

# Investigations into Flexible Operational Paradigms to Mitigate Variability

by

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Come to me, all you who are weary and burdened, and I will give you rest. Take my yoke upon you and learn from me, for I am gentle and humble in heart, and you will find rest for your souls. For my yoke is easy and my burden is light.

Matthew 11:28-30 (NIV)

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I dedicate this entire dissertation to my Lord and Savior Jesus Christ who, by His grace, used my time in Ann Arbor as a graduate student to bring me into life saving relationship with Him. Jesus paid it all for me, therefore all to Him I owe.

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# ABSTRACT

Investigations into Flexible Operational Paradigms to Mitigate Variability

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The work of this dissertation is concerned with the study of the effectiveness of paradigms of production flexibility to either improve system performance or mitigate system risk. A brief introduction to the concept of operational flexibility is provided in Chapter I. In Chapter II, we consider a cross-trained workforce on a serial production line, and we introduce a new strategy of worker cross training called a “fixed task zone chain” (FTZC) as a special type of zone based cross training. This new approach seeks to maximize the performance of a production line, in the same fashion as standard two skill chain, but with a significant reduction in the number of skills that must be cross trained. This allows a firm to maintain nearly the same levels of throughput, but at a fraction of the cross-training and implementation costs. Our approach shows that less than complete multi-functionality is often times sufficient. Our model focuses on a twelve station, four worker, and “constant work in process” (CONWIP) structure to show the value of the proposed paradigm: fixed task zone chaining. We provide an easily implemented and highly effective policy, the fixed first max shared (FFMS), for worker coordination and demonstrate its superiority versus a number of reasonable policies. With the FFMS policy, the FTZC achieved on average only 5% throughput

loss when compared with a set of benchmark structures. For most systems, 5% throughput loss is worth it to achieve the reduced skill cross-training the FTZC requires.

Chapter III targets our study of operational flexibility on the field of supply chain management. A flexible supply chain design is useful to mitigate the effect of stochastic supplier disruptions on operations and, especially, financial cash flows. The work of this chapter develops mitigation strategies for a firm to use in sourcing from flexible suppliers and demonstrates the conditions under which flexibility in the firm's supply chain is necessary. Consider a firm sourcing from two flexible but unreliable suppliers for a single product that is sold during a finite selling season. The firm is exposed to supplier defaults that can adversely affect its revenue during the selling season in two ways: 1) supply defaults that are idiosyncratic to each supplier, 2) a system-wide default that simultaneously disrupts both suppliers. Our objective function is to maximize total expected profit. That is, it must prevent finished goods inventory depletion while optimizing inventory holding cost. In addition to identifying the value of a supplier's diversification strategy, we also find the firm's optimal inventory order quantities in both the single and dual source environments. Within the single source case we analyzed that model and found that a single sourcing strategy for the firm is best under high marginal profit per unit conditions where disruption risk, both idiosyncratic and system wide, was low. Conversely, as disruption risk increases, the need to source with a second supplier also increases. Also, we learned the importance of the firm's cost structure, particularly as it relates to inventory holding cost.

Finally, to assist with the understanding of the flexibility paradigms, we have designed an instrument to promote the understanding of flexibility with respect to cross-training as well as to assist in its implementation. That is, we develop a hands-on, active learning experience placing participants "on the job" in a serial production line of cross-trained workers where participants can: 1) learn basic concepts of oper-

ations management, production control, and workforce agility; 2) understand system responsiveness and what can be done to improve it; 3) generate creative thinking and discussion on the value of flexibility; and 4) experience first-hand foundational factory physics concepts like cycle time, throughput, and Work In Process (WIP). In the Worker Agility Game (WAG), participants form teams and work together as cross-trained workers on a serial production line to create a product from raw materials to a finished good. Each team participates in a different worker cross-training paradigm for its production line, and simple time studies are completed of worker efficiency to ascertain how the cross-training policies work in comparison to each other.



## CHAPTER I

### Introduction

Operations managers must consistently deal with the inevitable consequences of variability on their systems. Although all variability is not bad, this work is concerned with the variability that harms system performance and threatens productivity. For example, consider unexpected demand changes, machine breakdowns, employee illness, or order cancelations which if not identified and controlled, can render an operation uncompetitive. In a global marketplace, characterized by incessant change, managerial insight is needed to increase a firm's responsiveness to change. Those firms which can adapt the best to ever changing circumstances will find themselves winners in global business competition. Moreover, for the business systems we study, increased adaptability goes hand in hand with increased efficiency, although this is not generally the case. Increased efficiency typically targets cost savings and an improvement of the business' profit margin.

The research interest that supports this work begins with the interaction between variability, efficiency, and profit. By understanding those dependence relationships, we have contributed to an approach that diminishes variability and increases profit. This approach is often referred to as operational flexibility, which is concerned with increasing capacity and responsiveness at the operations level of an organization. Operational flexibility seeks to improve systems that are susceptible to variability

by enabling them to adapt to changes cost-effectively with minimal interruption to system flow.

Hopp and Spearman [52] describe three basic elements operations manager's must design and control to respond to variability: (1) inventory, (2) capacity and (3) time. The research of this dissertation is intended to show that the implementation of operational flexibility, through the development of an agile workforce system and a flexible supply chain system, will increase the effectiveness of capacity and potentially inventory with respect to variability buffering, and these increases will provide a cost effective buffer to variability.

To achieve the agile workforce system, worker cross-training is targeted as a source for operational flexibility. A new cross-training paradigm is developed, the Fixed Task Zone Chain (FTZC), to increase system capacity and greatly reduce the cost of cross-training workers without a loss in system performance. To quantify the robustness of this new zone chain design, a metric of variation in work content is derived to measure variability in the process times of stations on a serial production line. The performance of the FTZC is measured against commonly used benchmark cross-training designs from the literature to demonstrate its efficiency across a wide range of variable production lines. We also provide a novel teaching and training role-play simulation that helps individuals and organizations to more deeply understand the FTZC and other paradigms for workforce agility. This can greatly assist in the adoption and effective deployment of workforce agility solutions.

The supply chain system compares and contrasts a firm's operational flexibility approach to mitigate the risk of supply disruptions. In our model, the firm can choose to single or dual source and carry inventory. The goal is to understand what type of operational flexibility best suits the firm's needs for a given set of conditions. Each flexibility decision of the firm provides increased inventory capacity and responsiveness, however it comes with a cost. Thus, our model maximizes profit to identify

which flexibility decision is best. Unique to this work is the model of unreliability of both suppliers in the firm's supply chain. These investigations into operational flexibility provide managerial insights into variability mitigation. Moreover, they are easily implementable making them sufficient for practice, and provide novel additions to the current state of the art.

## CHAPTER II

# Zone Training as an Effective Strategy for Inexpensive Cross-training in Serial Lines

### 2.1 Introduction

The need for flexibility in systems has never been greater as firms are faced with challenges such as increased demand uncertainty, process variability, supply network defaults, financial fluctuations, and credit disruptions. To counteract the adverse effects of demand uncertainty and process variability in manufacturing systems, labor cross-training offers capacity flexibility as a powerful variability buffering mechanism. The positive effects of a cross-trained workforce - including shortened cycle times, reduced inventory, and increased throughput - are well documented for a wide variety of systems (see, for example, Gurumurthi and Benjaafar [44] and Andradottir, Ayhan, and Down [4], where the relationship between throughput and flexibility is quantified. Research has also shown that lost productivity can be recaptured from reducing worker idle time via cross-training (see Hopp and Van Oyen [53] for a survey).

However, in most systems, all workers can not be cost-effectively fully cross-trained on all skills, even if cross-training is in use. When workers are cross-trained on serial lines, workers are often taught additional skills needed directly upstream or downstream of bottleneck stations. These skills enable workers to provide immediate

relief to problematic stations that are reducing the line’s throughput (see Hopp, Tekin, and Van Oyen [54]). However, as systems have become more complex, high labor training costs as well as skills that are very costly to cross-train, prevent a firm from freely adding additional skills to its workers’ skill sets. Therefore, the cost of cross-trained workers is one of the motivating factors of this work, and, as such, our goal is to develop labor cross-training structures that require minimal use of cross-training and are easily implementable for serial constant work-in-process (CONWIP) lines.

To accomplish this goal, the research described in this paper shows that zoned cross-training is an inexpensive form of cross-training in serial lines. *Zoned cross-training*, in our terminology, cross-trains the workers by assigning to each worker a zone, a group of consecutive work stations, that may or may not overlap. First, in overlapping zones, a worker’s skill set is classified as a zone and “neighboring workers are cross-trained so that they can share responsibility for machines where the zones overlap” (see McClain, Schultz, and Thomas [64]). Overlapping zones greatly increase the efficiency of serial production lines by avoiding worker idle time. We extend this concept of overlapping zones developed by McCain, Schultz, and Thomas by adding the second labor cross-training advance: skill chaining. It must be acknowledged that not all applications will permit the completion of the chain compared to a traditional straight line. It is far more practical in a U-shaped or similar shape that places the first and last stations in close proximity. We proceed under the assumption that the same worker can attend the input and output stations without a long walk time (as we ignore all walk times). Our application of their skill chaining, introduced in Hopp, Tekin, and Van Oyen [54] based on the work of Jordan and Graves [60], adds cross-trained skills to the edge (far upstream/downstream end) of a worker’s primary zone to create overlapping zones, and to indirectly reduce bottlenecks by “completing the chain”. We call this paradigm the Fixed Task Zone Chain (FTZC). The FTZC zone training merges two advances in labor cross-training: overlapping zones and skill

chaining in a setting in which there are significantly more stations than workers. In completing the chain the workers create a connected graph with two skills (arcs) for all the stations (nodes) in the serial line. As shown in the work of Hopp, Tekin, and Van Oyen [54] a two-skill chain design that connected the graph at two skills to “complete the chain” performed extremely well across a variety of policies and created the greatest throughput for a given WIP level. Whereas overlapping zones eradicates idle times, skill chaining shifts capacity indirectly between stations, as shown by Hopp, Tekin, and Van Oyen. Simultaneously, applying both these benefits, low idle time and ability to indirectly shift capacity, leads to our expanded concept of zoned cross-training. Going forward in this paper, we will refer to our expanded concept of zoned cross-training as zone chaining to represent the merger of these two advances, overlapping zones and skill chaining.

In this paper we leverage the existing cross-training paradigms to invent the FTZC. We contribute a deeper understanding of how the chain structure in zone chaining can achieve very high throughput with a limited amount of worker cross-training thus minimizing training costs and benchmark the performance. Furthermore, this new paradigm is applicable in a wider range of scenarios because each worker may have multiple “fixed” tasks that only a single worker can perform. By maintaining relatively high throughput and keeping labor training costs relatively low, zone chaining offers a new, flexible design for sharing work on a serial line.

Our scope is limited to establishing the effectiveness of a canonical fixed task zone chain. We leave it to further research to develop effective methods to tailor a FTZC implementation to a particular environment. Our work provides insight into the stability regions and dynamic line-balancing characteristics of our zone chain structure. It also determines the optimal fraction of time that a worker should be at a work station to balance the line for maximum throughput. In total, this paper suggests that implementing zone chaining in serial CONWIP lines improves upon the

current labor cross-training practices.

## 2.2 Literature Survey

Hopp and Van Oyen [53] and Treleven [86] both provide extensive surveys of the literature that examines workforce agility. Specifically, Hopp and Van Oyen [53] provide a framework for evaluating a flexible workforce. This framework describes key mechanisms by which cross-training can support the strategy of a firm, but they also identify the factors necessary to guide the selection of a worker coordination policy to implement workforce agility. Worksharing, how task-types are allocated on a line, is an important topic in literature on workforce agility. Askin and Chen [10] describe two types of worksharing as dynamic assembly line balancing (DLB) and moving worker modules (MWM). DLB generally has an equal number of machines and workers, where workers remain at their respective machines and either complete the work of their machine (i.e. fixed task-type), or complete the work of their upstream/downstream adjacent worker (i.e. shared task-type). Although this paper does not wholly fit in to general DLB models, we detail some of the relevant literature.

Downey and Leonard [32] studied fully cross-trained assembly lines and showed that it's best to assign workers to stations with the maximum workload. McClain, Thomas, and Sox [63] considered such lines without buffers and demonstrated that, even without buffers, DLB can increase efficiency. Van Oyen, Gel, and Hopp [89] also investigated fully cross-trained lines. Van Oyen, Gel, and Hopp showed that workers could collaborate on a single job and that throughput is maximized by an “expedite policy” where workers are assigned to the same jobs from start to finish. Throughput maximizing policies were also specified by Andradottir, Ayhan, and Down [3] and Andradottir and Ayhan [2] for two stage systems with two and three collaborative servers, respectively. Askin and Chen [10] further considered a two-stage DLB model under partial cross-training and defined the tradeoff between investing in WIP and

investing in workers.

A typical MWM line has more machines than workers. Workers attend the line within their zone and workers can be cross-trained when zones overlap and workers share machines. MWM lines can include, but are not limited to: 1) Toyota sewn product systems (TSS), for a review of TSS systems see Hopp, Tekin, and Van Oyen [54] and 2) the bucket brigade system (BBS) of Bartholdi and Eisenstein [13]. Bartholdi and Eisenstein show that when workers are sorted from slowest to fastest, with deterministic processing time, a BBS will always divide task-types optimally and the line will balance itself. (Note that Bischack [20] and McClain, Schultz, and Thomas [64] consider similar systems with random processing times.)

Gel, Hopp, and Van Oyen [38] developed the “fixed before shared” principle in their analysis of WIP constrained flow lines staffed by partially cross-trained workers. This principle prioritizes a flexible worker to a fixed task-type before a shared task-type. Saghafian, Van Oyen, and Kolfal [75] showed the optimality of the same principle in a “w” shaped parallel queueing structure under some conditions. Elements of optimal policies for more general cross-training patterns in serial lines were derived by Ahn and Righter [1]. McClain, Schultz, and Thomas [64] focused on zoned cross-training where the concept of “overlapping zones” is formally introduced. McClain, Schultz, and Thomas show the characteristics that influence the size of the overlap (i.e. the amount of cross-training). Building upon the work of McClain, Schultz, and Thomas, our work uses overlapping zones; however our work prescribes skill chaining a specific paradigm to develop greater insight into high-performance implementations that are especially appropriate in U-shaped lines. Although our paradigm is well suited to open lines, we emphasize closed (CONWIP) line to obtain stronger and more insightful results.

The principle of chaining, cross-training workers in a line to link all task-types into a chain, was introduced by Jordan and Graves [59] to achieve production flexibility



for plants that manufacture a portfolio of products. Brusco and Johns [22] applied chaining to cross-training by developing an integer linear programming model that minimized staffing costs subject to labor requirements. In their work, they observed cross-training that includes chaining structures that perform quite well. Jordan, Inman, and Blumenfield [60] studied chained cross-trained workers who perform maintenance tasks in a manufacturing plant. Jordan, Inman, and Blumenfield also found the robustness of chaining as a cross-training strategy in that “it performs well even if there are major changes to or uncertainty in system parameters.”

In general, the papers described above used simulation models and analysis to study the performance of their respective systems and to find the best worker coordination policy. The work of McClain, et. al [64], Hopp, et al. [54], and Jordan et al. [60], most closely relates to our work described here. To our knowledge, our work is unique in the development of zone chain structures for serial CONWIP production lines and provides managers of such lines with an easily implementable heuristic policy that offers an inexpensive alternative to cross-training.

The remainder of this paper is organized as follows. Section 2.3 describes the problem in detail and develops our basic zone chaining model. Section 2.4 presents analytical insights into our fixed-task zone chain (FTZC) structure with a CONWIP release policy. In Section 2.5 we apply our analytical insights to develop heuristic policies for our FTZC structure, the best of which we call the “fixed first, max shared policy” (FFMS). Our method of investigation is also detailed in this section. Section 2.6 compares the FTZC structure, under the FFMS policy, to three benchmark structures: full cross training, a new structure called a two-skill zone chain, and a new approximating model that we call the classic CONWIP approximation. Finally, in Section 4.6, we offer preliminary ideas about extensions of this work.

## 2.3 Problem Description

Our base model consists of twelve stations and four workers in a U-shaped serial production line. Raw materials begin at station one and proceed sequentially through to station twelve where they exit the line as finished goods. In this base model, a

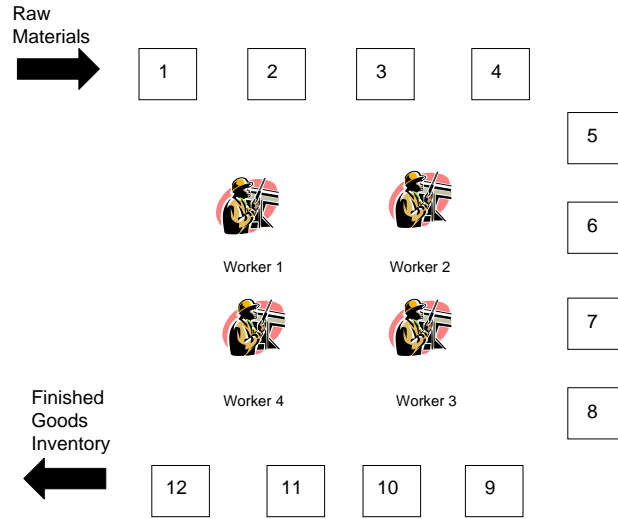


Figure 2.1: Base Model

“typical serial worksharing system” as described by McClain, Schultz, and Thomas [64], there are fewer workers than stations, and each worker is assigned to a zone which may overlap the zone of one or more other workers.

Each station of our base model accomplishes a unique task-type, that requires one worker to be available to complete a task. The model assumes workers are constantly available with no cost or changeover time needed to switch between stations. In addition, only one worker at a time can complete a specific job, but  $n$  workers can work at a station provided at least  $n$  jobs are available and the workers possess the skill for that station. Once a worker starts a task-type it cannot be preempted. The processing times of a task-type  $i$  at each station are exponentially distributed with mean  $T_i$ , while all four workers work at identical speeds (i.e., the standard worker model). A CONstant Work-In-Process (CONWIP) release policy is used to release

raw materials to the line. This release policy constrains the arrival of a new job to the line by the departure of an old one and allows us to mitigate variability in the line and to quantify the tradeoff between WIP and zone chaining. Our base model is large enough to be realistic, and to make our point, yet small enough to be computationally tractable.

A useful best benchmark for labor cross-training is *full cross-training*, where each worker is trained on every station. Applying full cross-training to the base model yields the structure shown in Figure 2.2 (left), which would provide maximum flex-

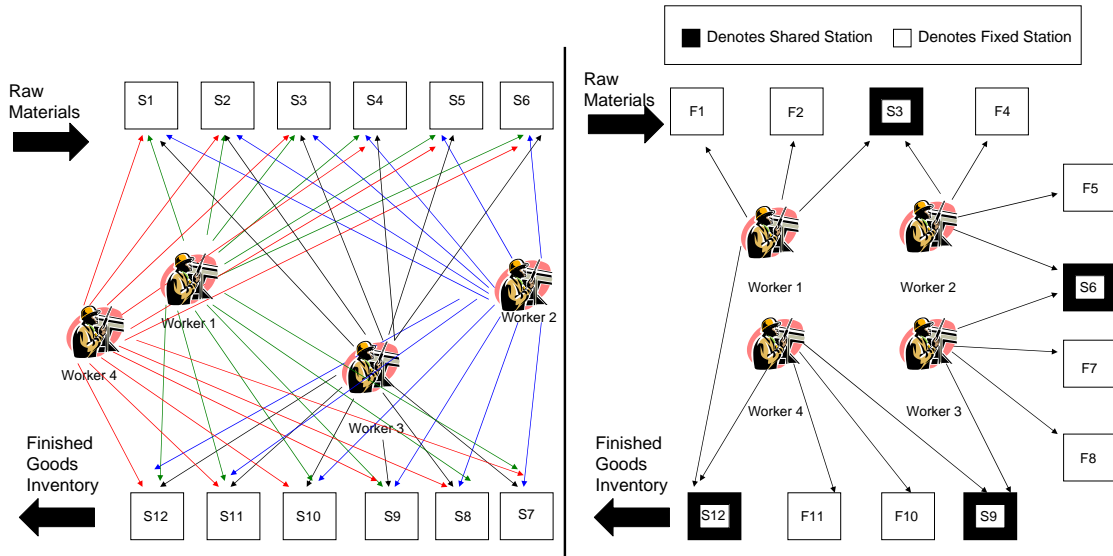


Figure 2.2: (left) Full Cross-training Structure; (right) Fixed Task Zone Chain (FTZC)

ibility (i.e., maximum throughput at any level of WIP) at maximum training cost. However, too much cross-training can be counterproductive as shown by Nembhard and Prichanont [67]. They showed that increasing the number of cross-trained skills eventually impedes performance, as well as diminishing returns, as workers forget how to perform task-types they've previously learned. To drastically reduce labor training costs while retaining high flexibility, we introduce our zone chaining structure of Figure 2.2 (right), the Fixed Task Zone Chain (FTZC).

FTZCs reduce the number of skills to be cross-trained while maintaining enough robustness and efficiency to achieve high levels of throughput. Stations within the FTZC are classified as being either *fixed* or *shared*. A fixed station is run by one worker who is the only one possessing the skill(s) to complete work at that station. A shared station is one that can be run by two workers with the skill to complete work at that station. This paper focuses on an instance of the FTZC paradigm for which each of four workers is responsible for two fixed stations (one upstream and one downstream) and two shared stations (one upstream and one downstream) called their zone. If we let  $Z_j$  denote the set of stations in the zone of worker  $j$ , then letting  $F$  and  $S$  denote fixed or shared for a given station,  $Z_1 = \{S12, F1, F2, S3\}$ ,  $Z_2 = \{S3, F4, F5, S6\}$ ,  $Z_3 = \{S6, F7, F8, S9\}$ , and  $Z_4 = \{S9, F10, F11, S12\}$ . Notice that the FTZC consists of only 4 zones,  $\{S3, S6, S9, S12\}$ , where the zones overlap and cross-training is needed. This dramatically reduces the number of cross trained skills from full cross-training (Full XT) as evidenced by Table 2.1.

	Total No. of Skills	No. of Cross-Trained Skills
Full XT	48	36
FTZC	16	4

Table 2.1: FTZC Skill Reduction

Table 2.1 shows that our FTZC reduces the number of cross-trained skills by 80% from full cross-training. This significant reduction in cross-training can significantly reduce labor training cost savings for a firm, provided the efficiency loss is manageable. Similar to Hopp, Tekin, and Van Oyen [54] we have “completed the chain” via the shared stations at the boundary of each zone, subsequently connecting all workers to all stations. With these connections in the FTZC, all workers can directly or indirectly assist with bottleneck stations. Because FTZC is designed to reduce labor cross training, in Section 2.6 we will analyze the efficiency of FTZC and determine its flexibility.

**Primary Zones:** To analyze the FTZC, (and to specify our test suite), we intro-

duce the term *primary zone* to further categorize the stations. The primary zone is a subset of a worker’s four stations and consists of three stations: the two fixed stations (upstream and downstream) and the third station is the downstream shared station at the end of the worker’s zone. For example, for the zone of worker 1,  $Z_1 = \{S_{12}, F1, F2, S3\}$ . The corresponding primary zone,  $PZ_1$ , for worker 1 would be  $PZ_1 = \{F1, F2, S3\}$ . The dotted lines of Figure 2.3 show that the most upstream

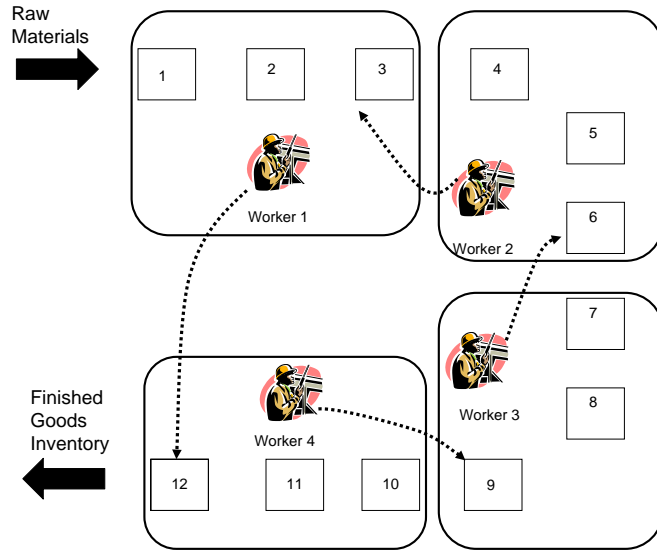


Figure 2.3: FTZC Primary Zones

shared station of a worker is excluded from that worker’s primary zone to avoid having the same station in multiple primary zones (i.e., we have formed a partition of the set of stations).

## 2.4 Analytical Insights

We analyzed our FTZC model with a CONWIP release policy. As an additional benefit, we gained insights into the server scheduling problem and used them to develop our heuristics. Specifically, we examined the FTZC structure, (See Figure 2.2(right)), to analyze stability, the optimal control/scheduling problem, and critical WIP.

The following notation, which is more general than the stylized FTZC model emphasized, will be used throughout this section:

1.  $N$  - total number of stations in a line
2.  $\mathcal{N}$  - the set of stations in the line,  $\mathcal{N} = \{1, \dots, N\}$
3.  $\oplus$  - mod  $N$  addition defined as: For  $i, j \in \mathcal{N}$

$$i \oplus j = i + j \quad \text{if } i + j \leq N; \quad \text{otherwise } i \oplus j = i + j - N$$

4.  $\ominus$  - mod  $N$  subtraction defined as: For  $i, j \in \mathcal{N}$

$$i \ominus j = i - j \quad \text{if } i - j \geq 1; \quad \text{otherwise } i \ominus j = i - j + N$$

5.  $W$  - total number of workers in a line
6.  $\mathcal{W}$  - the set of workers in the line,  $\mathcal{W} = \{1, \dots, W\}$
7.  $\mathcal{C}$  - set of worker capacities where  $\mathcal{C} = \{C_k, k \in \mathcal{W}\}$ . Our model is general for the sake of stability analysis, but for the rest of the paper  $C_k = 1$  for all  $k$ .
8.  $\mathcal{C}^s$  - set of worker capacities (or speeds) in units of work content per unit time worked for just shared task-types where  $\mathcal{C}^s = \{C_k^s, k \in \mathcal{W}\}$
9.  $\mathcal{F}_k$  - set of fixed stations that worker  $k$  is trained for where  $k \in \mathcal{W}$
10.  $\mathcal{S}_k$  - set of shared stations that worker  $k$  is trained for where  $k \in \mathcal{W}$ . Note that  $\mathcal{F}_k \cup \mathcal{S}_k$  is the skill set of worker  $k$ .
11.  $Z$  - total number of zones in a line
12.  $\mathcal{Z}$  - the set of zones in the line,  $\mathcal{Z} = \{1, \dots, Z\}$

13.  $\mathcal{S}$  - set of shared stations for the FTZC where  $\mathcal{S} = \{3, 6, 9, 12\}$
14.  $M$  - the number of fixed task-types per worker
15.  $T_n$  - mean processing time of the  $n^{th}$  station where  $n \in \mathcal{N}$
16.  $T_0$  - raw process time, the sum of the long term average process times of all stations. For the FTZC  $T_0 = \sum_{i \in \mathcal{N}} T_i$
17.  $T'_s$  - mean processing time of the  $s^{th}$  shared station where  $s \in \mathcal{S}$ ;  $T'_s = T_{3s}$  for  $s \in \mathcal{S}$
18.  $r_b$  - bottleneck rate, the rate of the station(s) having the longest mean processing time and the highest utilization (jobs/unit time)
19.  $D$  - the minimum number of cross-trained skills necessary in a symmetric D-skill chain to balance the line (as defined in Hopp, Tekin, and Van Oyen [54])

### 2.4.1 Stability

Similar to Hopp, Tekin, and Van Oyen [54] we define a line's capacity,  $\hat{\lambda}$ , as the maximum throughput rate it can achieve given full cross-training and infinite WIP. For the FTZC, since each worker is a standard worker then,  $\hat{\lambda} = W/T_0$ . Our zone chain is then a *balanced* line if the line has sufficient cross-training to achieve throughput of  $\hat{\lambda}$  as WIP tends to infinity. The following Theorem defines the stability parameters of our model. Our emphasis in this paper is FTZC with one shared task on each end of the zone, but we analyze a more general model in which the first D-1 stations of the downstream primary zones are cross-trained, where  $D \geq 2$ . (So  $D \leq 4$  in our 12 station line).

*Theorem 2.4.1.* The minimum value of  $D$  needed to balance the FTZC with standard

workers is  $D = \Delta + 1$  where

$$\Delta = \max_{i,j} \delta_{ij}, \quad \text{and}$$

$$\delta_{ij} = \min_{\delta=0,1,2,3} \left\{ \delta : \sum_{n=i}^{i \oplus (j-1)} (\hat{\lambda} T_{3n} - C_{n \oplus 1}^s) - \sum_{n=i \ominus \delta}^{i \ominus 1} C_{n \oplus 1}^s \leq 0 \right\}$$

**Proof.** See Appendix 2.8.1.1. ■

One important stability factor of our FTZC is that without fixed task-types, our FTZC reduces to the strongly connected network of shared task-types called a 2-skill chain by Hopp, et al. [54] and Iravani, Van Oyen, and Sims [56]. This network creates a pattern of cross-trained skills that offer two benefits: each worker can either directly or indirectly shift capacity to shared stations if bottlenecks occur at those shared stations or workers are freed from their shared stations. This added flexibility is quite attractive given the minimal amount of cross-training needed to achieve it.

## 2.4.2 Heuristic Policy Insight

In this section, we examine characteristics of optimal policies and identify properties to exploit in developing new heuristic policies that will be presented in Section 2.5. Our approach is based on sample path coupling methodology.

For any realization  $\omega$ , fixing the sequence of job arrival and service times, we denote by  $D_t^\pi(\omega)$  the number of completed jobs by time  $t$  under a worker allocation strategy  $\pi$ . Theorems 2.4.3 - 2.4.5 determine policy properties that maximize job completion along any sample path; that is, these theorems characterize a policy  $\pi^*$  such that

$$D_t^{\pi^*}(\omega) \geq D_t^\pi(\omega) \quad \forall \quad t, \pi, \omega.$$

This pathwise, finite-horizon is a highly demanding type of throughput metric. Of course, such a policy  $\pi^*$  also maximizes long-run throughput permit time. The results



of Theorems 2.4.3 and 2.4.4 hold for arbitrary service times and worker speeds, while Theorem 2.4.5 holds only for standard workers. Theorem 2.4.3 applies to workers 2 through 4 in our four-worker FTZC, while Theorems 2.4.4 and 2.4.5 apply to all workers. Theorems 2.4.3 and 2.4.4 generalize Lemma 3.1 of Ahn and Righter [1], which holds for domestic processing times.

One important outcome from these Theorems is the significance of the *Last Buffer First Served* (LBFS) rule, which we will define next, to determine the priority of a worker's task-type. Note that for worker 1, station 12 is prioritized over station 3, due to the closed network structure.

*Definition 2.4.2.* Last Buffer First Served (LBFS) rule - the priority order of job completion within a worker's set of stations is to first check for an available job at the most downstream station in their zone, which is the downstream shared station. Any job at that station always has first priority. If no job is present in the most downstream station, the worker moves upstream in the reverse order the stations are entered into by jobs and completes the task-type at the first station where a job is present. After completing the task-type the worker returns to the most downstream shared station and repeats the process.

*Theorem 2.4.3.* Consider worker  $i$ ,  $i \in \{2, 3, 4\}$ . When no jobs are waiting at the worker's downstream shared station, the job completion process is maximized along every sample path by a policy that (i) does not idle worker  $i$  when fixed task-types are present, and (ii) follows the LBFS rule among the worker's set of task-types.

**Proof.** See Appendix 2.8.1.2. ■

The above result reveals the effectiveness of the LBFS rule for a given worker's zone following our heuristic policies. Intuition indicates that to maximize throughput within the line, priority should be given to jobs that are closer to finished goods (downstream) than raw materials (upstream). The result confirms the intuition.

However, with our FTZC, a worker has both fixed task-type stations and shared task-type stations. To find the best policy, we analyzed policies that either prioritize jobs independent of the station type or policies that prioritize jobs dependent on the station type. The following Theorem helps to address this issue.

*Theorem 2.4.4.* A policy that follows the LBFS rule maximizes the job completion process along every sample path within the class of policies that give priority to each worker’s fixed task-types versus its shared tasks.

**Proof.** See Appendix 2.8.1.3. ■

Having shown that workers maximize FTZC throughput when they follow the LBFS rule; it is important to observe that this rule is easy to implement.

Now we consider situations where two workers are competing for one task-type at shared station  $\mathcal{S}$ , at time  $t$ , because only one job is available in their neighboring zones (but other jobs are present in other zones). Let  $\{U_{\mathcal{S}}, D_{\mathcal{S}}\}$  denote the upstream and downstream workers, respectively. We determine that the optimal policy gives preference to the downstream worker.

*Theorem 2.4.5.* The downstream worker,  $D_{\mathcal{S}}$ , is preferred when two workers are competing for a single task-type available to both of them.

**Proof.** See Appendix 2.8.1.4. ■

### 2.4.3 Critical WIP

As defined by Hopp and Spearman [52], the critical WIP level,  $\omega_0$ , is defined as the WIP level at which a line without congestion achieves maximum throughput with minimum cycle time. Critical WIP is useful as a way of quantifying how lean a system is operating via the ratio of WIP to  $\omega_0$ .

*Theorem 2.4.6.* In the FTZC structure, assuming equal and deterministic processing times at each station and a symmetric structure of skill chaining, the critical WIP is equal to the number of zones,  $\omega_0 = Z$ .

**Proof.** See Appendix 2.8.1.5 ■

The critical WIP value for the FTZC structure of Figure 2 (right) is four (as that is the number of zones). In practice, however, (and in our analysis of the FTZC) the line is not deterministic; it does experience congestion, and thus higher multiples of critical WIP must be explored.

## 2.5 Design of Heuristic Policies for FTZC

To determine a heuristic that can be a recommended policy for the FTZC structure, we developed and compared many policies via simulation, and we present the best (or most instructive) ones here. Determining a high-performance policy is necessary to determine the efficiency and robustness of the FTZC structure, in addition to providing a policy for practice.

### 2.5.1 Fixed First Max Shared (FFMS)

We now develop a novel policy, that combines four ideas, termed Fixed First Max Shared (FFMS), which to our knowledge has not been suggested before. First, it gives preference to the worker's fixed stations before the shared stations. In other words, the worker will only complete a job at shared stations if no jobs need to be completed at either fixed station. The phenomenon is called *fixed first*. Second, within the fixed stations, the LBFS rule is applied such that the downstream fixed station is preferred to the upstream fixed station. Third, if no jobs are available at the fixed stations, the worker applies a maximum queue policy at the shared stations. This

phenomenon is called *max shared*. Fourth, if two workers of the same shared station are simultaneously available, preference is given to the most downstream worker.

The FFMS policy is effective in part because it is easy to implement; although it must also perform well to be useful. The basis for the FFMS policy came from the following insights derived from our analytical results of Section 4.

*Insight 2.5.1.* A preference for fixed task-types will help to minimize worker starvation.

As previously stated, one of the goals of the FTZC structure is to maximize throughput and increase flexibility so that it will provide high throughput with low WIP and a small investment in cross-training and staffing (i.e., run lean) under as wide a range of operating conditions as possible. Throughput loss arises from worker starvation. Downstream workers will be starved when there are upstream bottlenecks (or long service time realizations) at *fixed task-type stations*, because these are the stations where the system has no flexibility. In their study of zoned cross-training, Koole and Righter [61], focused on systems with no shared tasks and showed that the LBFS policy is optimal in many cases. Thus, their work and our Theorem 2.4.4 supports the implementation of an LBFS rule among the fixed task-type stations so as to maximize the job completion process for the line.

*Insight 2.5.2.* Throughput is enhanced at a shared station when the downstream worker of the shared station pulls work, as opposed to the upstream worker of the same shared station pushing work.

Once a job is completed at a shared station it proceeds downstream through two fixed task-type stations to the next shared station unless the job is at station twelve, where it will immediately proceed to finished goods inventory. Given this, Theorem 2.4.5 supports our understanding that throughput is maximized when preference is given to the downstream worker of a shared station. We believe this insight has greater impact in open models, especially in light traffic, than is demonstrated in this

CONWIP model.

### 2.5.2 Fixed First Threshold Shared (FFTS)

We now build upon the FFMS policy to develop a more analytically-based heuristic, termed the Fixed First Threshold Shared (FFTS) policy. The FFTS policy is similar to FFMS in that FFTS gives preference to fixed stations over shared stations, and, within the fixed stations, FFTS applies the LBFS rule. However, the priority of the two shared stations attended by a worker is determined by comparing two threshold indices, each of which is computed from the observed queue length,  $Q_n(t)$ . The threshold index for each station incorporates queue length indicators such as optimal worker service time and average WIP level at that station. To describe the threshold index we define the following:

- $\{i', i' \oplus 3\}$  - the set of shared stations for any worker  $j$  where  $i'$  is the upstream shared station of worker  $j$  and  $i' \oplus 3$  is the downstream shared station
- $Q_{ij}$  - the observed queue length of worker  $j$ 's station  $i$
- $y_{ij}$  - the proportion of time worker  $j$  should spend at station  $i$ . In addition to the notation of Section 2.4, we define the following:

- $\Theta$ : line throughput
- $\mathcal{T}_j$ : skill set worker  $j$  is trained for
- $\mathcal{W}_i$ : set of workers trained for shared station  $i$
- $I_j$ : total idle time of worker  $j$

To avoid the problem of infinitely many solutions that are generated by the linear program at the heart of this procedure, we implement the following

quadratic program with “throughput penalty”  $\alpha$  and “sharing freedom restriction”  $\beta$  (where  $\alpha$  is a small positive number and  $\beta$  is slightly larger than 1):

$$\max \Theta - \alpha \sum_{j \in \mathcal{W}} (y_{i'j} - y_{i' \oplus 3j})^2 \quad (2.5.1)$$

*s. t.*

$$(Fixed\ Stations) \Theta \leq \frac{y_{ij}}{T_i} \leq \beta\Theta \quad \forall j \in \mathcal{W}, i \in \mathcal{F}_j \quad (2.5.2)$$

$$(Shared\ Stations) \Theta \leq \sum_{j \in \mathcal{W}_i} \frac{y_{ij}}{T_i} \leq \beta\Theta \quad \forall i \in \mathcal{S}_j \quad (2.5.3)$$

$$(Idle\ Times) I_j + \sum_{i \in \mathcal{I}_j} y_{ij} = 1 \quad j \in \mathcal{W} \quad (2.5.4)$$

$$\Theta \geq 0, y_{ij} \geq 0, I_j \geq 0.$$

This quadratic program approximates the optimal proportion of time a worker should spend at each station within the zone. The objective function (2.5.1), seeks to maximize throughput; however, to ensure the uniqueness of the solutions, the function includes a small throughput penalty,  $\alpha = .001$ , to slightly penalize throughput if the allocations of workers attending a shared station differ. This penalty drives the workers’ effort allocation at the shared stations to be as close to equal as possible, thus yielding unique solutions that are intuitively desirable and perform well. The linear program at its heart can be seen by setting  $\alpha = 0$  and removing the  $\leq \beta\Theta$  constraints in (5.2) and (5.3). The fixed and shared station constraints, (2.5.2) and (2.5.3) respectively, are bounded below by the near-optimal throughput and bounded above by a 102% freedom restriction,  $\beta$ . The  $\leq \beta\Theta$  constraint shifts a worker’s excess capacity into the idle time decision variables to provide a clear and near-optimal picture of the worker’s utilization. This shift occurs because the idle time constraint introduced in equation 2.5.4 allows  $y_{ij}$  to approximate the minimal fraction of

time that worker  $j$  should spend on station  $i$ , so that these quantities can be used in our FFTS heuristic.

To derive the average WIP level, we introduce a new model called the *Classic CONWIP Approximation* (CCA) to serve as a lower bound on the performance to be expected of the FTZC paradigm. We first consider the serial line of our base model with 12 stations each with unique exponential processing times. However, instead of 4 workers with zones, our classic CONWIP approximation has 12 dedicated workers, 1 worker per station, where each worker only attends one station. Thus, it is a standard CONWIP model. In Section 2.6 we will discuss the performance of this system as it relates to our FTZC structure, but here we need only solve for  $V_{ij}(k)$  to be used in the FFTS heuristic. That is, let  $V_{ij}(k)$  denote the average WIP level at worker  $j$ 's station  $i$  for a given WIP level  $k$ . We apply the mean value analysis algorithm of Hopp and Spearman [52]. See Appendix 2.8.2.1 for specific details on the algorithm. We are now ready to fully define that FFTS policy introduced above.

*Lemma 2.5.3.* For the FFTS policy, between worker  $j$ 's two shared stations, priority is given to the downstream station,  $(i' \oplus 3)$ , if

$$\left( \frac{Q_{(i' \oplus 3)j}}{V_{(i' \oplus 3)j}(k)} \right) y_{(i' \oplus 3)j} \geq \left( \frac{Q_{i'j}}{V_{i'j}(k)} \right) y_{i'j} \quad \forall j \in \mathcal{W}, \quad (2.5.5)$$

otherwise, priority is given to the upstream station.

The ratio of  $Q_{ij}$  to  $V_{ij}(k)$  in (2.5.5) scales the queue lengths of a worker's shared stations to account for the fact that asymmetrical lines will have the average station WIP level vary by station. To incorporate an estimate of the effort to be applied at each station, the  $y_{ij}$  terms further scale the ratios for an apples-to-apples comparison. We expect that the FFTS policy will outperform the other policies in our set, given

the increased information that the threshold index provides the FFTS policy to help the worker make a “better” decision when choosing between the shared stations. To check this hypothesis, we next define our method of investigation and then provide the heuristic policy benchmarking results.

### 2.5.3 Method of Investigation

To compare the set of policies, we developed a discrete event simulation in C++ code. We analyzed system throughput for various mean process times configurations and eight CONWIP levels. Each test case utilized 50 simulation replications initialized with an empty line; included a warm-up period of 3,000 jobs; and stopped after 8,000 jobs of finished goods inventory had exited the line. To reduce the variance in estimating the performance differences across policies we employed the technique known as “common random numbers” (see Law and Kelton [62]).

#### 2.5.3.1 Heuristic Policies

In addition to FFMS and FFTS, we report on three additional policies for prioritizing jobs from the literature on serial production lines: Random, Last Buffer First Served, and Maximum Queue (also known as Longest Queue).

- *Random (RND)* - This policy allows each worker to choose, at random but with equal probability, any station with an unfinished job. The queue lengths of the stations are not compared and preference is not given to any particular station. This policy in no way exploits the structure of the FTZC paradigm; thus, it can be considered as a lower bound upon which a good heuristic should improve.
- *Last Buffer First Served (LBFS)* - As stated in definition 2.4.2, the order in which a worker schedules jobs in his/her zone is: 1) check for an available job at the downstream shared station; 2) then check upstream stations in reverse



order and select the first nonempty station. This policy is derived from the craft mode of production termed “Pick and Run” in Van Oyen, et al. 2001 [89].

- *Maximum Queue (MaxQ)* - This policy assigns worker  $w$  to station  $n$  such that

$$n = \max_{n \in \{\mathcal{F}_w \cup \mathcal{S}_w : Q_n(t) > 0\}} Q_n(t),$$

where  $Q_n(t)$  is the observed queue length at workstation  $n$  at time  $t$ .

### 2.5.3.2 Test Suite

To compare the FTZC heuristic policies we study a wide variety of problem instances and check for robustness, by developing a test suite of 768 unique problem instances each having a unique mean processing time vector.

In each of the 768 problem instances the location, frequency, and magnitude of the bottleneck(s) are varied. The test suite is based on two multipliers, an across primary zone (APZ) multiplier and a within primary zone (WPZ) multiplier. To construct each instance, we begin with the standard, uniform line, all mean processing times for each of the 12 stations are  $\frac{1}{3}$ . Then, each station’s mean processing time is multiplied by both the APZ and WPZ multipliers of Tables 2.2 and 2.3. These multipliers are carefully chosen to ensure that although the means per station vary, the total raw processing time of the line stays equal to the number of workers, thus providing a maximum possible throughput of 1 in every instance.

APZ	PZ1	PZ2	PZ3	PZ4
<b>A</b>	1	1	1	1
<b>B</b>	1.42	0.86	0.86	0.86
<b>C</b>	1.42	0.58	1.42	0.58

Table 2.2: Across Primary Zone (APZ) Multipliers

Each problem in the test suite is denoted by a five character code (e.g. *B2134*) that identifies the multipliers used to create that problem instance. The first character

WPZ	U. Fixed	D. Fixed	D. Shared
<b>1</b>	1	0.9	1.1
<b>2</b>	1	1	1
<b>3</b>	1.1	0.9	1
<b>4</b>	1	1.1	0.9

Table 2.3: Within Primary Zone (WPZ) Multipliers

- either A, B, or C - represents the APZ multiplier used for that problem instance. For example, the problem instance  $B2134$ , the “B” APZ multiplier is used. The APZ multipliers alter the line processing times across the primary zones ( $PZ1 - PZ4$ ). Specifically, the **A** group of APZ multipliers serves as the standard line, the **B** group has one bottleneck in  $Z1$ , and the **C** group has two bottlenecks in  $Z1$  and  $Z3$  with two fast zones in  $Z2$  and  $Z4$ . As shown in Table 2.2, the multipliers corresponding to a given letter (A, B, or C) apply to each station within a primary zone.

The second through fifth characters of the code are the WPZ cases ranging from 1 to 4. (These identifiers generate the 768 codes in the test suite that range sequentially from  $A1111$  to  $C4444$ ). Each of the four numbers in the code, reading from left to right, represents the multiplier for the stations within that primary zones (see Figure 2.3). That is to say, the first number (second character in the code, which is a 2 in our  $B2134$  example) corresponds to  $PZ1$ , the second number (third character) corresponds to  $PZ2$ , and so forth. See Appendix 2.8.3.1 for a detailed example of how the multipliers are used.

Primary zone  $j$  consists of worker  $j$ 's upstream fixed station (**U. Fixed**), downstream fixed station (**D. Fixed**), and downstream shared station (**D. Shared**).

We test each problem instance for eight CONWIP levels of  $\{4, 6, 8, 10, 12, 24, 36, 48\}$ . These CONWIP levels were chosen because they range from  $\omega_0$  to  $12\omega_0$  and represent a realistic and instructive range to study for our system with a critical WIP,  $\omega_0 = 4$ . The performance for the extremely low CONWIP levels of 4 and 6 reveal the variability buffering power of the FTZC. We also identify the asymptotic behavior for high

levels, 36 and 48. Between these extremes, we are most interested in the “practical” CONWIP levels of  $2\omega_0$  to  $6\omega_0$ .

### 2.5.4 Comparison of Policies

Given the sophistication of the FFTS policy, and the amount of effort required to develop it, we expected it to perform the best; however, after comparing the performance of the heuristics in our FTZC policy set across our test suite, we were pleasantly surprised by what we learned.

*Observation 1.* The FFMS policy is the best policy of our policy set; i.e., it results in the highest average throughput across the entire test suite, it achieves the highest throughput for each tested CONWIP level.

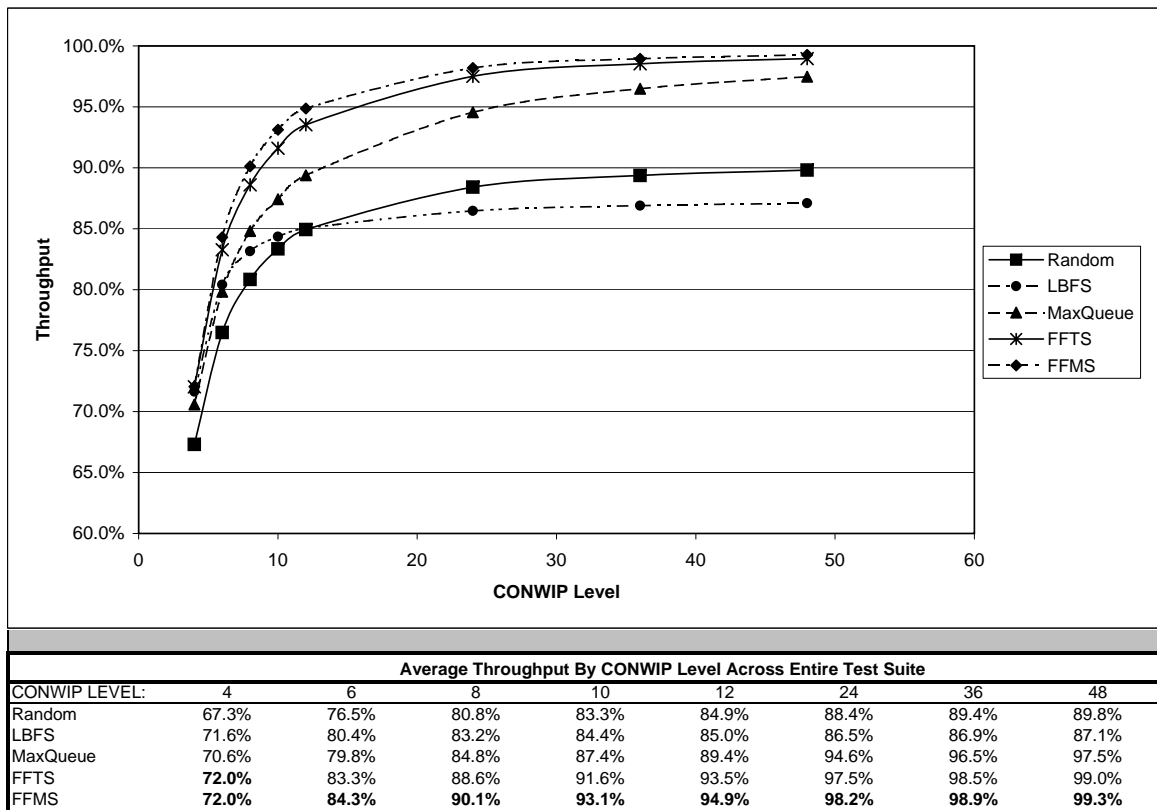


Figure 2.4: FTZC Heuristic Policy Performance for Entire Test Suite

Figure 2.4 shows that when throughput is averaged across the test suite for each policy, the FFMS consistently provides the highest throughput, particularly for “practical” CONWIP levels of  $2\omega_0$  to  $6\omega_0$ . We call this excellent performance of FFMS a pleasant surprise because having a policy that is both simple to implement and highly effective makes it very powerful. As expected, the random and max queue policies provided significantly lower throughput.

