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Final Report

STRUCTURAL BEHAVIOR OF THE 325-FT STEEL STACK FOR UNIT NO. 6 - GLEN LYN PLANT

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Project 2371

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ABSTRACT

The results of an investigation of the dynamic behavior of the proposed 325-ft steel stack for Unit No. 6 of the Glen Lyn Plant at Glen Lyn, Virginia, have been summarized in this report. The analytical study includes the determination of the vibration characteristics for both first and second modes of vibration. Both numerical calculations and solutions obtained from an analog computer have been used.

The resonant wind velocities are given for both completed and partially completed stacks and the maximum stresses determined for various critical wind conditions. Special stress problems that are encountered at the gas-inlet opening and the base connections, and in providing adequate ring stiffners have been solved. Recommendations are given for the structural arrangement and method of erection.

OBJECT

The object of this project is to make an analytical study by special methods, including the solution of the fundamental equations by use of an analog computer, which will provide information to determine the adequacy of the proposed structural arrangement to withstand aerodynamic vibration.

INTRODUCTION

On March 31, 1955, the American Gas and Electric Service Corporation, acting for the Appalachian Electric Power Company, approved a contract with The University of Michigan for a study of the structural behavior of a proposed 325-ft-high steel stack for Unit No. 6 of the Glen Lyn Plant at Glen Lyn, Virginia.

The general scope of the project was discussed at a meeting in New York on March 15, 1955 between Mr. E. A. Kammer, Chief of Design Division, and Mr. G. W. Bice, Assistant Mechanical Engineer, both of the American Gas and Electric Service Corporation, and Professor L. C. Maugh of The University of Michigan. It was agreed at that time that the primary objective was a study of the dynamic behavior of the stack for various wind velocities. More specifically, the investigation covered the following steps:

1. Study of the dynamic properties of the stack and the determination of displacements and unit stresses in the steel shell for resonant wind velocities. A comparison of static and dynamic values is also included in this investigation.

2. Study of the structural arrangement that is necessary to keep displacements and stresses within safe limits.

3. A detailed study of the stresses in the base details and at the gas-inlet opening.

4. Study of the required stiffening elements for the steel shell.

5. Study of possible erection procedures that will provide reasonable safety during construction without the installation of temporary cable guys.

The above work was under the general supervision of L. C. Maugh, Professor of Civil Engineering. Assisting in the investigation were

Mr. Wadi Rumman, Instructor in Civil Engineering, and Mr. Nelson Isada, Graduate Research Assistant, who made most of the numerical calculations. Mr. Isada arranged and carried out the calculations for the dynamic properties of the stack when vibrating in the first or second mode.

DISCUSSION OF RESULTS

FUNDAMENTAL FREQUENCIES AND CRITICAL VELOCITIES

The natural frequency of the stack for the first mode of vibration was determined for both the partially completed and the completed stacks. The second mode of vibration was computed for the completed stack only. The dimensions and properties used in making these calculations are shown in Fig. 1, page 3. These frequencies and related values are summarized in Table I below.

TABLE I

Stage of Construction	Frequency cycles/sec 1st mode	Frequency cycles/sec 2nd mode	Period sec/cycle 1st mode	Period sec/cycle 2nd mode	Resonant Wind Velocity mph(Eq. 3) 1st mode	Resonant Wind Velocity mph(Eq. 3) 2nd mode
a Completed stack, 1-1/2-in.lining	0.70	3.33	1.43	0.30	50.7	242
b Completed stack, 2-1/2-in.lining	0.61	3.10	1.64	0.32	44.3	224
c *Partially completed stack	2.16		0.46		156.8	
d **Partially completed stack	1.10		0.91		79.8	
e ***Partially completed stack	0.75		1.33		54.5	
f 325-ft stack—unlined	1.02		0.98		74.9	

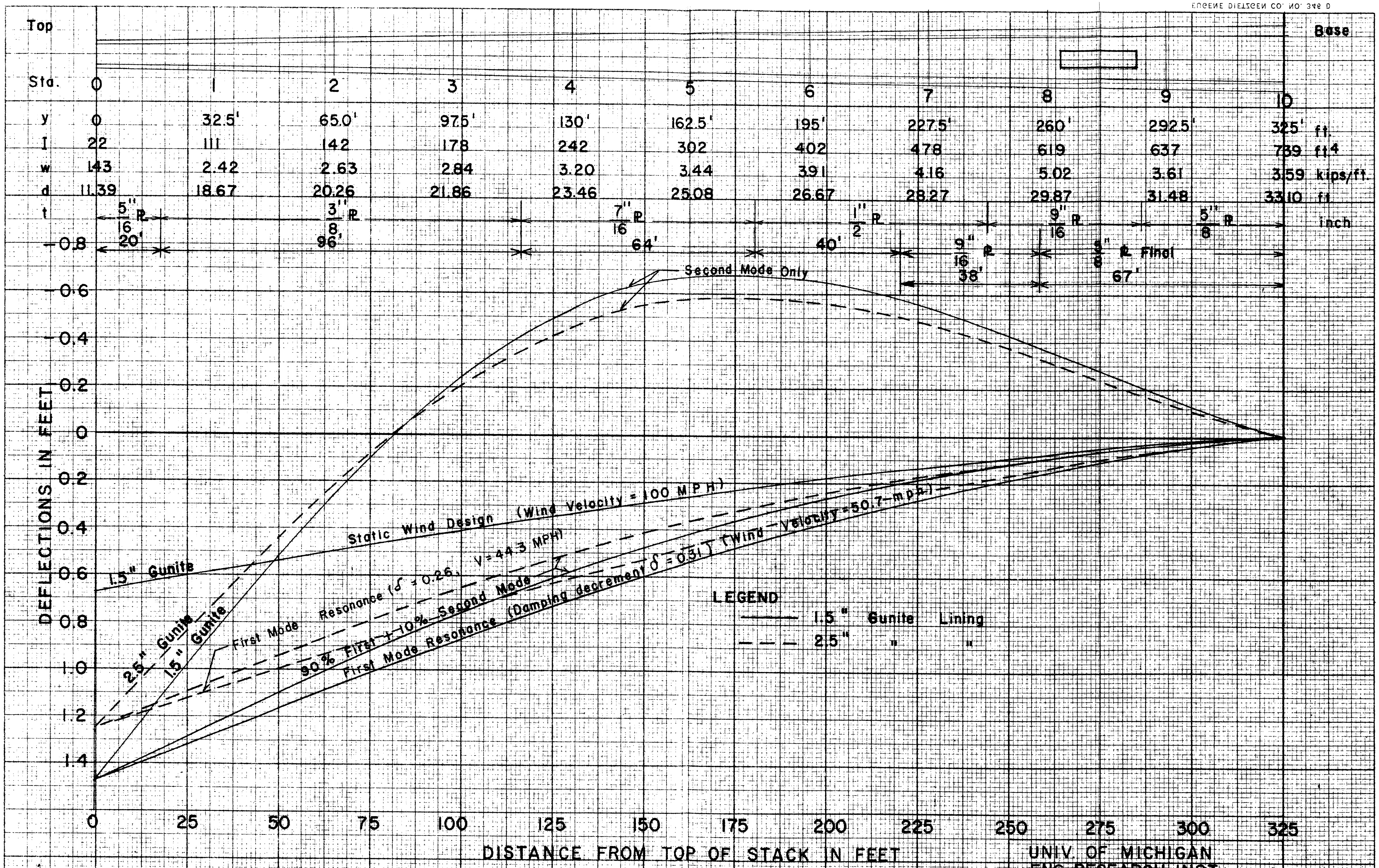
*195-ft high - no lining
**Lined to 195 ft - unlined to 260 ft
***Lined to 260 ft - unlined to 325 ft

