Dark matter capture in the first stars: a power source and limit on stellar mass

Katherine Freese[1,2], Douglas Spolyar[3] and Anthony Aguirre[3]

[1] Michigan Center for Theoretical Physics, Physics Department,
University of Michigan, Ann Arbor, MI 48109, USA
[2] Perimeter Institute for Theoretical Physics, 31 Caroline Street, North
Waterloo, ON, N2L 2Y5, Canada
[3] Physics Department, University of California, Santa Cruz, CA 95060, USA
E-mail: ktfreese@umich.edu, dspolyar@physics.ucsc.edu and
aguirre@scipp.ucsc.edu

Abstract. The annihilation of weakly interacting massive particles can provide
an important heat source for the first (Pop III, 'Pop' standing for 'population')
stars, potentially leading to a new phase of stellar evolution known as a 'dark
star'. When dark matter (DM) capture via scattering off baryons is included, the
luminosity from DM annihilation may dominate over the luminosity due to fusion,
depending on the DM density and scattering cross section. The influx of DM due
to capture may thus prolong the dark star phase of stellar evolution as long as
the ambient DM density is high enough. Comparison of DM luminosity with
the Eddington luminosity for the star may constrain the stellar mass of zero-
metallicity stars. Alternatively, if sufficiently massive Pop III stars are found,
they might be used to bound dark matter properties.

JCAP11(2008)014

Contents

1. Introduction

The first stars in the Universe mark the end of the cosmic dark ages, reionize the Universe, and provide the enriched gas required for later stellar generations. They may also be important as precursors to black holes that coalesce and power bright early quasars. The first stars are thought to form inside dark matter halos of mass $10^5 M_\odot$–$10^6 M_\odot$ at redshifts $z = 10$–50 [2, 3]. These halos consist of 85% dark matter and 15% baryons in the form of metal free hydrogen and helium gas. Theoretical calculations indicate that the baryonic matter cools and collapses via molecular hydrogen cooling [4]–[6] into a single small protostar [7] at the center of the halo (for reviews see e.g. [8]–[10]).

In this paper we continue our previous work [1] examining the effect of dark matter (DM) on the particles on the first stars. We focus on weakly interacting massive particles (WIMPs), which are the favorite dark matter candidates of many physicists because they automatically provide approximately the right amount of dark matter, i.e. 24% of the current energy density of the Universe. Many WIMP candidates are their own antiparticles, in which case they can annihilate with themselves in the early Universe with weak interaction cross sections, leaving behind this relic density. Probably the best example of a WIMP is the neutralino, which in many models is the lightest supersymmetric particle[4]. The neutralino, the supersymmetric partner of the W, Z, and Higgs bosons, has the required weak interaction cross section and mass $\sim$GeV–TeV to give the correct amount of dark matter and would play an important role in the first stars. For reviews of SUSY and other dark matter candidates see [25]–[27].

This same annihilation process is the basis of the work that we consider here. WIMP self-annihilation is relevant wherever the WIMP density is sufficiently high. Such regimes

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[4] If the lightest supersymmetric particle were the axino or gravitino, it would not have weak interaction annihilations and would not produce the phenomena described in this paper.

include those of the early Universe, in galactic halos today [11, 12], in the Sun [13] and the Earth [14, 15], and in the first stars. As our canonical values, we will use the standard value for the annihilation cross section

$$\langle \sigma v \rangle_{\mathrm{ann}} = 3 \times 10^{-26} \ \mathrm{cm^3 \ s^{-1}}, \tag{1}$$

as this gives the right WIMP relic density today, as well as taking $m_\chi = 100$ GeV for our canonical value of the WIMP particle mass but will also consider a broader range of WIMP masses (1 GeV–10 TeV) and cross sections.

The interaction strengths and masses of the neutralino depend on a large number of model parameters. In the minimal supergravity model, experimental and observational bounds restrict m_χ to 50 GeV–2 TeV, while $\langle \sigma v \rangle$ lies within an order of magnitude of 3×10^{-26} cm^3 s^{-1} (except at the low end of the mass range where it could be several orders of magnitude smaller) [17, 18]. Non-thermal particles can have annihilation cross sections that are many orders of magnitude larger (e.g. [16]) and would have even more drastic effects. Given the present state of the field there are many types of DM candidates which could apply [19]; the effects that we find apply equally well to other WIMP candidates with comparable cross sections for self-annihilation and scattering off nucleons.

In this paper, we consider the effects of WIMP annihilation on the first stars. In a previous paper (hereafter, paper I) [1] two of us (together with P Gondolo) considered the effects of dark matter annihilation on the formation of the first stars. We found that a crucial transition takes place when the gas number density of the collapsing protostar exceeds a critical value (10^{13} cm^{-3} for a 100 GeV WIMP mass): at this point WIMP annihilation heating dominates over all cooling mechanisms and prevents the further collapse of the star. We suggested that the very first stellar objects might be 'dark stars', a new phase of stellar evolution in which the DM—while only supplying 1% of the mass density—provides the power source for the star through DM annihilation. We are as yet uncertain of the lifetime of these theoretical objects. Once the DM contained inside the star runs out, the star could contract and heat up to the point where fusion becomes possible. However, as the star reaches nuclear density, there is an additional mechanism for repopulating the DM inside the star: capture of more DM from the ambient medium. This new source of DM can extend the lifetime of the dark star. Indeed the capture process can continue as long as there is enough ambient DM passing through the star, causing DM annihilation inside the star to continue, possibly even to today.

Dark stars, powered by DM annihilation, require three key ingredients, as shown in paper I: (1) high dark matter density, (2) annihilation products remain trapped in the star, and (3) DM heating dominates over the other sources of heating. The first stars exist at the right place and at the right time to have the best chance of achieving the first criterion: they exist in the high density centers of dark matter halos, and they form at high redshifts (density scales as $(1 + z)^3$). These densities are still not enough: paper I showed that the DM density must be driven up still further in order for the DM heating to become important: as the baryons condense to form stars, they come to dominate the potential well and pull the DM in with them and drive up the density. The resultant DM densities were computed in paper I from adiabatic contraction. Once the DM due to this effect runs out, the dark star phase might end. On the other hand, there is another effect that can repopulate the DM inside the stars and allow the dark star phase to continue: capture of more DM from the ambient medium. To get a significant amount of captured

material into the star requires the enhanced densities due to adiabatic contraction. We will estimate the required ambient DM density for capture to be important, and find that it is somewhat lower than what is required in the earlier protostellar phase.