For extreme cases of high and low WIP levels, some policies perform as good or better than the FFMS policy. For example, the LBFS policy, for some problem instances, outperforms the FFMS at a CONWIP level equal to the critical WIP value. Intuitively, with a low WIP level, the throughput is low and the CONWIP system becomes closer to an open problem instance, for which LBFS is optimal for all but worker 1. Given that these alternative policies sometimes outperform FFMS, we also studied policy performance at a micro level rather than only at a grand average over the entire suite. Specifically, we divide the whole suite (768 problem instances) into three “sub-suites” of equal size (256 problem instances), denoted by the APZ multiplier groups (A, B, or C). We then develop and apply a system classification metric, *Degree Of Imbalance*, to study how the FFMS policy performs under specific conditions of work content imbalance that would make it relatively easy or difficult for the FTZC structure.

#### 2.5.4.1 Degree of Imbalance

To gain deeper insight into the effectiveness of the FTZC and also the other zone chain structures of Section 2.6, we develop an index called *degree of imbalance* ( $DOI(m)$ ), that denotes the magnitude of imbalance in a given problem instance,  $m \in \{1...768\}$ . We do not want the  $DOI(m)$  metric to be tuned specifically to our test suite. Rather, in the interest of generality, we define an intuitively justified metric and show that it is useful for our test suite.

We calculate our  $DOI(m)$  index for each problem instance as the sum of two components: (1) the degree of imbalance across the primary zone ( $DOI^{APZ}(m)$ ) and (2) within the primary zone ( $DOI^{WPZ}(m)$ ). For example, each multiplier in Table 2.2 corresponds to a specific *group*,  $x \in \{A, B, C\}$ , and a specific primary zone,  $z \in \{1, 2, 3, 4\}$ . To calculate  $DOI^{APZ}(m)$ , we compute the sample standard deviation,  $\sigma^x(m)$ , of the four APZ multipliers that correspond to problem instance  $m$ .

$$DOI^{APZ}(m) = \sigma^x(m) \quad \forall m \in \{1 \dots 768\}. \quad (2.5.6)$$

To calculate  $DOI^{WPZ}(m)$  we first compute the standard deviation,  $\sigma_z^T(m)$ , of the processing times for each of the four zones,  $z$ , within a particular problem instance,  $m$ . We then find the mean,  $\mu_z^T(m)$ , for the same set of processing times. Then we can define,

$$DOI^{WPZ}(m) = \sum_{z=1}^4 \left( \frac{\sigma_z^T(m)}{\mu_z^T(m)} \right) \quad \forall m \in \{1 \dots 768\}. \quad (2.5.7)$$

Thus the degree of imbalance index is given by,

$$DOI(m) = DOI^{APZ}(m) + DOI^{WPZ}(m). \quad (2.5.8)$$

A higher DOI index corresponds to greater system imbalance, which we expect will result in lower throughput. (The DOI index is 0 for the standard problem instance (*A2222*) in our test suite, which is the problem instance where all station processing times are identical, a perfectly balanced line.) Dividing the test suite into three smaller suites, groups  $x \in \{A, B, C\}$ , results in the following average DOI for the set of problem instances in the sub-suites shown in Table 2.4. Because the FTZC structure is parsimonious in its use of cross-training, it is interesting to investigate how it performs as a function of line imbalance and the sensitivity of FFMS. Due to

Sub-suite	Average DOI
A	.300
B	.580
C	.785

Table 2.4: Average DOI for Test Sub-Suites

the fact that the test suite is formed from the product of APZ and WPZ factors, it is useful to analyze the operating environments of low (suite A), moderate (suite B), and high (suite C) line imbalance in Figures 2.5, 2.6, and 2.7, respectively.

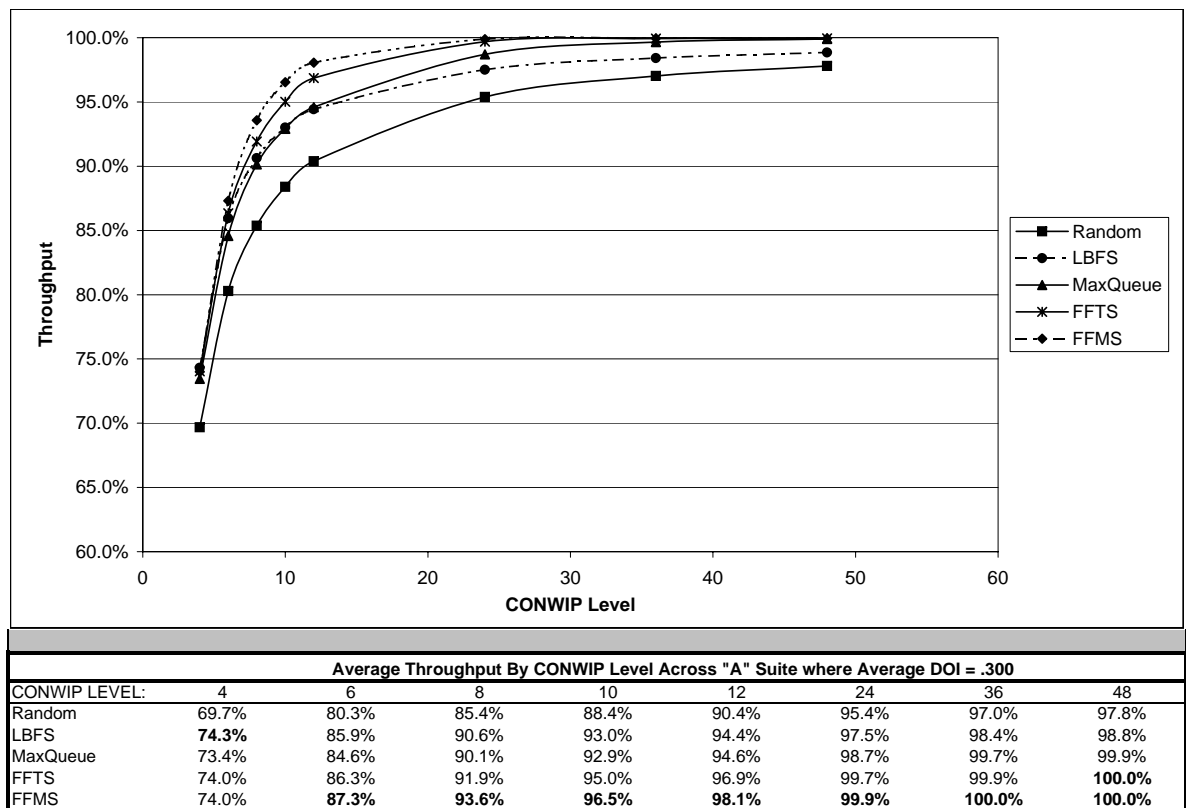


Figure 2.5: FTZC Heuristic Policy Performance under Low Line Imbalance (A Suite)

In Figure 2.5 (and the figures to follow) the bolded throughput values in the table below the graph represent the best performing policy for that CONWIP level. Under low line imbalance, LBFS at a CONWIP level of 4 performs the best by .3%, and FFMS and FFMS at CONWIP levels of 24 or higher perform about the same. Overall,

however, FFMS still performs the best in this environment, particularly across the “practical” CONWIP levels. For both suite B (see Figure 2.6) and suite C (see Figure

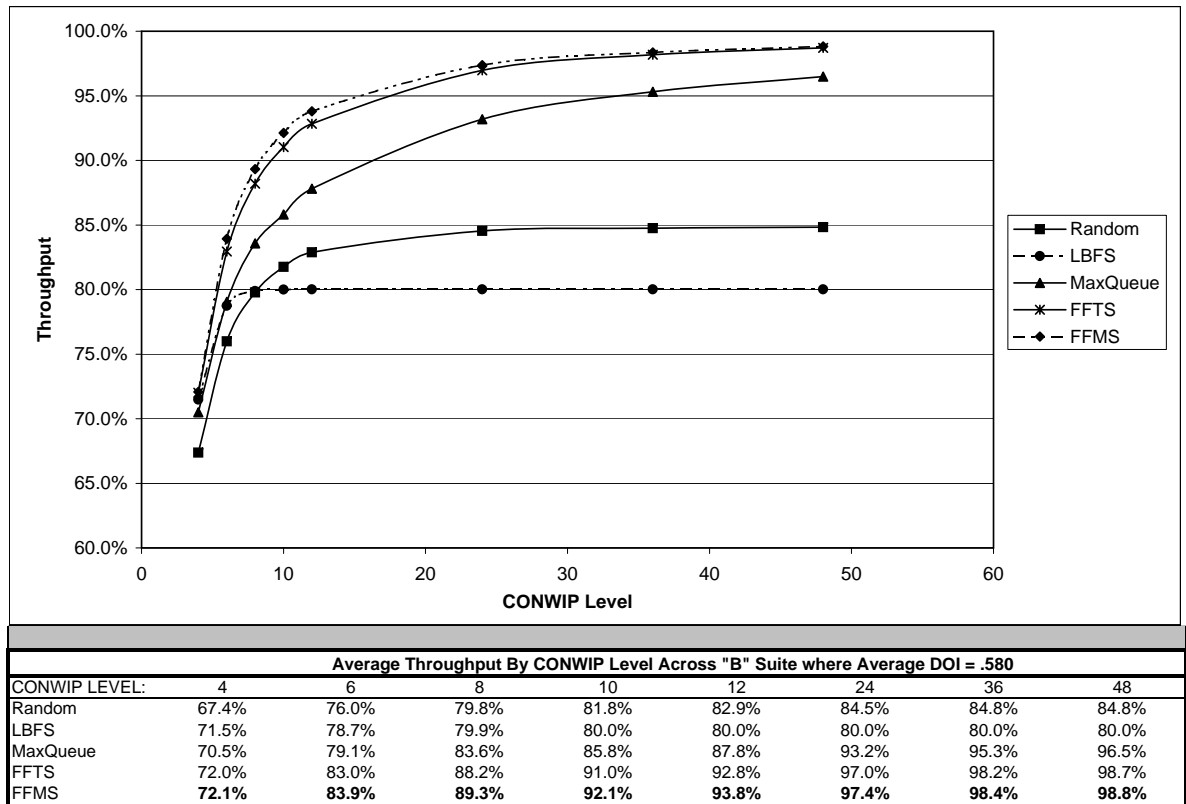


Figure 2.6: FTZC Heuristic Policy Performance under Moderate Line Imbalance (B Suite)

2.7) the FFMS policy dominates as well; therefore, we observe the following

*Observation 2.* In operating environments of low, moderate or high DOI line imbalance, the FFMS policy is the best policy of our policy set.

In general, at low DOI values, which represent relatively stable lines, other policies may be as good or a little better than FFMS; however, FFMS is the most effective policy over a wide range of operating conditions. The effectiveness of the FFMS policy is further confirmed in Figure 2.8, which shows the percent of cases where the FFMS policy is the best. In the entire test suite, in *all* 768 problem instances tested for the “practical” CONWIP levels of {6, 8, 10, 12, 24}, FFMS performed the best in

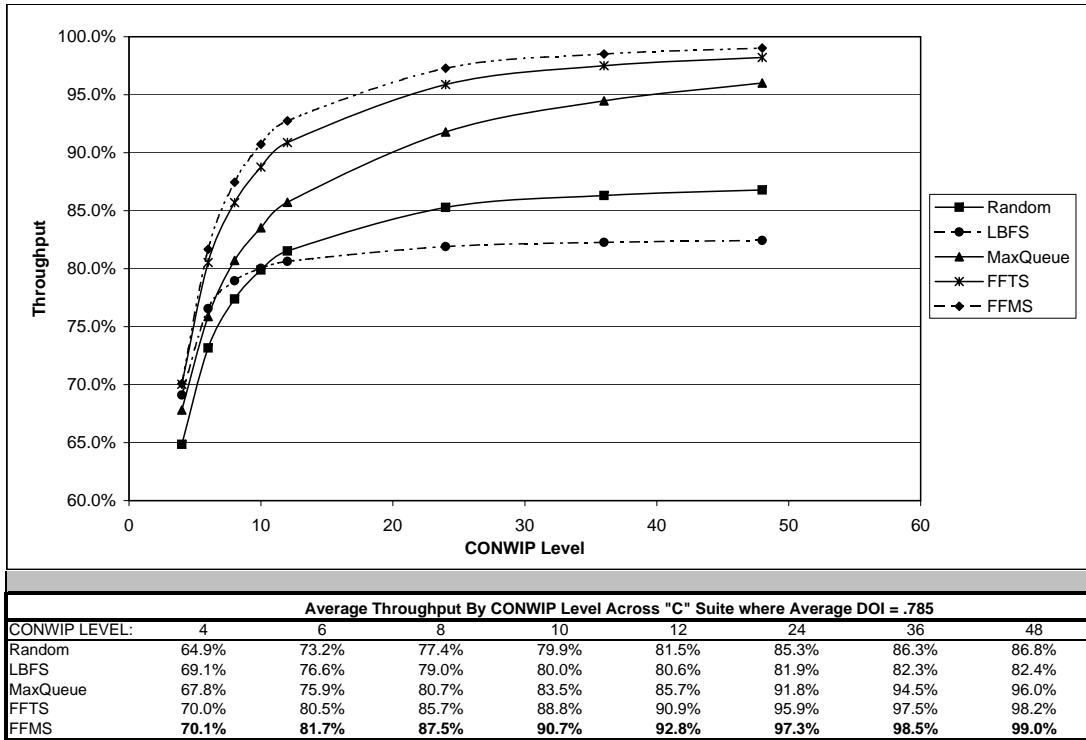


Figure 2.7: FTZC Heuristic Policy Performance under High Line Imbalance (C Suite)

our policy set. Given the magnitude of the FFMS policy superiority, we can make the following observation.

*Observation 3.* For “practical CONWIP levels” of  $2\omega_0$  to  $6\omega_0$  in our FTZC structure, the FFMS policy is superior for our policy set.

It is reassuring, to confirm that the FFMS policy dominates despite the considerably more sophisticated analysis needed to define its nearest competitor, the FFTS policy. The FFMS is also a simple policy, which makes this a very significant finding: a very simple and practical policy dominates the best alternatives we have been able to devise. Since the FFMS policy is the best policy for our FTZC structure next we examine the resulting performance of FTZC as a structure.



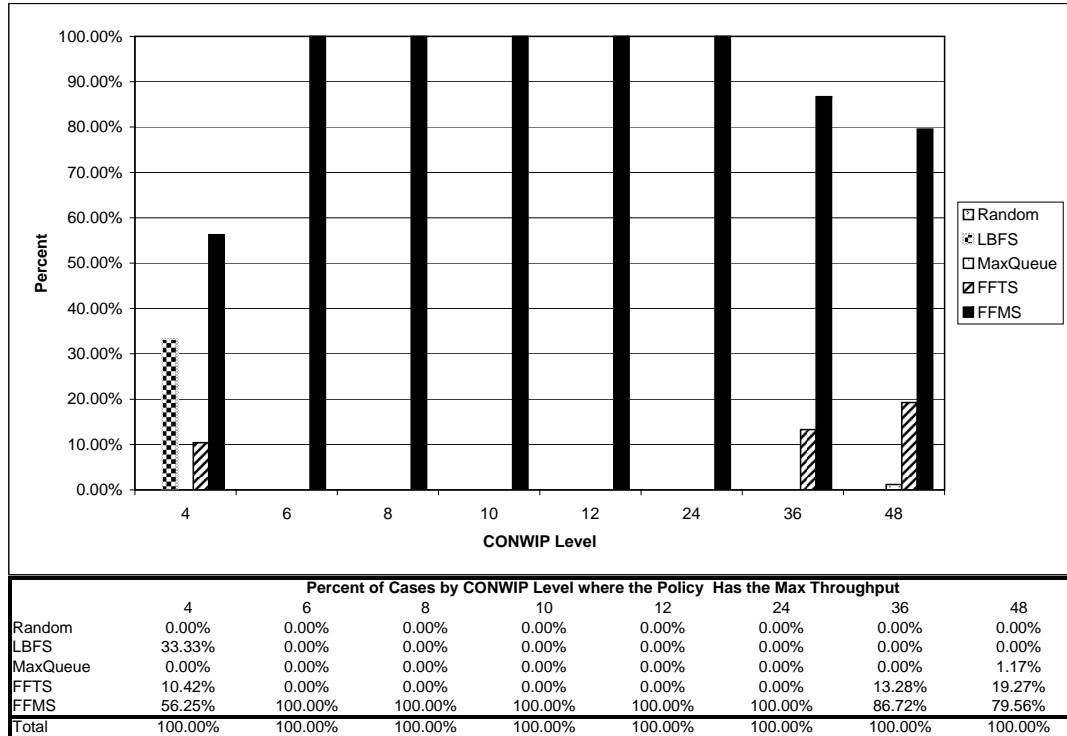


Figure 2.8: Percent of Cases of Maximum Throughput by Policy

## 2.6 Analysis of Zone Chain Structures

To demonstrate the effectiveness of zone training as an inexpensive alternative to full cross-training, we will test our FTZC structure, using the FFMS heuristic policy against three benchmark structures.

### 2.6.1 Benchmark Structures

The three structures used to benchmark the performance of our FTZC structure are: Classic CONWIP Approximation, Full Cross Training, and Two-Skill Zone Chain.

- Classic CONWIP Approximation (CCA) - This “classic” CONWIP model serves as the lower bound performance benchmark (see Figure 2.9 for an illustration). Its setup consists of one worker per station with no shared stations (i.e., no

flexibility). The workers are identical and observe the standard FCFS service discipline.

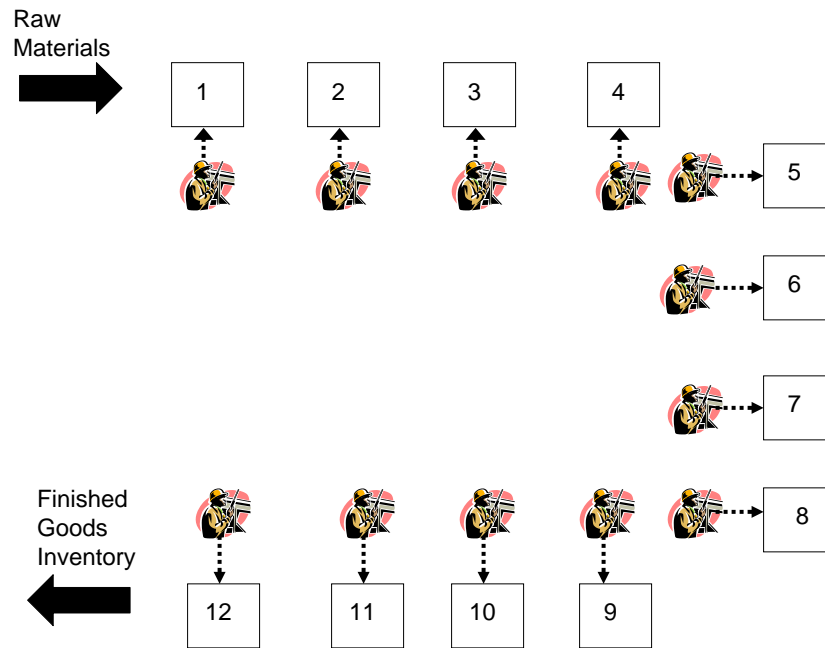


Figure 2.9: Classic CONWIP Model

- Full Cross-Training (Full XT) - This structure serves as the upper bound performance benchmark. Applying this structure in our base model of 12 stations and 4 standard workers each of the 4 workers is fully trained on all 12 stations. Every worker follows a job from station 1 through to completion (a craft or “pick and run” policy) to complete the job, then the worker returns to station 1 and repeats the process.
- Two-Skill Zone Chain (2SZC) - Following the spirit of the two-skill chain as described by Hopp, Tekin, and Van Oyen [54], the 2SZC structure cross-trains each of the four workers such that two workers cover each station in our model (see Figure 2.10). The 2SZC becomes a zone chaining adaptation of the two-skill chain model to a setting in which there are more stations than workers. All workers use the “maximum queue policy”, which Hopp, Tekin, and Van Oyen

[54] showed to be the best policy for the two-skill chain model. In this policy, two adjacent workers share the work at each station, meaning that each of the four workers is trained on six total stations so that a worker's zone (skill-set) is overlapped by both the adjacent worker upstream as well as downstream. These overlapping zones offer the 2SZC structure extensive flexibility. Intuitively, this structure serves as an upper bound benchmark for the FTZC; however, this cannot be rigorously established, because the policies employed are not truly optimal.

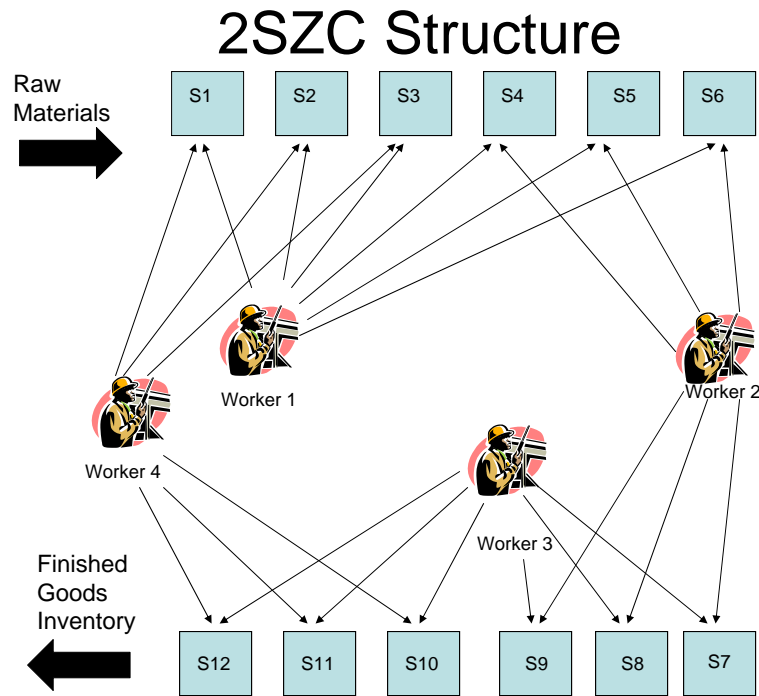


Figure 2.10: Two-Skill Zone Chain Structure

For the above four structures, along with our FTZC structure, Table 2.5 shows the relative skill requirements from full cross-training down to FTZC. It shows that the FTZC structure reduces cross-trained skills 67% from the 2SZC. This reduction in skills can result in significant labor training savings, provided the loss in efficiency is acceptable to a firm.

	Total No. of Skills	No. of Cross-Trained Skills
Full XT	48	36
2SZC	24	12
<i>FTZC</i>	<i>16</i>	<i>4</i>
CCA	12	0

Table 2.5: Cross-Training Skill Reduction

The 2SZC structure still requires a large total number of skills resulting in high labor training costs relative to FTZC. Another downside of the 2SZC is that real systems often have specialized task-types for which cross-training is impractical. In contrast, the FTZC was designed to minimize costs and maximize throughput for a wide range of applications.

To test for efficiency loss and compare these four structures - FullXT, 2SZC, FTZC, and CCA - we continue to analyze throughput using the method of investigation described in Section 2.5.3.

### 2.6.2 Structure Comparison Results

Using FFMS as the best policy for the FTZC structure, we compare the FTZC structure with 1) full cross-training (FullXT), 2) 2-skill zone chaining (2SZC), and 3) the new classic CONWIP approximation (CCA). The flexibility of the 2SZC structure enables it to serve as a zone chaining upper bound on the FTZC, and the FTZC is not expected to perform well below the 2SZC, given that the FTZC reduces the number of skills cross-trained by 67%. We want to determine relative efficiency by using the *Percent of Throughput Loss* (PTL or  $\%_{TL}$ ) of performance measure.  $\%_{TL}(2SZC \text{ vs. FTZC})$ , represents the percent of throughput lost by implementing the FTZC instead of the 2SZC.

We begin by averaging throughput, as a function of CONWIP level, over the entire test suite and present the results in Figure 2.11. The classic CONWIP structure is a conservative lower bound and its performance reveals the dramatic performance

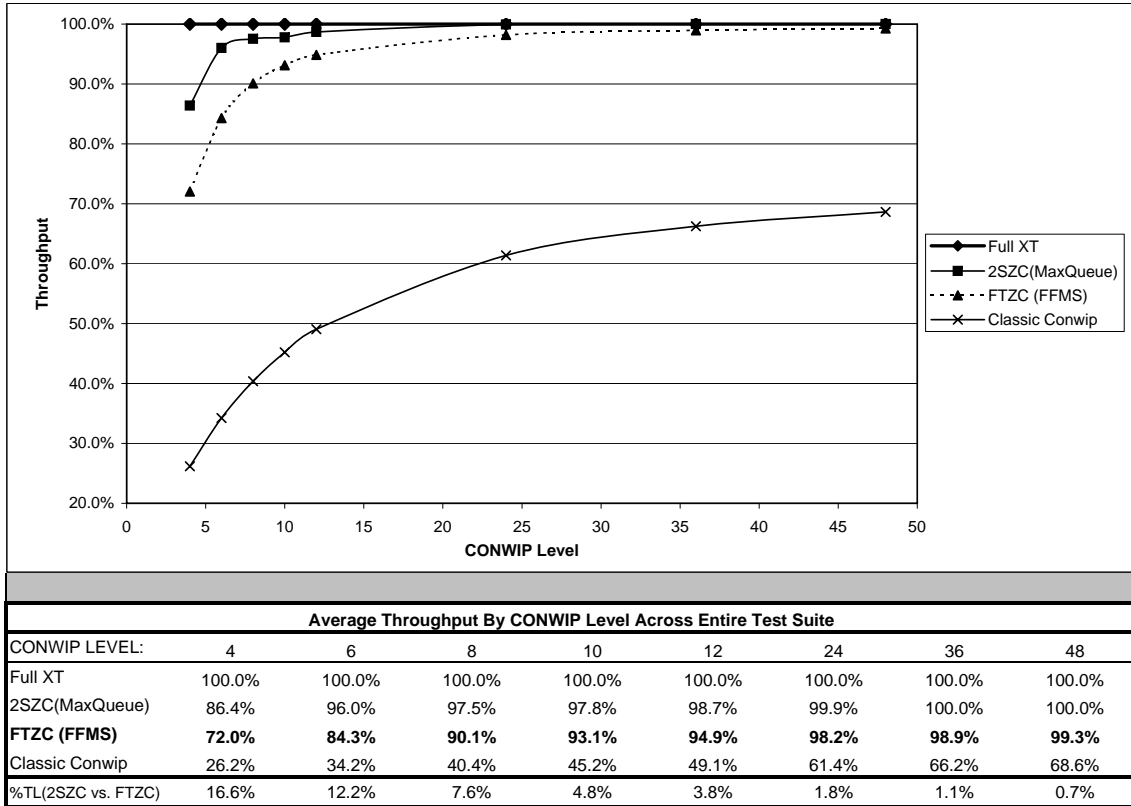


Figure 2.11: Structure Performance Comparison - Entire Suite

benefit that results from the flexibility of the FTZC. FTZC rapidly approaches the system capacity for WIP levels above 8. We interpret the results as strong evidence that when operated under the FFMS policy, the FTZC structure is a very effective paradigm. Because the (1) FTZC requires only four cross-trained skills, instead of 12 (2SZC) and 36 (FullXT) and (2) 50% of the FTZC’s available skills are dedicated to fixed task-types, the evidence strongly supports the claim that FTZC is a “high value” paradigm: good performance at low cost.

*Observation 4.* Across the entire test suite, for CONWIP levels of 10 or greater, the FTZC averages at least 95% of the throughput of the 2SZC. For the “practical” CONWIP levels of  $2\omega_0$  to  $6\omega_0$  the average  $\%_{TL}(2SZC \text{ vs. FTZC})$  is 4.5% .

To further study the effect of line imbalance on the FTZC, we again examine the

A, B, C sub-suites. Table 2.6 shows the adverse effect of line imbalance on the  $\%_{TL}$ .

CONWIP Level:	4	6	8	10	12	24	36	48
Entire Suite	16.6%	12.2%	7.6%	4.8%	3.8%	1.8%	1.1%	0.7%
A Suite	14.9%	10.0%	4.9%	2.2%	1.3%	0.1%	0.0%	0.0%
B Suite	16.5%	12.3%	8.3%	5.9%	5.0%	2.6%	1.6%	1.2%
C Suite	18.4%	14.3%	9.6%	6.4%	5.2%	2.6%	1.5%	1.0%

Table 2.6:  $\%_{TL}$ (2SZC vs. FTZC) By CONWIP Level

Table 2.6 shows that the  $\%_{TL}$ (2SZC vs. FTZC) generally increases as the DOI increases (from A suite to C suite) with minor anomalies at very high WIP. Low CONWIP levels allows the 2SZC to benefit from its flexibility advantage over the FTZC, as can be seen in Figure 2.12 for the C suite. The graphs for the A suite and B suite have been omitted to avoid redundancy with Figure 2.12 (see Appendix 2.8.4 for the details).

The FTZC structure, even when demonstrating double-digit throughput loss versus 2SZC, is still quite flexible. The above information, particularly Table 2.6, shows that throughput, CONWIP level and DOI for the FTZC performance interact. As our DOI index increases, so does  $\%_{TL}$ , which is intuitive as increased variability (in the sense of imbalance here) typically degrades performance. Table 2.6 shows that high imbalance exaggerates this relative dominance at low WIP levels, especially in the practical WIP range of  $2\omega_0$  to  $6\omega_0$ . Similarly, WIP is a form of buffering of variability and as CONWIP level increases,  $\%_{TL}$  decreases as shown in Figure 2.13.

While it is impossible to set a universally acceptable or unacceptable level of throughput loss we conservatively select 5% throughput loss as a target and use Figure 2.13 to make the following assertion:

*Observation 5.* The FTZC structure operated under the FFMS policy incurs a throughput loss of at most 5% when the CONWIP levels are at least approximately: (1)  $2\omega_0$  for the low line imbalance (A Suite); (2)  $3\omega_0$  for moderate line imbalance (B Suite); and (3)  $6\omega_0$  for high line imbalance (C Suite).

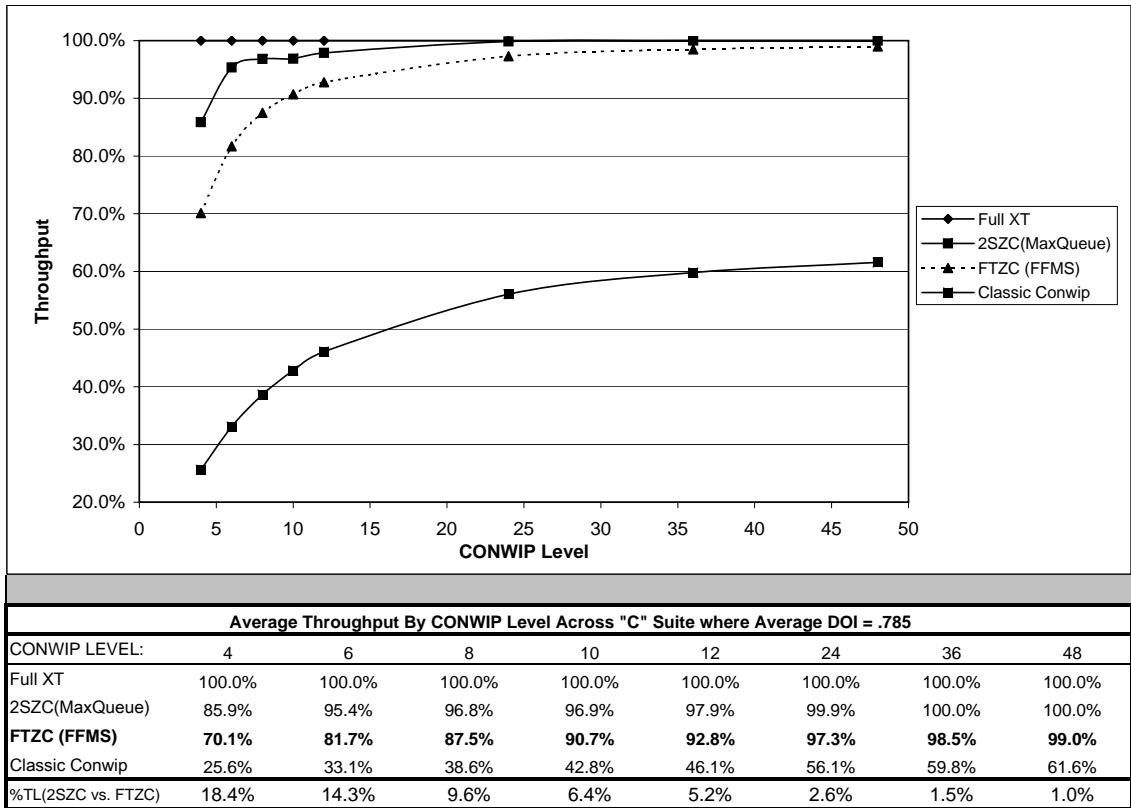


Figure 2.12: Performance Comparison across Paradigms - C Suite

Observation 5 helps clarify the robustness of the FTZC structure in various operating environments. Further, Figure 2.13 revealed that for CONWIP levels of  $2.5\omega_0$  or higher, the FTZC can maintain an average of at least 95% of the efficiency of the 2SZC. Because Observation 5 shows that only a WIP level of  $2\omega_0$  or more is required for 95% efficiency, we see that FTZC is very effective with low imbalance such as Sub-suite A, with an average DOI of 0.3. Figure 2.13 does also show that the WIP level drives efficiency more than the DOI; however, to keep cycle times short, low WIP is needed. Thus, the DOI can be used as a metric to support line-balancing effort in implementation to achieve a target performance.

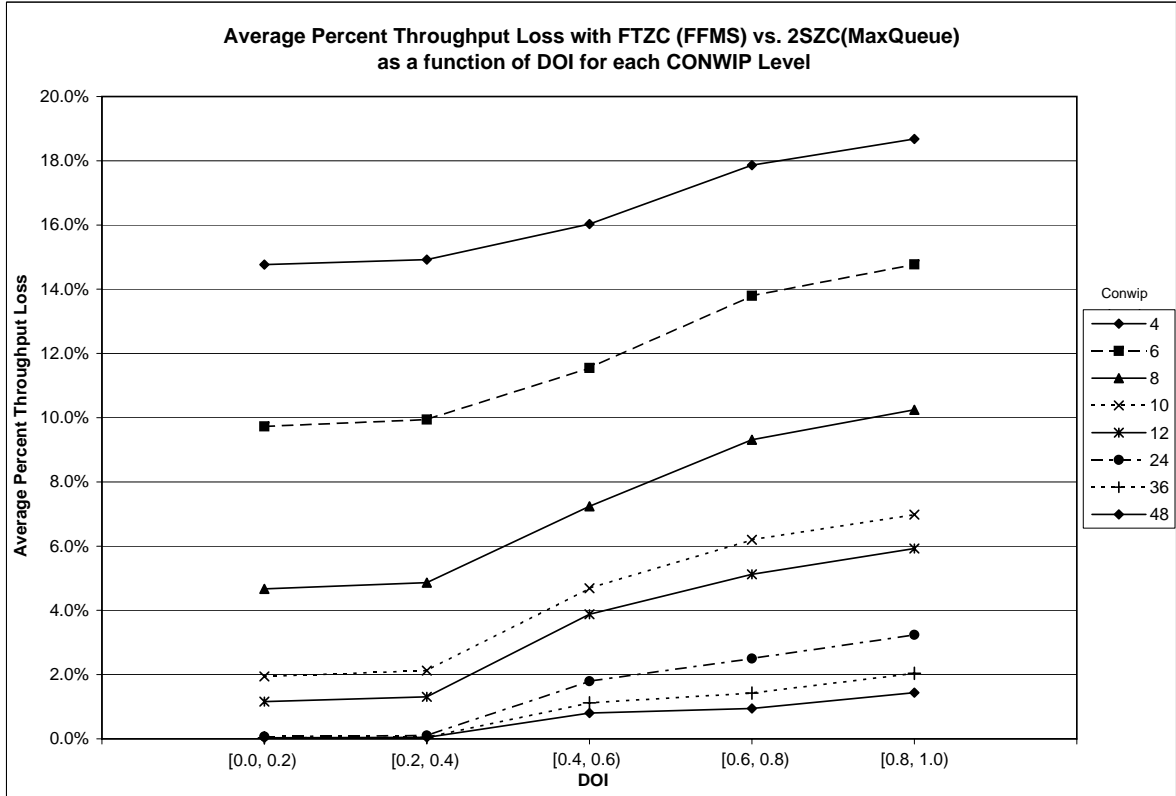


Figure 2.13: Average  $\%_{TL}$  as a function of DOI for each CONWIP level

## 2.7 Conclusions & Future Work

In conclusion, this paper identified a low cost, easily implementable labor cross-training structure for serial CONWIP lines that uses a little flexibility to great effect: Fixed Task Zone Chaining (FTZC). The FTZC structure needs only four cross-trained skills, for example, in a line of 12 stations and 4 workers. Zone chaining is used as the underlying structure from which FTZC is derived benefitting from past research that has shown the broad effectiveness of two-skill chains as well as the practicality of zoned structures of cross-training. In addition we also developed a zone-chaining version of the two-skill chain, themed the 2-Skill Zone Chain (2SZC), as an even more flexible chaining based paradigm at the cost of additional cross-training. As shown in this paper, the superiority of the FFMS policy for our FTZC structure is



important because the FFMS policy is easy to implement and it is superior to even the best complex alternative we could devise- FFTS policy. With the FFMS policy, the FTZC achieved on average only 5% throughput loss when compared with the 2SZC for CONWIP levels between 2 times and 6 times “critical WIP”. For most systems, 5% throughput loss is worth it to achieve the reduced skill cross-training required and the benefit that  $\frac{2}{3}$  of the tasks need not be cross trained at all.

Insight into the effectiveness and appropriate application of the FTZC paradigm and the FFMS policy was gained via (1) stability analysis (Section 2.4.1), (2) development of a Degree of Imbalance (DOI) line metric, and its use in benchmarking, and (3) analytical structural results of optimal policies in Section 2.4.2. These results demonstrate the broad effectiveness of zone training when integrated with skill chaining and provide insight into the value of implementing a FTZC as an inexpensive cross-training alternative. Looking towards the future, one could extend this work by investigating optimal zone designs. Further work could consider how throughput can be increased even more if the overlap of the zones is constructed optimally (as opposed to all zones being the same size). This would yield a manager’s decision support tool to identify the best policy to implement with their line, and the best zone design.

## 2.8 Appendix

### 2.8.1 Proofs of Section 2.4 Theorems

#### 2.8.1.1 Proof of Theorem 2.4.1

The minimum value of  $D$  needed to balance the FTZC with standard workers is  $D = \Delta + 1$  where

$$\Delta = \max_{i,j} \delta_{ij}$$

and

$$\delta_{ij} = \min_{\delta=0,1,2,3} \left\{ \delta : \sum_{n=i}^{i \oplus (j-1)} (\hat{\lambda} T_{3n} - C_{n \oplus 1}^s) - \sum_{n=i \ominus \delta}^{i \ominus 1} C_{n \oplus 1}^s \leq 0 \right\}$$

**Proof.** In the FTZC model, each worker has the same number of fixed task-types and the workers are identical standard workers, which results in a symmetrical cross-training structure. Given this symmetrical cross-training, we can define each worker's residual flex capacity for shared task-types as

$$C_k^s = (C_k - \hat{\lambda} \sum_{i \in \mathcal{F}_k} T_i) \quad \forall k \in \mathcal{W}. \quad (2.8.1)$$

For each worker we subtract the capacity required for that worker to perform fixed task-types from the worker's total capacity to determine the set of effective shared task-type capacities for the workers. We then observe that without fixed task-types, the workers are no longer identical, standard workers but rather workers who vary in speed, because they now have different capacities. Without the fixed task of the FTZC model, the problem is reduced to a skill chain as defined by Hopp, Tekin, and Van Oyen [54] because the fixed tasks (to which workers are dedicated) are completely eliminated by the computation in Eqn. 2.8.1 of the residual flex capacity. We then can directly follow the proof of their Theorem 2 with  $C_k^s$  from equation 2.8.1 serving

as the worker speed factors, denoted by  $v$  in that paper, and  $N = 4$ . This reduction of our FTZC and the loss of standardized workers is possible because fixed task-types are unique to each worker (i.e. they are not shared). ■

### 2.8.1.2 Proof of Theorem 2.4.3

*Consider worker  $i$ ,  $i \in \{2, 3, 4\}$ . When no jobs are waiting at the worker's downstream shared station, the job completion process is maximized along every sample path by a policy that (i) does not idle worker  $i$  when fixed task-types are present, and (ii) follows the LBFS rule among the worker's set of task-types.*

**Proof.** We prove part (ii) first. Let  $\tau$  be a time instant when worker  $i$  becomes available with at least two stations in her zone having waiting jobs, and  $k$  be the most downstream such non-empty station. For any policy  $\pi$  that assigns worker  $i$  in some other station  $l$ , where  $l < k$ , we construct an alternative policy  $\tilde{\pi}$  that assigns worker  $i$  in station  $k$  and is such that

$$D_t^{\tilde{\pi}}(\omega) = D_t^{\pi}(\omega)$$

along every sample path  $\omega$ , which proves the statement of the Theorem.

Because service times are task-type and worker dependent, sample path  $\omega$  is defined by sequences  $S_j^w(\mathcal{L}, \omega)$ ,  $\mathcal{L} = 1, 2, \dots$ , where  $S_j^w(\mathcal{L}, \omega)$  is the time duration of the  $\mathcal{L}$ -th service performed by worker  $w$  in station  $j$ . We construct policy  $\tilde{\pi}$  to be identical to  $\pi$  except that at time  $\tau$  worker  $i$  is assigned in station  $k$  and then follows the sequence of actions taken under  $\pi$  after time  $\tau$ ; the first action in that sequence is a service in station  $l$ , and the rest (that may include idle periods) are determined from the realization of the arrival and service processes. Policy  $\tilde{\pi}$  is well defined as it can always mimic  $\pi$  with respect to actions taken by workers other than  $i$  because at any time all stations where these workers can be assigned (the ones not

fixed to worker  $i$ ) have at least as many jobs under  $\tilde{\pi}$  as under  $\pi$ . The two policies are coupled at the time of the first service completion under  $\pi$  in station  $k$ . Time periods  $[\tau + S_k^i(1, \omega), \tau_c(\omega) - S_k^i(1, \omega)]$  under  $\pi$  and  $[\tau + S_k^i(1, \omega) + S_l^i(1, \omega), \tau_c(\omega)]$  under  $\tilde{\pi}$  include the same sequence of actions by worker  $i$ . Comparing the two policies, we see that we have the same number of job completions.

The proof of part (i) is based on a similar construction of the alternative policy  $\tilde{\pi}$ . After the service completion in station  $k$  worker  $i$  idles for the same amount of time she would have idled under  $\pi$ , say  $I(\omega)$ , which can be calculated by observing the sample path realization. Then she replicates her actions under  $\pi$  until the two policies are coupled the first time she completes a service in station  $k$ . The evolution of policies  $\pi$  and  $\tilde{\pi}$  along some sample path  $\omega$  is depicted in Figure 2.14, where  $I(\omega)$  plays the role of  $S_l^i(1, \omega)$ . ■

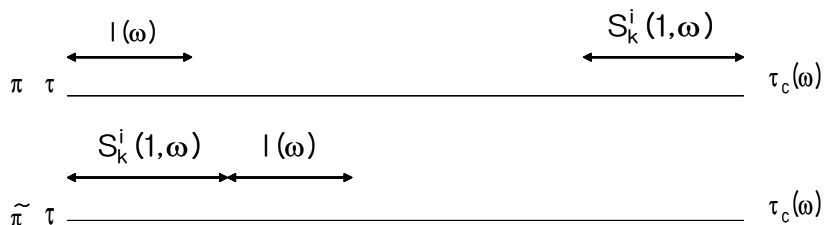


Figure 2.14: Worker Actions Under Policies  $\pi$  and  $\tilde{\pi}$  for part (i)

### 2.8.1.3 Proof of Theorem 2.4.4

*A policy that follows the LBFS rule maximizes the job completion process along every sample path within the class of policies that give priority to each worker's fixed task-types versus its shared tasks.*

**Proof.** Follows directly by the exchange argument used in the proof of Theorem 2.4.3. ■

#### 2.8.1.4 Proof of Theorem 2.4.5

*The downstream worker,  $D_S$ , is preferred when two workers are competing for a single task-type available to both of them.*

**Proof.** Let  $\pi$  be a policy that assigns  $U_S$  to  $\mathcal{S}$ . We define an alternative policy  $\tilde{\pi}$  that assigns  $D_S$  to  $\mathcal{S}$ , keeps  $U_S$  idle until job  $\mathcal{S}$  is served at the shared task-type, and is identical to  $\pi$  afterwards. Because the two workers have equal speeds, the service time of task-type  $\mathcal{S}$  is equal under policies  $\pi$  and  $\tilde{\pi}$  along any sample path. Along any sample path it is not possible for  $\pi$  to assign  $D_S$  to some task-type during the service of  $\mathcal{S}$ . First, no job can arrive at station  $\mathcal{S}$  from upstream because  $U_S$  is busy at  $\mathcal{S}$ , and second, no jobs can become available for  $D_S$  downstream of  $\mathcal{S}$  because  $D_S$  has only one shared station with his downstream neighbor (this station is either empty or occupied by that other worker). The two policies are coupled at the time of service completion at  $\mathcal{S}$  implying that they have the same performance. This shows that we can give the single job to  $D_S$  and not lose anything. ■

#### 2.8.1.5 Proof of Theorem 2.4.6

*In the FTZC structure, assuming equal and deterministic processing times at each station and a symmetric structure of skill chaining, the critical WIP is equal to the number of zones,  $\omega_0 = Z$ .*

**Proof.** Our FTZC includes  $N$  stations and  $W$  workers where all workers have two shared task-types, and an LBFS policy is followed at every queue in the zone. Given that each worker is attending a task-type with the same probability, each worker covers  $\frac{N}{Z}$  stations. This station allocation yields a total work content of  $(\frac{N}{Z})(\frac{T_0}{N}) = \frac{T_0}{Z}$  for every worker. We have the ideal (no congestion) scenario because each station has a deterministic process time of  $(\frac{T_0}{N})$ . There is no wait, so the cycle time is  $T_0$ ;

further each worker achieves a throughput of  $r_b = \frac{Z}{T_0}$ . By Little's law, the critical WIP,  $\omega_0 = r_b T_0 = (\frac{Z}{T_0}) T_0 = Z$ . ■

## 2.8.2 Classic CONWIP Approximation

### 2.8.2.1 Mean Value Analysis Algorithm

For our classic CONWIP approximation the mean value analysis algorithm calculates three performance measures given WIP level  $k$  and station  $i$ : (1) the cycle time ( $CT_i(k)$ ), (2) the throughput ( $\Theta(k)$ ), (3) and the per station  $i$  WIP level ( $V_{ij}(k)$ ).

The algorithm is initialized by setting  $V_{ij}(0) = 0$  and  $\Theta(0) = 0$ . Then, in an iterative fashion, the following equations are applied, as described in detail in Hopp and Spearman [52], for every desired WIP level:

$$CT_i(k) = [1 + V_{ij}(k - 1)]T_i$$

$$CT(k) = \sum_{i \in \mathcal{N}} CT_i(k)$$

$$\Theta(k) = \frac{k}{CT(k)}$$

$$V_{ij}(k) = \Theta(k)CT_i(k).$$

The  $V_{ij}(k)$  are derived iteratively as a function of the above recursion. This algorithm is exact in the case of exponential process times. Moreover, the reliability of this algorithm has been demonstrated in Buzacott and Shanthikumar [23], which tested these equations against simulation results for an extensive suite of their model parameters and found this approximation to be accurate for lines with a squared coefficient of variation on all process times between 0.5 and 2.0.

### 2.8.3 Test Suite

#### 2.8.3.1 Example of Test Code B4321 Mean Process Times Calculations

The following is a step-by-step example of how the mean processing times are calculated for the B4321 problem instance in our test suite. Recall the multipliers:

APZ	Z1	Z2	Z3	Z4
<b>A</b>	1	1	1	1
<b>B</b>	1.42	0.86	0.86	0.86
<b>C</b>	1.42	0.56	1.42	0.56

WPZ	D. Fixed	U. Fixed	D. Shared
<b>1</b>	1	0.9	1.1
<b>2</b>	1	1	1
<b>3</b>	1.1	0.9	1
<b>4</b>	1	1.1	0.9

First, we begin with a standard line where all of the station processing times are the same and assumed to be equal  $\frac{1}{3}$  unit of time. This is done because there are three stations for every zone, and every zone has one standard worker who has one unit of time to complete the work in his/her zone.

Then we apply the APZ multiplier for each station of a zone. For this example, we choose the B Sub-suite multiplier. For each balanced station processing time of .333 we multiply by 1.42 for each station in Z1, 0.86 for each station in Z2, 0.86 for each station in Z3, and 0.86 for each station in Z4. The new process time results become:

	Zone 1			Zone 2		
Station	1	2	3	4	5	6
Proc. Time*APZ	.333*1.42	.333*1.42	.333*1.42	.333*0.86	.333*0.86	.333*0.86
New Proc. Time	.473	.473	.473	.286	.286	.286

	Zone 3			Zone 4		
Station	7	8	9	10	11	12
Proc. Time*APZ	.333*0.86	.333*0.86	.333*0.86	.333*0.86	.333*0.86	.333*0.86
New Proc. Time	.286	.286	.286	.286	.286	.286

The test case *B4321* must be scaled by the WPZ multipliers, 4,3,2, and 1 in that order for the 4 primary zones. *Z1* is assigned the type 4 WPZ multipliers, *Z2* is assigned the type 3 WPZ multipliers, *Z3* is assigned the type 2 WPZ multipliers, and *Z4* is assigned the type 1 WPZ multipliers. For each station within a line, we multiply the WPZ value with the intermediate process time we have from first multiplying the APZ. Thus, for the *B4321* line the twelve station process times become:

	Zone 1			Zone 2		
Station	1	2	3	4	5	6
New Proc. Time*WPZ	.473*1	.473*1.1	.473*0.9	.286*1.1	.286*0.9	.286*1
<i>B4321</i> Proc. Time	.473	.520	.426	.315	.257	.286

	Zone 3			Zone 4		
Station	7	8	9	10	11	12
New Proc. Time*WPZ	.286*1	.286*1	.286*1	.286*1	.286*0.9	.286*1.1
<i>B4321</i> Proc. Time	.286	.286	.286	.286	.257	.315

For each APZ multiplier (A,B, or C), the test sub-suite includes all four digit combinations of 1111 to 4444.



