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Project "Wizard"

Aerodynamic Heating of a Supersonic Missile During Accelerated Flight

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INTRODUCTION

The air in the boundary layer of a high speed missile is heated by viscous retardation. The resulting high temperature of the missile skin is a controlling factor in the final choice of skin material and thickness. The importance of skin temperature problems in guided missile work was realized and a preliminary study was initiated in the summer of 1946. As a result of that study, University of Michigan Memorandum #3 was issued in March 1947 to provide a conservative estimate of the order of magnitude of the skin equilibrium temperature to be expected. Inasmuch as the equilibrium temperature is usually not realized in the case of accelerated flight, the method of the present report has been expanded to include the temperature lag due to the heat capacity of the skin. The development is based upon quasi-steady methods, because the influence of acceleration upon heat transfer is unknown.

Further, it has been noted that a different form of the Reynold's number was used in the original German work. This change greatly affects the numerical results of all calculation methods based upon the German tests.

The method of this report can be used to predict the maximum temperatures and rates of heating to be used for tests instudies of the strength of structural materials at elevated temperatures.

SUMMARY

The skin heating investigation which was begun in UMM-3 has been continued to include the transient effects of the heat capacity of the missile skin. In this report, the method of computation has been reduced to a standard form and the accuracy of prediction improved by rewriting the expression given in UMM-3 for the average heat transfer coefficient. The method of computation is completely developed in the Appendix. An example of the technique of computation is included, with a comparison with flight test measurements made on the V-2 missile at White Sands, N. M. The method gives good agreement with tests and, at this stage of progress, may be considered sufficient for the evaluation of skin temperatures. However, further experimental work is necessary before conclusions of satisfactory agreement and validity of assumptions can be derived.

SYMBOLS

A	=	Surface area, sq ft
a	=	Local speed of sound, ft per sec
c_p	=	Specific heat at constant pressure, BTU per lb per degree F
c_v	=	Specific heat at constant volume, BTU per lb per degree F
c_{skin}	=	Specific heat of skin material, BTU per lb per degree F
g	=	Acceleration of gravity, ft per sec per sec
h	=	Local heat transfer coefficient, BTU per sec per sq ft per degree F
$\bar{h}$	=	Average heat transfer coefficient, BTU per sec per sq ft per degree F
H	=	Altitude, ft
κ	=	Thermal conductivity, BTU per sec per ft per degree F
ℓ	=	Characteristic length, ft
M	=	Mach number, $\frac{v}{a}$
Nu	=	Nusselt number, $\frac{\ell}{\kappa} h$
Pr	=	Prandtl number $\frac{c_p \mu g}{\kappa}$
Q	=	Quantity of heat, BTU
Re	=	Reynolds number, $\frac{\rho v \ell}{\mu}$
T	=	Temperature, degrees Rankine
T_{amb}	=	Ambient temperature, degrees Rankine
T_{BL}	=	Boundary layer temperature, degrees Rankine
T_{skin}	=	Skin temperature, degrees Rankine
t	=	Time, sec
v	=	Velocity, ft per sec
w	=	Weight of skin material, lb per sq ft

β = Total cone angle degrees

γ = Ratio of specific heats, $\frac{c_p}{c_v}$

ε = Emissivity: Ratio of emissive power of a surface to that of a black body

μ = Coefficient of viscosity, slug per ft sec

ρ = Mass density, slug per cu ft

σ = Stefan-Boltzmann constant, 4.8×10^{-13} BTU per sec per sq ft per degree R^4

^oF = Degrees Fahrenheit

^oR = Degrees Rankine

DISCUSSION

In Report No. UMM-3 (Reference 1) the equilibrium temperature was defined as the temperature at which the heat radiated to space exactly equalled the heat received by the skin. For a missile in accelerated flight, the equilibrium temperature is usually not reached, due to the rather large heat capacity of the skin material. The transient temperature of the missile skin is then a function of the net heat added and the heat capacity of the skin material. An expression for the skin temperature can be found from the definition of the specific heat of the skin.

$$c_{skin} = \frac{\Delta Q}{\Delta T_{skin}} (wA)^{-1}$$

$$\Delta T_{skin} = \frac{\Delta Q}{Awc_{skin}}$$

where ΔT_{skin} is the change of skin temperature when an amount of heat ΔQ is added. Further, the transient condition can be expressed in the differential form,

$$\frac{dT_{skin}}{dt} = (Awc_{skin})^{-1} \frac{dQ}{dt} \qquad (1)$$

Now, for any flight path, an evaluation of the term $\frac{dQ}{dt}$ will be needed for a solution. At least six factors should be considered in the initial analysis.

