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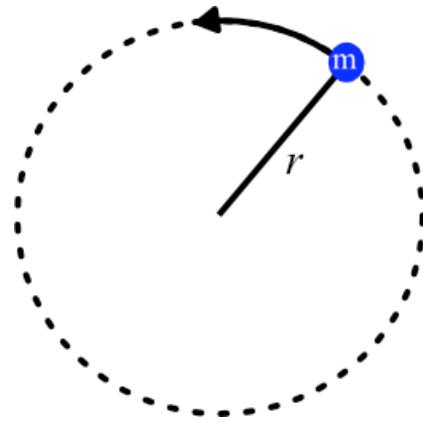


# Physics 140 – Fall 2007

lecture #15 : 25 Oct

## Ch 9 topics:

- rotational kinematics
- rotational kinetic energy
- moment of inertia

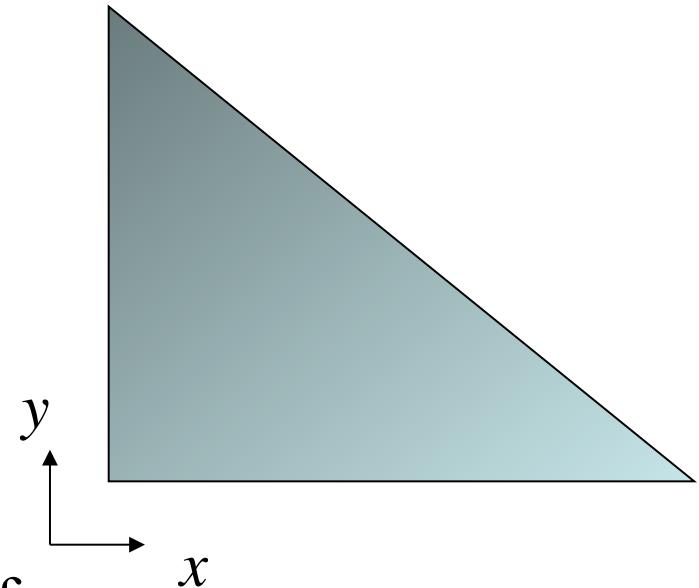


- exam #2 is next Thursday, 1 November, 6:00-7:30pm
- covers Chapters 6-8
- practice exam on CTools site -> Exams & Grading  
bring two 3x5 notecards, calculator, #2 pencils
- review next Monday evening, 29 October, 8:00-9:30pm

## Center of Mass of an extended, non-uniform object

An object with surface mass density  $\sigma(\mathbf{r})$ , shown here in 2D, has a total mass given by the integral

