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SI 563 Lecture 5

Repeated Games and Reputation

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Some material in this lecture drawn from http://gametheory.net/lectures/level.pl

Agenda

- Finitely repeated games
- Infinitely repeated games
- Folk Theorems
  - Minmax
  - Nash-threat
- Fun project: ad auction (Next Class)

Repeated Games and Reputation

(Watson Chapter 22)

Repeated Interaction

- Empirical observations
  - People often interact in ongoing relationships
  - Your behavior today might influence actions of others in the future
- New dimension: time
- Questions
  - What if interaction is repeated?
  - What strategies can lead players to cooperate?

Definitions

- **Repeated game:**
  played over discrete periods of time (period 1, period 2, and so on)
  - **t: any given period**
  - **T: total number of periods**
- **In each period, players play a static stage game**
- **History of play: sequence of action profiles**

A Two-Period Repeated Game

1 \ 2	X	Y	Z
A	$\underline{4}$,3	0,0	$\underline{1}$,$\underline{4}$
B	0,0	$\underline{2}$,$\underline{1}$	0,0

Stage game, repeated once (T = 2)

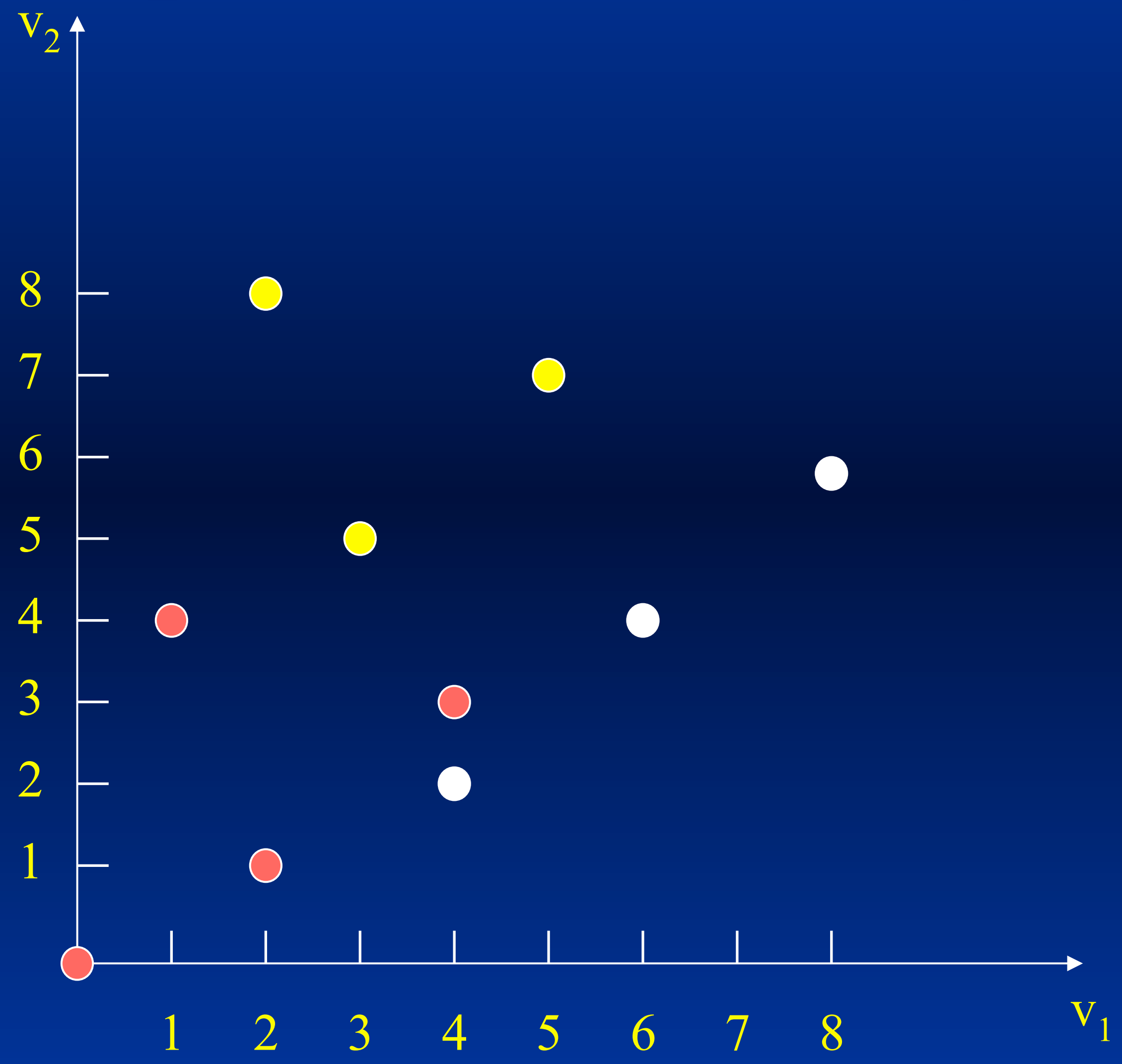
Stage game NE: (A, Z), (B, Y)