In determining the resonant wind velocity for forced vibration in the first and second modes, a value of .2 was adopted for the Strouhal number S in the formula

$$V = \frac{fd}{S} , \qquad (1)$$

where

V = resonant velocity in feet per second,
f = frequency in cycles per second,
d = diameter of the stack in feet, and



DEFLECTION CURVES

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FIG. 1

S = Strouhal number.

For S = .2, the resonant velocity becomes

$$V = 5fd. \tag{2}$$

As the stack is tapered throughout its height, the resonant velocity has been calculated for the average outside diameter of the top half, which is 21.28 ft. The resonant velocity in miles per hour is therefore equal to

$$V = (5)(21.28)\left(\frac{2600}{5280}\right)f = 72.6 \text{ f mph}, \tag{3}$$

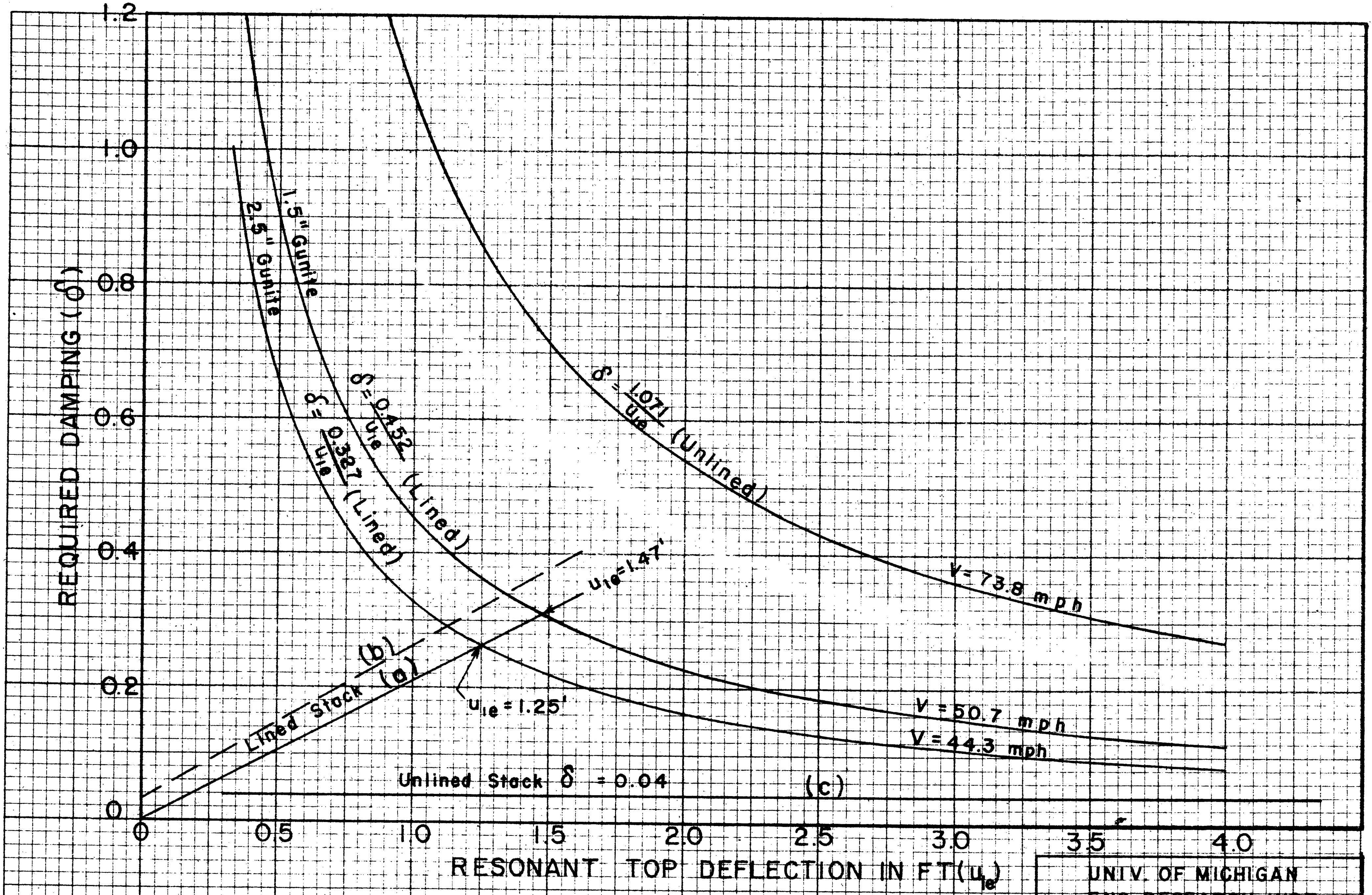
where f is the frequency in cycles per second. It is recognized that these resonant velocities are subject to some variations, due to the uncertain base conditions and to the effect of turbulent wind conditions on the aerodynamic forces. The rotation at the base of the stack due to anchor-bolt and column deformation has been considered, but the exact magnitude of the lateral wind forces and the manner in which they are applied to the stack are as yet somewhat uncertain. For this reason, the displacements and forces on the partially unlined stacks of cases (c) and (d) have also been considered for a wind velocity of 70 mph, although the theoretical wind velocities are 156 and 79.8 mph, respectively.

