SI 563 Lecture 1

Introduction and Representation of Games

Professor Yan Chen
Fall 2008

Agenda

- Game Theory
  - » History and applications
  - » Definitions and overview
- Representation: Extensive forms
- Strategies
- Representation: Normal forms

Introduction

Game Theory and Applications
(Watson Chapter 1)

It's Your Move

It's Your Move

Source: "eBay", ebay.com

Source: “The ESP Game”, http://www.gwap.com/gwap/gamesPreview/espgame/

THE **ESP GAME**

TWO-PLAYER ONLINE GAME

PARTNERS DON'T KNOW EACH OTHER AND CAN'T COMMUNICATE

OBJECT OF THE GAME:
TYPE THE SAME WORD

THE ONLY THING IN COMMON IS
AN IMAGE

THE **ESP GAME**

PLAYER 1

GUESSING: CAR

GUESSING: HAT

GUESSING: KID

SUCCESS!

YOU AGREE ON CAR

PLAYER 2

GUESSING: BOY

GUESSING: CAR

SUCCESS!

YOU AGREE ON CAR

What is a game?

- A game is being played whenever people *interact* with each other
  - Bidding in an auction
  - Pricing: amazon.com
  - Adoption of a new standard
  - Cuban missile crisis
- Interdependence
  - One person's behavior affect another's well-being
- What is not a game?
  - N=1: monopoly
  - N= infinity: perfect competition

Three Major Tensions of Strategic Interaction

- Game theory: a theory of strategic interaction
  - Conflict
  - Cooperation
- Three major tensions
  - Conflict between individual and group interests
  - Strategic uncertainty
  - Insufficient coordination

Game Theory: A Short History

- Cournot (1838) and Edgeworth (1881)
- Zermelo (1913): chess-like games can be solved in a (large!) finite number of moves
- von Neumann and Morgenstern (1944)
- Nash, Harsanyi, Selten: 1994 Nobel Prize for solution concepts in non-cooperative game theory
- Aumann and Schelling : 2005 Nobel Prize for game theoretic analysis of conflict and cooperation

Noncooperative vs. Cooperative Game Theory

- Noncooperative game theory
  - Individual decision making
  - Group decision making: specify procedures leading individual decisions to group outcomes
  - Solution concepts: prescriptions and predictions about the outcomes of games
- Cooperative game theory
  - Model joint actions

Applications of Game theory

- Game theory has been applied to sociology, economics, political science, decision theory, law, evolutionary biology, experimental psychology, military strategy, anthropology …
- School of information
  - Incentive-centered design
  - Information policy
  - Social computing
  - HCI and CSCW
  - ARM and LIS

Representing Games

An Overview

Representing Games

- A list of players
- A complete description of what players can do
- A description of what the players know when they act
- A specification of how player actions lead to outcomes
- A specification of player preferences over outcomes

Extensive- and Normal-Form Games

- Two basic types of interactions
  - Sequential: players make alternating moves
  - Simultaneous: players act at the same time
- In most cases interactions are partly sequential and partly simultaneous
- Can be modeled in two ways
  - Extensive-form games
  - Normal-form games

SI 563 Overview

- Games of complete information
  - Normal form games: Nash equilibrium
  - Extensive form games: SPNE
    - » Static
    - » Repeated
- Games of incomplete information
  - Normal form games: Bayesian Nash equilibrium
  - Extensive form games: perfect Bayesian equilibrium

Representing Games

The Extensive Form
(Watson Chapter 2)

Image of
Peanuts
comic removed

Link to football Peanuts comic:
http://comics.com/peanuts/1952-11-16/

- Set of players
  - CB
  - L
- Set of strategies
  - CB: {accept, reject}
  - L: {pull, not pull}
- Sequence of actions
- Outcomes
  - CB falls
  - CB kicks the ball
  - Nothing happens

Extensive Form Representation

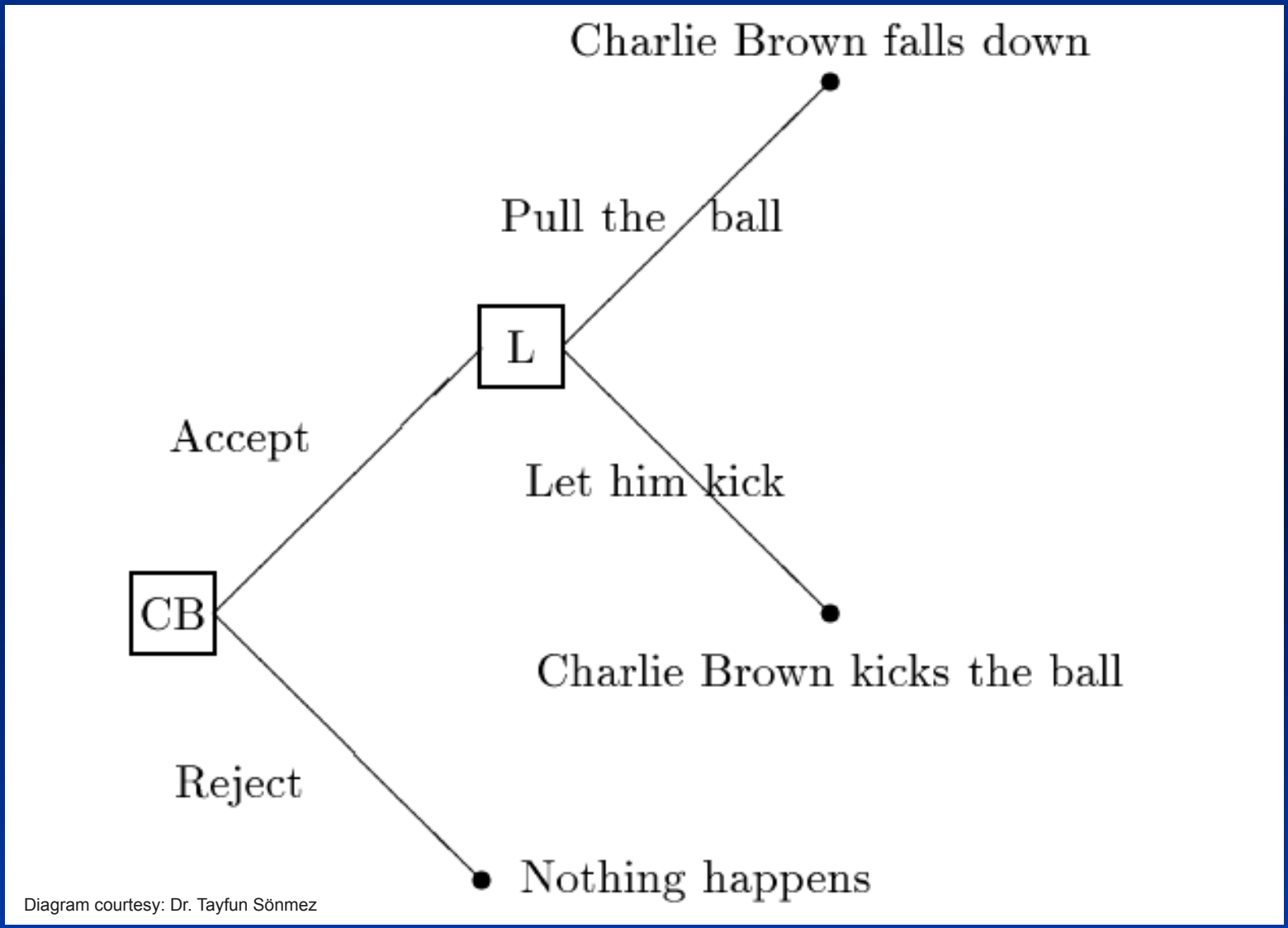


Diagram courtesy: Dr. Tayfun Sönmez

A game tree consists of:

- A series of nodes linked in a sequence
  - Non-terminal node: not an endpoint
  - Terminal node: indicates that game is over
- Branches represent actions

Note: loops (i.e. cycles) are not allowed in game trees.