In this paper, we consider the effects of DM annihilation on early zero-metallicity (Pop III) stars, once they do have fusion inside their cores. These stars live inside a reservoir of WIMPs; as the WIMPs move through the stars, some of the WIMPs are captured by the stars. The captured DM sinks to the center of the stars, where the DM can annihilate very efficiently. This has the effect of dramatically increasing the annihilation rate inside of a star, compared to DM annihilation without scattering. The annihilation provides a heat source for the stars, and we compare this DM annihilation luminosity to the fusion luminosity of the stars, as well as to the Eddington luminosity. Of course, the presence of DM (with high densities due to adiabatic contraction or capture) would already become important during (and seriously affect) the formation and earlier stages of these stars. Here, it is simply our intent to show that under some circumstances DM capture makes the importance of DM heating in Pop III stars unavoidable. Thus, even if DM annihilation fails to stop a Pop III star from forming (as was considered in [1]), a dark star powered by DM annihilation may exist at the high nuclear densities where fusion can take place. Again, we wish to note that the DM supplies less than one percent of the mass of the dark star and yet may be responsible for its luminosity.

The two key uncertainties in this work are: (i) the scattering cross section must be at (or near) the experimental limit, and (ii) the ambient DM density must be high enough for capture to take place. Whereas the annihilation cross section is fixed to be close to equation (1), on theoretical grounds the scattering cross section can vary across many orders of magnitude. As discussed later, it is, however, constrained by experimental bounds. For scattering to matter in the first stars, the scattering cross section must be within a few orders of magnitude of the current experimental limits. Such a cross section should be experimentally accessible in the next round of DM detection experiments. The second criterion is likely to be true for a while after the star is created, but it is not clear for how long. Once the halo containing the dark star merges with other objects, it is not clear how long the central DM in the halo remains undisturbed, and it is not clear how long the dark star remains at this central point. In principle the capture could continue indefinitely, so a dark star could still exist today, but this is very unclear.

Previous work on DM annihilation powering stars has also been done in the context of high DM densities near the supermassive black holes in galactic centers, e.g. WIMP burners [20] and more generally [21, 22].

Just as we were preparing to submit our paper, a very similar work was submitted by [23]. We have carried the analysis further. We agree with [23] in the conclusion that DM annihilation may dominate over fusion, and we illustrate the DM densities required for this conclusion. In addition, we go one step further and discuss the possibility that the DM power source may exceed the Eddington luminosity and prevent the first stars from growing beyond a limited mass. This would affect the IR background, the re-ionization of the Universe, the number of supernovae, and potentially the nature of supernovae of the first stars; this will be addressed in a separate publication [29].

We begin by discussing the equilibrium WIMP abundance in the first stars, by computing the number of WIMPs captured by the first stars, and equating this with the annihilation rate of WIMPs in the first stars. A discussion of adiabatic contraction,

which may drive up the DM density near the baryons, follows. Then, we compute the DM annihilation luminosity and compare with fusion luminosity. Finally, we compare the DM luminosity with the Eddington luminosity and find a maximum stellar mass as a function of DM density.

2. WIMP abundance

As WIMPs travel through a star, they can scatter off the nuclei in the star with the scattering cross section σ_c. Although most of the WIMPs travel right through the star, some of them lose enough energy to be captured. We call the capture rate C (s^{-1}). The WIMPs then sink to the center of the star, where they can annihilate with one another with the annihilation rate Γ_A (s^{-1}). This process was previously noticed as important for the Sun by [13] and in the Earth by [14, 15]. DM could be detected through neutrinos produced from the DM annihilation products in the Sun or Earth with experiments such as Super-Kamiokande [32] and AMANDA (which did not find a signal and placed bounds) and ICECUBE (which is starting to take data) [30]. Other indirect searches such as GLAST and PAMELA could detect the gamma-ray and positron annihilation products respectively from DM annihilating in the Milky Way halo (the gamma-rays and positrons from DM annihilation in the Sun or Earth would be trapped inside the objects and would not make it to our detectors).

The number of WIMPs N in the star is then determined by a competition between capture and annihilation via the differential equation

$$\dot{N} = C - 2\Gamma_A \equiv C - C_A N^2, \tag{2}$$

where Γ_A is the annihilation rate (and the factor of 2 appears because two particles are annihilated in each event), and

$$C_A = 2\Gamma_A / N^2 \tag{3}$$

is defined as an N-independent annihilation coefficient. Solving this equation, we find $\Gamma_A = \frac{1}{2} C \tanh^2(t/\tau)$, where

$$\tau = (C C_A)^{-1/2} \tag{4}$$

is the equilibration timescale. Equilibrium corresponds to a balance between the capture and annihilation rates, i.e.

$$\Gamma_A = \frac{1}{2} C. \tag{5}$$

As we will show, for the case of Pop III stars, equilibrium is quickly reached ($t \gg \tau$) and we may use equation (5).

2.1. Annihilation rate

The annihilation rate

$$\Gamma_A = \int d^3 r \, n_\chi(r)^2 (\sigma v)_{\text{ann}}, \tag{6}$$

where $n_\chi(r)$ is the density of captured DM at a point r inside of the star. In this section we will show that the WIMPs quickly thermalize with the core of the star, so we can treat

the DM density distribution as isothermal, and then we will compute the annihilation rate. Our work closely follows the approach previously given by [24].

