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Abstract: This paper proposes a theoretical growth model where seigniorage can be used to finance productive public spending, and show the existence of nonlinear effects between seigniorage and economic growth. Empirical evidence based on panel regression techniques provides some support for these nonlinear effects on a sample of OECD countries over the 1978-2005 period.

Keywords: economic growth, nonlinear effects of monetary policy

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Non-technical summary

The aim of this paper is to develop a theoretical model allowing for the seigniorage financing of productive public spending. In line with numerous recent empirical stylized facts (Kim & Willett, 2000, Black *et al.*, 2001, Bolton & Alexander, 2001), we emphasize the presence of nonlinearities between seigniorage and economic growth. Empirical evidence based on panel regression techniques on a sample of 22 OECD countries using annual data over the 1978-2005 period support the predictions of our theoretical model. We also test for a structural equation to investigate the existence of a joint inverted-U relation between both seigniorage and taxes, and growth, which is empirically confirmed by data.

1. Introduction

Early theoretical growth models conclude that inflation is harmful or at best neutral to economic growth, as Palivos & Yip (1995). Empirical work in Alexander (1997) emphasizes similar conclusions, but Paul *et al.* (1997) and Arai *et al.* (2004) question the robustness of this result. Further contributions isolate a negative correlation between inflation and economic growth, but only for high inflation (Black *et al.*, 2001, or Bolton & Alexander, 2001), suggesting that the relation between inflation and growth is probably nonlinear (Kim & Willett, 2000).

The aim of this paper is to emphasize the presence of nonlinearities between monetary policy and economic growth. For this matter, we develop in the next section a theoretical model allowing for nonlinear effects of seigniorage on growth. The empirical validity of these nonlinear effects is demonstrated in section 3 for a sample of OECD countries using panel regression techniques over the 1978-2005 period. Concluding remarks are reported in section 4.

2. The model

We consider a closed economy with a representative agent, a government and monetary authorities. The agent maximizes intertemporal utility, with a log-utility based on consumption ($c_t > 0$) and $\beta > 0$ the subjective discount rate[1]:

$$W = \int_0^\infty Log(c_t)\exp(-\beta t)dt \ , \qquad (1)$$

[1] Results are not modified for a more general isoelastic function $v(c_t) = (c^{1-\sigma} - 1)/(1-\sigma)$, with $\sigma > 0$ the inverse of the constant elasticity of substitution (see Minea & Villieu, 2007).

Output y_t is produced with private capital k_t and the flow of productive public spending g_t, with $0 < \varepsilon < 1$ the elasticity of output to private capital and we assume no congestion, as in Barro (1990) (all variables are expressed per capita):

$$y_t = k_t^{\varepsilon} g_t^{1-\varepsilon} \tag{2}$$

Household budget constraint is, in real variables ($\dot{x} \equiv dx/dt, \forall x$):

$$\dot{k}_t + \dot{m}_t = (1-\tau) y_t - c_t - \delta k_t - \pi_t m_t \tag{3}$$

Households use their income (y_t) to consume (c_t), invest $(z_t = \dot{k}_t + \delta k_t)$, with δ the private capital depreciation rate, and pay flat-rate taxes on output (τy_t), as in Barro (1990). We depart from Barro (1990) by assuming that agents hold money. The real balance stock is $m_t = M_t / P_t$, with M_t the nominal money stock and P_t the price level. $\pi_t = \dot{P}_t / P_t$ is the inflation rate, hence real money stock depreciation per unit of time is $\pi_t m_t$. To motivate a money demand, we introduce a cash-in-advance (CIA) constraint on all spending[2]:

$$c_t + z_t + g_t = m_t \tag{4}$$

Monetary authorities supply the nominal money stock M_t. Equilibrium on the money market determines the price level $P_t = M_t / m_t$. We are interested in monetary

[2] With a CIA on consumption only, raising money is always growth enhancing (Turnovsky, 1996).

policies that set an exogenous growth rate for money supply $\dot{M}_t / M_t = \theta$. Monetary authorities collect real seigniorage $\theta M_t / P_t = \theta m_t$ and transfer it to government:

$$g_t = \tau y_t + \theta m_t \qquad (5)$$

Relation (5) departs from the Barro (1990) budget constraint $(g_t = \tau y_t)$, since seigniorage can be used for government finance, as in Palivos & Yip (1995). However, Palivos & Yip (1995) consider exogenous unproductive public spending, while they are endogenous in our framework.