TOP DISPLACEMENT u_e AND DAMPING DECREMENT δ

When a stack is subjected to a resonant wind velocity for the first mode, its displacement is largely controlled by the amount of damping that is present. The amount of damping is conveniently expressed in terms of the log decrement δ (Reference 1), which can be determined only by actual field tests on existing stacks. Transferring this information to the design of new stacks and extrapolating the test results for displacements beyond those measured cannot be avoided. Unfortunately, the extrapolated data cannot be checked until a stack is built.

The relation between the amount of damping δ and the displacement of the top of the stack at the resonant wind velocity for the first mode is shown in Figs. 2 and 3 for various stack conditions. This relationship was established by equating the work performed by the wind forces acting through the displacement of the stack to the energy absorbed by the damping forces. Various points on the curves in Figs. 2 and 3 were checked by results obtained from solutions on the analog computer.

In determining the value of the lateral wind forces a coefficient of C_L equal to 1.0 in the formula



RESONANT FIRST MODE CURVES

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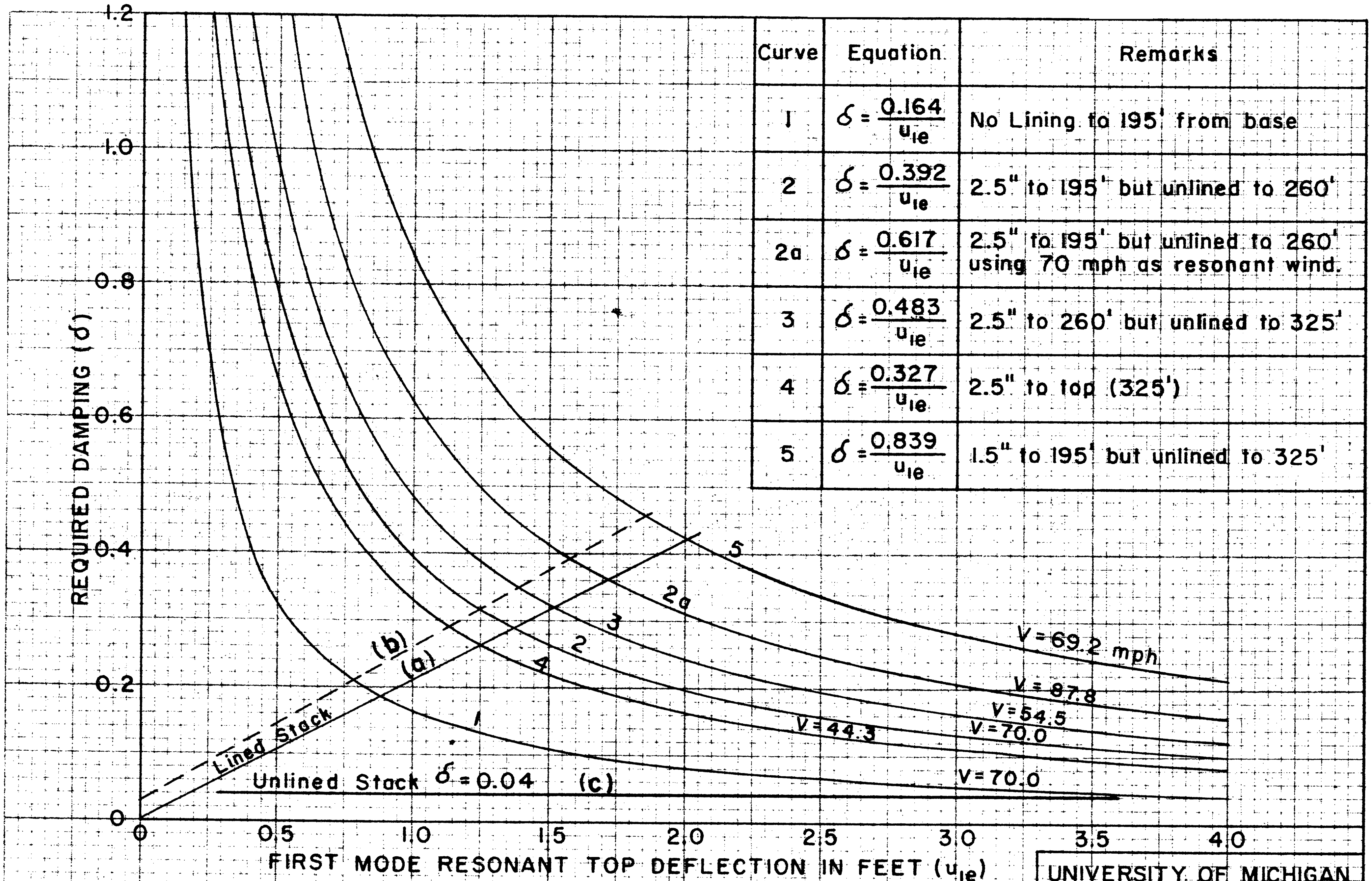
FIG 2



Curve	Equation	Remarks
1	$\delta = \frac{0.164}{u_{le}}$	No Lining to 195' from base
2	$\delta = \frac{0.392}{u_{le}}$	2.5" to 195' but unlined to 260'
2a	$\delta = \frac{0.617}{u_{le}}$	2.5" to 195' but unlined to 260' using 70 mph as resonant wind.
3	$\delta = \frac{0.483}{u_{le}}$	2.5" to 260' but unlined to 325'
4	$\delta = \frac{0.327}{u_{le}}$	2.5" to top (325')
5	$\delta = \frac{0.839}{u_{le}}$	1.5" to 195' but unlined to 325'

CONSTRUCTION RESONANT FIRST MODE CURVES

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FIG. 3

$$P = C_L \frac{\rho v^2}{2} \qquad (4)$$

was used, where

P = pressure in pounds per square foot,
v = velocity of wind in feet per second,
ρ = density of air, and
C_L = experimental coefficient.

