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COMMENTS ON THE USE OF SEMI-RIGID CONNECTIONS IN STEEL FRAMES

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LIST OF FIGURES

INTRODUCTION

For almost thirty years the effect of the deformations in the connections of structural members, particularly beams to columns, upon the stresses in steel frames has been investigated both experimentally and analytically. Although considerable progress has been indicated by the laboratory and research reports (see references) on methods for determining the physical characteristics of various riveted and welded connections, the uncertainties that are involved when variations occur in both beams, columns, and connecting elements still plague the structural designer. Now with the increased use of plastic or ultimate methods of design, it becomes even more essential to consider the influence of the connecting elements in the structural behavior of the frame. If the design assumes elastic deformations in both members and connections, then only certain types of connections will qualify.

It is therefore important that the engineer should test the particular connections that are likely to be used and that he should be able to make such tests quickly and with reasonable accuracy. In this paper a relatively simple laboratory procedure that requires no sensitive equipment or experienced personnel is described and the use of the data in obtaining the necessary beam coefficients is discussed. Once these coefficients are determined, the analysis can proceed as for any frame with variable moment of inertia.

Test Procedure For Measuring Beam Connection Properties.

The following laboratory procedures and interpretation of the data has been found to be convenient and sufficiently accurate for measuring the rotational restraint of beam connections.

1. The elastic portion a of a typical test specimen as shown in Figure 1 is assumed to extend within three inches of the edge of the connection. Strain gage measurements have shown that between this section and the face of the column, the strain distribution across any transverse section is non-linear. This region is called inelastic although such a description is open to question.

2. The resultant angle changes φ, Figure 2, for the inelastic range is assumed to occur at each face of the column for purposes of reference.

3. A laboratory test specimen as shown in Figure 1 is therefore divided into an elastic zone a and an inelastic zone L-a which contains the connecting elements and column section.

4. The specimen is loaded as shown in Figure 1 and the only measurement needed besides the central load P is the displacement Δ at the center of the span. This displacement can be measured with ordinary dial gages.

5. The numerical value of the rotation φ for the inelastic zone can now be determined by subtracting the calculated displacement Δ_e at the center due to the strain in the elastic portions a from the total measured deflection Δ. Thus, from Figure 2, the following relations can be established.

