used here the counting rate was limited to 5000 cps except in unavoidable special cases such as during reactivity traverses. The dead time correction for a random source at this rate would be about 2 per cent.

As criticality is approached the multiplication of the reactor source increases. If there is a limit on counting rate there is therefore a limit on how close critical may be approached. The difficulty cannot be alleviated by removing the counter too far since this decreases the detector efficiency and hence the $s^2/\bar{c}$ ratio. Because of the large residual spontaneous fission source in ZPR-V the closest approach to critical under counting rate restrictions was $(1-K) = 1.42 \times 10^{-3}$. ZPR-IV' without the large spontaneous source permitted an approach to $(1-K) = 2.14 \times 10^{-4}$. The saving fact here is that the statistics depend on the prompt multiplication and not the total multiplication. The prompt multiplication is related to the total multiplication by $(1-K') = (1-K) + \beta K$ and therefore begins to saturate as $(1-K)$ becomes the order of β. The full prompt multiplication can thus be effectively reached without being critical.

The reactivities at which the various counts were made was measured by taking input data for the Argonne RE-31 computer code. This required measuring the neutron level as a function of time during rod insertion. On ZPR-V a special instrument channel was utilized. A compensated chamber positioned on the cover of the fast core supplied a signal proportional to the neutron density to a vibrating reed electrometer. The output of this device was then recorded. The P-1 electrometer channel of ZPR-IV' was used for RE-31 measurements on that reactor. The uncompensated chamber of this channel was replaced for these measurements with a compensated chamber which was placed on top of the thermal column and against the face of the reactor.

DATA TREATMENT AND RESULTS

Formation of $\bar{c}$, s^2, and $s^2/\bar{c}$.

The end product of a given count was a number, 101 or 525, of gate values c_i. These c_i were in printed form on adding machine tape where they had been printed either directly by the recorder or by transcribing from a dictation record. In preparation for machine computation the c_i were punched on IBM cards six words to a card in fields suitable for processing by an IBM-650 program written in Bell Code. The punched cards containing the raw data are in storage at the Argonne National Laboratory and are available for use. A Bell Code program was used which formed the sample mean $\bar{c}$ in the conventional manner.

$$\bar{c} = \frac{1}{n} \sum_{i=1}^{n} c_i \qquad (53)$$

The program calculated the unbiased sample variance from:

$$s^2 = \frac{n \sum_{i=1}^{n} c_i^2 - \left(\sum_{i=1}^{n} c_i\right)^2}{n(n-1)} \qquad (54)$$

The ratio of sample variance to sample mean, $s^2/\bar{c}$, was obtained by dividing (54) by (53). After checking the accuracy of the punching operation all the data, about 180,000 numbers, were reduced by this machine program. The detailed results of these computations are listed in Appendix III. Graphs of the results appear in Appendix IV. The fitting of the points will be discussed in the next section.

Theory Fit

The theory as represented by Equation (14) was fitted to the data for the purpose of evaluating the unknown parameters of the theory and hence of the multiplying systems studied. It was desired to perform a least squares fit but it was not possible to reduce Equation (14) to a linear or rapidly convergent polynomial form susceptible to direct least squares attack. A more devious scheme was therefore resorted to. In a given gate time traverse all variables other than gate time remained fixed. Under these circumstances Equation (14) may be rewritten as:

$$W = \frac{V_c{}^T}{M_c} - 1 = \frac{EK'}{(1 - K)^2}\left[\frac{\overline{\nu^2} - \bar{\nu}}{\bar{\nu}}\right]\left[1 - \frac{(1 - e^{-aT})}{aT}\right] = Z\,X\,(aT) \qquad (55)$$

The constant Z is the combination of fixed variables (constants) which multiply the gate-time-dependent bracket which itself is replaced by X above. Now, if the value of "a" is fixed (55) becomes a linear relation between W and X upon which a least squares fit may be immediately performed to find a least squares Z. Furthermore, at some value of "a" there will be a "best" least squares fit in the sense that the sum of the squares of the deviations of the data points from the fitted curve is a minimum. A Bell Code IBM-650 program was utilized to fit the data in this manner.

The input to the fit program consisted of the measured W_i and the gate times, T_i, at which these measurements were made. A trial "a", a_j, was chosen and the machine calculated a least squares Z from:

$$Z_j = \frac{\sum_i W_i\, X\,(a_j T_i)}{\sum_i X^2\,(a_j T_i)} \qquad (56)$$

The sum of the squares of the deviations of the points from this fitted curve was then determined.

$$M_j = \sum_i \left[W_i - Z_j \, X \, (a_j T_i) \right]^2 \qquad (57)$$

The entire procedure was repeated to accumulate a table of values.

a_1	Z_1	M_1
a_2	Z_2	M_2
a_3	Z_3	M_3
.	.	.
.	.	.
.	.	.
a_j	Z_j	M_j

The square deviations, M, were then plotted as a function of "a". The minimum in M determines the least squares value of "a" and simultaneously a least squares value of Z, since there is a Z corresponding to each "a". Such a plot for Series N appears in Figure 7.

A fit was made only to the 525 gate count series and was carried out to three significant figures for "a". The points indicated by filled in circles on the graphs of Appendix IV which did not follow the general trend of the data were omitted from the fit. The values of "a" and Z determined for these series are displayed in Table III. The fitted curves appear in Appendix IV.

Prompt Neutron Lifetime

The prompt neutron lifetimes of the assemblies may be calculated from each of the individual fitted "a" values since "a" by definition equals $\frac{1 - K'}{\tau}$. For such a calculation 1 - K' is formed from the

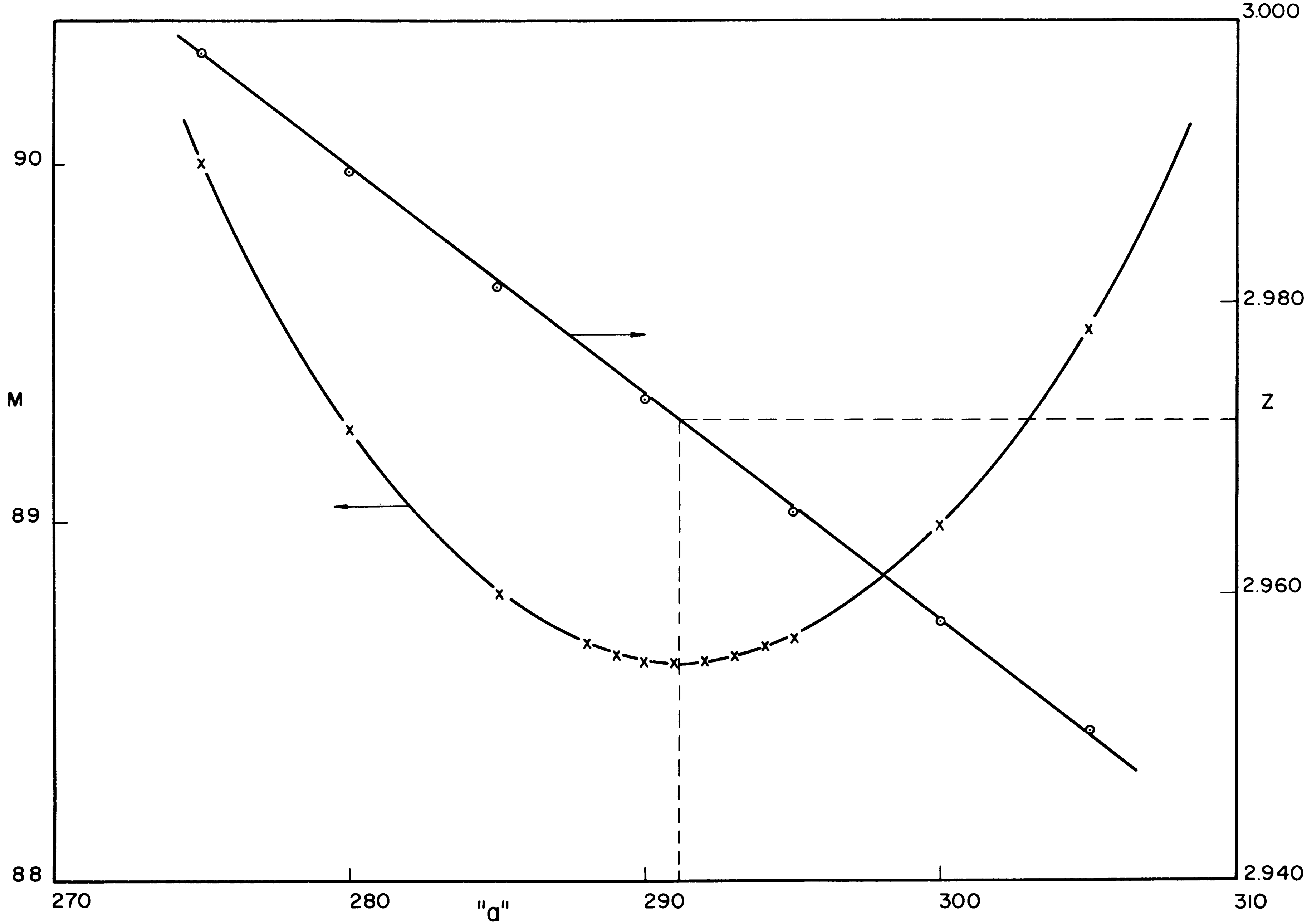

Figure 7. **Least** Squares Fit for "a" and Z, Series N

Table III

Results of Theory Fit

Series	ZPR	"a"	Z
P	IV'	322	3.782
QQ		324	5.263
R		223	8.189
S		172	14.685
T		144	25.668
U		92.7	38.255
V		99.8	13.793
O	V	502	2.037
X		437	2.301
N		291	2.972
Y		217	3.672
L		214	4.519
W		152	5.633

measured values of (1-K) and the chosen value of β. But by definition "a" is a linear function of (1-K'). Therefore, all the data may be combined to obtain a value of τ by fitting this relation. Such a procedure was followed and the value of τ was determined from the least squares equation:

$$\tau = \frac{\sum_i (1 - K')_i^2}{\sum_i a_i (1 - K')_i} \qquad (58)$$

The fitted curves using β = .00700 are given in Figures 8 and 9.