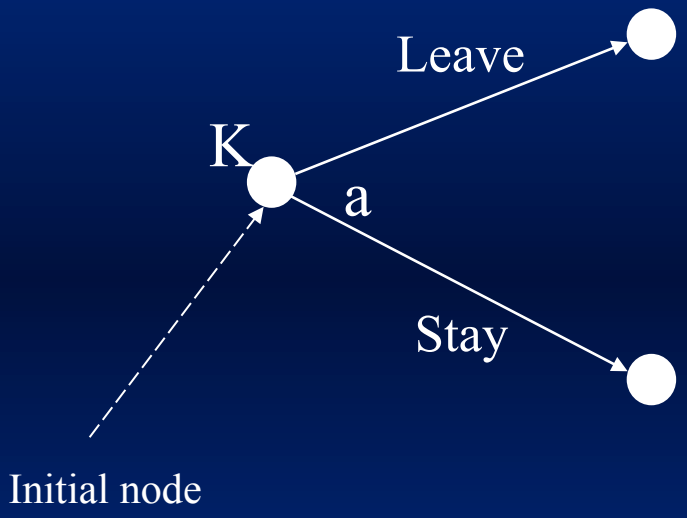
Two Crucial Elements of Extensive-Form Games

- **Timing** of actions that players may take
- **Information** they have when they must take those actions
  - Information sets

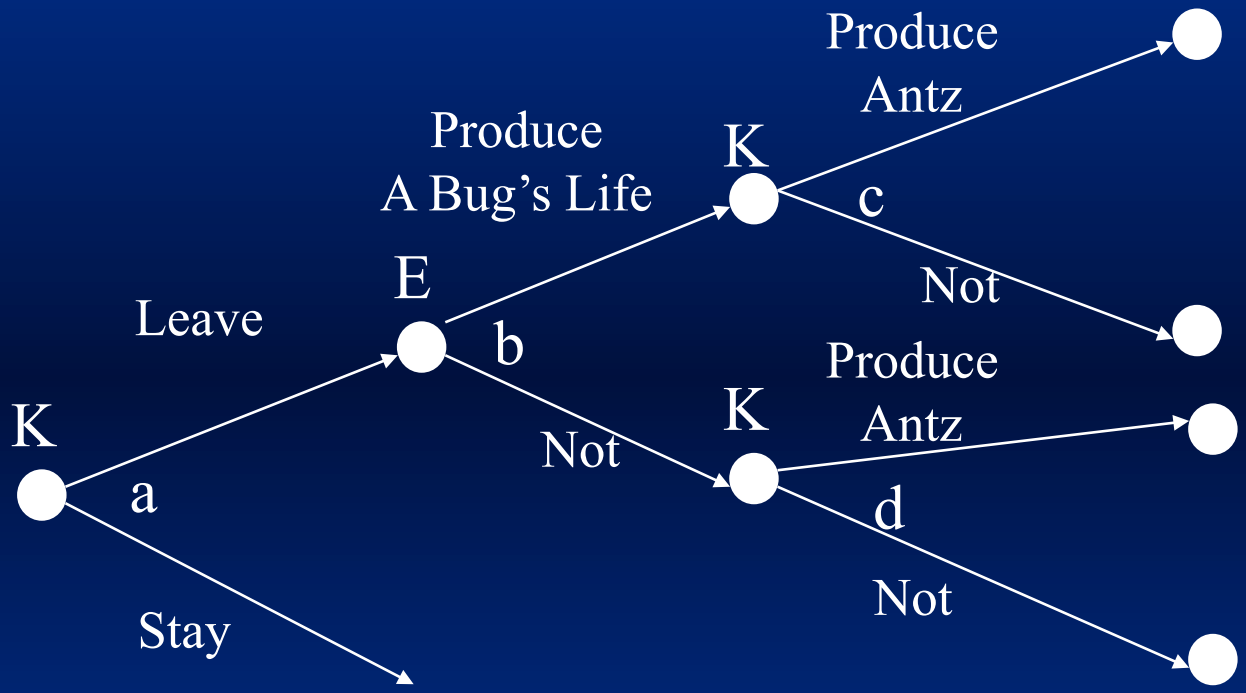
Example: the Bug Game

- A tale of two films (1998)
  - Disney: *A bug's life*
  - Dreamwork: *Antz*
- A model
  - Set of players
    - » Jeffrey Katzenberg
    - » Michael Eisner (Disney CEO)
  - Set of actions for each player, etc.

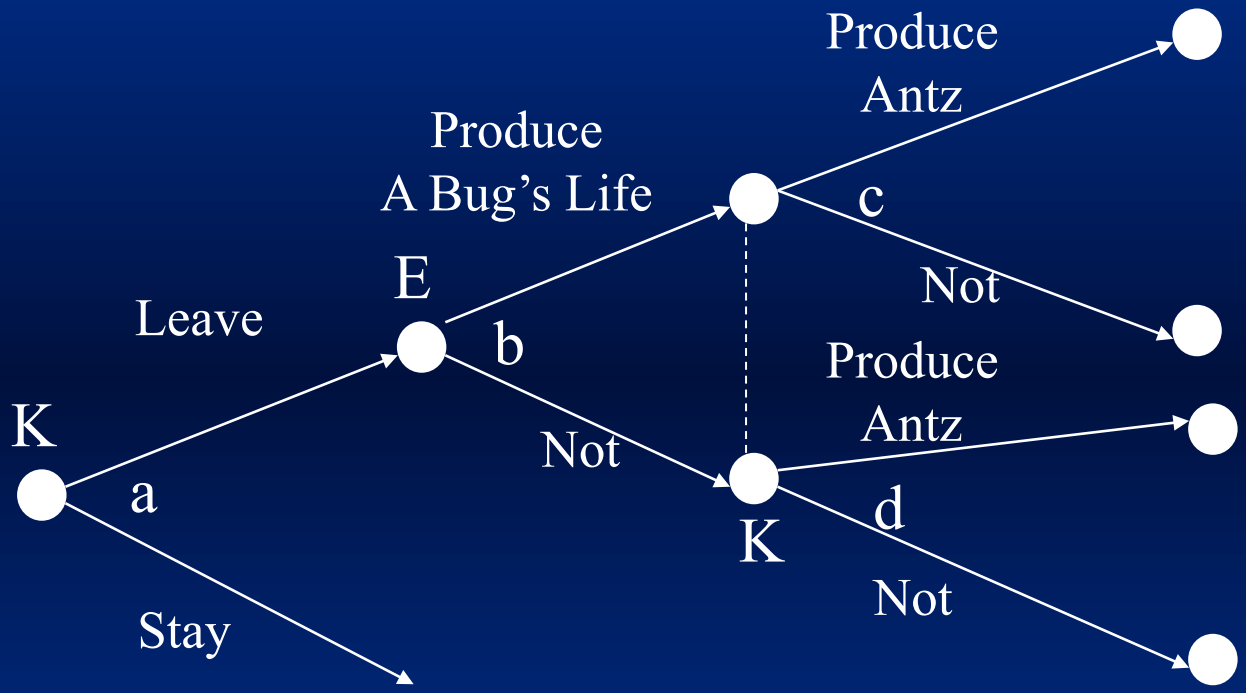
Building an extensive form: Katzenberg's first move