Subgame Following (A, Z)

1 \ 2	X	Y	Z
A	**5,7**	**1,4**	**2,8**
B	**1,4**	**3,5**	**1,4**

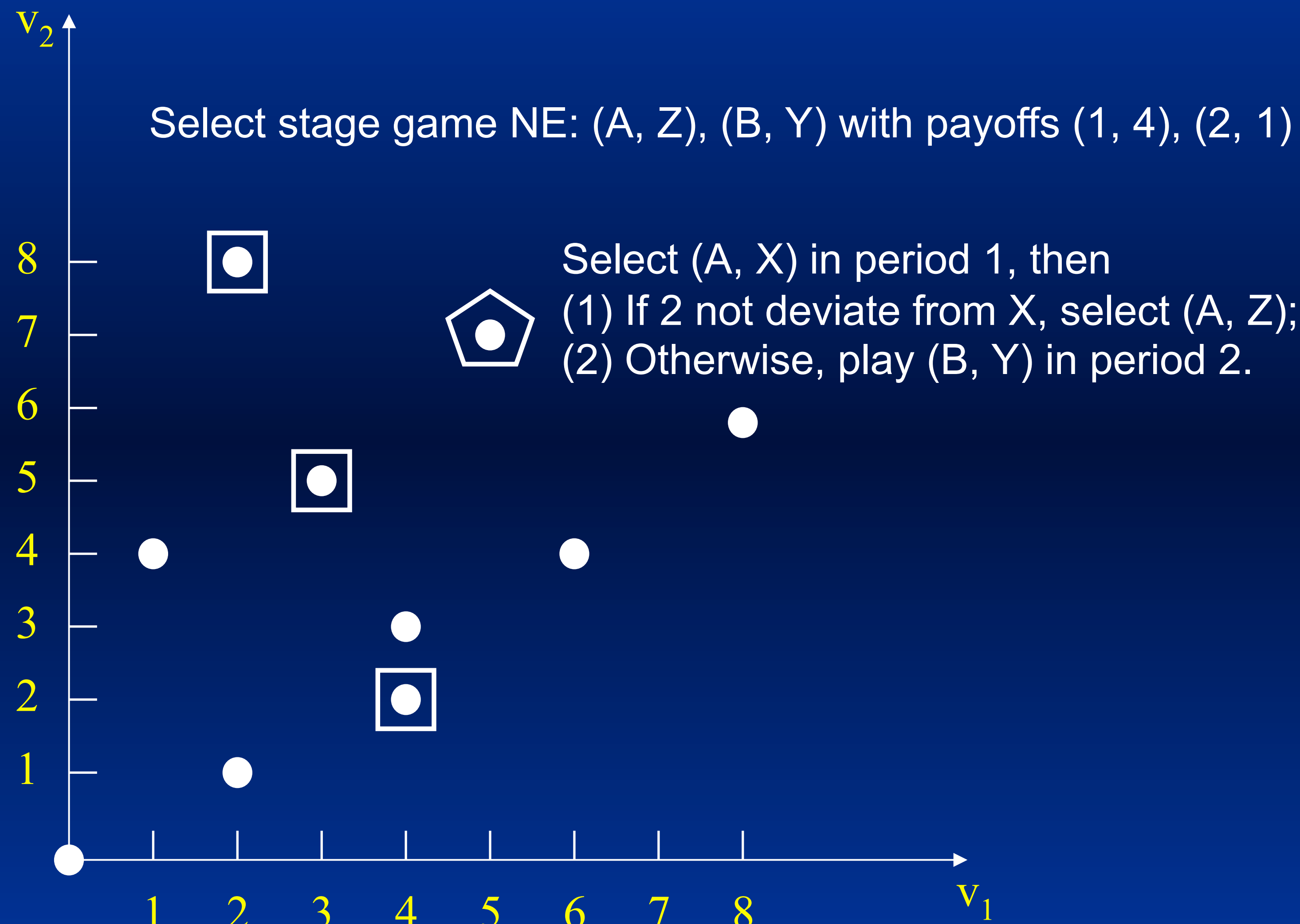
The subgame following (A,Z), with payoffs (1, 4)