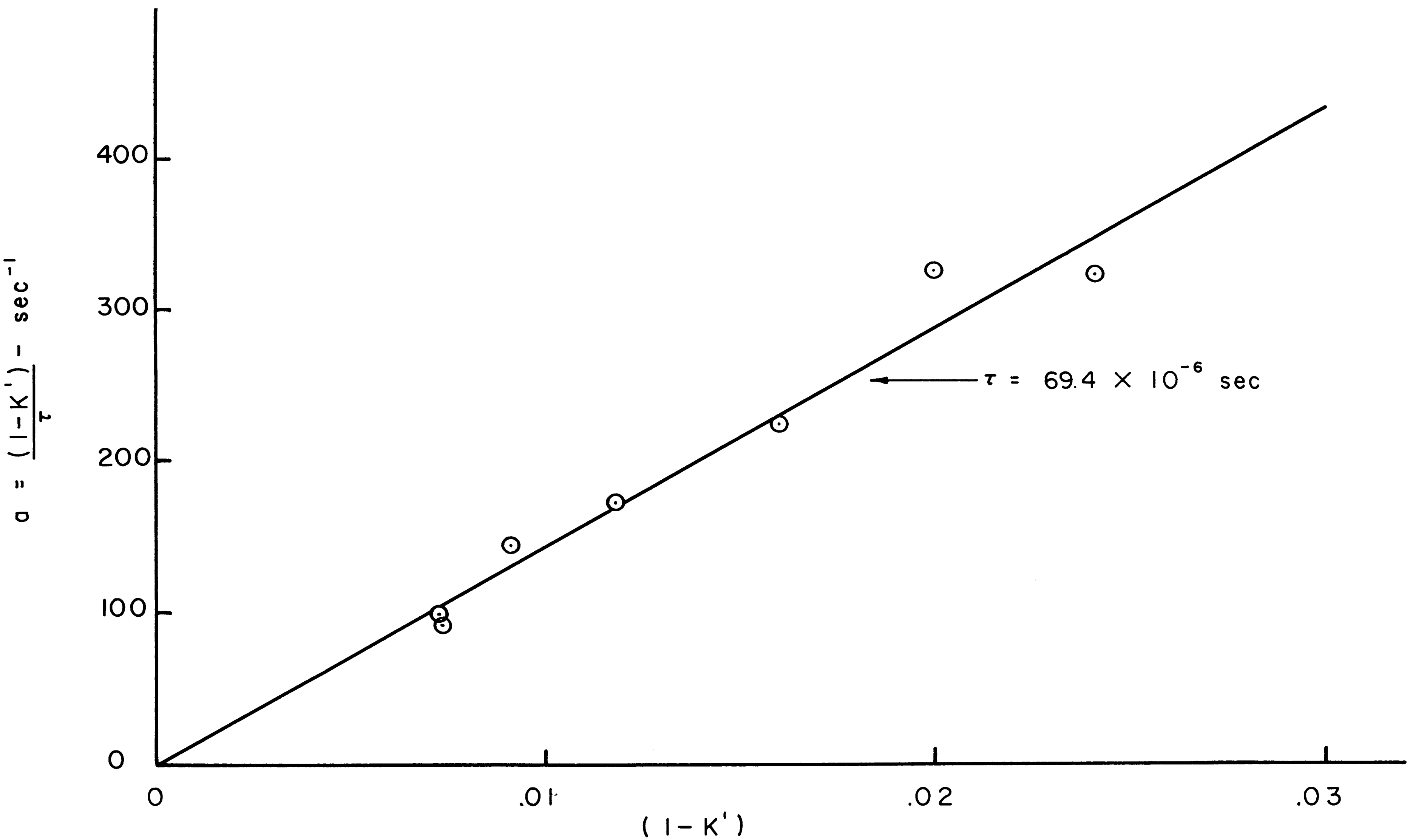

Figure 8. Prompt Neutron Periods, ZPR-IV'
β = .00700

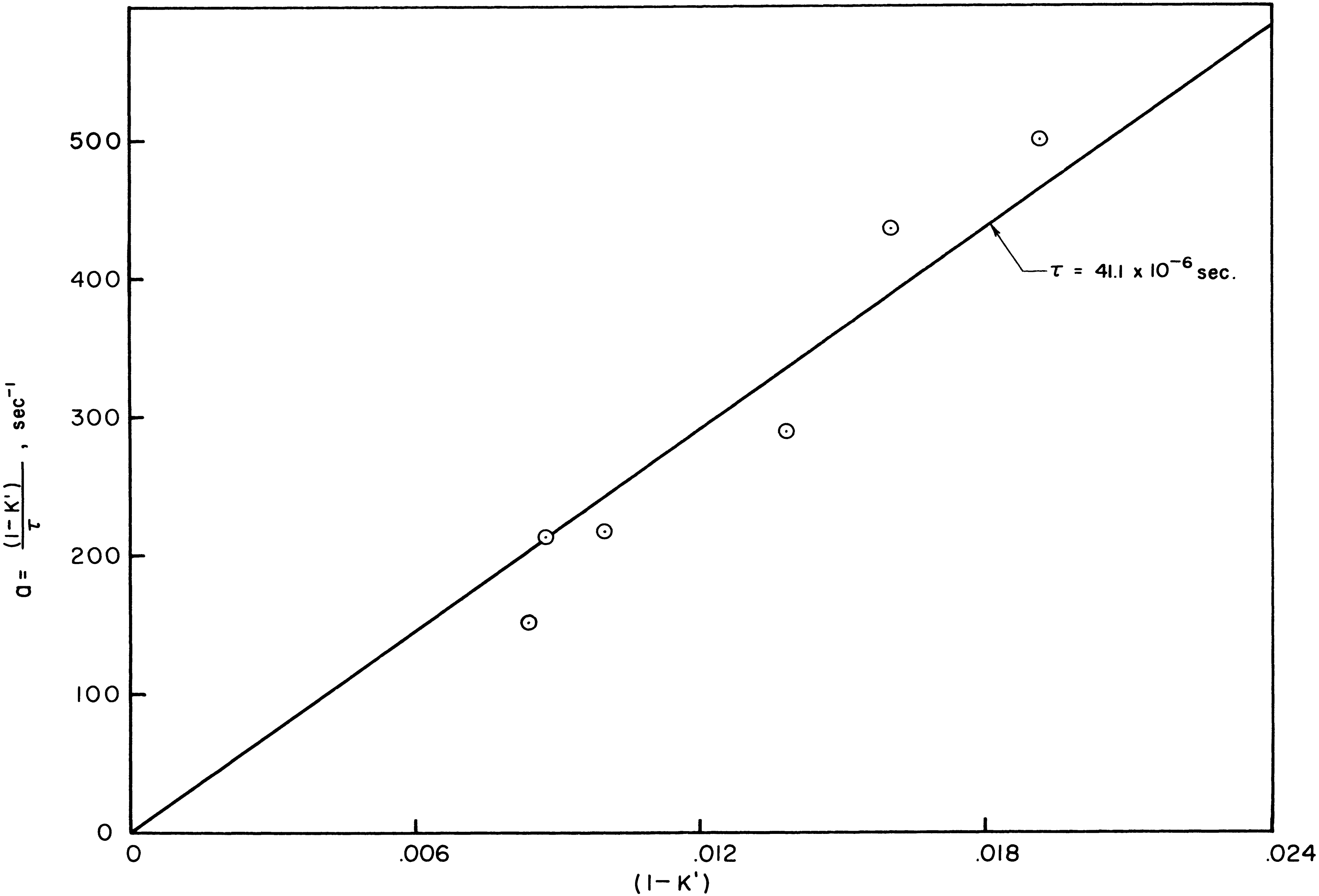

Figure 9. Prompt Neutron Periods, ZPR-V
β = .00700

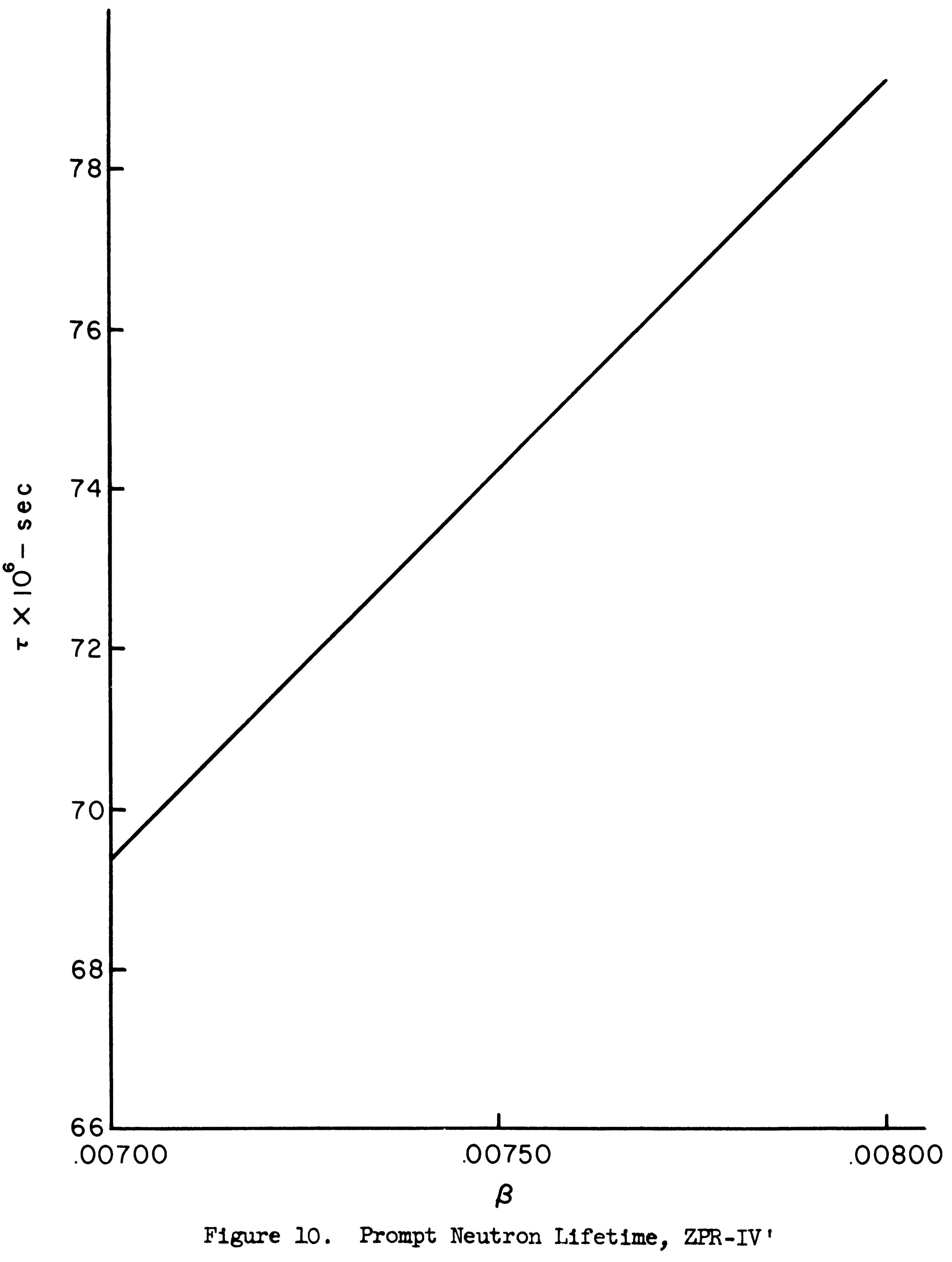

Figure 10. Prompt Neutron Lifetime, ZPR-IV'

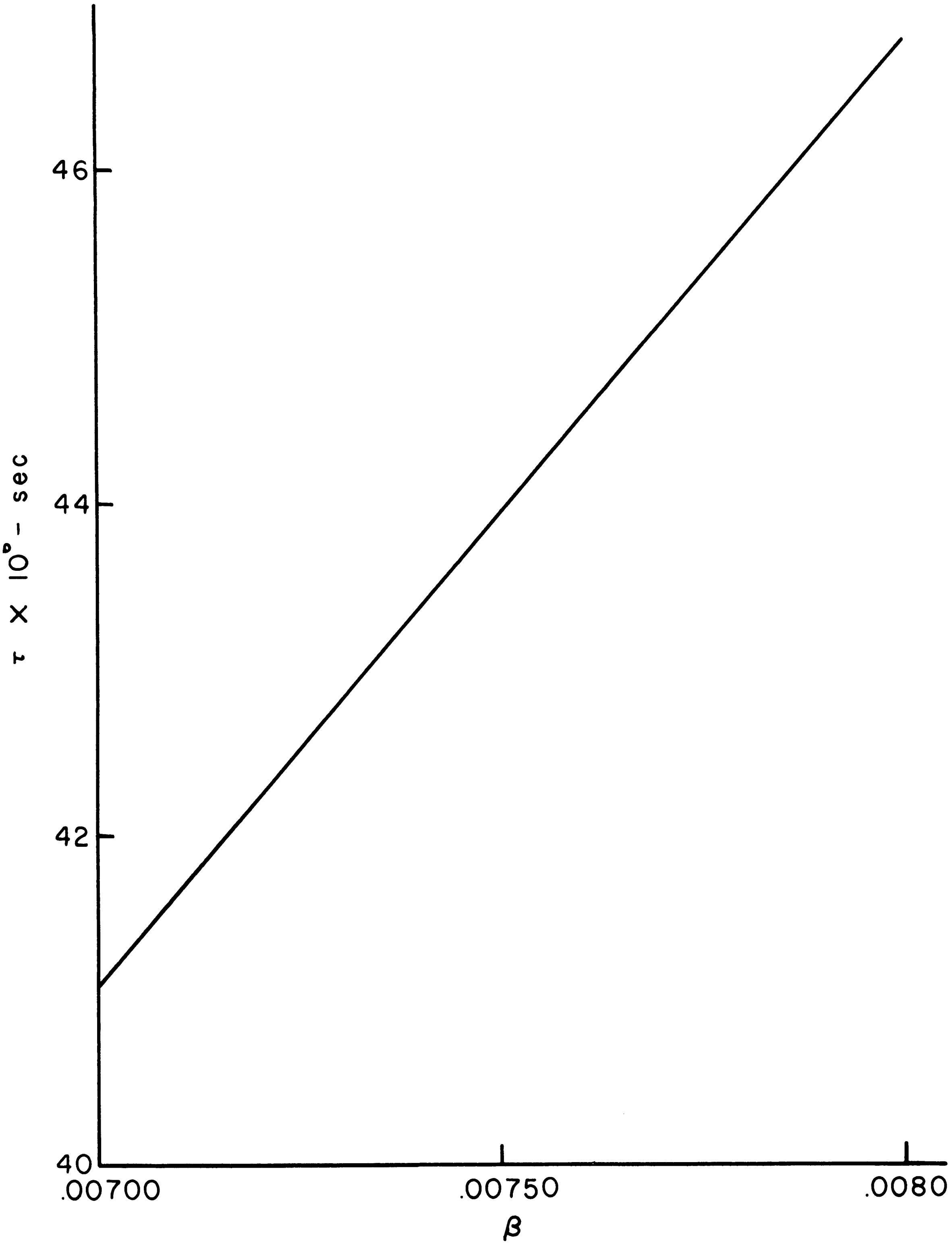

Figure 11. Prompt Neutron Lifetime, ZPR-V.

The values of the prompt neutron lifetime obtained depend on the value of the effective fraction of delayed neutrons used due to the influence of β on $(1 - K)$ and $(1 - K')$. The data were recomputed using various values of β to yield Figures 10 and 11.

Detector Efficiency and Power Level

The absolute detector efficiency, E, was calculated directly from the fitted Z values, the measured reactivities, and an independently measured (6) value of $\frac{\overline{\nu^2} - \bar{\nu}}{\bar{\nu}}$.

$$E = \frac{Z\,(1 - K')^2}{K'\left[\frac{\overline{\nu^2} - \bar{\nu}}{\bar{\nu}}\right]} \tag{59}$$

The mean counting rate $\frac{\overline{dc}}{dt}$ was measured during the experiment. The counting rate is related to the neutron loss rate or, in the steady state, production rate, F, through the absolute detector efficiency. E is the counts in the detector per neutron lost from or, in the steady state, born in the entire system.

$$F = \frac{\frac{\overline{dc}}{dt}}{E} \tag{60}$$

Once the absolute neutron production rates were calculated it was a simple matter to convert to absolute power levels. The readings of channel A-2 on ZPR-V and channel P-1 on ZPR-IV' were noted during the measurements. A correspondence between absolute power level and instrument readings was therefore established. The correspondence is made more useful by normalizing the power levels to a given instrument reading. These calculations are summarized in Tables IV and V.

Confidence Limits

Confidence limits may be placed on the τ and Z values if certain assumptions are made about the distribution of the data. The confidence limits for τ may be calculated under the assumptions that the "a" values are independently and normally distributed with common variance about a true regression line a = b (1-K'). Under these conditions

$$t = \left(\frac{1}{\tau} - b\right) \sqrt{\frac{(n-2)\,\Sigma\,(1-K')^2}{\Sigma\left[a - \frac{(1-K')}{\tau}\right]^2}} \qquad (61)$$

possesses a Student's t distribution (19) with n-2 degrees of freedom. The 50 per cent confidence intervals calculated for τ using β = .00700 are: ZPR-V, 39.2 - 42.8 x 10^{-6} seconds and ZPR-IV', 67.5 - 72.0 x 10^{-6} seconds.

TABLE IV. ABSOLUTE DETECTOR EFFICIENCY AND POWER LEVEL ZPR-IV'

Series	E* x 10^3	$\overline{dc}$/dt**	F x 10^{-6}	P-1 Per Cent	P-1 Scale	Normalized*** Power Level watts x 10^5
P	1.15	5403	4.70	23.00	Lo-1	26.7 ≠
QQ	1.09	4176	3.83	17.00	Hi-100 ≠	2.94
R	1.09	5225	4.79	15.75	Hi-100 ≠	3.98
S	1.05	4378	4.17	13.00	Hi-100 ≠	4.20
T	1.10	4871	4.43	11.75	Hi-100 ≠	4.92
U	1.07	5373	5.02	11.25	Hi-100 ≠	5.83
V	0.368	5495	14.95	20.00	Hi-100 ≠	9.77

*β = .00700, $\frac{\overline{\nu^2} - \bar{\nu}}{\bar{\nu}}$ = 1.9635.

**based on $\bar{c}$ of 100 ms gates.

***7.65 x 10^{10} n/s per watt, normalized to P-1 reading of 10 per cent on Hi-100 scale.

$\neq$ difference between Hi and Lo scales was factor of 10^3.

$\neq\neq\neq$ This series immediately followed a run at full power. The results for power level are unreliable due to the effect of decay gammas on the uncompensated P-1 chamber.

TABLE V. ABSOLUTE DETECTOR EFFICIENCY AND POWER LEVEL ZPR-V

Series	E* x 10^4	$\frac{\overline{dc}}{dt}$**	F x 10^{-7}	A-2 Per Cent	A-2 Scale	Normalized*** Power Level watts x 10^4
O	3.92	2708	0.690	7.00	10^{-10}	1.29
X	3.07	3760	1.22	7.50	10^{-10}	2.13
N	2.96	3734	1.26	9.75	10^{-10}	1.69
Y	1.93	7503	3.88	16.50	10^{-10}	3.08
L	1.79	12130	6.77	33.00	10^{-10}	2.68
W	2.05	5093	2.48	11.25	10^{-10}	2.88

*β = .00700, $\frac{\overline{\nu^2} - \overline{\nu}}{\overline{\nu}}$ = 1.9635.

**based on $\bar{c}$ of 100 ms gates.

***7.65 x 10^{10} n/s per watt, normalized to A-2 reading of 10 per cent on 10^{-10} scale.

The same method may be employed to define confidence limits for Z based on the regression Equation (55). In this instance the W values are assumed to be independently and normally distributed with common variance about a true regression line W = b X(aT). The Student's t distributed variable is then:

$$t = (Z-b)\sqrt{\frac{(n-2)\,\Sigma\, X^2(aT)}{\Sigma\,[W-ZX(aT)]^2}} \qquad (62)$$

Typical 50 per cent confidence limits calculated for Z values are: Series V, Z = 13.793 $\pm$ .086 and Series W, Z = 5.633 $\pm$ .066. These confidence intervals may then be carried through the calculations to determine their influence on the normalized power levels. However, the confidence intervals on the Z are so small compared to the size of other areas of uncertainty in the determination of power level that this was not done.