$$\varphi\,(L-d) = \Delta - \Delta_e = \Delta - \frac{Pa^3}{6EI}$$

or

$$\varphi = \frac{\Delta - \frac{Pa^3}{6EI}}{L-d} \qquad (1)$$

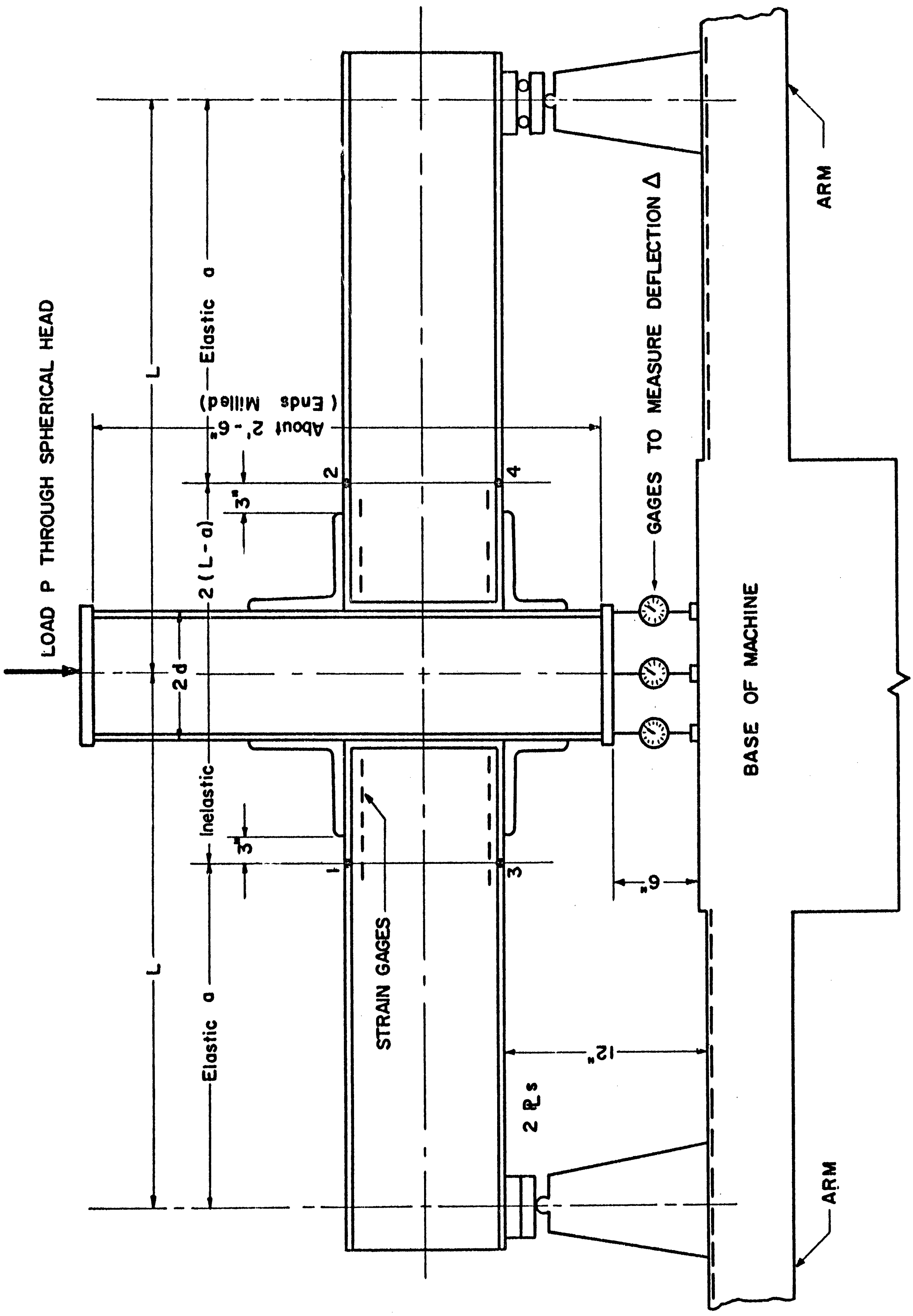

Figure 1. Arrangement of Specimens in Testing Machine.

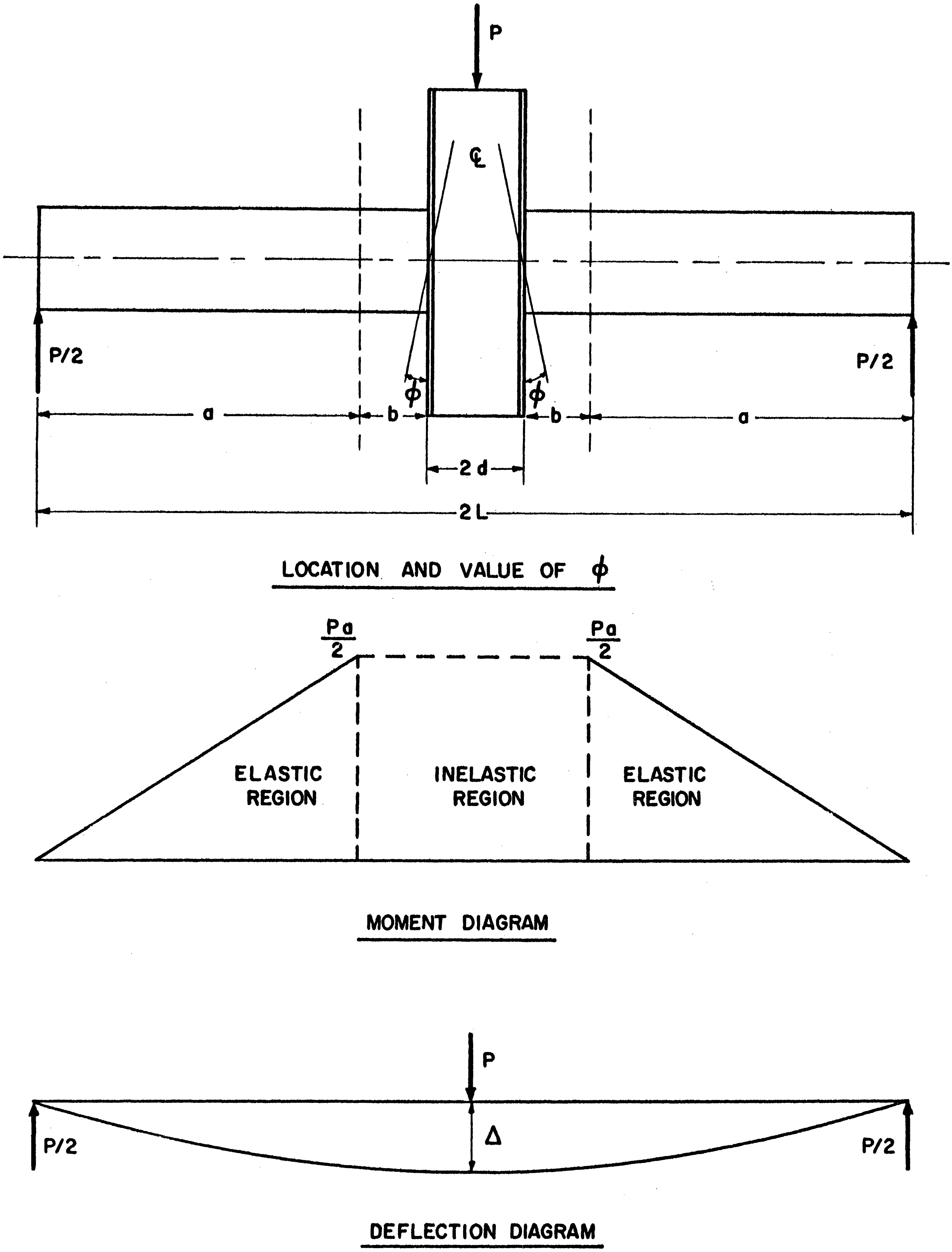

Figure 2. Location of Angle φ and Displacement Δ.

where

P = total load on the specimen

E = modulus of elasticity

I = moment of inertia

a = length of elastic portion

L = 1/2 span of specimen

d = 1/2 width of column

6. The values of φ that are determined from the measured value of Δ by means of Equation (1) when plotted as abscissae against the moment at the face of the column as ordinate provide typical moment-rotation (M, φ) curves as shown in Figures 3 and 4. In the diagrams are also shown corresponding rotations φ (see broken lines) which are obtained from the horizontal movement between two reference points that were established in each flange at the edges of the inelastic zone. These gage distances are shown by points 1, 2, 3, and 4 in Figure 1. The sum of the horizontal displacements between points 1 and 2 and between 3 and 4 divided by the vertical distance between the points was used to check the value of φ.