Repeated Game Payoffs



All possible repeated game payoffs: larger set

Stage Nash Profile and SPNE



Result: Any sequence of stage Nash profiles can be supported as the outcome of a SPNE. And there are more SPNE!

A Two-Period Repeated Game: Reputational Equilibrium as SPNE

- Reputational equilibrium:
  - Nonstage Nash profile in 1st period
  - Stage Nash profile in 2nd period
  - 2nd period actions contingent on outcome in first period (whether players cheat or not)
- Example:
  - Select (A, X) in 1st period
  - If player 2 chooses X in 1st period, select (A, Z) in 2nd period
  - If player 2 chooses Y or Z in 1st period, select (B, Y) in 2nd period

Infinitely Repeated Games

- Discounting (δ):
  future payoffs not as valuable as current payoffs
  - A fixed known chance of game's ending after each round, p
  - Interest rate, r
  - $\delta = 1 - p = 1/(1+r)$

Aside: Discounting

- Discounting:
  - Present-day value of future profits is less than value of current profits

- *r* is the interest rate
  - Invest \$1 today → get \$(1+r) next year
  - Want \$1 next year → invest \$1/(1+r) today
  - Annuity paying \$1 today and \$1 every year has a net present value of \$ 1+1/r

Aside: Infinite Sums

$$1+\frac{1}{(1+r)}+\frac{1}{(1+r)^2}+\frac{1}{(1+r)^3}+\frac{1}{(1+r)^4}+\ldots=1+\frac{1}{r}$$

or:

$$1+\delta+\delta^2+\delta^3+\ldots=\frac{1}{(1-\delta)}$$

- **Why?**

$$s=1+\delta+\delta^2+\delta^3+\ldots$$
$$s=1+\delta s$$
$$s=\frac{1}{1-\delta}$$

The Prisoner's Dilemma

Equilibrium: $54 K

		Firm 2	
		Low	High
Firm 1	Low	$\underline{54}$, $\underline{54}$	$\underline{72}$, 47
	High	47 , $\underline{72}$	60 , 60

Cooperation: $60 K

Source: Mike Shor, gametheory.net

Prisoner's Dilemma

- Private rationality ➔ collective irrationality
  - » The equilibrium that arises from using dominant strategies is worse for every player than the outcome that would arise if every player used her dominated strategy instead
- Goal:
  - » To sustain mutually beneficial cooperative outcome overcoming incentives to cheat

Source: Mike Shor, gametheory.net

Moving Beyond the Prisoner's Dilemma

- **Why does the dilemma occur?**
  - Interaction
    - » No fear of punishment
    - » Short term or myopic play
  - Firms:
    - » Lack of monopoly power
    - » Homogeneity in products and costs
    - » Overcapacity
    - » Incentives for profit or market share
  - Consumers
    - » Price sensitive
    - » Price aware
    - » Low switching costs

Source: Mike Shor, gametheory.net

Altering Interaction

- Interaction
  - No fear of punishment
    - Exploit repeated play
  - Short term or myopic play
    - Introduce repeated encounters
    - Introduce uncertainty

Source: Mike Shor, gametheory.net

Long-Term Interaction

- **No last period, so no backward induction**
- **Use history-dependent strategies**
- *Trigger strategies:*
  - » **Begin by cooperating**
  - » **Cooperate as long as the rivals do**
  - » **Upon observing a defection:**

    immediately revert to a period of punishment of specified length in which everyone plays non-cooperatively

Two Trigger Strategies

- Grim trigger strategy
  - Cooperate until a rival deviates
  - Once a deviation occurs, play non-cooperatively for the rest of the game

- Tit-for-tat
  - Cooperate if your rival cooperated in the most recent period
  - Cheat if your rival cheated in the most recent period

Trigger Strategy Extremes

- Tit-for-Tat is
  - most forgiving
  - shortest memory
  - proportional
  - credible
    but lacks deterrence

Tit-for-tat answers:

"Is cooperation easy?"