First let us examine the WIMP thermalization timescale inside the star. The amount of energy ΔE lost by a WIMP in a scattering event with a nucleus (proton mass $M_{\rm p}$) in the star ranges over $0 \leq \Delta E/E \leq m_\chi M_{\rm p}/[(m_\chi + M_{\rm p})/2]^2$. Assuming a flat distribution and taking $M_{\rm p} \ll m_\chi$, the average energy loss is $2M_{\rm p}/m_\chi$. Roughly, thermalization requires $\Delta E/E \sim 1$; i.e. there must be $m_\chi/2M_{\rm p}$ scatters. Thus, for $m_\chi \gg 1$ GeV, the timescale for thermalization can be estimated as

$$\tau_{\rm th} \approx \frac{1}{\sigma_{\rm c} v_{\rm esc} n_{\rm H}} \frac{M_\chi}{2M_{\rm H}}, \tag{7}$$

where $v_{\rm esc}$ is the velocity of escape of a DM particle from the surface of the star, and $n_{\rm H}$ is the average density of the star. For a hundred GeV WIMP, $\sigma_{\rm c} = 10^{-39}$ cm^{-2}, and $n_{\rm H} = 10^{24}$ cm^{-3}, we find the thermalization timescale to be very short, roughly three months.

Thus we can use an isothermal distribution for the DM:

$$n(r)_\chi = n_{\rm c} {\rm e}^{-m_\chi \phi/kT}, \tag{8}$$

where $n_{\rm c}$ is the central number density of DM and T is the central temperature of the star,

$$\phi(r) = \int_0^r \frac{GM(r)}{r} \, {\rm d}r \tag{9}$$

is the gravitational potential at radius r with respect to the center, and $M(r)$ is the mass interior to r. One can define effective volumes

$$V_j = 4\pi \int_0^{R_*} r^2 {\rm e}^{-{\rm j} m_\chi \phi/T} {\rm d}r. \tag{10}$$

Upon integration this gives

$$V_j = [3m_{\rm pl}^2 T/(2{\rm j} m_\chi \rho_{\rm c})]^{3/2}, \tag{11}$$

where $m_{\rm pl}$ is the Planck mass, and $\rho_{\rm c}$ is the core mass density of the star. The name 'effective volume' is suggestive since we have $N = n_{\rm c} V_1$ and also the total annihilation rate is given as $\Gamma = \langle \sigma v \rangle_{\rm ann} n_0^2 V_2$.

One can then solve equation (2) to find the N-independent annihilation coefficient defined in equation (3):

$$C_A = \langle \sigma v \rangle_{\rm ann} \frac{V_2}{V_1^2}. \tag{12}$$

In table 1, we list the computed C_A for Pop III stars of different masses. From Woosley [28], we have obtained the properties of zero-metallicity stars when they are halfway through hydrogen burning on the main sequence. In the table we have used our canonical values in equation (1) for the annihilation cross section and mass ($m_\chi = 100$ GeV); the results can easily be scaled to other values since $C_A \propto m_\chi^{1.5} \langle \sigma v \rangle_{\rm ann}$.

Table 1. Central temperature T (K) and central baryon density ρ (g cm^{-3}) for various masses of metal free stars halfway through hydrogen burning [28]. The effective volume V_2/V_1^2 is also shown. The DM annihilation rate is $\Gamma_A = \frac{1}{2}C_A N^2$ where N is the number of WIMPs in the star. Please note that the entry marked 'Sun' refers to the present day Sun for comparison. We have used the fiducial values $m_\chi = 100$ GeV and $\langle\sigma v\rangle_{\mathrm{ann}} = 3 \times 10^{-26}$ cm^3 s^{-1} as needed.

$M_\star$ ($M_\odot$)	T (K)	ρ (g cm^{-3})	V_2/V_1^2 (cm^{-3})	C_A (s^{-1})
Sun	—	—	1.72×10^{-28}	5.16×10^{-54}
10	9.55×10^7	225.8	1.77×10^{-29}	5.31×10^{-55}
50	1.13×10^8	48.63	1.38×10^{-30}	4.14×10^{-56}
100	1.18×10^8	31.88	6.86×10^{-31}	2.06×10^{-56}
250	1.23×10^8	19.72	3.11×10^{-31}	9.33×10^{-57}

2.2. Capture rate

WIMP interactions with nuclei are of two kinds: spin independent, which scale as A^2 (where A is the number of nucleons in the nucleus), and spin dependent, which require the nucleon to have a spin. Currently, the experimental bounds on elastic scattering are the weakest for the spin-dependent (SD) contribution (to be precise we are considering only scattering off protons since the stars are comprised primarily of hydrogen). In figure 2 [31] we illustrate bounds on the SD component from direct detection as well as from Super-Kamiokande [31, 32, 39, 33], [35]–[37]. The latter, which are the most constraining, assume that a significant fraction of the annihilation energy goes into neutrinos (as is likely for SUSY particles); if the neutrino component is small then the SD cross section could be several orders of magnitude higher. As our fiducial value, in this paper, we use the spin-dependent cross section

$$\sigma_{\mathrm{c}} = 10^{-39} \text{ cm}^2, \tag{13}$$

which is consistent with all experimental bounds, but we will always show the dependence of any result on the value of σ_{c}. The bound on the spin-independent (SI) scattering is much tighter, $\sigma_{\mathrm{SI}} \leq 10^{-42}$ cm^2 for $m_\chi = 100$ GeV [38, 39]. The first stars are made only of hydrogen or helium, so the A^2 enhancement for the spin-independent contribution is not substantial. In this paper, we consider only the spin-dependent contribution, though in principle for any specific candidate WIMP one should self-consistently include both.

The capture rate per unit volume at a distance r from the center of the star, for an observer at rest with respect to the WIMP distribution (as should be a good approximation here), is [41, 40]

$$\frac{\mathrm{d}C}{\mathrm{d}V}(r) = \left(\frac{6}{\pi}\right)^{1/2} n(r) n_\chi(r) (\sigma_{\mathrm{c}}\bar{v}) \frac{v(r)^2}{\bar{v}^2} \left[1 - \frac{1 - \exp(-B^2)}{B^2}\right], \tag{14}$$

where n is the number density of nucleons (here, we only consider hydrogen because we are not considering spin independent scattering and helium does not have a spin), n_χ is the WIMP number density, $v(r)$ is the velocity of escape of WIMPs from the star at a given radius r, $\bar{v}^2 \equiv 3kT_\chi/M$ is a 'velocity dispersion' of WIMPs in the DM halo, and

$$B^2 \equiv \frac{3}{2} \frac{v(r)^2}{\bar{v}^2} \frac{\mu}{\mu_-^2}, \tag{15}$$

where $\mu = m_\chi/M_N$ is the ratio of WIMP to nucleon mass and $\mu_- = (\mu + 1)/2$. For an observer moving with respect to the WIMPs, the quantity in square brackets in equation (14) becomes a more complicated function of B and the relative velocity as shown in equation (2.24) of [40].