## 2.8.4 Additional Structure Comparison Results

### 2.8.4.1 Structure Comparison by CONWIP Level for A Suite

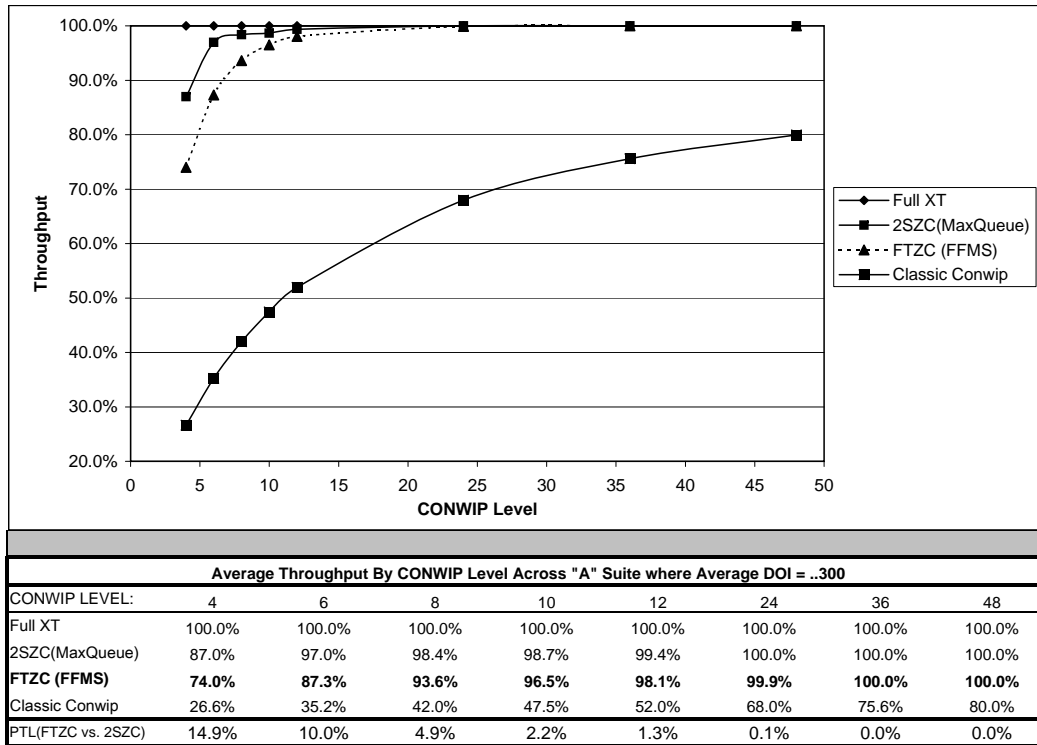


Figure 2.15: Structure Performance Comparison - A Suite

### 2.8.4.2 Structure Comparison by CONWIP Level for B Suite

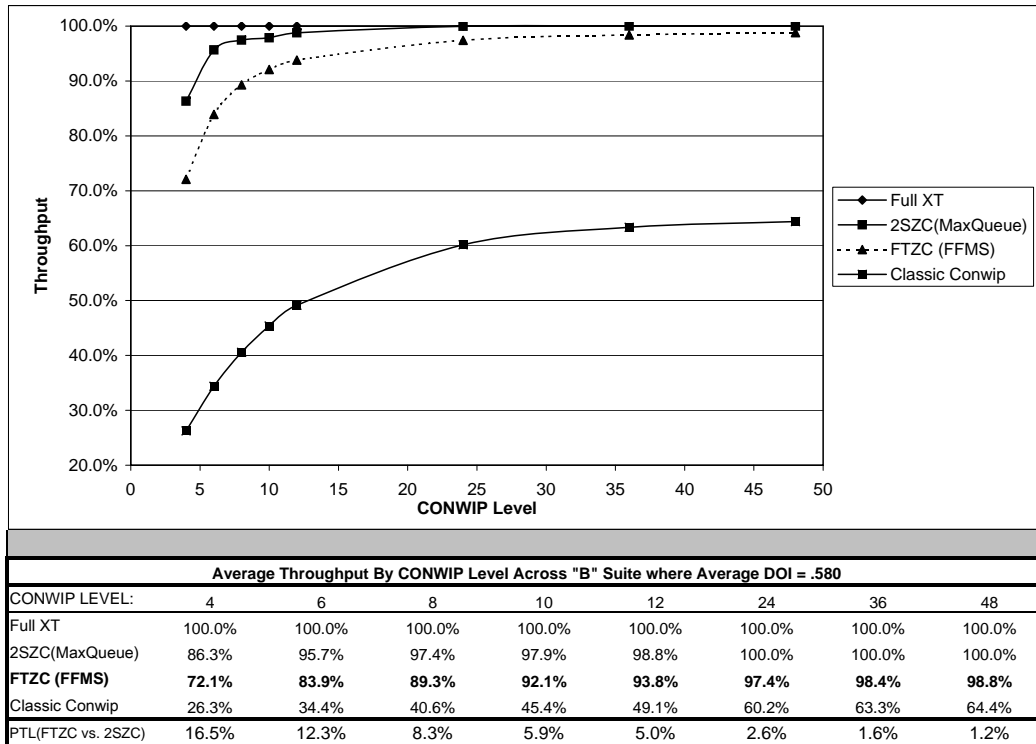


Figure 2.16: Structure Performance Comparison - B Suite

## CHAPTER III

# Modeling the Operational and Financial Impact of Supply Disruption with Correlated Defaults

### 3.1 Introduction

The importance of supply chain management has been quite visible due to the recent spate of newsworthy supply disruptions and global supply chain developments (see Tomlin [84]). From the vast field of supply chain management, various sub-fields of study have arisen including supplier reliability. Supplier reliability focuses on the operational and financial decision making processes within a firm's supply chain. This decision making is a result of the positive probability that the suppliers the firm has contracted with will be unable to deliver a portion or all of a given service or product. The ramifications of supplier unreliability have been well documented and caused suppliers and firms alike hundreds of millions of dollars in lost revenue. The introductions of Babich, Burnetas and Ritchken [11] and Saghafian and Van Oyen [74] provide detailed examples of these losses and their negative financial implications to firms. In light of these recent examples and the potential financial benefits of proactive responses to these risks, research in this area is well motivated. Sufficient impetus for the study of supplier reliability, particularly as it relates to this work, is motivated by the need to determine what can be done to help firms prepare for these costly risks.

In practice, most firms are currently more focused on lowering costs throughout the supply chain with little emphasis on mitigating the risks of disruption. There is little proactive push within most companies to address and adapt to the dangers inherent in supply chain disruption risk [80]. This work seeks to change that lack of emphasis and highlight its importance by developing models that link the disruption risk of a firm directly to their income statement.

This problem of supplier unreliability speaks to a larger issue of supply chain design in a global marketplace. For example, given our new business world “without borders”, a more expensive and more reliable supplier on a different continent may provide a better contract to a firm than a less expensive and less reliable supplier in the firm’s own country (or vice versa). The decision of who to purchase supply from, and how much supply to purchase, is a requirement to achieving good macro supply chain performance. Sourcing decisions at the operational level provide necessary insight at the managerial level to answer supply chain design questions. Knowing how much to order and whom to order it from, is important information to a firm. Moreover this knowledge provides firms with the tools to understand and accurately quantify the effects of supplier reliability on their supply chains.

This proposed research quantifies the financial impact of flexibility in a firm’s sourcing decision. With regard to flexibility we are referring to the sourcing options a firm has when ordering supply. In the face of supplier disruption where a supplier may disappear for the rest of the planning horizon, we seek to show that it is cost effective to have flexibility in sourcing decisions. This work examines a model of supplier reliability with a single firm contracting with two unreliable suppliers who provide substitutable products in a duopoly framework during a continuous, finite selling season. The duopoly framework resolves the lack of flexibility in a firm having a single supplier, yet its results are often times generalizable to a multiple (greater than two) supplier frameworks. Our model contains several attractive and interesting features

that when combined, to the author’s knowledge, have not been studied simultaneously before:

- The models allow inventory build up during the selling season to serve as a disruption mitigation strategy for the firm (in addition to dual-sourcing).
- Two types of supplier default exist, idiosyncratic and system wide, to allow the study of correlated disruptions using a salient driver of disruption correlation.
- The use of exponentials for the distribution time to default fits the unpredictability of disruption

A flexible supply chain design is needed to mitigate the effect of stochastic supplier disruptions on operations and especially financial cash flows. This work develops mitigation strategies for a firm to use in sourcing from unreliable suppliers and demonstrates the conditions under which flexibility in the firm’s supply chain is necessary. Our objective is to emphasize how supply decisions are impacted by parameters such as per unit profit, holding cost, and two unique types of default risk.

### 3.2 Literature Survey

Consider a firm contracting with  $n$  suppliers ( $n \geq 2$ ) where one or both of the suppliers is unreliable. The firm is operating in a newsvendor framework (see Dada, Petruzzi, and Schwarz [30]) in the sense that all of their suppliers provide substitutable products and the firm must first decide who to contract with and secondly how much to contract for. This basic framework is the basis for much of the supplier unreliability literature. Within the supplier unreliability sub-field, of literature, lines of distinction have been drawn in the current literature that specify exactly what type of supplier unreliability an author is speaking of. These types of unreliability include: yield uncertainty, random capacity, and supply disruption. Toiling [84] provides a concise,

yet thorough, review of the literature along these three distinctions. This current work fits into the literature within the supply disruption group, however for completeness all three types of work will briefly be described here.

In yield uncertainly models, the firm experiences randomness when their order is placed, as to how much of that order will be delivered by the supplier. Yano and Lee [91] provide a superb review of the yield uncertainty literature.

The literature on random capacity fits into the supplier unreliability sub-field in that the upper-bound on the supplier's production capacity is random in each period of the production horizon. The works of Henig and Gerchak [47] and Ciarallo, Akella, and Morton [27] provide a strong foundation in random capacity. Henig and Gerchak study a single period model where they find the surprising result that the existence of a reorder point is independent of the yield being random. Ciarallo, Akella, and Morton extend their work to show that in a multi-period setting order-up-to policies are optimal and dependent on the random yield.

The work of supply disruption consists of models of suppliers with a binary state of either up (active) or down (inactive). Generally, when the supplier is up, all of a given order is delivered on time, and when the supplier is down, none of a given order is supplied. By contrast to yield uncertainty and random capacity, in most of the supply disruption literature the state of a supplier is most often known by the firm prior to placing an order in a particular period. When the disruptions are going to occur and the length of a disruption when it does occur generally serve as the sources of randomness. Previous works that explore supply disruption in the duopoly framework of this proposed research are by Parlar and Perry [69] and Gurler and Parlar [43]. In Parlar and Perry [69] the authors consider supply disruption for a class of EOQ models where a single firm, facing deterministic demand, contracts with two infinite capacity suppliers with equal cost structures. The reliability of the supplier's is an alternating poisson process. When the optimal reorder point inventory level of the

firm is reached, and a supplier is active, the order is received. If both suppliers are inactive the firm must wait until one (or both) becomes active. An ordering policy for the firm is solved numerically, however the author's found it to be suboptimal. Gurler and Parlar [43] extend Parlar and Perry's work with erlang inter-failure times and general repair times. Again the ordering policy is solved numerically as well as the optimal inventory in the special case of erlang-2 inter-failures and exponential repair times.

Anupindi and Akella [6], in model one of their paper, examine a single delivery contract where an order is guaranteed to be fulfilled either in the current period, with probability  $\beta$ , or in the subsequent period with probability  $1 - \beta$ . They are able to analytically derive the optimal ordering policies that minimize total costs with backlogging in both a single and multi-period setting. Their results provide a firm with optimal solutions for a given set of model parameters, but little insight has been provided into the effects, to the firm, of the interaction of those parameters when two or more suppliers in the supply chain are unreliable. The models of this proposed work seek to address that.

Now the attention is turned to the supply disruption literature on correlated defaults. Babich, Burnetas and Ritchken [11] study the effects of competition and diversification on a supply chain with unreliable suppliers. They analyze the firms tradeoff between diversification and price competition in the dual and multiple supplier frameworks where the channel experiences correlated supply shocks. In their model they note that it is feasible to estimate risk-neutral default probabilities of supplier's from financial default events. The reason for this, they reason, is that pricing models for various types of defaultable claims contain information on the parameters of the default processes. Giesecke [40] has studied an easily implemented intensity-based exponential model for correlated defaults. The intuition and analysis of these papers provides the foundation of this correlated default study.

### 3.3 The Model

Consider a firm using dual-sourcing with two unreliable suppliers for supply of a substitutable product during a continuous, finite selling season of length  $T$ , with a constant demand rate,  $\xi$  (see Figure 3.1). The fashion industry is one example of a finite selling season. At the beginning of the selling season the firm contracts with each supplier, referred to as supplier 1 and supplier 2, for a constant order rate,  $(\gamma_1, \gamma_2)$ . We assume the firm contracts with its suppliers via an order minimum for at least the amount of demand such that,  $\gamma_i \geq \xi$  where  $i \in \{1, 2\}$ . Each order is purchased from the supplier's at a fixed wholesale per unit order cost of  $c$ . There is no supply lead time and during each time unit we assume instantaneous delivery of the firm's order. A per unit revenue of  $r$  is received during each time unit where demand is met by the firm. The firm's total order,  $\gamma_1 + \gamma_2$ , from their supplier's may exceed demand and any excess inventory is stored by the firm at a per unit holding cost rate of  $h$ .

The necessity of inventory is a result of two different types of supply disruptions. Both types of disruptions cause a supplier to become inactive, unable to supply the firm's order, and remain inactive for the rest of the finite selling season. The first are idiosyncratic disruptions that are unique to each supplier. When an idiosyncratic disruption occurs only one supplier is affected and the disruption is independent of the other supplier. For example, consider a fire that shuts down operations at supplier 1. This event would disrupt the supply that the firm ordered from supplier 1, but would have no affect on supplier 2. Both idiosyncratic disruptions are exponentially distributed and occur with rate  $\lambda_i$  corresponding to supplier  $i$  where  $i \in \{1, 2\}$ . The second type is a system-wide disruption that upon occurrence will cause both suppliers to become inactive simultaneously. For example, consider two suppliers in the same city where a natural disaster occurs causing their operations to be suspended. Also, a large exchange rate can drive both suppliers out of business if they are in the same



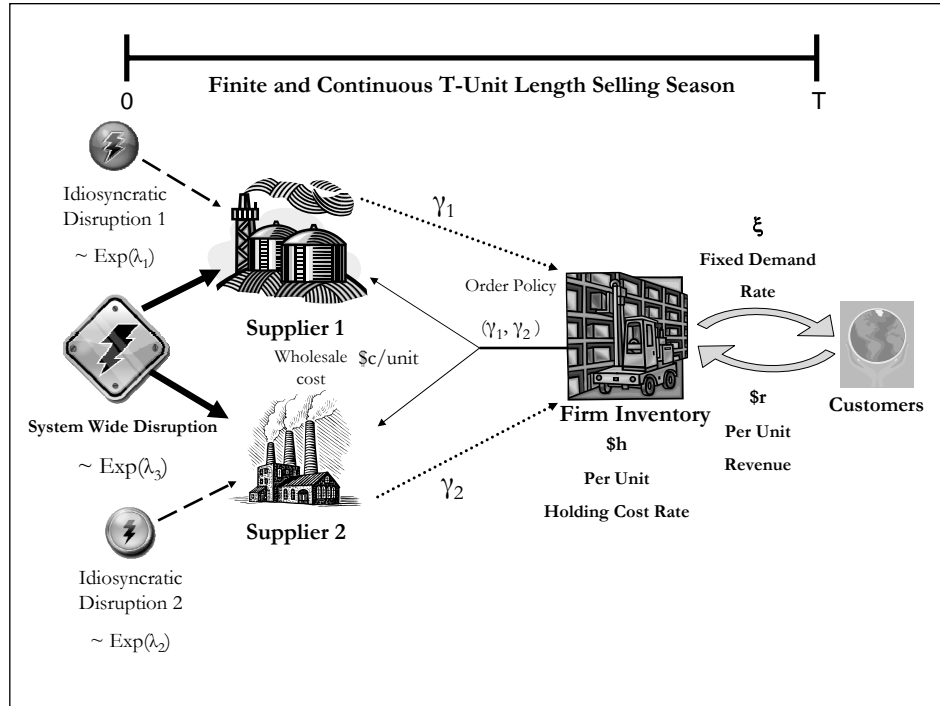


Figure 3.1: Dual Supplier Model

country. System wide disruptions are also exponentially distributed with rate  $\lambda_3$ . Our objective is to maximize total expected profit by the end of the selling season by determining the optimal order rates from suppliers 1 and 2.

### 3.3.1 Notation

To accomplish this objective consider the following notation:

- $r$  : firm's per unit revenue for meeting demand
- $c$  : firm's per unit purchasing cost from the supplier
- $h$  : firm's per unit holding cost for carrying inventory from the supplier
- $\xi$  : deterministic demand rate
- $\gamma$  : firm's order quantity rate from the supplier

- $I(t)$  : firm's inventory level at time  $t$ . If  $t = 0$ , then  $I(0) = 0$  is the firm's initial inventory at the beginning of the selling season
- $\tau_{\xi}(I(t))$  : r.v. denoting the time to inventory depletion at demand rate  $\xi$  given inventory  $I(t)$  and no further replenishment  

$$= \begin{cases} \frac{I(t)}{\xi} & I(t) > 0 \end{cases}$$
- $\tau_{(\xi-\gamma_i)}(I(t))$  : r.v. denoting the time to inventory depletion at demand rate  $(\xi - \gamma_i)$  with inventory  $I(t)$  and no further replenishment  

$$= \begin{cases} \frac{I(t)}{(\xi-\gamma_i)} \end{cases}$$
- $T$  : time horizon length (i.e., selling season)
- $T_i$  : time to disruption of type  $i \in \{1, 2, 3\}$ .  $\sim \text{Exp}(\lambda_i)$

$$i = \begin{cases} 1 : \text{Idiosyncratic to Supplier 1} \\ 2 : \text{Idiosyncratic to Supplier 2} \\ 3 : \text{System-wide to both Suppliers} \end{cases}$$

- $T'_i$  : with one supplier  $i$  up, the minimum of the length of time to disruption of the up supplier or the system-wide disruption:  $T'_i = \min\{T_i, T_3\}$ , which is a pair of competing exponential random variables.
- $T_{first}$  : time of either the first disruption or the end of the horizon when both supplier's are up. (Note: In the single supplier model  $T_{first} = 0$ )
- $T_{last}$  : Given that an idiosyncratic disruption has left one supplier up, this is the remaining time to either the last disruption or the end of the horizon
- $T'_{last}$  : This is the remaining time to inventory depletion when one supplier is up, under the condition that the replenishment rate is less than  $\xi$

- $T_{min}$  : With both suppliers down, this is the absolute time until, the minimum of the time to inventory depletion or the end of the time horizon

The flow chart of Figure 3.2 depicts the different sample paths of supplier disruption that must be considered to calculate the firm's total expected profit. The

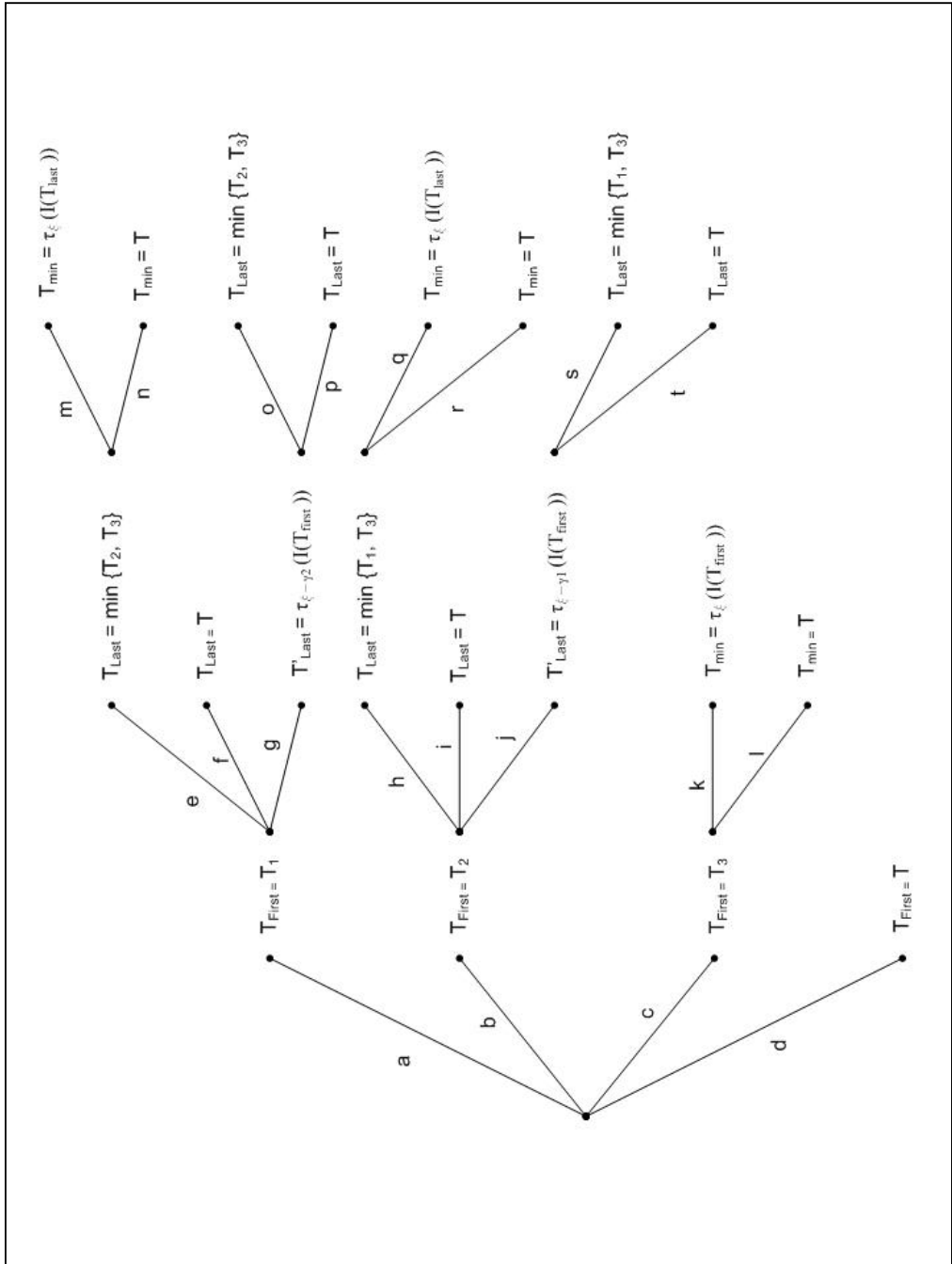


Figure 3.2: Disruption Sample Paths

importance of the sample paths is that the firm's profit rate changes depending on which arc of the flow chart is being considered. The total expected profit is the sum of the expected values of these arcs and multiple profit rate functions are required for this expectation.

### 3.3.2 Profit Rate Functions

These functions calculate the firm's profit, per unit time, under two conditions: 1) supplier availability ( $\ddot{\pi}$  = both suppliers available or active,  $\dot{\pi}$  = one supplier available or active,  $\pi$  = no supplier's available or active) and 2) inventory level (+: inventory level is increasing). It is important to be aware of the distinction we have made between  $\Pi$ , the firm's TOTAL profit function, and  $\pi$ , the firm's profit RATE function.

The *profit rate* functions for the firm, by supplier availability are:

- No Supplier Available. Consider the case when both suppliers are inactive either from two idiosyncratic disruptions or from a system wide disruption. Demand is met and profit is generated at each time instant from the depletion of the firm's inventory. Suppose no suppliers are available, supplier  $i$  was the last supplier to default (either by idiosyncratic or system wide disruption but in the latter case, the choice of  $i$  is arbitrary), and inventory was increasing up to the time of the last supplier going down,  $T_{last}$ . The instantaneous profit rate at time  $z$  after  $T_{first} + T_{last}$  is:

$$\pi_i^+(z) = r\xi - h[(I(T_{first}) + (\gamma_i - \xi)T_{last}) - \xi z].$$

When no suppliers are active due to the system wide disruption occurring while both suppliers were up:

$$\pi(z) = r\xi - h[(\gamma_1 + \gamma_2 - \xi)T_{first} - \xi z].$$

- Single Supplier Available - consider the case where one supplier is inactive due to an idiosyncratic disruption and the other supplier is still active:

$$\dot{\pi}_i^+(z) = r\xi - c\gamma_i - h[I(T_{first}) + (\gamma_i - \xi)z].$$

- Dual Suppliers Available - Consider the initial state of the system where both supplier's are active and no disruptions have occurred:

$$\ddot{\pi}^+(z) = r\xi - c(\gamma_1 + \gamma_2) - h[(\gamma_1 + \gamma_2) - \xi]z].$$

Each state of supplier availability, as the entire system is modeled, is a function of the previous state. First we model the no supplier state, followed by the single and dual supplier states. For each state of supplier availability we can use the above profit rate functions to model the total expected profit. Each model is dependent on the random length of time the system is in that particular state of supplier availability and the state it transitions to once disruption occurs (see Figure 3.2 for state transitions). The total expected profit is determined when each model is integrated over all possible time lengths and their corresponding probability distributions.

### 3.3.3 No Supplier

The no supplier state of availability considers two cases. The cases are dependent upon which state the system was in immediately prior to the transition of no supplier active. Case *N1* assumes the previous state was single supplier active and either an idiosyncratic or system wide disruption occurred. Case *N2* assumes the previous state was both suppliers active and a system wide disruption occurred.

### 3.3.3.1 Case N1

Case N1 assumes  $T_{first} = T_3$ . The total profit of Case N1 is dependent on whether the time until inventory depletes  $[\tau_\xi(I(T_3))]$  is before or after the end of the selling season  $T$  since both suppliers are down. This profit can be calculated directly without expectation, thus let  $\Pi^{N1}(T_3)$  be the total profit in the no supplier model under case N1.

$$\Pi^{N1}(T_3) = \begin{cases} \int_{s=T_3}^{\tau_\xi(I(T_3))} \pi(s) ds & \text{if } \tau_\xi(I(T_3)) < T - T_3 \\ \int_{s=T_3}^T \pi(s) ds & \text{if } \tau_\xi(I(T_3)) \geq T - T_3 \end{cases}$$

$$\text{where } \tau_\xi(I(T_3)) = \frac{I(T_3)}{\xi} = \frac{[(\gamma_1 + \gamma_2 - \xi)T_3]}{\xi}.$$

### 3.3.3.2 Case N2

Case N2 assumes  $T_{last} = T'_i$ . The total profit of Case N2 is dependent on whether the time until inventory depletes  $[\tau_\xi(I(T'_i))]$  is before or after the end of the selling season  $T$ . This profit also can be calculated directly without expectation. Let  $\Pi^{N2}(T'_i)$  be the total profit in the no supplier model under case N2.

$$\Pi^{N2}(T'_i) = \begin{cases} \int_{s=T'_i}^{T'_i + \tau_\xi(I(T'_i))} \pi_i^+(s) ds & \text{if } T'_i + \tau_\xi(I(T'_i)) < T - T_{first} - T'_i \\ \int_{s=T_{first} + T'_i}^T \pi_i^+(s) ds & \text{if } T'_i + \tau_\xi(I(T'_i)) \geq T - T_{first} - T'_i \end{cases}$$

$$\text{where } \tau_\xi(I(T'_i)) = \frac{I(T'_i)}{\xi} = \frac{[(\gamma_1 + \gamma_2 - \xi)T_{first} + (\gamma_i - \xi)(T'_i - T_{first})]}{\xi}.$$

Both no supplier cases are used strictly to help solve the single and dual supplier cases. Independently they offer no insight in to sourcing and inventory mitigation strategies for the firm.

### 3.3.4 Single Supplier

Let  $\dot{\Pi}^S(T_{first})$  be the total expected profit of the single supplier case. When the single supplier case is embedded within the model of the dual supplier case the resulting profit function is dependent on  $T_{first}$ , the time of the first disruption. When the single supplier case is considered as an independent model, the analysis stays the same and  $T_{first} = 0$ .



Lemma 3.3.1. The firm's total expected profit for the single supplier case is  $\mathbb{E}[\dot{\Pi}^S(T_{first})] =$

$$\begin{aligned}
& \frac{1}{2} \left( -e^{-T(\lambda_1 + \lambda_3)} T (2c\gamma_1 + hT(\gamma_1 - \xi) - 2r\xi) + \frac{1}{(\lambda_1 + \lambda_3)^2} \right. \\
& \left( 1 - e^{-T(\lambda_1 + \lambda_3)} \right) \left( -h(2\gamma_1 - 2\xi) - 2(\lambda_1 + \lambda_3)(c\gamma_1 - r\xi) + \right. \\
& \left. e^{-T(\lambda_1 + \lambda_3)} (2(\lambda_1 + \lambda_3)(1 + T(\lambda_1 + \lambda_3))(c\gamma_1 - r\xi) + h(\gamma_1(2 + T(\lambda_1 + \lambda_3) \right. \\
& \left. (2 + T(\lambda_1 + \lambda_3))) - 2\xi - T(\lambda_1 + \lambda_3)(2 + T(\lambda_1 + \lambda_3))\xi) \right) + \\
& \frac{1}{(\lambda_1 + \lambda_3)^2} (1 - e^{-T(\lambda_1 + \lambda_3)}) \left( e^{-T(\lambda_1 + \lambda_3)} (2r(\lambda_1 + \lambda_3)(2 + T(\lambda_1 + \lambda_3))\xi - \right. \\
& \left. h(2\gamma_1(4 + T(\lambda_1 + \lambda_3)(3 + T(\lambda_1 + \lambda_3))) - 8\xi - \right. \\
& \left. T(\lambda_1 + \lambda_3)(8 + 3T(\lambda_1 + \lambda_3))\xi) + \frac{1}{(\gamma_1 + \xi)^2} e^{-\frac{T(\lambda_1 + \lambda_3)\xi}{\gamma_1 + \xi}} \right. \\
& \left( -2r(\lambda_1 + \lambda_3)\xi(\gamma_1 + \xi)(\gamma_1(2 - T(\lambda_1 + \lambda_3)) + 2\xi + T(\lambda_1 + \lambda_3)\xi) + \right. \\
& \left. h(2\gamma_1^3(4 - T(\lambda_1 + \lambda_3)) - (8 + T(\lambda_1 + \lambda_3)(8 + 3T(\lambda_1 + \lambda_3)))\xi^3 - \right. \\
& \left. 2\gamma_1\xi(4\xi + T(\lambda_1 + \lambda_3)(1 - 2T(\lambda_1 + \lambda_3))\xi) - \right. \\
& \left. \gamma_1^2(-8\xi + T(\lambda_1 + \lambda_3)(-4 + T(\lambda_1 + \lambda_3))\xi) \right) \left. \right) - \\
& \frac{1}{(\lambda_1 + \lambda_3)^2 \xi} (1 - e^{-T(\lambda_1 + \lambda_3)}) \left( -2r(\lambda_1 + \lambda_3)(\gamma_1 - \xi)\xi + \right. \\
& \left. h(2\gamma_1^2 - 8\gamma_1\xi + 6\xi^2) + \frac{1}{(\gamma_1 + \xi)^2} e^{-\frac{T(\lambda_1 + \lambda_3)\xi}{\gamma_1 + \xi}} \right. \\
& \left( 2r(\lambda_1 + \lambda_3)\xi(\gamma_1 + \xi)(\gamma_1^2 + T\gamma_1(\lambda_1 + \lambda_3)\xi - (1 + T(\lambda_1 + \lambda_3))\xi^2) - \right. \\
& \left. h(2\gamma_1^4 + 2\gamma_1^3(-2 + T(\lambda_1 + \lambda_3))\xi + 3(2 + T(\lambda_1 + \lambda_3)(2 + T(\lambda_1 + \right. \\
& \left. \lambda_3))\xi^4 + \gamma_1^2\xi(-8\xi + T(\lambda_1 + \lambda_3)(-6 + T(\lambda_1 + \lambda_3))\xi) + \right. \\
& \left. 2\gamma_1\xi^2(2\xi - T(\lambda_1 + \lambda_3)(1 + 2T(\lambda_1 + \lambda_3))\xi) \right) \left. \right) \left. \right)
\end{aligned}$$

(3.3.1)

**Proof.** In the single supplier case the total expected profit of the firm can be calculated by conditioning on the time of the last disruption before, either idiosyncratic or system wide, before the system enters the no supplier state.

$$\mathbb{E}[\dot{\Pi}^S(T_{first})] = \mathbb{E}[\mathbb{E}[\dot{\Pi}^S(T_{first})|T_{last}]] \quad (3.3.2)$$

$$\begin{aligned} &= \mathbb{E}[\dot{\Pi}^S(T_{first})|T_{last} = T'_i] \mathbb{P}(T_{last} = T'_i) \\ &\quad + [\dot{\Pi}^S(T_{first})|T_{last} = (T - T_{first})] \mathbb{P}(T_{last} = T - T_{first}) \end{aligned} \quad (3.3.3)$$

$$\begin{aligned} &= \mathbb{E} \left( \left[ \int_{s=T_{first}}^{T'_i+T_{first}} \dot{\pi}^+(s) ds \right] + \Pi^{N1}(T'_i) \right) \mathbb{P}(T'_i < T - T_{first}) \\ &\quad + \left( \int_{s=T_{first}}^T \dot{\pi}^+(s) ds \right) \mathbb{P}(T'_i \geq T - T_{first}) \end{aligned} \quad (3.3.4)$$

$$\begin{aligned} &= \left( \left[ \int_{t'_i=0}^T \left( \int_{s=T_{first}}^{t'_i+T_{first}} \dot{\pi}^+(s) ds \right) + \Pi^{N1}(t'_i) \right] f_{T'_i}(t'_i) dt'_i \right) \\ &\quad + \int_{t'_i=T}^{\infty} \left( \int_{s=T_{first}}^T \dot{\pi}^+(s) ds \right) f_{T'_i}(t'_i) dt'_i, \end{aligned} \quad (3.3.5)$$

where  $f_{T'_i}(t'_i) = (\lambda_i + \lambda_3)e^{-(\lambda_i+\lambda_3)t'_i}$  is the distribution on the competing exponentials that remain by the idiosyncratic disruption of the active supplier and the system wide disruption. The rest is algebra and calculus.  $\blacksquare$

The following result confirms our intuition on the behavior of the single supplier profit function. We expect that in the absence of supplier disruption the firm would simply order the demand rate quantity. This expectation is confirmed in Proposition 3.3.2.

*Proposition 3.3.2.* As the idiosyncratic disruption rate and the system disruption rate vanish (i.e.,  $\lambda_1 \rightarrow 0$  and  $\lambda_3 \rightarrow 0$ ), for the single supplier expected profit function,  $\mathbb{E}[\dot{\Pi}^S(T_{first})]$ , the firm's optimal order quantity is  $\gamma_1 = \xi$ .

**Proof.** Consider Equation 3.3.1 for the single supplier case with  $T_{first} = 0$ ; taking the limit yields:

$$\lim_{(\lambda_1, \lambda_3) \rightarrow 0} \mathbb{E}[\dot{\Pi}^S(0)] = (r - c)T \max[\gamma_1, \xi] - hI_0T - \frac{hT^2(\gamma_1 - \xi)}{2} \quad (3.3.6)$$

Recall that based upon the firm's ordering restrictions of this model the firm's order rate must be at least the demand rate,  $\gamma_1 \geq \xi$ , and therefore  $\gamma_1 = \xi$  maximizes the above. This is because, in the limit, the firm's profit is linear in their order rate,  $\gamma_1$  and the profit peaks at  $\xi$ . The peak is caused by the two holding cost terms of Equation 3.3.6. The first term,  $hI_0T$  is constant in  $\gamma_1$  and the second term  $\frac{hT^2(\gamma_1 - \xi)}{2}$  is decreasing in  $\gamma_1$  as  $\gamma_1 > \xi$ . However, when  $\gamma_1 = \xi$  this term is zero and thus maximizes the function. ■

### 3.3.5 Dual Supplier

Let  $\mathbb{E}[\ddot{\Pi}^D]$  be the total expected profit of the dual supplier case. Many months of effort on the dual supplier case has revealed that this case is analytically intractable. Thus, we employ numerical analysis to obtain insights. The key is to embed the solution of the single supplier problem into the dual supplier case. The details of deriving the dual supplier profit function consist of conditioning on  $T_{first}$  and are provided below.

$$\mathbb{E}[\ddot{\Pi}^D] = \mathbb{E}[\mathbb{E}[\ddot{\Pi}^D | T_{first}]] \quad (3.3.7)$$

$$= \mathbb{E}[\ddot{\Pi}^D | T_{first} = T_1] \mathbb{P}(T_{first} = T_1) \quad (3.3.8)$$

$$+ \mathbb{E}[\ddot{\Pi}^D | T_{first} = T_2] \mathbb{P}(T_{first} = T_2) \quad (3.3.9)$$

$$+ \mathbb{E}[\ddot{\Pi}^D | T_{first} = T_3] \mathbb{P}(T_{first} = T_3) \quad (3.3.10)$$

$$+ \mathbb{E}[\ddot{\Pi}^D | T_{first} = T] \mathbb{P}(T_{first} = T) \quad (3.3.11)$$

$$= \mathbb{E} \left( \int_{s=0}^{T_1} \ddot{\pi}^+(s) ds + \mathbb{E}[\dot{\Pi}^S(T_1)] \right) \mathbb{P}(T_1 < \min\{T_2, T_3, T\}) \quad (3.3.12)$$

$$+ \mathbb{E} \left( \int_{s=0}^{T_2} \ddot{\pi}^+(s) ds + \mathbb{E}[\dot{\Pi}^S(T_2)] \right) \mathbb{P}(T_2 < \min\{T_1, T_3, T\}) \quad (3.3.13)$$

$$+ \mathbb{E} \left( \int_{s=0}^{T_3} \ddot{\pi}^+(s) ds + \mathbb{E}[\dot{\Pi}^{N^2}(T_3)] \right) \mathbb{P}(T_3 < \min\{T_1, T_2, T\}) \quad (3.3.14)$$

$$+ \left( \int_{s=0}^T \ddot{\pi}^+(s) ds \right) \mathbb{P}(T < \min\{T_1, T_2, T_3\}) \quad (3.3.15)$$

$$= \int_{t_1=0}^T \left( \int_{s=0}^{t_1} \ddot{\pi}^+(s) ds + \mathbb{E}[\dot{\Pi}^S(t_1)] \right) f_{[T_1|T_1 < \min\{T_2, T_3\}]}(t_1) dt_1 \quad (3.3.16)$$

$$+ \int_{t_2=0}^T \left( \int_{s=0}^{t_2} \ddot{\pi}^+(s) ds + \mathbb{E}[\dot{\Pi}^S(t_2)] \right) f_{[T_2|T_2 < \min\{T_1, T_3\}]}(t_2) dt_2 \quad (3.3.17)$$

$$+ \int_{t_3=0}^T \left( \int_{s=0}^{t_3} \ddot{\pi}^+(s) ds + \mathbb{E}[\dot{\Pi}^{N^2}(t_3)] \right) f_{[T_3|T_3 < \min\{T_1, T_2\}]}(t_3) dt_3 \quad (3.3.18)$$

$$+ \int_{t=0}^T \left( \int_{s=0}^t \ddot{\pi}^+(s) ds \right) \bar{f}_{T_{all}}(t) dt \quad (3.3.19)$$

A numerical study of models for both the single and dual supplier case was completed for parametric analysis of the models and to understand how inventory was used as a default risk mitigation strategy. Specifically, as it relates to the value of flexibility among the firm's suppliers, we seek to learn how the ability to hold inventory impacts the firm's ordering decision.

## 3.4 Single Supplier Results

### 3.4.1 Numerical Study

The single and dual supplier models were programmed using Mathematica to study the results of optimal ordering decisions by the firm and maximum profit

achieved across a test suite of parameter values. Test parameters were then compared two at a time across high and low values, while the non-test parameters were held constant at their low values, to gain insight on the effect the parameters had on the model. The following parameters were held constant throughout the study: horizon length ( $T = 10$ ), demand rate ( $\xi = 5$ ), and wholesale order cost ( $c = 1$ ). The test parameters our study investigated included: idiosyncratic default rates ( $\lambda_1, \lambda_2$ ), system wide default rate ( $\lambda_3$ ), marginal profit per unit ( $r - c$ ), and holding cost  $h$ . Each of the test parameters were tested over a series of values out of which specific values were chosen for insight.

The idiosyncratic default rate values were  $(\lambda_1, \lambda_2) \in \{\frac{1}{10}, \frac{1}{5}, \frac{1}{2}\}$ . Since the time horizon was set constant at 10, we tested infrequent idiosyncratic disruptions averaging from one per time horizon to frequent disruptions of 5 per time horizon. In the single supplier case only  $\lambda_1$  is considered as a degenerate case of the two supplier model. The values for the system wide default rate were  $(\lambda_3) \in \{\frac{1}{20}, \frac{1}{7}, \frac{1}{5}, \frac{1}{3}, \frac{1}{1}\}$ . Order cost was held constant at  $c = 1$ , so the revenue  $r$  was tested over several values,  $r \in \{1.1, 1.5, 5, 20\}$ , to change the marginal profit per unit,  $(r - c)$ . Lastly, the values for the holding cost were  $h \in \{0, 0.1, 0.2, 0.5\}$ . For ease of explanation we consider marginal profit per unit to be  $(r - c)$  without factoring in holding cost because  $r$  and  $c$  are per product bought and sold, whereas holding cost is per item, per unit time. Thus, we compare the holding cost against the profit for its effect. The values for the single supplier case yield a 240 case test suite.

We began with a base case of  $[\lambda_1 = \frac{1}{10}, \lambda_3 = \frac{1}{20}, r = 1.5, h = 0.2, c = 1, \xi = 5, T = 10]$ .

Then perturbing the parameters two at a time, we tested the interactions according to the high, medium, and low values of Table 3.1. Since we are highly interested in the effect of correlated defaults, the system wide disruption is considered for a high-med-low value, whereas  $\lambda_1$  has two levels.

	Low	Med.	High
$\lambda_1$	$\frac{1}{10}$	N/A	$\frac{1}{2}$
$\lambda_3$	$\frac{1}{20}$	$\frac{1}{5}$	1
r	1.5	N/A	20
h	0.1	N/A	0.5

Table 3.1: Test Parameter Values

### 3.4.2 Parameter Analysis

#### Holding Cost

We begin our parameter analysis by considering the holding cost of the single supplier as it relates to the maximum profit. As the holding cost rate increases, we expect to see the firm's total expected profit decrease. We are interested to learn what effect this phenomenon will have on the firm's order quantities.

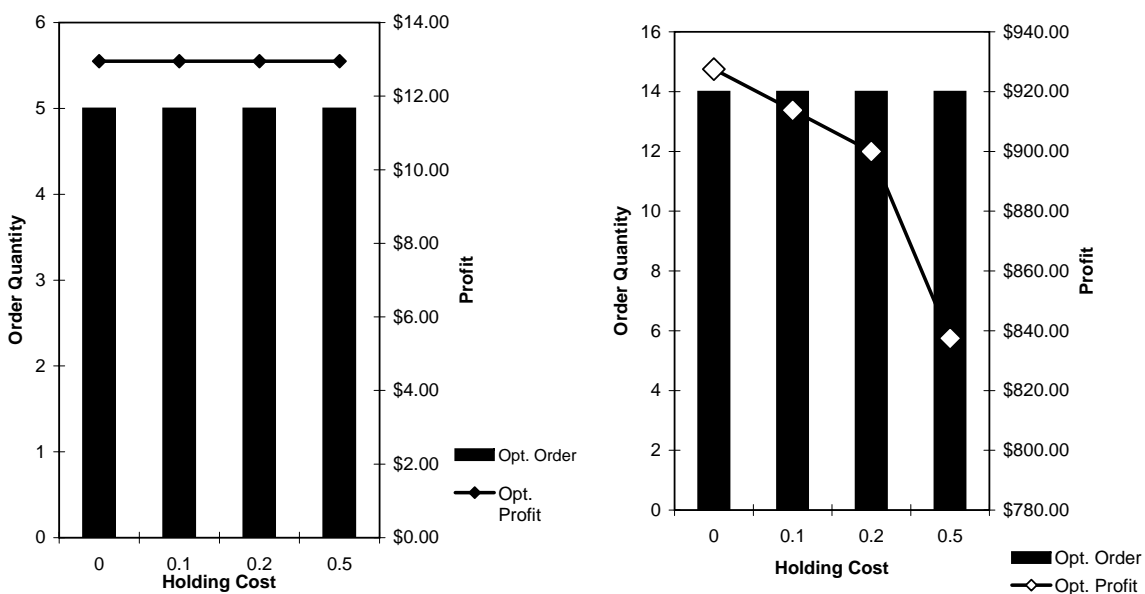


Figure 3.3: Order Qty and Max Profit vs. Holding Cost with Low Marginal Profit Per Unit(left) and High Marginal Profit Per Unit(right)

*Observation 6.* When marginal profit per unit ( $r-c$ ) is less than or equal to two times holding cost, inventory mitigation for non-zero default rates, is not desirable.

This is surprising given that one would expect the firm to use inventory mitigation as a strategy anytime per unit revenue is greater than per unit order and holding cost. However, what we find from Figure 3.3(left) that there is a consistent minimum order by the firm even as the holding cost increases. Even in the absence of holding cost the firm still orders the minimum. This can be attributed to the relative reliability of the supplier since both the idiosyncratic and system wide default rates are at their low level. At the high level of marginal profit per unit (see Figure 3.3(right)) we see the ordering increase from the lower level, however the ordering is still independent of the holding cost. The inventory mitigation at the high level is higher than the low level due to the default rates being at their low levels. As expected, once inventory mitigation is employed the maximum profit of the system becomes inversely proportional to the holding cost.

Figure 3.4 demonstrates the use of inventory mitigation as the idiosyncratic default rate increases. However, we learn that the use of inventory mitigation becomes dependent on the holding cost.

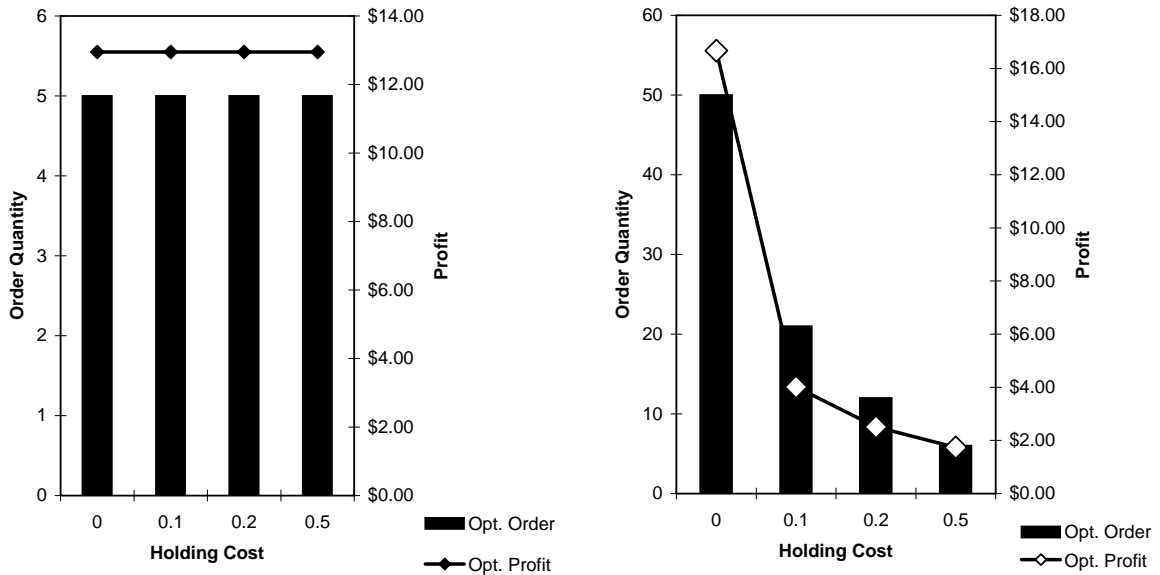


Figure 3.4: Order Qty and Max Profit vs. Holding Cost with Low Idiosyncratic Default Rate(left) and High Idiosyncratic Default Rate (right)

*Observation 7.* The firm's use of inventory mitigation is independent of the holding cost when the idiosyncratic disruption rate is low and becomes inversely proportional to the holding cost when the disruption rate is high.

At the low levels of idiosyncratic and system wide disruption, the firm treats the supplier as a reliable supplier (expecting 100% yield on orders). However as the probability of disruption increases the firm begins to accumulate inventory as a strategy to help mitigate the risk of a default. From Figure 3.4 (right) we see that the degree of use of this strategy decreases as the holding cost increases, which is intuitive. At the low level of holding cost, under high idiosyncratic disruption risk the firm orders the total demand that can be sold prior to the end of horizon since there is no penalty for excess inventory. However, at the high level of holding cost, the firm reverts back to ordering the minimum needed to serve demand even though the idiosyncratic disruption rate risk is high. When the holding cost was considered with low, medium, and high system wide default rates the results for the idiosyncratic disruption were the same. Since there is only one supplier in this model the we expect the system wide and idiosyncratic default rates to be symmetrical. Therefore, Observation 7 holds for not only the idiosyncratic disruptions, but the system wide disruptions as well and leads us to the following conclusion on the holding cost.

*Observation 8.* Holding cost bounds the firm's ordering between carrying inventory and not carrying inventory for high levels of disruption rate risk. For low levels of disruption rate risk the firm's ordering is independent of the holding cost.

### **Marginal Profit Per Unit**

Next we consider the revenue parameter which perturbs the firm's marginal profit per unit( $r - c$ ). As the values for  $(r - c)$  increase we expect the effects of holding cost to vanish as increased marginal profit per unit will insulate the firm from the adverse effects of holding cost. Given this, we can hold the holding cost static at its low level and look at how the firm orders when marginal profit per unit changes over low and



high levels of default risk.

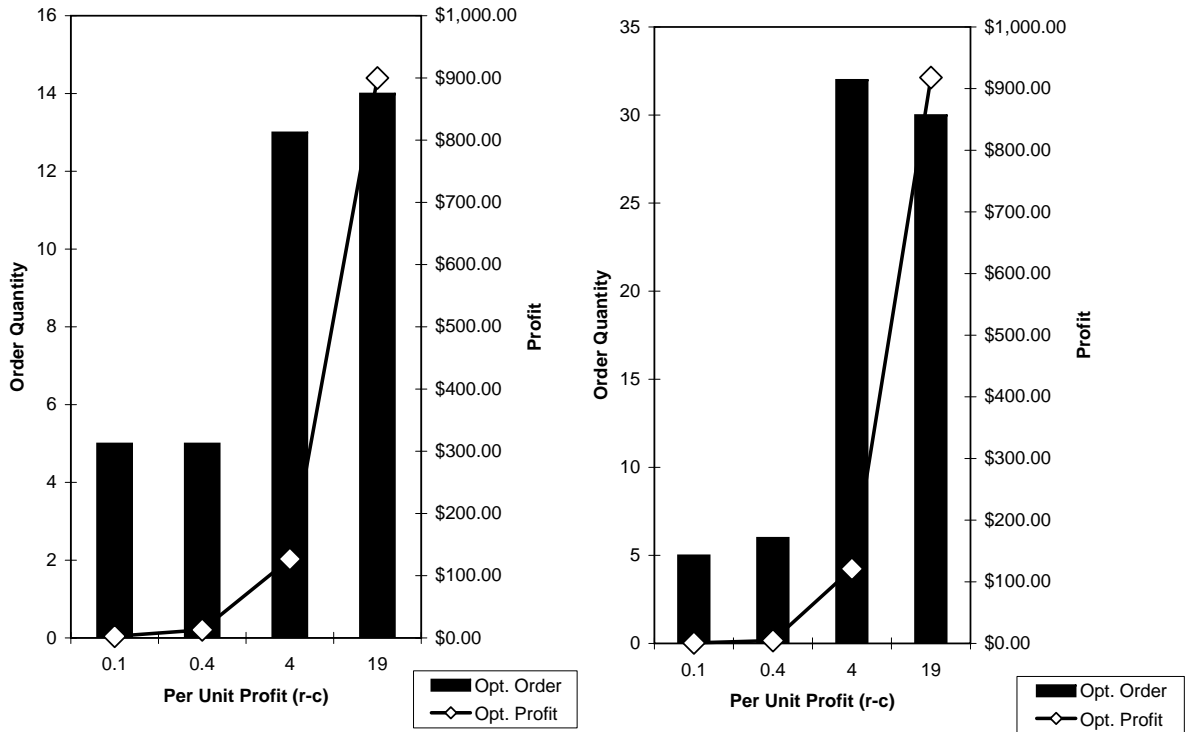


Figure 3.5: Order Qty and Max Profit vs. Marginal Profit Per Unit ( $r-c$ ) with Low Idiosyncratic Default Rate(left) and High Idiosyncratic Default Rate (right)

*Observation 9.* For the low values of marginal profit per unit, the ordering quantity stays nearly identical for low and high values of idiosyncratic disruption; very close to the demand rate  $\xi$ . For the high values of marginal profit per unit, however the ordering quantity is much more sensitive to the idiosyncratic disruption rate and is quite high.