Adding the production decisions



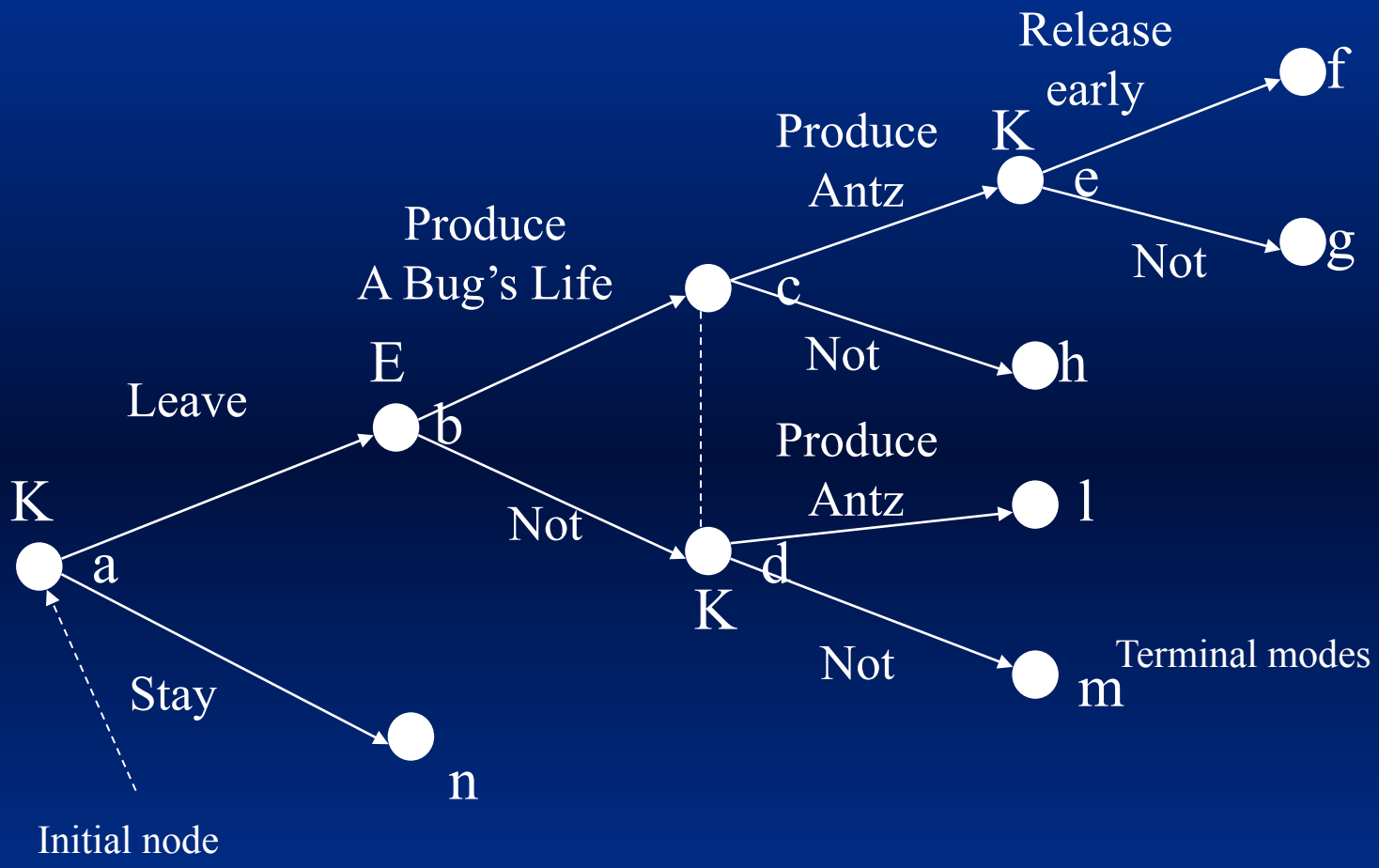
Capturing lack of information



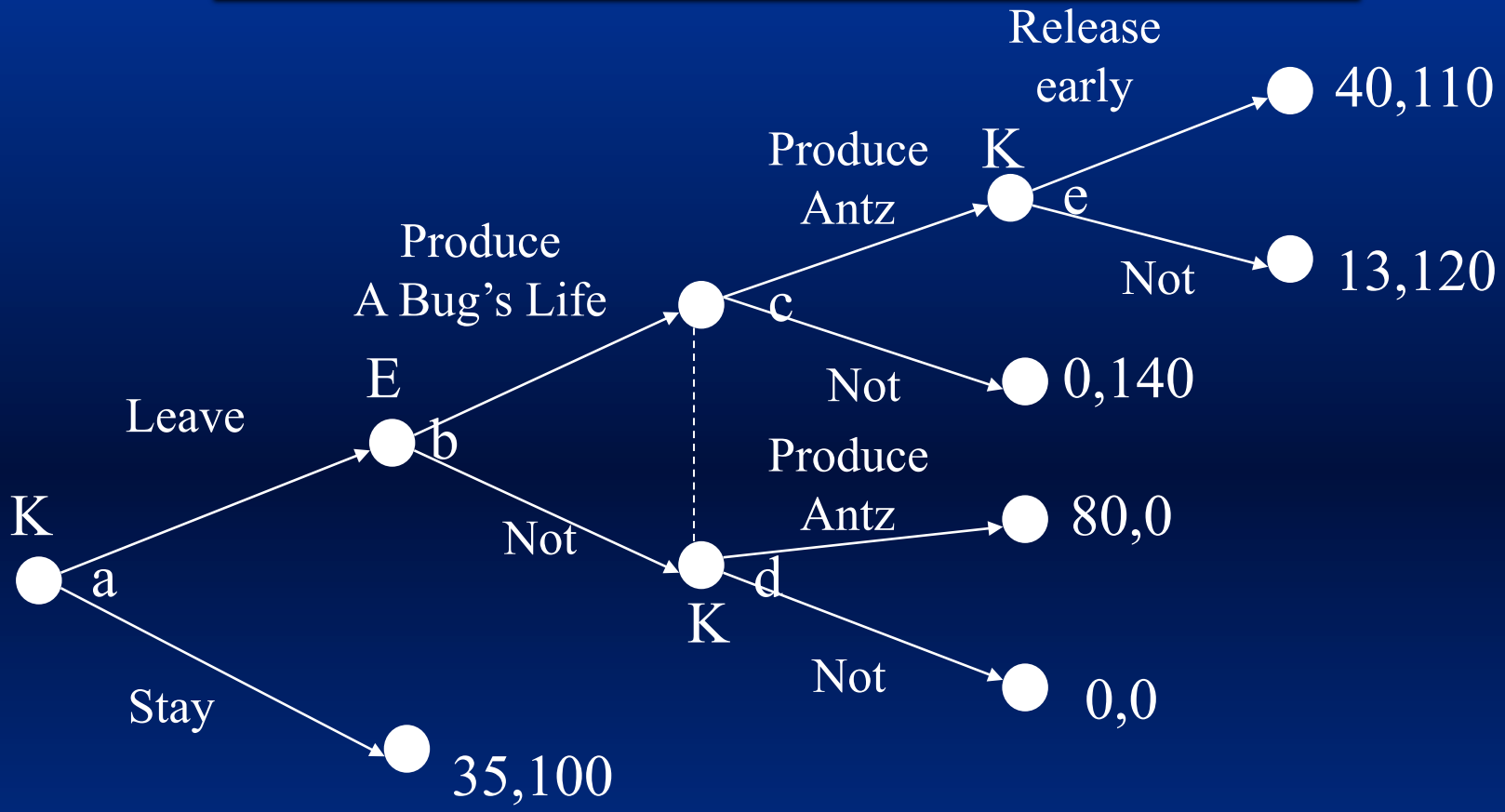
Information Sets

- Information sets summarize a player's knowledge of prior moves when she must decide
- If there are more than one nodes in an information set, a player knows that she is in one of the nodes in the information set (but does not know which one)
- Information sets containing only one node are referred to as singletons

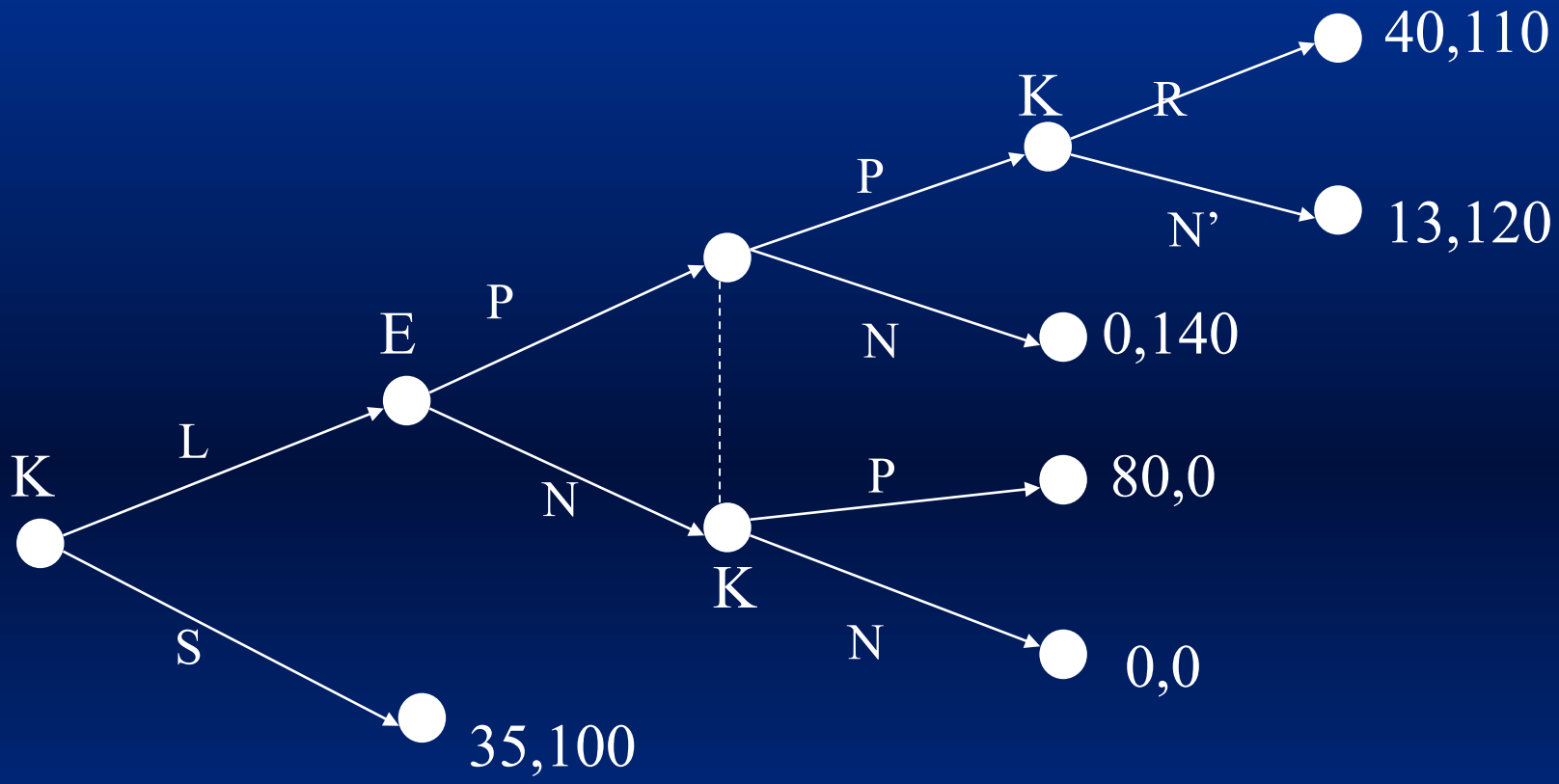
Adding terminal nodes



The Full Extensive Form



A more compact representation



Labeling branches:
-Differentiate between N and N'
-Conformity within an information set

Example: Cuban Missile Crisis

- Why did the Soviet Union attempt to place offensive missiles in Cuba?
- Why did US respond with a blockade of Cuba?
- Why did the Soviet Union decide to withdraw the missiles?