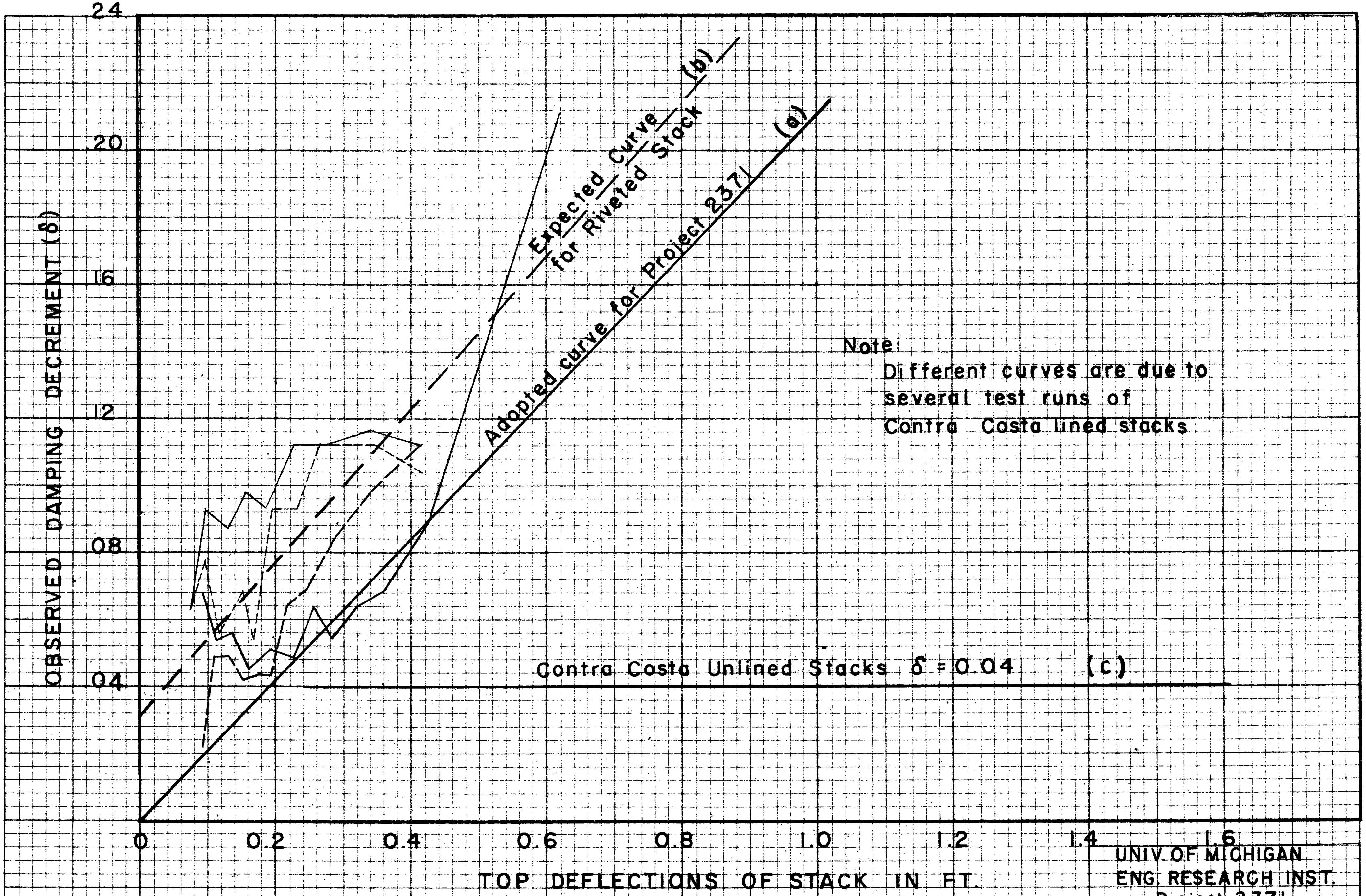
As values of C_L as low as .66 have been employed by other investigators, the value of $C_L = 1.0$ which has been used should provide conservative results. The pressure is assumed to vary with time from zero to the maximum value as given by Equation 4, according to a sinusoidal variation.

The curves in Fig. 2 show that it requires a damping decrement δ of .26 for the 2-1/2-in. gunite lining and .35 for 1-1/2-in. gunite lining to keep the top displacement equal or less than 1.25 ft. The actual amount of damping varies greatly with the structural arrangement of the stack, that is, whether riveted or welded, the amount and nature of the lining, the base conditions, and the amplitude of the displacements. The results of a number of tests on the damping decrement δ that were made on the welded stacks of the Contra Costa Plant on the Pacific coast (Reference 2) are shown in Fig. 4. These values were also compared with results obtained from field tests made by the Detroit Edison Company for both welded and riveted stacks before the solid lines shown in Figs. 2, 3, and 4 were adopted. The broken line marked (b) shows the relation that might be expected if further experimental results from other sources were included. From the intersection of the solid line marked (a) in Figs. 2 and 3 with the theoretical curves, the following maximum displacements (Table II) are obtained for various stack conditions.

TABLE II

Condition of Stack	Max. Expected Top Displacement at Resonance in 1st Mode-ft	Expected Damping Decrement
1-1/2-in. gunite lining, completed	1.47	.31
2-1/2-in. gunite lining, completed	1.25	.26

In view of the importance of controlling the top displacement for resonance vibration in the fundamental mode, it is desirable to increase the thickness of the gunite lining to 2-1/2 in. For this thickness the necessary damping decrement of .26 can be expected in a riveted stack when the displacement is 1.25 ft.



OBSERVED DAMPING DECREMENT CURVES

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FIG. 4

The behavior of the structure during construction was studied by means of the data shown in Fig. 3 which gives the expected displacements due to resonant wind conditions. From a study of these curves it was decided that the construction of the stack should proceed in the following manner:

(a) Construct the unlined steel shell to a height of about 195 ft above the base.

(b) The stack should then be lined with gunite to 195 ft and the steel shell carried unlined to 260 ft.