Observation 9 is also applicable to system wide disruptions (see Figure 3.6). It appears that carrying inventory is not advantageous to the firm until certain thresholds of marginal profit per unit are reached. In the case of high disruption risk one would expect the firm simply to carry more inventory. Observation 9 shows us that even in the face of high disruption risk, a firm's decision to order more than demand

is dependent on more factors than just the desire to be able to meet future demand, it also depends on the profit paradigms.

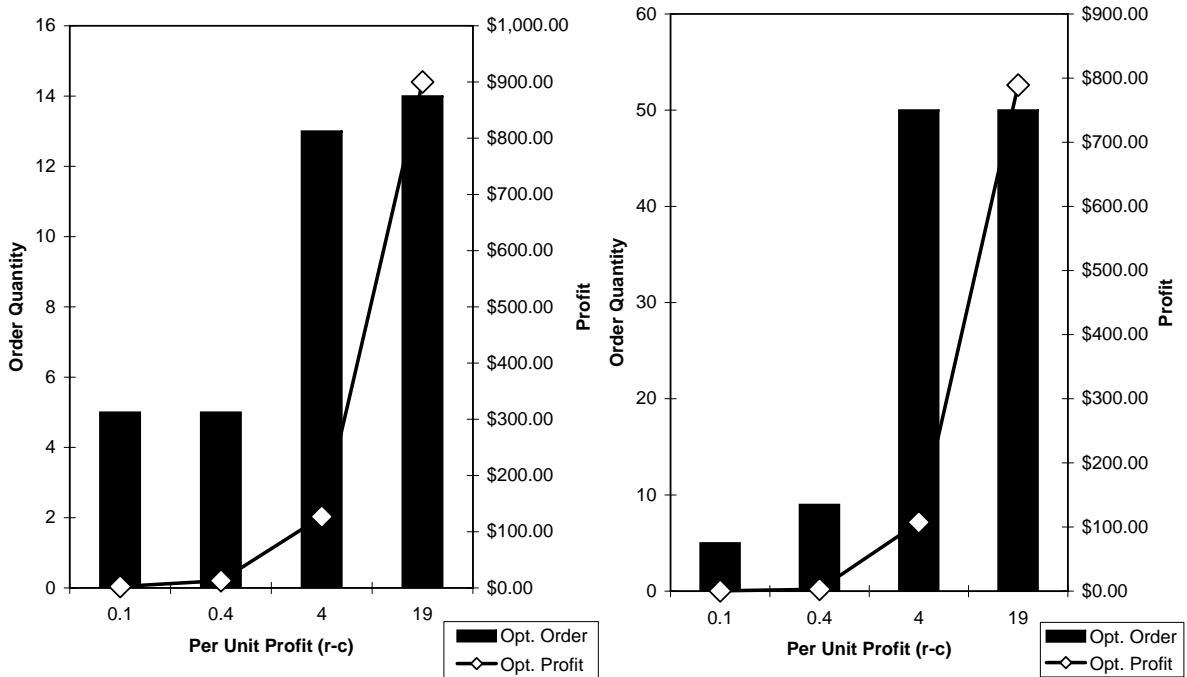


Figure 3.6: Order Qty and Max Profit vs. Marginal Profit Per Unit (r-c) with Low system wide Default Rate(left) and High system wide Default Rate (right)

### Disruption Risk

Our model consists of two disruption risk parameters. For the single supplier model the idiosyncratic and system wide disruptions rates serve as exponentials competing for which one will disrupt first and cause the firm to meet the demand rate via available inventory and the remaining supplier. Consider Figure 3.7 which compares the firm's optimal ordering across different system wide default rates under both low and high idiosyncratic default conditions.

*Observation 10.* Under low idiosyncratic default conditions, the firm's order quantities are independent of the system wide default rate, because the likelihood of supply disruption has not yet increased enough to make it profitable to hold inventory.

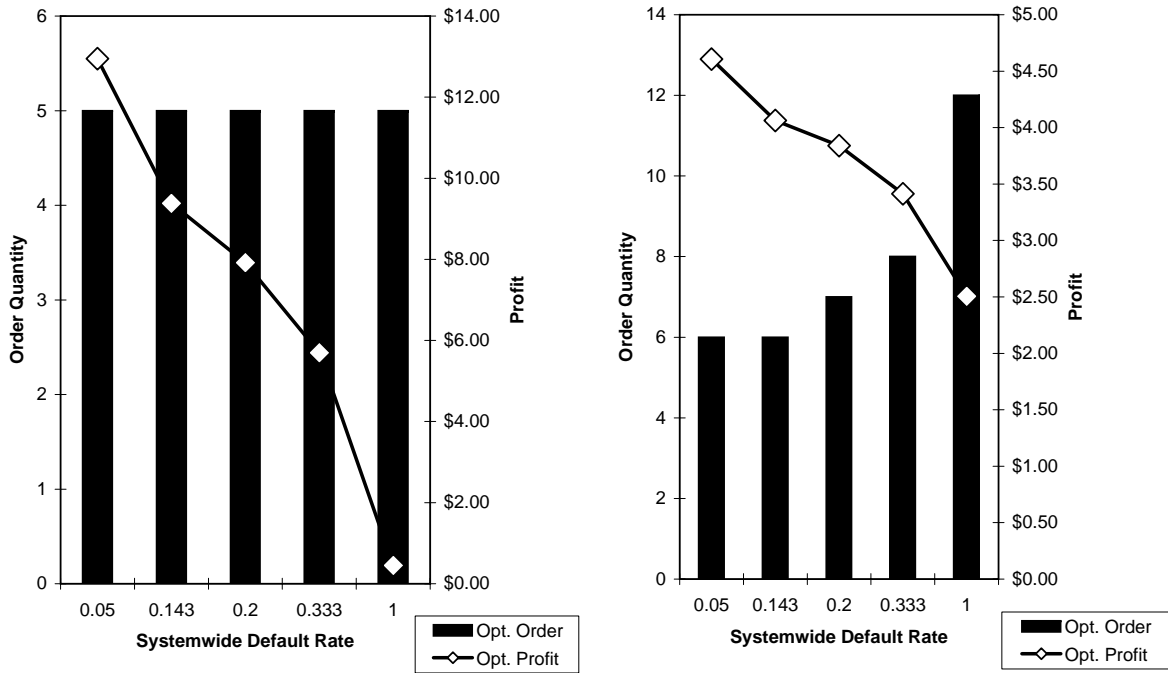


Figure 3.7: Order Qty and Max Profit vs. System wide Default Rate with Low Idiosyncratic Default Rate(left) and High Idiosyncratic Default Rate (right)

As the system wide default increases, the firm's total expected profit decreases, as expected. However, the independence of the firm's ordering from the system wide default rate increase, as shown in Figure 3.7 (left) is unexpected. However, if the firm wishes to order less than the order minimum then one can understand why the insensitivity occurs. Moreover, when the same parameters are tested under high idiosyncratic default rate conditions there is a modest increase in the ordering quantity to provide inventory mitigation. In contrast with Figure 3.6 the impact of the marginal profit per unit is clearly seen. When tested at its high level under high system wide default rate, the marginal profit per unit yields a high order quantity to fill inventory and sustain the firm throughout the horizon after the firm disrupts. This leads us to the following conclusion:

*Observation 11.* Order quantity is increasing in marginal profit per unit under high

disruption rate conditions.

This shows that for inventory to be used as a mitigation strategy against supplier disruption, there exists a minimum marginal profit per unit that is necessary for it to be effective. This holds true, for Figure 3.8, when the idiosyncratic disruption rate is tested under low and high marginal profit per unit conditions.

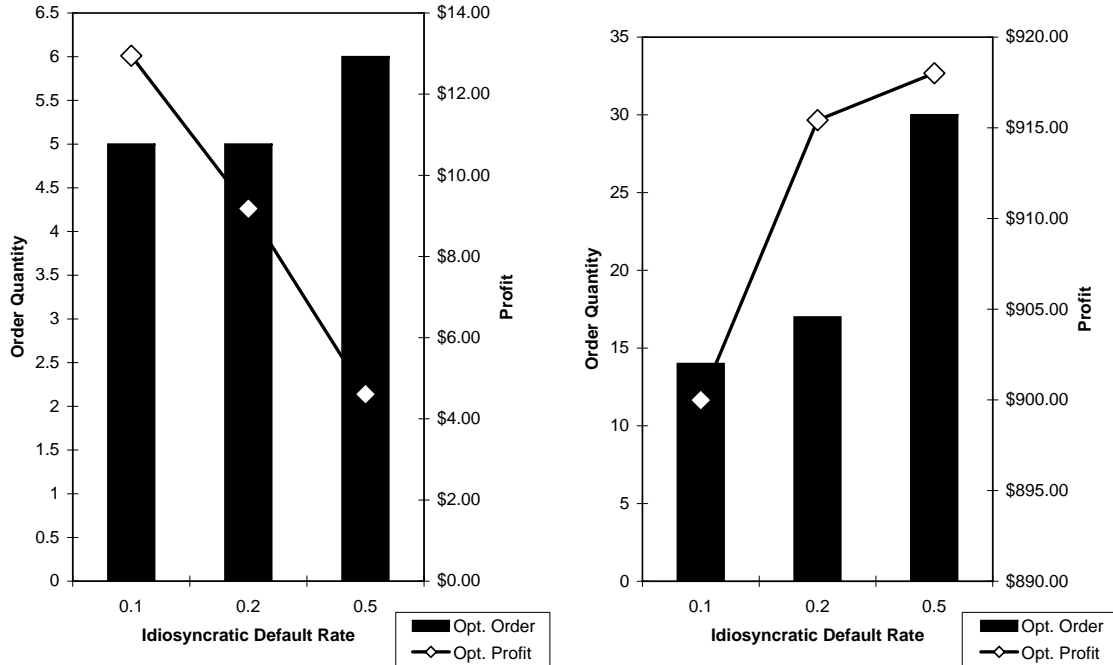


Figure 3.8: Order Qty and Max Profit vs. Idiosyncratic Default Rate with Low Marginal Profit Per Unit(left) and High Marginal Profit Per Unit (right)

Figure 3.8 (right) confirms our observations and demonstrates that inventory can be an effective strategy against default risk. What we find is that an important parameter to justify using inventory as a mitigation strategy is marginal profit per unit. Notice that, the firm is able to turn a decreasing profit, as a function of disruption rate, into an increasing profit by increased ordering and decreased total holding cost.

The single supplier case can only use inventory as a mitigation strategy against correlated default risk. We found that strategy to be effective only in high marginal profit per unit conditions. As we transition to the dual supplier case, both inventory

and sourcing from a second supplier can be used as mitigation strategies with the tradeoff being holding cost verses wholesale order cost. Interestingly, we seek to find the conditions which guide the firm to a dual sourcing strategy over single sourcing with inventory.

## 3.5 Dual Supplier Results

The same numerical study of Section 3.4.1 was used for the dual supplier case. For the dual supplier cases we seek to understand the value of dual sourcing as a strategy to mitigate disruption risk. Specifically, we are looking for insight into what range of disruption values lead to a dual sourcing strategy over single sourcing.

Embedded within the dual supplier case is the single supplier case with a nonzero value for  $T_{first}$ . In calculation, the dual supplier case simplifies to the single supplier case, with initial inventory, after the first disruption. The ordering restrictions,  $\gamma_1, \gamma_2 \geq \xi$ , allow for both inventory accumulation and dual sourcing; providing the firm with greater flexibility.

### 3.5.1 Flexible Sourcing

First we consider the effect holding cost has on dual supplier ordering under the low and high values of both disruptions risks. Dual sourcing, particularly under low disruption risk, will lead to an increase in the accumulation of inventory over single sourcing. Figure 3.9 shows dual supplier ordering to be independent of holding cost. This lack of effect can be attributed to the low value of the profit per unit for the given order minimum constraint. The single supplier case demonstrated that the positive effect of holding inventory only occurs under high profit per unit as single supplier ordering was insensitive to holding cost under low values as well. To confirm this intuition, we consider Figures 3.10 (left) and 3.11 (left) under the low and high values of the system wide disruption, and we find that this relationship continues:

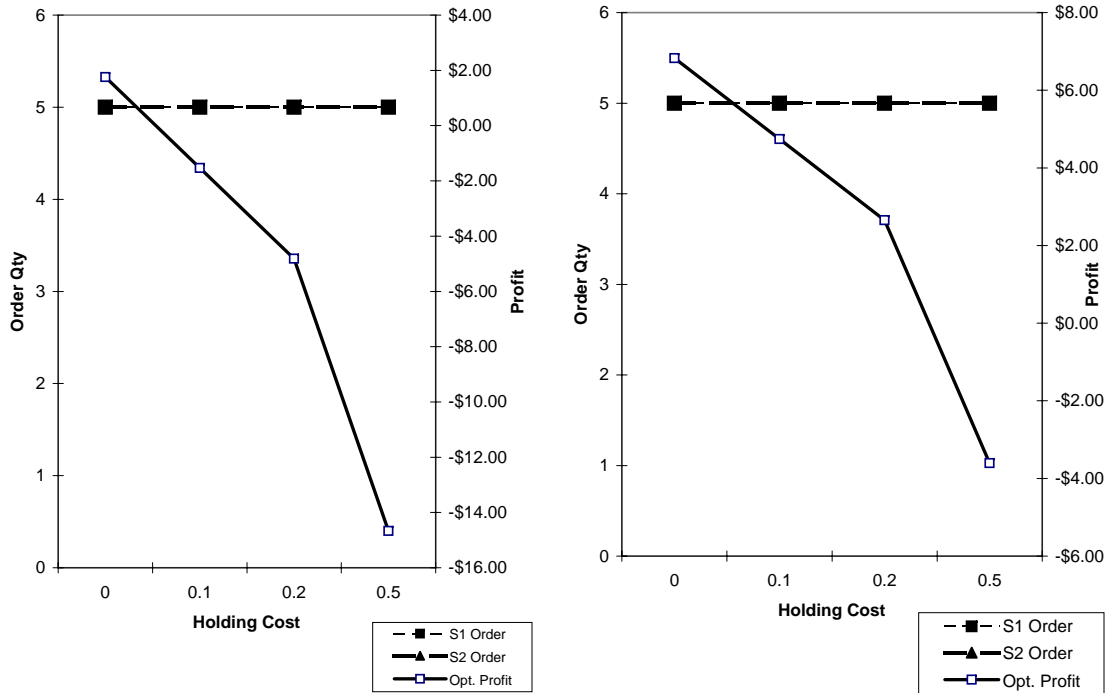


Figure 3.9: Dual Supplier Order Qty vs. Holding Cost vs. Max Profit with Low Idiosyncratic Default Rate(left) and High Idiosyncratic Default Rate (right)

*Observation 12.* To consider dual sourcing as a disruption risk mitigation strategy the system must have a high profit per unit.

Under low profit per unit, holding cost has too much of an effect on profit and thus drives ordering to the minimum quantities. This observation directs our attention to cases of the test suite, under the high profit per unit condition. A review of those cases, comparing the single supplier to the dual supplier, finds that flexible sourcing with a second supplier becomes quite important to the firm when contracting with a nearly reliable supplier (low idiosyncratic disruption) and an unreliable supplier (high idiosyncratic disruption).

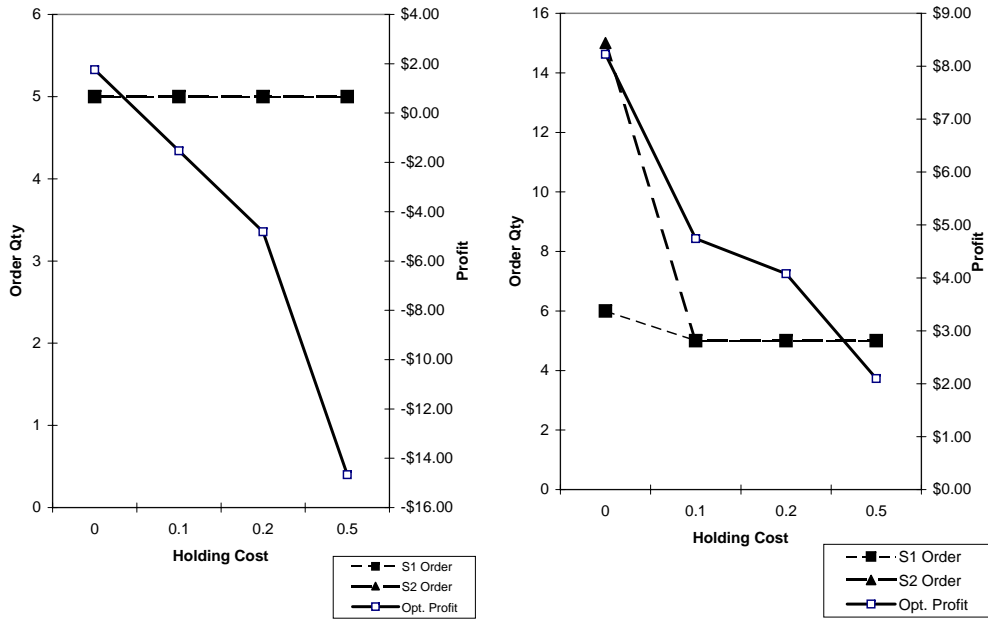


Figure 3.10: Order Qty vs. Holding Cost vs. Max Profit with Low system wide Default Rate(left) and Medium system wide Default Rate (right)

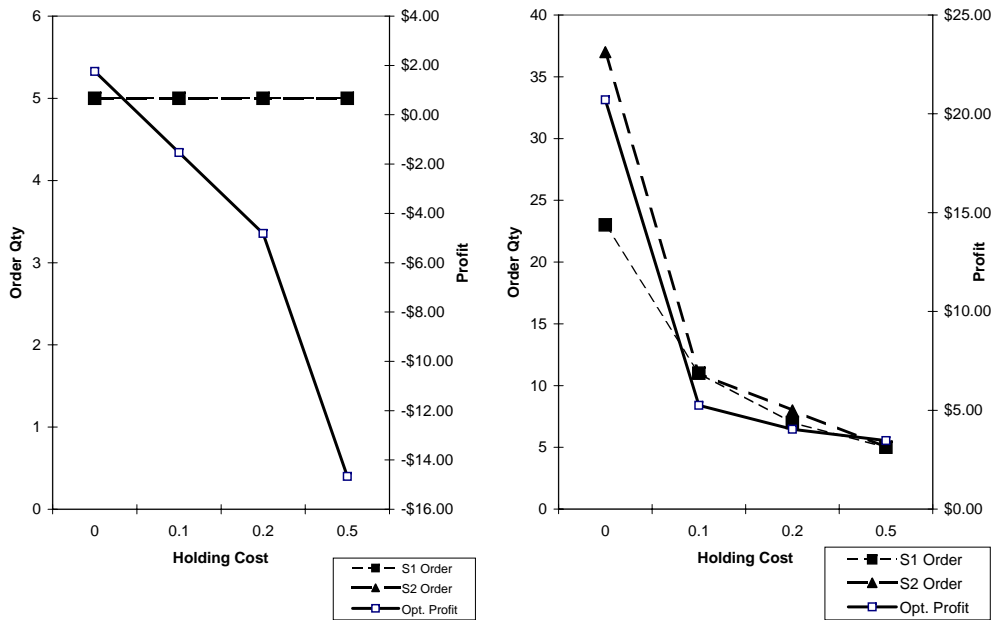


Figure 3.11: Order Qty vs. Holding Cost vs. Max Profit with Low system wide Default Rate(left) and High system wide Default Rate (right)

Figure 3.12 considers the average profit of cases under the high profit per unit condition where  $(\lambda_1 = 0.5, \lambda_2 = 0.1, \lambda_3 \in \{0.2, 0.5\})$ . We find that dual sourcing is

advantageous to single sourcing for all tested values of holding cost.

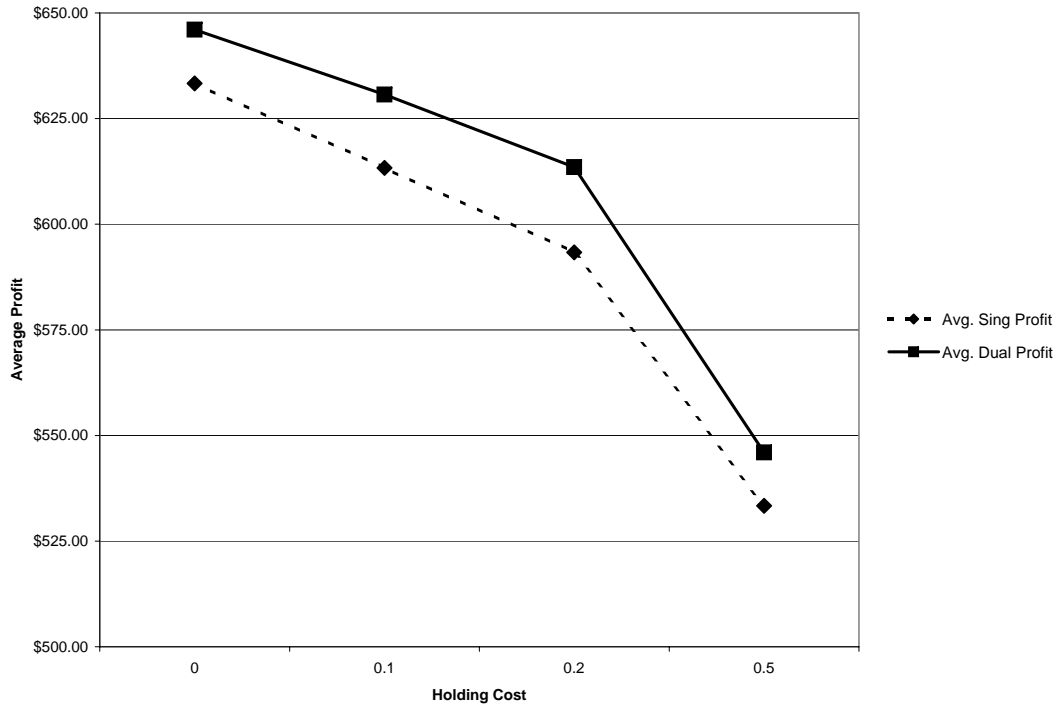


Figure 3.12: Average Profit vs. Holding Cost for both cases

To learn more about the effects under the system wide disruption we consider cases under the medium to high system wide disruption condition to see if dual sourcing is still advantageous to the firm.

Although not as clear a difference as seen in Figure 3.12, there does exist a difference between the average profit of the dual supplier cases, and the average profit of the single supplier cases under the high profit per unit and low holding cost conditions. (Note: The values above(below) the data point represent the dual supplier(single supplier) average profit and the values underneath represent the single supplier average profit.) Figures 3.12 and 3.13 yield the following conclusion:

*Observation 13.* Dual sourcing is a better disruption risk mitigation strategy than single sourcing with inventory under high profit per unit, high system wide disruption, and with one nearly reliable supplier and one unreliable supplier.



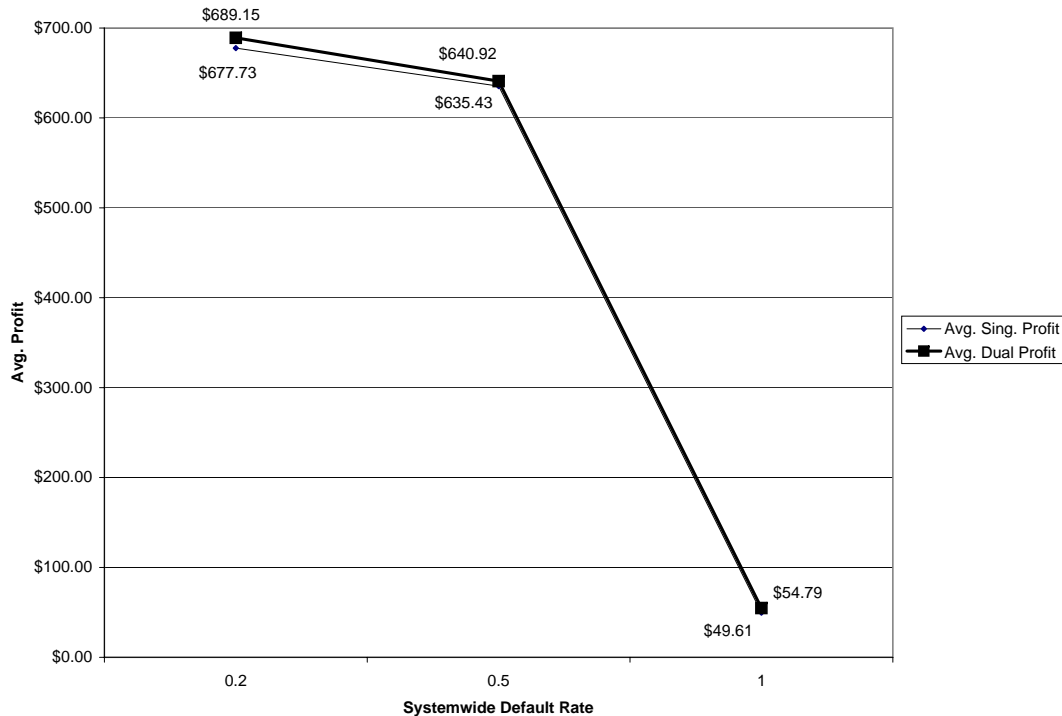


Figure 3.13: Average Profit vs. system wide Disruption for both cases

Intuitively the high profit and high disruption conditions of Observation 13 are reasonable when one would expect that as the risk of correlated defaults decreases the need for a second supplier decreases.

### 3.6 Conclusions and Future Work

Previous research has demonstrated the need for disruption risk mitigation strategies that go beyond the standard approach of using one supplier and not building up inventory to anticipate disruption. Two important strategies can include single sourcing with inventory or dual sourcing. In this paper we have sought to characterize observations about a firm's decision to single source or dual source when all of their available sourcing options are unreliable. Within the single source case we analyzed that model and found that a single sourcing strategy for the firm is best under high marginal profit per unit conditions where disruption risk, both idiosyn-

cratic and system wide, was low. Conversely, as disruption risk increases, the need to source with a second supplier also increases. Also, we learned the importance of the firm's cost structure, particularly as it relates to inventory holding cost. These insights provide managers with a foundation from which to consider the structure of their supply chains.

As it relates to extensions of this work, relaxing the assumption on the firm's ordering quantities yields one additional model for the single supplier case ( $\gamma_1 < \xi$ ) and three additional models for the dual supplier case  $\{(\gamma_1 < \xi, \gamma_2 \geq \xi), (\gamma_1 \geq \xi, \gamma_2 < \xi), (\gamma_1 < \xi, \gamma_2 < \xi)\}$ . Moreover, using real world financial data, (e.g. supplier credit ratings), to determine the idiosyncratic disruption risk would make the distributions on the time to disruption more consistent with real world experience. Given the great amount of future work that can be done in this area this work serves as a strong foundation in understanding a firm's sourcing decision when its suppliers are unreliable and there exists both idiosyncratic and system wide defaults. This work will help to strengthen supply chains as they combat the daily risk of supply default.

## 3.7 Appendix

### 3.7.1 Dual Supplier Profit Function

The subsequent pages represent the total expected profit,  $\mathbb{E}[\ddot{\Pi}^D]$ , for the dual supplier case.

$$\begin{aligned}
& e^{T(-\lambda_1-\lambda_2-\lambda_3)} \left( -c T \gamma_1 - \frac{1}{2} h T^2 \gamma_1 - c T \gamma_2 - \frac{1}{2} h T^2 \gamma_2 + r T \xi + \frac{1}{2} h T^2 \xi \right) - \\
& \frac{1}{2(\lambda_1 + \lambda_2) \lambda_3} \\
& e^{-T \lambda_3} \left( 1 - e^{-T \lambda_3} \right) \left( h \left( -2 + 2 e^{T \lambda_3} - 2 T \lambda_3 - T^2 \lambda_3^2 \right) (\gamma_1 + \gamma_2 - \xi) - \right. \\
& \quad \left. 2 \lambda_3 \left( 1 - e^{T \lambda_3} + T \lambda_3 \right) \left( c (\gamma_1 + \gamma_2) - r \xi \right) \right) - \\
& \frac{1}{2(\lambda_1 + \lambda_2) \lambda_3 \xi} \left( 1 - e^{-T \lambda_3} \right) (\gamma_1 + \gamma_2 - \xi) \\
& \left( 2 h (\gamma_1 + \gamma_2 - 3 \xi) - 2 r \lambda_3 \xi + e^{-\frac{T \lambda_3 \xi}{\gamma_1 + \gamma_2}} \left( 2 r \lambda_3 \xi \left( 1 + \frac{T \lambda_3 \xi}{\gamma_1 + \gamma_2} \right) - \right. \right. \\
& \quad \left. \left. h (\gamma_1 + \gamma_2 - 3 \xi) \left( 2 + \frac{2 T \lambda_3 \xi}{\gamma_1 + \gamma_2} + \frac{T^2 \lambda_3^2 \xi^2}{(\gamma_1 + \gamma_2)^2} \right) \right) \right) + \\
& \frac{1}{2(\lambda_1 + \lambda_2) \lambda_3} \left( 1 - e^{-T \lambda_3} \right) \left( -2 e^{-T \lambda_3} (-r \lambda_3 \xi + \right. \\
& \quad \left. h (2 \gamma_2 + T \gamma_2 \lambda_3 + \gamma_1 (2 + T \lambda_3) - 3 \xi - 2 T \lambda_3 \xi) \right) + \frac{1}{(\gamma_1 + \gamma_2)^2} \\
& e^{-\frac{T \lambda_3 \xi}{\gamma_1 + \gamma_2}} \left( 2 r (\gamma_1 + \gamma_2) \lambda_3 \xi (\gamma_1 (-1 + T \lambda_3) + \gamma_2 (-1 + T \lambda_3) - \right. \\
& \quad T \lambda_3 \xi) - h \left( 2 \gamma_1^3 (-2 + T \lambda_3) + 2 \gamma_2^3 (-2 + T \lambda_3) + \right. \\
& \quad \gamma_2^2 (6 - 6 T \lambda_3 + T^2 \lambda_3^2) \xi + 2 T \gamma_2 \lambda_3 (3 - 2 T \lambda_3) \xi^2 + 3 T^2 \\
& \quad \lambda_3^2 \xi^3 + \gamma_1^2 (6 \gamma_2 (-2 + T \lambda_3) + (6 - 6 T \lambda_3 + T^2 \lambda_3^2) \xi) + \\
& \quad \left. \left. 2 \gamma_1 (3 \gamma_2^2 (-2 + T \lambda_3) + \gamma_2 (6 - 6 T \lambda_3 + T^2 \lambda_3^2) \right. \right. \\
& \quad \left. \left. \xi + T \lambda_3 (3 - 2 T \lambda_3) \xi^2) \right) \right) \right) + \\
& \frac{1}{\lambda_1 + \lambda_3} \left( 1 - e^{-T \lambda_2} \right) \lambda_2^2 \left( \frac{1}{2(-2 \lambda_1 + \lambda_2 - 2 \lambda_3)^3 (\lambda_1 + \lambda_3)^2} \right. \\
& \quad \left( 24 e^{-2 T (\lambda_1 + \lambda_3)} h \gamma_1 \lambda_1^2 - 16 c e^{-2 T (\lambda_1 + \lambda_3)} \gamma_1 \lambda_1^3 + \right. \\
& \quad 16 e^{-2 T (\lambda_1 + \lambda_3)} h T \gamma_1 \lambda_1^3 - 8 c e^{-2 T (\lambda_1 + \lambda_3)} T \gamma_1 \lambda_1^4 + \\
& \quad 4 e^{-2 T (\lambda_1 + \lambda_3)} h T^2 \gamma_1 \lambda_1^4 - 24 e^{-2 T (\lambda_1 + \lambda_3)} h \gamma_1 \lambda_1 \lambda_2 + \\
& \quad 2 e^{-2 T (\lambda_1 + \lambda_3)} h \gamma_2 \lambda_1 \lambda_2 + 12 c e^{-2 T (\lambda_1 + \lambda_3)} \gamma_1 \lambda_1^2 \lambda_2 - \\
& \quad 16 e^{-2 T (\lambda_1 + \lambda_3)} h T \gamma_1 \lambda_1^2 \lambda_2 + 8 c e^{-2 T (\lambda_1 + \lambda_3)} T \gamma_1 \lambda_1^3 \lambda_2 - \\
& \quad \left. \left. 4 e^{-2 T (\lambda_1 + \lambda_3)} h T^2 \gamma_1 \lambda_1^3 \lambda_2 + 6 e^{-2 T (\lambda_1 + \lambda_3)} h \gamma_1 \lambda_2^2 - \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 2 c e^{-2 T(\lambda_1+\lambda_3)} \gamma_1 \lambda_1 \lambda_2^2 + 4 e^{-2 T(\lambda_1+\lambda_3)} h T \gamma_1 \lambda_1 \lambda_2^2 - \\
& 2 c e^{-2 T(\lambda_1+\lambda_3)} T \gamma_1 \lambda_1^2 \lambda_2^2 + e^{-2 T(\lambda_1+\lambda_3)} h T^2 \gamma_1 \lambda_1^2 \lambda_2^2 + \\
& 48 e^{-2 T(\lambda_1+\lambda_3)} h \gamma_1 \lambda_1 \lambda_3 - 48 c e^{-2 T(\lambda_1+\lambda_3)} \gamma_1 \lambda_1^2 \lambda_3 + \\
& 48 e^{-2 T(\lambda_1+\lambda_3)} h T \gamma_1 \lambda_1^2 \lambda_3 - 32 c e^{-2 T(\lambda_1+\lambda_3)} T \gamma_1 \lambda_1^3 \lambda_3 + \\
& 16 e^{-2 T(\lambda_1+\lambda_3)} h T^2 \gamma_1 \lambda_1^3 \lambda_3 - 24 e^{-2 T(\lambda_1+\lambda_3)} h \gamma_1 \lambda_2 \lambda_3 + \\
& 2 e^{-2 T(\lambda_1+\lambda_3)} h \gamma_2 \lambda_2 \lambda_3 + 24 c e^{-2 T(\lambda_1+\lambda_3)} \gamma_1 \lambda_1 \lambda_2 \lambda_3 - \\
& 32 e^{-2 T(\lambda_1+\lambda_3)} h T \gamma_1 \lambda_1 \lambda_2 \lambda_3 + 24 c e^{-2 T(\lambda_1+\lambda_3)} T \gamma_1 \lambda_1^2 \lambda_2 \lambda_3 - \\
& 12 e^{-2 T(\lambda_1+\lambda_3)} h T^2 \gamma_1 \lambda_1^2 \lambda_2 \lambda_3 - 2 c e^{-2 T(\lambda_1+\lambda_3)} \gamma_1 \lambda_2^2 \lambda_3 + \\
& 4 e^{-2 T(\lambda_1+\lambda_3)} h T \gamma_1 \lambda_2^2 \lambda_3 - 4 c e^{-2 T(\lambda_1+\lambda_3)} T \gamma_1 \lambda_1 \lambda_2^2 \lambda_3 + \\
& 2 e^{-2 T(\lambda_1+\lambda_3)} h T^2 \gamma_1 \lambda_1 \lambda_2^2 \lambda_3 + 24 e^{-2 T(\lambda_1+\lambda_3)} h \gamma_1 \lambda_3^2 - \\
& 48 c e^{-2 T(\lambda_1+\lambda_3)} \gamma_1 \lambda_1 \lambda_3^2 + 48 e^{-2 T(\lambda_1+\lambda_3)} h T \gamma_1 \lambda_1 \lambda_3^2 - \\
& 48 c e^{-2 T(\lambda_1+\lambda_3)} T \gamma_1 \lambda_1^2 \lambda_3^2 + 24 e^{-2 T(\lambda_1+\lambda_3)} h T^2 \gamma_1 \lambda_1^2 \lambda_3^2 + \\
& 12 c e^{-2 T(\lambda_1+\lambda_3)} \gamma_1 \lambda_2 \lambda_3^2 - 16 e^{-2 T(\lambda_1+\lambda_3)} h T \gamma_1 \lambda_2 \lambda_3^2 + \\
& 24 c e^{-2 T(\lambda_1+\lambda_3)} T \gamma_1 \lambda_1 \lambda_2 \lambda_3^2 - 12 e^{-2 T(\lambda_1+\lambda_3)} h T^2 \gamma_1 \lambda_1 \lambda_2 \lambda_3^2 - \\
& 2 c e^{-2 T(\lambda_1+\lambda_3)} T \gamma_1 \lambda_2^2 \lambda_3^2 + e^{-2 T(\lambda_1+\lambda_3)} h T^2 \gamma_1 \lambda_2^2 \lambda_3^2 - \\
& 16 c e^{-2 T(\lambda_1+\lambda_3)} \gamma_1 \lambda_3^3 + 16 e^{-2 T(\lambda_1+\lambda_3)} h T \gamma_1 \lambda_3^3 - \\
& 32 c e^{-2 T(\lambda_1+\lambda_3)} T \gamma_1 \lambda_1 \lambda_3^3 + 16 e^{-2 T(\lambda_1+\lambda_3)} h T^2 \gamma_1 \lambda_1 \lambda_3^3 + \\
& 8 c e^{-2 T(\lambda_1+\lambda_3)} T \gamma_1 \lambda_2 \lambda_3^3 - 4 e^{-2 T(\lambda_1+\lambda_3)} h T^2 \gamma_1 \lambda_2 \lambda_3^3 - \\
& 8 c e^{-2 T(\lambda_1+\lambda_3)} T \gamma_1 \lambda_3^4 + 4 e^{-2 T(\lambda_1+\lambda_3)} h T^2 \gamma_1 \lambda_3^4 - \\
& 30 e^{-2 T(\lambda_1+\lambda_3)} h \lambda_1^2 \xi - 4 e^{-2 T(\lambda_1+\lambda_3)} r \lambda_1^3 \xi - \\
& 28 e^{-2 T(\lambda_1+\lambda_3)} h T \lambda_1^3 \xi - 8 e^{-2 T(\lambda_1+\lambda_3)} h T^2 \lambda_1^4 \xi + \\
& 26 e^{-2 T(\lambda_1+\lambda_3)} h \lambda_1 \lambda_2 \xi + 6 e^{-2 T(\lambda_1+\lambda_3)} r \lambda_1^2 \lambda_2 \xi + \\
& 26 e^{-2 T(\lambda_1+\lambda_3)} h T \lambda_1^2 \lambda_2 \xi + 8 e^{-2 T(\lambda_1+\lambda_3)} h T^2 \lambda_1^3 \lambda_2 \xi - \\
& 6 e^{-2 T(\lambda_1+\lambda_3)} h \lambda_2^2 \xi - 2 e^{-2 T(\lambda_1+\lambda_3)} r \lambda_1 \lambda_2^2 \xi - \\
& 6 e^{-2 T(\lambda_1+\lambda_3)} h T \lambda_1 \lambda_2^2 \xi - 2 e^{-2 T(\lambda_1+\lambda_3)} h T^2 \lambda_1^2 \lambda_2^2 \xi - \\
& 60 e^{-2 T(\lambda_1+\lambda_3)} h \lambda_1 \lambda_3 \xi - 12 e^{-2 T(\lambda_1+\lambda_3)} r \lambda_1^2 \lambda_3 \xi - \\
& 84 e^{-2 T(\lambda_1+\lambda_3)} h T \lambda_1^2 \lambda_3 \xi - 32 e^{-2 T(\lambda_1+\lambda_3)} h T^2 \lambda_1^3 \lambda_3 \xi + \\
& 26 e^{-2 T(\lambda_1+\lambda_3)} h \lambda_2 \lambda_3 \xi + 12 e^{-2 T(\lambda_1+\lambda_3)} r \lambda_1 \lambda_2 \lambda_3 \xi + \\
& 52 e^{-2 T(\lambda_1+\lambda_3)} h T \lambda_1 \lambda_2 \lambda_3 \xi + 24 e^{-2 T(\lambda_1+\lambda_3)} h T^2 \lambda_1^2 \lambda_2 \lambda_3 \xi - \\
& 2 e^{-2 T(\lambda_1+\lambda_3)} r \lambda_2^2 \lambda_3 \xi - 6 e^{-2 T(\lambda_1+\lambda_3)} h T \lambda_2^2 \lambda_3 \xi - \\
& 4 e^{-2 T(\lambda_1+\lambda_3)} h T^2 \lambda_1 \lambda_2^2 \lambda_3 \xi - 30 e^{-2 T(\lambda_1+\lambda_3)} h \lambda_3^2 \xi - \\
& 12 e^{-2 T(\lambda_1+\lambda_3)} r \lambda_1 \lambda_3^2 \xi - 84 e^{-2 T(\lambda_1+\lambda_3)} h T \lambda_1 \lambda_3^2 \xi - \\
& 48 e^{-2 T(\lambda_1+\lambda_3)} h T^2 \lambda_1^2 \lambda_3^2 \xi + 6 e^{-2 T(\lambda_1+\lambda_3)} r \lambda_2 \lambda_3^2 \xi + \\
& 26 e^{-2 T(\lambda_1+\lambda_3)} h T \lambda_2 \lambda_3^2 \xi + 24 e^{-2 T(\lambda_1+\lambda_3)} h T^2 \lambda_1 \lambda_2 \lambda_3^2 \xi -
\end{aligned}$$

$$\begin{aligned}
& 2 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T}^2 \lambda_2^2 \lambda_3^2 \xi - 4 e^{-2T(\lambda_1+\lambda_3)} \mathbf{r} \lambda_3^3 \xi - \\
& 28 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T} \lambda_3^3 \xi - 32 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T}^2 \lambda_1 \lambda_3^3 \xi + \\
& 8 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T}^2 \lambda_2 \lambda_3^3 \xi - 8 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T}^2 \lambda_3^4 \xi) + \\
& \frac{1}{\lambda_2^3 (\lambda_1 + \lambda_3)^2 \xi} \left( -\mathbf{h} \gamma_1^2 \lambda_1^2 - 2 \mathbf{h} \gamma_1 \gamma_2 \lambda_1^2 - \mathbf{h} \gamma_2^2 \lambda_1^2 - \right. \\
& \mathbf{h} \gamma_1^2 \lambda_1 \lambda_2 - \mathbf{h} \gamma_1 \gamma_2 \lambda_1 \lambda_2 - \mathbf{h} \gamma_1^2 \lambda_2^2 - 2 \mathbf{h} \gamma_1^2 \lambda_1 \lambda_3 - \\
& 4 \mathbf{h} \gamma_1 \gamma_2 \lambda_1 \lambda_3 - 2 \mathbf{h} \gamma_2^2 \lambda_1 \lambda_3 - \mathbf{h} \gamma_1^2 \lambda_2 \lambda_3 - \mathbf{h} \gamma_1 \gamma_2 \lambda_2 \lambda_3 - \\
& \mathbf{h} \gamma_1^2 \lambda_3^2 - 2 \mathbf{h} \gamma_1 \gamma_2 \lambda_3^2 - \mathbf{h} \gamma_2^2 \lambda_3^2 + 2 \mathbf{h} \gamma_1 \lambda_1^2 \xi + 3 \mathbf{h} \gamma_2 \lambda_1^2 \xi + \\
& 3 \mathbf{h} \gamma_1 \lambda_1 \lambda_2 \xi + \mathbf{h} \gamma_2 \lambda_1 \lambda_2 \xi + \mathbf{r} \gamma_1 \lambda_1^2 \lambda_2 \xi - \mathbf{c} \gamma_2 \lambda_1^2 \lambda_2 \xi + \\
& \mathbf{r} \gamma_2 \lambda_1^2 \lambda_2 \xi + 3 \mathbf{h} \gamma_1 \lambda_2^2 \xi - \mathbf{c} \gamma_1 \lambda_1 \lambda_2^2 \xi + \mathbf{r} \gamma_1 \lambda_1 \lambda_2^2 \xi + \\
& 4 \mathbf{h} \gamma_1 \lambda_1 \lambda_3 \xi + 6 \mathbf{h} \gamma_2 \lambda_1 \lambda_3 \xi + 3 \mathbf{h} \gamma_1 \lambda_2 \lambda_3 \xi + \mathbf{h} \gamma_2 \lambda_2 \lambda_3 \xi + \\
& 2 \mathbf{r} \gamma_1 \lambda_1 \lambda_2 \lambda_3 \xi - 2 \mathbf{c} \gamma_2 \lambda_1 \lambda_2 \lambda_3 \xi + 2 \mathbf{r} \gamma_2 \lambda_1 \lambda_2 \lambda_3 \xi - \\
& \mathbf{c} \gamma_1 \lambda_2^2 \lambda_3 \xi + \mathbf{r} \gamma_1 \lambda_2^2 \lambda_3 \xi + 2 \mathbf{h} \gamma_1 \lambda_3^2 \xi + 3 \mathbf{h} \gamma_2 \lambda_3^2 \xi + \\
& \mathbf{r} \gamma_1 \lambda_2 \lambda_3^2 \xi - \mathbf{c} \gamma_2 \lambda_2 \lambda_3^2 \xi + \mathbf{r} \gamma_2 \lambda_2 \lambda_3^2 \xi - \mathbf{h} \lambda_1^2 \xi^2 - \\
& 2 \mathbf{h} \lambda_1 \lambda_2 \xi^2 - \mathbf{r} \lambda_1^2 \lambda_2 \xi^2 - 2 \mathbf{h} \lambda_2^2 \xi^2 - 2 \mathbf{h} \lambda_1 \lambda_3 \xi^2 - \\
& 2 \mathbf{h} \lambda_2 \lambda_3 \xi^2 - 2 \mathbf{r} \lambda_1 \lambda_2 \lambda_3 \xi^2 - \mathbf{h} \lambda_3^2 \xi^2 - \mathbf{r} \lambda_2 \lambda_3^2 \xi^2) + \\
& \frac{1}{2 (-\lambda_1 + \lambda_2 - \lambda_3)^3 (\lambda_1 + \lambda_3)^2 \xi} \left( 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1^2 \lambda_1^2 + \right. \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1 \gamma_2 \lambda_1^2 + 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_2^2 \lambda_1^2 - \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1^2 \lambda_1 \lambda_2 + 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1 \gamma_2 \lambda_1 \lambda_2 + \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1^2 \lambda_2^2 + 4 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1^2 \lambda_1 \lambda_3 + \\
& 4 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1 \gamma_2 \lambda_1 \lambda_3 + 4 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_2^2 \lambda_1 \lambda_3 - \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1^2 \lambda_2 \lambda_3 + 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1 \gamma_2 \lambda_2 \lambda_3 + \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1^2 \lambda_3^2 + 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1 \gamma_2 \lambda_3^2 + \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_2^2 \lambda_3^2 - 6 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1 \lambda_1^2 \xi - \\
& 4 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_2 \lambda_1^2 \xi + 8 \mathbf{c} e^{-T(\lambda_1+\lambda_3)} \gamma_1 \lambda_1^3 \xi - \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T} \gamma_1 \lambda_1^3 \xi + 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \gamma_2 \lambda_1^3 \xi + \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T} \gamma_2 \lambda_1^3 \xi - 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T}^2 \gamma_1 \lambda_1^4 \xi + \\
& 18 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1 \lambda_1 \lambda_2 \xi - 4 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_2 \lambda_1 \lambda_2 \xi - \\
& 12 \mathbf{c} e^{-T(\lambda_1+\lambda_3)} \gamma_1 \lambda_1^2 \lambda_2 \xi + 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \gamma_1 \lambda_1^2 \lambda_2 \xi + \\
& 6 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T} \gamma_1 \lambda_1^2 \lambda_2 \xi - 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \gamma_2 \lambda_1^2 \lambda_2 \xi - \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T} \gamma_2 \lambda_1^2 \lambda_2 \xi + 4 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T}^2 \gamma_1 \lambda_1^3 \lambda_2 \xi - \\
& 12 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1 \lambda_2^2 \xi + 4 \mathbf{c} e^{-T(\lambda_1+\lambda_3)} \gamma_1 \lambda_1 \lambda_2^2 \xi -
\end{aligned}$$



