open.michigan

UNIVERSITY OF MICHIGAN

SI 563 Lecture 4a

Bargaining

Professor Yan Chen
Fall 2008

Some material in this lecture drawn from http://gametheory.net/lectures/level.pl

Bargaining Problems

(Watson Chapter 18)

Bargaining: Value Creation and Division

- **Value creation**
  - Trade creates value
  - Gains from trade
- **Value division**
  - Parties jointly decide how to divide the value
  - Bargaining strengths
  - Negotiation procedures
  - Greater contracting environment

Representation of Bargaining Problems

- Example: partnership formation
  - Players 1, 2
  - If form partnership, payoff vector (4, 6)
  - If not, payoff vector (2, 2)
- Bargaining set: set of alternatives for a given bargaining problem

  V= {(4, 6), (2, 2)}
- Default outcome (or disagreement point)

  d = (2, 2)

Representation

- Monetary transfer, t
- Outcome, z

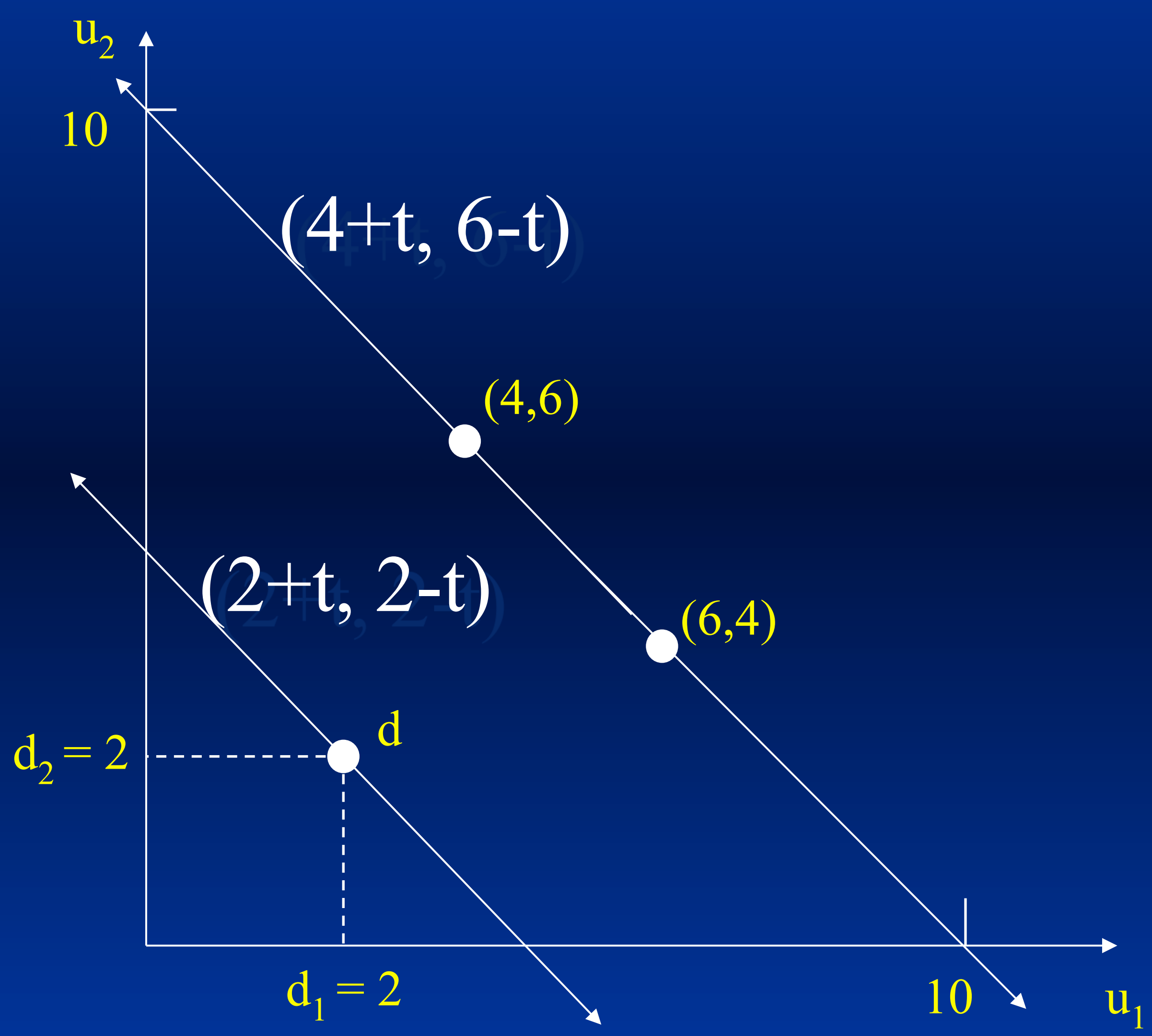
  z=1: forming partnership

  z=0: no partnership
- Transferable utility

  $u_1 = v_1(z) + t$

  $u_2 = v_2(z) - t$
- Efficient outcomes: max joint value

The bargaining set in the partnership example



Joint Value and Surplus

- For any z and t, *joint value* is

  $[v_1(z) + t] + [v_2(z) - t] = v_1(z) + v_2(z)$

- *Surplus* of an agreement is defined as the difference between the joint value of the contract and the default:

  $v_1(z) + v_2(z) - d_1 - d_2$

- *Bargaining power*: bargaining weight

π_i: proportion of surplus obtained by player i

Standard Bargaining Solution

- **Efficient outcome:**

  maximum payoff $v^* = v_1(z) + v_2(z)$

- **Players negotiate over the surplus:**
  $v^* - d_1 - d_2$

- **Standard bargaining solution (Nash)**
  $u_1 = d_1 + \pi_1(v^* - d_1 - d_2)$

Simple Bargaining Games

Using Noncooperative Game Theory (Watson Chapter 19)

What determines a player's bargaining power (weight)?

- Importance of rules:

  The rules of the game determine the outcome

- Diminishing pies:

  The importance of patience

- Estimating payoffs:

  Trust your intuition

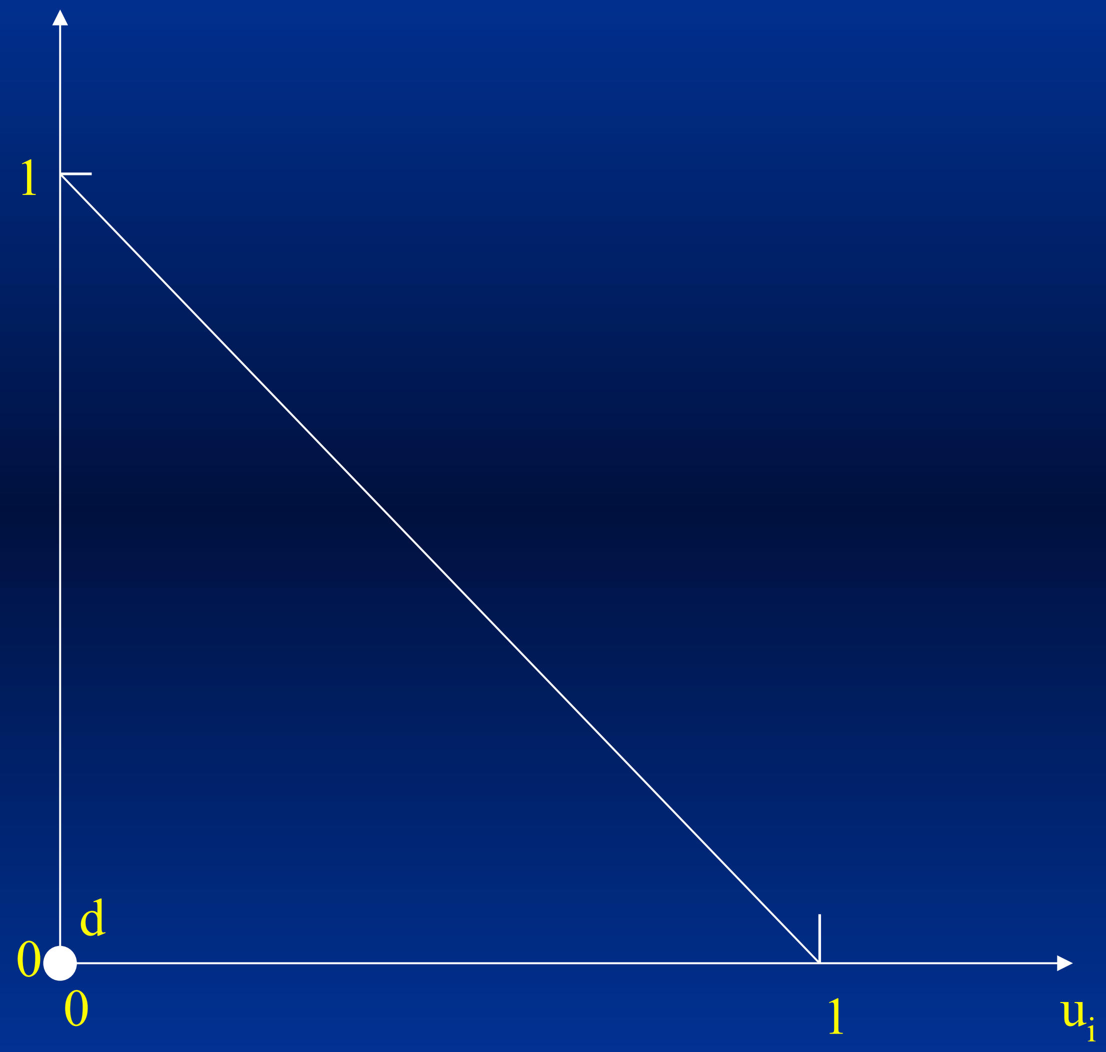
Ultimatum Games: Power to the Proposer

- Consider the following bargaining game (over a cake):
- I name a take-it-or-leave-it split.
- If you accept, we trade
- If you reject, no one eats!
- Under perfect information, there is a simple SPNE

Ultimatum Bargaining: a discrete version