Results of Typical Tests.

The test procedure as described above was used on the three specimens whose details are shown in Figures 5, 6, and 7. An initial load was applied through a movable head with a spherical bearing and removed several times before the final measurements were made.

The load-deflection curve for specimen number 2 is shown in Figure 8. From such diagrams the angle change φ in the inelastic zone was calculated for each specimen by means of Equation (1). The values of φ are shown in

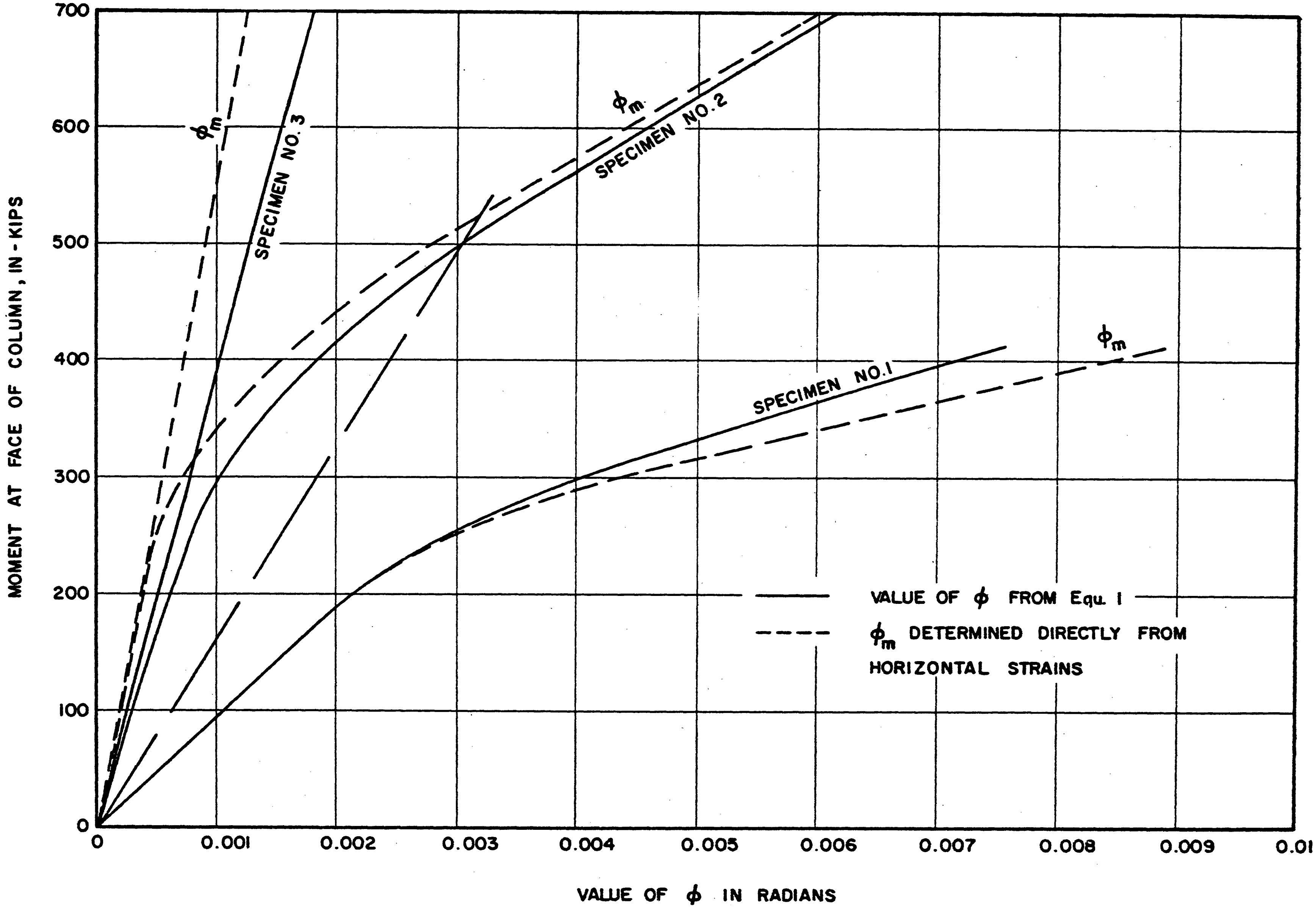

Figure 3. M-φ Diagram Determined from Displacement Δ.