The capture rate for the entire star is then

$$C = \int_0^{R_\star} 4\pi r^2 \, \mathrm{d}r \, \frac{\mathrm{d}C}{\mathrm{d}V}(r), \qquad (16)$$

where $R_\star$ is the radius of the star. To obtain a conservative and fairly accurate estimate of the capture rate[5], we may take

$$v(r)^2 = v(R_\star)^2 = \frac{2GM_\star}{R_\star} \equiv v_{\mathrm{esc}}^2 \qquad (17)$$

for all r, assume the term in square brackets in equation (14) is very close to 1 (justified below), and take a uniform dark matter density. In this case the integral simplifies to give

$$C = \left(\frac{6}{\pi}\right)^{1/2} \left(\frac{M_\star}{m_{\mathrm{p}}} f_{\mathrm{H}}\right) (\sigma_{\mathrm{c}} \bar{v}) \left(\frac{v_{\mathrm{esc}}}{\bar{v}}\right)^2 \frac{\rho_\chi}{m_\chi}, \qquad (18)$$

where $M_\star$ is the stellar mass, m_{p} is the proton mass and f_{H} is the fraction of the star in hydrogen. (Note that hydrogen has spin while helium generally does not. We could, in principle, consider the spin-independent contribution of scattering off hydrogen and helium in the stars; we have not done so because we believe this contribution to be subdominant. In any case the current work is conservative in considering only the spin-dependent scattering.)

To estimate $\bar{v}$ as per [43], we take the virial velocity of the DM halo:

$$\langle \bar{v}^2 \rangle = \frac{|W|}{M_{\mathrm{halo}}}, \qquad (19)$$

where

$$W = -4\pi G \int \rho_{\mathrm{halo}} M_{\mathrm{halo}}(r) r \, \mathrm{d}r \qquad (20)$$

and the typical DM halo containing a Pop III star has $M_{\mathrm{halo}} = 10^5$–$10^6 M_\odot$.

We use a Navarro, Frenk and White (NFW) profile [46] for the DM,

$$\rho_{\mathrm{halo}} = \frac{\rho_0}{(r/r_{\mathrm{s}})(1 + r/r_{\mathrm{s}})^2}, \qquad (21)$$

where r_{s} is the scale radius. The normalization ρ_0, known as the central density, depends on the concentration parameter C_{vir} and on the redshift when the halo virializes Z_{vir}. These parameters range from $C_{\mathrm{vir}} = (1$–$10)$ [42] and $Z_{\mathrm{vir}} = 10$–50 [2, 3]. With r_{s} in the range $(15$–$100)$ pc, we find $\bar{v} = (1$–$15)$ km s^{-1}. As our fiducial value, we will take

$$\bar{v} = 10 \text{ km s}^{-1}. \qquad (22)$$

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[5] We have subsequently obtained much more accurate values for $v(r)$ by treating the Pop III star as a polytrope with index $n = 3$ (a good approximation), and confirmed that the conservative capture rate presented here differs (it is too low) by only a factor of a few.

Table 2. Stellar mass ($M_\star$), radius ($R_\star$), and surface escape velocity (V_{esc}) in solar units, for metal free stars halfway through hydrogen burning. We have also calculated the capture rate C using our fiducial values $\rho_\chi = 10^9$ GeV cm^{-3}, $m_\chi = 100$ GeV, and $\sigma_c = 10^{-39}$ cm^2, and calculated τ using C_A from table 1. As in table 1, the entry marked Sun refers to the present day Sun and not a zero-metallicity star. The capture rate for the Sun still uses the fiducial values; in the true present day Sun the true capture rate is much smaller ($C \approx 10^{24}$ s^{-1}), mostly due to the much lower DM densities in the solar neighborhood.

$M_\star$ ($M_\odot$)	$R_\star$ ($R_\odot$)	V_{esc} ($V_\odot$)	C (s^{-1})	τ (yr)
Sun	1	1	4.9×10^{34}	63
10	1.16	2.49	8.1×10^{34}	152
50	4.76	3.24	6.8×10^{35}	190
100	7.04	3.77	1.9×10^{36}	160
250	11.8	4.60	6.9×10^{36}	126

For $B \gg 1$, the term in square brackets in equation (14) is very close to 1, and this holds for all stellar and WIMP masses that we are considering. (For example, for a $1 M_\odot$ star, we have $v_{esc} = 618$ km s^{-1}, which is much larger than $\bar{v} = 10$ km s^{-1}; then for $m_\chi = 100$ GeV we find $B \sim 100$.) Thus we may ignore the term in square brackets. The bracketed term changes for a star moving through the WIMP halo (rather than being stationary as we have assumed), but the factor that replaces the term in square brackets is O(1). (For example, for a 1 $M_\odot$ star, with $m_\chi = 100$ GeV, and with a velocity $\bar{v}$, we find that the factor is 0.66.) We note that this factor is much more important in today's stars than in the first stars because $\bar{v}$ is much larger today (due to the fact that today's galactic halos are much larger, e.g. $10^{12} M_\odot$).