A Simple Model of the Contest

- Set of players
  - Challenger: player CH
  - Defender: player D
- Preferences
  - Challenger (best to worst)
    - » Concession
    - » Status quo
    - » Back down
    - » war
  - Defender
    - » Backdown
    - » Status quo
    - » Concession
    - » war

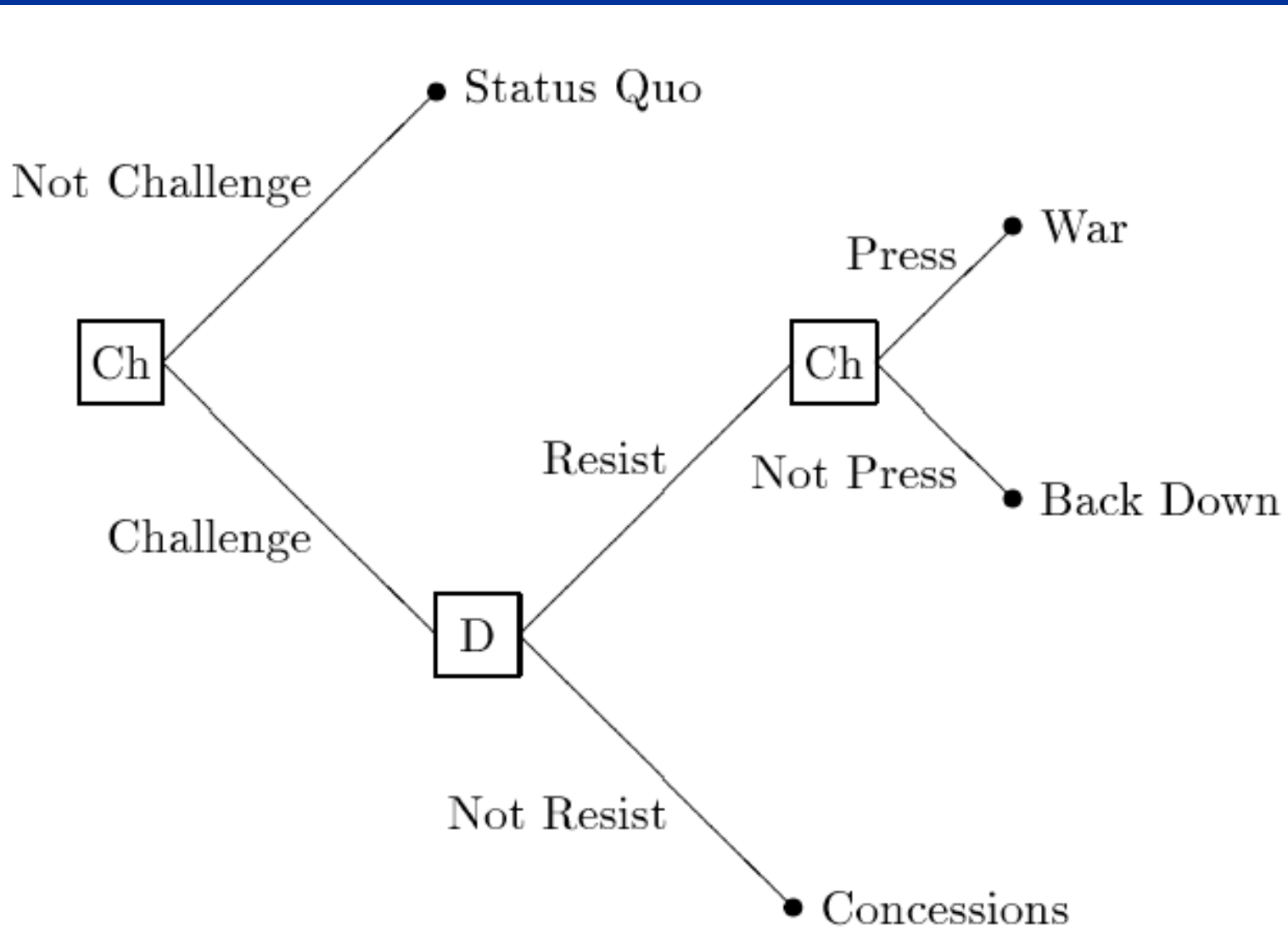


Diagram courtesy: Dr. Tayfun Sönmez

Adding Uncertainty

- If there is uncertainty, we model this by adding Nature (or Chance) as another player
  - It does not have payoffs
  - It chooses different types
- Example: two types of Defenders
  - Resolute type: prefers War to Concession
  - Irresolute type: prefers Concession to War

If Challenger can observe Defender's type

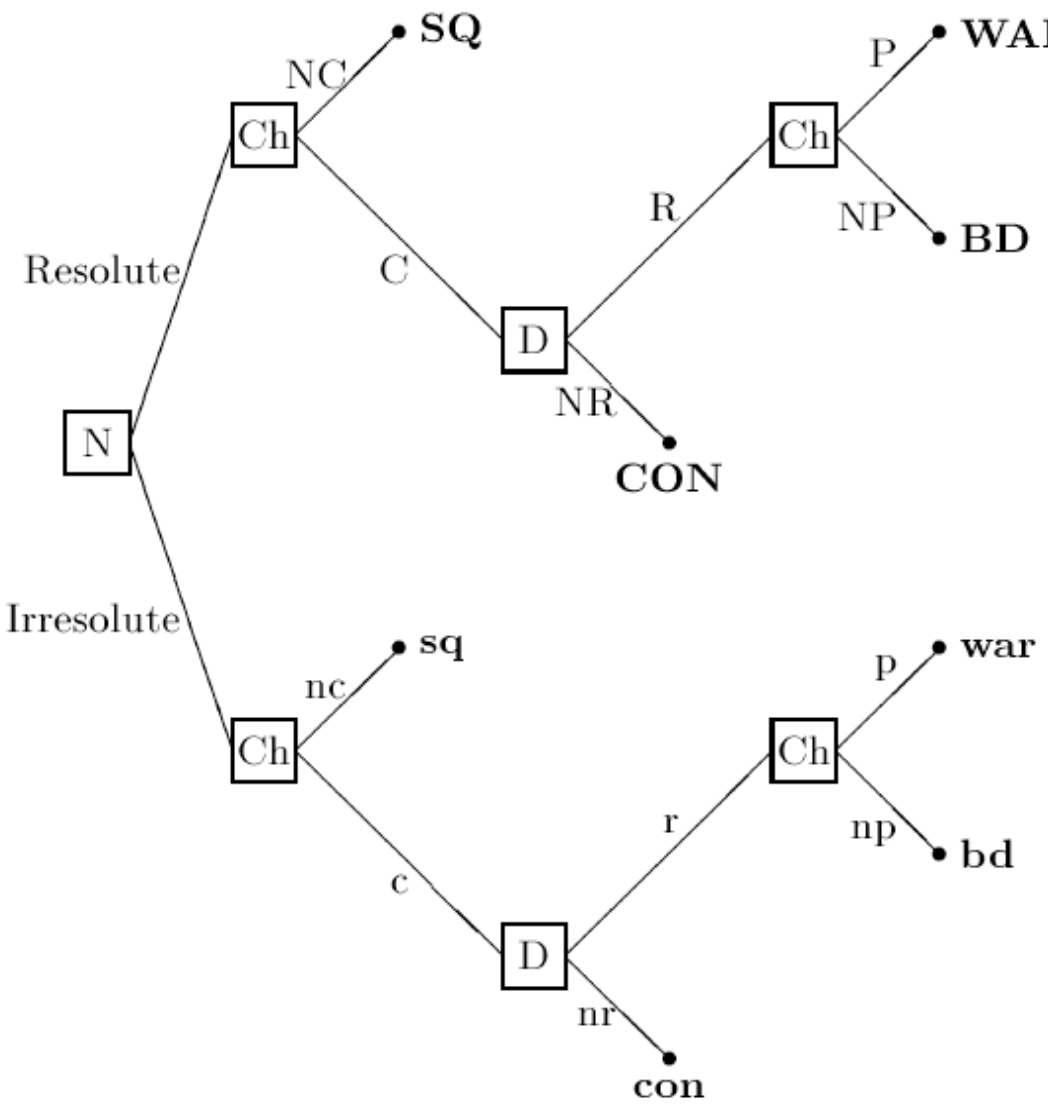


Diagram courtesy: Dr. Tayfun Sönmez

If Challenger can't observe Defender's type:

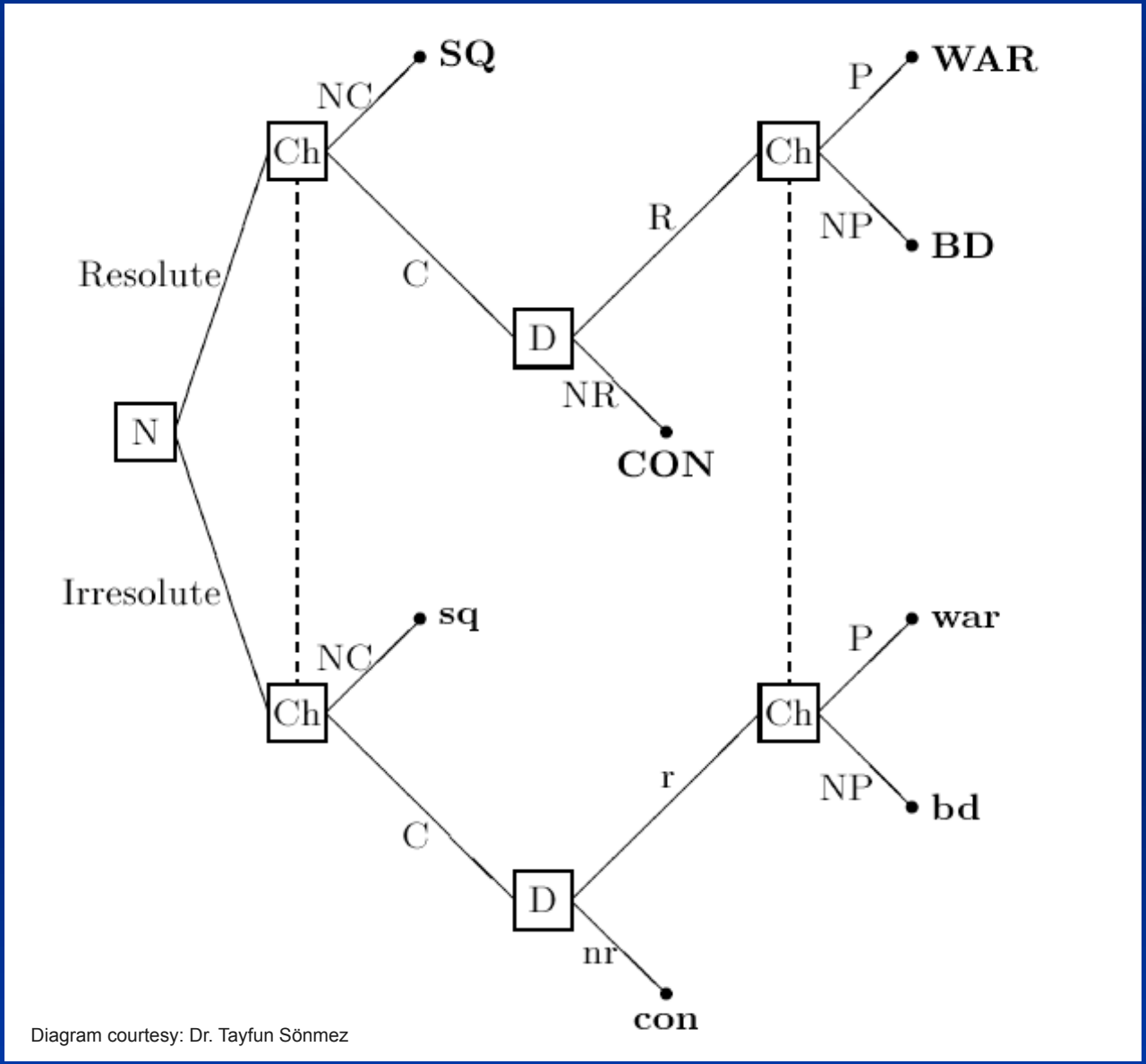


Diagram courtesy: Dr. Tayfun Sönmez

Example: Rock, Paper, Scissors

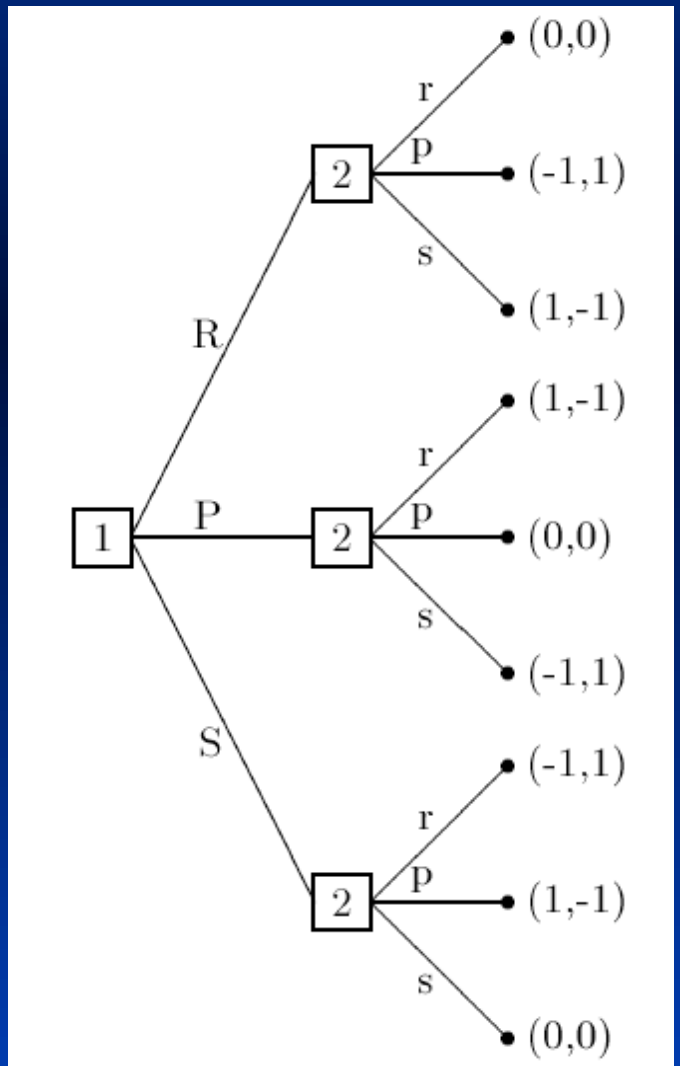
- **Simultaneous move game**
- **Normal-form representation:**

		Player 2		
		Rock	Paper	Scissors
Player 1	ROCK	0,0	-1,1	1,-1
	PAPER	1,-1	0,0	-1,1
	SCISSORS	-1,1	1,-1	0,0

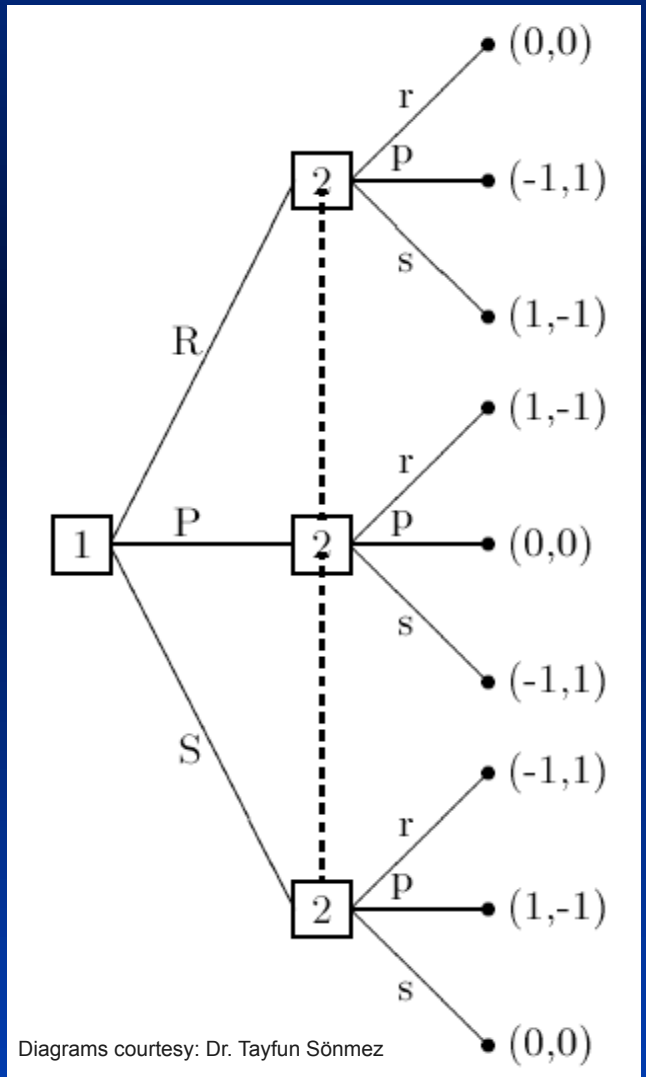
Diagram courtesy: Dr. Tayfun Sönmez

Rock, Paper, Scissors: Extensive Forms

- Sequential moves



- Simultaneous moves



Diagrams courtesy: Dr. Tayfun Sönmez

What if 2 can observe if 1 chooses Rock, but not otherwise?