- Grim trigger is
  - least forgiving
  - longest memory
  - adequate deterrence
    but lacks credibility

Grim trigger answers:

"Is cooperation possible?"

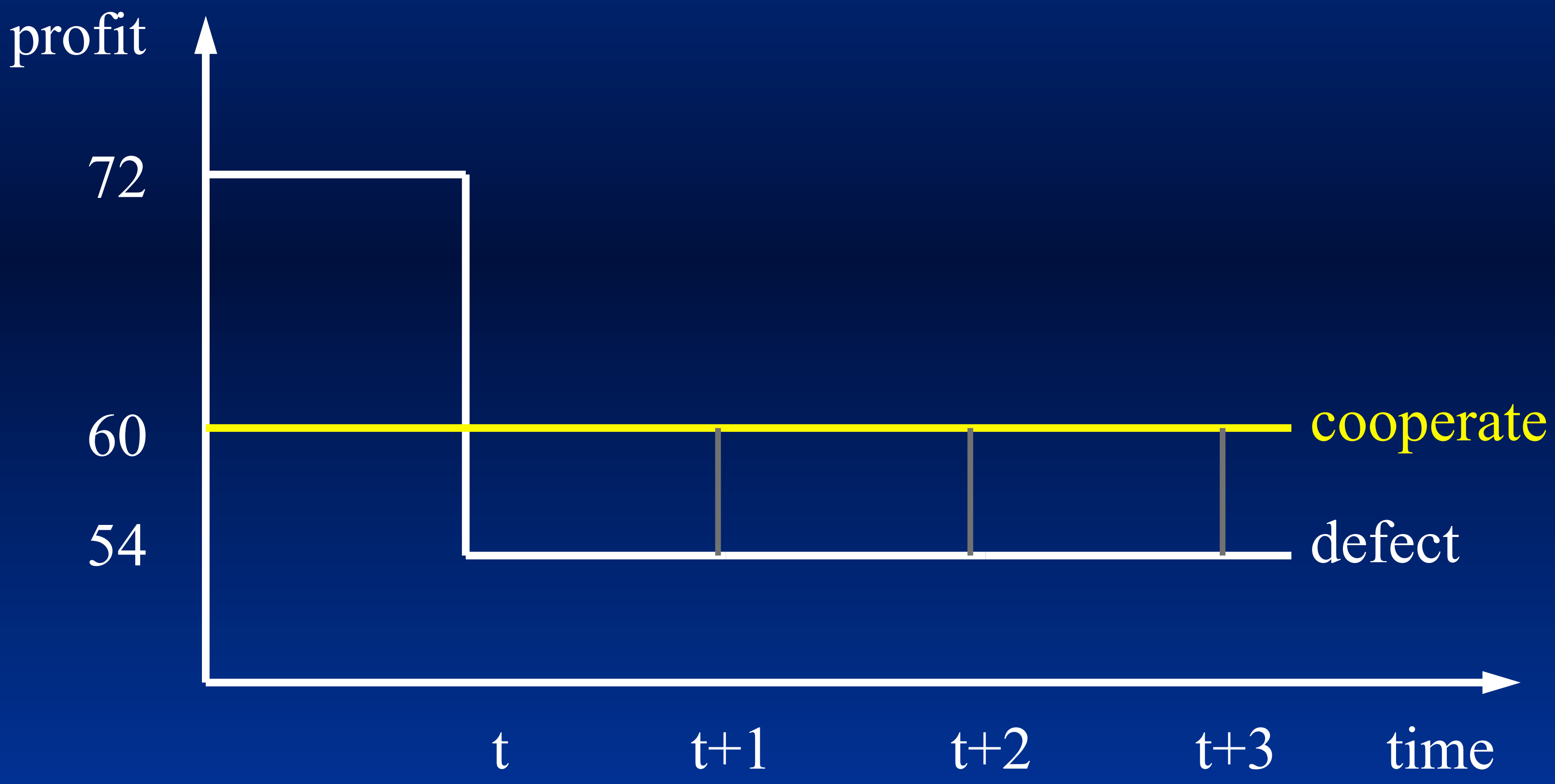
Why Cooperate (Against Grim Trigger Strategy)?