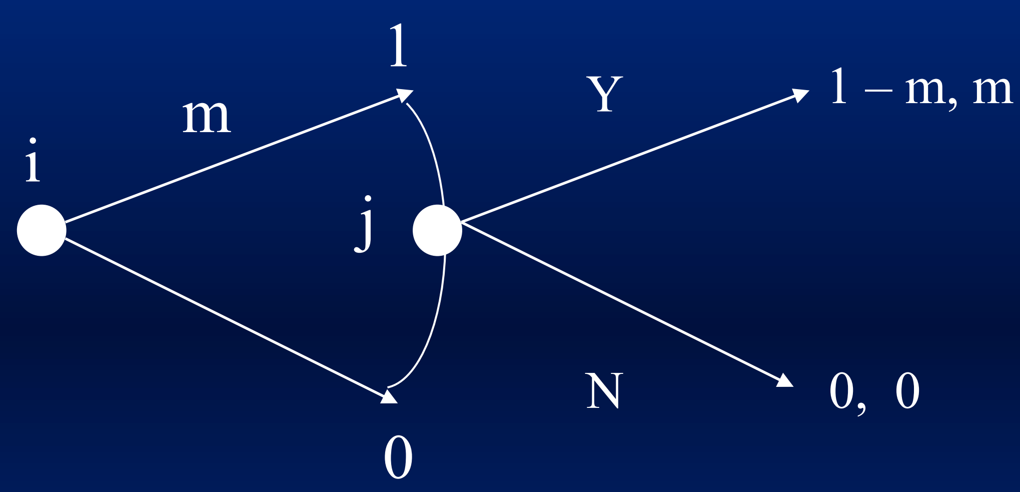
- Suppose I can only propose three divisions, (my share, your share):
  - (¼, ¾)
  - (½, ½)
  - (¾, ¼)
- Draw the extensive form
- Solve for the SPNE

Ultimatum Bargaining: continuous version



Bargaining set; disagreement point

Ultimatum Bargaining: continuous version



(Player i's payoff is listed first.)