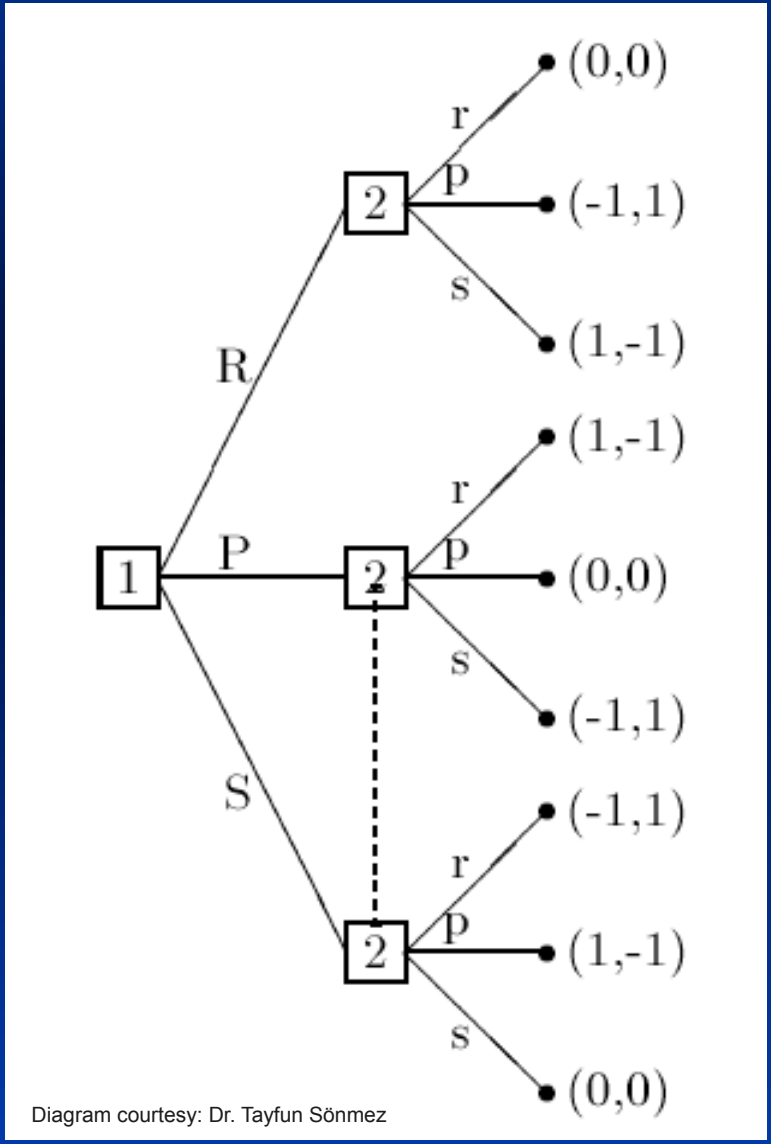


Diagram courtesy: Dr. Tayfun Sönmez

Example: The Truth Game

- An uneven coin: Heads 80% of the times
- Two players: 1 and 2
- Player 1 flips the coin and observes the results
- Player 1 announces H or T
- Player 2 hears 1's announcement but cannot observe results of the actual coin flip. 2 announces h or t
- Payoffs
  - 2 receives $10 if answer is true, $0 otherwise
  - 1 receives $20 if 2 announces heads, and an additional $10 if 1 tells the truth about the coin flip