- Cooperate if the present value of cooperation is greater than the present value of defection

		Firm 2	
		Low	High
Firm 1	Low	54 , 54	72 , 47
	High	47 , 72	60 , 60

- Cooperate: 60 today, 60 next year, 60 … 60
- Defect: 72 today, 54 next year, 54 … 54

Payoff Stream (GTS)



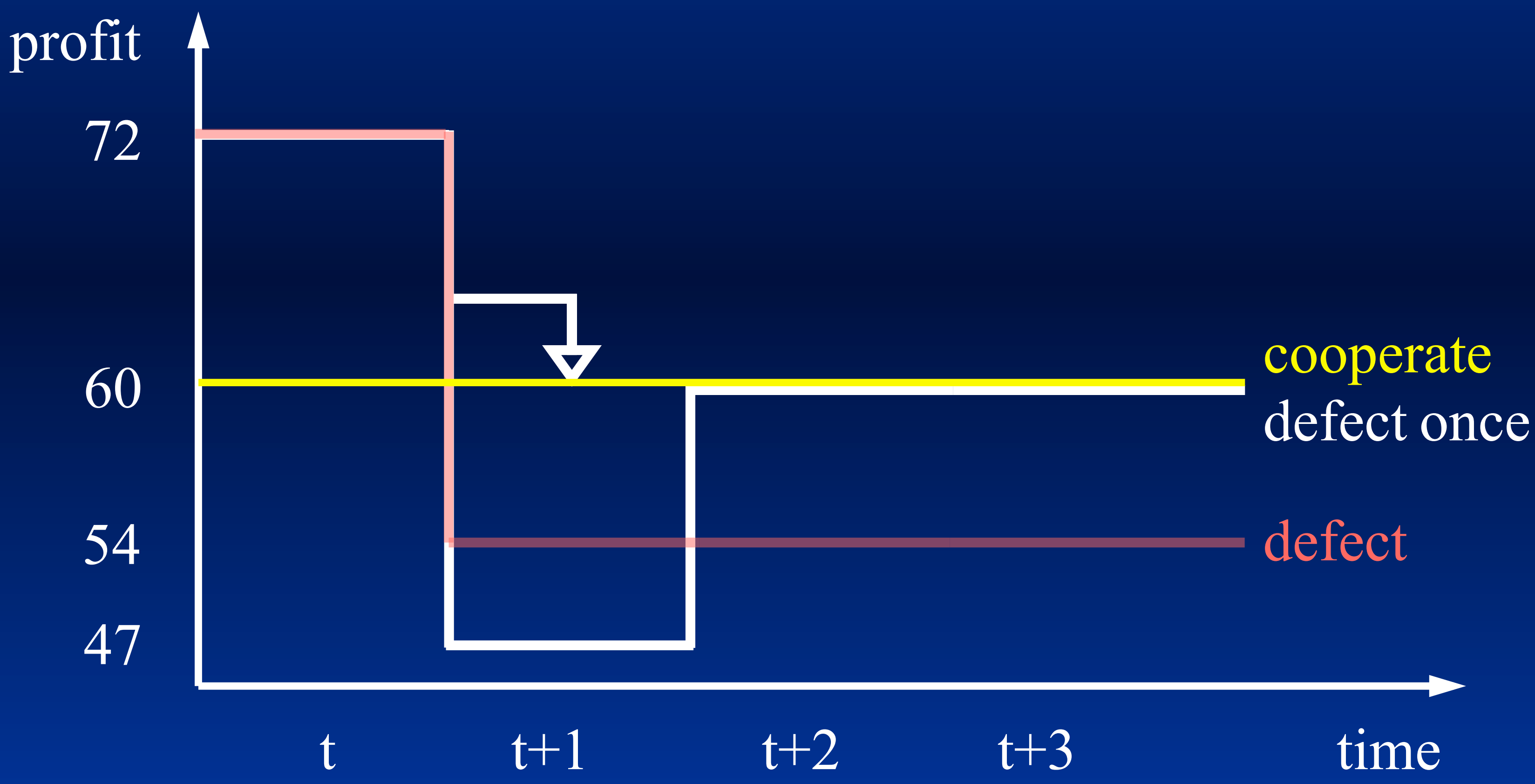
Calculus of GTS

- **Cooperate if**

PV(cooperation)	>	PV(defection)
60…60…60…60…	>	72…54…54…54…
$60/(1-\delta)$	>	$72 + 54\delta/(1-\delta)$
18δ	>	12
δ	>	$2/3$

- **Cooperation is sustainable using grim trigger strategies as long as $\delta > 2/3$**