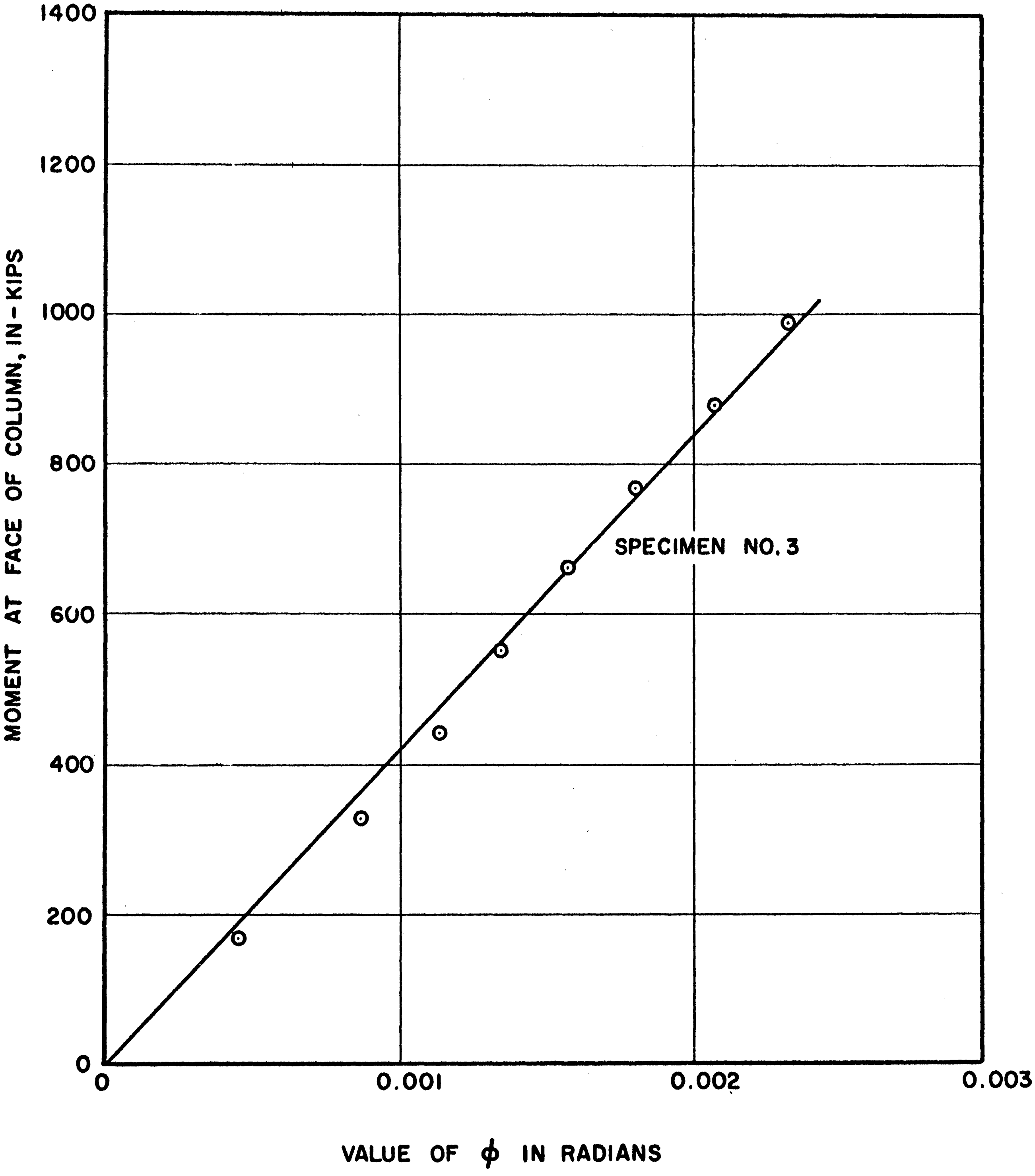

Figure 4. M-φ Diagram for Specimen No. 3.

Figures 3 and 4. The magnitudes of the inelastic portions a are 46, 43, and 46 inches for specimens 1, 2, and 3, respectively. L is equal to 60 inches and d is 5 inches for all specimens.

Use of M, φ Curves in Design.

If the actual M, φ curves in Figure 3 are approximated by a straight line the moment M can be expressed in terms of the rotation φ by the relation

$$M = \psi \varphi \qquad (2)$$

where ψ is the slope of the M, φ diagram. The quantity $\frac{M}{\psi}$ is therefore equivalent to $\frac{Mdx}{EI}$ in a beam and can be treated as such in the calculations. In any frame where the steel girders support a reinforced concrete floor, the actual EI value of the beam is uncertain. However, when only the steel members are considered in determining the beam coefficients, the following assumptions are recommended for design calculations:

(a) Consider the beam as a member with constant EI except at the ends where a concentrated angle change of $\frac{M}{\psi}$ occurs.

(b) When the connection stiffness ψ is the same for both ends of the beam, the coefficients 4 and 2, and the fixed-end moments M_{Fab} and M_{Fba} in the slope-deflection equations