$$M = \iint_{\text{object}} dx dy \sigma(\mathbf{r})$$



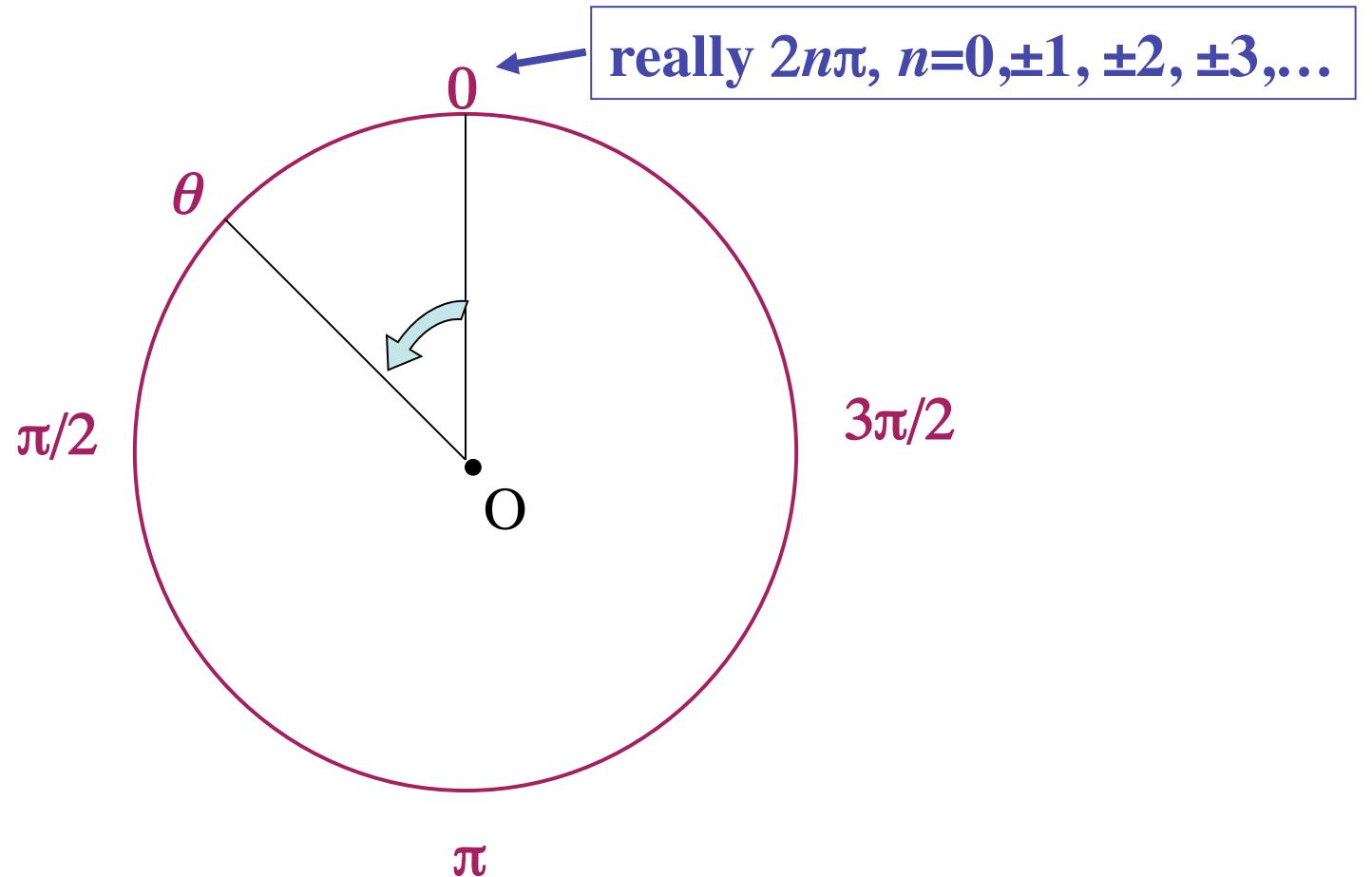
Its center of mass is defined by integrals of the mass-weighted positions:

$$x_{\text{com}} = \frac{1}{M} \iint_{\text{object}} dx dy \sigma(\mathbf{r}) x$$

$$y_{\text{com}} = \frac{1}{M} \iint_{\text{object}} dx dy \sigma(\mathbf{r}) y$$

# Rotational kinematics

To describe rotational motion, we begin with the **angular position  $\theta$**  (in radians) measured relative to an (arbitrary) reference angle.



## Rotational kinematics

A change in angular position,  $\Delta\theta$ , during a time interval  $\Delta t$  implies a non-zero average *angular velocity*

$$\omega_{\text{avg}} = \Delta\theta / \Delta t$$

A change in angular velocity,  $\Delta\omega$ , defines an average *angular acceleration*

$$\alpha_{\text{avg}} = \Delta\omega / \Delta t$$

The limit  $\Delta t \rightarrow 0$  defines instantaneous measures for these

$$\omega = d\theta / dt$$

$$\alpha = d\omega / dt$$

The snapshots of the ball are shown at fixed time intervals. Note how the angular displacement between images grows in time, implying an angular acceleration  $\alpha > 0$ .



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# Relations to translational kinematics

The magnitudes of translational quantities - displacement, velocity and tangential acceleration ( $l, v, a_{\tan}$ ) - are equal to the angular equivalent measures ( $\theta, \omega, \alpha$ ) multiplied by the distance  $r$  from the rotation axis.