1. Heat received through the boundary layer.
2. Heat lost by radiation to space.
3. Heat received by radiation from the sun.
4. Heat received by radiation from the earth.

5. Heat transmitted to the interior of the missile.

6. Heat received from the combustion chamber.

It is to be noted that all of the above factors can be taken into account in Equation (1). Factors (3) and (4) are, however, negligible by comparison with factors (1) and (2)*, and factors (5) and (6) must be omitted because a means for their evaluation has not been determined. It is conservative to assume that no heat is transmitted to the interior, while the neglect of factor (6) implies a well insulated combustion chamber.

It will be necessary, then, to evaluate the term $\frac{dQ}{dt}$ only for the heat received through the boundary layer and for the heat radiated to space. The Stefan-Boltzmann Law is used to find the amount of heat radiated.

$$\left\{\frac{\partial Q}{\partial t}\right\}_R = \sigma A \varepsilon T^4_{skin} \tag{2}$$

It will be desirable to radiate as much heat as possible, therefore, the missile should be painted or coated with a substance having a high emissivity. For this analysis, it has been assumed that $\varepsilon = 1$.

For the consideration of the heat received through the boundary layer, the heat transfer equation is written in the form,

$$\left\{\frac{\partial Q}{\partial t}\right\}_{BL} = Ah(T_{BL} - T_{skin}) \tag{3}$$

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* The maximum amount of heat received from the sun is found to be 0.11 BTU per sq ft per sec, from Reference 7. The heat received by radiation from the earth is found directly from the Stefan-Boltzmann Law to be 0.02 BTU per sq ft per sec. (Effective radiating temperature of the earth = 443°R)

The value of the heat transfer coefficient is found from the definition of the Nusselt number.

$$Nu = \frac{\ell}{K} h$$
$$h = \frac{K}{\ell} Nu \qquad (4)$$

Experiments by Eber (Reference 2) indicate that for turbulent flow over a cone

$$\overline{Nu} = 0.0224 \, \beta^{\frac{1}{3}} \, Re^{.80} \qquad (5)$$

where

$$Re = \frac{\rho_0 v_0}{\mu_{BL}}$$

$\overline{Nu}$ is understood to indicate the average value of the Nusselt number ahead of the place where the Reynold's number is computed. The expression for the Reynold's number in Equation 5 is based upon free stream values ρ_0 and v_0 (ahead of the leading shock), and the value of μ_{BL} taken at the boundary layer.* It was also reported in Reference 2 that if $\beta > 10°$, Equation 5 can be replaced by the expression

$$\overline{Nu} = 0.0107 \, Re^{.82}$$

where

$$Re = \ell \frac{\rho_1 v_1}{\mu_{BL}} \qquad (6)$$

Here the Reynold's number is based upon values of ρ_1 and v_1 behind the leading shock and the value of μ_{BL} again based upon the conditions within the boundary layer.

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* According to information received from Dr. Eber. In the past it has been generally assumed that the value of μ also was to be taken from the free stream condition.

When the average value of the Nusselt number is used in Equation 4, the heat transfer coefficient will be replaced by the average coefficient, $\bar{h}$. Consequently, any skin temperature which is computed by means of Equation 6 will represent an average temperature ahead of the point where the Reynold's number is computed.

Now the differential equation of skin heating can be written in the form of Equation 1 by the use of Equations 2 and 3.

$$\frac{dT_{skin}}{dt} = (wc_{skin})^{-1} \left\{ \bar{h} \, (T_{BL} - T_{skin}) - \sigma T^4_{skin} \right\} \qquad (7)$$

Upon integration, Equation 7 yields the value of the skin temperature as a function of the time. Unfortunately, Equation 7 cannot be integrated exactly, because T_{BL}, $\bar{h}$, and T_{skin} are all functions of time and the equation is non-linear.

REFERENCES

1. J. D. Schetzer and D. W. Lueck, "Curves for the Calculation of the Order of Magnitude of Skin Heating Due to Friction of a Missile in Steady Flight in the Atmosphere." UMM-3, University of Michigan, Ann Arbor, Michigan, March, 1947.