Reactivity Measurement

Reactivity was adjusted using the control and safety rods of the reactors. The reactivities thus obtained were measured by taking input data for the Argonne National Laboratory computer code RE-31. This program generates a numerical solution of the point reactor kinetic equations:

$$\frac{dN}{dt} = \frac{(K-1)N}{\tau} - \sum_i \frac{dC_i}{dt} \tag{63}$$

and:

$$\frac{dC_i}{dt} = \frac{\beta_i KN}{\tau} - \lambda_i C_i \tag{64}$$

The required problem input consists of $N(\Delta t)$, the parameters λ_i, β_i, τ, and Δt, and the initial conditions $N(0)$, $(K-1)_o$, and the asymptotic period at time zero.

Input data were taken by following the neutron level as a function of time while the rod whose reactivity it was desired to evaluate was incrementally inserted. The RE-31 runs were started with the reactor critical at high power. The initial asymptotic pile periods were therefore infinite, $(K-1)_o = 0$, and the level was high enough so that its decay could be followed for some time. The selected rod was then inserted to a predetermined position and held there for about 10 Δt, then inserted to the next position and held. This step insertion was continued until the neutron level decayed below the range of the measuring instrument.

From its input the RE-31 code calculates (K-1), $\frac{d(K-1)}{dt}$, $\frac{d^2(K-1)}{dt^2}$, and $\int_0^t N(y)\,dy$ at each Δt interval.

Since rod position was correlated with time while taking input data the RE-31 results constitute a calibration of the rods. At Δt intervals when the rods were in motion the RE-31 answers indicate a finite derivative and changing values of (K-1). For Δt intervals at which the rod position was held constant the derivatives are zero and the (K-1) values fluctuate about a mean value. The mean value of (K-1) at the fixed rod positions was calculated and taken as the measured worth of the rod at that position starting from the given critical point.

The delayed neutron parameters of Keepin(21,22) were used.

TABLE VI. DELAYED NEUTRON PARAMETERS

Group	λ_i	β_i
1	.01276	.000252
2	.03194	.001470
3	.11808	.001344
4	.31795	.002863
5	1.5068	.000945
6	5.3319	.000126
		.007000

While taking data the rods were moved slowly enough so that the resulting transients were controlled by the delayed neutron properties and not the prompt lifetime. The sensitivity of the RE-31 results to large changes in the prompt lifetime was investigated by calculating one ZPR-V run at lifetimes of 30, 50, and 70 x 10^{-6}. The results in (K-1) were the same to four significant figures. Furthermore, the results of several V and IV' runs were essentially the same whether the τ's were 60 and 40 x 10^{-6} or 6 and 4 x 10^{-6} sec. The rod calibrations were calculated using lifetimes of 40 x 10^{-6} for ZPR-V and 60 x 10^{-6} for ZPR-IV'. Due to the low sensitivity of the results to the input τ the results are valid even though the final measured τ's differ. In any case, the exact results can be obtained by an interative process.

The uncertainty in the value of the effective delayed neutron fraction β prompted exploration of the RE-31 results as a function of β. The increase in delayed neutron effectiveness derives from the lower birth energy of the delays compared to the prompts. Since the energies of the various delay groups (1) are about the same it was assumed that the effectiveness of each group would be increased by the same factor. The β_i were thus corrected to β = .00750 and β = .00800 by multiplying each β_i by .00750/.00700 and .00800/.00700. Many of the V and IV' problems were then rerun with the two new sets of β_i.

It was found that the (K-1) results for β = .00750 and .00800 were the same as those obtained by directly multiplying the β = .00700 results by the ratio of the betas. The proportionality of reactivity to delayed neutron fraction is further evidence that the basic data were taken from transients controlled by the delayed neutrons.

Rigorous Theory Fit Problem

In the above paragraphs the fitting of the data to obtain estimates of the parameters "a" and Z was described. An unambiguous but arbitrary method was used. All data points were assigned equal weights and a least squares criterion was applied. While it is obvious that the estimates so obtained are not rigorous maximum-likelihood-estimates(19) it is equally apparent that the fitted curves and the parameters which define them describe the data pretty well. Application of rigorous maximum likelihood methods requires knowledge of the frequency function of the basic count distribution. This frequency function is not known and only the first and second moments of the distribution have been used in this investigation.

The assigning of weights to the points is equivalent to defining confidence limits for the population variance to mean ratio, V_c/M_c, based on the corresponding measured sample value $s^2/\overline{c}$. This also requires knowledge of the c distribution.

Theoretically it is apparent that the c distribution is not normal since the variance is proportional to the mean from Equation (14). In a true normal distribution the variance and mean are independent. Experimentally, the form of the distribution is seen to change as a function of sample mean which is determined by gate time at a constant counting rate. At short gate times the mean is small and since the gate values are restricted by the zero axis the distribution appears Poissonian. For long gate times and large means the distribution appears to approach normal. This behavior was examined by classifying into intervals five source and five reactor counts with means ranging from 3.7 to 182. The $s^2/\overline{c}$ ratios of the reactor counts varied from 1.6 to 6.2. The six distributions in this group with the larger means were tested for normality with the chi-square test. Only the source count with largest mean was accepted as normal at a critical level of 5 per cent.

The above discussion also applies to the fit from which the value of τ was estimated. However, the values of "a" are undoubtedly of more nearly equal weight and probably approach being normally distributed.

In the same vein, the restrictions of normality and uniform variance required to find confidence intervals are undoubtedly not rigorously satisfied by the $s^2/\overline{c}$ distributions. The results of Series I are an example of a $s^2/\overline{c}$ distribution. There are listed the variance to mean ratios of a number of counts taken at fixed settings. The "a" distributions should again come closer to rigorously meeting the required conditions. On the other hand, it is felt that both distributions come near enough to satisfying the basic criteria to permit practical use of the calculated confidence intervals.

Dead Time Correction

Dead time losses effectively reduce the detection efficiency of the measuring system. Procedures for correcting for these losses in the case of a random source are well established(2). The necessity for and the method of applying a dead time correction in the present measurement is not so apparent.

The detection efficiency E is measured directly by the neutron statistics analysis. This efficiency includes in a straightforward manner such factors as neutron spatial distribution, detector size, and detector sensitivity. The efficiency also contains in some manner the effect of dead time losses. The question arises whether dead time losses are properly accounted for in the measured efficiencies or whether a correction is necessary. The answer to this question lies in the derivation of the theory.

In deriving the compound distribution of Equation (21) which was later used to describe the effect of a detector in Equation (37) a basic condition imposed was that the detector response be independent of the number of neutrons presented to it. This condition is obviously not satisfied if the efficiency of the detection system is dependent on the rate at which neutrons are registered. Therefore, although the measured efficiencies include dead time effects they are not included in a manner consistent with the theory and should be corrected for.

Random source correction procedures cannot be utilized since the count distribution is not random. The development of a suitable correction method requires knowledge of the count distribution and this is not available.

Dead time corrections were not made here but the error in this omission was kept small by limiting the counting rate for most counts to 5000 cps. For the measured dead time of 5 us this would give a random source dead time loss of 2 per cent or less. In any case, since dead time losses depend on counting rate and the counting rate was constant for a given series the fitted value of "a" and hence τ should not be affected. The magnitude of the measured $s^2/\bar{c}$ as reflected in the fitted values of Z would, however, require correction.

DISCUSSION

The measurement of neutron statistics by this counting type experiment proved to be fully as uncomplicated a method as was anticipated. The conventionality of the equipment and technique should be of interest to many groups.

The most difficult equipment problem was noise. Once this was solved in the manner described previously the measuring system was quite trouble-free. The apparatus could be set up at a new location and checked out all within an hour.

Data taking with the properly functioning measuring system was routine. Data treatment was not so routine, however, even with the extensive use of the computer. Difficulties centered on the hand operations, such as punching, where human errors were inevitable. It is recommended that for future work more automation be employed if large amounts of data are to be taken. This would avoid human errors and increase confidence in the results. Many schemes such as direct punching on cards instead of printout are conceivable.

The results of the gate time traverses as graphically depicted in Appendix IV verify the theoretically predicted behavior of the statistics as a function of counting or gate time. The fitted curves describe the data quite well except for a few "wild" points at the long gate times of many runs. These points have the disconcerting feature of not appearing in every series. Any physical explanation is therefore open to question. On the other hand, the measurement is least susceptible to equipment malfunction, such as gate fluctuation, at long gate times. It is thought that these wild values reflect the contributions of the delayed neutrons for long measuring (not gate) times. The measuring times for the long gate time counts approached 15 minutes. If this explanation is valid it has implications in the further application of this method which will be discussed later.

It should be pointed out before discussing the values of lifetime and power level obtained that the system measured actually consisted of the reactor plus the neutron counter. The effect of the counter was to increase the total absorption of the core and to cause a perturbation of the neutron flux. It has been noted previously that the counter absorption was compensated for by the addition of fuel. The absorption of the counter perturbs the reactor prompt neutron lifetime and the measurement of reactor power level. The magnitude of this effect may be evaluated from the measured detector efficiencies. The detector efficiency may be interpreted as the fraction of the neutrons lost from the system by absorption and leakage which are absorbed by the detector. The efficiencies measured were of the order of 10^{-3} and 10^{-4} so the absorption in the counter is quite a small fraction of the total absorption and leakage. Therefore, the lifetime and power level of the reactor plus counter should not vary significantly, due to the increased absorption, from the lifetime and power level of the reactor alone. However, the effect of the flux perturbation on the power level indicated by reactor power monitors can be quite large as will be discussed.

The measured values of the prompt neutron period, "a", are in reasonably linear relation as they should be with the prompt multiplication (1-K'). This relationship is illustrated in Figures 8 and 9. The values of the mean prompt neutron lifetimes of ZPR-V and ZPR-IV' deduced from the prompt periods compare favorably with previously measured and predicted values. The ZPR-V lifetime of 39.2-42.8 μs compares with the value measured by Toppel(31) of 39 ±3 μs . A lifetime of 67.5-72.0 μs was determined here for ZPR-IV'. Martens(24) predicted a lifetime of 59 μs for the basic IV' core. The lifetime of the system which included the graphite thermal column would be longer than that of the basic core alone due to the long lifetime of neutrons in graphite. C.E.Cohn recently measured 66.4 μs as a preliminary value of the lifetime of ZPR-IV'. His measurement was made in the presence of less graphite than the measurement detailed here.

The calculation of mean prompt neutron lifetimes from prompt periods requires the use of the delayed neutron parameters. The work reported here is based on the 1955 Keepin and Wimett(22) values, Table VI. More recent results of Keepin, Wimett and Zeigler(23) are now available. The increased effectiveness of the delayed neutrons over the prompts due to their lower birth energy must also be accounted for. This effect can introduce a correction of several per cent. These same considerations apply to the power level calibration yet to be discussed. Neutron lifetimes and power levels quoted here are based on an effective delayed neutron fraction of .00700. However, the basic reactivity data is such that it can be recalculated using any delayed neutron parameters preferred.

The measured detector efficiences are tabulated in Tables IV and V. They are in the range of rough estimates made based on the known detector size and sensitivity. The detector exhibited an order of magnitude higher efficiency in the small ZPR-IV' reactor than in the relatively large ZPR-V system. This is as it should be. In order to decrease the counting rate during one IV' series the counter was withdrawn to a position of decreased efficiency as is evident in Series V. Even in a fixed position the counter efficiency was not constant. Small changes can be accounted for by such factors as varying counter voltage and discriminator setting. It is evident, however, that the efficiency was subject to gross perturbation.

The variation of detector efficiency may best be explained by reference to the ZPR-V results. Examination of Figure 5 will reveal that control rod #1, which was used for reactivity adjustments, and the counter were situated on opposite sides of the core. Movement of the control rod changed the flux pattern over the core. When the rod was inserted, the flux in the neighborhood of the rod was depressed and the general flux pattern was shifted across the core and, in this case, onto the detector. As the rod was withdrawn towards its critical position the flux rose on the side by the control rod and shifted away from the counter until it assumed its symmetrical unperturbed shape on the core. While the flux was shifted onto the detector it was in a position of higher efficiency than when the flux was symmetric on the core since the efficiency is weighted directly by the neutron density which the counter samples. The decreasing counter efficiency as the rod was withdrawn toward criticality is quite apparent in the ZPR-V results. The perturbation was not so large in ZPR-IV'. Here the counter was located at the center of the core between the two rods which were used for reactivity adjustment. In its central position it was evidently not subjected to such widely varying flux levels as it would have been on the edge of the core.

The power level normalized to a given reading of the reactor instrument channels is also displayed in these tables. If the detectors, A-2 and P-1, of these channels had seen the reactors as point sources and had thus been unaffected by changing flux patterns the normalized power levels would have been constant. This was not the case. The increasing trend of the normalized power level is again explainable in terms of shifting flux patterns as the rods were withdrawn. Consider ZPR-V. A-2 detector was located in the same relative position as the experimental counter. As criticality was approached by withdrawing control rod #1 the flux shifted back onto the core and the A-2 efficiency decreased as did that of the counter. Therefore, a higher power level was required to produce a given A-2 channel reading. The magnitude of this effect was also significant in ZPR-IV' even though the counter efficiency did not vary much in this machine. This is because P-1 is located at the edge of the core where the flux level changes more drastically and where P-1 may even be shielded by control rod #2. There was a change at Series V of the same magnitude as those experienced on rod withdrawal since the counter which was withdrawn was a strong absorber itself.

The magnitude of the perturbation of the instrument channel readings was not anticipated and created obvious difficulties in the correlation of power level with the reactor instruments. Nevertheless, the full power levels of both machines were calculated for comparison with other values. This was done by assuming that the A-2 and P-1 channels were linear from the low powers at which these measurements were taken up to full power. The full power level of ZPR-V was thus calculated to be 127 watts at an A-2 reading of 44 per cent on the 10^{-5} scale* based on the normalized power of Series W. The full power level of ZPR-IV' was estimated to be 8.8 watts for a P-1 reading of 90 per cent on the Low - 1000 scale based on the normalized power level of Series V.

Fisher(14) predicted the full power of ZPR-V to be 450 watts

*44 per cent on 10^{-5} of A-2 = 130,000 counts per minute on south monitor counter.

from fission traverses and a radiochemical determination of the absolute fission rate. It is felt that this value may be too high. Martens(24) gave 19 watts as an estimate of the absolute full power of ZPR-IV'. Unfortunately, flux or fission traverses of IV' are not available and cannot now be made since the core is completely enclosed.

The absolute powers measured here are of the same order as those predicted independently. All corrections for instrument perturbations would tend to increase the measured values as would a smaller delayed neutron fraction. The effect of the experimental counter, unaccounted for here, can be quite large as indicated by IV', Series V. There is no theoretical reason for expecting consistently low power estimations from this method although there is an unknown correction for system size and heterogeneity which for ZPR-V might be reasonably large. In summary, this investigation supports but does not establish the validity of the neutron statistics method of absolute power level calibration. It does point up the precautions which must be taken in correlating power level with instrument readings both for this method and in general practice.

Power level calibration by this method is based on the fitted Z values. Any other successful investigation which utilizes the Z values therefore adds support to the general method. Feynman, de Hoffmann and Serber(13) successfully determined the dispersion of fission neutrons, $\frac{\overline{\nu^2}-\overline{\nu}}{\overline{\nu}}$, by measurement of Z. This is further evidence that the difficulty here lies in detector placement rather than with the theory.

APPLICATION AND LIMITATIONS

It would appear, based on experience gained here, that future measurements need not involve the large amount of data taken here. If the reactor has a small residual source only one count series taken as close to critical as counting rate considerations permit is required. The reactivity would then not have to be measured accurately since $(1-K')$ approaches the effective delay fraction β as criticality is approached. The mean prompt lifetime would be calculated from the fitted "a" and a value of β. The absolute power level would be deduced from the fitted Z. Based on the experience gained, special care should be taken to correlate the power level in terms of a detector which sees the reactor as a point source. If the measurements are taken near critical, rod perturbations should be minor. Counter perturbation should be minimized by placing it in the position of least efficiency which will still give $s^2/\bar{c}$ ratios acceptably greater than the random value of 1.