$$M_{ab} = \frac{EI}{L} (4\theta_a + 2\theta_b) + M_{Fab} \qquad (3a)$$

$$M_{ba} = \frac{EI}{L} (2\theta_a + 4\theta_b) + M_{Fba} \qquad (3b)$$

can be replaced by

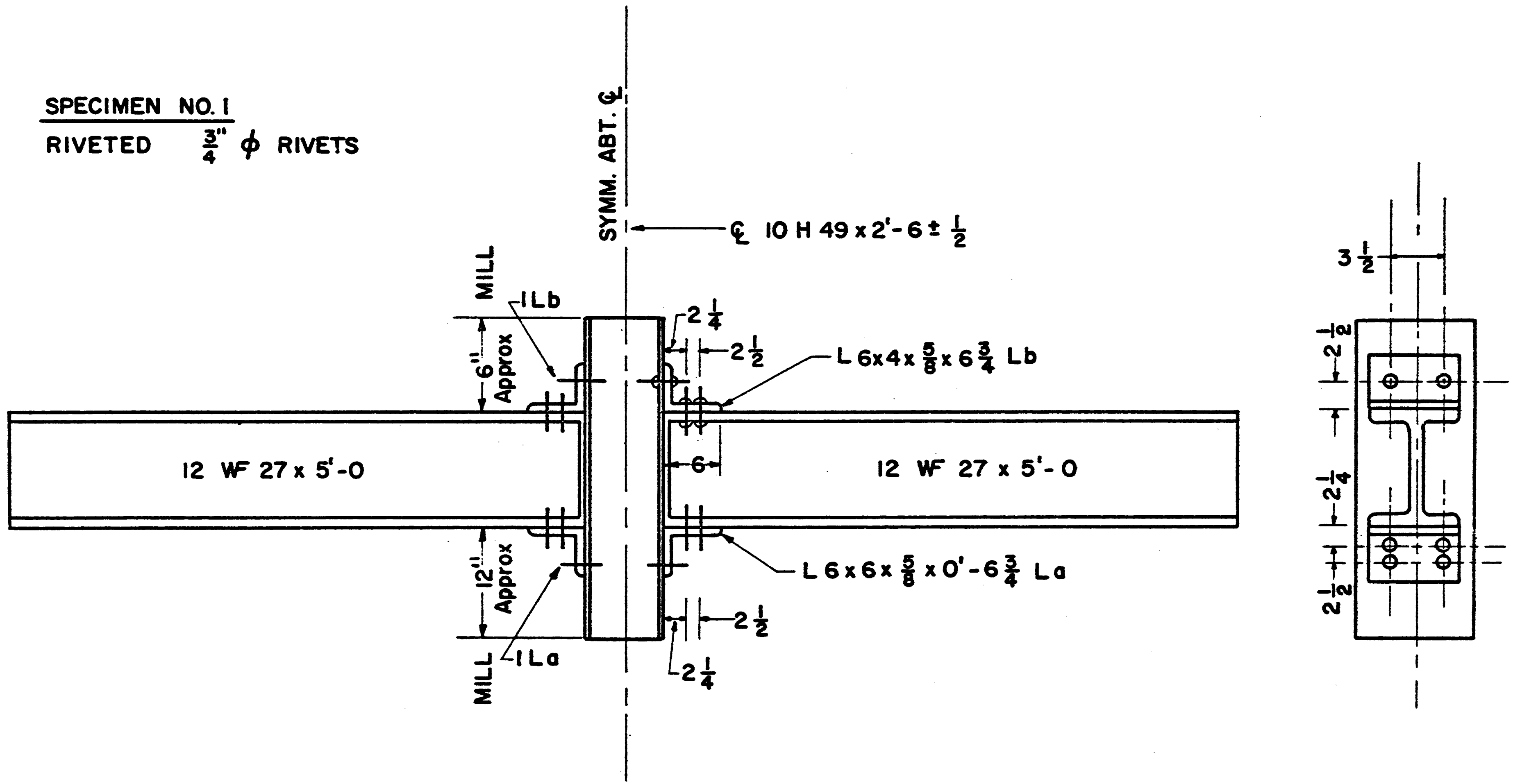

Figure 5. Connection Details of Specimen No. 1.