Payoff Stream (TFT)



Calculus of TFT

- **Cooperate if**

$$PV(\text{cooperation}) > PV(\text{defection})$$

and

$$PV(\text{cooperation}) > PV(\text{defect once})$$

$$60\ldots60\ldots60\ldots60\ldots > 72\ldots47\ldots60\ldots60\ldots$$

$$60 + 60\,\delta > 72 + 47\,\delta$$

$$13\,\delta > 12$$

$$\delta > 12/13$$

- **Much harder to sustain than grim trigger**
- **Cooperation may not be likely**

Trigger Strategies

- **Grim Trigger and Tit-for-Tat are extremes**
- **Balance two goals:**

Deterrence

- » **GTS is adequate punishment**
- » **Tit-for-tat might be too little**

Credibility

- » **GTS hurts the punisher too much**
- » **Tit-for-tat is credible**

Axelrod's Simulation

- R. Axelrod, *The Evolution of Cooperation*
- Prisoner's Dilemma repeated 200 times
- Game theorists submitted strategies
- Pairs of strategies competed
- Winner: Tit-for-Tat
- Reasons:
    - »Forgiving, Nice, Provocable, Clear

Main Ideas from Axelrod

- Not necessarily tit-for-tat
  - » Doesn't always work
- Don't be envious
- Don't be the first to cheat
- Reciprocate opponent's behavior
  - » Cooperation and defection
- Don't be too clever

Summary

- Cooperation
  » Struggle between high profits today and a lasting relationship into the future
- Deterrence
  » A clear, provocable policy of punishment
- Credibility
  » Must incorporate forgiveness
- Looking ahead:
  » How to be credible?

Another PD Example

1 \ 2	C	D
C	**4,4**	**-2,6**
D	**6,-2**	**0,0**

When cooperation can be sustained: grim trigger

Conditions under which cooperation can be sustained:
We check whether Grim Trigger can form a SPNE:
Suppose j plays GT. If i also plays GT, her payoff is

$$4 + 4\delta + 4\delta^2 + 4\delta^3 + ... = \frac{4}{1-\delta}.$$

If i defects, she gets 6 in period of defection, and 0 afterwards.
Player i has an incentive to cooperate if

$$\frac{4}{1-\delta} \geq 6, \;\; or \;\; \delta \geq \frac{1}{3}$$

Modified Grim Trigger

- Players alternate between (C, C) and (D, C) over time, starting with (C, C)
- If either or both deviates from the alternating strategy, both will revert to the stage Nash profile, (D, D)
- Can MGT be supported as a SPNE?

Suppose 2 plays MGT. If 1 also plays MGT, 1's payoff is

$$\begin{aligned}\mathrm{PV}_1 &= 4 + 6\delta + 4\delta^2 + 6\delta^3 + \ldots \\ &= 4(1 + \delta^2 + \delta^4 + \ldots) + 6\delta(1 + \delta^2 + \delta^4 + \ldots) \\ &= \frac{4 + 6\delta}{1 - \delta^2}.\end{aligned}$$

If 2 plays MGT, 2's payoff is

$$\begin{aligned}\mathrm{PV}_2 &= 4 - 2\delta + 4\delta^2 - 2\delta^3 + \ldots \\ &= 4(1 + \delta^2 + \delta^4 + \ldots) - 2\delta(1 + \delta^2 + \delta^4 + \ldots) \\ &= \frac{4 - 2\delta}{1 - \delta^2}.\end{aligned}$$

(1) If 2 defects in an odd-numbered period,
her payoff is 6 in this round, and 0 after:
2 has no incentive to deviate in any odd-numbered period, if

$$\frac{4-2\delta}{1-\delta^2} \geq 6, \quad or \quad 3\delta^2 - \delta - 1 \geq 0, \quad or \quad \delta \geq 0.77.$$