In table 2, we give the capture rate evaluated using equation (18), again using properties (including stellar radius) of Pop III stars from [28]. In obtaining these numbers, we have used $\rho_\chi = 10^9$ GeV cm^{-3}, $m_\chi = 100$ GeV, and $\sigma_c = 10^{-39}$ cm^2; the result can easily be scaled to other values since $C \propto \rho_\chi \sigma_c / m_\chi$. For general values we find that

$$C = 4.9 \times 10^{34} \text{ s}^{-1} \left(\frac{M_\star}{M_\odot} \right) \left(\frac{v_{esc}}{618 \text{ km s}^{-1}} \right)^2 \left(\frac{\bar{v}}{10 \text{ km s}^{-1}} \right)^{-1} \left(\frac{\rho_\chi}{10^9 \text{ GeV cm}^{-1}} \right)$$
$$\times \left(\frac{m_\chi}{100 \text{ GeV}} \right)^{-1} \left(\frac{\sigma_c}{10^{-39} \text{ cm}^2} \right). \tag{23}$$

Further, using equation (17) and noting that for the Pop III models of [28] it is roughly true that $R_\star \propto M_\star^{0.45}$, we find that approximately

$$C \approx 4.9 \times 10^{34} \text{ s}^{-1} \left(\frac{M_\star}{M_\odot} \right)^{1.55} \left(\frac{\bar{v}}{10 \text{ km s}^{-1}} \right)^{-1} \left(\frac{\rho_\chi}{10^9 \text{ GeV cm}^{-1}} \right) \left(\frac{m_\chi}{100 \text{ GeV}} \right)^{-1}$$
$$\times \left(\frac{\sigma_c}{10^{-39} \text{ cm}^2} \right). \tag{24}$$

Table 2 also shows the equilibrium timescale given by equation (4) using the capture and annihilation rates determined above. We can see that τ is extremely short, compared to the lifetime of a star. Hence for most of the lifetime of the zero-metallicity stars, $t \gg \tau$ and we may use equation (5).

2.3. Dark matter density

To study the effects of dark matter on the first stars, we need to know the density of the DM passing through the stars to determine the capture rate. Simulations have unfortunately not (as yet) resolved this issue. Below we will use a variety of DM densities, since these numbers are unknown.

2.3.1. Dark matter density before capture is included. First we need estimates of the DM density in the region where the star forms, prior to including the effects of capture. In a previous paper [1], two of us (with P Gondolo) used adiabatic contraction [45] to obtain estimates of the DM profile. Prior to this contraction, we assumed an overdense region of $10^5 M_\odot$–$10^6 M_\odot$ with a Navarro–Frenk–White (NFW) profile [46] for both DM and gas, where the gas contribution is 15%. (For comparison, we also used a Burkert profile [47], which has a DM core. A Burkert profile has been shown to be a good fit for the dynamics of today's galaxies [49, 48].) As the gas collapses, we allowed the DM to respond to the changing baryonic gravitational potential, where the gas density profiles were taken from simulations of [50, 51]). The final DM density profiles were computed with adiabatic contraction ($M(r)r = $ constant). After contraction, we found a DM density at the outer edge of the baryonic core of roughly $\rho_\chi \simeq 5 \ (n/\mathrm{cm}^{-3})^{0.81} \ \mathrm{GeV/cm}^{-3}$ which scales as $\rho_\chi \propto r^{-1.9}$ outside the core (see figure 1 in [1]). Our adiabatically contracted NFW profiles match the DM profile obtained numerically in [50] (see their figure 2). They present their earliest (gas core density $n \sim 10^3 \ \mathrm{cm}^{-3}$) and latest ($n \sim 10^{13} \ \mathrm{cm}^{-3}$) DM profiles, as far inward as 5×10^{-3} pc and 0.1 pc. The slope of these two curves is the same as ours. If one extrapolates them inward to smaller radii, one obtains the same DM densities as with our adiabatic contraction approach. The highest DM density found by [50] was $10^8 \ \mathrm{Gev \ cm}^{-3}$. Should the adiabatic contraction continue all the way to the small stellar cores at $n \sim 10^{22} \ \mathrm{cm}^{-3}$ (which we doubt), the DM density would be as high as $10^{18} \ \mathrm{GeV \ cm}^{-3}$. We note that [52] obtained DM density profiles in galaxies and found that adiabatic contraction produces densities that are too high by only a factor of 2 or 3, even when radial orbits are included, or in the presence of bars, or in the absence of spherical symmetry. Below we will use a variety of DM densities due to the uncertainties. To give a sense of the numbers, we compute that in a $10 M_\odot$ dark star, before capture is taken into account, there are roughly 10^{16} baryons in the star for every WIMP particle.

2.3.2. Dark matter density including capture. The amount of dark matter in the dark star obviously increases significantly due to capture. We find that (again for a $10 M_\odot$ star) there are 10^{12} baryons for every WIMP particle; i.e., the fraction of WIMP particles has grown by a factor of 10^4 due to capture. Of course the density of WIMPs is more centrally concentrated, so the ratio of WIMPs to baryons near the center of the star, where the annihilation rate peaks, is higher than these numbers. Still, as we will show in the next section, it is remarkable that particles which are 10^{-12} as numerous as the baryons can provide the dominant heat source for the star.

3. Luminosity due to WIMP annihilation

We may now compute the luminosity due to WIMP annihilation,

$$L_{\mathrm{DM}} = f \Gamma_{\mathrm{A}}(2 m_\chi), \tag{25}$$

Table 3. Luminosity $L_\star$ due to fusion versus luminosity L_{DM} due to DM annihilation in zero-metallicity stars (as defined in previous tables), using fiducial values for DM properties. In the final column we vary ρ_χ to determine the value at which L_{DM} exceeds $L_\star$. We stress that the DM densities listed here are those in the ambient medium (NFW plus adiabatic contraction) rather than the densities after capture; it is these ambient densities that determine the capture rate and thus the equilibrium luminosity.