Player j: accept if m > 0;
Player i: offer the smallest possible m.
SPNE: {m=0; accept all offers}
Proposer keeps all profits.

Cake Cutting: changing the rules

- Suppose I get to cut the cake in one of three different ways (as before)
- And you get to pick which part is yours
- Draw the extensive form
- Solve for the SPNE

Two-Period Alternating Offer Games: Power to the Patient

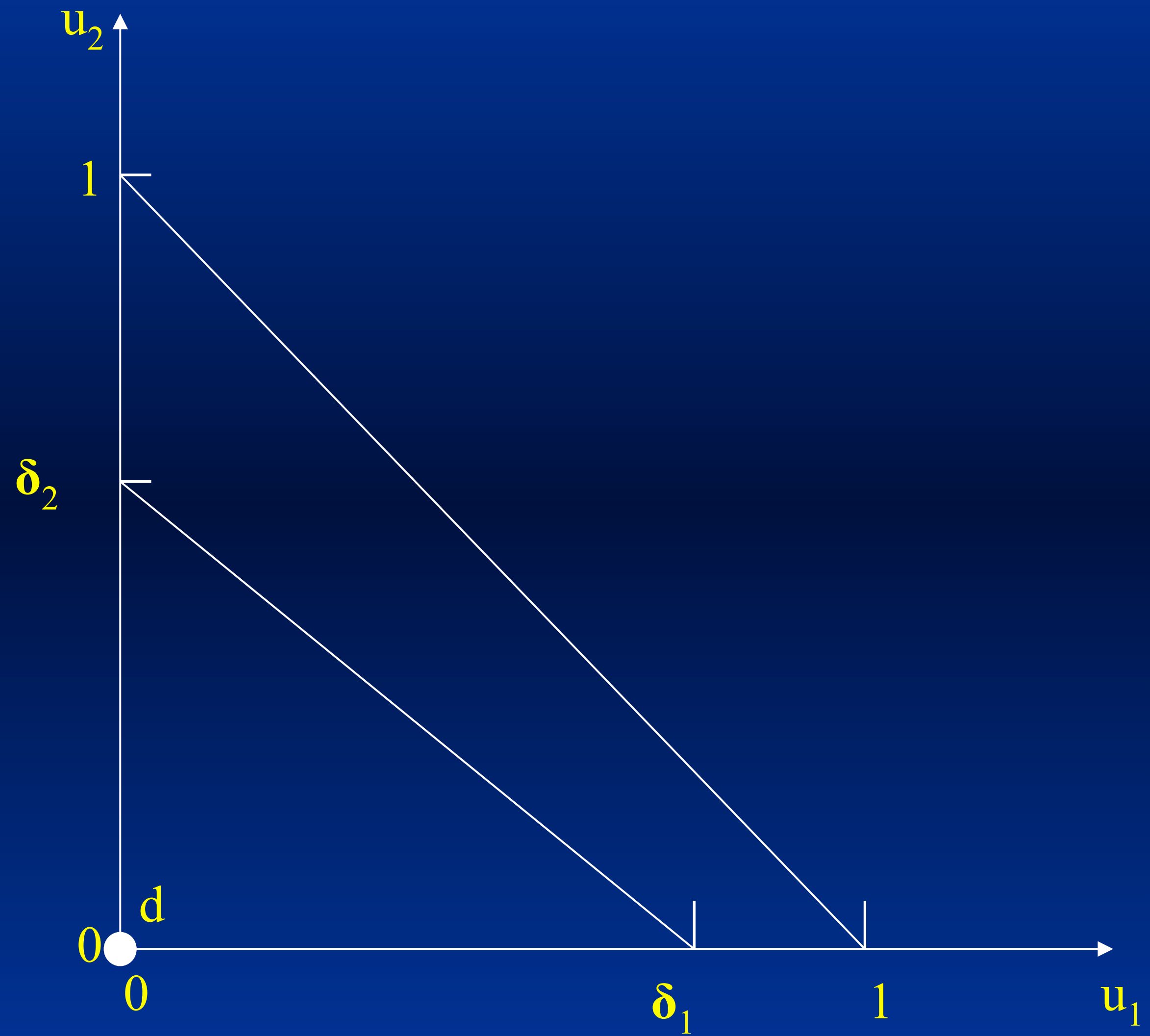
- In general, bargaining takes on a "take-it-or-counteroffer" procedure
- If time has value, both parties prefer trade earlier to trade later
- E.g. Labor negotiations –