$$\begin{aligned}
& 4 e^{-T(\lambda_1+\lambda_3)} \mathbf{r}^T \lambda_1 \lambda_2^2 \lambda_3 \xi^2 + 6 e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_1 \lambda_2^2 \lambda_3 \xi^2 + \\
& 8 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \lambda_3^2 \xi^2 - 6 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \lambda_1 \lambda_3^2 \xi^2 + \\
& 18 e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_1 \lambda_3^2 \xi^2 + 12 e^{-T(\lambda_1+\lambda_3)} \mathbf{r}^T \lambda_1^2 \lambda_3^2 \xi^2 + \\
& 18 e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_1^2 \lambda_3^2 \xi^2 - 12 e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_2 \lambda_3^2 \xi^2 - \\
& 12 e^{-T(\lambda_1+\lambda_3)} \mathbf{r}^T \lambda_1 \lambda_2 \lambda_3^2 \xi^2 - 18 e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_1 \lambda_2 \lambda_3^2 \xi^2 + \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{r}^T \lambda_2^2 \lambda_3^2 \xi^2 + 3 e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_2^2 \lambda_3^2 \xi^2 - \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \lambda_3^3 \xi^2 + 6 e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_3^3 \xi^2 + \\
& 8 e^{-T(\lambda_1+\lambda_3)} \mathbf{r}^T \lambda_1 \lambda_3^3 \xi^2 + 12 e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_1 \lambda_3^3 \xi^2 - \\
& 4 e^{-T(\lambda_1+\lambda_3)} \mathbf{r}^T \lambda_2 \lambda_3^3 \xi^2 - 6 e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_2 \lambda_3^3 \xi^2 + \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{r}^T \lambda_3^4 \xi^2 + 3 e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_3^4 \xi^2) + \\
& \left( e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h} \gamma_1^4 \lambda_1 + e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h} \gamma_1^3 \gamma_2 \lambda_1 - e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h} \gamma_1^4 \lambda_2 + \right. \\
& e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h} \gamma_1^4 \lambda_3 + e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h} \gamma_1^3 \gamma_2 \lambda_3 - \\
& e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1^3 \lambda_1^2 \xi - e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1^2 \gamma_2 \lambda_1^2 \xi - \\
& 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h} \gamma_1^3 \lambda_2 \xi + e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1^3 \lambda_1 \lambda_2 \xi - \\
& 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1^3 \lambda_1 \lambda_3 \xi - 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1^2 \gamma_2 \lambda_1 \lambda_3 \xi + \\
& e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1^3 \lambda_2 \lambda_3 \xi - e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1^3 \lambda_3^2 \xi - \\
& e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1^2 \gamma_2 \lambda_3^2 \xi - 3 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h} \gamma_1^2 \lambda_1 \xi^2 - \\
& 3 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h} \gamma_1 \gamma_2 \lambda_1 \xi^2 - 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1^2 \lambda_1^2 \xi^2 - \\
& e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h}^T \gamma_1^2 \lambda_1^2 \xi^2 - 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1 \gamma_2 \lambda_1^2 \xi^2 - \\
& e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h}^T \gamma_1 \gamma_2 \lambda_1^2 \xi^2 + 3 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1^2 \lambda_1 \lambda_2 \xi^2 + \\
& e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h}^T \gamma_1^2 \lambda_1 \lambda_2 \xi^2 - 3 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h} \gamma_1^2 \lambda_3 \xi^2 - \\
& 3 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h} \gamma_1 \gamma_2 \lambda_3 \xi^2 - 4 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1^2 \lambda_1 \lambda_3 \xi^2 - \\
& 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h}^T \gamma_1^2 \lambda_1 \lambda_3 \xi^2 - 4 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1 \gamma_2 \lambda_1 \lambda_3 \xi^2 - \\
& 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h}^T \gamma_1 \gamma_2 \lambda_1 \lambda_3 \xi^2 + 3 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1^2 \lambda_2 \lambda_3 \xi^2 + \\
& e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h}^T \gamma_1^2 \lambda_2 \lambda_3 \xi^2 - 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1^2 \lambda_3^2 \xi^2 - \\
& e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h}^T \gamma_1^2 \lambda_3^2 \xi^2 - 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1 \gamma_2 \lambda_3^2 \xi^2 - \\
& \left. e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h}^T \gamma_1 \gamma_2 \lambda_3^2 \xi^2 - 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h} \gamma_1 \lambda_1 \xi^3 - \right)
\end{aligned}$$



$$\begin{aligned}
& 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h \gamma_2 \lambda_1 \xi^3 - e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} r \gamma_1 \lambda_1^2 \xi^3 - \\
& e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h T \gamma_1 \lambda_1^2 \xi^3 - e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} r \gamma_2 \lambda_1^2 \xi^3 - \\
& e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h T \gamma_2 \lambda_1^2 \xi^3 + 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h \gamma_1 \lambda_2 \xi^3 + \\
& 3 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} r \gamma_1 \lambda_1 \lambda_2 \xi^3 + 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h T \gamma_1 \lambda_1 \lambda_2 \xi^3 - \\
& 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h \gamma_1 \lambda_3 \xi^3 - 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h \gamma_2 \lambda_3 \xi^3 - \\
& 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} r \gamma_1 \lambda_1 \lambda_3 \xi^3 - 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h T \gamma_1 \lambda_1 \lambda_3 \xi^3 - \\
& 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} r \gamma_2 \lambda_1 \lambda_3 \xi^3 - 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h T \gamma_2 \lambda_1 \lambda_3 \xi^3 + \\
& 3 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} r \gamma_1 \lambda_2 \lambda_3 \xi^3 + 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h T \gamma_1 \lambda_2 \lambda_3 \xi^3 - \\
& e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} r \gamma_1 \lambda_3^2 \xi^3 - e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h T \gamma_1 \lambda_3^2 \xi^3 - \\
& e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} r \gamma_2 \lambda_3^2 \xi^3 - e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h T \gamma_2 \lambda_3^2 \xi^3 + \\
& e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h \lambda_2 \xi^4 + e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} r \lambda_1 \lambda_2 \xi^4 + e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h T \lambda_1 \lambda_2 \xi^4 + \\
& e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} r \lambda_2 \lambda_3 \xi^4 + e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h T \lambda_2 \lambda_3 \xi^4 \Big) /
\end{aligned}$$

$$\begin{aligned}
& \left( (\lambda_1 + \lambda_3)^2 \xi (\gamma_1 \lambda_1 + \gamma_2 \lambda_1 - \gamma_1 \lambda_2 + \gamma_1 \lambda_3 + \gamma_2 \lambda_3 - \lambda_2 \xi) \right. \\
& \left. (-\gamma_1 \lambda_1 - \gamma_2 \lambda_1 + \gamma_1 \lambda_2 - \gamma_1 \lambda_3 - \gamma_2 \lambda_3 + \lambda_2 \xi) \right) -
\end{aligned}$$

$$\begin{aligned}
& \left( -2 e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h \gamma_1^4 \lambda_1 - e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h \gamma_1^3 \gamma_2 \lambda_1 + \right. \\
& e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h \gamma_1^4 \lambda_2 - 2 e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h \gamma_1^4 \lambda_3 - \\
& e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h \gamma_1^3 \gamma_2 \lambda_3 - 2 e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h \gamma_1^3 \lambda_1 \xi + \\
& 2 e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} r \gamma_1^3 \lambda_1^2 \xi + e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} r \gamma_1^2 \gamma_2 \lambda_1^2 \xi + \\
& 2 e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h \gamma_1^3 \lambda_2 \xi - e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} r \gamma_1^3 \lambda_1 \lambda_2 \xi - \\
& 2 e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h \gamma_1^3 \lambda_3 \xi + 4 e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} r \gamma_1^3 \lambda_1 \lambda_3 \xi + \\
& 2 e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} r \gamma_1^2 \gamma_2 \lambda_1 \lambda_3 \xi - \\
& e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} r \gamma_1^3 \lambda_2 \lambda_3 \xi + 2 e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} r \gamma_1^3 \lambda_3^2 \xi + \\
& e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} r \gamma_1^2 \gamma_2 \lambda_3^2 \xi + 3 e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h \gamma_1^2 \lambda_1 \xi^2 + \\
& 3 e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h \gamma_1 \gamma_2 \lambda_1 \xi^2 + 5 e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} r \gamma_1^2 \lambda_1^2 \\
& \left. \xi^2 + 2 e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h T \gamma_1^2 \lambda_1^2 \xi^2 + 2 e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \right)
\end{aligned}$$





$$\begin{aligned}
& e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \gamma_1 \lambda_1 \lambda_2 \lambda_3 + 24 c e^{-2T(\lambda_1+\lambda_3)} T \gamma_1 \lambda_1^2 \lambda_2 \lambda_3 - \\
& 12 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \gamma_1 \lambda_1^2 \lambda_2 \lambda_3 - 2 c e^{-2T(\lambda_1+\lambda_3)} \gamma_1 \lambda_2^2 \lambda_3 + \\
& 4 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \gamma_1 \lambda_2^2 \lambda_3 - 4 c e^{-2T(\lambda_1+\lambda_3)} T \gamma_1 \lambda_1 \lambda_2^2 \lambda_3 + \\
& 2 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \gamma_1 \lambda_1 \lambda_2^2 \lambda_3 + 24 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1 \lambda_3^2 - \\
& 48 c e^{-2T(\lambda_1+\lambda_3)} \gamma_1 \lambda_1 \lambda_3^2 + 48 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \gamma_1 \lambda_1 \lambda_3^2 - \\
& 48 c e^{-2T(\lambda_1+\lambda_3)} T \gamma_1 \lambda_1^2 \lambda_3^2 + 24 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \gamma_1 \lambda_1^2 \lambda_3^2 + \\
& 12 c e^{-2T(\lambda_1+\lambda_3)} \gamma_1 \lambda_2 \lambda_3^2 - 16 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \gamma_1 \lambda_2 \lambda_3^2 + 24 c \\
& e^{-2T(\lambda_1+\lambda_3)} T \gamma_1 \lambda_1 \lambda_2 \lambda_3^2 - 12 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \gamma_1 \lambda_1 \lambda_2 \lambda_3^2 - \\
& 2 c e^{-2T(\lambda_1+\lambda_3)} T \gamma_1 \lambda_2^2 \lambda_3^2 + e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \gamma_1 \lambda_2^2 \lambda_3^2 - \\
& 16 c e^{-2T(\lambda_1+\lambda_3)} \gamma_1 \lambda_3^3 + 16 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \gamma_1 \lambda_3^3 - \\
& 32 c e^{-2T(\lambda_1+\lambda_3)} T \gamma_1 \lambda_1 \lambda_3^3 + 16 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \gamma_1 \lambda_1 \lambda_3^3 + \\
& 8 c e^{-2T(\lambda_1+\lambda_3)} T \gamma_1 \lambda_2 \lambda_3^3 - 4 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \gamma_1 \lambda_2 \lambda_3^3 - \\
& 8 c e^{-2T(\lambda_1+\lambda_3)} T \gamma_1 \lambda_3^4 + 4 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \gamma_1 \lambda_3^4 - \\
& 30 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h} \lambda_1^2 \xi - 4 e^{-2T(\lambda_1+\lambda_3)} \mathbf{r} \lambda_1^3 \xi - \\
& 28 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_1^3 \xi - 8 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_1^4 \xi + \\
& 26 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h} \lambda_1 \lambda_2 \xi + 6 e^{-2T(\lambda_1+\lambda_3)} \mathbf{r} \lambda_1^2 \lambda_2 \xi + \\
& 26 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_1^2 \lambda_2 \xi + 8 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_1^3 \lambda_2 \xi - \\
& 6 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h} \lambda_2^2 \xi - 2 e^{-2T(\lambda_1+\lambda_3)} \mathbf{r} \lambda_1 \lambda_2^2 \xi - \\
& 6 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_1 \lambda_2^2 \xi - 2 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_1^2 \lambda_2^2 \xi - \\
& 60 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h} \lambda_1 \lambda_3 \xi - 12 e^{-2T(\lambda_1+\lambda_3)} \mathbf{r} \lambda_1^2 \lambda_3 \xi - \\
& 84 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_1^2 \lambda_3 \xi - 32 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_1^3 \lambda_3 \xi + \\
& 26 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h} \lambda_2 \lambda_3 \xi + 12 e^{-2T(\lambda_1+\lambda_3)} \mathbf{r} \lambda_1 \lambda_2 \lambda_3 \xi + \\
& 52 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_1 \lambda_2 \lambda_3 \xi + 24 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_1^2 \lambda_2 \lambda_3 \xi - \\
& 2 e^{-2T(\lambda_1+\lambda_3)} \mathbf{r} \lambda_2^2 \lambda_3 \xi - 6 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_2^2 \lambda_3 \xi - \\
& 4 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_1 \lambda_2^2 \lambda_3 \xi - 30 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h} \lambda_3^2 \xi - \\
& 12 e^{-2T(\lambda_1+\lambda_3)} \mathbf{r} \lambda_1 \lambda_3^2 \xi - 84 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_1 \lambda_3^2 \xi - \\
& 48 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_1^2 \lambda_3^2 \xi + 6 e^{-2T(\lambda_1+\lambda_3)} \mathbf{r} \lambda_2 \lambda_3^2 \xi + \\
& 26 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_2 \lambda_3^2 \xi + 24 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_1 \lambda_2 \lambda_3^2 \xi - \\
& 2 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_2^2 \lambda_3^2 \xi - 4 e^{-2T(\lambda_1+\lambda_3)} \mathbf{r} \lambda_3^3 \xi - \\
& 28 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_3^3 \xi - 32 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_1 \lambda_3^3 \xi + \\
& 8 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_2 \lambda_3^3 \xi - 8 e^{-2T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_3^4 \xi \Big) +
\end{aligned}$$

$$\begin{aligned}
& e^{-T\lambda_2} \left( \frac{\mathbf{h} T^2 (\gamma_1^2 + 2\gamma_1\gamma_2 + \gamma_2^2 - 2\gamma_1\xi - 3\gamma_2\xi + \xi^2)}{2\lambda_2\xi} - \right. \\
& \frac{1}{\lambda_2^2(\lambda_1 + \lambda_3)\xi} T \left( -\mathbf{h}\gamma_1^2\lambda_1 - 2\mathbf{h}\gamma_1\gamma_2\lambda_1 - \mathbf{h}\gamma_2^2\lambda_1 - \right. \\
& \quad \mathbf{h}\gamma_1^2\lambda_2 - \mathbf{h}\gamma_1\gamma_2\lambda_2 - \mathbf{h}\gamma_1^2\lambda_3 - 2\mathbf{h}\gamma_1\gamma_2\lambda_3 - \mathbf{h}\gamma_2^2\lambda_3 + \\
& \quad 2\mathbf{h}\gamma_1\lambda_1\xi + 3\mathbf{h}\gamma_2\lambda_1\xi + 3\mathbf{h}\gamma_1\lambda_2\xi + \mathbf{h}\gamma_2\lambda_2\xi + \\
& \quad \mathbf{r}\gamma_1\lambda_1\lambda_2\xi - \mathbf{c}\gamma_2\lambda_1\lambda_2\xi + \mathbf{r}\gamma_2\lambda_1\lambda_2\xi + 2\mathbf{h}\gamma_1\lambda_3\xi + \\
& \quad 3\mathbf{h}\gamma_2\lambda_3\xi + \mathbf{r}\gamma_1\lambda_2\lambda_3\xi - \mathbf{c}\gamma_2\lambda_2\lambda_3\xi + \mathbf{r}\gamma_2\lambda_2\lambda_3\xi - \\
& \quad \mathbf{h}\lambda_1\xi^2 - 2\mathbf{h}\lambda_2\xi^2 - \mathbf{r}\lambda_1\lambda_2\xi^2 - \mathbf{h}\lambda_3\xi^2 - \mathbf{r}\lambda_2\lambda_3\xi^2) - \\
& \frac{1}{\lambda_2^3(\lambda_1 + \lambda_3)^2\xi} \left( -\mathbf{h}\gamma_1^2\lambda_1^2 - 2\mathbf{h}\gamma_1\gamma_2\lambda_1^2 - \mathbf{h}\gamma_2^2\lambda_1^2 - \right. \\
& \quad \mathbf{h}\gamma_1^2\lambda_1\lambda_2 - \mathbf{h}\gamma_1\gamma_2\lambda_1\lambda_2 - \mathbf{h}\gamma_1^2\lambda_2^2 - 2\mathbf{h}\gamma_1^2\lambda_1\lambda_3 - 4\mathbf{h}\gamma_1 \\
& \quad \gamma_2\lambda_1\lambda_3 - 2\mathbf{h}\gamma_2^2\lambda_1\lambda_3 - \mathbf{h}\gamma_1^2\lambda_2\lambda_3 - \mathbf{h}\gamma_1\gamma_2\lambda_2\lambda_3 - \mathbf{h}\gamma_1^2 \\
& \quad \lambda_3^2 - 2\mathbf{h}\gamma_1\gamma_2\lambda_3^2 - \mathbf{h}\gamma_2^2\lambda_3^2 + 2\mathbf{h}\gamma_1\lambda_1^2\xi + 3\mathbf{h}\gamma_2\lambda_1^2\xi + \\
& \quad 3\mathbf{h}\gamma_1\lambda_1\lambda_2\xi + \mathbf{h}\gamma_2\lambda_1\lambda_2\xi + \mathbf{r}\gamma_1\lambda_1^2\lambda_2\xi - \mathbf{c}\gamma_2\lambda_1^2\lambda_2\xi + \\
& \quad \mathbf{r}\gamma_2\lambda_1^2\lambda_2\xi + 3\mathbf{h}\gamma_1\lambda_2^2\xi - \mathbf{c}\gamma_1\lambda_1\lambda_2^2\xi + \mathbf{r}\gamma_1\lambda_1\lambda_2^2\xi + \\
& \quad 4\mathbf{h}\gamma_1\lambda_1\lambda_3\xi + 6\mathbf{h}\gamma_2\lambda_1\lambda_3\xi + 3\mathbf{h}\gamma_1\lambda_2\lambda_3\xi + \mathbf{h}\gamma_2\lambda_2\lambda_3\xi + \\
& \quad 2\mathbf{r}\gamma_1\lambda_1\lambda_2\lambda_3\xi - 2\mathbf{c}\gamma_2\lambda_1\lambda_2\lambda_3\xi + 2\mathbf{r}\gamma_2\lambda_1\lambda_2\lambda_3\xi - \\
& \quad \mathbf{c}\gamma_1\lambda_2^2\lambda_3\xi + \mathbf{r}\gamma_1\lambda_2^2\lambda_3\xi + 2\mathbf{h}\gamma_1\lambda_3^2\xi + 3\mathbf{h}\gamma_2\lambda_3^2\xi + \\
& \quad \mathbf{r}\gamma_1\lambda_2\lambda_3^2\xi - \mathbf{c}\gamma_2\lambda_2\lambda_3^2\xi + \mathbf{r}\gamma_2\lambda_2\lambda_3^2\xi - \mathbf{h}\lambda_1^2\xi^2 - \\
& \quad 2\mathbf{h}\lambda_1\lambda_2\xi^2 - \mathbf{r}\lambda_1^2\lambda_2\xi^2 - 2\mathbf{h}\lambda_2^2\xi^2 - 2\mathbf{h}\lambda_1\lambda_3\xi^2 - \\
& \quad \left. 2\mathbf{h}\lambda_2\lambda_3\xi^2 - 2\mathbf{r}\lambda_1\lambda_2\lambda_3\xi^2 - \mathbf{h}\lambda_3^2\xi^2 - \mathbf{r}\lambda_2\lambda_3^2\xi^2 \right) + \\
& e^{-T\lambda_2+T(\lambda_1+\lambda_3)} \left( -\frac{e^{-T(\lambda_1+\lambda_3)} \mathbf{h} T^2 (-\gamma_1^2 - 2\gamma_1\gamma_2 - \gamma_2^2 + 4\gamma_2\xi)}{2(\lambda_1 - \lambda_2 + \lambda_3)\xi} + \right. \\
& \left. T \left( e^{-T(\lambda_1+\lambda_3)} \mathbf{h}\gamma_1\gamma_2\lambda_1 + e^{-T(\lambda_1+\lambda_3)} \mathbf{h}\gamma_2^2\lambda_1 + e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \right. \right. \\
& \quad \gamma_1^2\lambda_2 + e^{-T(\lambda_1+\lambda_3)} \mathbf{h}\gamma_1\gamma_2\lambda_2 + e^{-T(\lambda_1+\lambda_3)} \mathbf{h}\gamma_1\gamma_2\lambda_3 + \\
& \quad e^{-T(\lambda_1+\lambda_3)} \mathbf{h}\gamma_2^2\lambda_3 + 3e^{-T(\lambda_1+\lambda_3)} \mathbf{h}\gamma_1\lambda_1\xi - 2e^{-T(\lambda_1+\lambda_3)} \\
& \quad \mathbf{h}\gamma_2\lambda_1\xi + 2\mathbf{c}e^{-T(\lambda_1+\lambda_3)} \gamma_1\lambda_1^2\xi + e^{-T(\lambda_1+\lambda_3)} \mathbf{r}\gamma_1\lambda_1^2\xi + \\
& \quad e^{-T(\lambda_1+\lambda_3)} \mathbf{h} T \gamma_1\lambda_1^2\xi + e^{-T(\lambda_1+\lambda_3)} \mathbf{r}\gamma_2\lambda_1^2\xi + e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \\
& \quad T \gamma_2\lambda_1^2\xi - 3e^{-T(\lambda_1+\lambda_3)} \mathbf{h}\gamma_1\lambda_2\xi - 2e^{-T(\lambda_1+\lambda_3)} \mathbf{h}\gamma_2\lambda_2\xi - \\
& \quad \left. 2\mathbf{c}e^{-T(\lambda_1+\lambda_3)} \gamma_1\lambda_1\lambda_2\xi - e^{-T(\lambda_1+\lambda_3)} \mathbf{r}\gamma_1\lambda_1\lambda_2\xi - \right.
\end{aligned}$$

$$\begin{aligned}
& e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \gamma_1 \lambda_1 \lambda_2 \xi - e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \gamma_2 \lambda_1 \lambda_2 \xi - \\
& e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \gamma_2 \lambda_1 \lambda_2 \xi + 3 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1 \lambda_3 \xi - \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_2 \lambda_3 \xi + 4 \mathbf{c} e^{-T(\lambda_1+\lambda_3)} \gamma_1 \lambda_1 \lambda_3 \xi + \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \gamma_1 \lambda_1 \lambda_3 \xi + 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \gamma_1 \lambda_1 \lambda_3 \xi + \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \gamma_2 \lambda_1 \lambda_3 \xi + 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \gamma_2 \lambda_1 \lambda_3 \xi - \\
& 2 \mathbf{c} e^{-T(\lambda_1+\lambda_3)} \gamma_1 \lambda_2 \lambda_3 \xi - e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \gamma_1 \lambda_2 \lambda_3 \xi - \\
& e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \gamma_1 \lambda_2 \lambda_3 \xi - e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \gamma_2 \lambda_2 \lambda_3 \xi - \\
& e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \gamma_2 \lambda_2 \lambda_3 \xi + 2 \mathbf{c} e^{-T(\lambda_1+\lambda_3)} \gamma_1 \lambda_3^2 \xi + \\
& e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \gamma_1 \lambda_3^2 \xi + e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \gamma_1 \lambda_3^2 \xi + e^{-T(\lambda_1+\lambda_3)} \\
& \mathbf{r} \gamma_2 \lambda_3^2 \xi + e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \gamma_2 \lambda_3^2 \xi - e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \lambda_1 \xi^2 - \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \lambda_1^2 \xi^2 + e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \lambda_2 \xi^2 + 2 e^{-T(\lambda_1+\lambda_3)} \\
& \mathbf{r} \lambda_1 \lambda_2 \xi^2 - e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \lambda_3 \xi^2 - 4 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \lambda_1 \lambda_3 \xi^2 + \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \lambda_2 \lambda_3 \xi^2 - 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \lambda_3^2 \xi^2) \Big) / \\
& ((-\lambda_1 + \lambda_2 - \lambda_3) (\lambda_1 + \lambda_3) (\lambda_1 - \lambda_2 + \lambda_3) \xi) - \\
& \frac{1}{2 (-\lambda_1 + \lambda_2 - \lambda_3)^3 (\lambda_1 + \lambda_3)^2 \xi} \left( 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1^2 \lambda_1^2 + \right. \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1 \gamma_2 \lambda_1^2 + 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_2^2 \lambda_1^2 - \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1^2 \lambda_1 \lambda_2 + 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1 \gamma_2 \lambda_1 \lambda_2 + \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1^2 \lambda_2^2 + 4 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1^2 \lambda_1 \lambda_3 + \\
& 4 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1 \gamma_2 \lambda_1 \lambda_3 + 4 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_2^2 \lambda_1 \lambda_3 - \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1^2 \lambda_2 \lambda_3 + 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1 \gamma_2 \lambda_2 \lambda_3 + \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1^2 \lambda_3^2 + 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1 \gamma_2 \lambda_3^2 + 2 e^{-T(\lambda_1+\lambda_3)} \\
& \mathbf{h} \gamma_2^2 \lambda_3^2 - 6 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1 \lambda_1^2 \xi - 4 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_2 \lambda_1^2 \xi + \\
& 8 \mathbf{c} e^{-T(\lambda_1+\lambda_3)} \gamma_1 \lambda_1^3 \xi - 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \gamma_1 \lambda_1^3 \xi + \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \gamma_2 \lambda_1^3 \xi + 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \gamma_2 \lambda_1^3 \xi - \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \gamma_1 \lambda_1^4 \xi + 18 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1 \lambda_1 \lambda_2 \xi - \\
& 4 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_2 \lambda_1 \lambda_2 \xi - 12 \mathbf{c} e^{-T(\lambda_1+\lambda_3)} \gamma_1 \lambda_1^2 \lambda_2 \xi + \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \gamma_1 \lambda_1^2 \lambda_2 \xi + 6 e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \gamma_1 \lambda_1^2 \lambda_2 \xi - \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \gamma_2 \lambda_1^2 \lambda_2 \xi - 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \gamma_2 \lambda_1^2 \lambda_2 \xi + \\
& 4 e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \gamma_1 \lambda_1^3 \lambda_2 \xi - 12 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1 \lambda_2^2 \xi + \\
& 4 \mathbf{c} e^{-T(\lambda_1+\lambda_3)} \gamma_1 \lambda_1 \lambda_2^2 \xi - 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \gamma_1 \lambda_1 \lambda_2^2 \xi - \\
& 4 e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \gamma_1 \lambda_1 \lambda_2^2 \xi - 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \gamma_1 \lambda_1^2 \lambda_2^2 \xi - \\
& 12 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1 \lambda_1 \lambda_3 \xi - 8 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_2 \lambda_1 \lambda_3 \xi + \\
& 24 \mathbf{c} e^{-T(\lambda_1+\lambda_3)} \gamma_1 \lambda_1^2 \lambda_3 \xi - 6 e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \gamma_1 \lambda_1^2 \lambda_3 \xi +
\end{aligned}$$

$$\begin{aligned}
& 6 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \gamma_2 \lambda_1^2 \lambda_3 \xi + 6 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T} \gamma_2 \lambda_1^2 \lambda_3 \xi - \\
& 8 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T}^2 \gamma_1 \lambda_1^3 \lambda_3 \xi + 18 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1 \lambda_2 \lambda_3 \xi - \\
& 4 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_2 \lambda_2 \lambda_3 \xi - 24 c e^{-T(\lambda_1+\lambda_3)} \gamma_1 \lambda_1 \lambda_2 \lambda_3 \xi + \\
& 4 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \gamma_1 \lambda_1 \lambda_2 \lambda_3 \xi + 12 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T} \gamma_1 \lambda_1 \lambda_2 \lambda_3 \xi - \\
& 4 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \gamma_2 \lambda_1 \lambda_2 \lambda_3 \xi - 4 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T} \gamma_2 \lambda_1 \lambda_2 \lambda_3 \xi + \\
& 12 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T}^2 \gamma_1 \lambda_1^2 \lambda_2 \lambda_3 \xi + 4 c e^{-T(\lambda_1+\lambda_3)} \gamma_1 \lambda_2^2 \lambda_3 \xi - \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \gamma_1 \lambda_2^2 \lambda_3 \xi - 4 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T} \gamma_1 \lambda_2^2 \lambda_3 \xi - \\
& 4 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T}^2 \gamma_1 \lambda_1 \lambda_2^2 \lambda_3 \xi - 6 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_1 \lambda_3^2 \xi - \\
& 4 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \gamma_2 \lambda_3^2 \xi + 24 c e^{-T(\lambda_1+\lambda_3)} \gamma_1 \lambda_1 \lambda_3^2 \xi - \\
& 6 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T} \gamma_1 \lambda_1 \lambda_3^2 \xi + 6 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \gamma_2 \lambda_1 \lambda_3^2 \xi + \\
& 6 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T} \gamma_2 \lambda_1 \lambda_3^2 \xi - 12 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T}^2 \gamma_1 \lambda_1^2 \lambda_3^2 \xi - \\
& 12 c e^{-T(\lambda_1+\lambda_3)} \gamma_1 \lambda_2 \lambda_3^2 \xi + 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \gamma_1 \lambda_2 \lambda_3^2 \xi + \\
& 6 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T} \gamma_1 \lambda_2 \lambda_3^2 \xi - 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \gamma_2 \lambda_2 \lambda_3^2 \xi - \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T} \gamma_2 \lambda_2 \lambda_3^2 \xi + 12 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T}^2 \gamma_1 \lambda_1 \lambda_2 \lambda_3^2 \\
& \xi - 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T}^2 \gamma_1 \lambda_2^2 \lambda_3^2 \xi + 8 c e^{-T(\lambda_1+\lambda_3)} \gamma_1 \lambda_3^3 \xi - \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T} \gamma_1 \lambda_3^3 \xi + 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \gamma_2 \lambda_3^3 \xi + \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T} \gamma_2 \lambda_3^3 \xi - 8 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T}^2 \gamma_1 \lambda_1 \lambda_3^3 \xi + \\
& 4 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T}^2 \gamma_1 \lambda_2 \lambda_3^3 \xi - 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T}^2 \gamma_1 \lambda_3^4 \xi + \\
& 8 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \lambda_1^2 \xi^2 - 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \lambda_1^3 \xi^2 + \\
& 6 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T} \lambda_1^3 \xi^2 + 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \mathbf{T} \lambda_1^4 \xi^2 + \\
& 3 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T}^2 \lambda_1^4 \xi^2 - 18 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \lambda_1 \lambda_2 \xi^2 - \\
& 12 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T} \lambda_1^2 \lambda_2 \xi^2 - 4 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \mathbf{T} \lambda_1^3 \lambda_2 \xi^2 - \\
& 6 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T}^2 \lambda_1^3 \lambda_2 \xi^2 + 10 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \lambda_2^2 \xi^2 + \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \lambda_1 \lambda_2^2 \xi^2 + 6 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T} \lambda_1 \lambda_2^2 \xi^2 + \\
& 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \mathbf{T} \lambda_1^2 \lambda_2^2 \xi^2 + 3 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T}^2 \lambda_1^2 \lambda_2^2 \xi^2 + \\
& 16 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \lambda_1 \lambda_3 \xi^2 - 6 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \lambda_1^2 \lambda_3 \xi^2 + \\
& 18 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T} \lambda_1^2 \lambda_3 \xi^2 + 8 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \mathbf{T} \lambda_1^3 \lambda_3 \xi^2 + \\
& 12 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T}^2 \lambda_1^3 \lambda_3 \xi^2 - 18 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \lambda_2 \lambda_3 \xi^2 - \\
& 24 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T} \lambda_1 \lambda_2 \lambda_3 \xi^2 - 12 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \mathbf{T} \lambda_1^2 \lambda_2 \lambda_3 \xi^2 - \\
& 18 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T}^2 \lambda_1^2 \lambda_2 \lambda_3 \xi^2 + 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \lambda_2^2 \lambda_3 \xi^2 + \\
& 6 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T} \lambda_2^2 \lambda_3 \xi^2 + 4 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \mathbf{T} \lambda_1 \lambda_2^2 \lambda_3 \xi^2 + \\
& 6 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T}^2 \lambda_1 \lambda_2^2 \lambda_3 \xi^2 + 8 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \lambda_3^2 \xi^2 - \\
& 6 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \lambda_1 \lambda_3^2 \xi^2 + 18 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T} \lambda_1 \lambda_3^2 \xi^2 + \\
& 12 e^{-T(\lambda_1+\lambda_3)} \mathbf{r} \mathbf{T} \lambda_1^2 \lambda_3^2 \xi^2 + 18 e^{-T(\lambda_1+\lambda_3)} \mathbf{h} \mathbf{T}^2 \lambda_1^2 \lambda_3^2 \xi^2 -
\end{aligned}$$

$$\begin{aligned}
& 12 e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_2 \lambda_3^2 \xi^2 - 12 e^{-T(\lambda_1+\lambda_3)} \mathbf{r}^T \lambda_1 \lambda_2 \lambda_3^2 \xi^2 - \\
& 18 e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_1 \lambda_2 \lambda_3^2 \xi^2 + 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{r}^T \lambda_2^2 \lambda_3^2 \xi^2 + \\
& 3 e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_2^2 \lambda_3^2 \xi^2 - 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{r}^T \lambda_3^3 \xi^2 + \\
& 6 e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_3^3 \xi^2 + 8 e^{-T(\lambda_1+\lambda_3)} \mathbf{r}^T \lambda_1 \lambda_3^3 \xi^2 + \\
& 12 e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_1 \lambda_3^3 \xi^2 - 4 e^{-T(\lambda_1+\lambda_3)} \mathbf{r}^T \lambda_2 \lambda_3^3 \xi^2 - \\
& 6 e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_2 \lambda_3^3 \xi^2 + 2 e^{-T(\lambda_1+\lambda_3)} \mathbf{r}^T \lambda_3^4 \xi^2 + \\
& 3 e^{-T(\lambda_1+\lambda_3)} \mathbf{h}^T \lambda_3^4 \xi^2 \Big) + e^{\frac{T(\gamma_1+\gamma_2)(\lambda_1+\lambda_3)}{\gamma_1+\xi}}
\end{aligned}$$

$$\begin{aligned}
& \left( e^{-T\lambda_2} \left( \frac{e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h}^T (\gamma_1 + \gamma_2) (\gamma_1 + \xi)}{(\lambda_1 + \lambda_3) (\gamma_1 \lambda_1 + \gamma_2 \lambda_1 - \gamma_1 \lambda_2 + \gamma_1 \lambda_3 + \gamma_2 \lambda_3 - \lambda_2 \xi)} - \right. \right. \\
& \left. \left( e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h} \gamma_1^4 \lambda_1 + e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h} \gamma_1^3 \gamma_2 \lambda_1 - e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h} \right. \right. \\
& \left. \left. \gamma_1^4 \lambda_2 + e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h} \gamma_1^4 \lambda_3 + e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h} \gamma_1^3 \gamma_2 \lambda_3 - \right. \right. \\
& \left. \left. e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1^3 \lambda_1^2 \xi - e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1^2 \gamma_2 \lambda_1^2 \xi - \right. \right. \\
& \left. \left. 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h} \gamma_1^3 \lambda_2 \xi + e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1^3 \lambda_1 \lambda_2 \xi - \right. \right. \\
& \left. \left. 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1^3 \lambda_1 \lambda_3 \xi - 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1^2 \gamma_2 \lambda_1 \lambda_3 \xi + \right. \right. \\
& \left. \left. e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1^3 \lambda_2 \lambda_3 \xi - e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1^3 \lambda_3^2 \xi - \right. \right. \\
& \left. \left. e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1^2 \gamma_2 \lambda_3^2 \xi - 3 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h} \gamma_1^2 \lambda_1 \xi^2 - \right. \right. \\
& \left. \left. 3 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h} \gamma_1 \gamma_2 \lambda_1 \xi^2 - 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1^2 \lambda_1^2 \xi^2 - \right. \right. \\
& \left. \left. e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h}^T \gamma_1^2 \lambda_1^2 \xi^2 - 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1 \gamma_2 \lambda_1^2 \xi^2 - \right. \right. \\
& \left. \left. e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h}^T \gamma_1 \gamma_2 \lambda_1^2 \xi^2 + 3 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1^2 \lambda_1 \lambda_2 \xi^2 + \right. \right. \\
& \left. \left. e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h}^T \gamma_1^2 \lambda_1 \lambda_2 \xi^2 - 3 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h} \gamma_1^2 \lambda_3 \xi^2 - \right. \right. \\
& \left. \left. 3 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h} \gamma_1 \gamma_2 \lambda_3 \xi^2 - 4 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1^2 \lambda_1 \lambda_3 \xi^2 - \right. \right. \\
& \left. \left. 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h}^T \gamma_1^2 \lambda_1 \lambda_3 \xi^2 - 4 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1 \gamma_2 \lambda_1 \lambda_3 \xi^2 - \right. \right. \\
& \left. \left. 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h}^T \gamma_1 \gamma_2 \lambda_1 \lambda_3 \xi^2 + 3 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1^2 \lambda_2 \lambda_3 \xi^2 + \right. \right. \\
& \left. \left. e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h}^T \gamma_1^2 \lambda_2 \lambda_3 \xi^2 - 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1^2 \lambda_3^2 \xi^2 - \right. \right. \\
& \left. \left. e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h}^T \gamma_1^2 \lambda_3^2 \xi^2 - 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1 \gamma_2 \lambda_3^2 \xi^2 - \right. \right.
\end{aligned}$$



$$\begin{aligned}
& e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h T \gamma_1 \gamma_2 \lambda_3^2 \xi^2 - 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h \gamma_1 \lambda_1 \xi^3 - \\
& 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h \gamma_2 \lambda_1 \xi^3 - e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} r \gamma_1 \lambda_1^2 \xi^3 - \\
& e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h T \gamma_1 \lambda_1^2 \xi^3 - e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} r \gamma_2 \lambda_1^2 \xi^3 - \\
& e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h T \gamma_2 \lambda_1^2 \xi^3 + 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h \gamma_1 \lambda_2 \xi^3 + \\
& 3 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} r \gamma_1 \lambda_1 \lambda_2 \xi^3 + 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h T \gamma_1 \lambda_1 \lambda_2 \xi^3 - \\
& 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h \gamma_1 \lambda_3 \xi^3 - 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h \gamma_2 \lambda_3 \xi^3 - \\
& 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} r \gamma_1 \lambda_1 \lambda_3 \xi^3 - 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h T \gamma_1 \lambda_1 \lambda_3 \xi^3 - \\
& 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} r \gamma_2 \lambda_1 \lambda_3 \xi^3 - 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h T \gamma_2 \lambda_1 \lambda_3 \xi^3 + \\
& 3 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} r \gamma_1 \lambda_2 \lambda_3 \xi^3 + 2 e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h T \gamma_1 \lambda_2 \lambda_3 \xi^3 - \\
& e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} r \gamma_1 \lambda_3^2 \xi^3 - e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h T \gamma_1 \lambda_3^2 \xi^3 - \\
& e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} r \gamma_2 \lambda_3^2 \xi^3 - e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h T \gamma_2 \lambda_3^2 \xi^3 + \\
& e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h \lambda_2 \xi^4 + e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} r \lambda_1 \lambda_2 \xi^4 + e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h T \lambda_1 \\
& \lambda_2 \xi^4 + e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} r \lambda_2 \lambda_3 \xi^4 + e^{-\frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} h T \lambda_2 \lambda_3 \xi^4 \Big) / \\
& \left( (\lambda_1 + \lambda_3)^2 \xi (\gamma_1 \lambda_1 + \gamma_2 \lambda_1 - \gamma_1 \lambda_2 + \gamma_1 \lambda_3 + \gamma_2 \lambda_3 - \lambda_2 \xi) \right. \\
& \left. (-\gamma_1 \lambda_1 - \gamma_2 \lambda_1 + \gamma_1 \lambda_2 - \gamma_1 \lambda_3 - \gamma_2 \lambda_3 + \lambda_2 \xi) \right) + \\
& e^{-T\lambda_2 + T(\lambda_1 + \lambda_3)} \left( - \left( e^{-T(\lambda_1 + \lambda_3)} - \frac{T(\lambda_1 + \lambda_3)\xi}{\gamma_1 + \xi} h T (\gamma_1 + \gamma_2) (\gamma_1 + \xi) \right) / \right. \\
& \left( (\lambda_1 + \lambda_3) (2\gamma_1 \lambda_1 + \gamma_2 \lambda_1 - \gamma_1 \lambda_2 + 2\gamma_1 \lambda_3 + \gamma_2 \lambda_3 + \right. \\
& \left. \lambda_1 \xi - \lambda_2 \xi + \lambda_3 \xi) \right) + \left( -2 e^{-T(\lambda_1 + \lambda_3)} - \frac{T(\lambda_1 + \lambda_3)\xi}{\gamma_1 + \xi} h \gamma_1^4 \lambda_1 - \right. \\
& e^{-T(\lambda_1 + \lambda_3)} - \frac{T(\lambda_1 + \lambda_3)\xi}{\gamma_1 + \xi} h \gamma_1^3 \gamma_2 \lambda_1 + e^{-T(\lambda_1 + \lambda_3)} - \frac{T(\lambda_1 + \lambda_3)\xi}{\gamma_1 + \xi} h \\
& \gamma_1^4 \lambda_2 - 2 e^{-T(\lambda_1 + \lambda_3)} - \frac{T(\lambda_1 + \lambda_3)\xi}{\gamma_1 + \xi} h \gamma_1^4 \lambda_3 - e^{-T(\lambda_1 + \lambda_3)} - \frac{T(\lambda_1 + \lambda_3)\xi}{\gamma_1 + \xi} \\
& h \gamma_1^3 \gamma_2 \lambda_3 - 2 e^{-T(\lambda_1 + \lambda_3)} - \frac{T(\lambda_1 + \lambda_3)\xi}{\gamma_1 + \xi} h \gamma_1^3 \lambda_1 \xi + \\
& 2 e^{-T(\lambda_1 + \lambda_3)} - \frac{T(\lambda_1 + \lambda_3)\xi}{\gamma_1 + \xi} r \gamma_1^3 \lambda_1^2 \xi + e^{-T(\lambda_1 + \lambda_3)} - \frac{T(\lambda_1 + \lambda_3)\xi}{\gamma_1 + \xi} \\
& r \gamma_1^2 \gamma_2 \lambda_1^2 \xi + 2 e^{-T(\lambda_1 + \lambda_3)} - \frac{T(\lambda_1 + \lambda_3)\xi}{\gamma_1 + \xi} h \gamma_1^3 \lambda_2 \xi - \\
& e^{-T(\lambda_1 + \lambda_3)} - \frac{T(\lambda_1 + \lambda_3)\xi}{\gamma_1 + \xi} r \gamma_1^3 \lambda_1 \lambda_2 \xi - 2 e^{-T(\lambda_1 + \lambda_3)} - \frac{T(\lambda_1 + \lambda_3)\xi}{\gamma_1 + \xi}
\end{aligned}$$



$$\begin{aligned}
& \mathbf{h}^T \gamma_1 \lambda_1 \lambda_3 \xi^3 + 2 e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_2 \lambda_1 \lambda_3 \xi^3 + \\
& 2 e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h}^T \gamma_2 \lambda_1 \lambda_3 \xi^3 - 3 e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \\
& \mathbf{r} \gamma_1 \lambda_2 \lambda_3 \xi^3 - 2 e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h}^T \gamma_1 \lambda_2 \lambda_3 \xi^3 + 4 \\
& e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_1 \lambda_3^2 \xi^3 + 3 e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h}^T \gamma_1 \\
& \lambda_3^2 \xi^3 + e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \gamma_2 \lambda_3^2 \xi^3 + e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \\
& \mathbf{h}^T \gamma_2 \lambda_3^2 \xi^3 + e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h} \lambda_1 \xi^4 + \\
& e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \lambda_1^2 \xi^4 + e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h}^T \lambda_1^2 \xi^4 - \\
& e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h} \lambda_2 \xi^4 - e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \lambda_1 \lambda_2 \xi^4 - \\
& e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h}^T \lambda_1 \lambda_2 \xi^4 + e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h} \lambda_3 \xi^4 + \\
& 2 e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \lambda_1 \lambda_3 \xi^4 + 2 e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \\
& \mathbf{h}^T \lambda_1 \lambda_3 \xi^4 - e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{r} \lambda_2 \lambda_3 \xi^4 - \\
& e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h}^T \lambda_2 \lambda_3 \xi^4 + e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \\
& \mathbf{r} \lambda_3^2 \xi^4 + e^{-T(\lambda_1+\lambda_3) - \frac{T(\lambda_1+\lambda_3)\xi}{\gamma_1+\xi}} \mathbf{h}^T \lambda_3^2 \xi^4 \Big) / \\
& \left( (\lambda_1 + \lambda_3)^2 \xi (-2 \gamma_1 \lambda_1 - \gamma_2 \lambda_1 + \gamma_1 \lambda_2 - 2 \gamma_1 \lambda_3 - \right. \\
& \left. \gamma_2 \lambda_3 - \lambda_1 \xi + \lambda_2 \xi - \lambda_3 \xi)^2 \right) \Big) \Big) +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{\lambda_2 + \lambda_3} (1 - e^{-T\lambda_1}) \lambda_1^2 \left( \frac{1}{2 (\lambda_1 - 2 \lambda_2 - 2 \lambda_3)^3 (\lambda_2 + \lambda_3)^2} \right. \\
& (6 e^{-2T(\lambda_2+\lambda_3)} \mathbf{h} \gamma_2 \lambda_1^2 + \\
& 2 e^{-2T(\lambda_2+\lambda_3)} \mathbf{h} \gamma_1 \lambda_1 \lambda_2 - \\
& 24 e^{-2T(\lambda_2+\lambda_3)} \mathbf{h} \gamma_2 \lambda_1 \lambda_2 - \\
& 2 c e^{-2T(\lambda_2+\lambda_3)} \gamma_2 \lambda_1^2 \lambda_2 + \\
& 4 e^{-2T(\lambda_2+\lambda_3)} \mathbf{h}^T \gamma_2 \lambda_1^2 \lambda_2 + \\
& 24 e^{-2T(\lambda_2+\lambda_3)} \mathbf{h} \gamma_2 \lambda_2^2 + \\
& 12 c e^{-2T(\lambda_2+\lambda_3)} \gamma_2 \lambda_1 \lambda_2^2 - \\
& 16 e^{-2T(\lambda_2+\lambda_3)} \mathbf{h}^T \gamma_2 \lambda_1 \lambda_2^2 - \\
& \left. 2 c e^{-2T(\lambda_2+\lambda_3)} \mathbf{T} \gamma_2 \lambda_1^2 \lambda_2^2 + \right)
\end{aligned}$$

$$\begin{aligned}
& e^{-2T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_1^2 \lambda_2^2 - \\
& 16 c e^{-2T(\lambda_2+\lambda_3)} \gamma_2 \lambda_2^3 + \\
& 16 e^{-2T(\lambda_2+\lambda_3)} h T \gamma_2 \lambda_2^3 + \\
& 8 c e^{-2T(\lambda_2+\lambda_3)} T \gamma_2 \lambda_1 \lambda_2^3 - \\
& 4 e^{-2T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_1 \lambda_2^3 - \\
& 8 c e^{-2T(\lambda_2+\lambda_3)} T \gamma_2 \lambda_2^4 + \\
& 4 e^{-2T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_2^4 + \\
& 2 e^{-2T(\lambda_2+\lambda_3)} h \gamma_1 \lambda_1 \lambda_3 - \\
& 24 e^{-2T(\lambda_2+\lambda_3)} h \gamma_2 \lambda_1 \lambda_3 - \\
& 2 c e^{-2T(\lambda_2+\lambda_3)} \gamma_2 \lambda_1^2 \lambda_3 + \\
& 4 e^{-2T(\lambda_2+\lambda_3)} h T \gamma_2 \lambda_1^2 \lambda_3 + \\
& 48 e^{-2T(\lambda_2+\lambda_3)} h \gamma_2 \lambda_2 \lambda_3 + \\
& 24 c e^{-2T(\lambda_2+\lambda_3)} \gamma_2 \lambda_1 \lambda_2 \lambda_3 - \\
& 32 e^{-2T(\lambda_2+\lambda_3)} h T \gamma_2 \lambda_1 \lambda_2 \lambda_3 - \\
& 4 c e^{-2T(\lambda_2+\lambda_3)} T \gamma_2 \lambda_1^2 \lambda_2 \lambda_3 + \\
& 2 e^{-2T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_1^2 \lambda_2 \lambda_3 - \\
& 48 c e^{-2T(\lambda_2+\lambda_3)} \gamma_2 \lambda_2^2 \lambda_3 + \\
& 48 e^{-2T(\lambda_2+\lambda_3)} h T \gamma_2 \lambda_2^2 \lambda_3 + \\
& 24 c e^{-2T(\lambda_2+\lambda_3)} T \gamma_2 \lambda_1 \lambda_2^2 \lambda_3 - \\
& 12 e^{-2T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_1 \lambda_2^2 \lambda_3 - \\
& 32 c e^{-2T(\lambda_2+\lambda_3)} T \gamma_2 \lambda_2^3 \lambda_3 + \\
& 16 e^{-2T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_2^3 \lambda_3 + \\
& 24 e^{-2T(\lambda_2+\lambda_3)} h \gamma_2 \lambda_3^2 + \\
& 12 c e^{-2T(\lambda_2+\lambda_3)} \gamma_2 \lambda_1 \lambda_3^2 - \\
& 16 e^{-2T(\lambda_2+\lambda_3)} h T \gamma_2 \lambda_1 \lambda_3^2 - \\
& 2 c e^{-2T(\lambda_2+\lambda_3)} T \gamma_2 \lambda_1^2 \lambda_3^2 + \\
& e^{-2T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_1^2 \lambda_3^2 - \\
& 48 c e^{-2T(\lambda_2+\lambda_3)} \gamma_2 \lambda_2 \lambda_3^2 + \\
& 48 e^{-2T(\lambda_2+\lambda_3)} h T \gamma_2 \lambda_2 \lambda_3^2 + \\
& 24 c e^{-2T(\lambda_2+\lambda_3)} T \gamma_2 \lambda_1 \lambda_2 \lambda_3^2 - \\
& 12 e^{-2T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_1 \lambda_2 \lambda_3^2 - \\
& 48 c e^{-2T(\lambda_2+\lambda_3)} T \gamma_2 \lambda_2^2 \lambda_3^2 + \\
& 24 e^{-2T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_2^2 \lambda_3^2 - \\
& 16 c e^{-2T(\lambda_2+\lambda_3)} \gamma_2 \lambda_3^3 +
\end{aligned}$$