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**TABLE 8.2 Linear and angular kinematic parameters**

At fixed  $r$ , the components of:  
displacement  
tangential velocity  
tangential accel.  $a_{\tan}$

Equation	Units
$l = r\theta$	$l$ (m)
$v = r\omega$	$v$ (m/s)
$a_T = r\alpha$	$a_T$ (m/s <sup>2</sup> )

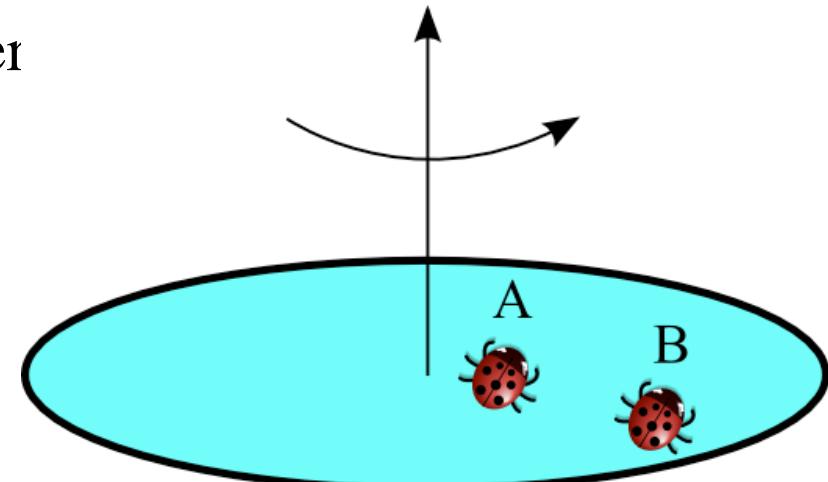
## Kinematic equations for rotation

The kinematic equations developed in Chapter 2 for translational motion also apply to rotational motion. (see Table 9.1 in Y&F)

Rotational Motion ( $\alpha = \text{constant}$ )	Linear Motion ( $a = \text{constant}$ )
$\omega = \omega_0 + \alpha t$	$v = v_0 + at$
$\theta = \frac{1}{2} (\omega_0 + \omega) t$	$x = \frac{1}{2} (v_0 + v) t$
$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$	$x = v_0 t + \frac{1}{2} at^2$
$\omega^2 = \omega_0^2 + 2\alpha\theta$	$v^2 = v_0^2 + 2ax$

Two ladybugs rest without slipping on a rotating platter that is increasing its angular velocity. Ladybug A is closer to the rotation axis than bug B.

Which statement correctly describes the relationship between the bugs' angular accelerations ( $\alpha$ ) and centripetal accelerations ( $a_{\text{rad}}$ )?



- 1)  $\alpha_A > \alpha_B$  and  $a_{\text{rad},A} > a_{\text{rad},B}$
- 2)  $\alpha_A < \alpha_B$  and  $a_{\text{rad},A} < a_{\text{rad},B}$
- 3)  $\alpha_A = \alpha_B$  and  $a_{\text{rad},A} < a_{\text{rad},B}$
- 4)  $\alpha_A = \alpha_B$  and  $a_{\text{rad},A} = a_{\text{rad},B}$
- 5)  $\alpha_A = \alpha_B$  and  $a_{\text{rad},A} > a_{\text{rad},B}$

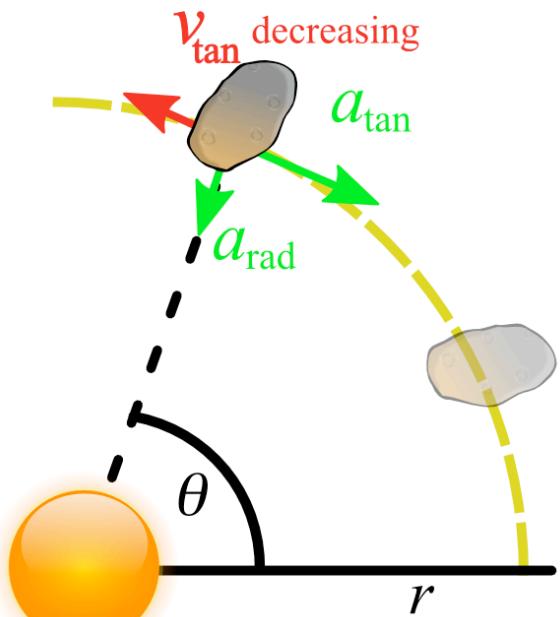


Negotiating circular motion at tangential speed  $v_t$  around a circular arc of radius  $r$  still requires a *radial component* of acceleration

$$a_{\text{rad}} = v_t^2 / r = \omega^2 r$$

directed towards the center of the circle.