2. G. R. Eber, "Experimentelle Untersuchung der Bremstemperature und des Wärmüberganges an Einfachen Köpern bei Ueberschall Geschwindigkeit", Peenemünde, Archiv Nr. 66/57g.

3. J. H. Keenan and J. Kaye, "Thermodynamic Properties of Air." John Wiley and Sons, Inc., New York, 1945.

4. H. J. Unger, "Thermal Effects on High Speed, Plastic, Ogival Missiles". APL-TP-13. Applied Physics Laboratory, Johns Hopkins University, Silver Spring, Maryland. January 1945.

5. W. S. Aiken Jr., "Standard Nomenclature for Airspeeds with Tables and Charts for Use in Calculation of Airspeed". NACA-TN-1120, Langley Field, Virginia, September 1946.

6. C. N. Warfield, "Tentative Tables for the Properties of the Upper Atmosphere." NACA-TN-1200, Langley Field, Virginia, January 1947.

7. Croft, "Thermodynamics", McGraw Hill Book Company, Inc., New York, 1938.

APPENDIX

Heat Transferred Through the Boundary Layer

The rate of heat transfer through the boundary layer is found from Equation 3

$$\left\{\frac{\partial Q}{\partial t}\right\}_{BL} = Ah\,(T_{BL} - T_{skin}) \qquad (3)$$

where the heat transfer coefficient is expressed by

$$h = \frac{K}{\ell}\,Nu \qquad (4)$$

Using Eber's data, which is expressed in Equation 6, Equation 4 is rewritten in the form

$$\bar{h} = 0.0107\,\frac{K}{\ell}\,Re^{.82} \qquad (8)$$

where $\bar{h}$ represents the average value of the heat transfer coefficient ahead of the point where the Reynold's number is computed. Here it is important to note that it was implied, in the original German report, that the Reynold's number to be used with Equation 8 is to be based upon the values of ρ and v in the free stream behind the shock wave, <u>and the value of μ is to be found from conditions within the boundary layer</u>. Inasmuch as this fact was not expressly stated in the German report, the free stream value of the viscosity has been used almost exclusively in this country, wherever Eber's data has been used. The magnitude of the error introduced into the value of $\bar{h}$ will be of the order of the ratio of the viscosities of the air in the boundary layer and in the free stream.

Equation 8 can be put into a more useful form by expanding the Reynold's number

$$\bar{h} = 0.0107 \left\{ \frac{K}{\mu_{BL}\, g\, c_{p_{BL}}} \right\} c_{p_{BL}} \frac{\rho v g}{Re^{.18}} \qquad (8a)$$

Recognizing the term in brackets to be the reciprocal of the Prandtl number, it is found that

$$\bar{h} = 0.0107 \left\{ \frac{c_p}{Pr} \right\}_{BL} \left\{ \frac{\rho v g}{Re^{.18}} \right\} \qquad (8b)$$

From the data given in Reference 3 it is found that

$$\left\{ \frac{c_p}{Pr} \right\}_{BL} = 0.4$$

is a good approximation; therefore, Equation 8b can be written

$$\bar{h} = 0.138 \frac{\rho v}{Re^{.18}} \qquad (8c)$$

The values of $\bar{h}$ obtained using Equation 8c approach infinity near the tip of the missile and the method breaks down. Experiments reported by Unger (Reference 4) indicate that the actual temperatures will fall below the calculated values in the stagnation region. This effect was attributed to the low thermal conductivity of stagnant air.

Returning to the method of this analysis, it will be remembered that Equation 8 was intended to be used with free stream values of ρ and v behind the leading shock. These results can be generalized somewhat by assuming the change in the product ρv to be small through the shock wave. Then, only the ambient density and the flight velocity are needed to solve

Equation 8c. This simplification leads to an unconservative error. The error is, however, blanketed by the difference between the average temperature and the actual temperature at a given point of the missile. The coefficient $\bar{h}$ was defined as the average value ahead of the point at which the Reynold's number was computed. Because the skin temperature decreases toward the rear of the missile as the Reynold's number increases, the temperature at a given point is less than the average temperature ahead of that point.

The boundary layer temperature is needed for a complete solution of Equation 3. The expression is usually written in the form

$$T_{BL} = T_{amb} \left(1 + c \frac{\gamma - 1}{2} M^2\right)$$

The value c = 0.9 is well established by experiment.