(2) If 2 defects in an even-numbered period,
her payoff is 0 in this round, and 0 after:
2 has no incentive to deviate in any even-numbered period, if

$$\frac{-2+4\delta}{1-\delta^2} \geq 0, \quad or \quad \delta \geq 0.5.$$

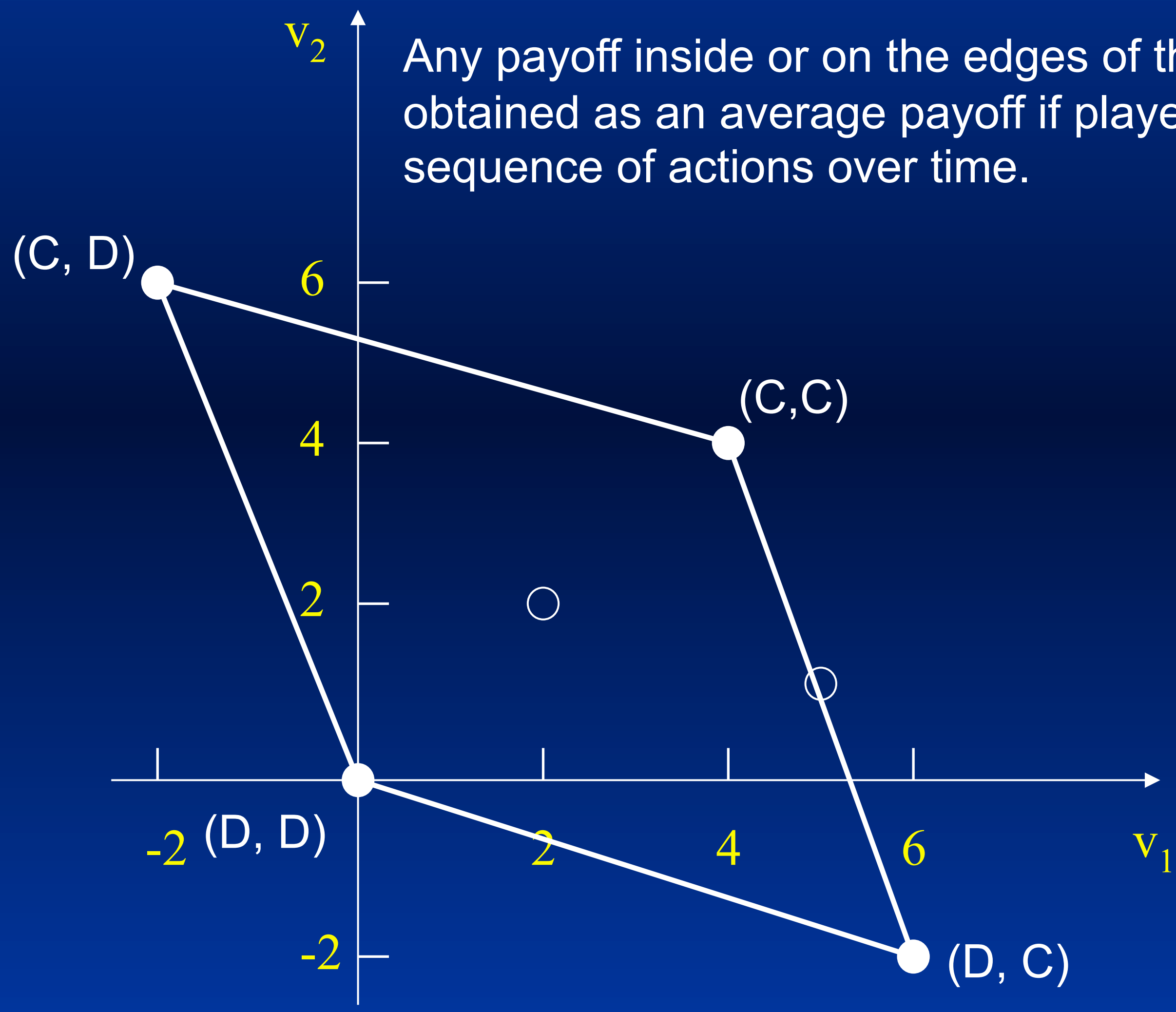
Therefore, MGT can be supported as SPNE if $\delta \geq 0.77$.