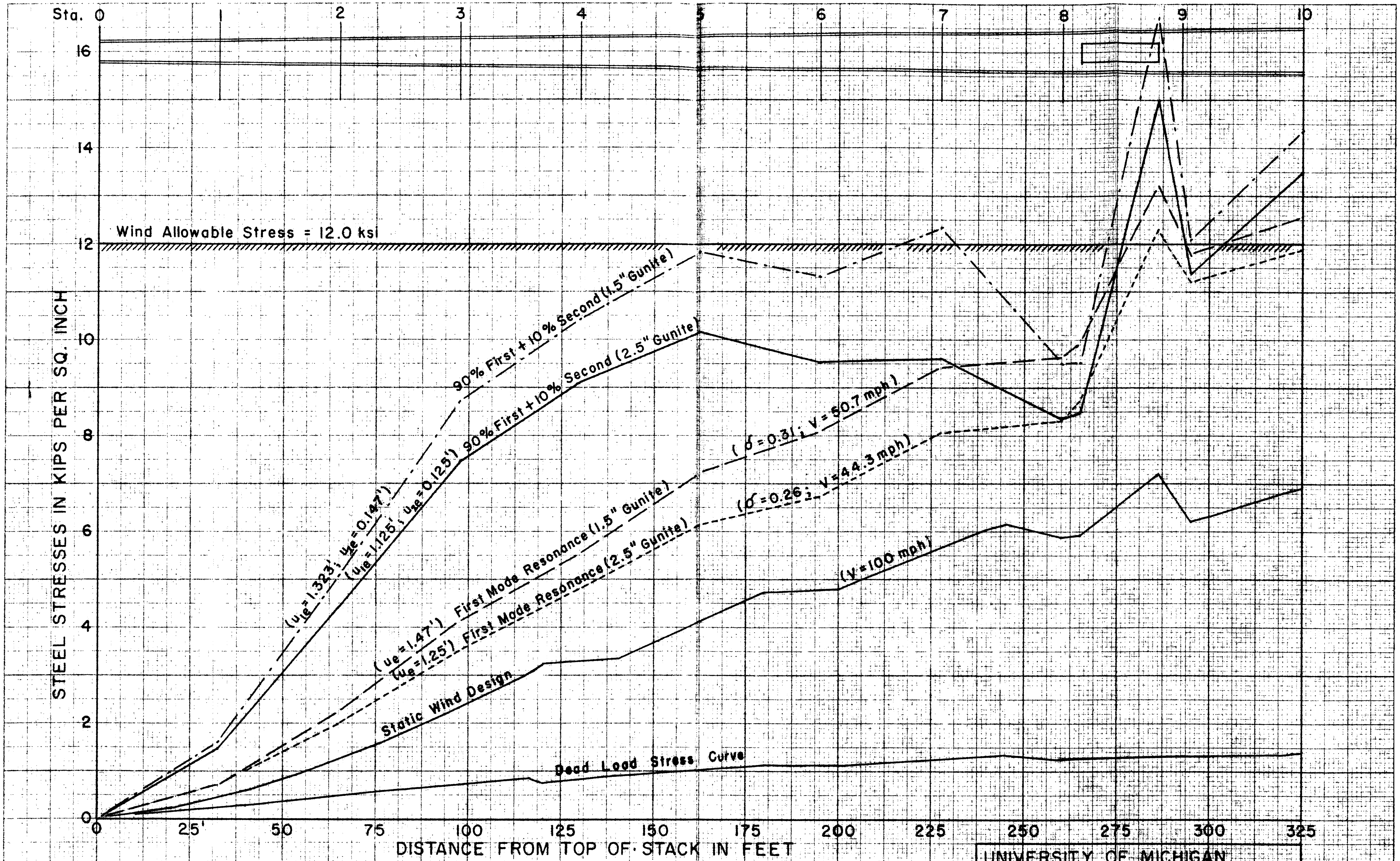
(c) Gunite lining is then carried to 260 ft and steel shell carried unlined to 325 ft.

(d) Gunite lining is then completed to top of stack.

COMPARISON OF STATIC AND DYNAMIC STRESSES IN THE STEEL SHELL

A comparison of the compressive stresses in the steel shell for a static wind force varying with height from 24 to 32 psf, in accordance with the specifications of the American Concrete Institute, and for the inertia forces when the stack is vibrating due to resonant wind conditions is shown in Fig. 5. One set of curves gives the dynamic stresses when the stack is vibrating in the first mode only with the top amplitude u_e as indicated. Another set of curves in Fig. 5 indicates the large increase in stress near the middle of the stack when only a relatively small percentage of the second mode is in phase with the first mode. The actual combination of the second mode with the first mode will depend upon the character of the wind forces and upon the amount and character of the damping forces in the stack. The compressive stresses due to wind action were calculated for the composite section of steel shell and the gunite lining. However, it is recognized that these stresses can increase at points where the lining might crack, but failure due to compression is extremely unlikely for such localized stresses. The allowable unit stress of 12.0 kips/sq in. is based on the resistance to general instability of the steel shell which depends upon the $\bar{r}/t$ ratio. The influence of the gunite lining is somewhat uncertain.

The sudden increase in unit stress at the bottom of the gas-inlet openings is primarily due to the additional bending stresses arising from the openings. As the assumptions in the calculation of these stresses were conservative, and as additional stiffeners have been added in this region, the maximum unit stress of 15.0 kips/sq in. for the stack with 2-1/2-in. gunite lining under extreme wind conditions seems justifiable.



STEEL STRESSES DUE TO WIND
(Including Effect of Dead Load)

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FIG. 5

In Fig. 6, the unit compressive stresses in the steel shell are shown for the dynamic action of the wind during the construction of the stack. As the resonant wind velocities for the partially completed stack are considerably higher than for the completed stack, high stresses are obtained at the base.

The maximum unit compressive stress shown in curve 2 with an unlined stack 195 ft high is about 19 kips/sq in. for a wind velocity of 70 mph, which is considered a more practical value than the theoretical resonant velocity of 87.8 mph. For a temporary condition during construction, this value of the unit stress at the base is considered safe.