The Heat Radiated to Space

The amount of heat radiated to space can be computed using Equation 2. Upon substituting the value for σ and assuming that $\varepsilon = 1$, it is found that

$$\left\{\frac{\partial Q}{\partial t}\right\}_R = 4.8(10^{-13})\, A T^4_{skin} \qquad (2a)$$

The Practical Integration of the Skin Heating Equation

It has already been noted that the differential equation of skin heating, Equation 7, cannot be integrated exactly. A stepwise integration can be performed by putting the equation in the incremental form.

$$\Delta T_{skin} = \frac{\Delta t}{wc_{skin}} \left\{\bar{h}_{av} \left(T_{BL_{av}} - T_{skin_{av}}\right) - 4.8(10^{-13}) T^4_{skin_{av}}\right\} \qquad (7a)$$

Here the flight path is broken up into several finite intervals for which average values, $(\ )_{av}$, are computed from the expressions previously determined. The change of temperature, ΔT_{skin}, is found for each interval and then added to the temperature at the end of the previous interval. The method will be demonstrated in the following example.

Example

Consider a V-2 missile flying in accordance with the flight path shown on Figure 1. The problem is to compute the transient skin temperture throughout the given path. The temperatures will be computed for a point 9.8 feet aft of the tip of the missile to conform with the position of the thermocouple used in the tests. At this point on the V-2, the steel skin is 0.5 mm thick. The quantities in Table I were computed according to the expressions already set up. Atmospheric data was taken from References 5 and 6, and the values of the thermodynamic coefficients where taken from Reference 3.

In Table I the flight path has been broken up into 10-second intervals. As an example of the technique of computation, Equation 7a will be used to compute the skin temperature at the end of 50 seconds of flight. It will be assumed that the temperature has been found to be 155°F (615°R) at the end of 40 seconds, from the computation of the previous interval.

From a plot of the values in Table I, select average values of T_{BL}, and $\bar{h}$, for the 40-50 second interval.

$$(T_{BL})_{av} = 750^{\circ}R$$

$$(h)_{av} = 0.0064 \ *$$

* If the error in the Reynold's number, which was noted in the development of the method, had been carried along; the value would be 0.0093.

and further assume

$$(T_{skin})_{av} = 650^{o}R$$

as a probable value, inasmuch as the temperature at the end of 40 seconds was 615 ^{o}R and the skin temperature must increase because the boundary layer temperature still exceeds the skin temperature.

The coefficient $\frac{\Delta t}{wc_{skin}}$ is computed from the physical properties of the skin and the length of the time interval.

$$\frac{\Delta t}{wc_{skin}} = 112$$

Then Equation 7b becomes

$$\Delta T_{skin} = 112 \left\{ 0.0064 \ (750\text{-}650) - 4.8(10^{-13})(650)^4 \right.$$

$$\Delta T_{skin} = 71^{o}$$

It is necessary to check the accuracy of the assumption for the average skin temperature, which was made before.

$$(T_{skin})_{av} = (T_{skin})_{40} + \tfrac{1}{2}\Delta T_{skin}$$

$$(T_{skin})_{av} = 651^{o}R$$

Having established the validity of the assumption, the temperature at the end of 50 seconds of flight is found directly from the expression

$$(T_{skin})_{50} = (T_{skin})_{40} + \Delta T_{skin}$$

$$(T_{skin})_{50} = 687^{o}R \quad (227^{o}F)$$

Comparison with Experiment

The process outlined in the previous section has been carried out for the complete flight path shown on Figure 1. The resulting skin temperatures are shown on Figure 2 as a solid line. Figure 1 represents the flight path of a V-2 rocket which was fired by the Naval Research Laboratory at White Sands, on 10 October, 1946. The crosses on Figure 2 represent thermocouple measurements which were read from the telemeter records of that flight. Telemeter accuracy was believed to be ± 10%.