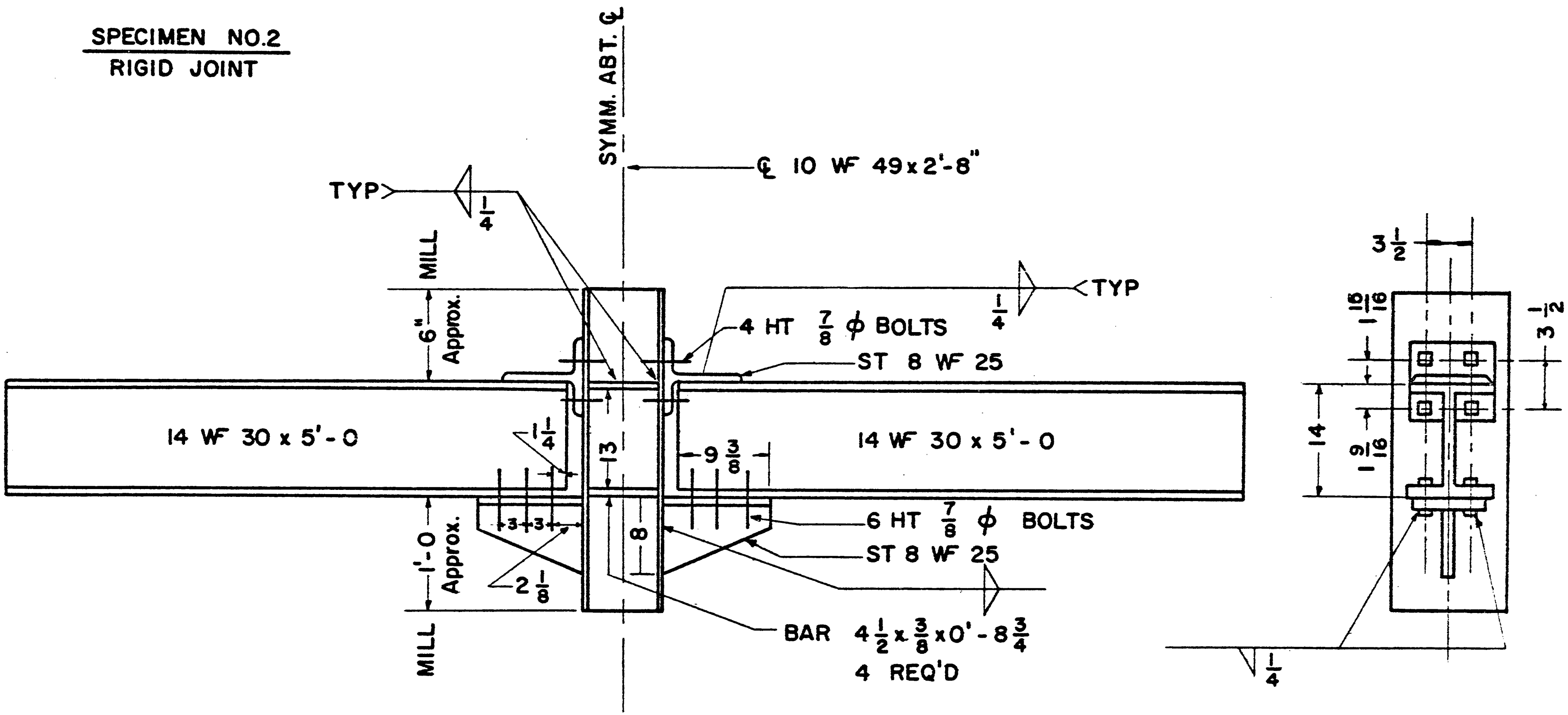

Figure 6. Connection Details of Specimen No. 2.

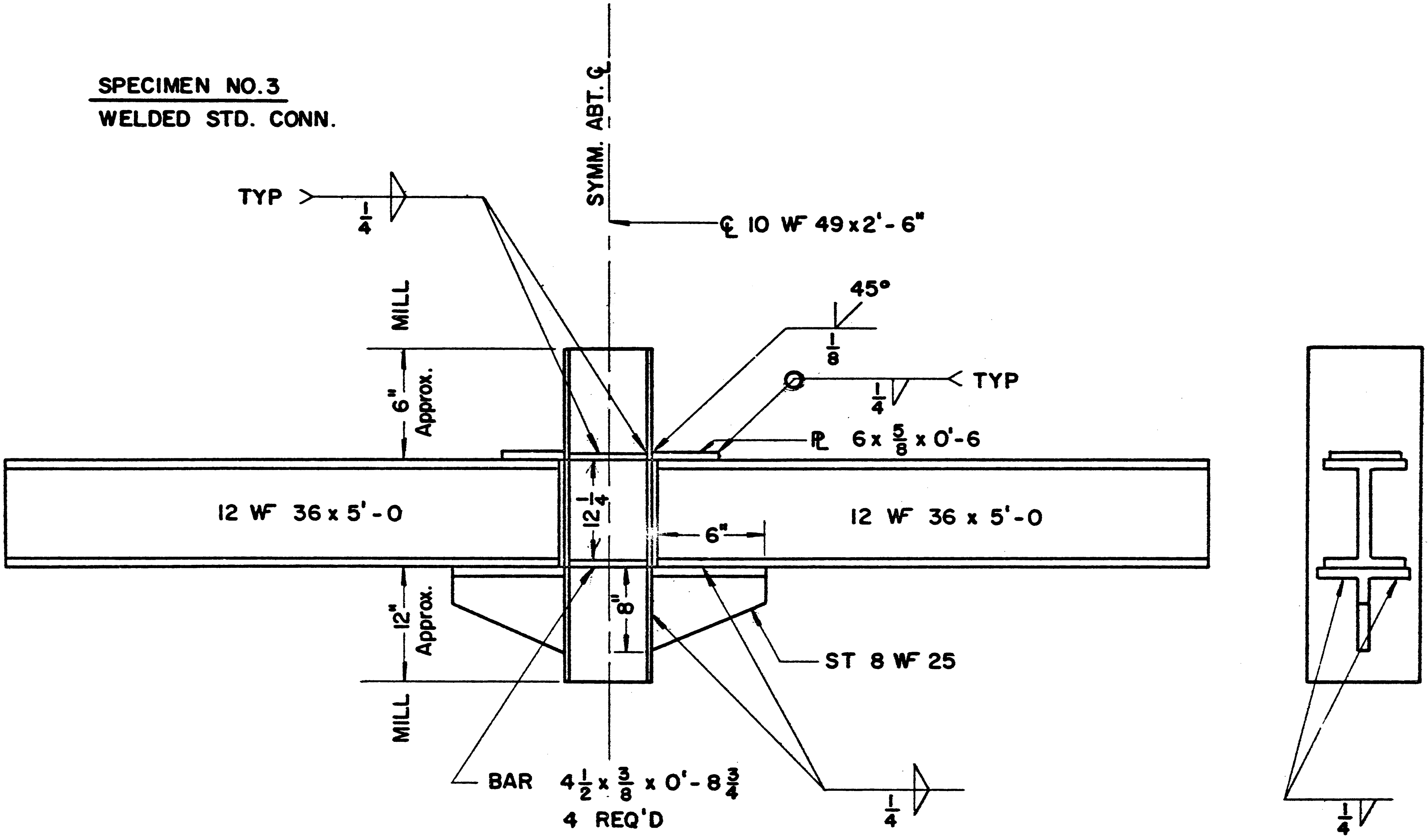

Figure 7. Connection Details of Specimen No. 3.