  Later agreements come at a price of strikes, work stoppages, etc.
- Delays imply less surplus left to be shared among the parties

Two Stage Bargaining

- Bargaining over division of a cake
- I offer a proportion, *m*, of the cake to you
- If rejected, you may counteroffer (and δ of the cake remains, the rest melts)
- Discount factor: δ
- Payoffs:
  - » In first period: 1-m, m
  - » In second period: δ(1-m), δm

Bargaining set and disagreement point for 2-stage game



Extensive Form

Backward Induction

- Since period 2 is the final period, this is just like a take-it-or-leave-it offer:
  - You will offer me the smallest piece that I will accept, leaving you with all of δ and leaving me with almost 0

- What do I do in the first period?

Backward Induction

- Give you at least as much surplus
- Your surplus if you accept in the first period is 1-m

- Accept if:

Your surplus in 1st period ≥ Your surplus in 2nd period

$$m \geq \delta$$

Backward Induction

- **If there is a second stage,
you get δ and I get 0.**
- **You will reject any offer in the first stage that does not offer you at least δ.**
- **In the first period, I offer you δ.**
- **Note: the *more* patient you are (the slower the cake melts) the *more* you receive** now!

First or Second Mover Advantage?

- **Are you better off being the first to make an offer, or the second?**

Example: Cold Day

- If $\delta=4/5$ (20% melts)

- Period 2: You offer a division of 1,0
  - » You get all of remaining cake = 0.8
  - » I get 0 = 0
- In the first period, I offer 80%
  - » You get 80% of whole cake = 0.8
  - » I get 20% of whole cake = 0.2

Source: Mike Shor, gametheory.net

Example: Hot Day

- If δ=1/5 (80% melts)
- Period 2: You offer a division of 1,0
  - » You get all of remaining cake = 0.2
  - » I get 0 = 0
- In the first period, I offer 20%
  - » You get 20% of whole cake = 0.2
  - » I get 80% of whole cake = 0.8

Source: Mike Shor, gametheory.net

First or Second Mover Advantage?

- **When players are impatient (hot day)**
  First mover is better off
  - Rejecting my offer is less credible since we both lose a lot

- **When players are patient (cold day)**
  Second mover better off
  - Low cost to rejecting first offer

- **Either way – if both players think through it, deal struck in period 1**

Source: Mike Shor, gametheory.net

Don't Waste Cake

- In any bargaining setting, strike a deal as early as possible!
- Why doesn't this happen?
  - Reputation building
  - Lack of information

Source: Mike Shor, gametheory.net

Uncertainty in Civil Trials

- Plaintiff sues defendant for $1M
- Legal fees cost each side $100,000
- If each agrees that the chance of the plaintiff winning is ½:
  - » Plaintiff: $500K - $100K = $ 400K
  - » Defendant: -$500K - $100K = -$600K
- If simply agree on the expected winnings, $500K, each is better off

Source: Mike Shor, gametheory.net

Uncertainty in Civil Trials

- What if both parties are too optimistic?
- Each thinks that his or her side has a ¾ chance of winning:
  - » Plaintiff: $750K - $100K = $ 650K
  - » Defendant: - $250K - $100K = $-350K
- No way to agree on a settlement!

Source: Mike Shor, gametheory.net

Lessons

- **Rules of the bargaining game uniquely determine the bargaining outcome**
- **Which rules are better for you depends on patience, information**
- **What is the smallest acceptable piece? Trust your intuition**
- **Delays are always less profitable: Someone must be wrong**

Source: Mike Shor, gametheory.net

Homework Assignment

- **Chapter 19: #1, 2, 7, 8**