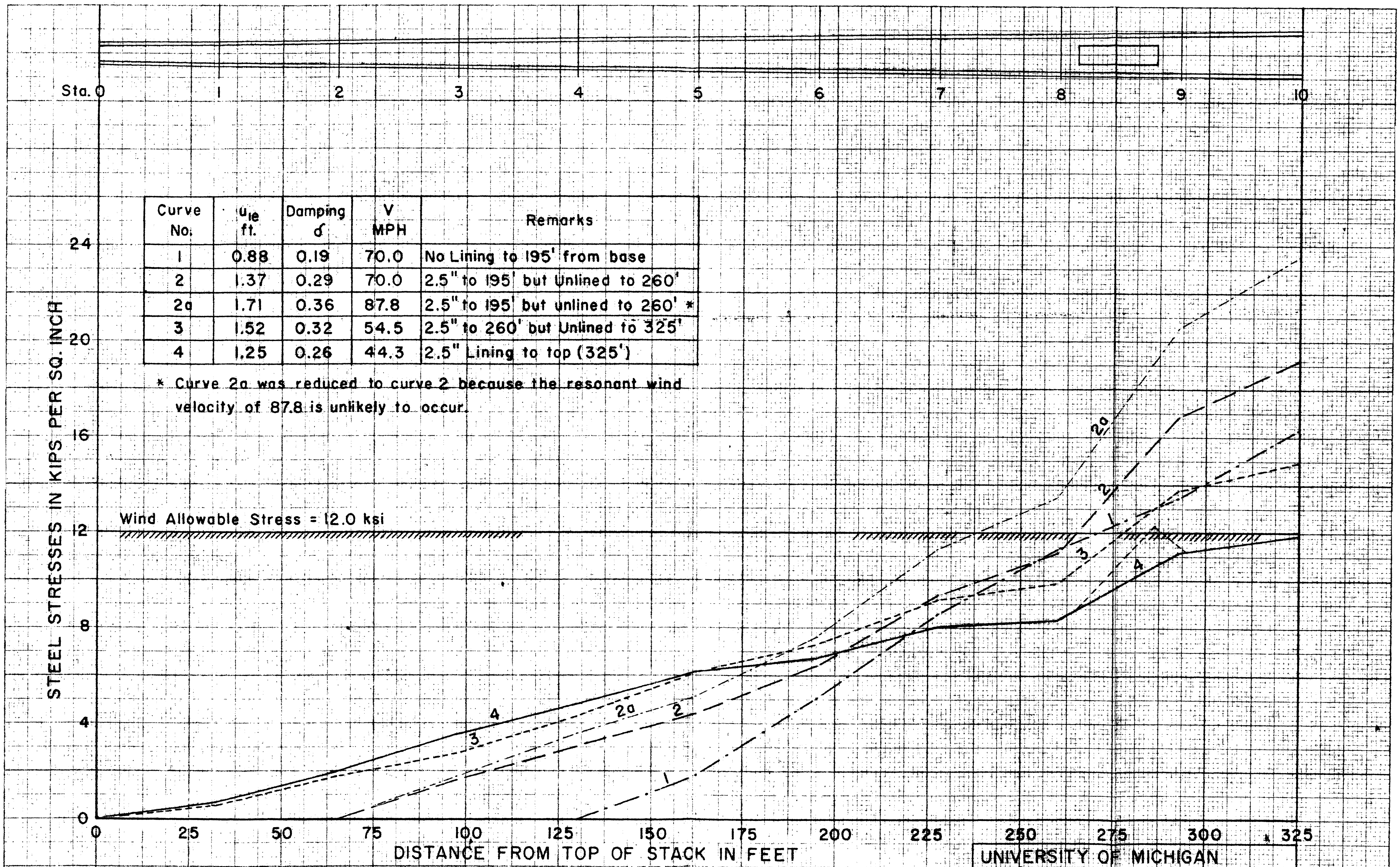
STRESSES IN ANCHOR BOLTS

The stresses in the anchor bolts were calculated for the condition that the base of the stack will always be in contact with the supporting girders. This condition assumes that sufficient initial extension will be placed in the bolts so that the stiffeners will always be subjected to some compression.

The maximum moment at the base of the completed stack with 2-1/2-in. gunite lining for a resonant wind velocity in the first mode of 44.3 mph and for a top displacement of 1.25 ft is 64,500 ft-kips. However, to provide for possible second mode vibration for extremely high steady wind conditions, it was decided to consider that 10 percent of the displacement could be attributed to second-mode vibration, while 90 percent was in the first mode. For this assumption, which is based on previous studies, the bending moment at the base of the stack when the top displacement is 1.25 ft in the first mode and -.125 ft in the second mode will equal

$$64{,}500 + 16{,}800 = 81{,}300 \text{ ft-kips.}$$

The initial tension required in the bolts to keep the stack base in contact with the supporting girders is 32.0 kips/sq in. This initial tension in the 3-1/4-in. diameter steel bolts will produce a compresive stress of 11.0 kips/sq in. in the stiffeners. The final maximum tensile stress in the bolts when a moment of 81,300 ft-kips is applied to the base is 43.0 kips/sq in. As the anchor bolts are to be fabricated from steel with a specified yield point of 80 kips/sq in., the above unit stresses are considered satisfactory.



Curve No.	u_{1e} ft.	Damping δ	V MPH	Remarks
1	0.88	0.19	70.0	No Lining to 195' from base
2	1.37	0.29	70.0	2.5" to 195' but Unlined to 260'
2a	1.71	0.36	87.8	2.5" to 195' but unlined to 260' *
3	1.52	0.32	54.5	2.5" to 260' but Unlined to 325'
4	1.25	0.26	44.3	2.5" Lining to top (325')

* Curve 2a was reduced to curve 2 because the resonant wind velocity of 87.8 is unlikely to occur.

FIRST MODE RESONANT STEEL STRESSES FOR CONSTRUCTION
(Including Effect of Dead Load)

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FIG. 6

DESIGN OF THE RING STIFFENERS

The selection of the size and spacing of the ring stiffeners presents a difficult problem as no existing design formulas apply to the structural behavior of a steel stack under bending. The functions of a ring stiffener in a steel stack are twofold: first, to reduce the distortion of the shell under the action of longitudinal compressive stresses due to bending and axial forces, and second, to help distribute the pressure of the wind into the steel shell. The first function will usually govern the design when bending stresses are fairly high, whereas in the upper quarter of the stack the second function may predominate.

To determine the size of the stiffeners that are required to reduce the distortion of the cross section, a special formula was developed to determine the maximum bending moment M and normal force N acting on the ring. This derivation, which is based on the distortion of the cross section due to flexure, gave the following values:

$$M_{max} = .00575 \, \mu \, f_x \, str \tag{5}$$

$$N = 0.2023 \, \mu \, f_x \, st, \tag{6}$$

in which

f_x = maximum unit longitudinal compressive stress at elevation of the ring,
s = spacing of the adjacent rings,
t = thickness of the steel shell,
r = radius of the shell, and
μ = Poisson's ratio or about 0.3.

If s, t, and r are taken in inch units and f_x is kips per square inch, then M will be obtained in inch-kips and N in kips. Figure 7 shows the section modulus that is required for the Glen Lyn stack as determined from Equation 5 for an allowable stress on the outer fiber of 16 kips/sq in.