Representation of the Truth Game

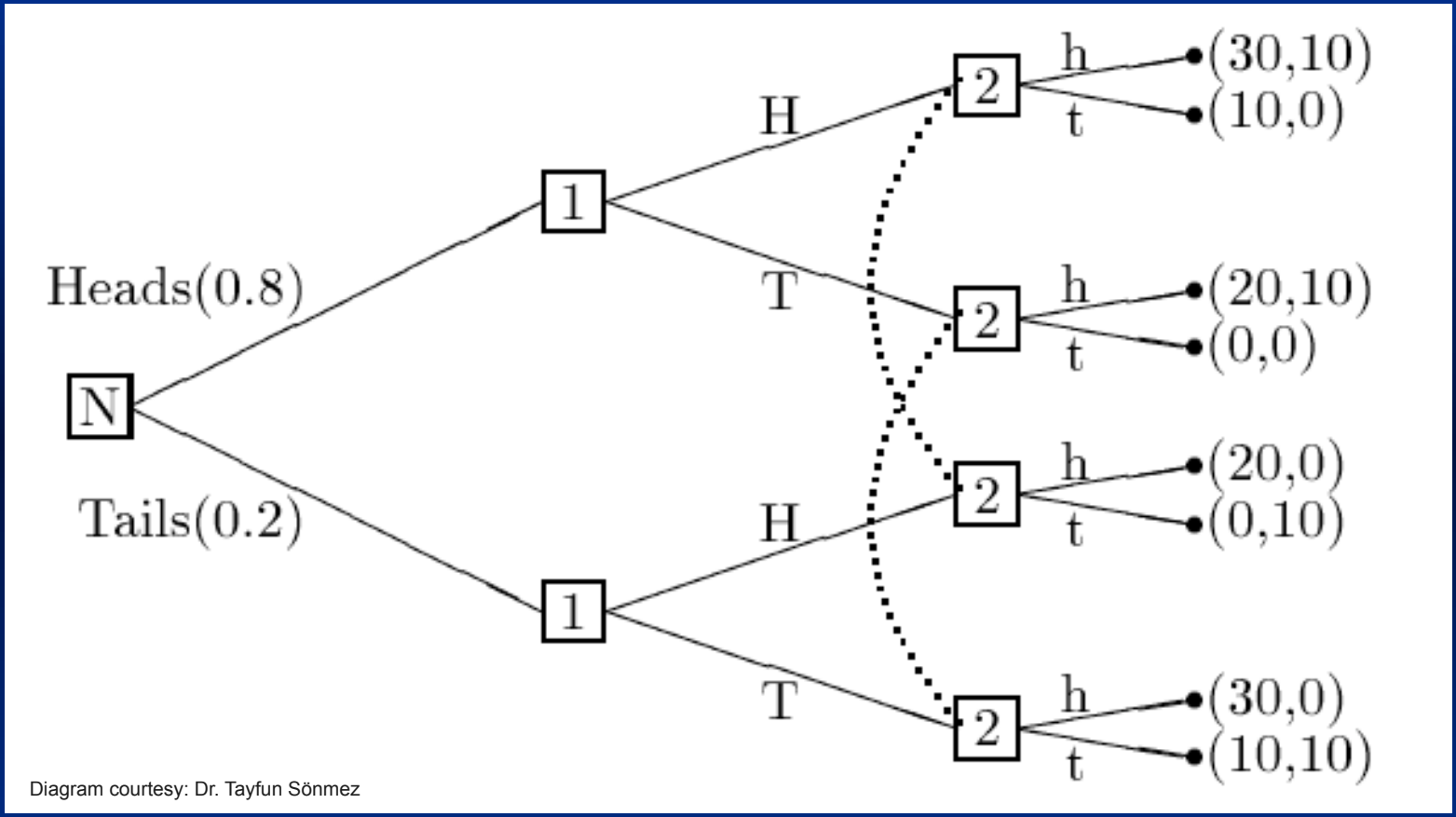


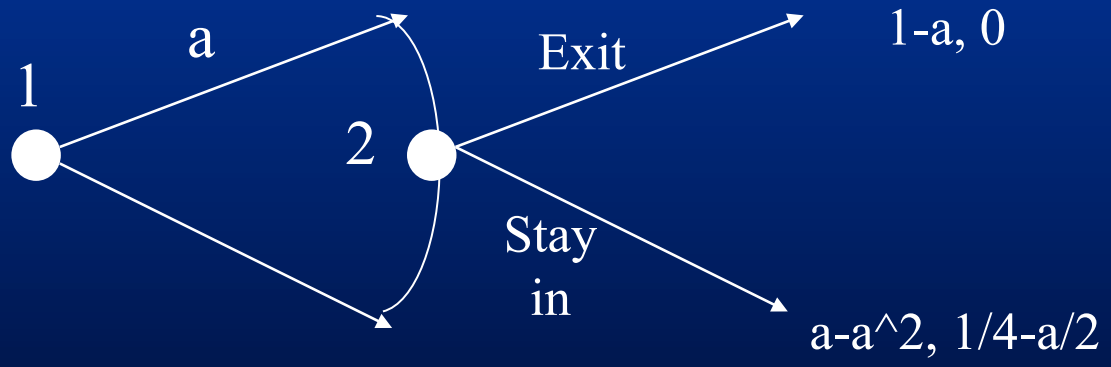
Diagram courtesy: Dr. Tayfun Sönmez

Payoffs

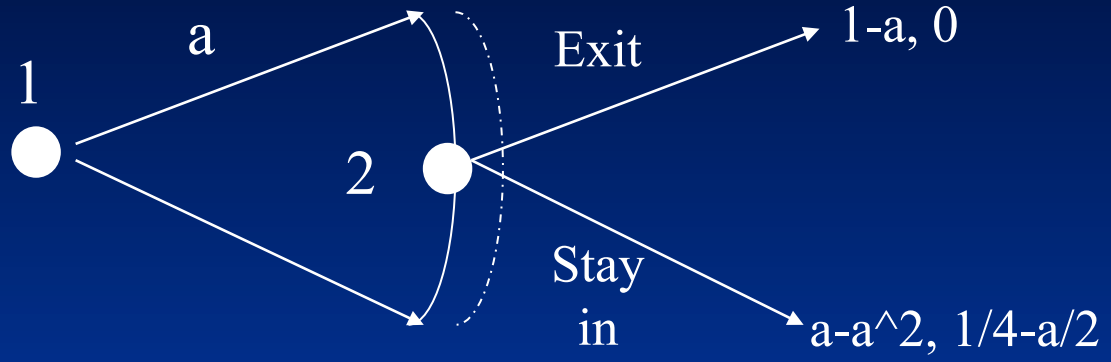
- **2 receives $10 if answer is true, $0 otherwise**
- **1 receives $20 if 2 announces heads, and an additional $10 if 1 tells the truth**

Example: Advertising/Exit

(a) 2 observes 1's actions:



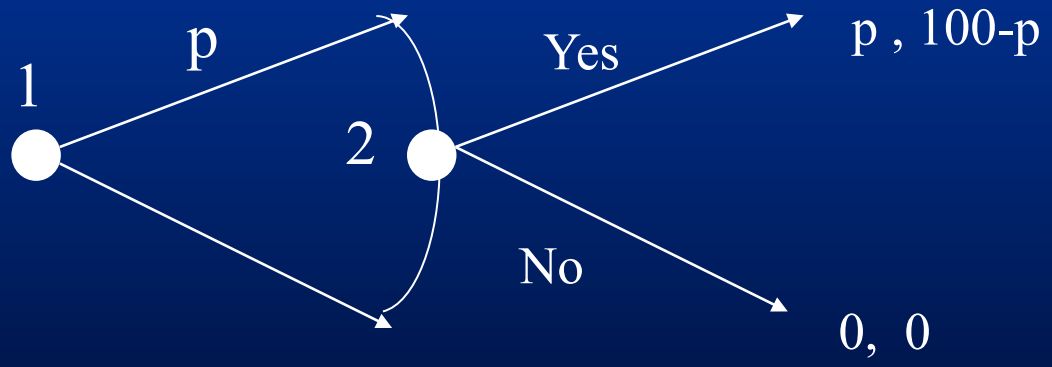
(b) 2 does not observe 1's actions:



Firm 1: how much to spend on advertising, [0, $1 million]

Firm 1: how much to spend on advertising, [0, $1 million]

Example: Ultimatum Bargaining



Player 1 wishes to sell a painting to player 2.
Painting is worth nothing to player 1, 100 to player 2.
Seller makes a take-it-or-leave-it offer.
If buyer accepts the price, trade at this price.
Otherwise, both parties obtain nothing.

Definition: an n-person extensive form game consists of:

- A finite game tree composed of nodes and branches
- A division of nodes over players, chance, and endpoints
- Probability distribution for each chance move
- A division of each player's nodes into information sets
- A set of outcomes and an outcome to each endpoint
- A payoff (or utility) function for each player over all outcomes

All this is common knowledge to all players

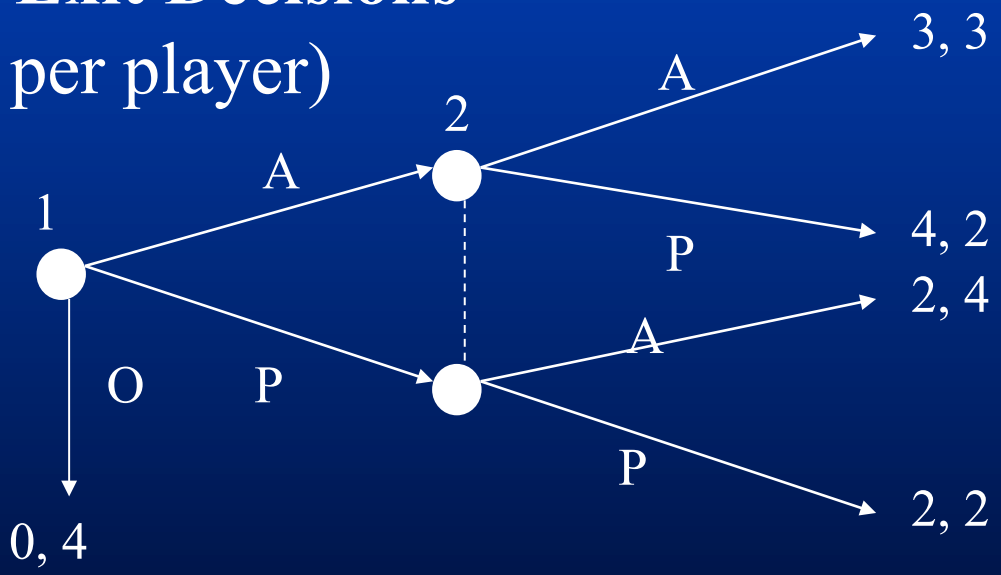
Strategies

(Watson Chapter 3)

Strategy: Definition

- A *strategy* is a complete contingent plan for a player in the game
  - Complete contingent: describes what she will do at each of her information sets
- Writing strategies for a player i:
  - Find every information set for player i
  - At each information set, find all actions
  - Find all combinations of actions at these information sets

Example: Exit Decisions
(1 info set per player)

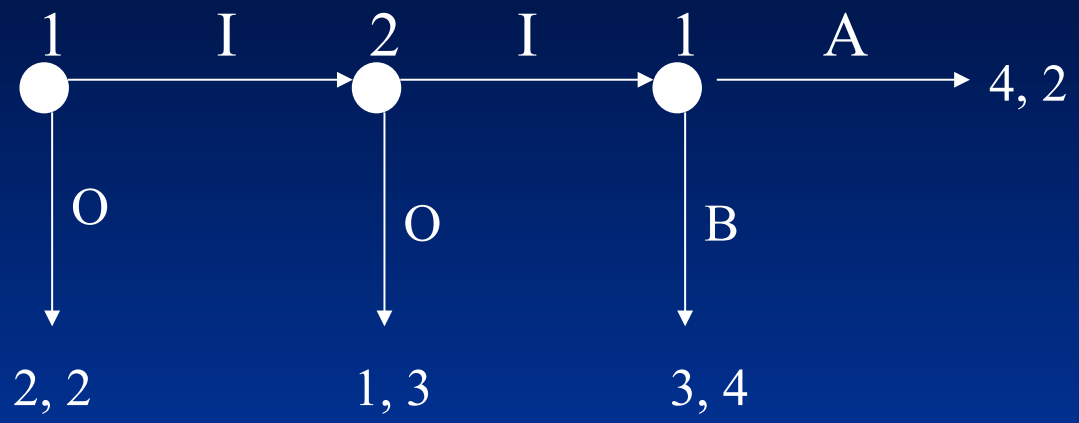


Firm 1: Aggressive (A), Passive (P) or Out (O)
Firm 2: Aggressive (A) or Passive (P)

Strategy Sets:
Firm 1: $S_1=\{A, P, O\}$
Firm 2: $S_2=\{A, P\}$

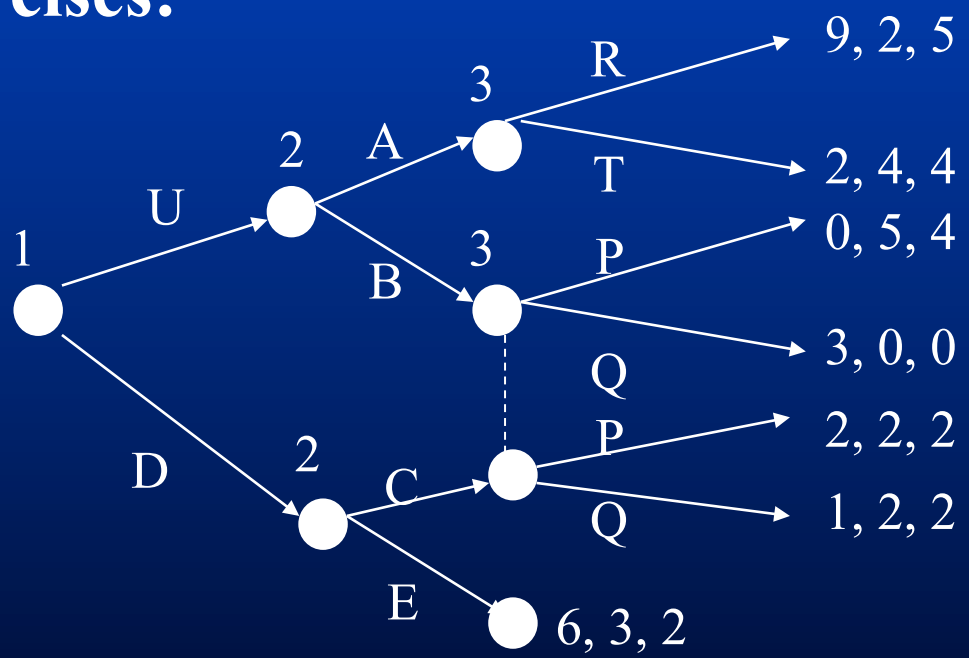
Exercise: finding strategies

1. Find number of Information sets for Players 1 and 2;
2. Find number of actions at each information set;
3. Write down the strategy set for each player.