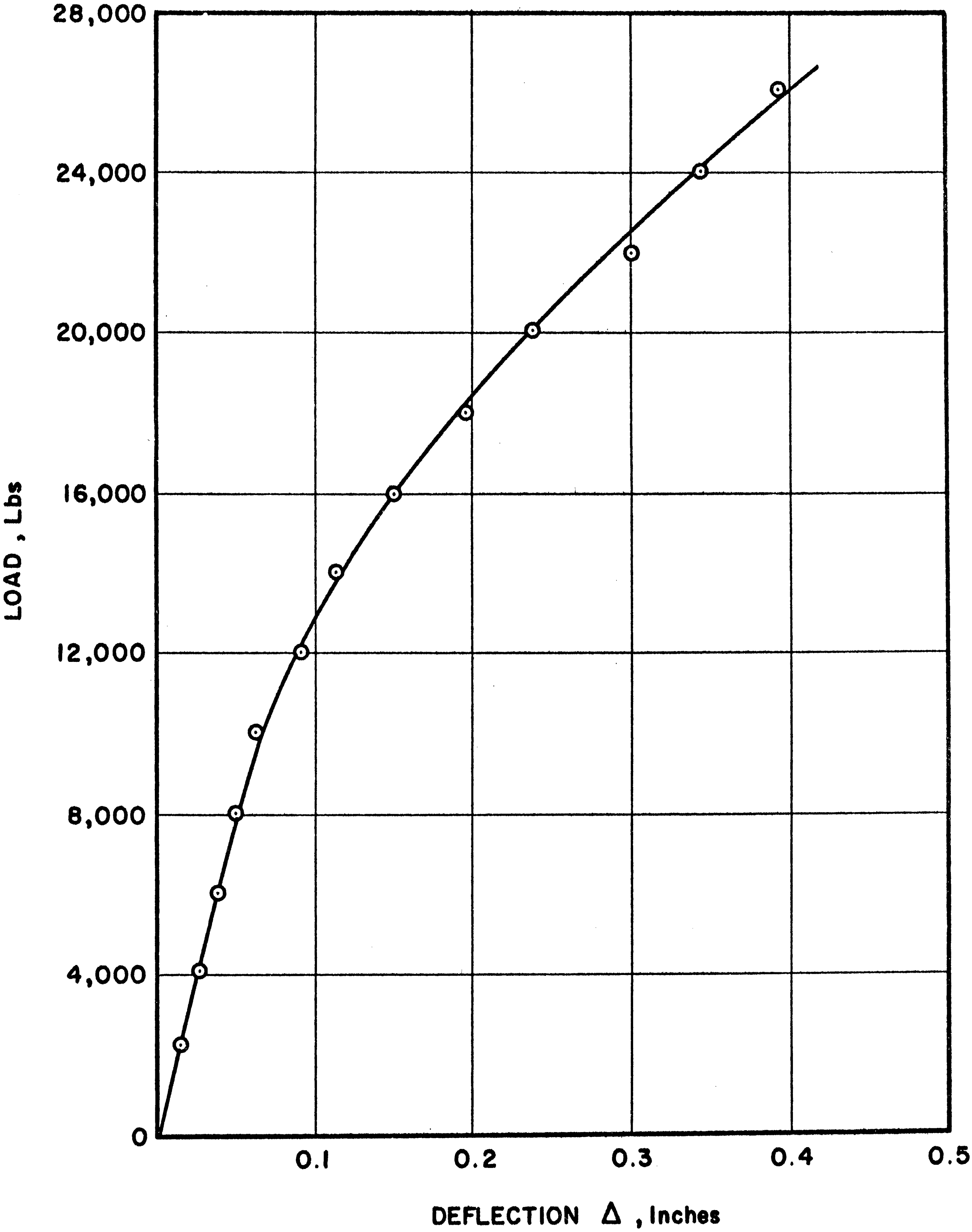

Figure 8. Load-Deflection Diagram for Specimen No. 2.

$$M_{ab} = \frac{EI}{L} (C_1\theta_a + C_2\theta_b) + M'_{Fab} \qquad (4a)$$

$$M_{ba} = \frac{EI}{L} (C_2\theta_a + C_1\theta_b) + M'_{Fba} \qquad (4b)$$

in which, assuming that $\psi_a = \psi_b = \psi$

$$C_1 = \frac{12A}{4A^2 - 1} \qquad (5a)$$

$$C_2 = \frac{6}{4A^2 - 1} \qquad (5b)$$

where

$$A = 1 + \frac{3EI}{L\psi} = 1 + \frac{3K}{\psi} \qquad (5c)$$

$$K = \frac{EI}{L}$$

in which L = distance center to center of columns.

Also,

$$M'_{Fab} = \frac{1}{6} [M_{Fab} (2C_1 - C_2) + M_{Fba} (2C_2 - C_1)] \qquad (6a)$$

$$M'_{Fba} = \frac{1}{6} [M_{Fab} (2C_2 - C_1) + M_{Fba} (2C_1 - C_2)] \qquad (6b)$$

where M_{Fab} and M_{Fba} are the usual fixed-end moments in Equations (3a) and (3b) that is for ψ equals infinity and A equal to one. For a symmetrical loading such that

$$M_{Fab} = - M_{Fba}$$

then

$$M'_{Fab} = \frac{1}{6} [M_{Fab} (2C_1 - C_2 - 2C_2 + C_1)]$$

or

$$M'_{Fab} = \frac{1}{2} (C_1 - C_2) M_{Fab}$$

Important Features of Semi-Rigid Connections.

As variations in the coefficients C_1, C_2, M'_{Fab}, and M'_{Fba} are important factors in a structural design, it is interesting to note that a particular end connection may provide considerable restraint for a beam with a small $\frac{K}{\psi}$ value, but relatively little if the beam has a large $\frac{K}{\psi}$ value. The variation of the coefficients C_1 and C_2 with respect to $\frac{K}{\psi}$ are shown in Figure 9. A disturbing feature of these diagrams is the rapid change that may occur in the values of C_1, C_2, and M'_F for small changes in $\frac{K}{\psi}$. The changes in the fixed-end moments M'_F are indicated in Figure 8 for a uniform load over the entire span. It is apparent that the fixed-end moments may change rapidly for even small changes in the $\frac{K}{\psi}$ values.