Equilibrium Payoff Set with Discounting

- **Depending on the discount factor, there are many SPNE in the repeated PD**
  - **(D, D) in every period**
  - **(GT, GT)**
  - **(TFT, TFT) etc.**

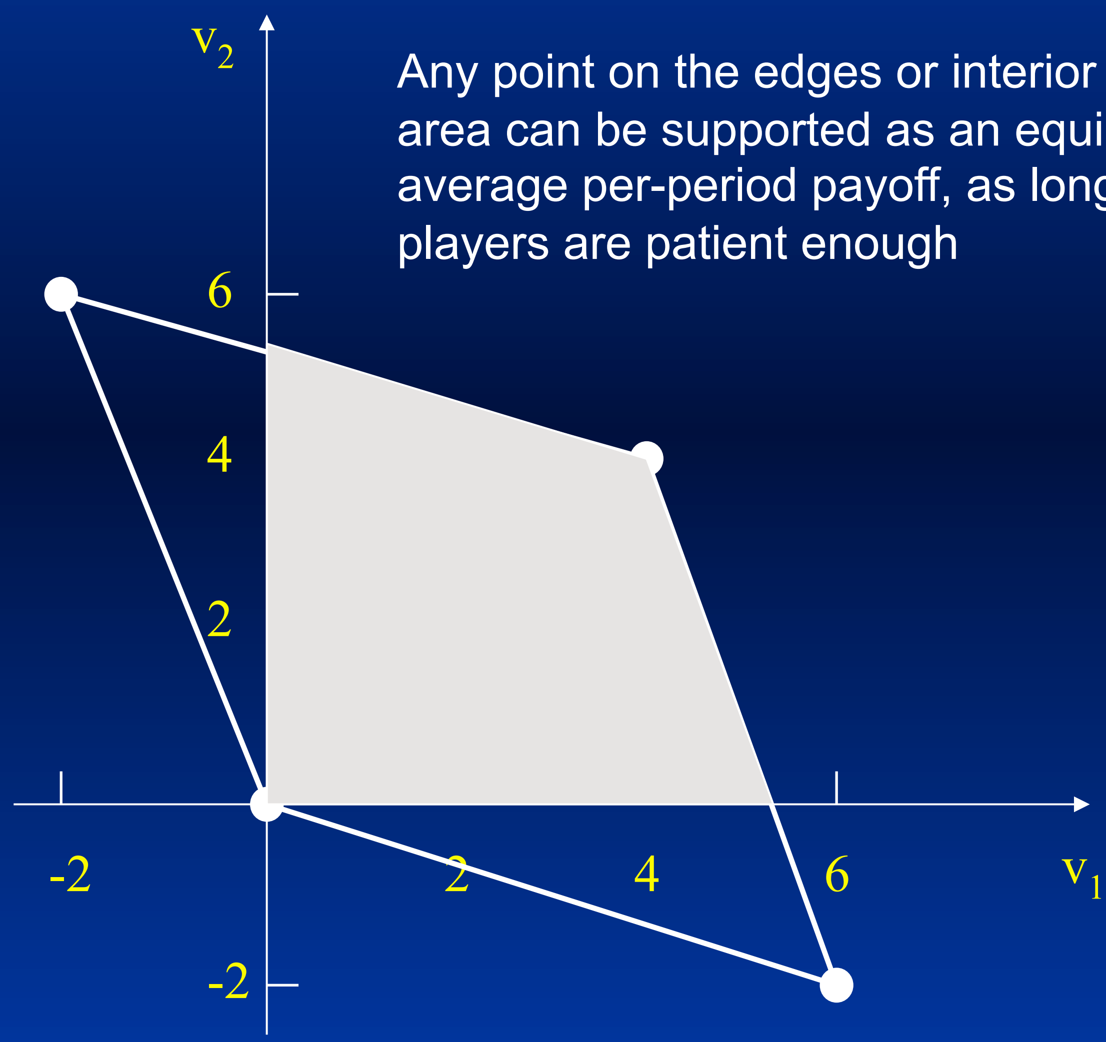
Possible Repeated Game Payoffs: Per Period

Any payoff inside or on the edges of the diamond can be obtained as an average payoff if players choose the right sequence of actions over time.



Equilibrium Per-Period Payoffs

Any point on the edges or interior of the shaded area can be supported as an equilibrium average per-period payoff, as long as the players are patient enough



Folk Theorems

- **The Nash-threat Folk Theorem:**

  For repeated games with stage game G, for any feasible payoffs (M) greater than or equal to the Nash equilibrium payoffs, and for sufficiently large discount factor, there is a SPNE that has payoffs M.

Applications

- Governing the Commons – The Evolution of Institutions for Collective Action by Elinor Ostrom
- International trade agreements
- eBay's reputation system

(Check out Chapter 23)

Highlights

- Finitely repeated games
- Infinitely repeated games
- Folk theorems
- Next week:
  - Games with Incomplete Information
- Fun exercise: ad words auction

Homework Assignment

- Chapter 22: #1, 2, 3, 5