This section modulus can be used to select a trial size for the ring stiffeners. The final design must be checked for both bending and axial stresses, that is, for both M and N. In the final analyses a portion of the steel shell of width b + 24t is assumed to act with the stiffener where b is the width of the stiffener flange and t is the thickness of the steel shell.

From the above procedures the following size stiffeners were selected with a constant spacing s of 96 in.

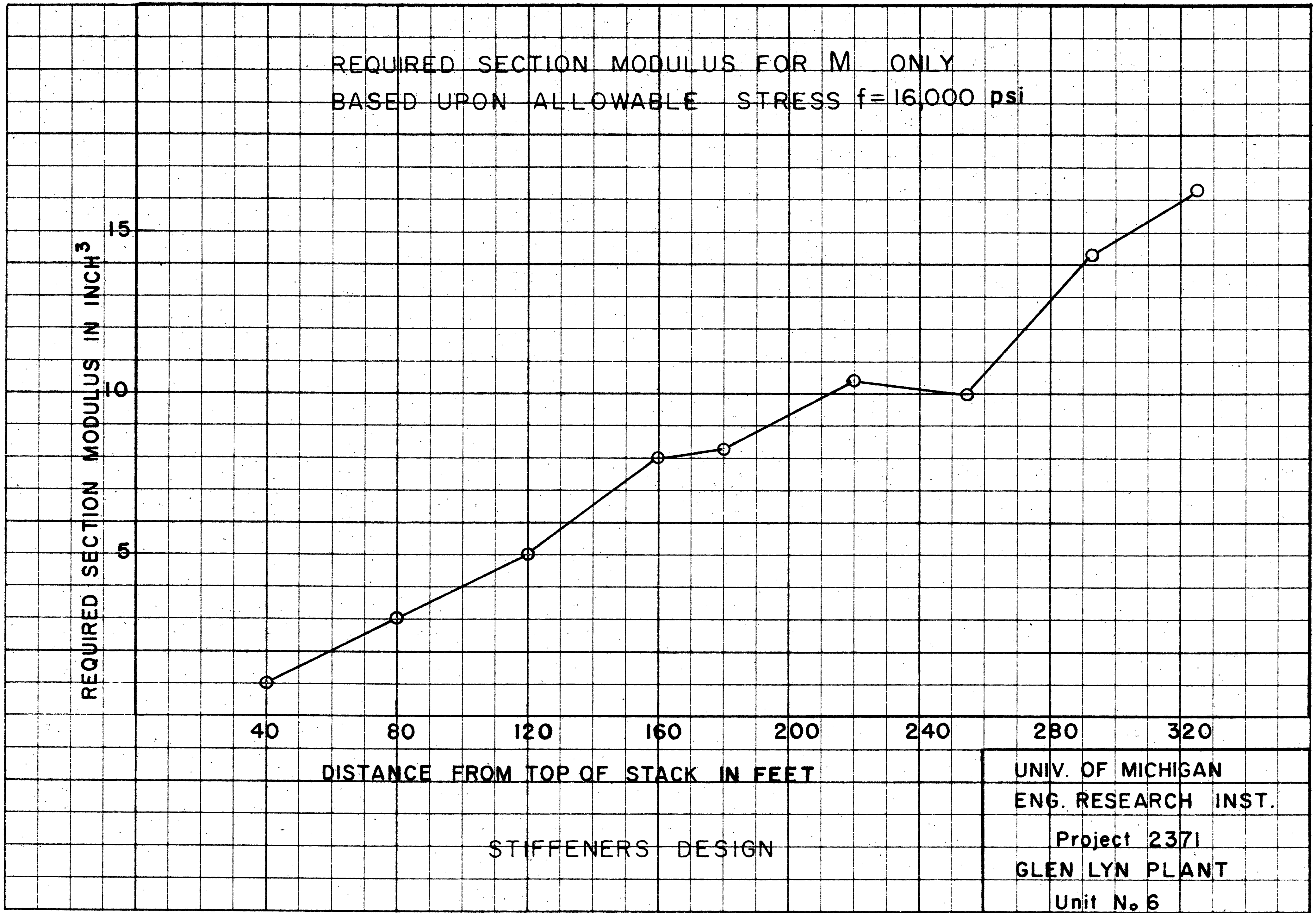
REQUIRED SECTION MODULUS FOR M ONLY
BASED UPON ALLOWABLE STRESS f = 16,000 psi
15
10
5
REQUIRED SECTION MODULUS IN INCH3
40
80
120
160
200
240
280
320
DISTANCE FROM TOP OF STACK IN FEET
STIFFENERS DESIGN
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FIG. 7

TABLE III

Distance Below Top ft	Longitudinal Stress f_x kips/sq in.	Size of Stiffeners
116	9.8	4 WF 13#
180	11.6	5 WF 16#
244	11.1	5 WF 18.5#
293	11.7	6 WF 20#
325	12.6	6 WF 25#

SPECIAL STRESS PROBLEMS

The stress conditions around the two 8'0" x 22'5" gas-inlet openings, due to the most severe combinations of moments and shears, were studied. The combination of moment and shear that was finally selected resulted from a vibration of the stack with a top displacement of 1.25 ft due to resonant wind velocity of 44.3 mph. Although this displacement is determined for a first-mode resonant condition, Fig. 1, it is believed that some allowance should be made for second-mode vibration, particularly when considering the middle half of the stack. From a consideration of all factors involved, 90 percent of the displacement has been attributed to first-mode and 10 percent to second-mode vibration. These two modes are considered to be in phase at certain times.

The above conditions give a maximum moment of 57,200 ft-kips and a shear of 573 kips at a section 286 ft from the top. The compressive stress at the base of the gas-inlet openings was calculated for the overall bending stress plus the additional localized bending stress in the two separate portions. The sum of these two stresses gives

$$9.43 + 5.57 = 15.00 \text{ kips/sq in.}$$

Although this combined stress is higher than the 12.0 kips/sq in. adopted for the maximum compression in the steel shell, this excess can be permitted at a localized point which has additional stiffening against local buckling. To keep the localized unit stress at 15 kips/sq in., however, it was necessary to extend the 5/8-in. shell plate to a section above the gas-inlet opening.