$M_\star$ ($M_\odot$)	$L_\star$ (erg s^{-1})	L_{DM} (erg s^{-1})	ρ_χ (GeV cm^{-3}) for which $L_\star = L_{\mathrm{DM}}$
Sun	3.9×10^{33}	5.2×10^{33}	7.5×10^{7}
10	4.2×10^{37}	3.2×10^{35}	1.3×10^{10}
50	2.0×10^{39}	2.8×10^{36}	7.5×10^{10}
100	6.45×10^{39}	7.6×10^{36}	8.3×10^{10}
250	2.31×10^{40}	2.8×10^{37}	8.5×10^{10}

where we take the energy per annihilation to be twice the WIMP mass; two WIMPS annihilate per annihilation. Here f is the fraction of annihilation energy that goes into the luminosity. Roughly 1/3 of the annihilation energy is lost to neutrinos that stream right out of the star, whereas the other 2/3 goes into electrons, positrons, and photons that are trapped in the star and have their energy thermalized. Hence we take [1]

$$f \sim 2/3. \tag{26}$$

For $t \gg \tau$, which is quickly reached, equation (25) may be rewritten using equation (5) as

$$L_{\mathrm{DM}} = \frac{f}{2} C(2m_\chi). \tag{27}$$

The WIMP luminosity is given in table 3 for a variety of stellar masses together with the ordinary fusion-powered stellar luminosity $L_\star$ provided by the models of [28] for these stars. Roughly, using equation (24) we may write

$$L_{\mathrm{DM}} = 5.2 \times 10^{33} \ \mathrm{erg \ s^{-1}} \left(\frac{M_\star}{M_\odot}\right)^{1.55} \left(\frac{\rho_\chi}{10^9 \ \mathrm{GeV \ cm^{-3}}}\right) \left(\frac{\sigma_{\mathrm{c}}}{10^{-39} \ \mathrm{cm^2}}\right) \left(\frac{\bar{v}}{10 \ \mathrm{km \ s^{-1}}}\right)^{-1}. \tag{28}$$

The WIMP luminosity depends linearly on the WIMP density passing through the stars. We have also computed the WIMP energy density $\rho_{\chi,\mathrm{crit}}$ that is required in order for the WIMP annihilation energy to equal the ordinary stellar luminosity, which will dramatically alter the properties of the first stars [29]. We stress that the DM densities listed here are those in the ambient medium (NFW plus adiabatic contraction) rather than the densities after capture; it is this ambient density that determines the capture rate, and also therefore the annihilation rate (in equilibrium), and consequently the luminosity. For any WIMP densities higher than this value, the star's luminosity is dominated by annihilation energy (rather than by ordinary fusion). We note that the luminosity due to fusion in the zero-metallicity stars [28] scales as $L_\star \propto M^2$ (see figure 1) whereas $L_{\mathrm{DM}} \propto M^{1.55}$, so DM heating is more important for lower mass stars. The DM luminosity is dominant for the values of $\rho_{\chi,\mathrm{crit}}$ given in the last column of table 3, and is dominant for
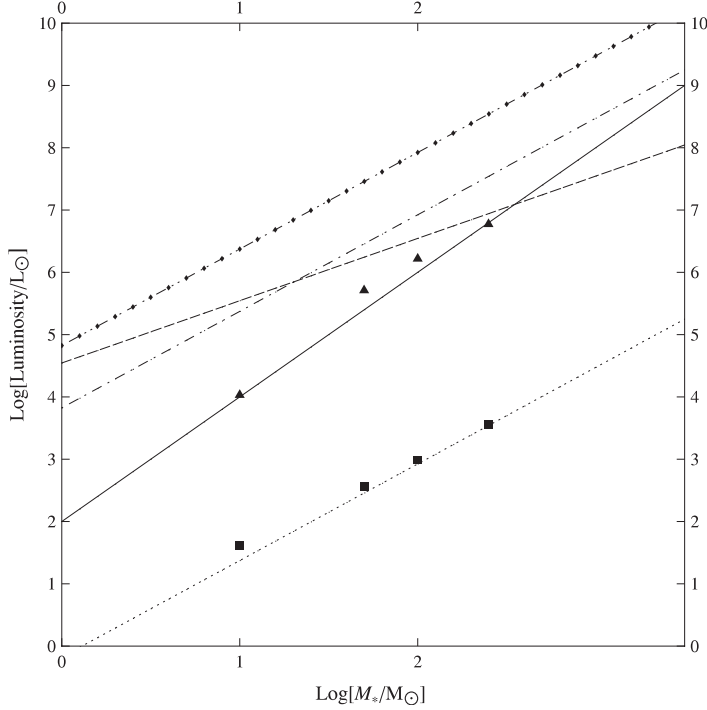
Figure 1. Log–log plot of stellar luminosity versus stellar mass (in solar units). The dashed line shows Eddington luminosity for Thompson scattering. Triangles show the data points for the luminosity of zero-metallicity stars from table 3; the solid line is a fit to these points. The remaining lines indicate the DM annihilation luminosity for different DM density assumptions. We stress that the DM densities listed here are those in the ambient medium (NFW plus adiabatic contraction) rather than the densities after capture; it is these densities that determine the capture rate and the luminosity. The dotted line is our fiducial example with $\rho_\chi = 10^9$ GeV cm^{-3} and $\sigma_c = 10^{-39}$ cm^2 (squares are the examples listed in the third column of table 3). If we keep the cross section fixed, the middle dot–dashed line corresponds to a DM density of 10^{12} GeV cm^{-3}, and the top diamond line would have a DM density of 10^{14} GeV cm^{-3}; in this final case, the DM luminosity would dominate over the Eddington luminosity for a star with a mass of $\leq 1 M_\odot$.

all relevant stellar masses for $\rho_\chi \gtrsim 9 \times 10^{11}$ GeV cm^{-3}; thus for dark matter densities in excess of this value, it will be the DM heating that determines stellar properties, and the star is a dark star. The two luminosities are plotted in figure 1 where they can be compared with one another. If the first stars are observed (e.g., by the James Webb Space Telescope) to have the properties predicted of fusion-driven stars, then one could use these results to constrain WIMP properties to make sure that DM annihilation remains subdominant.