The measurements reported here, on reactors with prompt lifetimes characteristic of light water moderated systems, were made using conventional equipment and techniques. It appears that more elaborate equipment and special data taking techniques may be required to investigate systems with much shorter or much longer lifetimes. Short lifetimes dictate short gate times and therefore high counting rates. This imposes severe dead time and gate time precision limitations which would probably require specially designed electronics. Long lifetimes mean long gate times and therefore long measuring times. Such conditions invite delayed neutron contributions which could not be correlated in terms of the simple prompt neutron theory. If the "wild" points reported here at long gate times are evidence of delayed neutron effects some modification of data taking technique may be called for in the case of long lifetimes. In order to cut the measuring time a continuous record technique such as that employed by Feynman, de Hoffmann and Serber(13) could be resorted to. Alternately, the data could be broken up into shorter measuring times and the long period fluctuations corrected for by a scheme similar to that employed by the above investigators.

CONCLUSIONS

The following conclusions are drawn from this study of the measurement of the statistics of neutron multiplying systems and their interpretation in terms of reactor parameters.

1. The neutron statistics of reactors with prompt neutron lifetimes characteristic of light water moderated systems may be measured with conventional counting instrumentation and employing conventional counting techniques.

2. The dependence of the prompt neutron statistics of such systems on counting time may be correlated in terms of several available theories(4,5).

3. The mean prompt neutron lifetime of such systems may be deduced from the statistics measurements subject to a choice of value for the effective fraction of delayed neutrons.

4. The measurements support but do not establish the validity of the statistics method as a means of evaluating the absolute power level of reactors.

5. Extreme caution must be exercised in correlating power level with instrument channel readings.

6. The counting type measurement of neutron statistics and reactor parameters shows promise of application in systems of shorter and longer neutron lifetime but probably not without changes in equipment and/or data-taking technique and treatment.

APPENDIX I

DETERMINATION OF MEASURING SYSTEM DEAD TIME

The dead time of the system is defined as that period following the registration of a pulse by the scaler during which a second neutron event in the detector will not be registered by the scaler. In a counting system such as described previously which consists of a proportional counter, fast amplifiers, and a scaler the dead time of the system is that of the first binary of the scaler. F. H. Martens suggested the following technique for measuring the dead time using a double pulse generator.*

The double pulse generator used supplied two pulses, one following the other in time. The time separation between the two pulses was adjustable as was the polarity, amplitude, and width of each pulse. If double pulses were fed to the scaler and the pulse separation between them decreased the scaler counting rate dropped by a factor of two at some pulse separation. At this separation the scaler had ceased counting both pulses and was counting only the first. When it was not counted, the second pulse was arriving during the dead time of the scaler. The minimum pulse separation when both pulses were counted was identified with the dead time.

The shape of the generator pulse was adjusted to be as similar as possible to the output pulses of the amplifier discriminator. That is, an effort was made to simulate the input to the scaler which it would receive if in place in the counting system. Reasonable simulation was achieved, however, the generator pulses had a little faster rise time and were more square. The double pulses were then fed into the DY-2500 and the

*Berkeley Double Pulse Generator, Model 903, Serial 479.

counting rate observed as a function of pulse separation with the results of Table VII.

TABLE VII. COUNTING RATE VS. PULSE SEPARATION

Pulse Separation μs	Counting Rate c/s
10	206
9	206
8	206
7	206
6	206
5.5	205
5.25	205
5	103
4	103
2	103
1	103
0	103

The cut point was quite sharp and reproducible. The dead time of the DY-2500 scaler was deduced from these measurements to be 5.1 microseconds.

To verify that the scaler did indeed dictate the dead time of the entire system this procedure was repeated driving the input of the amplifier and the input of the preamplifier with the double pulses in the subsequent tests. In each case, the measured dead time was 5.1 μs and was not extended by the inclusion of the additional components in the signal path.

The absolute accuracy of the pulse separation control was checked to the accuracy required in this application by cross calibration with an oscilloscope.

TABLE VIII. GENERATOR PULSE SEPARATION (μs)

Generator Dial	Oscilloscope Sweep*
10	10.0
9	9.3
8	8.2
7	7.2
6	6.1
5	5.1
4	4.1
3	3.0
2	2.0
1	1.0

*Tektronix, Type 545, Serial 1022

The scope was then utilized in a rough check of the dead time measurement. The counting system was driven with a Ra-Be source at a counting rate of about 12,000 cps. The output of the first binary of the scaler was then observed. With the scope set on internal sweep trigger, the minimum time between trigger of sweep and change to the second stable voltage level of the binary was noted to be 5.5 μs. This is good confirmation of the dead time measured using the double pulse generator.

APPENDIX II

Detailed Checkout Measurements

Table IX

Typical Discriminator Plateaus

Counter: N. C. Wood G10380, 40 cm. Hg BF_3 Counter Voltage: 1850 v
Amplifier Bandwidth: Delay Line
Amplifier Coarse Gain: 64
Amplifier Fine Gain: 1

Counts Per Second
Date or Series

Discriminator Level, volts	8/9/57	B	E	N	T	Y
100	12	12	15	19	37	44
90	24	28	56	42	79	80
80	44	137	174	107	210	217
70	315	317	287	569	1792	736
60	761	559	475	1920	3119	4023
50	1032	842	673	2717	3508	5933
40	1253	1173	892	3301	4358	7471
30	1326	1526	1151	3632	4908	8385
20	1468	1665	1445	4053	5526	9101
10	1525	1803	1659	4095	5278	9235

Table X

Random Source Statistics

100 Gates Per Counte

Gate Time = .100 sec. for all counts except as noted

Date or Series	$\bar{c}$	s^2	$s^2/\bar{c}$	Date or Series	$\bar{c}$	s^2	$s^2/\bar{c}$
8/8	36.9	35.1	0.95	L, N, and O	47.4	45.1	0.95
8/8	32.2	28.0	0.87	L, N, and O	47.5	43.9	0.93
8/8	31.8	32.2	1.01	9/3	61.5	49.1	0.80
8/8	30.7	30.8	1.00	9/3	63.1	66.5	1.05
8/9	50.6	39.9	0.79	9/3	63.2	60.3	0.95
8/9	40.7	46.1	1.13	9/3	63.6	68.0	1.07
8/13	51.9	53.4	1.03	9/3	62.3	60.7	0.97
8/13	34.3	34.0	0.99	P	41.7	41.5	1.00

Table X (continued)

Date or Series	$\bar{c}$	s^2	$s^2/\bar{c}$	Date or Series	$\bar{c}$	s^2	$s^2/\bar{c}$
A and B	46.5	49.9	1.07	P	41.7	42.2	1.01
A and B	47.7	45.7	0.96	Q	40.3	36.3	0.90
C and D	39.7	44.6	1.12	Q	42.9	38.3	0.89
C and D	37.3	48.0	1.29	QQ	43.0	40.4	0.94
E	36.2	39.6	1.09	QQ	44.2	35.3	0.80
E	35.8	32.9	0.92	R and S	43.3	43.3	1.00
F and G	42.7	41.3	0.97	RandS	43.9	36.5	0.83
F and G	42.0	45.7	1.09	UandT	39.5	34.3	0.87
H and I	49.3	44.3	0.90	U andT	40.3	34.3	0.85
H and I	47.9	38.3	0.80	V	29.1	29.3	1.01
J	43.7	51.6	1.18	V	28.5	24.0	0.84
J	43.2	38.5	0.89	9/11	30.1	23.9	0.80
K	49.6	47.4	0.96	9/11	30.4	28.7	0.94
K	51.7	55.0	1.06	W*	28.6	25.8	0.90
K	49.4	49.0	0.99	W*	27.4	22.8	0.83
				W*	27.6	25.3	0.92
L, N. and O	48.5	58.7	1.21	X and Y**	37.3	35.3	0.95
L, N, and O	46.8	65.9	1.41	X and Y**	36.8	47.6	1.29
L, N, and O	48.0	50.0	1.04				

*Gate = 1.000 sec.
**Gate = .030 sec.

Table XI

Random Source Statistics
About 525 Gates Per Count

Date or Series	Gate Time sec.	$\bar{c}$	s^2	$s^2/\bar{c}$
K	.001	3.751	3.979	1.061
K	.002	7.652	7.453	0.974
K	.003	11.23	11.31	1.007
K	.004	14.95	14.34	0.959
K	.005	18.41	17.82	0.968
K	.006	22.29	21.48	0.964
K	.008	29.48	28.73	0.974
K	.010	37.30	34.04	0.913
K	.013	47.69	41.48	0.870
K	.017	62.45	57.43	0.920
K	.020	72.81	69.52	.955
K	.030	109.5	120.5	1.101
K	.050	182.3	178.9	0.981
K	1.000	3639	3946	1.084
L, N and O	0.100	47.68	52.61	1.103
P	0.100	41.25	46.10	1.118
9/3	0.100	62.87	62.26	0.990
Q	0.100	41.92	39.65	0.946
QQ	0.100	43.86	42.65	0.972
R and S	0.100	43.92	45.56	1.037
U and T	0.100	40.78	42.43	1.041
V	0.100	28.87	28.99	1.004
9/11	0.100	30.41	29.33	0.965
W	1.000	27.75	25.98	0.936
X and Y	0.030	37.27	39.52	1.060

APPENDIX III

Results of Machine Computation of $\bar{c}$ s^2, and $s^2/\bar{c}$

Table XII

Date: 8/9/57 Series: —

Count Number	(1-K)	Source	$\bar{c}$	$s^2/\bar{c}$
1	2.11×10^{-2}	in	127	1.61
2	1.78×10^{-2}	in	143	2.11
3	1.05×10^{-2}	in	234	2.33
4	9.23×10^{-3}	in	262	2.36
5	7.03×10^{-3}	in	325	2.48
6	5.79×10^{-3}	in	391	3.70
7	4.46×10^{-3}	in	482	3.01
8	3.56×10^{-3}	in	596	3.87
9	2.30×10^{-3}	in	890	3.79
10	1.36×10^{-3}	in	1446	4.43
10'	1.36×10^{-3}	out	513	5.86
11	8.92×10^{-4}	out	722	5.48
12	5.01×10^{-4}	out	1260	6.39
13	2.98×10^{-4}	out	2126	4.09

Table XIII

Date: 8/13/57 Series: –

Gate Time ms	$\bar{c}$	$s^2/\bar{c}$
1	4.75	1.41
2	11.08	1.55
3	15.33	1.64
4	19.33	1.72
5	25.61	2.47
6	29.07	2.36
7	33.50	2.50
8	40.98	2.59
9	44.97	2.83
10	49.24	3.01
20	100.7	4.98
50	252.1	3.94
100	502.1	3.81
500	2511	4.05
1000	4986	5.10

Table XIV

Date: 8/15/57 Series: A

Gate Time ms	$\bar{c}$	$s^2/\bar{c}$
1	1.178	1.14
2	2.317	1.32
3	3.386	1.26
5	5.900	1.83
6	7.693	1.74
8	9.683	2.24
10	12.33	2.27
13	15.27	2.01
17	19.23	1.98
20	24.14	2.14
30	33.73	1.82
50	59.52	2.10
100	120.4	1.74
500	593.0	2.07
1000	1183	2.23

Table XV

Date: 8/15/57 Series: B

Gate Time ms	$\bar{c}$	
1	1.752	1.169
2	3.416	1.963
3	5.752	2.133
5	8.861	1.871
6	11.16	2.179
8	15.17	2.049
10	18.38	2.012
13	23.69	1.650
17	31.83	2.284
20	37.25	2.325
30	52.50	2.379
50	90.58	2.077
100	185.4	1.947
500	914.6	2.188
1000	1833	2.372

Table XVI

Date: 8/16/57 Series: C

Gate Time ms	$\bar{c}$	$s^2/\bar{c}$
1	1.188	1.224
2	2.624	1.737
3	3.604	1.760
5	6.130	2.157
6	7.594	1.528
8	10.55	1.126
10	13.52	1.733
13	15.45	1.965
17	21.86	1.983
20	24.62	2.187
30	35.91	1.739
50	62.38	2.516
100	124.1	2.194
500	616.1	1.715
1000	1235	2.589

Table XVII

Date: 8/16/57 Series: D

Gate Time ms	$\bar{c}$	$s^2/\bar{c}$
1	1.733	1.395
2	3.109	1.884
3	4.693	1.810
5	8.772	2.207
6	9.495	1.623
8	13.49	1.632
10	16.47	2.118
13	19.96	2.039
17	26.43	1.990
20	31.66	1.831
30	48.15	1.860
50	81.41	2.449
100	162.9	1.983
500	816.3	2.469
1000	1637	2.329

Table XVIII

Date: 8/19/57 Series: E

Gate Time ms	$\bar{c}$	$s^2/\bar{c}$
1	1.980	1.464
2	4.396	1.684
3	6.703	1.353
5	11.17	2.214
6	14.35	1.968
8	19.40	1.744
10	22.64	2.564
13	28.94	2.611
17	38.91	2.718
20	46.08	2.449
30	69.16	2.259
50	114.2	2.412
100	228.0	2.203
500	1143	2.946
1000	2297	2.445

Table XIX

Date: 8/20/57		Series: F
Gate Time ms	$\bar{c}$	$s^2/\bar{c}$
1	3.188	1.491
2	5.900	1.591
3	8.653	1.820
5	14.65	2.421
6	18.78	1.856
8	24.08	2.091
10	31.75	2.268
13	39.47	3.376
17	52.84	2.472
20	58.97	2.935
30	91.49	3.460
50	151.4	3.057
100	310.1	3.123
500	1524	3.674
1000	3050	5.924

Table XX

Date: 8/20/57		Series: G
Gate Time ms	$\bar{c}$	$s^2/\bar{c}$
1	5.020	1.171
2	9.465	1.362
3	15.45	2.030
5	24.98	2.729
6	29.20	2.291
8	39.58	3.968
10	52.62	2.238
13	65.83	3.747
17	84.40	3.552
20	97.83	4.393
30	146.0	3.290
50	243.8	2.888
100	485.7	3.606
500	2479	3.894
1000	4930	4.374

Table XXI

Date: 8/21/57		Series: H
Gate Time ms	$\bar{c}$	$s^2/\bar{c}$
1	10.67	1.289
2	24.38	1.442
3	34.18	1.809
5	57.71	2.716
6	69.46	2.612
8	90.61	2.527
10	112.1	3.165
13	147.0	3.196
17	194.9	4.453
20	233.0	3.737
30	347.3	3.817
50	571.0	4.813
100	1143	5.344
500	5754	5.139
1000	11408	7.558

Table XXII

Date: 8/21/57		Series: I
Count Number	$\bar{c}$	$s^2/\bar{c}$
1	606.5	3.792
2	602.8	3.834
3	584.6	4.768
4	587.4	3,570
5	594.9	3.717
6	590.8	3.870
7	598.2	4.778
8	595.0	3.403
9	593.9	4.467
10	592.1	4.065
11	586.5	4.922