Time	Altitude	Velocity	a	M	ρ X 10^6	T_{amb}	T_{BL}	μ_{BL} x10^7	ρ v	R_e X10^{-6}	$R_e^{.18}$	$\bar{h}$
sec.	ft.	$\frac{ft.}{sec.}$	$\frac{ft.}{sec}$		$\frac{slug}{ft^3}$	^oR	^oR	$\frac{slug}{ft.sec}$				$\frac{BTU}{ft^2 sec^o}$
0	—	—	–	–	–	520	520	3.73	0	0	0	-
10	4,500	400	1,100	0.32	1928	493	504	3.70	0.68	18.2	20.3	0.0046
20	9,500	750	1,080	0.69	1784	484	525	3.76	1.34	35.0	22.8	0.0081
30	19,000	1,420	1,040	1.37	1311	451	605	4.19	1.86	43.5	23.6	0.0109
40	34,000	1,950	978	1.99	765	397	678	4.64	1.40	29.6	22.1	0.0087
50	63,000	2,800	971	2.79	194	392	945	5.78	0.54	9.20	17.9	0.0042
60	100,000	4,100	971	4.22	33	392	1581	8.07	0.14	1.70	13.2	0.0014
70	137,000	5,200	1,124	4.63	5	525	2550	11.19	0.03	0.26	9.5	0.0004