If a semi-rigid connection such as in specimen 3 is used instead of a rigid connection than a constant value of ψ of 385×10^6 in-lbs is obtained from the slope of the M, φ curve in Figure 3. When this connection is used an a 12 WF 36 beam of 17 feet length the value of $\frac{K}{\psi}$ is

$$\frac{K}{\psi} = \frac{EI}{L\psi} = \frac{29 \times 10^6 \times 280.8}{17 \times 12 \times 385 \times 10^6} = .104$$

From Figure 8 or from Equations (5a, 5b, 5c) we obtain,

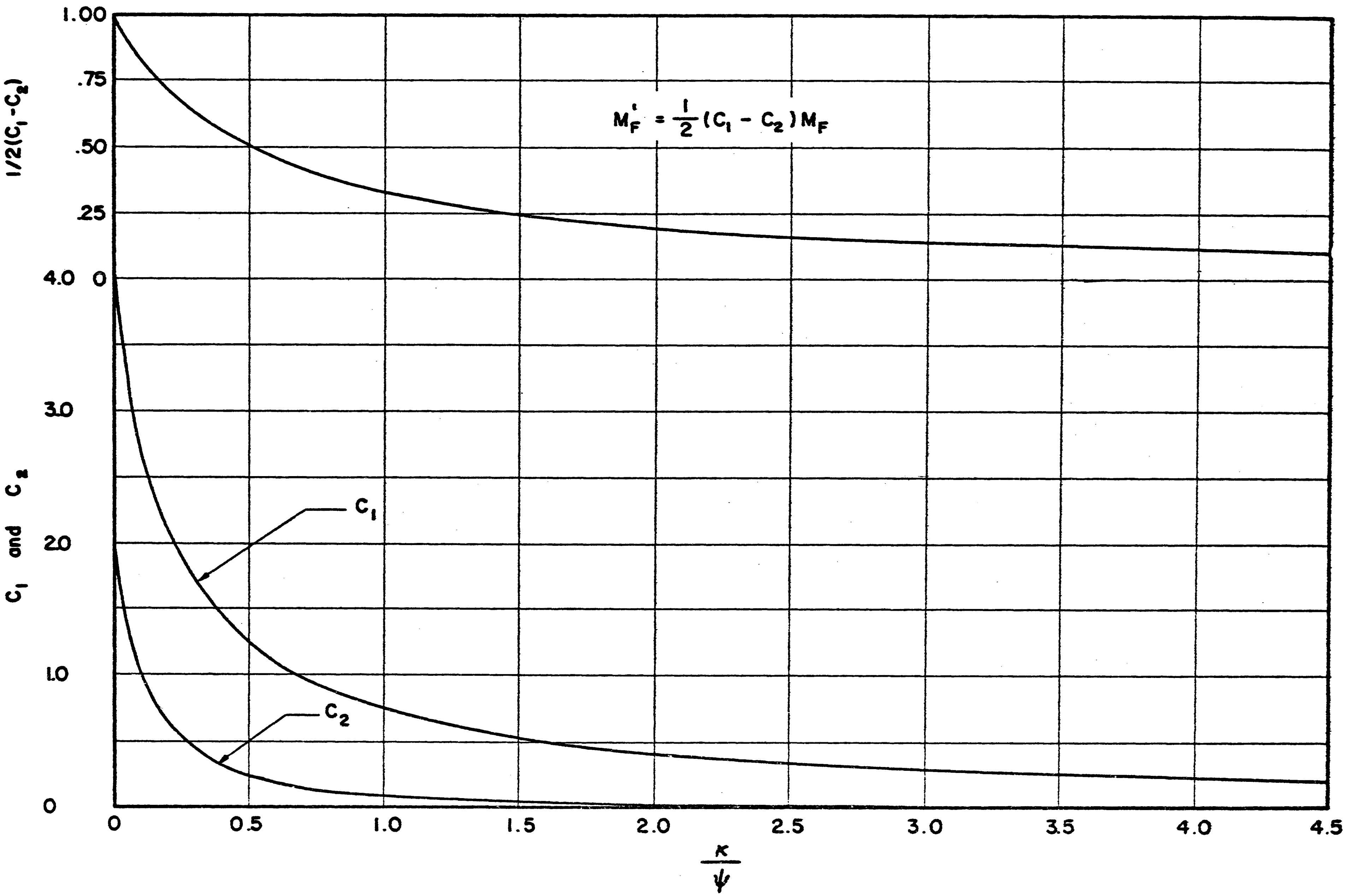

Figure 9. Values of C_1, C_2 and M_F'.

$C_1 = 2.68$ $C_2 = 1.02$

$M'_F = \frac{1}{2}(2.68 - 1.02)$ $M_F = (.83)(\frac{wL^2}{12}) = .069\ wL^2$

SUMMARY

In this paper the importance of considering the deformation of beam connections in the design of steel frames has again been emphasized. A laboratory procedure for determining the M, φ diagram for any type of beam connection has been discussed. This method involves measuring only a single vertical displacement by means of dial gages.

Analytical methods for incorporating the properties of the connections into the slope-deflection equations are presented. It has been shown that the stiffness of the beam and the magnitude of the end couples may be modified considerably by the rotational restraint factor ψ of the connections. Therefore the actual beam coefficients and fixed-end moments, for the particular beam and connection should be determined from test results and used in the structural analysis.

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