Table XXIII

Date: 8/22/57 Series: J

Count Number	(1-k)	$\bar{c}$	$s^2/\bar{c}$
1	2.47×10^{-2}	116.6	2.240
2	1.97×10^{-2}	137.3	1.687
2	1.43×10^{-2}	170.6	2.082
3	1.17×10^{-2}	224.0	2.302
4	8.73×10^{-3}	259.1	2.918
5	6.20×10^{-3}	331.5	3.343
6	5.40×10^{-3}	381.8	3.394
7	4.15×10^{-3}	472.2	3.894
8	3.25×10^{-3}	576.9	4.228
9	2.67×10^{-3}	676.0	4.447
10	2.30×10^{-3}	789.5	3.253
11	1.75×10^{-3}	1001	4.835
12	1.27×10^{-3}	1384	4.685
13	1.05×10^{-3}	1649	5.436
14	8.3×10^{-4}	2110	3.503
15	6.0×10^{-4}	2874	4.568
16	4.0×10^{-4}	3935	3.656

Table XXIV

Date: 8/29/57 Series: K

Gate Time ms	$\bar{c}$	$s^2/\bar{c}$
1	3.751	1.061
2	7.652	0.970
3	11.23	1.007
4	14.95	0.959
5	18.41	0.968
6	22.29	0.964
8	29.48	0.974
10	37.30	0.913
13	47.69	0.870
17	62.45	0.920
20	72.81	0.955
30	109.5	1.101
50	182.3	0.981
1000	3639	1.084

Table XXV

Date: 8/30/57		Series: L
Gate Time ms	$\bar{c}$	$s^2/\bar{c}$
1	12.44	1.454
2	24.18	1.801
3	36.21	2.166
4	47.94	2.705
5	61.02	2.654
6	72.58	3.001
7	83.84	3.107
8	94.71	3.054
9	108.9	3.380
10	123.2	3.426
15	182.9	4.778
20	241.0	4.447
30	366.0	5.388
50	601.6	4.625
70	845.3	5.269
100	1213	5.143
200	2390	6.023
300	3625	4.861
500	6021	6.106
700	8446	6.516
1000	12052	7.024

Table XXVI

Date: 8/30/57		Series: N
Gate Time ms	$\bar{c}$	$s^2/\bar{c}$
1	3.707	1.623
2	7.424	1.650
3	10.98	2.072
4	15.04	2.040
5	18.69	2.404
6	22.16	2.937
7	27.10	2.659
8	29.99	2.580
9	34.22	3.114
10	37.42	2.838
15	57.09	3.412
20	73.75	3.451
30	111.8	3.548
50	187.8	3.183
70	265.8	3.989
100	373.4	3.693
200	750.9	4.055
300	1128	4.037
500	1875	3.893
700	2625	4.102
1000	3746	4.200

Table XXVII

Date: 8/30/57		Series: O
Gate Time ms	$\bar{c}$	$s^2/\bar{c}$
1	2.916	1.487
2	5.510	1.833
3	8.448	1.862
4	11.27	1.943
5	13.74	2.203
6	16.35	2.360
7	19.04	2.648
8	21.49	2.666
9	24.03	2.851
10	27.59	2.495
15	40.60	2.872
20	53.60	2.849
30	80.79	2.758
50	135.6	2.983
70	191.0	2.848
100	270.8	3.073
200	543.4	2.982
300	813.1	3.279
500	1353	3.241
700	1888	3.388
1000	2697	3.407

Table XXVIII

Date: 9/4/57		Series: P
Gate Time ms	$\bar{c}$	$s^2/\bar{c}$
1	5.687	1.481
2	11.43	1.974
3	16.91	2.518
4	22.46	2.941
5	28.70	2.929
6	33.49	3.007
7	38.56	3.411
8	43.38	3.134
9	48.98	3.658
10	55.15	3.546
15	80.97	3.937
20	108.9	3.864
30	164.8	4.454
50	271.3	4.638
70	378.2	4.579
100	540.3	5.036
200	1077	4.653
300	1619	4.662
500	2700	5.662
700	3818	5.918
1000	5417	5.972

Table XXIX

Date: 9/5/57		Series: Q
Gate Time ms	$\bar{c}$	$s^2/\bar{c}$
1	4.729	1.630
2	8.703	2.056
3	13.48	2.918
4	17.76	3.335
5	21.53	3.830
6	27.07	4.290
7	31.35	4.324
8	36.22	4.746
9	41.22	4.500
10	44.12	4.233
15	66.15	4.816
20	87.84	5.711
30	130.4	5.602
50	219.4	6.247
70	309.5	6.306

Table XXX

Date: 9/5/57		Series: QQ
Gate Time ms	$\bar{c}$	$s^2/\bar{c}$
1	4.123	1.770
2	8.416	2.338
3	12.30	2.760
4	16.87	3.426
5	20.80	3.737
6	25.02	4.088
7	29.66	4.275
8	32.98	4.030
9	38.16	4.619
10	40.40	4.803
15	63.64	5.204
20	82.81	5.161
30	124.2	5.857
50	211.1	5.878
70	293.2	6.040
100	417.6	6.413
200	838.3	6.105
300	1260	6.094
500	2083	7.084
700	2926	7.262
1000	4186	6.905

Table XXXI

Date: 9/6/57		Series: R
Gate Time ms	$\bar{c}$	$s^2/\bar{c}$
1	5.426	1.919
2	10.59	2.547
3	15.72	3.473
4	20.95	3.710
5	26.11	4.204
6	31.03	4.429
7	35.73	4.613
8	41.72	5.537
9	47.40	5.426
10	52.36	6.262
15	77.59	7.533
20	102.8	6.814
30	159.3	8.185
50	258.9	8.492
70	364.7	8.599
100	522.5	8.573
200	1037	9.158
300	1567	8.396
500	2610	9.559
700	3650	8.825
1000	5195	9.677

Table XXXII

Date: 9/6/57		Series: S
Gate Time ms	$\bar{c}$	$s^2/\bar{c}$
1	4.222	2.047
2	8.540	3.463
3	12.48	4.320
4	17.68	6.015
5	21.15	5.546
6	25.94	6.703
7	29.76	7.320
8	36.50	7.536
9	38.70	7.525
10	44.49	8.933
15	68.05	10.772
20	88.69	11.303
30	130.7	12.321
50	215.6	14.274
70	308.8	13.432
100	437.8	14.838
200	867.4	15.523
300	1291	15.150
500	2183	16.578
700	3062	19.234
1000	4417	15.540

Table XXXIII

Date: 9/9/57		Series: T
Gate Time ms	$\bar{c}$	$s^2/\bar{c}$
1	4.905	2.852
2	9.378	4.718
3	14.90	6.193
4	19.80	7.225
5	25.20	9.242
6	29.11	9.245
7	34.69	11.595
8	38.56	11.000
9	45.09	11.812
10	50.31	13.796
15	73.58	15.157
20	95.00	17.120
30	148.9	22.071
50	239.0	20.902
70	352.4	25.009
100	487.1	25.074
200	982.2	27.009
300	1477	25.493
500	2415	32.704
700	3398	36.895
1000	4806	38.491

Table XXXIV

Date: 9/9/57		Series: U
Gate Time ms	$\bar{c}$	$s^2/\bar{c}$
1	5.163	2.654
2	10.53	4.489
3	15.65	6.420
4	22.16	7.657
5	25.84	9.292
6	29.30	10.462
7	35.66	13.000
8	36.67	11.730
9	42.89	16.334
10	51.15	15.612
15	77.44	16.688
20	97.89	21.831
30	150.1	22.944
50	273.9	32.206
70	350.22	30.026
100	537.3	34.747
200	1061	39.927
300	1600	35.334
500	2627	41.622
700	3606	51.591
1000	5197	48.944

Table XXXV

Date: 9/10/57		Series: V
Gate Time ms	$\bar{c}$	$s^2/\bar{c}$
1	5.598	1.583
2	10.43	2.349
3	15.84	2.992
4	21.79	3.746
5	28.30	3.918
6	33.51	4.493
7	40.38	4.256
8	47.16	5.341
9	49.06	6.003
10	51.08	6.699
15	78.49	7.306
20	96.03	8.838
30	157.4	10.302
50	284.3	11.593
70	390.5	13.189
100	549.5	12.668
200	1095	14.822
300	1575	14.320
500	2715	17.766
700	3661	25.279
1000	5475	19.529

Table XXXVI

DAte: 9/12/57		Series: W
Gate Time ms	$\bar{c}$	$s^2/\bar{c}$
1	5.041	1.592
2	10.52	1.828
3	15.16	2.285
4	20.68	2.772
5	25.56	2.600
6	30.27	3.534
7	34.81	3.064
8	39.55	3.407
9	45.18	3.700
10	50.48	3.602
15	74.00	4.170
20	101.2	4.748
30	148.9	5.354
50	253.2	5.227
70	353.1	5.607
100	509.3	6.264
200	1008	6.561
300	1516	6.935
500	2511	6.784
700	3515	6.843
1000	5060	8.011

Table XXXVII

Date: 9/13/57		Series: X
Gate Time ms	$\bar{c}$	$s^2/\bar{c}$
1	3.749	1.629
2	7.511	1.861
3	11.03	1.914
4	15.56	2.228
5	19.22	2.576
6	22.07	1.999
7	25.57	2.912
8	30.36	2.704
9	34.27	2.735
10	37.54	2.735
15	57.88	3.151
20	75.24	2.499
30	112.6	3.000
50	187.5	3.029
70	260.0	3.185
100	376.0	3.430
200	755.2	3.320
300	1135	3.405
500	1879	3.289
700	2626	3.508
1000	3741	4.111

Table XXXVIII

Date: 9/13/57 Series: Y

Gate Time ms	$\bar{c}$	$s^2/\bar{c}$
1	7.421	1.418
2	15.000	1.686
3	22.64	2.246
4	29.44	2.361
5	37.59	2.543
6	44.23	2.932
7	53.10	2.742
8	59.73	2.890
9	67.21	2.915
10	74.62	3.147
15	110.4	3.512
20	148.2	3.740
30	224.7	3.580
50	371.0	4.215
70	519.4	4.195
100	750.3	4.516
200	1504	4.335
300	2245	4.887
500	3742	4.803
700	5236	4.792
1000	7427	5.049