TABLE 1: Calculations for Sample Problem

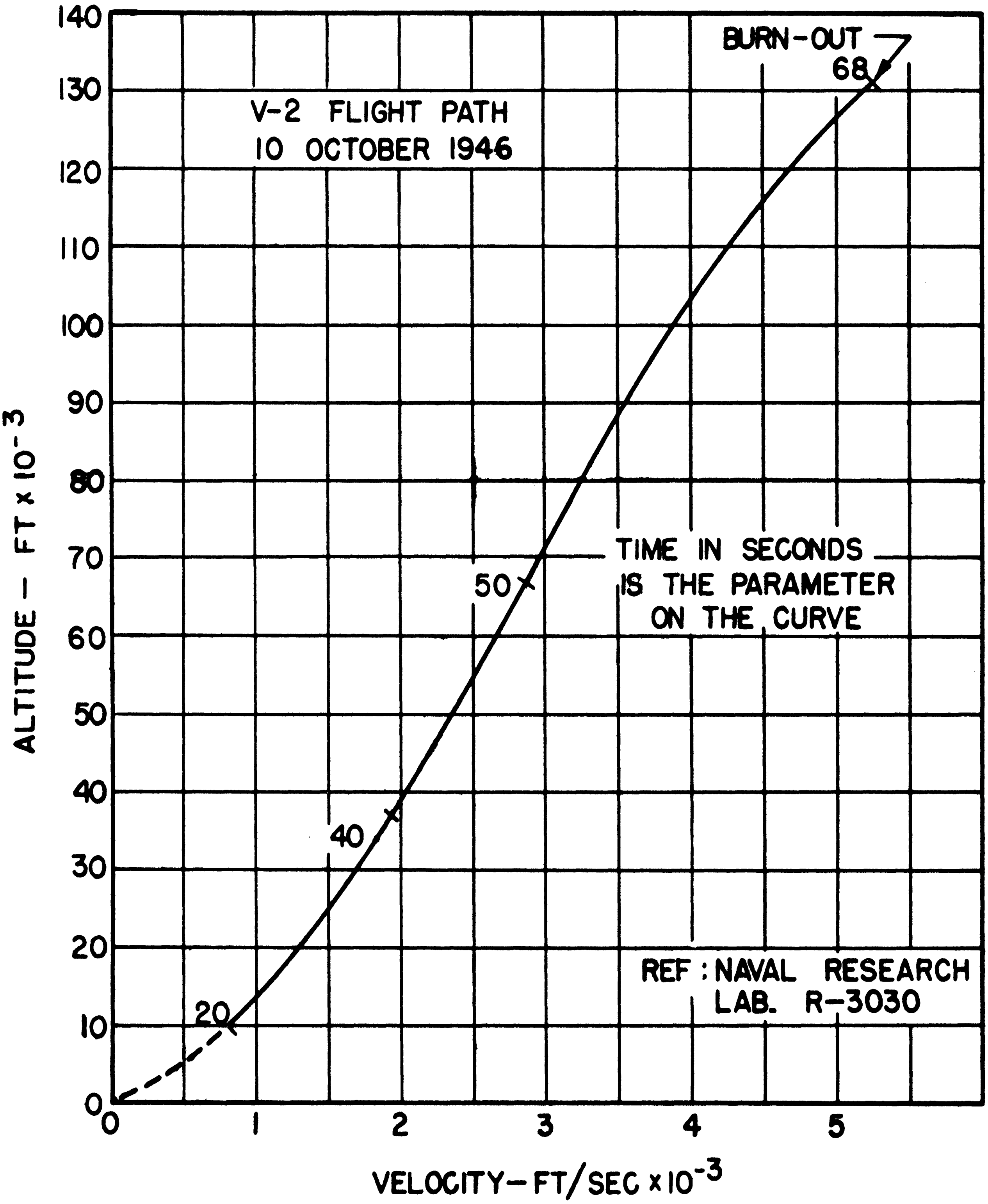

FIGURE 1: Flight Path of V-2 Rocket

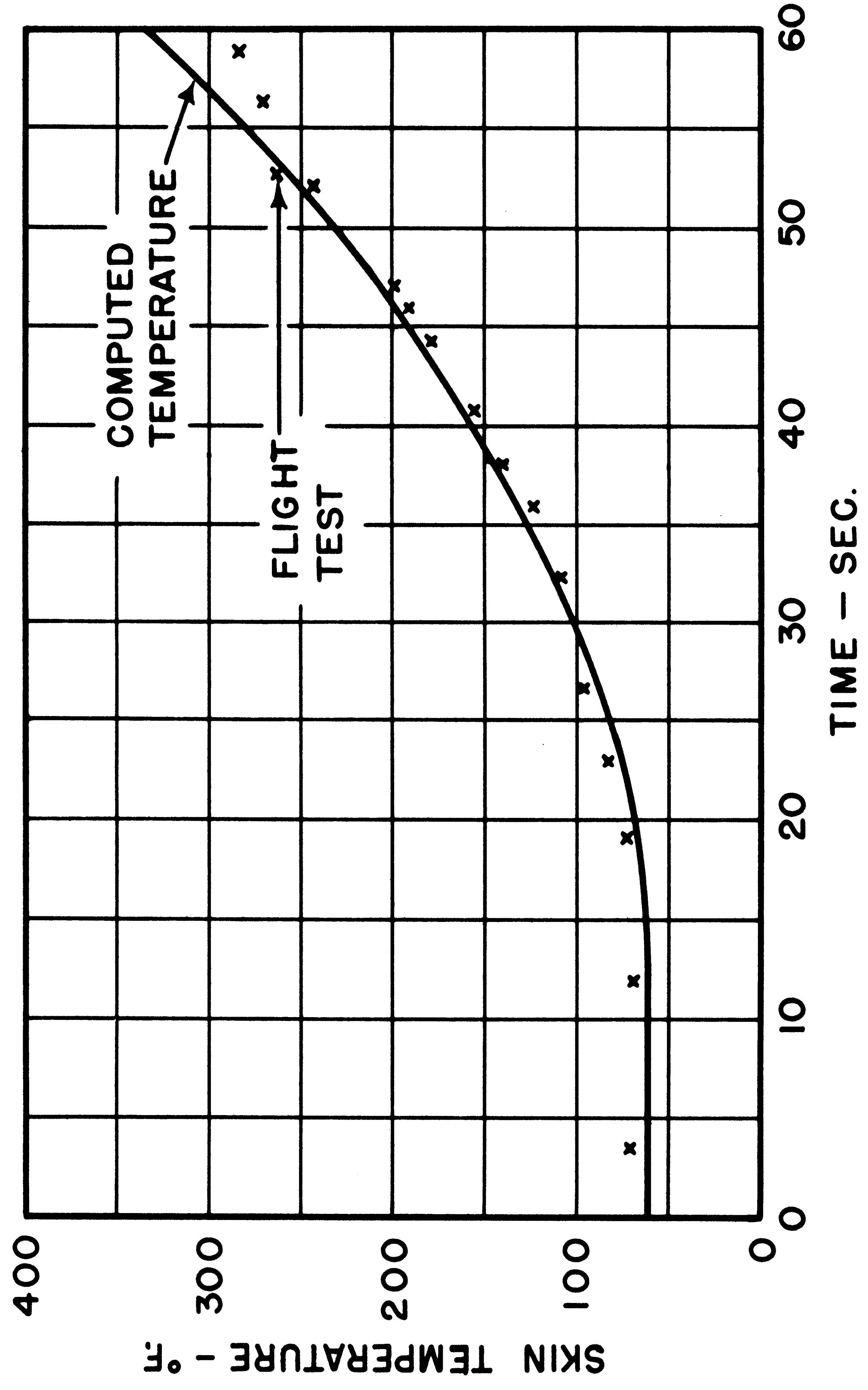

FIGURE 2: Skin Temperature of the V-2 During Flight Firing of 10 October, 1946

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