$$\begin{aligned}
& 16 e^{-2T(\lambda_2+\lambda_3)} h T \gamma_2 \lambda_3^3 + \\
& 8 c e^{-2T(\lambda_2+\lambda_3)} T \gamma_2 \lambda_1 \lambda_3^3 - \\
& 4 e^{-2T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_1 \lambda_3^3 - \\
& 32 c e^{-2T(\lambda_2+\lambda_3)} T \gamma_2 \lambda_2 \lambda_3^3 + \\
& 16 e^{-2T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_2 \lambda_3^3 - \\
& 8 c e^{-2T(\lambda_2+\lambda_3)} T \gamma_2 \lambda_3^4 + \\
& 4 e^{-2T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_3^4 - \\
& 6 e^{-2T(\lambda_2+\lambda_3)} h \lambda_1^2 \xi + 26 e^{-2T(\lambda_2+\lambda_3)} h \lambda_1 \lambda_2 \xi - \\
& 2 e^{-2T(\lambda_2+\lambda_3)} r \lambda_1^2 \lambda_2 \xi - \\
& 6 e^{-2T(\lambda_2+\lambda_3)} h T \lambda_1^2 \lambda_2 \xi - \\
& 30 e^{-2T(\lambda_2+\lambda_3)} h \lambda_2^2 \xi + 6 e^{-2T(\lambda_2+\lambda_3)} r \lambda_1 \lambda_2^2 \xi + \\
& 26 e^{-2T(\lambda_2+\lambda_3)} h T \lambda_1 \lambda_2^2 \xi - \\
& 2 e^{-2T(\lambda_2+\lambda_3)} h T^2 \lambda_1^2 \lambda_2^2 \xi - \\
& 4 e^{-2T(\lambda_2+\lambda_3)} r \lambda_2^3 \xi - 28 e^{-2T(\lambda_2+\lambda_3)} h T \lambda_2^3 \xi + \\
& 8 e^{-2T(\lambda_2+\lambda_3)} h T^2 \lambda_1 \lambda_2^3 \xi - \\
& 8 e^{-2T(\lambda_2+\lambda_3)} h T^2 \lambda_2^4 \xi + \\
& 26 e^{-2T(\lambda_2+\lambda_3)} h \lambda_1 \lambda_3 \xi - \\
& 2 e^{-2T(\lambda_2+\lambda_3)} r \lambda_1^2 \lambda_3 \xi - \\
& 6 e^{-2T(\lambda_2+\lambda_3)} h T \lambda_1^2 \lambda_3 \xi - \\
& 60 e^{-2T(\lambda_2+\lambda_3)} h \lambda_2 \lambda_3 \xi + \\
& 12 e^{-2T(\lambda_2+\lambda_3)} r \lambda_1 \lambda_2 \lambda_3 \xi + \\
& 52 e^{-2T(\lambda_2+\lambda_3)} h T \lambda_1 \lambda_2 \lambda_3 \xi - \\
& 4 e^{-2T(\lambda_2+\lambda_3)} h T^2 \lambda_1^2 \lambda_2 \lambda_3 \xi - \\
& 12 e^{-2T(\lambda_2+\lambda_3)} r \lambda_2^2 \lambda_3 \xi - \\
& 84 e^{-2T(\lambda_2+\lambda_3)} h T \lambda_2^2 \lambda_3 \xi + \\
& 24 e^{-2T(\lambda_2+\lambda_3)} h T^2 \lambda_1 \lambda_2^2 \lambda_3 \xi - \\
& 32 e^{-2T(\lambda_2+\lambda_3)} h T^2 \lambda_2^3 \lambda_3 \xi - \\
& 30 e^{-2T(\lambda_2+\lambda_3)} h \lambda_3^2 \xi + 6 e^{-2T(\lambda_2+\lambda_3)} r \lambda_1 \lambda_3^2 \xi + \\
& 26 e^{-2T(\lambda_2+\lambda_3)} h T \lambda_1 \lambda_3^2 \xi - \\
& 2 e^{-2T(\lambda_2+\lambda_3)} h T^2 \lambda_1^2 \lambda_3^2 \xi - \\
& 12 e^{-2T(\lambda_2+\lambda_3)} r \lambda_2 \lambda_3^2 \xi - \\
& 84 e^{-2T(\lambda_2+\lambda_3)} h T \lambda_2 \lambda_3^2 \xi + \\
& 24 e^{-2T(\lambda_2+\lambda_3)} h T^2 \lambda_1 \lambda_2 \lambda_3^2 \xi - \\
& 48 e^{-2T(\lambda_2+\lambda_3)} h T^2 \lambda_2^2 \lambda_3^2 \xi -
\end{aligned}$$

$$\begin{aligned}
& 4 e^{-2T(\lambda_2+\lambda_3)} r \lambda_3^3 \xi - 28 e^{-2T(\lambda_2+\lambda_3)} h T \lambda_3^3 \xi + \\
& 8 e^{-2T(\lambda_2+\lambda_3)} h T^2 \lambda_1 \lambda_3^3 \xi - \\
& 32 e^{-2T(\lambda_2+\lambda_3)} h T^2 \lambda_2 \lambda_3^3 \xi - \\
& 8 e^{-2T(\lambda_2+\lambda_3)} h T^2 \lambda_3^4 \xi) + \\
& \frac{1}{\lambda_1^3 (\lambda_2 + \lambda_3)^2 \xi} \left( -h \gamma_2^2 \lambda_1^2 - h \gamma_1 \gamma_2 \lambda_1 \lambda_2 - \right. \\
& h \gamma_2^2 \lambda_1 \lambda_2 - h \gamma_1^2 \lambda_2^2 - 2 h \gamma_1 \gamma_2 \lambda_2^2 - \\
& h \gamma_2^2 \lambda_2^2 - h \gamma_1 \gamma_2 \lambda_1 \lambda_3 - h \gamma_2^2 \lambda_1 \lambda_3 - \\
& 2 h \gamma_1^2 \lambda_2 \lambda_3 - 4 h \gamma_1 \gamma_2 \lambda_2 \lambda_3 - \\
& 2 h \gamma_2^2 \lambda_2 \lambda_3 - h \gamma_1^2 \lambda_3^2 - 2 h \gamma_1 \gamma_2 \lambda_3^2 - \\
& h \gamma_2^2 \lambda_3^2 + 3 h \gamma_2 \lambda_1^2 \xi + h \gamma_1 \lambda_1 \lambda_2 \xi + \\
& 3 h \gamma_2 \lambda_1 \lambda_2 \xi - c \gamma_2 \lambda_1^2 \lambda_2 \xi + \\
& r \gamma_2 \lambda_1^2 \lambda_2 \xi + 3 h \gamma_1 \lambda_2^2 \xi + 2 h \gamma_2 \lambda_2^2 \xi - \\
& c \gamma_1 \lambda_1 \lambda_2^2 \xi + r \gamma_1 \lambda_1 \lambda_2^2 \xi + \\
& r \gamma_2 \lambda_1 \lambda_2^2 \xi + h \gamma_1 \lambda_1 \lambda_3 \xi + \\
& 3 h \gamma_2 \lambda_1 \lambda_3 \xi - c \gamma_2 \lambda_1^2 \lambda_3 \xi + \\
& r \gamma_2 \lambda_1^2 \lambda_3 \xi + 6 h \gamma_1 \lambda_2 \lambda_3 \xi + \\
& 4 h \gamma_2 \lambda_2 \lambda_3 \xi - 2 c \gamma_1 \lambda_1 \lambda_2 \lambda_3 \xi + \\
& 2 r \gamma_1 \lambda_1 \lambda_2 \lambda_3 \xi + 2 r \gamma_2 \lambda_1 \lambda_2 \lambda_3 \xi + \\
& 3 h \gamma_1 \lambda_3^2 \xi + 2 h \gamma_2 \lambda_3^2 \xi - c \gamma_1 \lambda_1 \lambda_3^2 \xi + \\
& r \gamma_1 \lambda_1 \lambda_3^2 \xi + r \gamma_2 \lambda_1 \lambda_3^2 \xi - \\
& 2 h \lambda_1^2 \xi^2 - 2 h \lambda_1 \lambda_2 \xi^2 - h \lambda_2^2 \xi^2 - \\
& r \lambda_1 \lambda_2^2 \xi^2 - 2 h \lambda_1 \lambda_3 \xi^2 - 2 h \lambda_2 \lambda_3 \xi^2 - \\
& \left. 2 r \lambda_1 \lambda_2 \lambda_3 \xi^2 - h \lambda_3^2 \xi^2 - r \lambda_1 \lambda_3^2 \xi^2 \right) + \\
& \frac{1}{2 (\lambda_1 - \lambda_2 - \lambda_3)^3 (\lambda_2 + \lambda_3)^2 \xi} \\
& \left( 2 e^{-T(\lambda_2+\lambda_3)} h \gamma_2^2 \lambda_1^2 + 2 e^{-T(\lambda_2+\lambda_3)} h \gamma_1 \gamma_2 \lambda_1 \lambda_2 - \right. \\
& 2 e^{-T(\lambda_2+\lambda_3)} h \gamma_2^2 \lambda_1 \lambda_2 + 2 e^{-T(\lambda_2+\lambda_3)} h \gamma_1^2 \lambda_2^2 + \\
& 2 e^{-T(\lambda_2+\lambda_3)} h \gamma_1 \gamma_2 \lambda_2^2 + 2 e^{-T(\lambda_2+\lambda_3)} h \gamma_2^2 \lambda_2^2 + \\
& 2 e^{-T(\lambda_2+\lambda_3)} h \gamma_1 \gamma_2 \lambda_1 \lambda_3 - \\
& 2 e^{-T(\lambda_2+\lambda_3)} h \gamma_2^2 \lambda_1 \lambda_3 + 4 e^{-T(\lambda_2+\lambda_3)} h \gamma_1^2 \lambda_2 \lambda_3 + \\
& 4 e^{-T(\lambda_2+\lambda_3)} h \gamma_1 \gamma_2 \lambda_2 \lambda_3 + \\
& \left. 4 e^{-T(\lambda_2+\lambda_3)} h \gamma_2^2 \lambda_2 \lambda_3 + 2 e^{-T(\lambda_2+\lambda_3)} h \gamma_1^2 \lambda_3^2 + \right)
\end{aligned}$$

$$\begin{aligned}
& 2 e^{-T(\lambda_2+\lambda_3)} h \gamma_1 \gamma_2 \lambda_3^2 + 2 e^{-T(\lambda_2+\lambda_3)} h \gamma_2^2 \lambda_3^2 - \\
& 12 e^{-T(\lambda_2+\lambda_3)} h \gamma_2 \lambda_1^2 \xi - 4 e^{-T(\lambda_2+\lambda_3)} h \gamma_1 \lambda_1 \lambda_2 \xi + \\
& 18 e^{-T(\lambda_2+\lambda_3)} h \gamma_2 \lambda_1 \lambda_2 \xi + \\
& 4 c e^{-T(\lambda_2+\lambda_3)} \gamma_2 \lambda_1^2 \lambda_2 \xi - \\
& 2 e^{-T(\lambda_2+\lambda_3)} r \gamma_2 \lambda_1^2 \lambda_2 \xi - \\
& 4 e^{-T(\lambda_2+\lambda_3)} h^T \gamma_2 \lambda_1^2 \lambda_2 \xi - \\
& 4 e^{-T(\lambda_2+\lambda_3)} h \gamma_1 \lambda_2^2 \xi - 6 e^{-T(\lambda_2+\lambda_3)} h \gamma_2 \lambda_2^2 \xi - \\
& 2 e^{-T(\lambda_2+\lambda_3)} r \gamma_1 \lambda_1 \lambda_2^2 \xi - \\
& 2 e^{-T(\lambda_2+\lambda_3)} h^T \gamma_1 \lambda_1 \lambda_2^2 \xi - \\
& 12 c e^{-T(\lambda_2+\lambda_3)} \gamma_2 \lambda_1 \lambda_2^2 \xi + \\
& 2 e^{-T(\lambda_2+\lambda_3)} r \gamma_2 \lambda_1 \lambda_2^2 \xi + \\
& 6 e^{-T(\lambda_2+\lambda_3)} h^T \gamma_2 \lambda_1 \lambda_2^2 \xi - \\
& 2 e^{-T(\lambda_2+\lambda_3)} h^T \gamma_2 \lambda_1^2 \lambda_2^2 \xi + \\
& 2 e^{-T(\lambda_2+\lambda_3)} r \gamma_1 \lambda_2^3 \xi + 2 e^{-T(\lambda_2+\lambda_3)} h^T \gamma_1 \lambda_2^3 \xi + \\
& 8 c e^{-T(\lambda_2+\lambda_3)} \gamma_2 \lambda_2^3 \xi - 2 e^{-T(\lambda_2+\lambda_3)} h^T \gamma_2 \lambda_2^3 \xi + \\
& 4 e^{-T(\lambda_2+\lambda_3)} h^T \gamma_2 \lambda_1 \lambda_2^3 \xi - \\
& 2 e^{-T(\lambda_2+\lambda_3)} h^T \gamma_2 \lambda_2^4 \xi - 4 e^{-T(\lambda_2+\lambda_3)} h \gamma_1 \lambda_1 \lambda_3 \xi + \\
& 18 e^{-T(\lambda_2+\lambda_3)} h \gamma_2 \lambda_1 \lambda_3 \xi + \\
& 4 c e^{-T(\lambda_2+\lambda_3)} \gamma_2 \lambda_1^2 \lambda_3 \xi - \\
& 2 e^{-T(\lambda_2+\lambda_3)} r \gamma_2 \lambda_1^2 \lambda_3 \xi - \\
& 4 e^{-T(\lambda_2+\lambda_3)} h^T \gamma_2 \lambda_1^2 \lambda_3 \xi - \\
& 8 e^{-T(\lambda_2+\lambda_3)} h \gamma_1 \lambda_2 \lambda_3 \xi - 12 e^{-T(\lambda_2+\lambda_3)} h \gamma_2 \lambda_2 \lambda_3 \xi - \\
& 4 e^{-T(\lambda_2+\lambda_3)} r \gamma_1 \lambda_1 \lambda_2 \lambda_3 \xi - \\
& 4 e^{-T(\lambda_2+\lambda_3)} h^T \gamma_1 \lambda_1 \lambda_2 \lambda_3 \xi - \\
& 24 c e^{-T(\lambda_2+\lambda_3)} \gamma_2 \lambda_1 \lambda_2 \lambda_3 \xi + \\
& 4 e^{-T(\lambda_2+\lambda_3)} r \gamma_2 \lambda_1 \lambda_2 \lambda_3 \xi + \\
& 12 e^{-T(\lambda_2+\lambda_3)} h^T \gamma_2 \lambda_1 \lambda_2 \lambda_3 \xi - \\
& 4 e^{-T(\lambda_2+\lambda_3)} h^T \gamma_2 \lambda_1^2 \lambda_2 \lambda_3 \xi + \\
& 6 e^{-T(\lambda_2+\lambda_3)} r \gamma_1 \lambda_2^2 \lambda_3 \xi + \\
& 6 e^{-T(\lambda_2+\lambda_3)} h^T \gamma_1 \lambda_2^2 \lambda_3 \xi + \\
& 24 c e^{-T(\lambda_2+\lambda_3)} \gamma_2 \lambda_2^2 \lambda_3 \xi - \\
& 6 e^{-T(\lambda_2+\lambda_3)} h^T \gamma_2 \lambda_2^2 \lambda_3 \xi + \\
& 12 e^{-T(\lambda_2+\lambda_3)} h^T \gamma_2 \lambda_1 \lambda_2^2 \lambda_3 \xi - \\
& 8 e^{-T(\lambda_2+\lambda_3)} h^T \gamma_2 \lambda_2^3 \lambda_3 \xi -
\end{aligned}$$

$$\begin{aligned}
& 4 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \gamma_1 \lambda_3^2 \xi - 6 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \gamma_2 \lambda_3^2 \xi - \\
& 2 e^{-T(\lambda_2+\lambda_3)} \mathbf{r} \gamma_1 \lambda_1 \lambda_3^2 \xi - \\
& 2 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \mathbf{T} \gamma_1 \lambda_1 \lambda_3^2 \xi - \\
& 12 \mathbf{c} e^{-T(\lambda_2+\lambda_3)} \gamma_2 \lambda_1 \lambda_3^2 \xi + \\
& 2 e^{-T(\lambda_2+\lambda_3)} \mathbf{r} \gamma_2 \lambda_1 \lambda_3^2 \xi + \\
& 6 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \mathbf{T} \gamma_2 \lambda_1 \lambda_3^2 \xi - \\
& 2 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \mathbf{T}^2 \gamma_2 \lambda_1^2 \lambda_3^2 \xi + \\
& 6 e^{-T(\lambda_2+\lambda_3)} \mathbf{r} \gamma_1 \lambda_2 \lambda_3^2 \xi + \\
& 6 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \mathbf{T} \gamma_1 \lambda_2 \lambda_3^2 \xi + \\
& 24 \mathbf{c} e^{-T(\lambda_2+\lambda_3)} \gamma_2 \lambda_2 \lambda_3^2 \xi - \\
& 6 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \mathbf{T} \gamma_2 \lambda_2 \lambda_3^2 \xi + \\
& 12 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \mathbf{T}^2 \gamma_2 \lambda_1 \lambda_2 \lambda_3^2 \xi - \\
& 12 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \mathbf{T}^2 \gamma_2 \lambda_2^2 \lambda_3^2 \xi + \\
& 2 e^{-T(\lambda_2+\lambda_3)} \mathbf{r} \gamma_1 \lambda_3^3 \xi + 2 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \mathbf{T} \gamma_1 \lambda_3^3 \xi + \\
& 8 \mathbf{c} e^{-T(\lambda_2+\lambda_3)} \gamma_2 \lambda_3^3 \xi - 2 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \mathbf{T} \gamma_2 \lambda_3^3 \xi + \\
& 4 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \mathbf{T}^2 \gamma_2 \lambda_1 \lambda_3^3 \xi - \\
& 8 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \mathbf{T}^2 \gamma_2 \lambda_2 \lambda_3^3 \xi - \\
& 2 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \mathbf{T}^2 \gamma_2 \lambda_3^4 \xi + 10 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \lambda_1^2 \xi^2 - \\
& 18 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \lambda_1 \lambda_2 \xi^2 + 2 e^{-T(\lambda_2+\lambda_3)} \mathbf{r} \lambda_1^2 \lambda_2 \xi^2 + \\
& 6 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \mathbf{T} \lambda_1^2 \lambda_2 \xi^2 + 8 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \lambda_2^2 \xi^2 - \\
& 12 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \mathbf{T} \lambda_1 \lambda_2^2 \xi^2 + \\
& 2 e^{-T(\lambda_2+\lambda_3)} \mathbf{r} \mathbf{T} \lambda_1^2 \lambda_2^2 \xi^2 + \\
& 3 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \mathbf{T}^2 \lambda_1^2 \lambda_2^2 \xi^2 - \\
& 2 e^{-T(\lambda_2+\lambda_3)} \mathbf{r} \lambda_2^3 \xi^2 + 6 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \mathbf{T} \lambda_2^3 \xi^2 - \\
& 4 e^{-T(\lambda_2+\lambda_3)} \mathbf{r} \mathbf{T} \lambda_1 \lambda_2^3 \xi^2 - 6 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \mathbf{T}^2 \lambda_1 \lambda_2^3 \xi^2 + \\
& 2 e^{-T(\lambda_2+\lambda_3)} \mathbf{r} \mathbf{T} \lambda_2^4 \xi^2 + 3 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \mathbf{T}^2 \lambda_2^4 \xi^2 - \\
& 18 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \lambda_1 \lambda_3 \xi^2 + 2 e^{-T(\lambda_2+\lambda_3)} \mathbf{r} \lambda_1^2 \lambda_3 \xi^2 + \\
& 6 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \mathbf{T} \lambda_1^2 \lambda_3 \xi^2 + 16 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \lambda_2 \lambda_3 \xi^2 - \\
& 24 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \mathbf{T} \lambda_1 \lambda_2 \lambda_3 \xi^2 + \\
& 4 e^{-T(\lambda_2+\lambda_3)} \mathbf{r} \mathbf{T} \lambda_1^2 \lambda_2 \lambda_3 \xi^2 + \\
& 6 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \mathbf{T}^2 \lambda_1^2 \lambda_2 \lambda_3 \xi^2 - \\
& 6 e^{-T(\lambda_2+\lambda_3)} \mathbf{r} \lambda_2^2 \lambda_3 \xi^2 + 18 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \mathbf{T} \lambda_2^2 \lambda_3 \xi^2 - \\
& 12 e^{-T(\lambda_2+\lambda_3)} \mathbf{r} \mathbf{T} \lambda_1 \lambda_2^2 \lambda_3 \xi^2 -
\end{aligned}$$



$$\begin{aligned}
& 18 e^{-T(\lambda_2+\lambda_3)} \mathbf{h}^T \lambda_1 \lambda_2^2 \lambda_3 \xi^2 + \\
& 8 e^{-T(\lambda_2+\lambda_3)} \mathbf{r}^T \lambda_2^3 \lambda_3 \xi^2 + \\
& 12 e^{-T(\lambda_2+\lambda_3)} \mathbf{h}^T \lambda_2^3 \lambda_3 \xi^2 + \\
& 8 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \lambda_3^2 \xi^2 - 12 e^{-T(\lambda_2+\lambda_3)} \mathbf{h}^T \lambda_1 \lambda_3^2 \xi^2 + \\
& 2 e^{-T(\lambda_2+\lambda_3)} \mathbf{r}^T \lambda_1^2 \lambda_3^2 \xi^2 + \\
& 3 e^{-T(\lambda_2+\lambda_3)} \mathbf{h}^T \lambda_1^2 \lambda_3^2 \xi^2 - \\
& 6 e^{-T(\lambda_2+\lambda_3)} \mathbf{r} \lambda_2 \lambda_3^2 \xi^2 + 18 e^{-T(\lambda_2+\lambda_3)} \mathbf{h}^T \lambda_2 \lambda_3^2 \xi^2 - \\
& 12 e^{-T(\lambda_2+\lambda_3)} \mathbf{r}^T \lambda_1 \lambda_2 \lambda_3^2 \xi^2 - \\
& 18 e^{-T(\lambda_2+\lambda_3)} \mathbf{h}^T \lambda_1 \lambda_2 \lambda_3^2 \xi^2 + \\
& 12 e^{-T(\lambda_2+\lambda_3)} \mathbf{r}^T \lambda_2^2 \lambda_3^2 \xi^2 + \\
& 18 e^{-T(\lambda_2+\lambda_3)} \mathbf{h}^T \lambda_2^2 \lambda_3^2 \xi^2 - 2 e^{-T(\lambda_2+\lambda_3)} \mathbf{r} \lambda_3^3 \xi^2 + \\
& 6 e^{-T(\lambda_2+\lambda_3)} \mathbf{h}^T \lambda_3^3 \xi^2 - 4 e^{-T(\lambda_2+\lambda_3)} \mathbf{r}^T \lambda_1 \lambda_3^3 \xi^2 - \\
& 6 e^{-T(\lambda_2+\lambda_3)} \mathbf{h}^T \lambda_1 \lambda_3^3 \xi^2 + 8 e^{-T(\lambda_2+\lambda_3)} \mathbf{r}^T \lambda_2 \lambda_3^3 \xi^2 + \\
& 12 e^{-T(\lambda_2+\lambda_3)} \mathbf{h}^T \lambda_2 \lambda_3^3 \xi^2 + \\
& 2 e^{-T(\lambda_2+\lambda_3)} \mathbf{r}^T \lambda_3^4 \xi^2 + 3 e^{-T(\lambda_2+\lambda_3)} \mathbf{h}^T \lambda_3^4 \xi^2) + \\
& \left( -e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{h} \gamma_2^4 \lambda_1 + e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{h} \gamma_1 \gamma_2^3 \lambda_2 + \right. \\
& e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{h} \gamma_2^4 \lambda_2 + e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{h} \gamma_1 \gamma_2^3 \lambda_3 + \\
& e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{h} \gamma_2^4 \lambda_3 - 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{h} \gamma_2^3 \lambda_1 \xi + \\
& e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{r} \gamma_2^3 \lambda_1 \lambda_2 \xi - e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{r} \gamma_1 \gamma_2^2 \lambda_2^2 \xi - \\
& e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{r} \gamma_2^3 \lambda_2^2 \xi + e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{r} \gamma_2^3 \lambda_1 \lambda_3 \xi - \\
& 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{r} \gamma_1 \gamma_2^2 \lambda_2 \lambda_3 \xi - \\
& 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{r} \gamma_2^3 \lambda_2 \lambda_3 \xi - e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{r} \gamma_1 \gamma_2^2 \lambda_3^2 \xi - \\
& e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{r} \gamma_2^3 \lambda_3^2 \xi - 3 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{h} \gamma_1 \gamma_2 \lambda_2 \xi^2 - \\
& 3 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{h} \gamma_2^2 \lambda_2 \xi^2 + 3 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{r} \gamma_2^2 \lambda_1 \lambda_2 \xi^2 + \\
& e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{h}^T \gamma_2^2 \lambda_1 \lambda_2 \xi^2 - 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{r} \gamma_1 \gamma_2 \lambda_2^2 \xi^2 - \\
& e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{h}^T \gamma_1 \gamma_2 \lambda_2^2 \xi^2 - 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{r} \gamma_2^2 \lambda_2^2 \xi^2 - \\
& e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{h}^T \gamma_2^2 \lambda_2^2 \xi^2 - 3 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{h} \gamma_1 \gamma_2 \lambda_3 \xi^2 - \\
& 3 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{h} \gamma_2^2 \lambda_3 \xi^2 + 3 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{r} \gamma_2^2 \lambda_1 \lambda_3 \xi^2 + \\
& \left. e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{h}^T \gamma_2^2 \lambda_1 \lambda_3 \xi^2 - \right)
\end{aligned}$$

$$\begin{aligned}
& 4 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_1 \gamma_2 \lambda_2 \lambda_3 \xi^2 - \\
& 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_1 \gamma_2 \lambda_2 \lambda_3 \xi^2 - \\
& 4 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2^2 \lambda_2 \lambda_3 \xi^2 - 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_2^2 \lambda_2 \lambda_3 \xi^2 - \\
& 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_1 \gamma_2 \lambda_3^2 \xi^2 - e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_1 \gamma_2 \lambda_3^2 \xi^2 - \\
& 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2^2 \lambda_3^2 \xi^2 - e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_2^2 \lambda_3^2 \xi^2 + \\
& 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_2 \lambda_1 \xi^3 - 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_1 \lambda_2 \xi^3 - \\
& 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_2 \lambda_2 \xi^3 + 3 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2 \lambda_1 \lambda_2 \xi^3 + \\
& 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_2 \lambda_1 \lambda_2 \xi^3 - e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_1 \lambda_2^2 \xi^3 - \\
& e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_1 \lambda_2^2 \xi^3 - e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2 \lambda_2^2 \xi^3 - \\
& e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_2 \lambda_2^2 \xi^3 - 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_1 \lambda_3 \xi^3 - \\
& 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_2 \lambda_3 \xi^3 + 3 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2 \lambda_1 \lambda_3 \xi^3 + \\
& 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_2 \lambda_1 \lambda_3 \xi^3 - 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_1 \lambda_2 \lambda_3 \xi^3 - \\
& 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_1 \lambda_2 \lambda_3 \xi^3 - 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2 \lambda_2 \lambda_3 \xi^3 - \\
& 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_2 \lambda_2 \lambda_3 \xi^3 - e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_1 \lambda_3^2 \xi^3 - \\
& e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_1 \lambda_3^2 \xi^3 - e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2 \lambda_3^2 \xi^3 - \\
& e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_2 \lambda_3^2 \xi^3 + e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \lambda_1 \xi^4 + \\
& e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \lambda_1 \lambda_2 \xi^4 + e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \lambda_1 \lambda_2 \xi^4 + \\
& e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \lambda_1 \lambda_3 \xi^4 + e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \lambda_1 \lambda_3 \xi^4 \Big) / \\
& \left( (\lambda_2 + \lambda_3)^2 \xi (-\gamma_2 \lambda_1 + \gamma_1 \lambda_2 + \gamma_2 \lambda_2 + \gamma_1 \lambda_3 + \gamma_2 \lambda_3 - \lambda_1 \xi) \right. \\
& \left. (\gamma_2 \lambda_1 - \gamma_1 \lambda_2 - \gamma_2 \lambda_2 - \gamma_1 \lambda_3 - \gamma_2 \lambda_3 + \lambda_1 \xi) \right) - \\
& \left( e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_2^4 \lambda_1 - e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_1 \gamma_2^3 \lambda_2 - \right. \\
& 2 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_2^4 \lambda_2 - \\
& e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_1 \gamma_2^3 \lambda_3 - \\
& 2 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_2^4 \lambda_3 + \\
& \left. 2 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_2^3 \lambda_1 \xi - \right)
\end{aligned}$$

$$\begin{aligned}
& 2 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_2^3 \lambda_2 \xi - \\
& e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2^3 \lambda_1 \lambda_2 \xi + \\
& e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_1 \gamma_2^2 \lambda_2^2 \xi + \\
& 2 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2^3 \lambda_2^2 \xi - \\
& 2 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_2^3 \lambda_3 \xi - \\
& e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2^3 \lambda_1 \lambda_3 \xi + \\
& 2 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_1 \gamma_2^2 \lambda_2 \lambda_3 \xi + \\
& 4 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2^3 \lambda_2 \lambda_3 \xi + \\
& e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_1 \gamma_2^2 \lambda_3^2 \xi + \\
& 2 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2^3 \lambda_3^2 \xi + \\
& 3 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_1 \gamma_2 \lambda_2 \xi^2 + \\
& 3 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_2^2 \lambda_2 \xi^2 - \\
& 3 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2^2 \lambda_1 \lambda_2 \xi^2 - \\
& e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_2^2 \lambda_1 \lambda_2 \xi^2 + \\
& 2 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_1 \gamma_2 \lambda_2^2 \xi^2 + \\
& e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_1 \gamma_2 \lambda_2^2 \xi^2 + \\
& 5 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2^2 \lambda_2^2 \xi^2 + \\
& 2 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_2^2 \lambda_2^2 \xi^2 + \\
& 3 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_1 \gamma_2 \lambda_3 \xi^2 + \\
& 3 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_2^2 \lambda_3 \xi^2 - \\
& 3 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2^2 \lambda_1 \lambda_3 \xi^2 - \\
& e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_2^2 \lambda_1 \lambda_3 \xi^2 + \\
& 4 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_1 \gamma_2 \lambda_2 \lambda_3 \xi^2 + \\
& 2 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_1 \gamma_2 \lambda_2 \lambda_3 \xi^2 + \\
& 10 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2^2 \lambda_2 \lambda_3 \xi^2 +
\end{aligned}$$

$$\begin{aligned}
& 4 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_2^2 \lambda_2 \lambda_3 \xi^2 + \\
& 2 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_1 \gamma_2 \lambda_3^2 \xi^2 + \\
& e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_1 \gamma_2 \lambda_3^2 \xi^2 + \\
& 5 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2^2 \lambda_3^2 \xi^2 + \\
& 2 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_2^2 \lambda_3^2 \xi^2 - \\
& 2 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_2 \lambda_1 \xi^3 + \\
& 2 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_1 \lambda_2 \xi^3 + \\
& 4 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_2 \lambda_2 \xi^3 - \\
& 3 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2 \lambda_1 \lambda_2 \xi^3 - \\
& 2 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_2 \lambda_1 \lambda_2 \xi^3 + \\
& e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_1 \lambda_2^2 \xi^3 + \\
& e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_1 \lambda_2^2 \xi^3 + \\
& 4 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2 \lambda_2^2 \xi^3 + \\
& 3 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_2 \lambda_2^2 \xi^3 + \\
& 2 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_1 \lambda_3 \xi^3 + \\
& 4 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_2 \lambda_3 \xi^3 - \\
& 3 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2 \lambda_1 \lambda_3 \xi^3 - \\
& 2 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_2 \lambda_1 \lambda_3 \xi^3 + \\
& 2 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_1 \lambda_2 \lambda_3 \xi^3 + \\
& 2 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_1 \lambda_2 \lambda_3 \xi^3 + \\
& 8 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2 \lambda_2 \lambda_3 \xi^3 + \\
& 6 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_2 \lambda_2 \lambda_3 \xi^3 + \\
& e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_1 \lambda_3^2 \xi^3 + \\
& e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_1 \lambda_3^2 \xi^3 + \\
& 4 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2 \lambda_3^2 \xi^3 +
\end{aligned}$$

$$\begin{aligned}
& 3 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_2 \lambda_3^2 \xi^3 - \\
& e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \lambda_1 \xi^4 + e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \lambda_2 \xi^4 - \\
& e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \lambda_1 \lambda_2 \xi^4 - \\
& e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \lambda_1 \lambda_2 \xi^4 + \\
& e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \lambda_2^2 \xi^4 + e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \lambda_2^2 \xi^4 + \\
& e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \lambda_3 \xi^4 - e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \lambda_1 \lambda_3 \xi^4 - \\
& e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \lambda_1 \lambda_3 \xi^4 + \\
& 2 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \lambda_2 \lambda_3 \xi^4 + \\
& 2 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \lambda_2 \lambda_3 \xi^4 + \\
& e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \lambda_3^2 \xi^4 + \\
& e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \lambda_3^2 \xi^4 \Big) / ((\lambda_2 + \lambda_3)^2 \xi \\
& (\gamma_2 \lambda_1 - \gamma_1 \lambda_2 - 2 \gamma_2 \lambda_2 - \gamma_1 \lambda_3 - 2 \gamma_2 \lambda_3 + \lambda_1 \xi - \lambda_2 \xi - \lambda_3 \xi)^2) + \\
& e^{-T \lambda_1 + 2 T (\lambda_2 + \lambda_3)} \left( \frac{e^{-2 T (\lambda_2 + \lambda_3)} h T^2 (2 \gamma_1 - \xi)}{2 (-\lambda_1 + 2 \lambda_2 + 2 \lambda_3)} + \right. \\
& \left. (T (e^{-2 T (\lambda_2 + \lambda_3)} h \gamma_1 \lambda_1 + 2 c e^{-2 T (\lambda_2 + \lambda_3)} \gamma_2 \lambda_1 \lambda_2 - \right. \\
& \quad 4 c e^{-2 T (\lambda_2 + \lambda_3)} \gamma_2 \lambda_2^2 + 2 c e^{-2 T (\lambda_2 + \lambda_3)} \gamma_2 \lambda_1 \lambda_3 - \\
& \quad 8 c e^{-2 T (\lambda_2 + \lambda_3)} \gamma_2 \lambda_2 \lambda_3 - 4 c e^{-2 T (\lambda_2 + \lambda_3)} \gamma_2 \lambda_3^2 + \\
& \quad e^{-2 T (\lambda_2 + \lambda_3)} h \lambda_1 \xi - 3 e^{-2 T (\lambda_2 + \lambda_3)} h \lambda_2 \xi - e^{-2 T (\lambda_2 + \lambda_3)} r \\
& \quad \lambda_1 \lambda_2 \xi + e^{-2 T (\lambda_2 + \lambda_3)} h T \lambda_1 \lambda_2 \xi + 2 e^{-2 T (\lambda_2 + \lambda_3)} r \lambda_2^2 \xi - \\
& \quad 2 e^{-2 T (\lambda_2 + \lambda_3)} h T \lambda_2^2 \xi - 3 e^{-2 T (\lambda_2 + \lambda_3)} h \lambda_3 \xi - \\
& \quad e^{-2 T (\lambda_2 + \lambda_3)} r \lambda_1 \lambda_3 \xi + e^{-2 T (\lambda_2 + \lambda_3)} h T \lambda_1 \lambda_3 \xi + \\
& \quad 4 e^{-2 T (\lambda_2 + \lambda_3)} r \lambda_2 \lambda_3 \xi - 4 e^{-2 T (\lambda_2 + \lambda_3)} h T \lambda_2 \lambda_3 \xi + \\
& \quad \left. \left. 2 e^{-2 T (\lambda_2 + \lambda_3)} r \lambda_3^2 \xi - 2 e^{-2 T (\lambda_2 + \lambda_3)} h T \lambda_3^2 \xi) \right) / \right. \\
& \left. ((\lambda_1 - 2 \lambda_2 - 2 \lambda_3) (\lambda_2 + \lambda_3) (-\lambda_1 + 2 \lambda_2 + 2 \lambda_3)) - \right. \\
& \quad \left. \frac{1}{2 (\lambda_1 - 2 \lambda_2 - 2 \lambda_3)^3 (\lambda_2 + \lambda_3)^2} \right. \\
& \left. (6 e^{-2 T (\lambda_2 + \lambda_3)} h \gamma_2 \lambda_1^2 + 2 e^{-2 T (\lambda_2 + \lambda_3)} h \gamma_1 \lambda_1 \lambda_2 - \right. \\
& \quad 24 e^{-2 T (\lambda_2 + \lambda_3)} h \gamma_2 \lambda_1 \lambda_2 - 2 c e^{-2 T (\lambda_2 + \lambda_3)} \gamma_2 \lambda_1^2 \lambda_2 + \\
& \quad \left. 4 e^{-2 T (\lambda_2 + \lambda_3)} h T \gamma_2 \lambda_1^2 \lambda_2 + 24 e^{-2 T (\lambda_2 + \lambda_3)} h \gamma_2 \lambda_2^2 + \right.
\end{aligned}$$

$$\begin{aligned}
& 12 c e^{-2 T(\lambda_2+\lambda_3)} \gamma_2 \lambda_1 \lambda_2^2 - 16 e^{-2 T(\lambda_2+\lambda_3)} h T \gamma_2 \lambda_1 \lambda_2^2 - \\
& 2 c e^{-2 T(\lambda_2+\lambda_3)} T \gamma_2 \lambda_1^2 \lambda_2^2 + e^{-2 T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_1^2 \lambda_2^2 - \\
& 16 c e^{-2 T(\lambda_2+\lambda_3)} \gamma_2 \lambda_2^3 + 16 e^{-2 T(\lambda_2+\lambda_3)} h T \gamma_2 \lambda_2^3 + \\
& 8 c e^{-2 T(\lambda_2+\lambda_3)} T \gamma_2 \lambda_1 \lambda_2^3 - 4 e^{-2 T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_1 \lambda_2^3 - \\
& 8 c e^{-2 T(\lambda_2+\lambda_3)} T \gamma_2 \lambda_2^4 + 4 e^{-2 T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_2^4 + \\
& 2 e^{-2 T(\lambda_2+\lambda_3)} h \gamma_1 \lambda_1 \lambda_3 - 24 e^{-2 T(\lambda_2+\lambda_3)} h \gamma_2 \lambda_1 \lambda_3 - \\
& 2 c e^{-2 T(\lambda_2+\lambda_3)} \gamma_2 \lambda_1^2 \lambda_3 + 4 e^{-2 T(\lambda_2+\lambda_3)} h T \gamma_2 \lambda_1^2 \lambda_3 + \\
& 48 e^{-2 T(\lambda_2+\lambda_3)} h \gamma_2 \lambda_2 \lambda_3 + 24 c e^{-2 T(\lambda_2+\lambda_3)} \gamma_2 \lambda_1 \lambda_2 \lambda_3 - 32 \\
& e^{-2 T(\lambda_2+\lambda_3)} h T \gamma_2 \lambda_1 \lambda_2 \lambda_3 - 4 c e^{-2 T(\lambda_2+\lambda_3)} T \gamma_2 \lambda_1^2 \lambda_2 \lambda_3 + \\
& 2 e^{-2 T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_1^2 \lambda_2 \lambda_3 - 48 c e^{-2 T(\lambda_2+\lambda_3)} \gamma_2 \lambda_2^2 \lambda_3 + \\
& 48 e^{-2 T(\lambda_2+\lambda_3)} h T \gamma_2 \lambda_2^2 \lambda_3 + 24 c e^{-2 T(\lambda_2+\lambda_3)} T \gamma_2 \lambda_1 \lambda_2^2 \lambda_3 - \\
& 12 e^{-2 T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_1 \lambda_2^2 \lambda_3 - 32 c e^{-2 T(\lambda_2+\lambda_3)} T \gamma_2 \lambda_2^3 \\
& \lambda_3 + 16 e^{-2 T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_2^3 \lambda_3 + 24 e^{-2 T(\lambda_2+\lambda_3)} h \gamma_2 \lambda_3^2 + \\
& 12 c e^{-2 T(\lambda_2+\lambda_3)} \gamma_2 \lambda_1 \lambda_3^2 - 16 e^{-2 T(\lambda_2+\lambda_3)} h T \gamma_2 \lambda_1 \lambda_3^2 - \\
& 2 c e^{-2 T(\lambda_2+\lambda_3)} T \gamma_2 \lambda_1^2 \lambda_3^2 + e^{-2 T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_1^2 \lambda_3^2 - \\
& 48 c e^{-2 T(\lambda_2+\lambda_3)} \gamma_2 \lambda_2 \lambda_3^2 + 48 e^{-2 T(\lambda_2+\lambda_3)} h T \gamma_2 \lambda_2 \lambda_3^2 + 24 c \\
& e^{-2 T(\lambda_2+\lambda_3)} T \gamma_2 \lambda_1 \lambda_2 \lambda_3^2 - 12 e^{-2 T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_1 \lambda_2 \lambda_3^2 - \\
& 48 c e^{-2 T(\lambda_2+\lambda_3)} T \gamma_2 \lambda_2^2 \lambda_3^2 + 24 e^{-2 T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_2^2 \lambda_3^2 - \\
& 16 c e^{-2 T(\lambda_2+\lambda_3)} \gamma_2 \lambda_3^3 + 16 e^{-2 T(\lambda_2+\lambda_3)} h T \gamma_2 \lambda_3^3 + \\
& 8 c e^{-2 T(\lambda_2+\lambda_3)} T \gamma_2 \lambda_1 \lambda_3^3 - 4 e^{-2 T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_1 \lambda_3^3 - \\
& 32 c e^{-2 T(\lambda_2+\lambda_3)} T \gamma_2 \lambda_2 \lambda_3^3 + 16 e^{-2 T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_2 \lambda_3^3 - \\
& 8 c e^{-2 T(\lambda_2+\lambda_3)} T \gamma_2 \lambda_3^4 + 4 e^{-2 T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_3^4 - \\
& 6 e^{-2 T(\lambda_2+\lambda_3)} h \lambda_1^2 \xi + 26 e^{-2 T(\lambda_2+\lambda_3)} h \lambda_1 \lambda_2 \xi - \\
& 2 e^{-2 T(\lambda_2+\lambda_3)} r \lambda_1^2 \lambda_2 \xi - 6 e^{-2 T(\lambda_2+\lambda_3)} h T \lambda_1^2 \lambda_2 \xi - \\
& 30 e^{-2 T(\lambda_2+\lambda_3)} h \lambda_2^2 \xi + 6 e^{-2 T(\lambda_2+\lambda_3)} r \lambda_1 \lambda_2^2 \xi + \\
& 26 e^{-2 T(\lambda_2+\lambda_3)} h T \lambda_1 \lambda_2^2 \xi - 2 e^{-2 T(\lambda_2+\lambda_3)} h T^2 \lambda_1^2 \lambda_2^2 \xi - \\
& 4 e^{-2 T(\lambda_2+\lambda_3)} r \lambda_2^3 \xi - 28 e^{-2 T(\lambda_2+\lambda_3)} h T \lambda_2^3 \xi + \\
& 8 e^{-2 T(\lambda_2+\lambda_3)} h T^2 \lambda_1 \lambda_2^3 \xi - 8 e^{-2 T(\lambda_2+\lambda_3)} h T^2 \lambda_2^4 \xi + \\
& 26 e^{-2 T(\lambda_2+\lambda_3)} h \lambda_1 \lambda_3 \xi - 2 e^{-2 T(\lambda_2+\lambda_3)} r \lambda_1^2 \lambda_3 \xi - \\
& 6 e^{-2 T(\lambda_2+\lambda_3)} h T \lambda_1^2 \lambda_3 \xi - 60 e^{-2 T(\lambda_2+\lambda_3)} h \lambda_2 \lambda_3 \xi + \\
& 12 e^{-2 T(\lambda_2+\lambda_3)} r \lambda_1 \lambda_2 \lambda_3 \xi + 52 e^{-2 T(\lambda_2+\lambda_3)} h T \lambda_1 \lambda_2 \lambda_3 \xi - \\
& 4 e^{-2 T(\lambda_2+\lambda_3)} h T^2 \lambda_1^2 \lambda_2 \lambda_3 \xi - 12 e^{-2 T(\lambda_2+\lambda_3)} r \lambda_2^2 \lambda_3 \xi - \\
& 84 e^{-2 T(\lambda_2+\lambda_3)} h T \lambda_2^2 \lambda_3 \xi + 24 e^{-2 T(\lambda_2+\lambda_3)} h T^2 \lambda_1 \lambda_2^2 \lambda_3 \xi - \\
& 32 e^{-2 T(\lambda_2+\lambda_3)} h T^2 \lambda_2^3 \lambda_3 \xi - 30 e^{-2 T(\lambda_2+\lambda_3)} h \lambda_3^2 \xi +
\end{aligned}$$

$$\begin{aligned}
& \left. \begin{aligned}
& 6 e^{-2T(\lambda_2+\lambda_3)} r \lambda_1 \lambda_3^2 \xi + 26 e^{-2T(\lambda_2+\lambda_3)} h T \lambda_1 \lambda_3^2 \xi - \\
& 2 e^{-2T(\lambda_2+\lambda_3)} h T^2 \lambda_1^2 \lambda_3^2 \xi - 12 e^{-2T(\lambda_2+\lambda_3)} r \lambda_2 \lambda_3^2 \xi - \\
& 84 e^{-2T(\lambda_2+\lambda_3)} h T \lambda_2 \lambda_3^2 \xi + 24 e^{-2T(\lambda_2+\lambda_3)} h T^2 \lambda_1 \lambda_2 \lambda_3^2 \xi - \\
& 48 e^{-2T(\lambda_2+\lambda_3)} h T^2 \lambda_2^2 \lambda_3^2 \xi - 4 e^{-2T(\lambda_2+\lambda_3)} r \lambda_3^3 \xi - \\
& 28 e^{-2T(\lambda_2+\lambda_3)} h T \lambda_3^3 \xi + 8 e^{-2T(\lambda_2+\lambda_3)} h T^2 \lambda_1 \lambda_3^3 \xi - \\
& 32 e^{-2T(\lambda_2+\lambda_3)} h T^2 \lambda_2 \lambda_3^3 \xi - 8 e^{-2T(\lambda_2+\lambda_3)} h T^2 \lambda_3^4 \xi \Big) + \\
& e^{-T\lambda_1} \left( \frac{h T^2 (\gamma_1^2 + 2 \gamma_1 \gamma_2 + \gamma_2^2 - 3 \gamma_1 \xi - 2 \gamma_2 \xi + \xi^2)}{2 \lambda_1 \xi} - \right. \\
& \frac{1}{\lambda_1^2 (\lambda_2 + \lambda_3) \xi} T (-h \gamma_1 \gamma_2 \lambda_1 - h \gamma_2^2 \lambda_1 - h \gamma_1^2 \lambda_2 - \\
& 2 h \gamma_1 \gamma_2 \lambda_2 - h \gamma_2^2 \lambda_2 - h \gamma_1^2 \lambda_3 - 2 h \gamma_1 \gamma_2 \lambda_3 - h \gamma_2^2 \lambda_3 + \\
& h \gamma_1 \lambda_1 \xi + 3 h \gamma_2 \lambda_1 \xi + 3 h \gamma_1 \lambda_2 \xi + 2 h \gamma_2 \lambda_2 \xi - \\
& c \gamma_1 \lambda_1 \lambda_2 \xi + r \gamma_1 \lambda_1 \lambda_2 \xi + r \gamma_2 \lambda_1 \lambda_2 \xi + 3 h \gamma_1 \lambda_3 \xi + \\
& 2 h \gamma_2 \lambda_3 \xi - c \gamma_1 \lambda_1 \lambda_3 \xi + r \gamma_1 \lambda_1 \lambda_3 \xi + r \gamma_2 \lambda_1 \lambda_3 \xi - \\
& 2 h \lambda_1 \xi^2 - h \lambda_2 \xi^2 - r \lambda_1 \lambda_2 \xi^2 - h \lambda_3 \xi^2 - r \lambda_1 \lambda_3 \xi^2) - \\
& \frac{1}{\lambda_1^3 (\lambda_2 + \lambda_3)^2 \xi} (-h \gamma_2^2 \lambda_1^2 - h \gamma_1 \gamma_2 \lambda_1 \lambda_2 - h \gamma_2^2 \lambda_1 \lambda_2 - \\
& h \gamma_1^2 \lambda_2^2 - 2 h \gamma_1 \gamma_2 \lambda_2^2 - h \gamma_2^2 \lambda_2^2 - h \gamma_1 \gamma_2 \lambda_1 \lambda_3 - \\
& h \gamma_2^2 \lambda_1 \lambda_3 - 2 h \gamma_1^2 \lambda_2 \lambda_3 - 4 h \gamma_1 \gamma_2 \lambda_2 \lambda_3 - \\
& 2 h \gamma_2^2 \lambda_2 \lambda_3 - h \gamma_1^2 \lambda_3^2 - 2 h \gamma_1 \gamma_2 \lambda_3^2 - h \gamma_2^2 \lambda_3^2 + \\
& 3 h \gamma_2 \lambda_1^2 \xi + h \gamma_1 \lambda_1 \lambda_2 \xi + 3 h \gamma_2 \lambda_1 \lambda_2 \xi - c \gamma_2 \lambda_1^2 \lambda_2 \xi + \\
& r \gamma_2 \lambda_1^2 \lambda_2 \xi + 3 h \gamma_1 \lambda_2^2 \xi + 2 h \gamma_2 \lambda_2^2 \xi - c \gamma_1 \lambda_1 \lambda_2^2 \xi + \\
& r \gamma_1 \lambda_1 \lambda_2^2 \xi + r \gamma_2 \lambda_1 \lambda_2^2 \xi + h \gamma_1 \lambda_1 \lambda_3 \xi + 3 h \gamma_2 \lambda_1 \lambda_3 \xi - \\
& c \gamma_2 \lambda_1^2 \lambda_3 \xi + r \gamma_2 \lambda_1^2 \lambda_3 \xi + 6 h \gamma_1 \lambda_2 \lambda_3 \xi + \\
& 4 h \gamma_2 \lambda_2 \lambda_3 \xi - 2 c \gamma_1 \lambda_1 \lambda_2 \lambda_3 \xi + 2 r \gamma_1 \lambda_1 \lambda_2 \lambda_3 \xi + \\
& 2 r \gamma_2 \lambda_1 \lambda_2 \lambda_3 \xi + 3 h \gamma_1 \lambda_3^2 \xi + 2 h \gamma_2 \lambda_3^2 \xi - \\
& c \gamma_1 \lambda_1 \lambda_3^2 \xi + r \gamma_1 \lambda_1 \lambda_3^2 \xi + r \gamma_2 \lambda_1 \lambda_3^2 \xi - 2 h \lambda_1^2 \xi^2 - \\
& 2 h \lambda_1 \lambda_2 \xi^2 - h \lambda_2^2 \xi^2 - r \lambda_1 \lambda_2^2 \xi^2 - 2 h \lambda_1 \lambda_3 \xi^2 - \\
& \left. \left. 2 h \lambda_2 \lambda_3 \xi^2 - 2 r \lambda_1 \lambda_2 \lambda_3 \xi^2 - h \lambda_3^2 \xi^2 - r \lambda_1 \lambda_3^2 \xi^2 \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& e^{-T \lambda_1 + T (\lambda_2 + \lambda_3)} \left( - \frac{e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \mathbf{T}^2 (-\gamma_1^2 - 2 \gamma_1 \gamma_2 - \gamma_2^2 + 4 \gamma_1 \xi)}{2 (-\lambda_1 + \lambda_2 + \lambda_3) \xi} + \right. \\
& \left. \left( \mathbf{T} \left( e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \gamma_1 \gamma_2 \lambda_1 + e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \gamma_2^2 \lambda_1 + e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \right. \right. \right. \\
& \quad \gamma_1^2 \lambda_2 + e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \gamma_1 \gamma_2 \lambda_2 + e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \gamma_1^2 \lambda_3 + \\
& \quad e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \gamma_1 \gamma_2 \lambda_3 - 2 e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \gamma_1 \lambda_1 \xi - 3 e^{-T (\lambda_2 + \lambda_3)} \\
& \quad \mathbf{h} \gamma_2 \lambda_1 \xi - 2 e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \gamma_1 \lambda_2 \xi + 3 e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \gamma_2 \lambda_2 \xi - \\
& \quad e^{-T (\lambda_2 + \lambda_3)} \mathbf{r} \gamma_1 \lambda_1 \lambda_2 \xi - e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \mathbf{T} \gamma_1 \lambda_1 \lambda_2 \xi - \\
& \quad 2 \mathbf{c} e^{-T (\lambda_2 + \lambda_3)} \gamma_2 \lambda_1 \lambda_2 \xi - e^{-T (\lambda_2 + \lambda_3)} \mathbf{r} \gamma_2 \lambda_1 \lambda_2 \xi - \\
& \quad e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \mathbf{T} \gamma_2 \lambda_1 \lambda_2 \xi + e^{-T (\lambda_2 + \lambda_3)} \mathbf{r} \gamma_1 \lambda_2^2 \xi + \\
& \quad e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \mathbf{T} \gamma_1 \lambda_2^2 \xi + 2 \mathbf{c} e^{-T (\lambda_2 + \lambda_3)} \gamma_2 \lambda_2^2 \xi + \\
& \quad e^{-T (\lambda_2 + \lambda_3)} \mathbf{r} \gamma_2 \lambda_2^2 \xi + e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \mathbf{T} \gamma_2 \lambda_2^2 \xi - 2 e^{-T (\lambda_2 + \lambda_3)} \\
& \quad \mathbf{h} \gamma_1 \lambda_3 \xi + 3 e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \gamma_2 \lambda_3 \xi - e^{-T (\lambda_2 + \lambda_3)} \mathbf{r} \gamma_1 \lambda_1 \lambda_3 \xi - \\
& \quad e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \mathbf{T} \gamma_1 \lambda_1 \lambda_3 \xi - 2 \mathbf{c} e^{-T (\lambda_2 + \lambda_3)} \gamma_2 \lambda_1 \lambda_3 \xi - \\
& \quad e^{-T (\lambda_2 + \lambda_3)} \mathbf{r} \gamma_2 \lambda_1 \lambda_3 \xi - e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \mathbf{T} \gamma_2 \lambda_1 \lambda_3 \xi + \\
& \quad 2 e^{-T (\lambda_2 + \lambda_3)} \mathbf{r} \gamma_1 \lambda_2 \lambda_3 \xi + 2 e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \mathbf{T} \gamma_1 \lambda_2 \lambda_3 \xi + \\
& \quad 4 \mathbf{c} e^{-T (\lambda_2 + \lambda_3)} \gamma_2 \lambda_2 \lambda_3 \xi + 2 e^{-T (\lambda_2 + \lambda_3)} \mathbf{r} \gamma_2 \lambda_2 \lambda_3 \xi + \\
& \quad 2 e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \mathbf{T} \gamma_2 \lambda_2 \lambda_3 \xi + e^{-T (\lambda_2 + \lambda_3)} \mathbf{r} \gamma_1 \lambda_3^2 \xi + \\
& \quad e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \mathbf{T} \gamma_1 \lambda_3^2 \xi + 2 \mathbf{c} e^{-T (\lambda_2 + \lambda_3)} \gamma_2 \lambda_3^2 \xi + e^{-T (\lambda_2 + \lambda_3)} \\
& \quad \mathbf{r} \gamma_2 \lambda_3^2 \xi + e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \mathbf{T} \gamma_2 \lambda_3^2 \xi + e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \lambda_1 \xi^2 - \\
& \quad e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \lambda_2 \xi^2 + 2 e^{-T (\lambda_2 + \lambda_3)} \mathbf{r} \lambda_1 \lambda_2 \xi^2 - 2 e^{-T (\lambda_2 + \lambda_3)} \\
& \quad \mathbf{r} \lambda_2^2 \xi^2 - e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \lambda_3 \xi^2 + 2 e^{-T (\lambda_2 + \lambda_3)} \mathbf{r} \lambda_1 \lambda_3 \xi^2 - \\
& \quad \left. \left. \left. 4 e^{-T (\lambda_2 + \lambda_3)} \mathbf{r} \lambda_2 \lambda_3 \xi^2 - 2 e^{-T (\lambda_2 + \lambda_3)} \mathbf{r} \lambda_3^2 \xi^2 \right) \right) \right) / \\
& \left( (\lambda_1 - \lambda_2 - \lambda_3) (\lambda_2 + \lambda_3) (-\lambda_1 + \lambda_2 + \lambda_3) \xi \right) - \\
& \quad \frac{1}{2 (\lambda_1 - \lambda_2 - \lambda_3)^3 (\lambda_2 + \lambda_3)^2 \xi} \\
& \left( 2 e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \gamma_2^2 \lambda_1^2 + 2 e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \gamma_1 \gamma_2 \lambda_1 \lambda_2 - \right. \\
& \quad 2 e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \gamma_2^2 \lambda_1 \lambda_2 + 2 e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \gamma_1^2 \lambda_2^2 + \\
& \quad 2 e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \gamma_1 \gamma_2 \lambda_2^2 + 2 e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \gamma_2^2 \lambda_2^2 + \\
& \quad 2 e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \gamma_1 \gamma_2 \lambda_1 \lambda_3 - 2 e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \gamma_2^2 \lambda_1 \lambda_3 + \\
& \quad 4 e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \gamma_1^2 \lambda_2 \lambda_3 + 4 e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \gamma_1 \gamma_2 \lambda_2 \lambda_3 + \\
& \quad 4 e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \gamma_2^2 \lambda_2 \lambda_3 + 2 e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \gamma_1^2 \lambda_3^2 + \\
& \quad 2 e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \gamma_1 \gamma_2 \lambda_3^2 + 2 e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \gamma_2^2 \lambda_3^2 - \\
& \quad \left. 12 e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \gamma_2 \lambda_1^2 \xi - 4 e^{-T (\lambda_2 + \lambda_3)} \mathbf{h} \gamma_1 \lambda_1 \lambda_2 \xi + \right.
\end{aligned}$$