Maximizing (1) subject to (2)-(3)-(4), k_0 given and a standard transversality condition, yields the traditional Keynes-Ramsey relation (we further omit for the sake of simplicity time indexes) $\gamma \equiv \dot{c}/c = r - \beta$, with r the real interest rate. If investment is money-constrained, as in (4), the real interest rate becomes $r = y'(k)/(1+i) - \delta$, with i the nominal interest rate (Stockman, 1981). The return on private investment $y'(k)$ must be deflated by the monetary financing cost of new capitals $(1+i)$; hence r stands for (net of monetary financing costs) private capital productivity. Under the technology (2) and flat-rate taxes, the real interest rate is $r = \varepsilon(1-\tau)(g/k)^{1-\varepsilon}/(1+i) - \delta$. Using the government constraint (5), money market equilibrium $\dot{m}/m = \theta - \pi$ and the Fisher equation $i = r + \pi$, we find steady-state economic growth rate γ as:

$$\gamma = \frac{\varepsilon(1-\tau)(\tau+\theta)^{(1-\varepsilon)/\varepsilon}}{1+\theta+\beta} - \delta - \beta \qquad (6)$$

We can then demonstrate the following result:

Proposition 1:

(a) An inverted-U curve exists between money and economic growth;

(b) The optimal money growth rate is an increasing function of the tax rate.

(c) An inverted-U curve exists between taxes and growth.

Proof:

(a) and (b): using the first order condition $\partial\gamma(\tau,\theta)/\partial\theta = 0$ we get the growth-maximizing money growth rate $\theta^{*} = \frac{(1-\varepsilon)(1+\beta)-\varepsilon\tau}{2\varepsilon-1}$, which is inversely related to taxes.

(c): Using the first order condition $\partial\gamma(\tau,\theta)/\partial\tau = 0$, the growth maximizing flat-rate tax is $\tau^{*} = 1-\varepsilon-\varepsilon\theta$, with a similar explanation as in Barro (1990).

To enlighten *Proposition 1a,b*, remark that any increase in seigniorage is devoted to productive public expenditures that are growth-enhancing (numerator of (6)), but such an increase simultaneously raises the financing cost of private investment, which is harmful to growth (denominator of (6)). The trade-off between these two effects illustrates that productive public spending crowd-out private investment, results in the ceiling θ^{*}. As tax rate increases, the elasticity of public spending to seigniorage decreases, which explains why θ^{*} is inversely related to τ. Our findings reproduce numerous empirical results emphasizing the existence of threshold (nonlinear) effects between seigniorage or inflation[3] and growth. For instance,

[3] Generally, long-run inflation ($\pi = \theta - \gamma$) positively depends on seigniorage.

Thirlwall & Barton (1971) identify the positive effects of inflation rates inferior-to-8%, on growth and negative effects for inflation higher-than-10%. Gylfasson (1991) associates high-growth countries with lower-to-5% inflation rates, and low-growth economies to inflation higher than-20%, while Sarrel (1996) and Bolton & Alexander (2001) find a breakpoint in inflation to growth relation.