APPENDIX IV

GRAPHS OF THEORY FIT

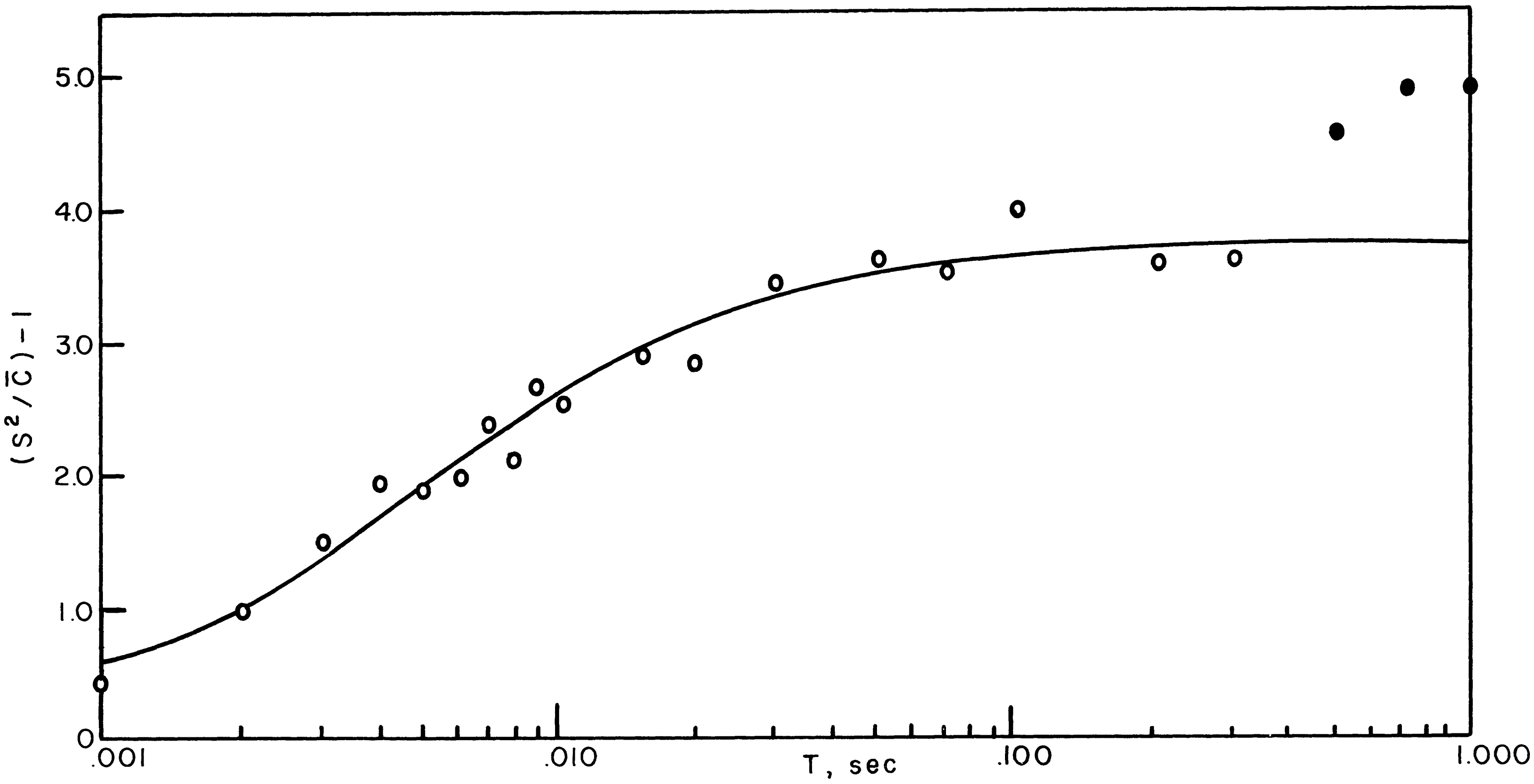

Figure 12   Gate Traverse   Series P

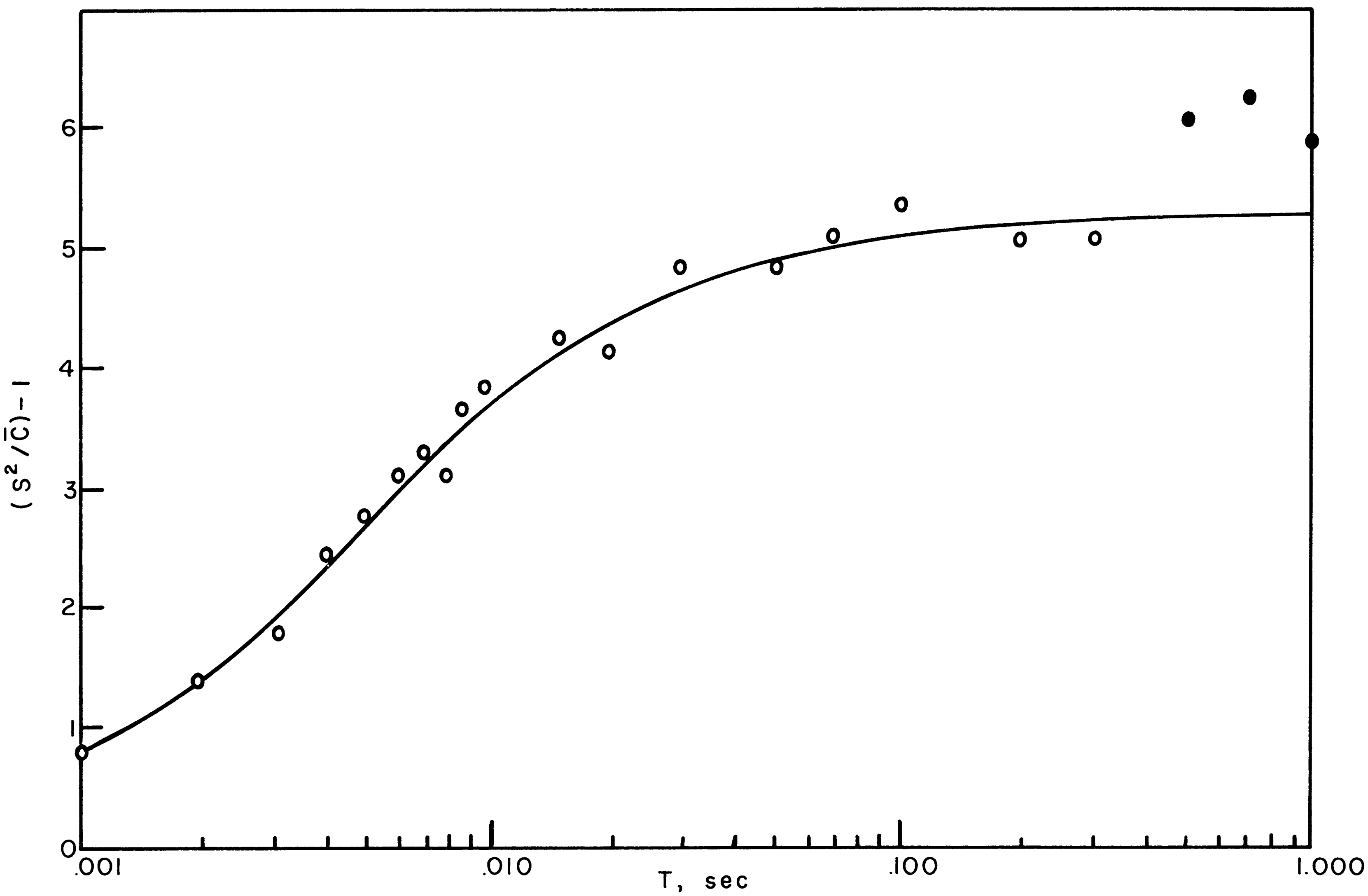

Figure 13 Gate Traverse Series QQ

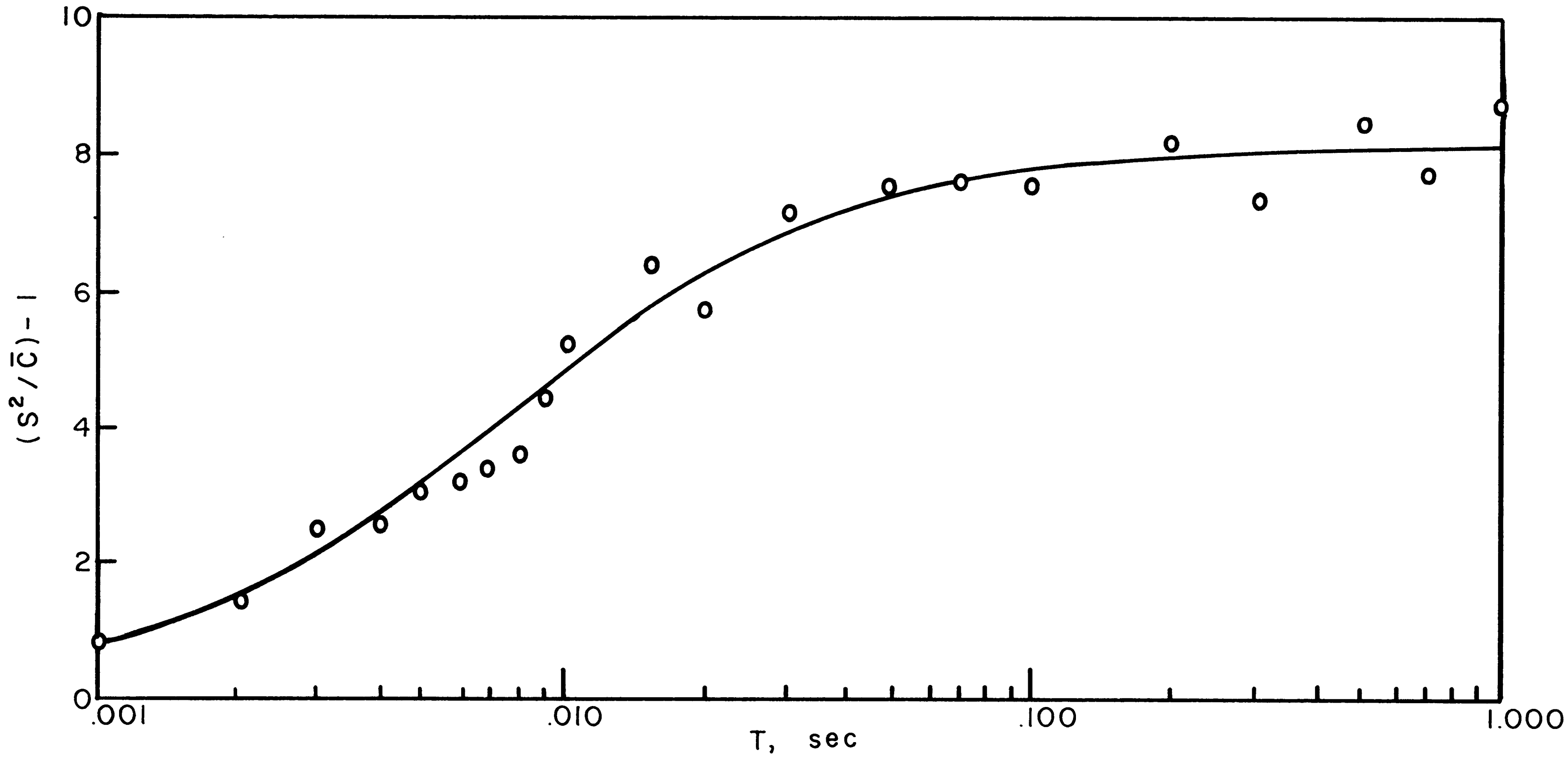

Figure 14 Gate Traverse Series R

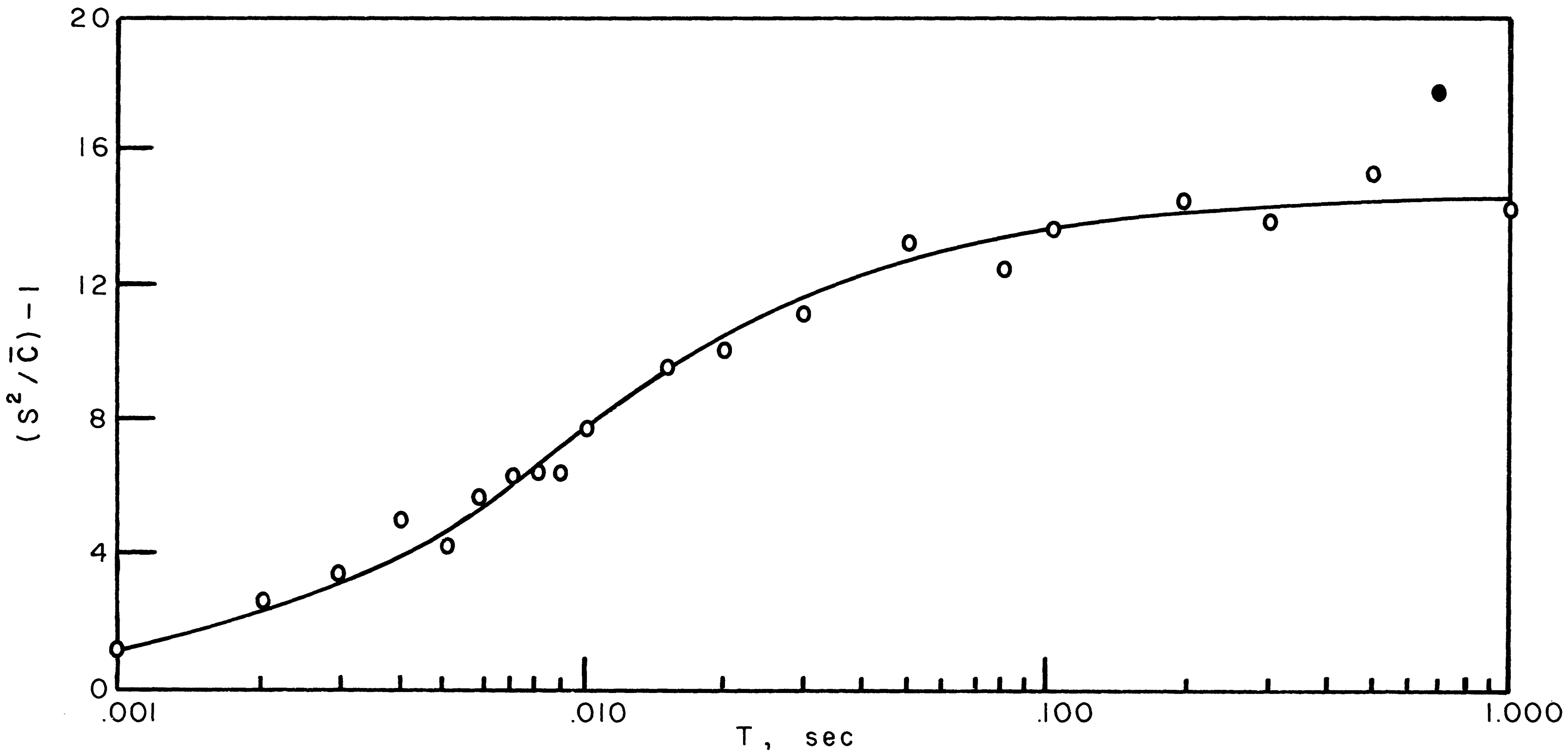

Figure 15 Gate Traverse Series S

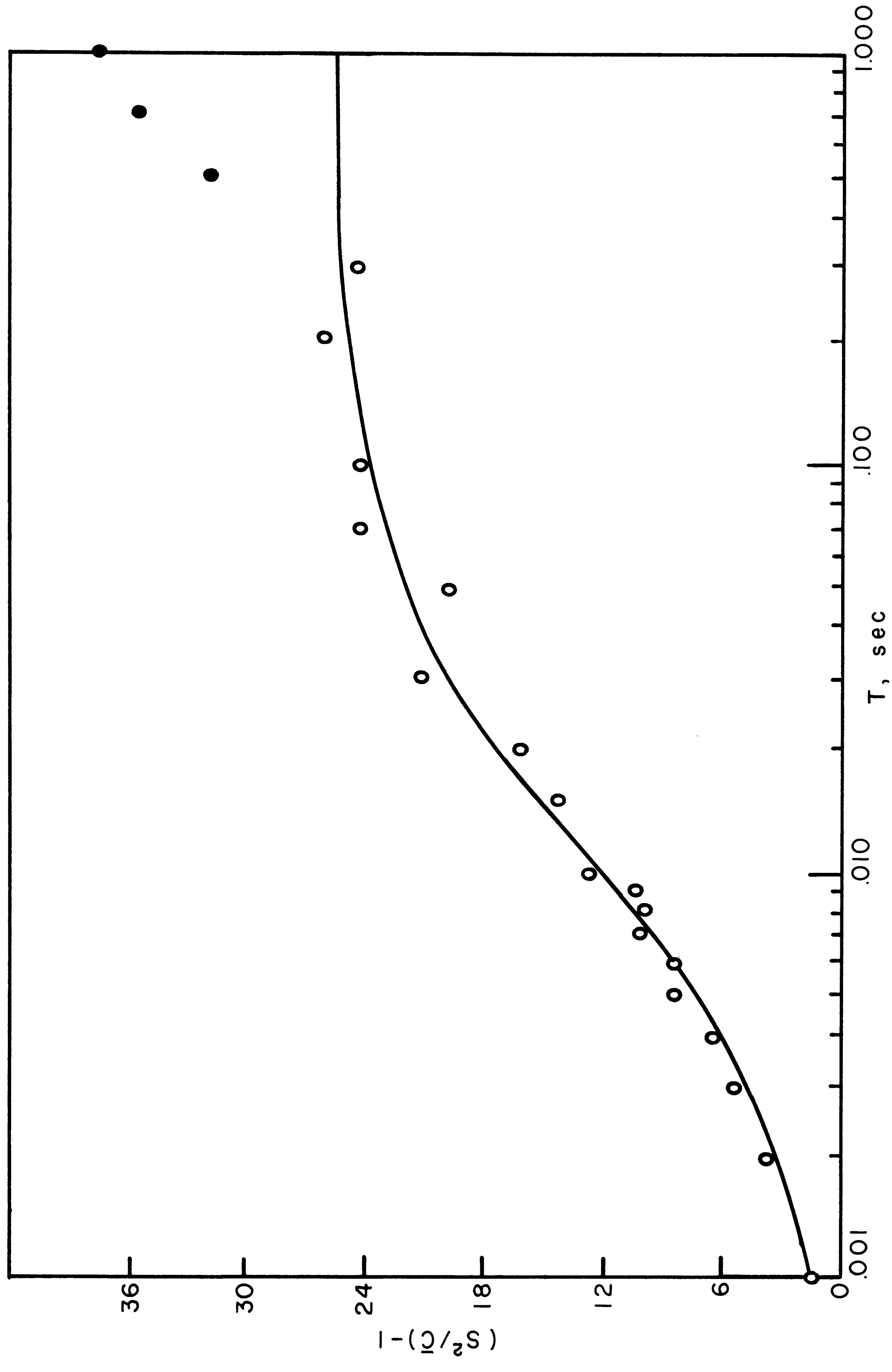

Figure 16 Gate Traverse Series T

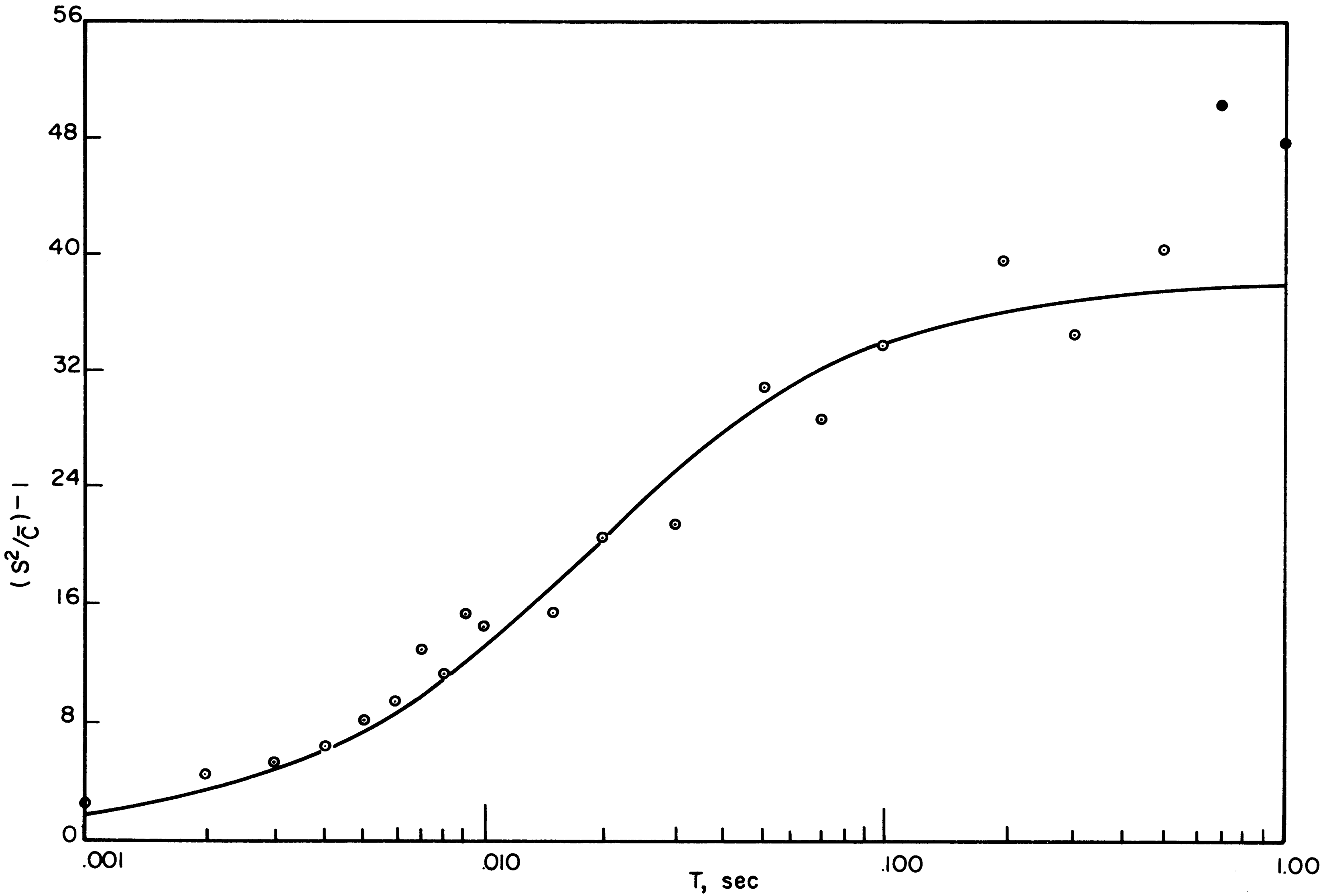

Figure 17 Gate Traverse Series U

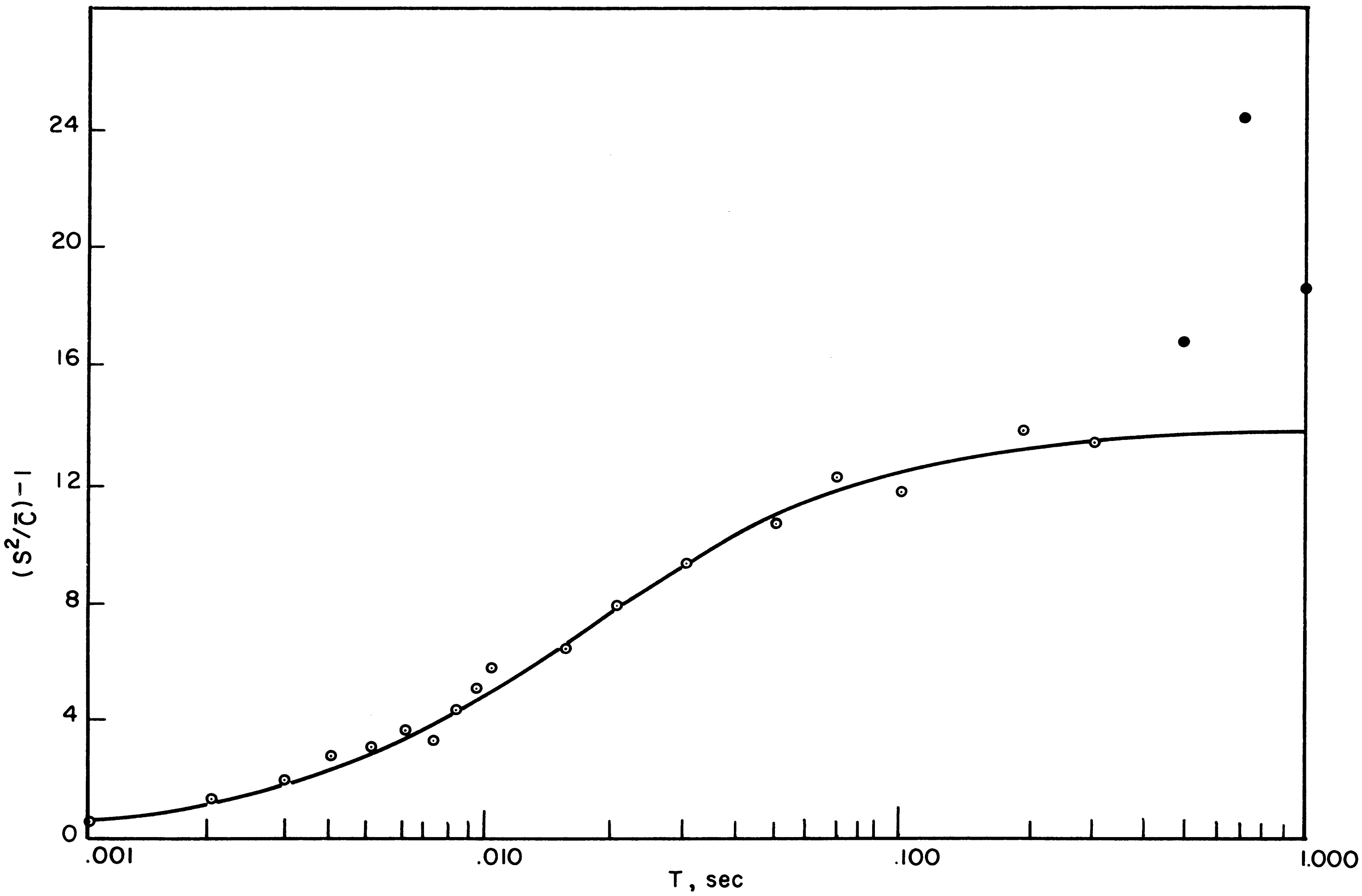

Figure 18 Gate Traverse Series V

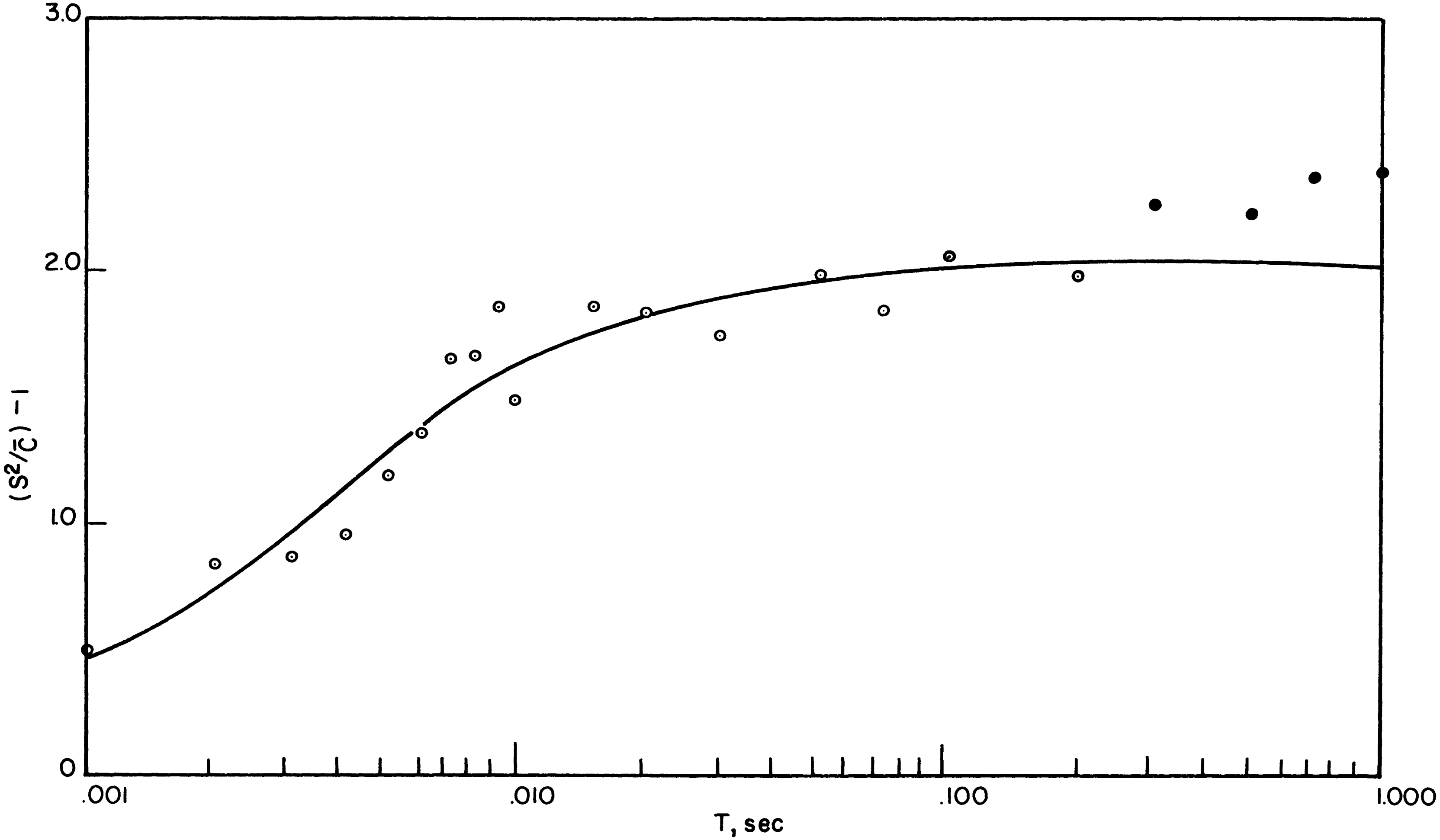

Figure 19 Gate Traverse Series 0

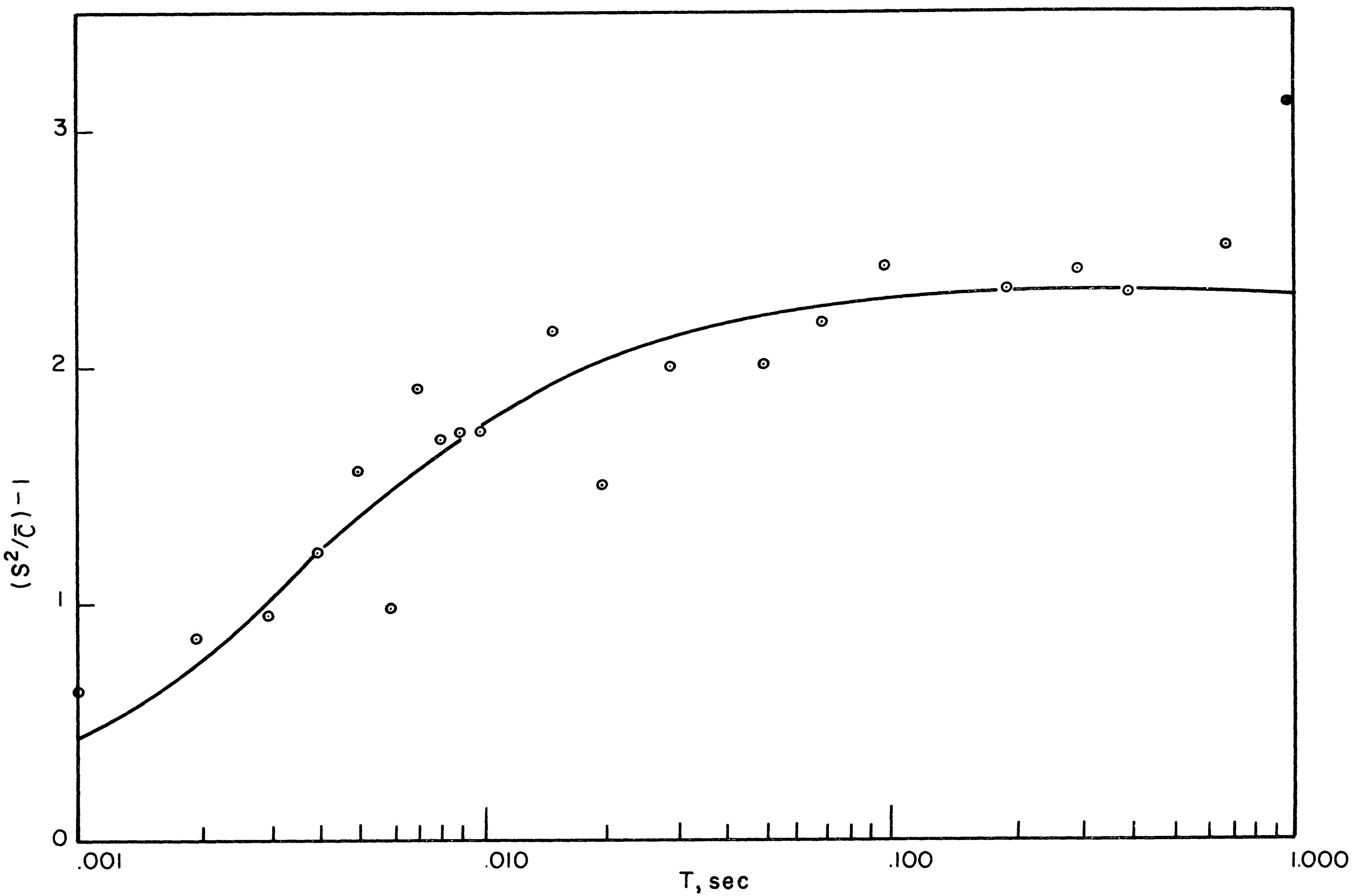

Figure 20 Gate Traverse Series X

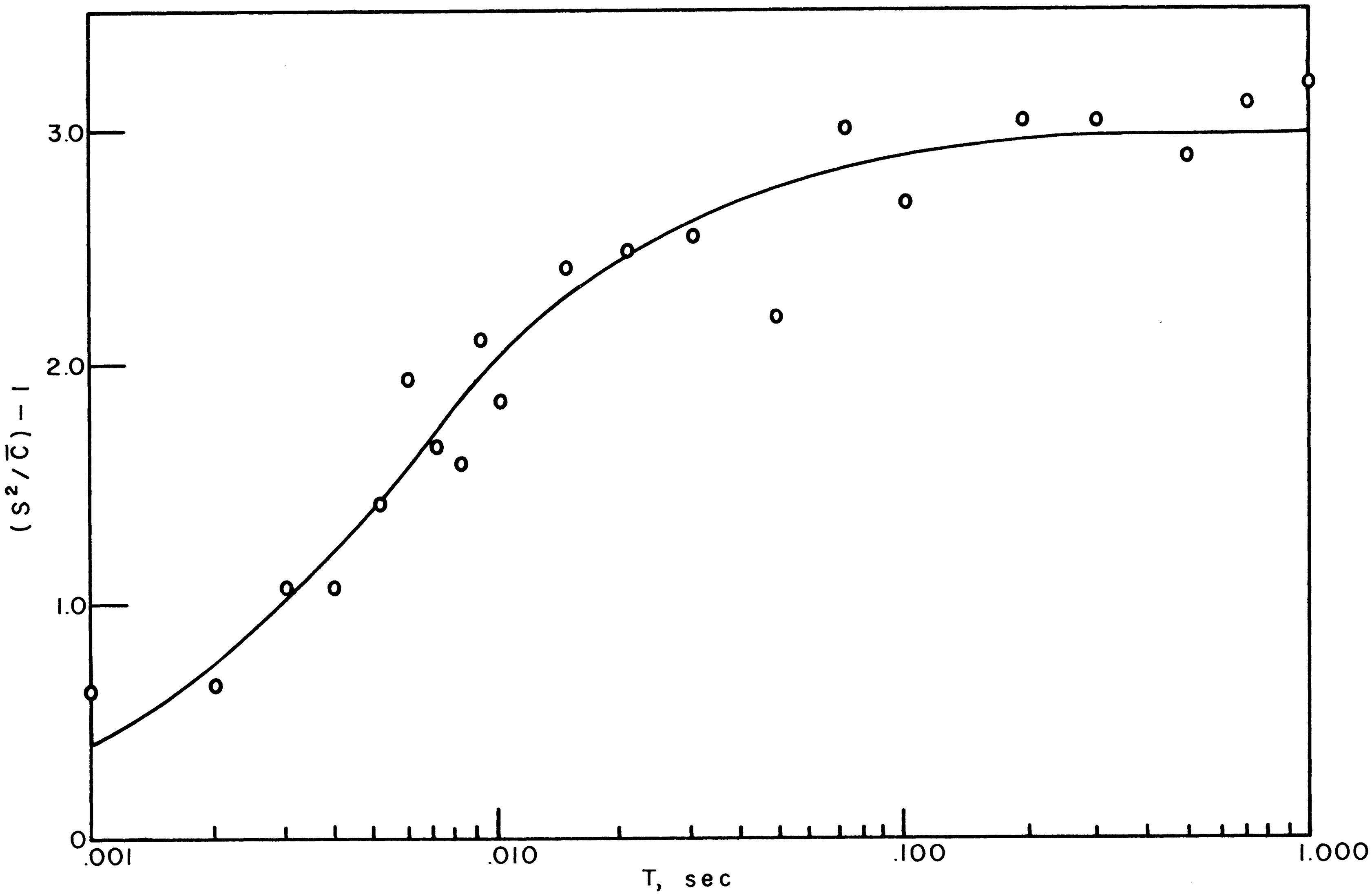

Figure 21 Gate Traverse Series N

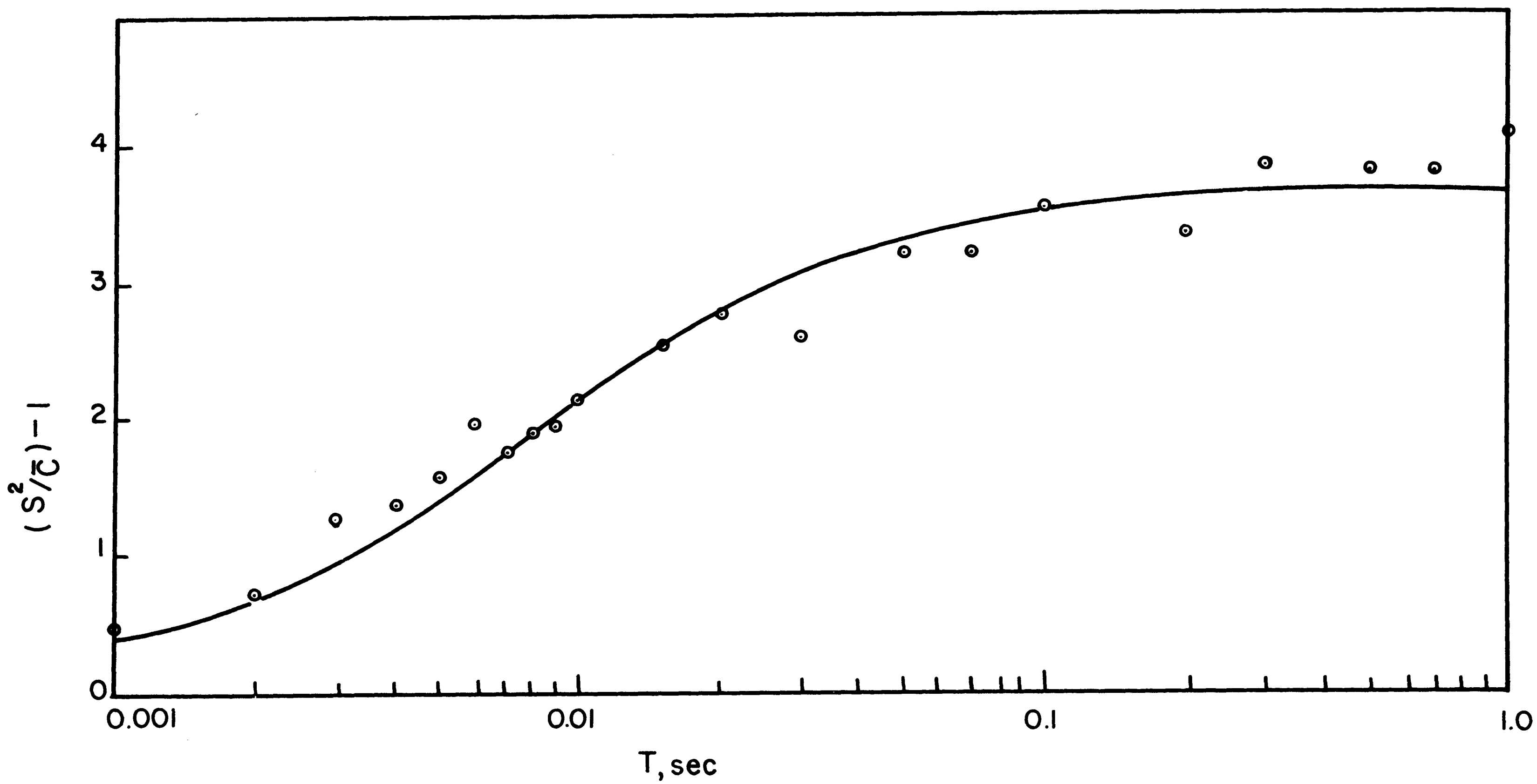

Figure 22. Gate Traverse Series Y

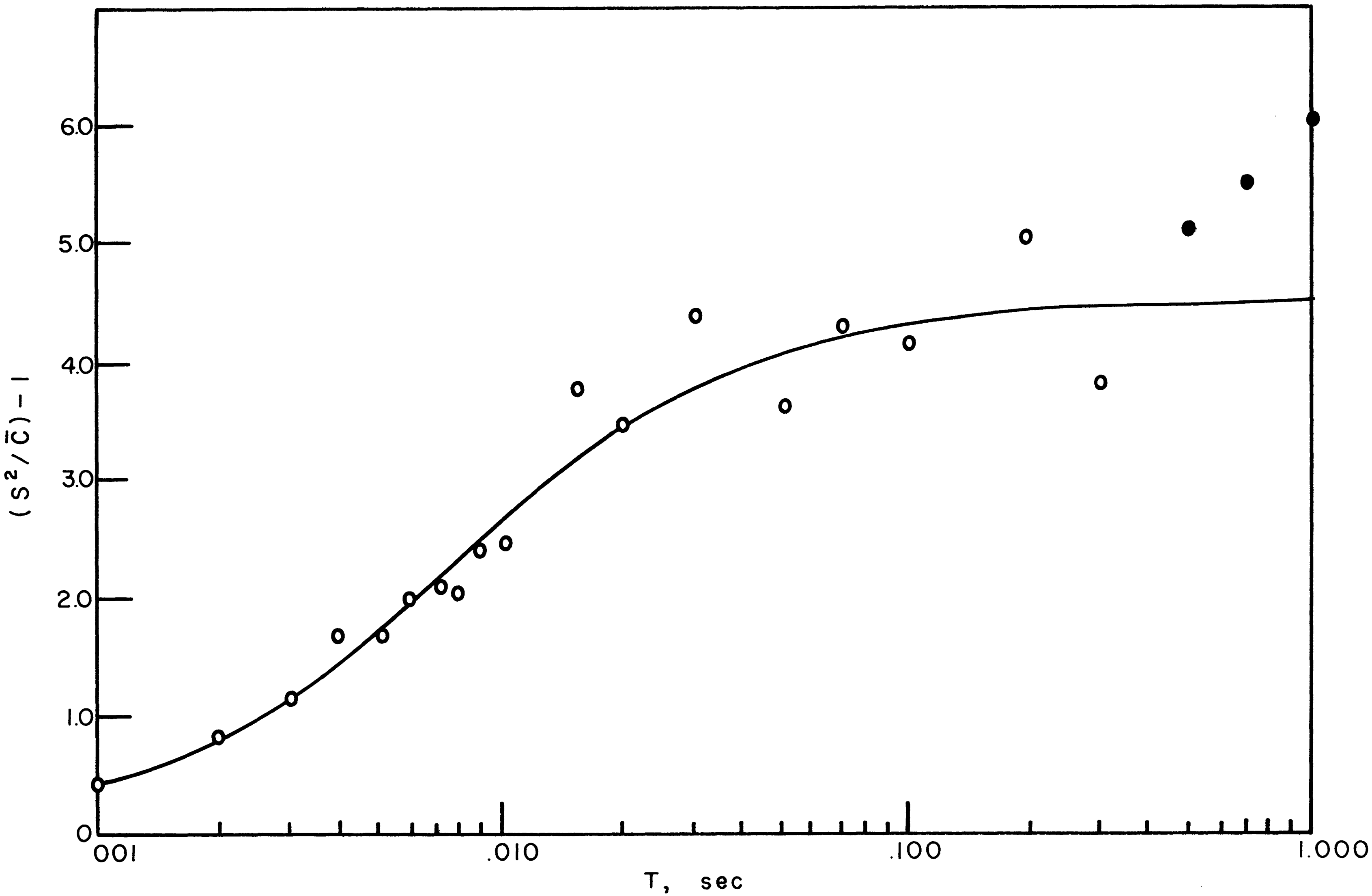

Figure 23 Gate Traverse Series L

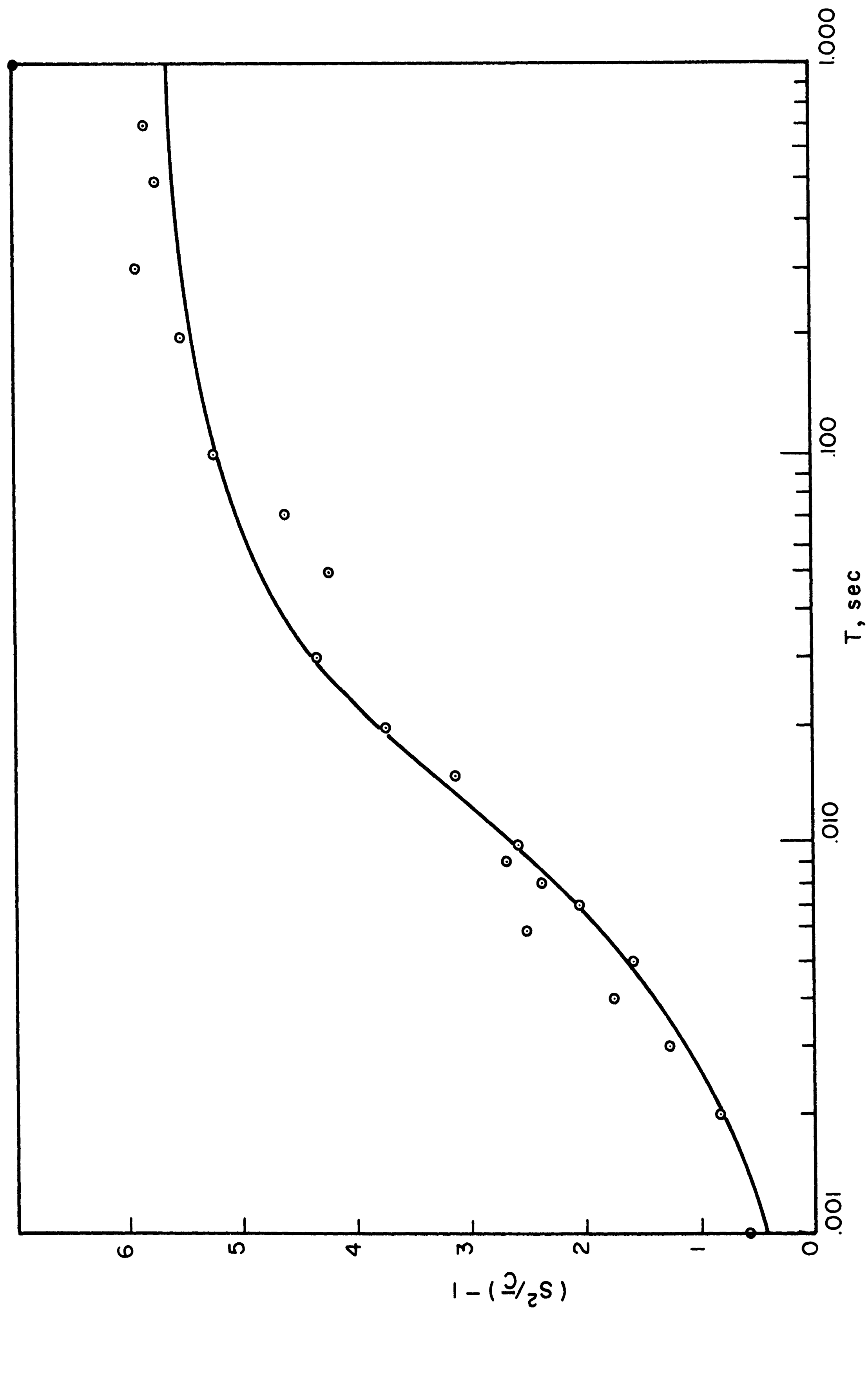

Figure 24 Gate Traverse Series W

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NOMENCLATURE