$$\begin{aligned}
& 18 e^{-T(\lambda_2+\lambda_3)} h \gamma_2 \lambda_1 \lambda_2 \xi + 4 c e^{-T(\lambda_2+\lambda_3)} \gamma_2 \lambda_1^2 \lambda_2 \xi - \\
& 2 e^{-T(\lambda_2+\lambda_3)} r \gamma_2 \lambda_1^2 \lambda_2 \xi - 4 e^{-T(\lambda_2+\lambda_3)} h T \gamma_2 \lambda_1^2 \lambda_2 \xi - \\
& 4 e^{-T(\lambda_2+\lambda_3)} h \gamma_1 \lambda_2^2 \xi - 6 e^{-T(\lambda_2+\lambda_3)} h \gamma_2 \lambda_2^2 \xi - \\
& 2 e^{-T(\lambda_2+\lambda_3)} r \gamma_1 \lambda_1 \lambda_2^2 \xi - 2 e^{-T(\lambda_2+\lambda_3)} h T \gamma_1 \lambda_1 \lambda_2^2 \xi - \\
& 12 c e^{-T(\lambda_2+\lambda_3)} \gamma_2 \lambda_1 \lambda_2^2 \xi + 2 e^{-T(\lambda_2+\lambda_3)} r \gamma_2 \lambda_1 \lambda_2^2 \xi + \\
& 6 e^{-T(\lambda_2+\lambda_3)} h T \gamma_2 \lambda_1 \lambda_2^2 \xi - 2 e^{-T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_1^2 \lambda_2^2 \xi + \\
& 2 e^{-T(\lambda_2+\lambda_3)} r \gamma_1 \lambda_2^3 \xi + 2 e^{-T(\lambda_2+\lambda_3)} h T \gamma_1 \lambda_2^3 \xi + \\
& 8 c e^{-T(\lambda_2+\lambda_3)} \gamma_2 \lambda_2^3 \xi - 2 e^{-T(\lambda_2+\lambda_3)} h T \gamma_2 \lambda_2^3 \xi + \\
& 4 e^{-T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_1 \lambda_2^3 \xi - 2 e^{-T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_2^4 \xi - \\
& 4 e^{-T(\lambda_2+\lambda_3)} h \gamma_1 \lambda_1 \lambda_3 \xi + 18 e^{-T(\lambda_2+\lambda_3)} h \gamma_2 \lambda_1 \lambda_3 \xi + \\
& 4 c e^{-T(\lambda_2+\lambda_3)} \gamma_2 \lambda_1^2 \lambda_3 \xi - 2 e^{-T(\lambda_2+\lambda_3)} r \gamma_2 \lambda_1^2 \lambda_3 \xi - \\
& 4 e^{-T(\lambda_2+\lambda_3)} h T \gamma_2 \lambda_1^2 \lambda_3 \xi - 8 e^{-T(\lambda_2+\lambda_3)} h \gamma_1 \lambda_2 \lambda_3 \xi - \\
& 12 e^{-T(\lambda_2+\lambda_3)} h \gamma_2 \lambda_2 \lambda_3 \xi - 4 e^{-T(\lambda_2+\lambda_3)} r \gamma_1 \lambda_1 \lambda_2 \lambda_3 \xi - \\
& 4 e^{-T(\lambda_2+\lambda_3)} h T \gamma_1 \lambda_1 \lambda_2 \lambda_3 \xi - 24 c e^{-T(\lambda_2+\lambda_3)} \gamma_2 \lambda_1 \lambda_2 \lambda_3 \xi + \\
& 4 e^{-T(\lambda_2+\lambda_3)} r \gamma_2 \lambda_1 \lambda_2 \lambda_3 \xi + 12 e^{-T(\lambda_2+\lambda_3)} h T \gamma_2 \lambda_1 \lambda_2 \lambda_3 \xi - \\
& 4 e^{-T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_1^2 \lambda_2 \lambda_3 \xi + 6 e^{-T(\lambda_2+\lambda_3)} r \gamma_1 \lambda_2^2 \lambda_3 \xi + \\
& 6 e^{-T(\lambda_2+\lambda_3)} h T \gamma_1 \lambda_2^2 \lambda_3 \xi + 24 c e^{-T(\lambda_2+\lambda_3)} \gamma_2 \lambda_2^2 \lambda_3 \xi - \\
& 6 e^{-T(\lambda_2+\lambda_3)} h T \gamma_2 \lambda_2^2 \lambda_3 \xi + 12 e^{-T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_1 \lambda_2^2 \lambda_3 \\
& \xi - 8 e^{-T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_2^3 \lambda_3 \xi - 4 e^{-T(\lambda_2+\lambda_3)} h \gamma_1 \lambda_3^2 \xi - \\
& 6 e^{-T(\lambda_2+\lambda_3)} h \gamma_2 \lambda_3^2 \xi - 2 e^{-T(\lambda_2+\lambda_3)} r \gamma_1 \lambda_1 \lambda_3^2 \xi - \\
& 2 e^{-T(\lambda_2+\lambda_3)} h T \gamma_1 \lambda_1 \lambda_3^2 \xi - 12 c e^{-T(\lambda_2+\lambda_3)} \gamma_2 \lambda_1 \lambda_3^2 \xi + \\
& 2 e^{-T(\lambda_2+\lambda_3)} r \gamma_2 \lambda_1 \lambda_3^2 \xi + 6 e^{-T(\lambda_2+\lambda_3)} h T \gamma_2 \lambda_1 \lambda_3^2 \xi - \\
& 2 e^{-T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_1^2 \lambda_3^2 \xi + 6 e^{-T(\lambda_2+\lambda_3)} r \gamma_1 \lambda_2 \lambda_3^2 \xi + \\
& 6 e^{-T(\lambda_2+\lambda_3)} h T \gamma_1 \lambda_2 \lambda_3^2 \xi + 24 c e^{-T(\lambda_2+\lambda_3)} \gamma_2 \lambda_2 \lambda_3^2 \xi - \\
& 6 e^{-T(\lambda_2+\lambda_3)} h T \gamma_2 \lambda_2 \lambda_3^2 \xi + 12 e^{-T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_1 \lambda_2 \lambda_3^2 \\
& \xi - 12 e^{-T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_2^2 \lambda_3^2 \xi + 2 e^{-T(\lambda_2+\lambda_3)} r \gamma_1 \lambda_3^3 \xi + \\
& 2 e^{-T(\lambda_2+\lambda_3)} h T \gamma_1 \lambda_3^3 \xi + 8 c e^{-T(\lambda_2+\lambda_3)} \gamma_2 \lambda_3^3 \xi - \\
& 2 e^{-T(\lambda_2+\lambda_3)} h T \gamma_2 \lambda_3^3 \xi + 4 e^{-T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_1 \lambda_3^3 \xi - \\
& 8 e^{-T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_2 \lambda_3^3 \xi - 2 e^{-T(\lambda_2+\lambda_3)} h T^2 \gamma_2 \lambda_3^4 \xi + \\
& 10 e^{-T(\lambda_2+\lambda_3)} h \lambda_1^2 \xi^2 - 18 e^{-T(\lambda_2+\lambda_3)} h \lambda_1 \lambda_2 \xi^2 + \\
& 2 e^{-T(\lambda_2+\lambda_3)} r \lambda_1^2 \lambda_2 \xi^2 + 6 e^{-T(\lambda_2+\lambda_3)} h T \lambda_1^2 \lambda_2 \xi^2 + \\
& 8 e^{-T(\lambda_2+\lambda_3)} h \lambda_2^2 \xi^2 - 12 e^{-T(\lambda_2+\lambda_3)} h T \lambda_1 \lambda_2^2 \xi^2 + \\
& 2 e^{-T(\lambda_2+\lambda_3)} r T \lambda_1^2 \lambda_2^2 \xi^2 + 3 e^{-T(\lambda_2+\lambda_3)} h T^2 \lambda_1^2 \lambda_2^2 \xi^2 - \\
& 2 e^{-T(\lambda_2+\lambda_3)} r \lambda_2^3 \xi^2 + 6 e^{-T(\lambda_2+\lambda_3)} h T \lambda_2^3 \xi^2 -
\end{aligned}$$

$$\begin{aligned}
& 4 e^{-T(\lambda_2+\lambda_3)} \mathbf{r}^T \lambda_1 \lambda_2^3 \xi^2 - 6 e^{-T(\lambda_2+\lambda_3)} \mathbf{h}^T \lambda_1 \lambda_2^3 \xi^2 + \\
& 2 e^{-T(\lambda_2+\lambda_3)} \mathbf{r}^T \lambda_2^4 \xi^2 + 3 e^{-T(\lambda_2+\lambda_3)} \mathbf{h}^T \lambda_2^4 \xi^2 - \\
& 18 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \lambda_1 \lambda_3 \xi^2 + 2 e^{-T(\lambda_2+\lambda_3)} \mathbf{r} \lambda_1^2 \lambda_3 \xi^2 + \\
& 6 e^{-T(\lambda_2+\lambda_3)} \mathbf{h}^T \lambda_1^2 \lambda_3 \xi^2 + 16 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \lambda_2 \lambda_3 \xi^2 - \\
& 24 e^{-T(\lambda_2+\lambda_3)} \mathbf{h}^T \lambda_1 \lambda_2 \lambda_3 \xi^2 + 4 e^{-T(\lambda_2+\lambda_3)} \mathbf{r}^T \lambda_1^2 \lambda_2 \lambda_3 \xi^2 + \\
& 6 e^{-T(\lambda_2+\lambda_3)} \mathbf{h}^T \lambda_1^2 \lambda_2 \lambda_3 \xi^2 - 6 e^{-T(\lambda_2+\lambda_3)} \mathbf{r} \lambda_2^2 \lambda_3 \xi^2 + \\
& 18 e^{-T(\lambda_2+\lambda_3)} \mathbf{h}^T \lambda_2^2 \lambda_3 \xi^2 - 12 e^{-T(\lambda_2+\lambda_3)} \mathbf{r}^T \lambda_1 \lambda_2^2 \lambda_3 \xi^2 - \\
& 18 e^{-T(\lambda_2+\lambda_3)} \mathbf{h}^T \lambda_1 \lambda_2^2 \lambda_3 \xi^2 + 8 e^{-T(\lambda_2+\lambda_3)} \mathbf{r}^T \lambda_2^3 \lambda_3 \xi^2 + \\
& 12 e^{-T(\lambda_2+\lambda_3)} \mathbf{h}^T \lambda_2^3 \lambda_3 \xi^2 + 8 e^{-T(\lambda_2+\lambda_3)} \mathbf{h} \lambda_3^2 \xi^2 - \\
& 12 e^{-T(\lambda_2+\lambda_3)} \mathbf{h}^T \lambda_1 \lambda_3^2 \xi^2 + 2 e^{-T(\lambda_2+\lambda_3)} \mathbf{r}^T \lambda_1^2 \lambda_3^2 \xi^2 + \\
& 3 e^{-T(\lambda_2+\lambda_3)} \mathbf{h}^T \lambda_1^2 \lambda_3^2 \xi^2 - 6 e^{-T(\lambda_2+\lambda_3)} \mathbf{r} \lambda_2 \lambda_3^2 \xi^2 + \\
& 18 e^{-T(\lambda_2+\lambda_3)} \mathbf{h}^T \lambda_2 \lambda_3^2 \xi^2 - 12 e^{-T(\lambda_2+\lambda_3)} \mathbf{r}^T \lambda_1 \lambda_2 \lambda_3^2 \xi^2 - \\
& 18 e^{-T(\lambda_2+\lambda_3)} \mathbf{h}^T \lambda_1 \lambda_2 \lambda_3^2 \xi^2 + 12 e^{-T(\lambda_2+\lambda_3)} \mathbf{r}^T \lambda_2^2 \lambda_3^2 \xi^2 + \\
& 18 e^{-T(\lambda_2+\lambda_3)} \mathbf{h}^T \lambda_2^2 \lambda_3^2 \xi^2 - 2 e^{-T(\lambda_2+\lambda_3)} \mathbf{r} \lambda_3^3 \xi^2 + \\
& 6 e^{-T(\lambda_2+\lambda_3)} \mathbf{h}^T \lambda_3^3 \xi^2 - 4 e^{-T(\lambda_2+\lambda_3)} \mathbf{r}^T \lambda_1 \lambda_3^3 \xi^2 - \\
& 6 e^{-T(\lambda_2+\lambda_3)} \mathbf{h}^T \lambda_1 \lambda_3^3 \xi^2 + 8 e^{-T(\lambda_2+\lambda_3)} \mathbf{r}^T \lambda_2 \lambda_3^3 \xi^2 + \\
& 12 e^{-T(\lambda_2+\lambda_3)} \mathbf{h}^T \lambda_2 \lambda_3^3 \xi^2 + 2 e^{-T(\lambda_2+\lambda_3)} \mathbf{r}^T \lambda_3^4 \xi^2 + \\
& 3 e^{-T(\lambda_2+\lambda_3)} \mathbf{h}^T \lambda_3^4 \xi^2 \Big) + e^{\frac{T(\gamma_1+\gamma_2)(\lambda_2+\lambda_3)}{\gamma_2+\xi}}
\end{aligned}$$

$$\begin{aligned}
& \left( e^{-T\lambda_1} \left( \frac{e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{h}^T (\gamma_1 + \gamma_2) (\gamma_2 + \xi)}{(\lambda_2 + \lambda_3) (-\gamma_2 \lambda_1 + \gamma_1 \lambda_2 + \gamma_2 \lambda_2 + \gamma_1 \lambda_3 + \gamma_2 \lambda_3 - \lambda_1 \xi)} - \right. \right. \\
& \left. \left( -e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{h} \gamma_2^4 \lambda_1 + e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{h} \gamma_1 \gamma_2^3 \lambda_2 + e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{h} \right. \right. \\
& \left. \left. \gamma_2^4 \lambda_2 + e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{h} \gamma_1 \gamma_2^3 \lambda_3 + e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{h} \gamma_2^4 \lambda_3 - \right. \right. \\
& \left. \left. 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{h} \gamma_2^3 \lambda_1 \xi + e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{r} \gamma_2^3 \lambda_1 \lambda_2 \xi - \right. \right. \\
& \left. \left. e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{r} \gamma_1 \gamma_2^2 \lambda_2^2 \xi - e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{r} \gamma_2^3 \lambda_2^2 \xi + \right. \right. \\
& \left. \left. e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{r} \gamma_2^3 \lambda_1 \lambda_3 \xi - 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{r} \gamma_1 \gamma_2^2 \lambda_2 \lambda_3 \xi - \right. \right. \\
& \left. \left. 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{r} \gamma_2^3 \lambda_2 \lambda_3 \xi - e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{r} \gamma_1 \gamma_2^2 \lambda_3^2 \xi - \right. \right. \\
& \left. \left. e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{r} \gamma_2^3 \lambda_3^2 \xi - 3 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{h} \gamma_1 \gamma_2 \lambda_2 \xi^2 - \right. \right. \\
& \left. \left. 3 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{h} \gamma_2^2 \lambda_2 \xi^2 + 3 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \mathbf{r} \gamma_2^2 \lambda_1 \lambda_2 \xi^2 + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_2^2 \lambda_1 \lambda_2 \xi^2 - 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_1 \gamma_2 \lambda_2^2 \xi^2 - \\
& e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_1 \gamma_2 \lambda_2^2 \xi^2 - 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2^2 \lambda_2^2 \xi^2 - \\
& e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_2^2 \lambda_2^2 \xi^2 - 3 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_1 \gamma_2 \lambda_3 \xi^2 - \\
& 3 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_2^2 \lambda_3 \xi^2 + 3 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2^2 \lambda_1 \lambda_3 \xi^2 + \\
& e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_2^2 \lambda_1 \lambda_3 \xi^2 - 4 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_1 \gamma_2 \lambda_2 \lambda_3 \xi^2 - \\
& 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_1 \gamma_2 \lambda_2 \lambda_3 \xi^2 - 4 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2^2 \lambda_2 \lambda_3 \xi^2 - \\
& 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_2^2 \lambda_2 \lambda_3 \xi^2 - 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_1 \gamma_2 \lambda_3^2 \xi^2 - \\
& e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_1 \gamma_2 \lambda_3^2 \xi^2 - 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2^2 \lambda_3^2 \xi^2 - \\
& e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_2^2 \lambda_3^2 \xi^2 + 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_2 \lambda_1 \xi^3 - \\
& 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_1 \lambda_2 \xi^3 - 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_2 \lambda_2 \xi^3 + \\
& 3 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2 \lambda_1 \lambda_2 \xi^3 + 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_2 \lambda_1 \lambda_2 \xi^3 - \\
& e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_1 \lambda_2^2 \xi^3 - e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_1 \lambda_2^2 \xi^3 - \\
& e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2 \lambda_2^2 \xi^3 - e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_2 \lambda_2^2 \xi^3 - \\
& 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_1 \lambda_3 \xi^3 - 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_2 \lambda_3 \xi^3 + \\
& 3 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2 \lambda_1 \lambda_3 \xi^3 + 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_2 \lambda_1 \lambda_3 \xi^3 - \\
& 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_1 \lambda_2 \lambda_3 \xi^3 - 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_1 \lambda_2 \lambda_3 \xi^3 - \\
& 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2 \lambda_2 \lambda_3 \xi^3 - 2 e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_2 \lambda_2 \lambda_3 \xi^3 - \\
& e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_1 \lambda_3^2 \xi^3 - e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_1 \lambda_3^2 \xi^3 - \\
& e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2 \lambda_3^2 \xi^3 - e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_2 \lambda_3^2 \xi^3 + \\
& e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \lambda_1 \xi^4 + e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \lambda_1 \lambda_2 \xi^4 + e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \lambda_1 \\
& \lambda_2 \xi^4 + e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \lambda_1 \lambda_3 \xi^4 + e^{-\frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \lambda_1 \lambda_3 \xi^4 \Big) / \\
& \left( (\lambda_2 + \lambda_3)^2 \xi (-\gamma_2 \lambda_1 + \gamma_1 \lambda_2 + \gamma_2 \lambda_2 + \gamma_1 \lambda_3 + \gamma_2 \lambda_3 - \right. \\
& \left. \lambda_1 \xi) (\gamma_2 \lambda_1 - \gamma_1 \lambda_2 - \gamma_2 \lambda_2 - \gamma_1 \lambda_3 - \gamma_2 \lambda_3 + \lambda_1 \xi) \right) + \\
& e^{-T \lambda_1 + T(\lambda_2 + \lambda_3)} \left( - \left( e^{-T(\lambda_2 + \lambda_3)} - \frac{T(\lambda_2 + \lambda_3)\xi}{\gamma_2 + \xi} h T (\gamma_1 + \gamma_2) (\gamma_2 + \xi) \right) / \right)
\end{aligned}$$



$$\begin{aligned}
& 2 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_1 \lambda_2 \xi^3 + 4 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \\
& h \gamma_2 \lambda_2 \xi^3 - 3 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2 \lambda_1 \lambda_2 \xi^3 - \\
& 2 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_2 \lambda_1 \lambda_2 \xi^3 + e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \\
& r \gamma_1 \lambda_2^2 \xi^3 + e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_1 \lambda_2^2 \xi^3 + \\
& 4 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2 \lambda_2^2 \xi^3 + 3 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \\
& h T \gamma_2 \lambda_2^2 \xi^3 + 2 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_1 \lambda_3 \xi^3 + \\
& 4 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \gamma_2 \lambda_3 \xi^3 - 3 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \\
& r \gamma_2 \lambda_1 \lambda_3 \xi^3 - 2 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_2 \lambda_1 \lambda_3 \xi^3 + \\
& 2 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_1 \lambda_2 \lambda_3 \xi^3 + 2 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \\
& h T \gamma_1 \lambda_2 \lambda_3 \xi^3 + 8 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2 \lambda_2 \lambda_3 \xi^3 + \\
& 6 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_2 \lambda_2 \lambda_3 \xi^3 + e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \\
& r \gamma_1 \lambda_3^2 \xi^3 + e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \gamma_1 \lambda_3^2 \xi^3 + \\
& 4 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \gamma_2 \lambda_3^2 \xi^3 + 3 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \\
& h T \gamma_2 \lambda_3^2 \xi^3 - e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \lambda_1 \xi^4 + \\
& e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \lambda_2 \xi^4 - e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \lambda_1 \lambda_2 \xi^4 - \\
& e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \lambda_1 \lambda_2 \xi^4 + e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \lambda_2^2 \\
& \xi^4 + e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \lambda_2^2 \xi^4 + e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h \\
& \lambda_3 \xi^4 - e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \lambda_1 \lambda_3 \xi^4 - e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \\
& h T \lambda_1 \lambda_3 \xi^4 + 2 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} r \lambda_2 \lambda_3 \xi^4 + \\
& 2 e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \lambda_2 \lambda_3 \xi^4 + e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} \\
& r \lambda_3^2 \xi^4 + e^{-T(\lambda_2+\lambda_3) - \frac{T(\lambda_2+\lambda_3)\xi}{\gamma_2+\xi}} h T \lambda_3^2 \xi^4 \Big) / \\
& \left( (\lambda_2 + \lambda_3)^2 \xi (\gamma_2 \lambda_1 - \gamma_1 \lambda_2 - 2 \gamma_2 \lambda_2 - \gamma_1 \lambda_3 - \right. \\
& \left. \left. \left. 2 \gamma_2 \lambda_3 + \lambda_1 \xi - \lambda_2 \xi - \lambda_3 \xi \right)^2 \right) \right) \Big) \Big)
\end{aligned}$$

## CHAPTER IV

# Contrasting paradigms for production control, cross-training, and compensation

### 4.1 Introduction

Competition on a global scale over the last half of a century has caused a shift in our American economy away from production where we make products, towards the service sector. The conventional wisdom at one time was that this shift was necessary as it defined American growth, ingenuity, and power. In fact, as late as 1985 in a report to Congress our country's leader President Ronald Reagan stated that, "The move from an industrial society toward a post-industrial service economy has been one of the greatest changes to affect the developed world since the Industrial Revolution. The progression of an economy such as America's from agriculture to manufacturing to services is a natural change." Unfortunately, the negative side effects of this "natural change" have had broad effects throughout various aspects of our economy. A few of these side effects include: growing concern from the poor safety regulations and port regulations of our imported food, domestic small businesses forced out of business as they are unable to stay cost competitive against international competition, and the tragic loss of jobs for our skilled labor workforce as many companies are outsourcing their jobs to the less expensive international workforce.

If American companies continue to internationally outsource more jobs and production, we will likely see the amount of our imports increase dramatically. There is an inherent danger for any country as their imports increase, because they can become severely dependent on other countries for their existence. For example, consider America's current dependence on other countries as it relates to petroleum imports. Presently our dependence on foreign oil is often times referred to in the news media as a "clear and present danger". Moreover, in addition to the effects of increased importing, many high paying service jobs, (e.g. engineers, consultants, etc.), are inextricably tied to production jobs. Therefore as we see a loss of jobs among blue collar workers, we will also see a loss of jobs among white collar workers. These occurrences highlight, for American society, a need for a generational paradigm shift whereby we raise and educate a new generation that not only understands the need for manufacturing and production, but also develops a genuine interest in their core ideas and concepts. Learning the core concepts of manufacturing and production helps to nurture interest in those fields and provides a foundation from which a new generation can expand the state of the art within those fields. To facilitate this education, at high school levels and beyond, there exists a need for simple yet effective learning tools in the classroom that support these interests. This work addresses this need through the development of the worker agility game (WAG), which offers a creatively designed tool that teaches the value of workforce flexibility in production systems.

In the literature, flexibility has been researched as it relates to manufacturing [56], production [21], process [59], and the workforce [64]. Workforce flexibility is one important concept within manufacturing and production; but in each of these areas, flexibility relates to increasing a system's capacity to efficiently improve responsiveness and the decision making of that system in the presence of uncertainty. Workforce agility speaks to the increased capacity that a particular system can gain from its workforce via cross-training (See Figure 4.1). The impact of an agile work-

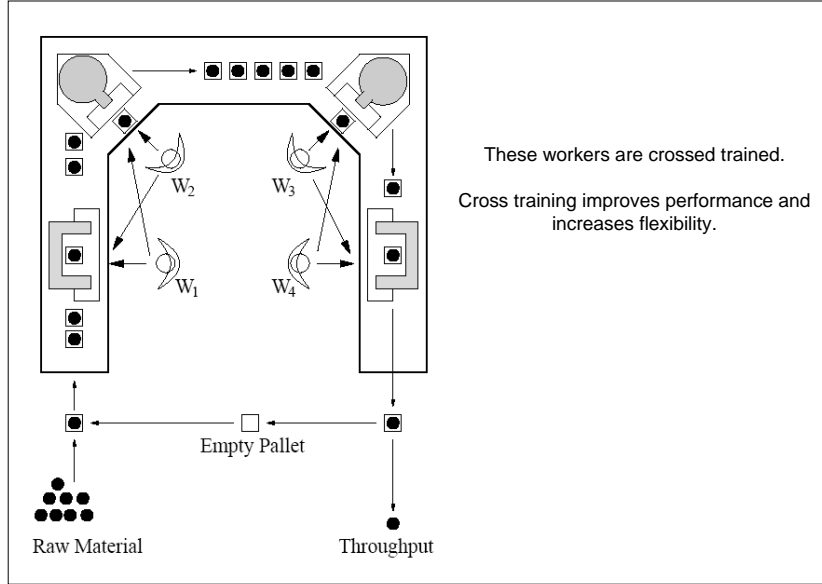


Figure 4.1: A Cross-trained Workforce

force and its positive effects have been thoroughly researched and documented for various systems (see Hopp and VanOyen [54], Gurusurthi and Benjaafar [44], and Andradottir, Ayhan, and Down [4]).

WAG is an example of a “simple game” for teaching described by Heineke and Meile [46] where they stated that “many complex and important concepts (of operations management) could be demonstrated in a simple game”. This activity offers a hands-on, active learning experience placing students “on the job” in a serial production line of cross-trained workers where participants can: 1) learn basic concepts of operations management, production control, and workforce agility; 2) understand system responsiveness and what can be done to improve it; 3) generate creative thinking and discussion on the value of flexibility; and 4) see first hand foundational factory physics concepts like cycle time, throughput, and Work In Process (WIP). In WAG, participants form teams and work together as cross-trained workers on a serial production line to create a product from raw materials to a finished good. Each team



forms a different worker cross-training paradigm for its production line, and simple time studies are completed of worker efficiency to ascertain how the cross-training policies work in comparison to each other. From two live implementations of WAG, one with tenth grade high school students and the other with senior level undergraduate and master's degree students at the authors' institutions, its effectiveness and impact on the students understanding was quite impressive. The students described in post-activity discussions how their understanding of worker cross-training had increased and the value of flexibility in production systems.

In this paper we will describe the WAG game, its relevance to the latest workforce agility literature, and its usefulness for classroom instruction. The remainder of this paper is organized as follows: Section 4.2 will explain the concepts behind the WAG game as well as survey the literature which supports the game; Section 4.3 describes how the game was developed and the simplicity of the tools needed for implementation; Sections 4.4 and 4.5 provide the necessary materials for instructors and students respectively; and Section 4.6 concludes with suggestions for future work and WAG expansion.

## **4.2 Concepts behind WAG**

Comprehensive surveys that examine the workforce agility literature have been supplied by Sethi and Sethi [79], Hopp and Van Oyen [54], and Treleven [86]. Each considers both serial and parallel production lines. For the purposes of this work we will confine our scope to workforce agility as it relates to serial production lines. More to the point, specific to this work there are four agile workforce coordination policies for serial lines which describe how workers on the production line are to complete their work: pick and run, bucket brigade, two skill chain, and worksharing with overlapping zones. These policies are used in WAG to highlight the basic concepts of production control in workforce agility. First we will detail these agile workforce

policies and the corresponding literature that has studied them. Then we will outline those basic concepts of operations management which are prevalent in each policy. For each policy, as it relates to WAG, consider a four station serial production line with four workers. An entity begins as raw materials at station one, and proceeds sequentially through to station four where it is completed as a finished good.

#### **4.2.1 The pick and run policy**

The pick and run policy is implemented to illustrate the case where all the workers on a line are fully cross-trained to complete all task-types at each station on the line. Given this full cross-training a worker can “pick” up an entity as raw materials at station one and process the entity through all stations until completion (“run”). Once the entity has been completed the worker can return to station one to repeat the process. Thus, fully cross-trained workers in this policy individually handle entities from start to finish and interaction between workers is non-existent. An example of this is the sandwich maker in a Subway restaurant during slow periods [54]. The pick and run was studied by both Iyer and Askin [58] and Van Oyen, Gel, and Hopp [89] who found it to be near optimal for demand constrained environments. However, in environments where there is, at a minimum, always one job per worker under constant WIP the pick and run was found to be optimal. Ahn and Righter [1] generalized results on the optimality of the pick and run under certain conditions in serial manufacturing systems.

#### **4.2.2 The bucket brigade**

The bucket brigade is similar to the pick and run in that a worker processes an entity at each sequential station, but only until the worker is either bumped or blocked. A worker is bumped when the entity they are processing is taken from them by an idle downstream worker. This process is known as entity preemption, and it

is enabled by full cross-training of the workers. Blocking occurs when a worker cannot process an entity at a given station because that station is being occupied by another worker. Varying worker speeds or station processing times where a downstream worker (or station) is slower than their adjacent upstream counterpart will cause blocking. Interaction between workers is significant with bucket brigades which is one of the main points of differentiation from the pick and run. The seminal work on bucket brigades is Bartholdi and Eisenstein [13]. They demonstrated that when workers are sequenced from slowest to fastest, with deterministic processing times, a bucket brigade will always divide work optimally and the line will balance itself. (Note that Bischack [20] and McClain, Schultz, and Thomas [63] consider similar systems with random processing times.) Further analysis on bucket brigades was also completed by Bartholdi and Eisenstein (see [14] and [15].)

### **4.2.3 The two skill chain**

The two skill chain considers each station of the serial production line to be a skill that a worker must learn to be able to process entities at that station. Each worker is then cross-trained on two skills in such a way that they “complete the chain”. This means that the skills of each worker are indirectly linked to provide worker skill set coverage for all stations. For example, consider four skills (S1, S2, S3, S4) and four workers (W1, W2, W3, W4). To complete the chain for a two skill chain, the workers would be cross-trained on skills as follows: W1 - (S1, S2); W2 - (S2, S3); W3 - (S3,S4); W4 - (S4, S1). In this configuration, the skills of the workers overlap in such a way that each worker can either directly or indirectly provide assistance to all other workers. Hopp, Tekin, and Van Oyen [54] introduced the concept of two skill chaining in serial production lines based upon the capacity chaining ideas of Jordan and Graves [59]. In their work they showed that the two skill chain was highly effective when idle workers served the maximum queue of their two skills. Jordan,

Inman, and Blumenfield [60] applied it to automotive production lines focused on maintenance and repair and were able to show the robustness of chaining.

#### 4.2.4 Worksharing with overlapping zones

Worksharing with overlapping zones classifies a workers skill set as a zone and neighboring workers are cross-trained so that they can share responsibility for machines at stations where the zones overlap (see McClain, Schultz, and Thomas [64]). The efficiency of serial production lines is greatly increased by using overlapping zones, thereby avoiding worker idle time. The conditions of zone overlap generally include systems with more stations than workers and cross-trained workers whose skills connect adjacent tasks between two zones. The importance of worksharing with overlapping zones is that it can limit inexpensive cross-training in serial lines. This reduced expense for training is a result of the fact that the number of skills that must be cross-trained with zones is much smaller than full cross-training of all workers since skills are only cross-trained at the overlapping portions of the zones. Williams et al. [90] introduced a new paradigm in worksharing with overlapping zones by merging together the concepts of overlapping zones and completing the chain. Their model for zone chaining is called the fixed task zone chain and it demonstrated the cost benefits of overlapping zones by performing quite robustly against models with more extensive cross-training.

In addition to these agile workforce policies there are also several basic operations management concepts that WAG participants will become familiar with and learn through participation in the activity. The details for all of these concepts can be found in Hopp and Spearman [52], however for completeness we will detail their connection to WAG.

The most fundamental operations management principle that WAG participants

will learn is the relationship between work in process (WIP), cycle time (CT), and throughput (TH). This fundamental relationship,  $WIP = TH * CT$ , is known as Little's Law. Interestingly enough, without ever studying queueing theory or learning Little's Law many of our WAG participants were able to understand the intuition described by Little's Law simply by participating in WAG. Upon completion of the WAG activity participants are able to calculate the average CT of their production line and then discuss issues of line efficiency in terms of worker utilization. This discussion of line efficiency allows the participants to speak more analytically about how well they believed their line performed, and what steps they would take to improve it. For example, worker utilization varies by team depending on the cross-training paradigm implemented for that specific team. For example, by being directly involved in the production, participants can speak to whether they experienced idleness waiting for their adjacent upstream worker to finish a task-type, or whether WIP accrued at the buffer at their station based upon their high utilization. This discussion on high utilization introduces the idea of unstable queues and other simple queueing theory concepts.

As participants discuss utilization the conversation focuses on the type of system a production line was: push versus pull. Pull systems involve adjacent downstream workers pulling work from upstream workers. If the downstream worker is cross-trained on the same skill set as the upstream worker then the downstream worker can pull incomplete entities, and thus employ the concept of preemption discussed with bucket brigades. Allowing participants to actually engage in preemption during the activity gives them hands on experience with this concept. Push systems allow workers to work as fast as they can on as much product as they can, and once they have completed service on an entity they can then push it into the queue of the adjacent downstream worker. When comparing and contrasting worker speeds, participants are able to recall how faster workers in pull systems could use preemption to assist

slower workers, however in push systems faster workers often times found themselves idle and under-utilized.

The final operations management concept that WAG participants are able to receive is that of worker compensation. Although worker compensation is not directly a part of the WAG activity, in the post-WAG discussion participants engage in lively discussions on how peer pressure and perceptions impacted their beliefs about effective pay incentives. Participants are able to consider, based upon their production, how they would want to be compensated and whether that compensation should be independent of or dependent on team productivity and performance. As can be expected, the better performing teams are more inclined to want team productivity based compensation and those teams are the ones with more flexibility. Participants gain an understanding as to how increased flexibility yields increased throughput for their production lines and thus with productivity based compensation they are able to learn about a manager's struggle to align the incentives of production with those of the employees. WAG makes for a rich discussion on these concepts and allows participants to gain a deeper understanding and interest into some components of an agile workforce in production.

### **4.3 Game development**

WAG was developed to fill the need for a workforce agility learning activity that is simple, yet effective. The cutting edge cross-training paradigms introduced in [54] and [90] can be communicated to senior level undergraduates and masters students who needed practical understanding of workforce flexibility prior to entering the engineering workforce. WAG closes the education gap that the authors found to exist. WAG has generated the type of thought provoking discussion amongst the students that leads to interest, competence, and innovation in the field.

WAG was designed to simulate a serial line production facility that used cross-

trained workers to create fruit baskets. For the purposes of classroom instruction, the simulated facility makes fruit baskets through stencil tracing on a piece of paper, called a job sheet. Each piece of paper that moves through the line represents one fruit basket and denotes on it the number of apples, pears, bananas, and cherries to be traced. The number of each fruit to be traced is randomly assigned for each job sheet making each fruit basket unique. The activity consists of multiple production lines that each make these fruit baskets where the differences among the lines are the cross-training policies (detailed in Section 4.2) to be used. Each production line consists of six participants who each hold one of three positions: release agent, line worker, or finished goods inventory supervisor.

At the beginning of each line the release agent holds all of the raw materials, which are the job sheets, and distributes them to line according to the job release instructions for that particular line. The release agent demonstrates to the participants how the flow of raw materials introduced into a production line is controlled. Next to the release agent, in sequential order, are four line workers, each with a unique skill set. The skill set of a worker is determined by which fruit they are skilled (i.e. permitted) to trace on the job sheet. Each of the fruits is traced with a different color, so a worker's skill set is identified by the set of colored markers they are given to trace with. For example, a worker who is given a red and a green marker has the skill set for two fruits, apples and pears, and can only trace apples and pears on a job sheet. Once the workers understand their skill sets, based upon the markers given, they are also told the type of system they are working in, including whether they can push, pull, or preempt work from upstream workers on the line with the same skill set. The final participant on the line is the finished goods inventory supervisor. The finished goods inventory supervisor oversees the work of the line, ensuring that workers are staying within their assigned skill set. Once a job sheet is completed, it is given to the finished goods inventory supervisor to be checked off when completed.

The finished goods inventory supervisor, at the end of the exercise, is responsible for tallying the total amount of work completed, the start and stop time of production, and the average WIP, TH, and CT.

The six participants of one production line make up a team. The activity consists of five teams with the following cross-training policies: 1) Team 1 is specialized - each worker only knows 1 skill. This is a traditional assembly line with a push work system. 2) Team 2 is a two skill chain, push work system, where each worker knows 2 skills, and the skills are assigned to the workers to “complete the chain”; 3) Team 3 is a pick and run pull system where all workers act independently (each worker knows all four skills). 4) Team 4 is a specialized push system, exactly like Team 1, except they are under an 8 job sheet CONWIP release policy; 5) Team 5 is a bucket brigade pull system where each worker knows the skills up to and including the station where they are assigned (e.g. W2 - (1,2), W4 - (1,2,3,4)); 6) Team 6 is a fixed task zone chain where each worker is assigned a zone of 3 stations where the skill for 2 of the stations is shared by an adjacent worker and 1 station is fixed meaning only that worker has the skill for that station. For each team the release agent and the finished goods inventory supervisor have the same job function.

Game play was devised to begin with a pre-game discussion over the basic instructions for each team. There is some vocabulary that may require explanation to the participants (see Section 4.4). Once participants understand their role, the pre-game discussion consists of student hypotheses about which team would be the most accurate and efficient. The participants justification for their hypotheses begins the process for them to learn the basic concepts of workforce agility. For example, most participants can immediately identify where bottlenecks will form within the specialized teams and how by knowing only one skill those teams are at a decided efficiency disadvantage to the cross-trained teams. Upon conclusion of the pre-game discussion the same number of randomly generated (to provide variability in job pro-



cess times) sheets is given to each team. The objective for each team is to complete their job sheets as accurately, quickly, and efficiently as possible.

The post game discussion has three main discussion topics: 1) calculations, 2) results analysis, and 3) worker incentives. Upon completion of the exercise the finished goods inventory supervisor will calculate on an inventory sheet the average WIP, TH, and CT for his/her team. These calculations allow the participants to receive a first hand understanding of the relationship Little's Law describes. The participants ability to actually see Little's Law at work during the simulated activity, prior to completing (or even perhaps learning) the calculation gives them a deeper understanding of the relationship once the calculation is done. Intuitively the participants generally understand that one method of increasing their throughput, and overall team performance, is to decrease their cycle time. As the discussion moves towards results analysis, ideas on how to accomplish this goal clearly demonstrate the participants comprehension of the value of flexibility.

Without knowledge of the performance of the other teams, participants are challenged to discuss why their team performed as well, or not so well, as they did. This challenge requests that the students consider their production line configuration and critically think about the pros and cons of their configuration and what they believe can be done to improve it. Further, participants consider the factors of the line that they cannot change such as worker speed (invariably one person will trace a stencil more accurately and quickly than another). Given that not all workers were standard, i.e., possessing the same speed and skill set, the value of how flexibility allows workers to directly or indirectly assist each other is shown. Finally, the participants discuss worker incentives as it relates to compensation. Participants are asked to consider three different compensation structures and which they would prefer for the cross-training paradigm in which they participated. The first compensation structure is a flat hourly wage that is independent of work completed and team performance.

The second compensation structure reduces the flat hourly wage of the first level, but then adds a group bonus based upon team productivity. Finally, the third compensation structure focuses solely on a commission that is based on team performance - the number of jobs completed. This conversation about compensation level is intended to drive the participants to think about improving the value of cross-training by aligning the incentives of the worker with the production goals of the line.

WAG was designed to provide the members involved in the activity with broad perspectives on cross-training. The activity itself provides the foundation upon which the pre and post game discussions can build an introductory, yet solid foundation of workforce agility as it relates to operations management. Moreover, the authors have been able to use these discussions to guide interested participants into degree fields (for high school students) and specific coursework (for undergraduate students) that will broaden and deepen their understanding of the concepts they learned through the activity. This learning tool does not replace standard classroom instruction, however it has been shown to greatly motivate, enhance, and support it.

#### **4.4 Materials for instructors**

This section outlines the materials that were developed for instructors or WAG facilitators and consists of three sub-sections including: 1) Game Instructions; 2) Examples; and 3) Software Instructions to use the Excel VBA game application to generate jobs for use in play.

#### 4.4.1 Game Instructions

Game Instructions

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## I. Overview

### A. Objectives and Expectations

The objective of the Worker Agility Game (WAG) is to involve participants as cross-trained workers in an organized, simulated serial production line. Through this hands-on experience we hope they will better understand basic concepts of operations management, production control, workforce agility, and the importance of manufacturing finished goods from raw materials. Participants will be actively involved in the transformation of raw materials into finished goods and will experience some elements of queueing theory as well as the benefits of cross-training. By being directly involved in how serial production lines can be organized, participants will explore theories of efficiency, cycle time, throughput, and compensation. The simulation of a production facility will help clarify concepts and help create educational connections for the participants.

### B. Game Requirements

To play a minimum of 36 participants is needed, broken into 6 teams (numbered 1 through 6) of 6 players each. If there are more than 36 participants, teams can be repeated (i.e. there can be two Team 1's or two Team 4's, etc.) A set of job sheets must be created for game play, and the number of job sheets determines the length of play. Once a set of job sheets is created, that set can be copied with each team receiving its own individual set. Lastly, 16 sets of colored markers are needed with each set consisting of a red, green, yellow, and blue markers. The sets of markers are distributed to the teams as follows: Teams 1 and 4 receive 1 set each, Team 2 receives 2 sets, and Teams 3,5, and 6 receive 4 sets each.

### C. Timing Guidelines

To plan the time for WAG, please consider the following four stages of the game: setup, pre-game discussion, WAG activity, post-game discussion. Setup time is minimal and truly depends on the number of teams as setup only involves providing the teams with their instructions, job sheets, and markers. The pre-game discussion generally takes about 20 minutes to review the game assignments and provide introductory information. We have found that allotting 1 minute per job sheet is a good estimation to determine how many job sheets to create for game play. For example, if you wish the actual activity to last 20 minutes then each team should be provided a set of 20 job sheets. The post-game discussion takes a minimum of approximately 30 minutes, however it can be shortened or lengthened according to the instructor's time constraints. In total, the average WAG experience was designed to take approximately 90 minutes (one class period) to complete.

## I. How to Play

### A. Task Description

The 6 participants on a team include one: release agent (RA), one finished goods inventory (FG), and four line workers (LW).

The RA holds the raw materials, which are the uncompleted job sheets, records the start time of production, and distributes the jobs to the team according to the job distribution rules of that team. A record of each job that is released is to be kept by the RA on the WAG Inventory Record Sheet that is maintained by the RA.

The FG collects the completed jobs from the production line and places them in inventory. Prior to placing the job sheet into inventory the FG completes a quality control check of the job sheet to ensure accuracy and completeness. Any job sheet that is found to fail quality control inspection is discarded. A record of each completed job, as well as the end time of production, is to be maintained by the FG on the WAG Inventory Sheet that is maintained by the FG, which is different from the sheet maintained by the RA. At the conclusion of the activity the FG is responsible for the

throughput (TH), cycle time (CT), and Work In Process (WIP) calculations for their team.

The LW's perform the actual production work of the production line. Each team in the activity is responsible for producing fruit baskets by tracing the predetermined number of pieces of fruit on the job sheet. On each job sheet there are four different pieces of fruit that can be traced: apples, pears, bananas, and cherries. Each piece of fruit must be traced in a different color. Each LW is given one or more marker colors, each color represents one skill in the skill set of a particular LW. LW's trace the fruit according the skills they've been assigned, ensuring to do "quality at the source" checks of their work as one element of quality control. Poorly traced pieces of fruit, or an incorrect number of traced pieces of fruit will fail the quality control inspection of the FG and thus cause that job sheet to be discarded. Each line worker may vary in the number of skills that they have. The four possible skills are:

Skill 1 - Trace with a red marker the number of apple outlines as specified next to the row of apples on the job sheet.

Skill 2 - Trace with a green marker the number of pear outlines as specified next to the row of pears on the job sheet.

Skill 3 - Trace with a yellow marker the number of banana outlines as specified next to the row of bananas on the job sheet.

Skill 5 - Trace with a blue marker the number of cherries outlined as specified next to the row of cherries on the job sheet.