4. Eddington luminosity

We, as yet, know little about the effect that significant dark matter heating would have on the structure of zero-metallicity stars, but we can nonetheless place an approximate upper bound on the stars' mass if we assume that they must be sub-Eddington. In the
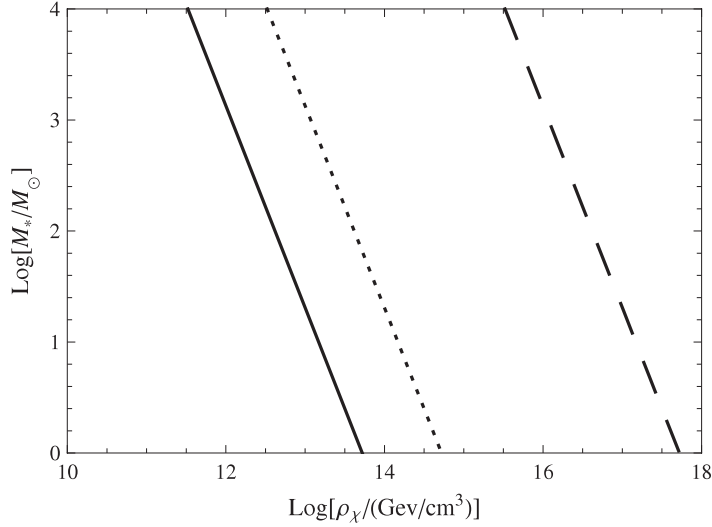
Figure 2. Maximum stellar mass due to the Eddington limit as a function of ρ_χ for a fixed cross section. The different lines correspond to different spin-dependent cross sections: $\sigma_{\rm c} = 10^{-39}$ cm^2 (solid line), $\sigma_{\rm c} = 10^{-41}$ cm^2 (dotted line), and $\sigma_{\rm c} = 10^{-43}$ cm^2 (dashed line).

current paper, we simply take the ZAMS stars, add the DM luminosity, and ask whether or not the resulting dark stars are self-consistent. In other words, is it possible for dark stars of this mass to exist? We compute the Eddington luminosity $L_{\rm Edd}$ for these objects, and ascertain whether or not the DM luminosity is in excess of this value. If $L_{\rm DM} > L_{\rm Edd}$, then a star of this mass cannot exist: the pressure from the star would be so large as to blow off some of the mass. In reality one should address a different problem: one should really follow the protostars and compute their structure as they accrete mass on their way to becoming ZAMSs. Such a calculation would essentially compare accretion luminosity (the value of which would depend on the nature of the accretion, e.g. spherical or disk accretion) to the Eddington luminosity; here we are comparing dark star luminosity to the Eddington luminosity.

The Eddington luminosity is defined (e.g. [44]) to be

$$L_{\rm Edd} = \frac{4\pi cGM_\star}{\kappa_{\rm p}}, \tag{29}$$

where G is Newton's constant, c is the speed of light, $M_\star$ is the mass of the star, and $\kappa_{\rm p}$ is the opacity of stellar atmosphere. Since the first stars' stellar atmospheres are hot and nearly metal free, the opacity is dominated by Thompson scattering, so we take

$$L_{\rm Edd} = 1.4 \times 10^{38} \text{ erg s}^{-1} (M_\star/M_\odot) = 3.5 \times 10^4 (M_\star/M_\odot) L_\odot. \tag{30}$$

In figure 1, we have plotted three luminosities as a function of stellar mass for zero-metallicity stars: the luminosity due to fusion, the luminosity due to WIMP annihilation (for a variety of WIMP densities in the star), and the Eddington luminosity. Since the Eddington luminosity scales as $L \propto M_\star$ whereas the DM luminosity scales as $L_{\rm DM} \propto M_\star^{1.55}$, for a large enough dark matter density the two curves will cross for some stellar mass. The lightest stellar mass for which $L_{\rm DM} > L_{\rm Edd}$ then constitutes an approximate upper
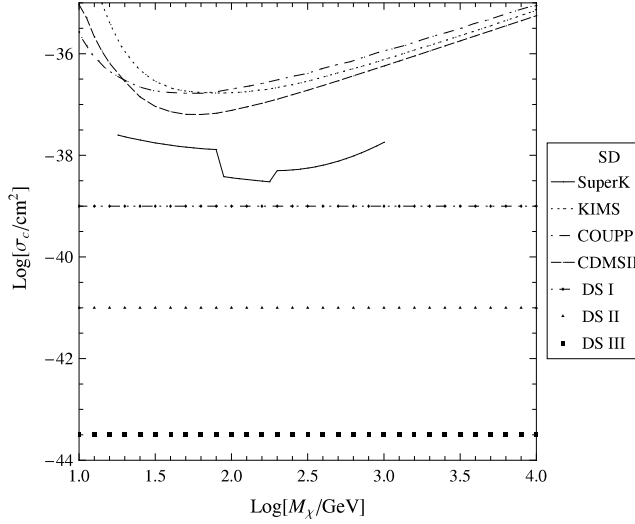
Figure 3. Bounds on spin-dependent scattering cross section as a function of WIMP mass from experiments, as well as potential bounds from the effect of WIMP annihilation on the first stars. Displayed limits include various direct and indirect detection experiments as labeled; the tightest present bounds on spin-dependent cross sections are from Super-Kamiokande (labeled Super-K), at around 10^{-38} cm^2 [31, 32], [35]–[37]. The three horizontal lines are bounds that would result from the discovery of Pop III stars of $1M_\odot$ for a variety of DM densities inside these stars (due to the fact that DM annihilation pressure would otherwise prevent their formation). The lines are labeled as follows: DS I corresponds to ambient density $\rho_\chi = 1.1 \times 10^{13}$ GeV cm^{-3} and would lead to the bound $\sigma_c \leq 10^{-39}$ cm^2; DS II corresponds to $\rho_\chi = 3 \times 10^{15}$ GeV cm^{-3} and would lead to $\sigma_c \leq 10^{-41}$ cm^2; and DS III corresponds to $\rho_\chi = 10^{18}$ GeV cm^{-3} and would lead to $\sigma_c < 3 \times 10^{-44}$ cm^2.

mass limit for the first stars, because the star of that mass is unable to accrete any further due to the radiation pressure from the WIMP annihilation. If one assumes that accretion efficiently drives up the mass of any Pop III star, then the mass of the first star will be uniquely determined by the properties of the DM. Using equations (28) and (29), we get

$$M_\star^{\mathrm{max}} = 1.1 \times 10^8 M_\odot \left(\frac{\rho_\chi}{10^9 \text{ GeV cm}^{-3}} \right)^{-1.8} \left(\frac{\sigma_c}{10^{-39} \text{ cm}^2} \right)^{-1.8}. \tag{31}$$

For example, we find that for a dark matter density of $\rho_\chi = 1.1 \times 10^{13}$ GeV cm^{-3}, the first stars cannot be more massive than $1M_\odot$. Figure 2 illustrates $M_\star^{\mathrm{max}}$ as a function of WIMP energy density for several values of the scattering cross section.