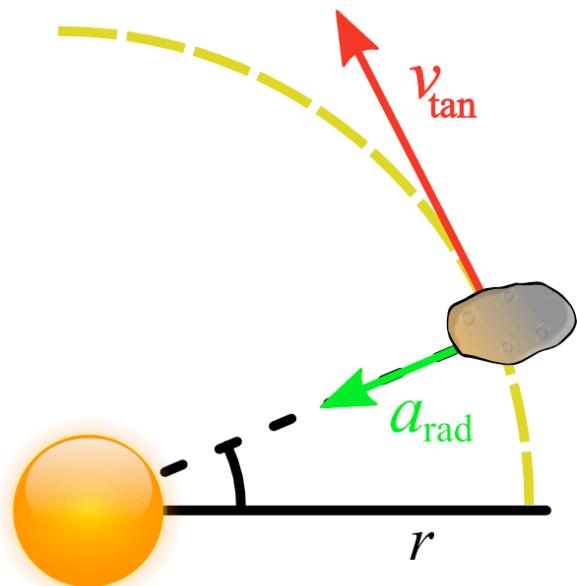
This component changes the direction of the velocity, keeping it tangent to the circle.



The *tangential component* of acceleration

$$a_{\text{tan}} = \alpha r$$

changes the speed,  $\frac{dv_t}{dt} = a_{\text{tan}}$ .



## Rotational kinetic energy and moment of inertia

A set of masses  $m_i$  uniformly rotating with angular velocity  $\omega$  about some fixed axis A possesses a kinetic energy defined by

$$K = \frac{1}{2} \sum_i m_i v_i^2 = \frac{1}{2} \sum_i m_i r_i^2 \omega^2$$

where  $r_i$  is the distance from the  $i^{\text{th}}$  mass to the rotation axis.