## B. Production Line Method Description

### 1. Push vs. Pull

Push or pull describes the way in which jobs move down the production line between adjacent workers. Upstream and downstream describe the relative positions of one work to another (See Figure 4.2). A worker  $i$  is considered to be

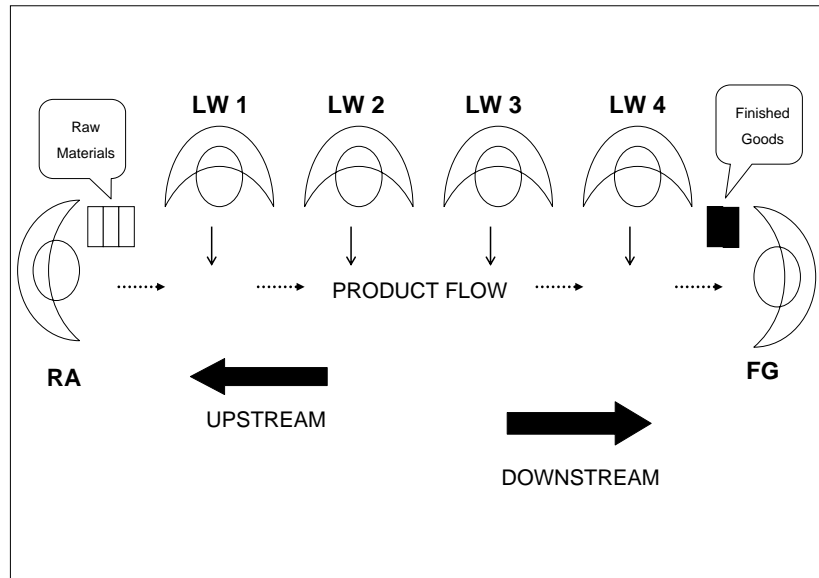


Figure 4.2: Upstream vs. Downstream

upstream(downstream) of an adjacent worker  $i + 1$  if they are closer to the beginning(end)of the line (i.e. closer to the RA (FG)). For any given team the method of job movement through the line is push if the upstream line worker with task  $n$  must push the job down the line to the adjacent downstream worker with task  $n + 1$ . If the job is not pushed, then it does not move. Conversely, if the downstream worker can take or pull the job from the upstream worker then the job movement through the line is a pull system. Given these two descriptions of job flow movement, each team must follow their unique set of instructions.

## 2. Team Configuration Descriptions

The cross training configuration of each team is unique and includes with it instructions for how jobs are released to the line and how jobs flow through the

line. Each team must follow their unique set of instructions.

### Team 1 - Specialized production

Team 1, see Figure 4.13, is a specialized, traditional assembly line where each line

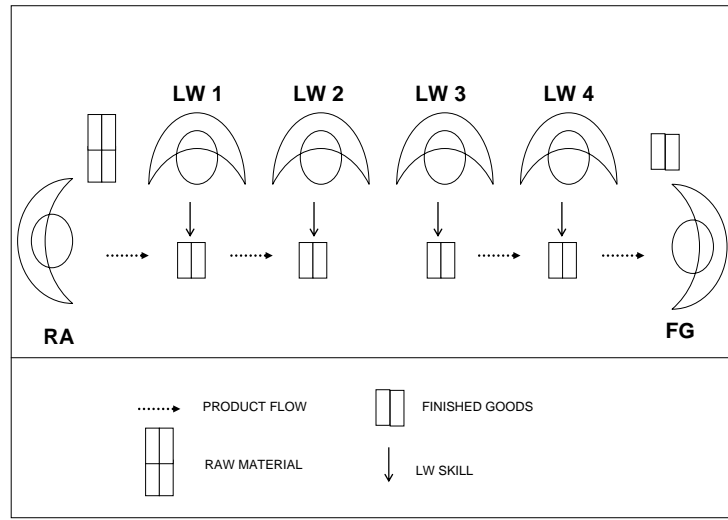


Figure 4.3: Team 1 - Specialized workers

worker is trained on one skill and jobs are pushed through the line from an upstream worker to their adjacent downstream worker. In the event an upstream worker has completed their task, is ready to push a job downstream, and the downstream worker has not completed their task; then a queue of uncompleted work builds between those two workers. The downstream worker removes jobs from the queue following a first in, first out policy.

The RA releases jobs only to LW 1 and the skills of the LW's are assigned as: LW 1 - Skill 1; LW 2 - Skill 2; LW 3 - Skill 3; LW 4 - Skill 4. The RA does not hold jobs if queues between workers are building, therefore each time LW 1 is idle they are given a new job sheet. The tasks for all jobs must be completed in sequential order with skill 1 being completed first and skill 4 being completed last. When a job is complete



LW 4 gives the job to the FG.

Team 2 - Two skill chaining

Team 2, see Figure 4.14, has a 2-skill chain configuration where each worker is cross-trained in two skills. The skills are cross-trained to “complete the chain”, which

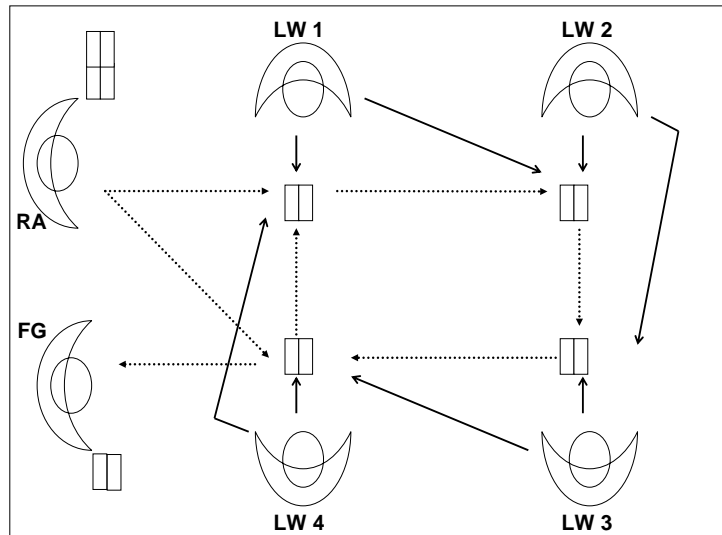


Figure 4.4: Team 2 - Two skill chaining

means that all skills are either directly or indirectly linked by the workers. Each skill has exactly two workers that are able to complete it. The skills are assigned to the LW’s as follows: LW 1 - Skills 1 and 2; LW 2 - Skills 2 and 3; LW 3 - Skills 3 and 4; LW 4 - Skills 4 and 1. Given this configuration, it is advantageous to seat the students in a U-shape so LW 1 and LW4 may assist each other as needed.

Each LW  $n$ , labeled  $n = 1, 2, 3$  does the following: LW  $n$  - Do skill  $n$ , in sequential order of skills 1 through 4, and then either do skill  $n+1$  if the adjacent downstream LW is busy or give the job, for skill  $n + 1$ , to the downstream LW if they are idle. LW 4 is to prioritize skill 4 and the job sheets in their queue before assisting LW 1. The RA releases new jobs to LW 1 or LW 4 only when they are idle. The FG can

only receive completed jobs from LW 4.

#### Team 3 - Craft production with pick and run

Team 3, see Figure 4.15, is a “pick and run” system where all LW’s operate independently because they each are cross-trained on all four skills. Each LW begins with

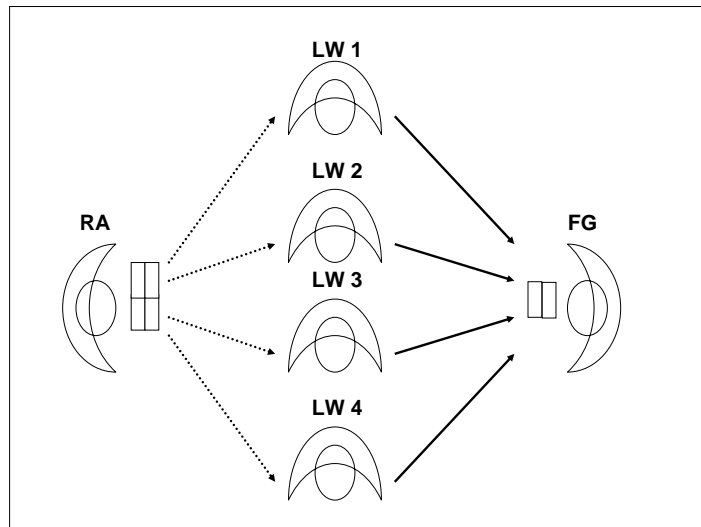


Figure 4.5: Team 3 - Craft production with pick and run

a fresh job sheet and completes all four skills on the job sheet themselves without interaction with the other LW’s on the production line. Once a job is complete the LW’s hand their jobs off to the FG and return to the RA to get a fresh job. Thus, the FG can receive a completed job from any LW and the RA can release a job to any LW. At any given time there should always be 4 jobs in WIP since there is no interaction between workers and workers do not have idle time.

#### Team 4 - Specialized production under a CONWIP release policy

The configuration of team 4 is exactly like the configuration of team 1. This is a specialized team where all LW’s are trained on only 1 skill. The difference between teams 1 and 4 is that team 4’s job release policy is governed by having a CONstant Work In Process (CONWIP) of 8 jobs. Initially, at the beginning of the activity the

RA releases 8 jobs to LW 1. The jobs proceed sequentially through the line until LW 4 completes the fourth task and hands the completed job off to the FG. Each completed job triggers the release of a new job to the queue in front of LW 1 such that there is a maximum of 8 jobs in the system at any one time.

Team 5 - Bucket brigade

Team 5, see Figure 4.17, is configured in a standard “bucket brigade”, which is a pull system based on the Toyota sewn production system, where each worker is cross-trained in all skills up to and including their assigned skill. The skill assignments

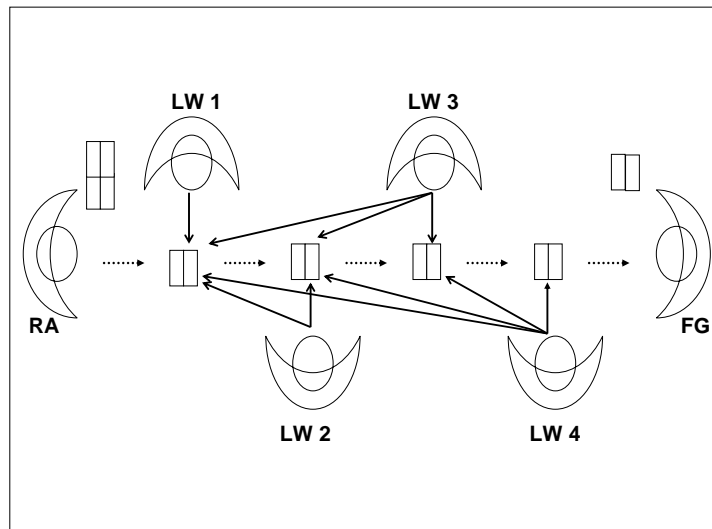


Figure 4.6: Team 5 - Bucket brigade

for the bucket brigade are: LW 1 - Skill 1; LW 2 - Skills 1 and 2; LW 3 - Skills 1,2, and 3; LW 4 - Skills 1,2,3,and 4. Workers complete their assigned task(s) on a given job until preempted by a downstream LW. Preemption works as follows: Once a downstream worker becomes idle, and there is no queue between them and their immediate upstream worker, that downstream worker can preempt the work of their adjacent upstream worker by taking an incomplete job from them and completing the task the upstream worker was in the middle of. Once preempted the upstream worker

immediately moves to the jobs in their queue or preempts their adjacent worker if their queue is empty. The RA only releases new jobs to LW 1 and then the process works as follows:

LW 1 begins work on skill 1 for a given job until preempted by LW 2. Once the preempted job is given to LW 2, LW 1 receives a fresh job from the RA.

LW 2 completes the remainder of skills 1 and 2 until preempted by LW 3. Once the preempted job is given to LW 3, LW 2 turns and again preempts LW 1 if there are no jobs in the queue between LW 1 and LW 2. If there are jobs in the queue LW 2 completes those jobs before preempting LW 1.

LW 3 completes the remainder of skills 1,2, and 3 until preempted by LW 4. Once the preempted job is given to LW 4, LW 3 turns and again preempts LW 2 if there are no jobs in the queue between LW 2 and LW 3. If there are jobs in the queue LW 3 completes those jobs before preempting LW 1.

LW 4 completes the remainder of skills 1,2,3, and 4. Once the job is completed LW 4 hands the job to the FG. Only worker 4 may hand jobs to the FG.

For maximum efficiency of the bucket brigade, if possible, sequence the workers from slowest (LW 1) to fastest (LW 4). All jobs must be completed in order from 1 to 4 and jobs can only be pulled down the line by preemption and not pushed. At any given time there should only be four jobs in the system.

#### Team 6 - Fixed task zone chain under a CONWIP release policy

Team 6, see Figure 4.18, is configured as a type of fixed task zone chain (FTZC) which is a unique worksharing system, within the class of worksharing systems, with overlapping zones. The FTZC of team 6 is broken into two serial lines operating in parallel, as opposed to just one serial line as with the other teams. The reason for this is to create lines with more stations than workers. Each serial line has two LW's where each LW is assigned a zone of three skills. The skill assignments for the FTZC are: LW 1 - Skills 1,2, and 3; LW 2 - Skills 2,3, and 4; LW 4 - Skills 1,2 and

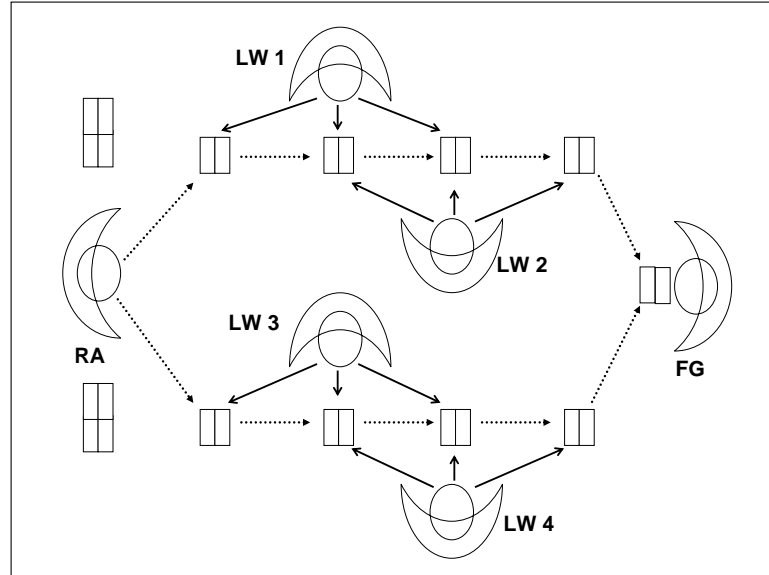


Figure 4.7: Team 6 - Fixed task zone chain

3; LW 4 - Skills 2,3,and 4. Notice that within each LW's zone there are two shared tasks, stations where another worker possesses the same skill, and one fixed task, station where only that worker possesses that skill. Workers on both lines complete their work by following a fixed first maximum shared policy (FFMS). The FFMS rule states that LW's give priority to completing work to their fixed stations. Only when the fixed stations are idle do workers complete work at their shared stations. Priority amongst the shared stations is given to the station with the maximum queue length. The RA only releases new jobs to LW 1 and LW 3 and the FG can only receive completed jobs from LW 2 and LW 4.

### C. Factor Design

#### 1. How to generate random job sheets

Random job sheets are generated using Microsoft Excel. Each worksheet within the Excel program is formatted with a random number generator so that the jobs sheets will each be unique in terms of the number of fruit that is traced each time. During the activity each team should have the exact same set of job sheets.

Please see the 'Software Instructions' as part of this instructor materials packet for further details on job sheet creation.

## 2. How to complete a job

Job completion occurs when all four tasks (fruit) have been traced accurately. Each fruit must have the correct number traced in their respective color as denoted by the number in the right hand column on the job sheet.

### III. Executing the Game

#### A. Pre-game Discussion

##### 1. Student vocabulary

The below terms are suggested vocabulary to go over with the participants prior to playing the game. Please keep in mind that WAG is designed that participants will be able to participate even if they do not fully understand all of the concepts below.

**Specialist** - a LW who is trained on one skill

**Raw materials** - material stocked at beginning of serial product line

**Finished good** - finished product held in inventory prior to shipping to the customer

**WIP** - inventory between the start and endpoints of a serial product line

**Queue** - a line of job sheets that accrues in front of a worker waiting to be processed by that worker

**Cross-training** - the training of a LW to be proficient at more than one skill

**Throughput** - is the average quantity of good (non-defective) jobs produced per unit time

**Cycle time** - time between release of the job at the beginning of the line until it reaches an inventory point at the end of the line

## 2. The Fruit Basket Story

It is imperative, for the purpose of making connections with the participants understanding, to make it clear to the participants that the processes and concepts they are being learned in the activity are “real world”, in that they are used in operational production facilities. Consider using the following brief story to introduce the activity:

Fruits, Inc. is a production facility that specializes in creating fruit baskets for Mother’s Day. A fruit basket is comprised of a specific number of four types of fruits: apples, bananas, pears, and cherries. As Mother’s Day approaches Fruits, Inc. has orders to fill for fruit baskets and those orders are given to each of the five production lines in their manufacturing facility. Each production line, or team, is responsible for filling as many orders as possible within the allotted time. Only baskets that have an accurate amount of fruit are considered good.

## 3. Student Hypotheses

Prior to beginning the activity a discussion can be facilitated with the students with regard to what they expect to occur, as it relates to team efficiency, given the team configurations. Consider posing the following questions to the participants:

- Which team do you expect to complete the most jobs during the game and why?
- Would you expect the teams with more cross-training to perform better than the teams without? Why?
- Which teams do you expect to have bottleneck problems and where do you believe the bottlenecks will occur?

### B. Setup Guidelines

#### 1. Team Creation

Each team has a total of 6 members: 4 LW's, 1 RA, and 1 FG. The most convenient way to seat each team, except Team 2, in a room is to have them seated in a straight line with the RA at one end, followed by the 4 LW's in order from LW 1 to LW 4, and then ended by the FG. This will help with the follow of job sheets, particularly ensuring that jobs are done in order. Team 2 should for a U-shaped line since LW 1 and LW 4 share a skill.

## 2. Instruction Review

Prior to playing the game it is important that the participants get familiar with their roles in the production line. If time permits, doing a practice run with several job sheets is quite helpful. This will alleviate work stoppages due to questions during the activity. Furthermore, allowing the teams to do their practice runs one at a time, will give the participants who are not on that team an opportunity to learn about the other teams.

## C. Post Game Discussion

### 1. Calculations

At the conclusion of the exercise the FG is responsible for the calculations on the WAG Inventory Sheet. First the FG should total the number of jobs completed by the production line and also total the number of jobs released by the RA. WIP can be calculated by subtracting the total number of jobs released by the RA from the total number of jobs completed by the production line. Throughput (TH) is calculated by dividing the total number of jobs completed by the total time spent for the activity (actual production time). Cycle Time (CT) can then be calculated from the Little's Law relationship of  $WIP = TH * CT$ . Each of the three calculations TH, WIP, and CT should be shared with each member of the team.

### 2. Result Analysis

The calculations of TH, WIP, and CT for each team should be put on display for all participants to view. Next, a discussion analyzing the results can be



facilitated around the following questions:

- Did the teams perform as you expected in the pre-game discussion? If not, why?
- How did worker's speed affect the performance of the specialized teams?
- Was cross-training beneficial?
- What do you think can be done to improve upon the cross-training that was done?

### 3. Worker Incentives

A discussion regarding worker incentives can be facilitated around the following questions:

- Did you experience peer pressure with respect to your work rate? If so, from whom?
- To the RA and FG: Could you tell which workers were faster on your line?
- Based upon your experience on the production line, how would you want to be paid - Flat hourly wage? Reduced flat hourly wage with a bonus based upon team performance? Solely based on team performance (e.g. Per number of jobs completed)?

#### 4.4.2 Examples

##### Examples

##### Table of Contents

I. Job Sheet Tracing Example

II. WAG Inventory Sheet Example

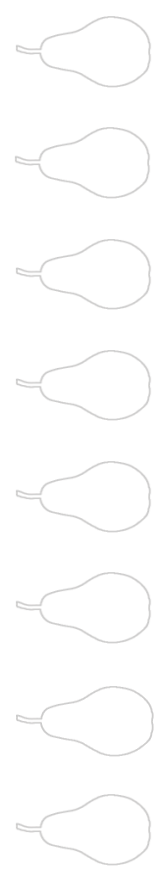
Instructions:  
Trace the correct number of fruit associated with your assigned skill only. The number of fruit to trace in each row is shown to the right of each  
Trace the apples in red. Trace the pears in green. Trace the bananas in yellow. Trace the cherries in blue.

Number of Fruit to trace in each row

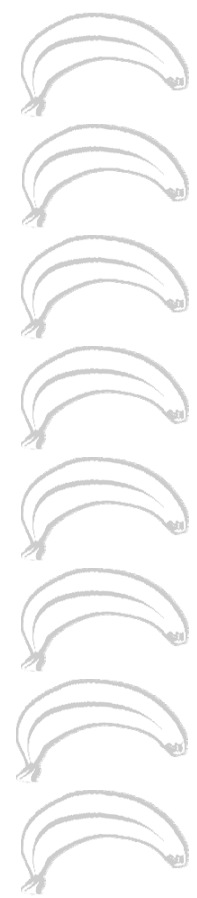
6



5



7



2



Figure 4.8: Job Sheet Tracing Example



### 4.4.3 Software Instructions

Software Instructions

Table of Contents

I. Job Sheet Generator Instructions

II. Example Data Worksheet Page

III. Blank Job Sheet

IV. Blank Inventory Sheet

## I. Job Sheet Generator Instructions

The set of job sheets to be used for the WAG activity are created via a random number generator in Microsoft Excel. Follow these steps to create the job sheets:

**STEP 1:** This step will enable to the random number generator to create unique job sheets. Upon opening the Excel file “job\_sheet\_generator\_TEMPLATE.xls”, a dialogue box will appear asking whether to enable or disable Macros. Select Enable Macros.

**STEP 2:** In this step you will save a separate file of the job sheet generator instructions so as not to disturb the template file from the generated sheets file, once the random number generator has been run. Click File on the top toolbar and choose Save As. A Save As dialogue box will open. Select the location you wish to save the file in and name it. Click Save.

There will be two worksheets at the bottom of the page, one labeled Data and the other labeled Template.

**STEP 3:** Click on the Data worksheet.

**STEP 4:** In the Data worksheet enter the appropriate data in cells A3 to A8. Cell A3 corresponds to the job number that will be generated at the top right hand corner of every job sheet. In this cell, please enter the number that you would like the sequence to begin with. Please note that most often this number will be 1. However, if you’ve previously generated 20 job sheets, for example, and you want to generate 20 more for the same set, then you can change this number to 21. Cells A4 to A7 denote the maximum number of fruit stencils in each row that are available to trace. Here you may enter any integer between 1 and 8. Cell A8 represents the total number of job sheets you would like created in your set.

**STEP 5:** To generate the job sheets follow the instructions given in the Data worksheet where it states ‘Instructor Action Required’.

**STEP 7:** Wait for the Macro to finish running and the job sheets to be created. Each job sheet will create a new worksheet in the Excel file. The number on the tab of each worksheet corresponds to the job created.

**STEP 8:** Print the entire set of job sheets. Click File on the top toolbar and select Print. {File → Print}

**STEP 9:** In the 'Print' dialog box go to the 'Print What' section select the 'Entire Workbook' option. The generated job sheets as well as the data sheet will print. Copies can be made of the set of job sheets so that each team has one set.

**STEP 10:** SAVE the file before closing.

## II. Example Data Worksheet Page

Please type in the data requested in the first column next to its associated description	
<i>1</i>	Start # (job number/label to start with)
<i>8</i>	Max # of Apple Fruits to trace
<i>8</i>	Max # of Pear Fruits to trace
<i>8</i>	Max # of Banana Fruits to trace
<i>8</i>	Max # of Cherry Fruits to trace
<i>60</i>	Number of sheets to generate

**INSTRUCTOR ACTION REQUIRED:**

To generate job sheet go through the menu sequence Tools --> Macro --> Macros, then highlight FruitGenerator and Macros in This Workbook, then click Run.

Figure 4.10: Data Worksheet Page



III. Blank Job Sheet

**Instructions:**  
 Trace the correct number of fruits associated with your assigned skill(s) only (1=Apples, 2=Pears, 3=Bananas, 4=Cherries).  
 The number to trace is shown to the right of each row of fruit (and changes from job to job).  
 Trace the apples in red, pears in green, bananas in yellow, and cherries in blue.

Number of Fruits to trace in each row



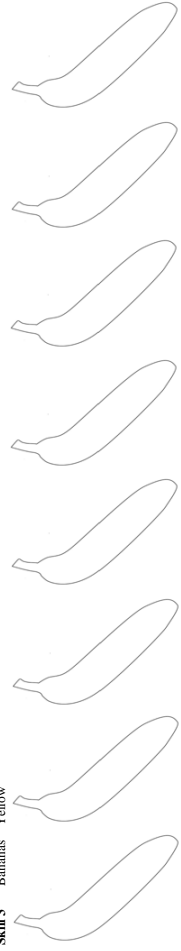
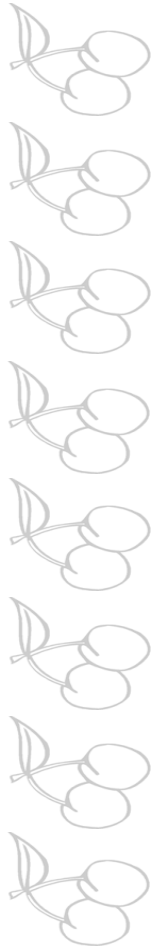
Skill 1 Apples	Red	
Skill 2 Pears	Green	
Skill 3 Bananas	Yellow	
Skill 4 Cherries	Blue	

Figure 4.11: Blank Job Sheet



## 4.5 Materials for students

This section outlines the WAG instructions that were developed for students or participants of the game. Since the instructions for each differ vary the team's instructions are separated into their individual subsections.

### 4.5.1 Team 1 Instructions: Specialized production

#### Overview

Each team's goal is to complete as many jobs as possible in the allotted time. A job is completed when all the tasks are finished on a particular job sheet. The tasks involve tracing each piece of fruit in a row the directed number of times as specified next to the row. Every worker on each team has a different skill set of the tasks that they can perform. Be sure to complete only your task as specified by your team number. If the number traced does not match the number specified next to the row, the job is no good and must be discarded. When time is called, STOP and put everything down. Do not keep working to finish the current job.

#### General Instructions

All tasks must be done in order from line worker 1 (LW 1) to line worker 4 (LW 4)

#### Skill Description

- Skill 1 - Trace the apple outline in red the number of times specified next to the row.
- Skill 2 - Trace the pear outline in green the number of times specified next to the row.
- Skill 3 - Trace the banana outline in yellow the number of times specified next to the row.

- Skill 4 - Trace the cherries outline in blue the number of times specified next to the row.

### **Team 1 Configuration**

The 6 team members should be arranged in a straight line in the following order: Release Agent (RA), LW 1, LW 2, LW 3, LW 4, Finished Goods Inventory Supervisor (FG) (see Figure 4.13). The following color markers should be given to each LW: LW 1 - red; LW 2 - green; LW 3 - yellow; LW 4 - blue.

### **Team Member Roles**

- RA: Give a new job sheet to LW 1, and only LW 1, when they are idle. Each time a job sheet is given to LW 1 check the corresponding job sheet number on the WAG inventory sheet.
- FG: Receive finished jobs from LW 4 and only LW 4. Inspect the job for completeness and accuracy. A job is complete if all the fruits are traced in the appropriate color, the appropriate number of times. A job is accurate if the stencil is outlined neatly. Discard any job that is not both complete and accurate. Upon receipt of good jobs, place a check next to the job sheet number on the WAG inventory sheet.
- LW 1: Complete skill 1 and only skill 1 for each job. Upon completion of skill 1 push the job on to LW 2 if they are idle. If LW 2 is not idle place the job in the inventory pile next to them. Receive a new job from the RA.
- LW 2: Complete skill 2 and only skill 2 for each job. Upon completion of skill 2 push the job on to LW 3 if they are idle. If LW 3 is not idle place the job in the inventory pile next to them. Then either work on the next job in your inventory pile or wait to receive a job from LW 1.

- LW 3: Complete skill 3 and only skill 3 for each job. Upon completion of skill 3 push the job on to LW 4 if they are idle. If LW 4 is not idle place the job in the inventory pile next to them. Then either work on the next job in your inventory pile or wait to receive a job from LW 2.
- LW 4: Complete skill 4 and only skill 4 for each job. Upon completion of skill 4 give the job to the FG. Then either work on the next job in your inventory pile or wait to receive a job from LW 3.

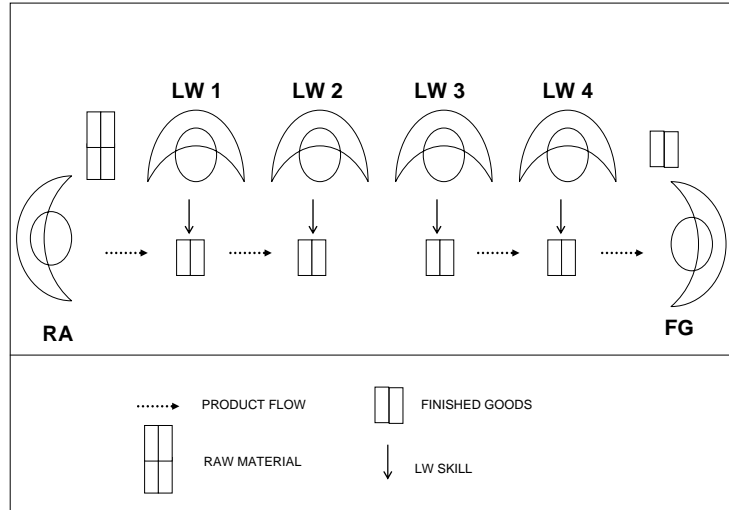


Figure 4.13: Team 1

## **4.5.2 Team 2 Instructions: Two skill chaining**

### **Overview**

Each team's goal is to complete as many jobs as possible in the allotted time. A job is completed when all the tasks are finished on a particular job sheet. The tasks involve tracing each piece of fruit in a row the directed number of times as specified next to the row. Every worker on each team has a different skill set of the tasks that they can perform. Be sure to complete only your task as specified by your team number. If the number traced does not match the number specified next to the row, the job is no good and must be discarded. When time is called, STOP and put everything down. Do not keep working to finish the current job.

### **General Instructions**

All tasks must be done in order from line worker 1 (LW 1) to line worker 4 (LW 4)

### **Skill Description**

- Skill 1 - Trace the apple outline in red the number of times specified next to the row.
- Skill 2 - Trace the pear outline in green the number of times specified next to the row.
- Skill 3 - Trace the banana outline in yellow the number of times specified next to the row.
- Skill 4 - Trace the cherries outline in blue the number of times specified next to the row.

### **Team 2 Configuration**

The 6 team members should be arranged in a U-Shaped line in the following order: Release Agent (RA), LW 1, LW 2, LW 3, LW 4, Finished Goods Inventory Supervisor

(FG) (see Figure ??). The U-Shape should put LW 1 and LW 4 in close proximity since they share a common skill. The following color markers should be given to each LW: LW 1 - red and green; LW 2 - green and yellow; LW 3 - yellow and blue; LW 4 - blue and red.

### **Team Member Roles**

- RA: Give a new job sheet to LW 1 or LW 4 when they are idle. Each time a job sheet is given check the corresponding job sheet number on the WAG inventory sheet.
- FG: Receive finished jobs from LW 4 and only LW 4. Inspect the job for completeness and accuracy. A job is complete if all the fruits are traced in the appropriate color, the appropriate number of times. A job is accurate if the stencil is outlined neatly. Discard any job that is not both complete and accurate. Upon receipt of good jobs, place a check next to the job sheet number on the WAG inventory sheet.
- LW 1: Complete skill 1 for each job. Upon completion of skill 1 if LW 2 is busy complete skill 2 on the job. If LW 2 is idle then push the job to LW 2. After completing skill 2 or pushing the job to LW 2 receive a new job sheet from the RA.
- LW 2: Check to see if skill 2 has been completed on the job and if not, complete it. Upon the completion of skill 2 if LW 3 is busy complete skill 3 on the job. If LW 3 is idle then push the job to LW 3. After completing skill 3 or pushing the job to LW 3 complete the next job in your inventory pile.
- LW 3: Check to see if skill 3 has been completed on the job and if not, complete it. Upon the completion of skill 3 if LW 4 is busy complete skill 4 on the job. If LW 4 is idle then push the job to LW 4. After completing skill 4 or pushing the job to LW 4 complete the next job in your inventory pile.

- LW 4: Check to see if skill 4 has been completed on the job and if not, complete it. Upon the completion of skill 4 give the job to the FG for inspection. After turning in the completed job first complete work in your inventory pile. If there is no work in your inventory pile, then receive a fresh job from the RA to complete skill 1. After completing skill 1 place the job in the inventory pile of LW 1.

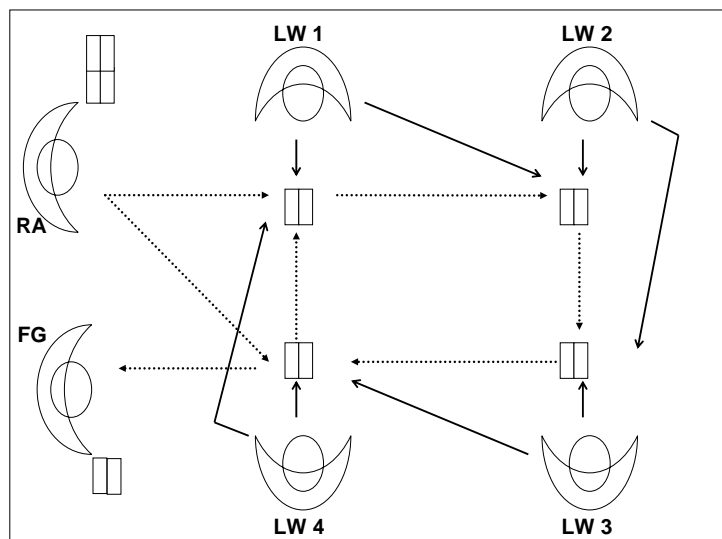


Figure 4.14: Team 2



### **4.5.3 Team 3 Instructions: Craft production with pick and run**

#### **Overview**

Each team's goal is to complete as many jobs as possible in the allotted time. A job is completed when all the tasks are finished on a particular job sheet. The tasks involve tracing each piece of fruit in a row the directed number of times as specified next to the row. Every worker on each team has a different skill set of the tasks that they can perform. Be sure to complete only your task as specified by your team number. If the number traced does not match the number specified next to the row, the job is no good and must be discarded. When time is called, STOP and put everything down. Do not keep working to finish the current job.

#### **General Instructions**

All tasks must be done in order from line worker 1 (LW 1) to line worker 4 (LW 4)

#### **Skill Description**

- Skill 1 - Trace the apple outline in red the number of times specified next to the row.
- Skill 2 - Trace the pear outline in green the number of times specified next to the row.
- Skill 3 - Trace the banana outline in yellow the number of times specified next to the row.
- Skill 4 - Trace the cherries outline in blue the number of times specified next to the row.

#### **Team 3 Configuration**

The team members do not need to be arranged in any particular order as all team members operate independently (see Figure 4.15). Each LW should receive four

colored markers: red, yellow, green, and blue.

### Team Member Roles

- RA: Give a new job sheet any LW when they are idle. Each time a job sheet is given check it off on the inventory sheet.
- FG: Receive finished jobs from any LW. Inspect the job for completeness and accuracy. A job is complete if all the fruits are traced in the appropriate color, the appropriate number of times. A job is accurate if the stencil is outlined neatly. Discard any job that is not both complete and accurate. Upon receipt of good jobs, place a check next to the job sheet number on the WAG inventory sheet.
- ALL LW's: Receive a new job sheet from the RA and complete all four skills. When finished, give the completed job to the FG and take a new job from the RA. We call this the pick and run policy.

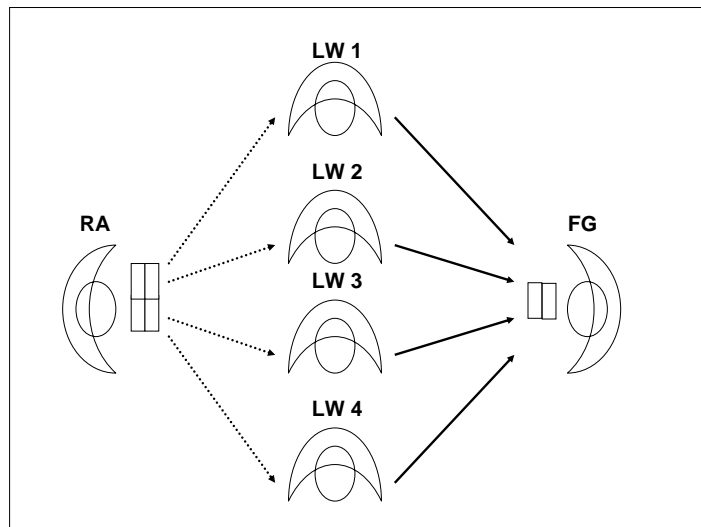


Figure 4.15: Team 3

#### **4.5.4 Team 4 Instructions: Specialized production under a CONWIP release policy**

##### **Overview**

Each team's goal is to complete as many jobs as possible in the allotted time. A job is completed when all the tasks are finished on a particular job sheet. The tasks involve tracing each piece of fruit in a row the directed number of times as specified next to the row. Every worker on each team has a different skill set of the tasks that they can perform. Be sure to complete only your task as specified by your team number. If the number traced does not match the number specified next to the row, the job is no good and must be discarded. When time is called, STOP and put everything down. Do not keep working to finish the current job.

##### **General Instructions**

All tasks must be done in order from line worker 1 (LW 1) to line worker 4 (LW 4)

##### **Skill Description**

- Skill 1 - Trace the apple outline in red the number of times specified next to the row.
- Skill 2 - Trace the pear outline in green the number of times specified next to the row.
- Skill 3 - Trace the banana outline in yellow the number of times specified next to the row.
- Skill 4 - Trace the cherries outline in blue the number of times specified next to the row.

##### **Team 4 Configuration**

The 6 team members should be arranged in a straight line in the following order: Release Agent (RA), LW 1, LW 2, LW 3, LW 4, Finished Goods Inventory Supervisor (FG) (see Figure 4.16). The following color markers should be given to each LW: LW 1 - red; LW 2 - green; LW 3 - yellow; LW 4 - blue.

### **Team Member Roles**

- RA: Begin by releasing 8 jobs to LW 1 and only LW 1. Check each of the 8 jobs off of the inventory sheet once released. Each time a job is completed and given to the FG release a job to LW 1. At any given time there should only be 8 released jobs out on the line.
- FG: Receive finished jobs from LW 4 and only LW 4. Inspect the job for completeness and accuracy. A job is complete if all the fruits are traced in the appropriate color, the appropriate number of times. A job is accurate if the stencil is outlined neatly. Discard any job that is not both complete and accurate. Upon receipt of good jobs, place a check next to the job sheet number on the WAG inventory sheet.
- LW 1: Complete skill 1 and only skill 1 for each job. Upon completion of skill 1 push the job on to LW 2 if they are idle. If LW 2 is not idle place the job in the inventory pile next to them. Receive a new job from the RA.
- LW 2: Complete skill 2 and only skill 2 for each job. Upon completion of skill 2 push the job on to LW 3 if they are idle. If LW 3 is not idle place the job in the inventory pile next to them. Then either work on the next job in your inventory pile or wait to receive a job from LW 1.
- LW 3: Complete skill 3 and only skill 3 for each job. Upon completion of skill 3 push the job on to LW 4 if they are idle. If LW 4 is not idle place the job in the inventory pile next to them. Then either work on the next job in your inventory pile or wait to receive a job from LW 2.

- LW 4: Complete skill 4 and only skill 4 for each job. Upon completion of skill 4 give the job to the FG. Then either work on the next job in your inventory pile or wait to receive a job from LW 3.

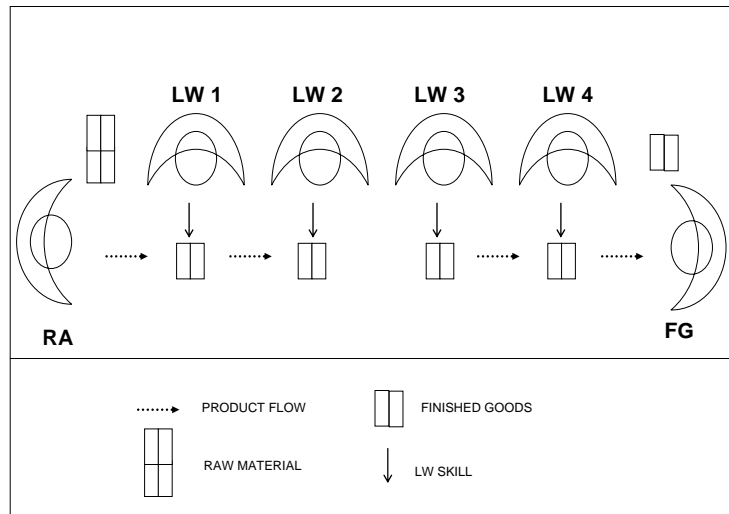


Figure 4.16: Team 4

#### **4.5.5 Team 5 Instructions: Bucket brigade**

##### **Overview**

Each team's goal is to complete as many jobs as possible in the allotted time. A job is completed when all the tasks are finished on a particular job sheet. The tasks involve tracing each piece of fruit in a row the directed number of times as specified next to the row. Every worker on each team has a different skill set of the tasks that they can perform. Be sure to complete only your task as specified by your team number. If the number traced does not match the number specified next to the row, the job is no good and must be discarded. When time is called, STOP and put everything down. Do not keep working to finish the current job.

##### **General Instructions**

All tasks must be done in order from line worker 1 (LW 1) to line worker 4 (LW 4)

##### **Skill Description**

- Skill 1 - Trace the apple outline in red the number of times specified next to the row.
- Skill 2 - Trace the pear outline in green the number of times specified next to the row.
- Skill 3 - Trace the banana outline in yellow the number of times specified next to the row.
- Skill 4 - Trace the cherries outline in blue the number of times specified next to the row.

##### **Team 5 Configuration**

The 6 team members should be arranged in a straight line in the following order: Release Agent (RA), LW 1, LW 2, LW 3, LW 4, Finished Goods Inventory Supervisor

(FG) (see Figure 4.17). The following color markers should be given to each LW: LW 1 - red; LW 2 - red and green; LW 3 - red, green, and yellow; LW 4 - red, green, yellow, and blue. If possible order the LW's from slowest (LW 1) to fastest (LW 4).

### **Team Member Roles**

- RA: Release a new job sheet to only LW 1 when they are idle. Each time a job is released check it off on the inventory sheet.
- FG: Receive finished jobs from LW 4 and only LW 4. Inspect the job for completeness and accuracy. A job is complete if all the fruits are traced in the appropriate color, the appropriate number of times. A job is accurate if the stencil is outlined neatly. Discard any job that is not both complete and accurate. Upon receipt of good jobs, place a check next to the job sheet number on the WAG inventory sheet.
- LW 1: Complete skill 1 and only skill 1 for each job. Work on skill 1 of your current job until LW 2 preempts or takes the job from you. Once the job has been preempted receive a new job from the RA.
- LW 2: Once idle preempt LW 1 by taking their current job from them. Do as much as you can for skill 1 and skill 2 on the current job until the job is preempted or taken from you by LW 3. After preemption from LW 3, immediately preempt LW 1 and repeat the process.
- LW 3: Once idle preempt LW 2 by taking their current job from them. Do as much as you can for skill 1, skill 2, and skill 3 on the current job until the job is preempted or taken from you by LW 4. After preemption from LW 4, immediately preempt LW 2 and repeat the process.
- LW 4: Once idle preempt LW 3 by taking their current job from them. Complete all skills on the job that are incomplete and submit the job to the FG. After

the job has been turned in, immediately preempt LW 3 and repeat the process.

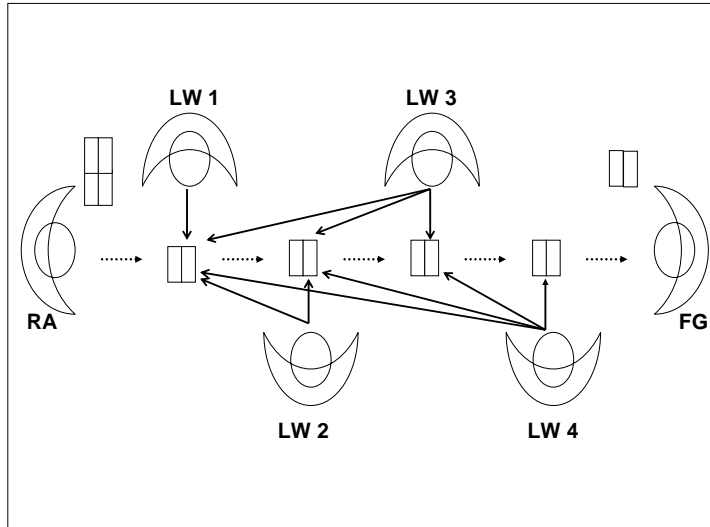


Figure 4.17: Team 5



#### **4.5.6 Team 6 Instructions: Fixed task zone chain under a CONWIP release policy**

##### **Overview**

Each team's goal is to complete as many jobs as possible in the allotted time. A job is completed when all the tasks are finished on a particular job sheet. The tasks involve tracing each piece of fruit in a row the directed number of times as specified next to the row. Every worker on each team has a different skill set of the tasks that they can perform. Be sure to complete only your task as specified by your team number. If the number traced does not match the number specified next to the row, the job is no good and must be discarded. When time is called, STOP and put everything down. Do not keep working to finish the current job.

##### **General Instructions**

All tasks must be done in order from line worker 1 (LW 1) to line worker 2 (LW 2) on the first line and line worker 3 (LW 3) to line worker 4 (LW 4) on the second line.

##### **Skill Description**

- Skill 1 - Trace the apple outline in red the number of times specified next to the row.
- Skill 2 - Trace the pear outline in green the number of times specified next to the row.
- Skill 3 - Trace the banana outline in yellow the number of times specified next to the row.
- Skill 4 - Trace the cherries outline in blue the number of times specified next to the row.

##### **Team 6 Configuration**

The 6 team members should be arranged in two serial lines operating in parallel (see Figure 4.18). The order of the first serial line should be: Release Agent (RA), LW 1, LW 2, FG. The order of the second serial line should be: Release Agent (RA), LW 3, LW 4, FG. (Note: The RA and FG are the same individuals for both lines as they are shared between the two lines. The following color markers should be given to each LW: LW 1 - red, green, yellow; LW 2 - green, yellow, blue; LW 3 - red, green, yellow; LW 4 - green, yellow, blue

### **Team Member Roles**

- RA: Release a new job sheet to only LW 1 and LW 3 when they are idle. Each time a job is released check it off on the inventory sheet.
- FG: Receive finished jobs from LW 2 and LW 4. Inspect the job for completeness and accuracy. A job is complete if all the fruits are traced in the appropriate color, the appropriate number of times. A job is accurate if the stencil is outlined neatly. Discard any job that is not both complete and accurate. Upon receipt of good jobs, place a check next to the job sheet number on the WAG inventory sheet.
- LW 1: Complete skill 1 for each job. If LW 2 is busy continue working on the job to work on skill 2, and if LW 2 is still busy go on to do skill 3. As soon as LW 2 becomes idle give them your job, if it is at station 2 or 3. Once your job is given to LW 2 immediately receive a new job from the RA. Note: At the beginning of play, LW 2 is initially idle, so after completing skill 1, immediately give them the job and return to the RA for a new job.
- LW 2: Prioritize completing skill 4 of jobs. If there is a job awaiting skill 4, always do it first. Otherwise, receive a new job from LW 1 after skill 1 has been completed on that job.

- LW 3 and LW 4 have their own line with LW 3 observing the rules for LW 1. Similarly LW 4 matches LW 2.

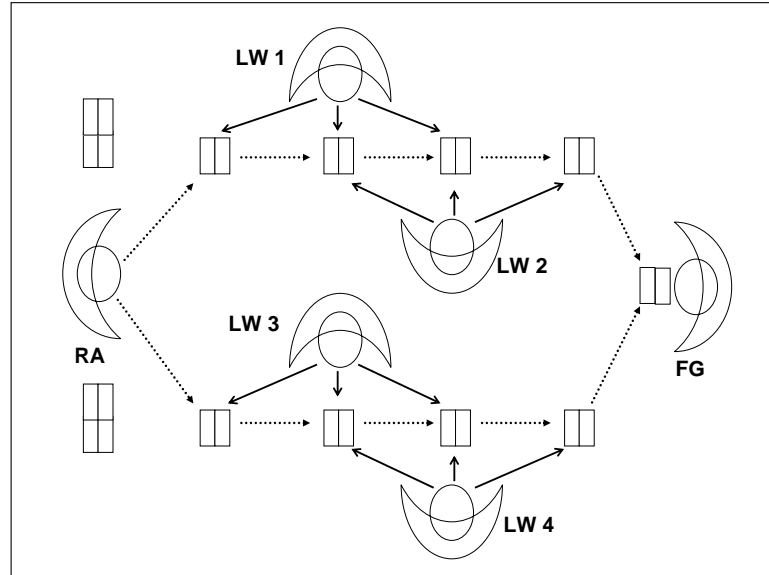


Figure 4.18: Team 6

## 4.6 Conclusions and Future Work

The results of implementing WAG in classroom settings were quite astounding. Students not only gained a deeper understanding of basic operations management concepts, in particular the value of flexibility, but they also offered insightful and innovative ideas on how to cross-train and configure production lines to increase throughput. The authors found this to be quite useful when teaching workforce agility in an undergraduate senior level operations management course. WAG gave the students an opportunity to immediately apply concepts they had recently learned and the students remarked how this helped them with retention of the material. Another academic benefit to the students of WAG is that it draws on components of introductory queueing theory that is taught during the junior year of the industrial and operations engineering curriculum at the authors' institution. This allows students to gain clarity with respect to previously learned material. In addition to academic benefits, WAG also provided the students with a hands on learning tool to creatively think about and discuss production control and manufacturing techniques. It is the author's hope that such discussion will help drive interest in manufacturing and production related fields.

Future expansions of WAG could include expounding on the quality control inspection done by the FG at the end of the line. This inspection could be expanded to include results of accept, rework, or discard for each completed job. Accepted jobs would immediately enter inventory, reworked jobs would be sent back into production, and discarded jobs would be thrown away. The inclusion of rework and a penalty for discarded jobs would add to the teams' simulation of a real production line.

## CHAPTER V

### Conclusion

In conclusion, the importance and effectiveness of operational flexibility has been demonstrated. This work provides operations managers with new and innovative methods for their systems. The Fixed Task Zone Chain (FTZC) as a labor cross-training structure is low cost and easy to implement. We have shown the minimal productivity loss that occurs when it is implemented with a Fixed First Maximum Shared (FFMS) policy against more expensive policies. Insight into the effectiveness and appropriate application of the FTZC paradigm and the FFMS policy was gained via (1) stability analysis (Section 2.4.1), (2) development of a Degree of Imbalance (DOI) line metric, and its use in benchmarking, and (3) analytical structural results of optimal policies in Section 2.4.2. Moreover, we developed a hands on learning tool intended to support the teaching of operational flexibility as it relates to cross-training workers. This tool enhances the foundation of understanding and interest in worker agility that will sponsor further innovation in the field. Finally, within the supply chain system, we modeled a single source and dual source unreliable supplier system for the firm. We learned how important several parameters are to the single source model, in particular, the per unit profit of the product. Then, in the dual source model, we identified conditions upon which a firm should create a contract with a second supplier, rather than simply rely on inventory mitigation. These investiga-

tions have contributed to academic understanding, but are also suited to advancing industrial practice in the mitigation of operational variability.

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