3. Empirical link between monetary policy and economic growth

3.1 The effects of seigniorage on economic growth

To investigate the empirical validity of our theoretical results, we perform panel regressions on a sample of 22 OECD countries[4] using annual data covering the period 1978-2005. Selected variables are real GDP growth (γ) and the tax rate (τ, computed as the fiscal and non-fiscal total revenues of public administration to GDP ratio) from *OECD Economic Perspectives*, with money growth θ from the *IMF* database *IFS*. *Table 1* exhibits results related to the estimation of a model including fixed effects in accordance with data properties.

Table 1 – The nonlinear relation between seigniorage and economic growth

	Dependent variable: real GDP growth rate		
	[i]	**[ii]**	**[iii]**
average dummy	0.026	0.026	0.025
θ	0.022 (0.012)*		0.024 (0.012)**
θ^2	-0.031 (0.009)***	-0.029 (0.010)***	
$\theta * \tau$		0.035 (0.029)	
$\theta^2 * \tau$			-0.062 (0.015)***

[4] Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, The Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom and the United States. Burdekin *et al.* (2004) suggest not to mix developed with developing countries when assessing inflation effects on output.

Observations (NT)	581	561	561
Countries	22	22	22
Adjusted R^2	0.2008	0.2032	0.2116
F Fisher	3.827 [0.00]	3.801 [0.00]	4.058 [0.00]

Notes:
a - standard errors are into parenthesis, p-values into brackets; we introduce country fixed effects using dummies; all dummies are significant; average dummy stands for the average country fixed effect.
b- ***1% significance; **5% significance; *10% significance.

Significant coefficients in regression [i] confirm the presence of nonlinearities between growth and seigniorage, describing an inverted-U shape (negative square-money growth coefficient) with positive economic growth-maximizing money growth (positive money growth coefficient).

While these results sustain our theoretical findings in *Proposition 1a*, we further give interest to *Proposition 1b*. For this matter, we specify a quadratic model in θ and allow the optimal money growth rate to linearly depend on the tax rate. In regressions [ii] $\gamma_{it} = \mu_i + \alpha_1 \theta_{it} \tau_{it} + \alpha_2 \theta_{it}^2 + u_{it}$ and [iii] $\gamma_{it} = \mu_i + \alpha_1 \theta_{it} + \alpha_2 \theta_{it}^2 \tau_{it} + u_{it}$, seigniorage and square-seigniorage respectively are multiplied by the tax rate.

Both models [ii] and [iii] exhibit inverted-U curves with positive optimal seigniorage values (see *Table 1*). Nevertheless, they imply opposite correlations between the optimal seigniorage value θ^* and the tax rate τ. In model [ii], the maximum $\hat{\theta}^* = \hat{\alpha}_1 \tau / (-2\hat{\alpha}_2)$ implies a positive correlation, while in model [iii] the maximum $\hat{\theta}^* = \hat{\alpha}_1 / (-2\hat{\alpha}_2 \tau)$ implies a negative correlation. However, as in model [iii] all estimated coefficients are significant, which is not the case for model [ii], we focus on what follows in model [iii]. As emphasized above, in this model the growth-maximizing estimated seigniorage rate is inversely related to taxes $\hat{\theta}^* = 0.194/\tau$, confirming the robustness of *Proposition 1b*.

3.2 Nonlinear joint effects between seigniorage, taxes and economic growth

In accordance with *Proposition 1*, both taxes and seigniorage exhibit nonlinear effects on economic growth. Next, we investigate the presence of a joint nonlinear relation between seigniorage, taxes and growth, in which both optimal money θ^* and taxes τ^* would depend on τ and θ respectively. Consequently, our regressions must enclose square-money growth and square taxes (for possible inverted-U curves), but also a multiple of $\tau * \theta$. *Table 2* summarizes results.