In principle, these arguments can be turned around to place a bound on the scattering cross section. Figure 3 is a plot of WIMP scattering cross section versus WIMP mass. Experimental bounds are shown. The horizontal lines indicate the values of σ_c that correspond to different values of ρ_χ at which a $1M_\odot$ star is Eddington limited by DM annihilation. (The scaling to other stellar masses is straightforward since $L_{\mathrm{DM}} \propto M_\star^{1.55}$.) Once the mass of the first stars is determined, then one could rule out any combination
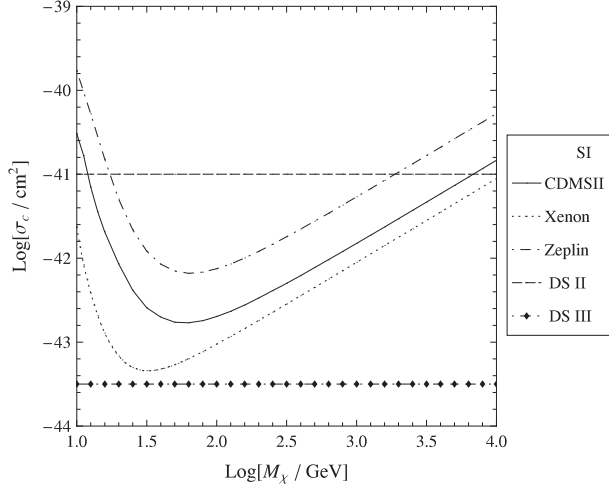
Figure 4. Same as the previous figure but for WIMPs with spin-independent interactions [38, 39, 34, 37].

of σ_c and ρ_χ for a given WIMP mass that would preclude stars of such a mass from forming. As an extreme example, if one were to believe that adiabatic contraction occurs all the way to the limit where the protostellar core has gas density $n \sim 10^{22}$ cm^{-3}, the WIMP density would reach $\rho_\chi = 10^{18}$ GeV cm^{-3}. In this case the Eddington limit would be reached for masses $\ll 1 M_\odot$ for our fiducial scattering cross section. If, instead, the first stars are observed to form with masses larger than one solar mass, and such an enormous WIMP density were found to be sensible, one could place a bound of $\sigma_c < 3 \times 10^{-44}$ cm^2 on WIMPs for almost any mass (the equilibrium DM luminosity is independent of m_χ), which would be the tightest known bound on the spin-dependent scattering cross section by several orders of magnitude; see figure 3. In this extreme case we could also put interesting limits on spin-independent scattering; see figure 4.

Obtaining bounds on the WIMP parameters in this way depends on detailed understanding of the ambient WIMP density within which the dark star resides. Clearly this is a very difficult problem which will not be solved in the near future. Indeed, it is likely that the $10^6 M_\odot$ halos containing the dark stars will merge with other halos and that the stars will not remain forever in regions of high DM density. However, simulations of structure formation are becoming ever better and one may hope someday to address this question.

It is of course true that we do not know exactly the ambient DM around any one DS, but one can construct statistical arguments. The background density will of course not be the same for all dark stars. Instead, for a given DS, the density depends on the detailed evolution and merger history of the halo within which the DS resides. However, in principle, from the simulations one can obtain the distribution of DM in the neighborhood of many dark stars. If one also had observations of many DS, one could perform a statistical comparison of the DS properties in the observations with what is learned from the simulations to learn about WIMP properties.

5. Conclusion

In summary, Pop III stars are expected to reside at the core of a $10^5-10^6 M_\odot$ dark matter halo. Previously two of us [1] discussed the importance of dark matter annihilation in the first stars, proposing the existence of 'dark stars' powered by DM heating. Even once the DM initially collapsing with the baryons into the star runs out as a source of fuel, it can be replenished by capture of more DM from the ambient medium (as the DM passes through the dark star). In this paper, we have estimated the rates of WIMP capture and self-annihilation in such stars. We have shown that when these rates are in equilibrium, the accompanying heating would provide an energy source that can rival nuclear fusion and prolong the dark star phase, if the ambient dark matter density is sufficiently high. Such densities seem plausible on the basis of analytic models of adiabatically contracted halos [1], suggesting that DM heating may radically affect the structure of the first stars.

The two key uncertainties in this work are: (i) the scattering cross section must be at (or near) the experimental limit, and (ii) the ambient DM density must be high enough for sufficient capture to take place. While this second criterion is likely to be true for a while after the star has formed, we wish to comment further here on the question of how long high enough DM densities last.

A potentially important consideration is the time for which a dark star will be able to burn DM at the calculated rate, given that it will annihilate its captured store of DM in a time of order τ as given by equation (4). The maximal burning time is determined by the total mass of DM in the region of phase space that intersects the star. This mass could be small if the star were fixed exactly at the central cusp of the DM halo. However, we expect that given the complexities of the collapse process, the star will have some non-zero velocity and thus 'wander' through a region of some radius significantly larger than that of the star. This makes much more DM mass available for burning, at the expense of somewhat lower average DM density. (It should then be noted that the fiducial DM density numbers that we assume are not those expected in the innermost core but those for over some significantly larger region.) Once the halo containing the dark star merges with other objects, it is not clear how long the central DM in the halo remains undisturbed, and it is not clear how long the dark star remains near the center. While, in principle, the dark star could continue to remain in a high density environment for a long time (even until now), estimation of the expected degree of wandering, the detailed DM density profile, and therefore the maximal burning time of the dark stars will be left for future and more detailed study.

For high enough DM density, DM heating will lower the Eddington stellar mass limit to provide an upper mass cutoff for Pop III; because DM heating might also affect the formation properties of Pop III stars, the gross properties of these objects may well be determined by the particle properties of dark matter. And conversely, inferred properties of Pop III stars (or even future direct observations) might be used to strongly constrain DM masses and interaction cross sections.

Acknowledgments

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JCAP11(2008)014