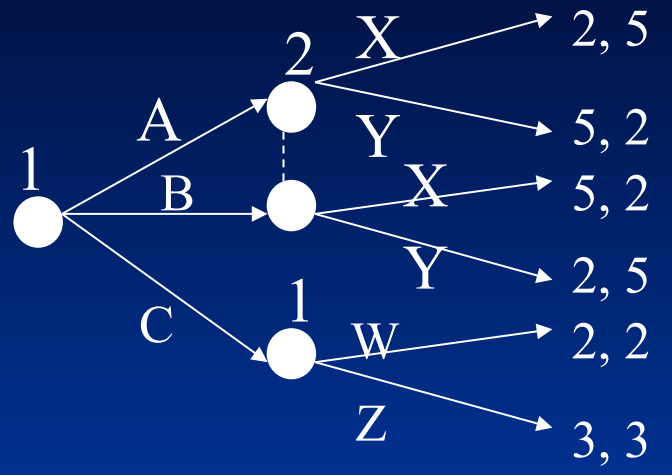
More Exercises:

(a)



$S_1=\{U,D\}$
$S_2=\{AC,AE,BC,BE\}$
$S_3=\{RP,RQ,TP,TQ\}$

(b)



$S_1=\{AW,BW,CW,AZ,BZ,CZ\}$
$S_2=\{X,Y\}$

Representing Games

The Normal Form
(Watson Chapter 3)

The Normal (Strategic) Form

- A game in *normal form* consists of
  - A set of players, {1, 2, …, n}
  - Strategy spaces for the players, $S_1, S_2, \ldots, S_n$
  - Payoff functions for the players, $u_1, u_2, \ldots, u_n$
- Compared to the extensive form, normal form can be
  - More compact
  - For each extensive form, there exists an equivalent normal form representation

Classic Normal-Form Games

- **Example: Prisoners' Dilemma**
  - Set of players: N = {Conductor, Tchaikovsky}
  - Timing: simultaneous move
  - Set of strategies: S_i = {Confess, Not Confess}
  - Set of payoffs:
    - » If one confesses, the other does not: 0, 15 years in jail
    - » If both confess: each gets 5 years in jail
    - » If neither confess: each gets 1 year in jail

PD: Write down the extensive form representation

Example: Prisoners' Dilemma

Tchaikovsky

Conductor	Confess	Not Confess
Confess	**-5, -5**	**0, -15**
Not Confess	**-15, 0**	**-1, -1**

Classical Games: Matching Pennies

1 \ 2	H	T
H	**1, -1**	**-1, 1**
T	**-1, 1**	**1, -1**

Zero-sum game: sum of payoffs in each cell is zero

Classic Game: Coordination

1 \ 2	A	B
A	**1, 1**	**0, 0**
B	**0, 0**	**1, 1**

Coordination: want to use the same strategy, (A, A) or (B, B)
Example: traffic rules

Classic Game: Pareto Coordination

1 \ 2	A	B
A	**2, 2**	**0, 0**
B	**0, 0**	**1, 1**

Coordination: want to select the same strategy;
Prefer to coordinate on A rather than on B.

Classic Game: Battle of the Sexes

1 \ 2	Opera	Movie
Opera	**2, 1**	**0, 0**
Movie	**0, 0**	**1, 2**

Coordination game: want to go to an event together, with slightly different preferences

Classic Game: Hawk-Dove/Chicken

1 \ 2	H	D
H	**0, 0**	**3, 1**
D	**1, 3**	**2, 2**

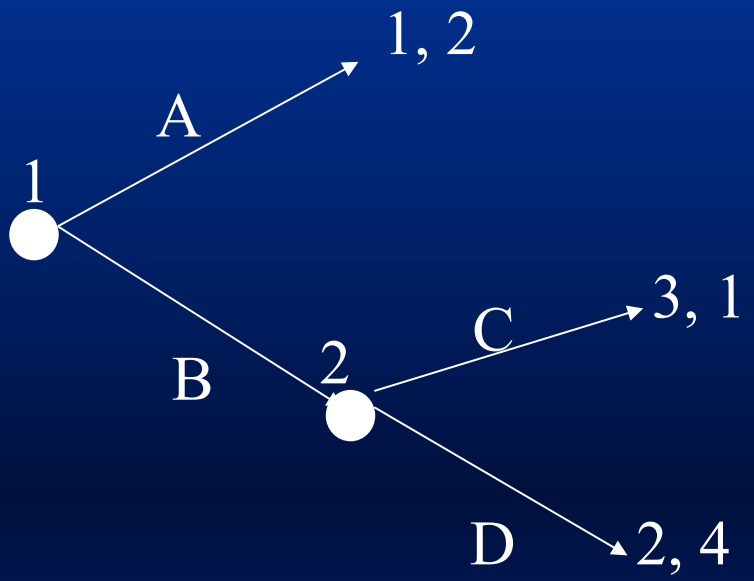
Coordination game: want to take different strategies

Classic Game: Pigs

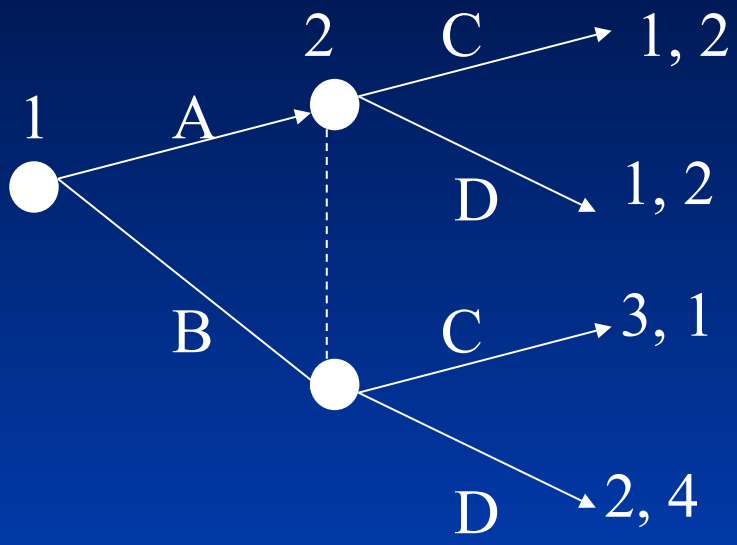
D \ S	P	D
P	**4, 2**	**2, 3**
D	**6, -1**	**0, 0**

D: dominant pig
S: submissive pig

Corresponding extensive and normal forms



1 \ 2	C	D
A	**1, 2**	**1, 2**
B	**3, 1**	**2, 4**



Beliefs, Mixed Strategies, and Expected Payoffs

(Watson Chapter 4)

Beliefs

- A player's assessment about the strategies of the others in the game
- Representing beliefs
  - Probabilities
  - Normal form games:
    - » probability distribution over the strategies of the other players
    - » Example: Prisoner's Dilemma

Example: Prisoners' Dilemma

		Tchaikovsky 0.25 Confess	 0.75 Not Confess
Conductor	**Confess**	**-5, -5**	**0, -15**
	Not Confess	**-15, 0**	**-1, -1**

Conductor's *expected payoff* from "Confess"
=0.25(-5)+0.75 (0) = -1.25

Example: Prisoners' Dilemma

Tchaikovsky

Conductor	Confess	Not Confess
Confess	**-5, -5**	**0, -15**
Not Confess	**-15, 0**	**-1, -1**

Highlights

- What is a game?
- What is a strategy?
- Key concepts
  - Extensive form
  - Normal form

Homework Assignment

- **Chapter 2: #1, 2, 5**
- **Chapter 3: #2, 3**