a	prompt neutron period (Rossi alpha) = $\frac{1-K'}{\tau}$
b	parameter
b_m	probability of event m
B	general probability generating function
c	neutron count
C	delayed neutron precursor population number
D	detector distribution probability generating function
E	detector efficiency, probability of a count in the detector per neutron lost from system
F	neutron production or loss rate, single generation probability generating function
G	general probability generating function
H	general probability generating function
j	index
k	coefficient of neutron period
K	total neutron reproduction factor
K'	prompt neutron reproduction factor = $K(1-\beta)$
m	index
ms	milli-second
M_i	population mean of i distribution (Equation (53) in the limit of as n approaches infinity)
n	neutrons born or lost over time interval, index
N	neutron population number
P	probability
P_ν	probability for the release of ν prompt neutrons on fission
s	index
s^2	sample variance (Equation (54) with n finite)

NOMENCLATURE CONT'D

S	extraneous source strength, general probability generating function
S_I	sum of I terms
t	time, Student's t distributed variable
T	gate or counting time
T_d	delay time
Tm	measuring time
Tr	resolving time
U	generating function of the sum of the neutrons in a chain
V	probability generating function of number of neutrons emitted on fission
V_i	population variance of i distribution (Equation (54) in the limit as n approaches infinity)
V_i^T	population variance of i distribution measured with resolving or gate time T over infinite or long measuring time
$V_{i\,Tm}^{\,Tr}$	population variance of i distribution measured with resolving time Tr over measuring time Tm
W	$s^2/\overline{c} - 1$
X	random variable
x	variable
y	variable
β	effective fraction of delayed neutrons
γ	neutron period
Δt	time interval
λ	decay constant of delayed neutron precursor
μs	micro-second
ν	number of prompt neutrons released on fission
$\overline{\nu}$	population mean of the number of neutrons emitted in fission

NOMENCLATURE CONT'D

$\overline{\nu^2}$	population mean square of the number of neutrons emitted in fission
τ	mean prompt neutron lifetime
$(\lambda_i \tau)^*$	effective generation time