Table 2 – The joint nonlinear relation between taxes, seigniorage and growth

	Dependent variable: real GDP growth rate				
	[A]	**[B]**	**[C]**	**[D]**	**[E]**
average dummy	0.013	0.026	0.042	0.025	0.039
θ	0.020 (0.012)*				0.054 (0.029)*
$\tau * \theta^2$		-0.054 (0.022)**		-0.015 (0.025)	-0.029 (0.029)
$\tau * \theta$			0.034 (0.029)	0.348 (0.114)***	
θ^2	-0.029 (0.009)***		-0.029 (0.010)***		
τ	0.132 (0.182)				-0.031 (0.034)
$\tau^2 * \theta$		-0.050 (0.067)		-0.719 (0.261)***	-0.213 (0.063)
τ^2	-0.231 (0.216)		-0.084 (0.033)***		
Obs. (NT)	561	561	561	561	561
Countries	22	22	22	22	22
Adj. R^2	0.2134	0.2053	0.2123	0.2189	0.2174
F Fisher	4.87 [0.00]	3.87 [0.00]	3.95 [0.00]	4.15 [0.00]	3.98 [0.00]

Notes:
a - standard errors are into parenthesis, p-values into brackets; we introduce country fixed effects using dummies; all dummies are significant; average dummy stands for the average country fixed effect.
b- ***1% significance; **5% significance; *10% significance.

Note first the presence of non-significant coefficients in all [A]-[E] regressions. Depending on the selected model, an inverted-U relation exists on either taxes or seigniorage, but never a joint significant one. These results may receive at least two interpretations. First, despite five different specifications, we may have been unable to avoid colinearity problems between variables. One solution would be to search for econometrical specifications that avoid these colinearities. Secondly, it may emphasize that models [B]-[E] are unable to vigorously approximate our theoretical relation. Precisely, quadratic form may well reproduce individual inverted-U curves, while less adapted to approximate joint inverted-U curves.

To deal with this issue, we directly consider equation (6). For this purpose, assuming δ and β sufficiently small, one can log-linearize (6) and get:

$$\log(\gamma) = \log(\varepsilon) + \log(1-\tau) + (1-\varepsilon)/\varepsilon * \log(\tau+\tau) + \log(1+\tau) \qquad (7)$$

, with $\alpha_0 \equiv \log(\varepsilon)$ and $(1-\varepsilon)/\varepsilon$ included in α_2.

We then estimate the following equation on the same panel data set of OECD countries:

$$\log(\gamma_{it}) = \alpha_{0i} + \alpha_1 \log(1-\tau_{it}) + \alpha_2 \log(\tau_{it} + \theta_{it}) + \alpha_3 \log(1+\theta_{it}) + u_{it} \quad (8)$$

Table 3 – $\gamma(\tau,\theta)$

	$Log(\gamma)$
average dummy	-0.396
$Log(1-\tau)$	3.299 (0.327)***
$Log(\tau+\theta)$	1.801 (0.315)***
$Log(1+\theta)$	-2.985 (0.693)***
Obs. (NT)	502
Countries (N)	22
Adjusted R^2	0.1982
F Fisher	3.273 [0.00]

Notes:
a - standard errors are into parenthesis, p-values into brackets; we introduce country fixed effects using dummies; all dummies are significant; average dummy stands for the average country fixed effect.
b- ***1% significance.

All coefficients are now significant with a sign in accordance with theoretical expectations (positive for $1-\tau$ and $\tau+\theta$ and negative for $1+\theta$). These econometric results provide evidence in favor of the theoretical model developed in section 2, and emphasize the empirical relevance of a joint inverted-U relation between taxes, seigniorage and growth.

4. Concluding remarks

We developed in this paper a theoretical model allowing for the seigniorage financing of productive public spending. In line with numerous recent empirical stylized facts (Kim & Willett, 2000, Black *et al.*, 2001, Bolton & Alexander, 2001), we emphasized the presence of nonlinearities between seigniorage and economic growth. Empirical evidence based on panel regression techniques on a sample of 22 OECD countries using annual data over the 1978-2005 period support the predictions of our theoretical model. We also tested for a structural equation to investigate the existence of a joint inverted-U relation between both seigniorage and taxes, and growth, which was empirically confirmed by data.

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