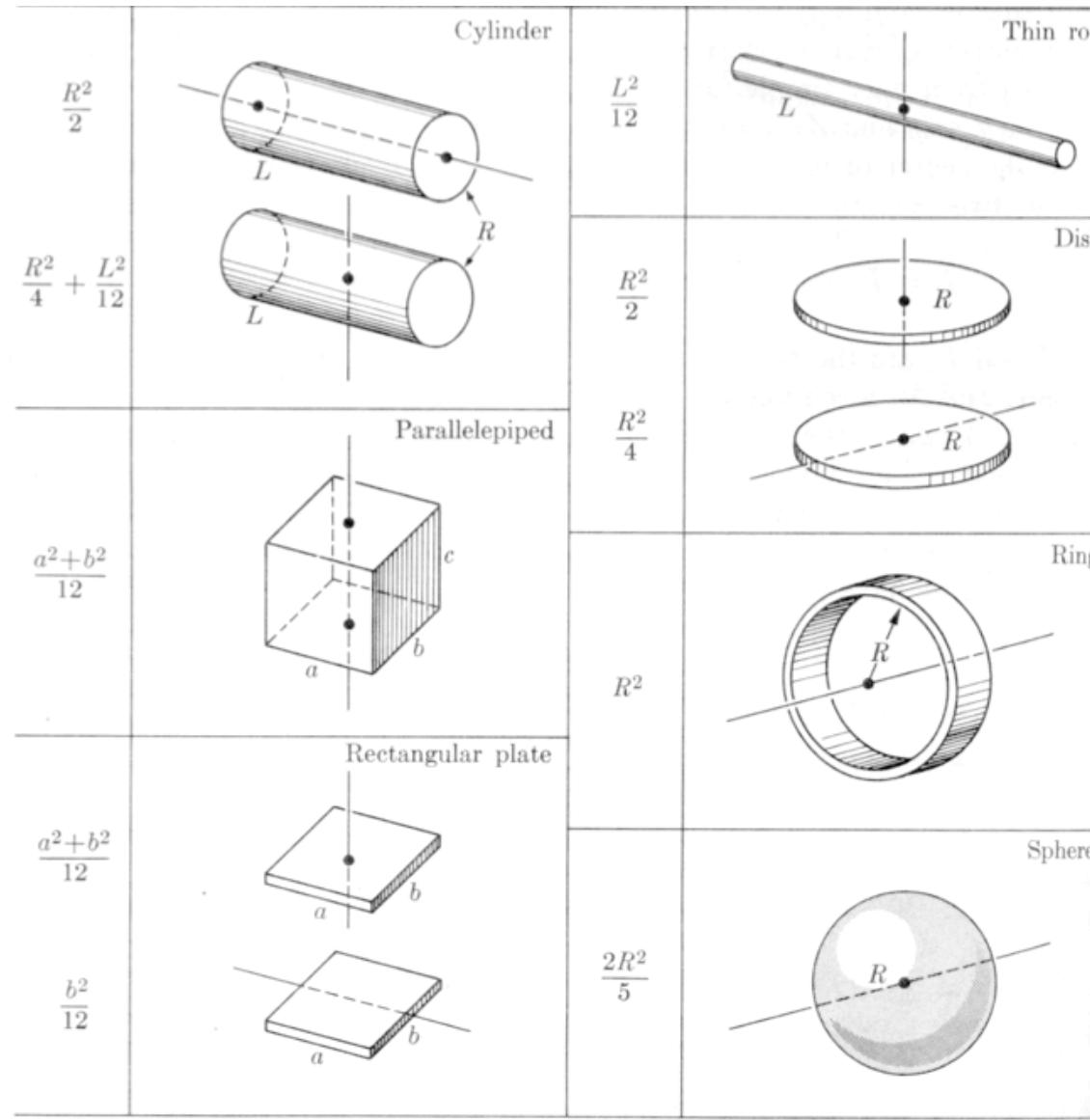
For such a set of mass, or for a continuous body, we define the *moment of inertia I about the specified axis A* as

$$I = \sum_i m_i r_i^2$$

Then the rotational kinetic energy can be written as

$$K = \frac{1}{2} I \omega^2$$

A given object has only one mass  $m$ , but **many** moments of inertia  $I$ , *depending on the location and orientation of the rotation axis*.

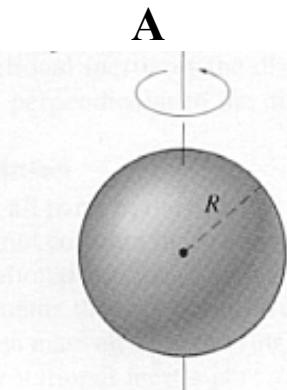


Note: this graphic assumes an object of unit mass ( $M=1$ ).

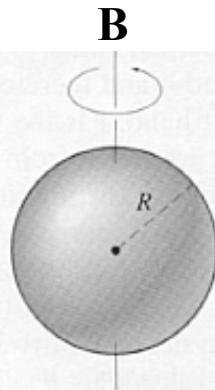
Refer to Table 9.2 in YF for a similar list.

Source: Undetermined

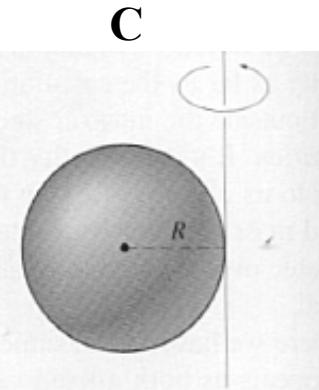
solid sphere



hollow sphere



solid sphere



Source: Undetermined

The three spheres above have the same mass  $M$  and the same radius  $R$ . Sphere B is hollow, A and C are solid. Sphere C rotates about an axis adjacent to its edge while spheres A and B rotate about their centers. All rotate at the same angular velocity. Rank the spheres according to their rotational kinetic energy, largest to smallest.

1. A, B, C
2. B, A, C
3. A, C, B
4. C, B, A