CONCLUSIONS AND RECOMMENDATIONS

To keep the top displacement of the lined stack within practical limits, that is, from 1.0 to 1.25 ft, the damping decrement δ should be about .4 for the original stack with 1-1/2-in. gunite lining. This value is difficult to obtain even with riveted butt splices. If the thickness of the gunite is increased to 2-1/2 in. and the splices are riveted, then the required damping decrement δ = .26 to keep the top displacement at 1.25 ft for a first-mode resonant wind velocity of 44.3 mph should be obtained. For this reason a 2-1/2-in. gunite lining is recommended.

The construction of the stack should be carried out in three stages. The steel shell should first be carried to about 195 ft above the base and the 2-1/2-in. gunite lining then placed to that height. The steel shell should then be continued to about 260 ft and the gunite lining placed to that height. The steel shell and gunite can then be completed. This procedure will eliminate the need for temporary exterior supports.

The thickness of the steel shell should be increased around the gas-inlet openings to allow for the increased stresses that result from carrying the large transverse shears past the openings. It is recommended that the 5/8-in. shell plate be extended to a point about 68 ft above the base. The 9/16-in. reinforcing plate can be left as shown. Even a small amount of second-mode vibration that is in phase with the first mode makes the stresses in this vicinity rather severe. The maximum compressive stress has been kept at 15.0 kips/sq in. at the base of the opening.

By means of the special formulas that were developed for the design of the circumferential stiffeners, the following changes are recommended:

3 x 9 x 5/16 angles replaced with 3 I 5.7#
4 x 4 x 3/8 angles replaced with 4 WF 13#
4 x 4 x 7/16 angles replaced with 5 WF 16#
4 x 4 x 1/2 angles replaced with 5 WF 18-1/2#
4 x 4 x 9/16 angles replaced with 6 WF 20#
6 x 4 x 1/2 angles replaced with 6 WF 25#

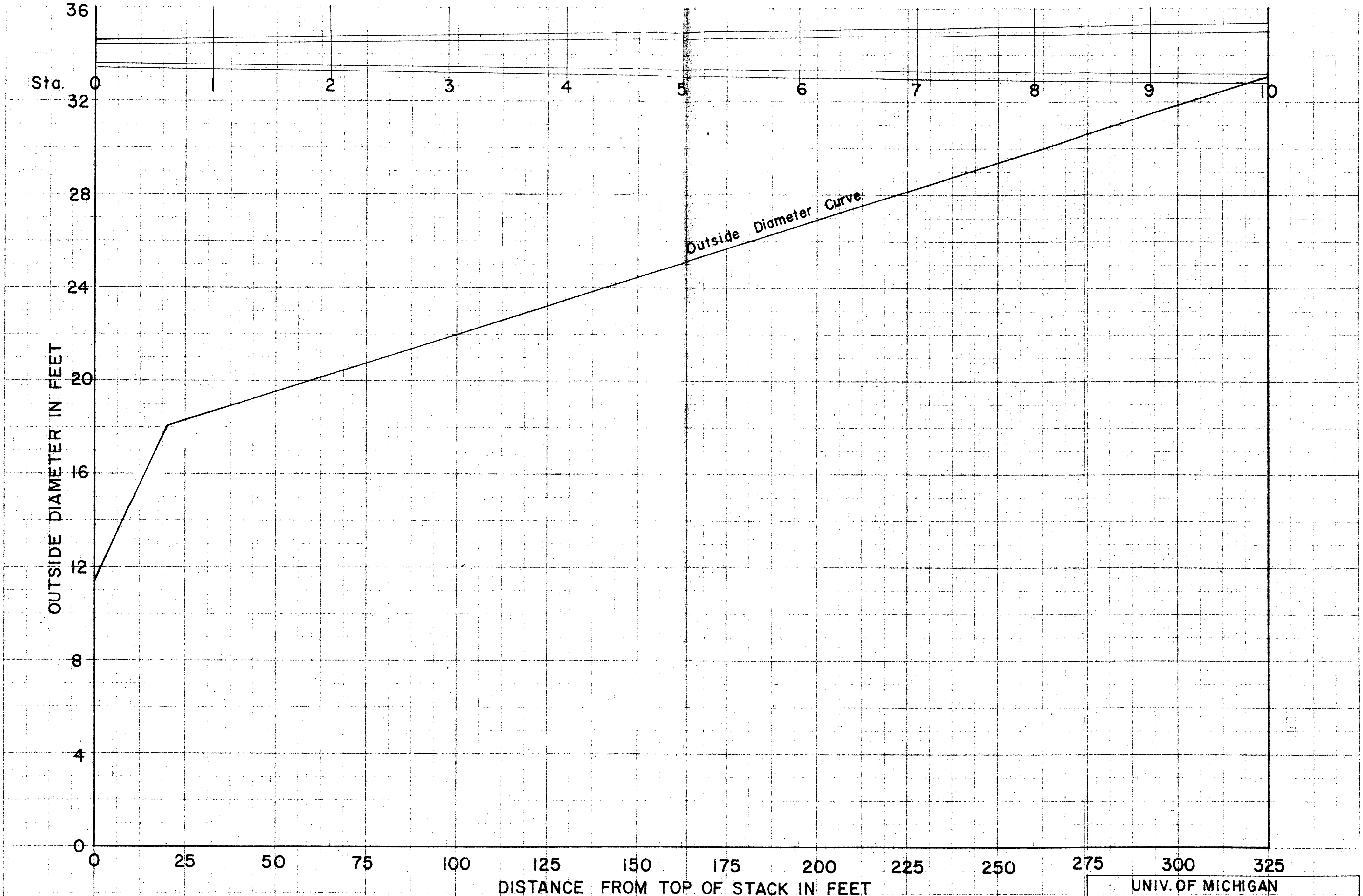
The spacing of these stiffeners is satisfactory as shown, that is, at 8 ft.

The present arrangement of the 3-1/4-in.-diameter anchor bolts is satisfactory if these bolts are made of the high-strength steel with

80-kips/sq in. yield point as now specified. To insure tightness at the base, these anchor bolts should be given an initial tension of 32 kips/sq in. The torque that is required to give this initial tension should be checked in the field by means of S-R.4 strain gages placed on the bolts. After two or three bolts have been checked, a measurement of the torque may then be sufficient if the measured results on the bolts do not vary by more than $\pm$ 5%. If an initial tension of 32 kips/sq in. is used in the anchor bolts, the maximum tensile stress will be approximately 43 kips/sq in. This value is determined from a conservative estimate of the moment at the base which is due to first- and second-mode vibration.

APPENDIX A

Design Data



OUTSIDE DIAMETER CURVE

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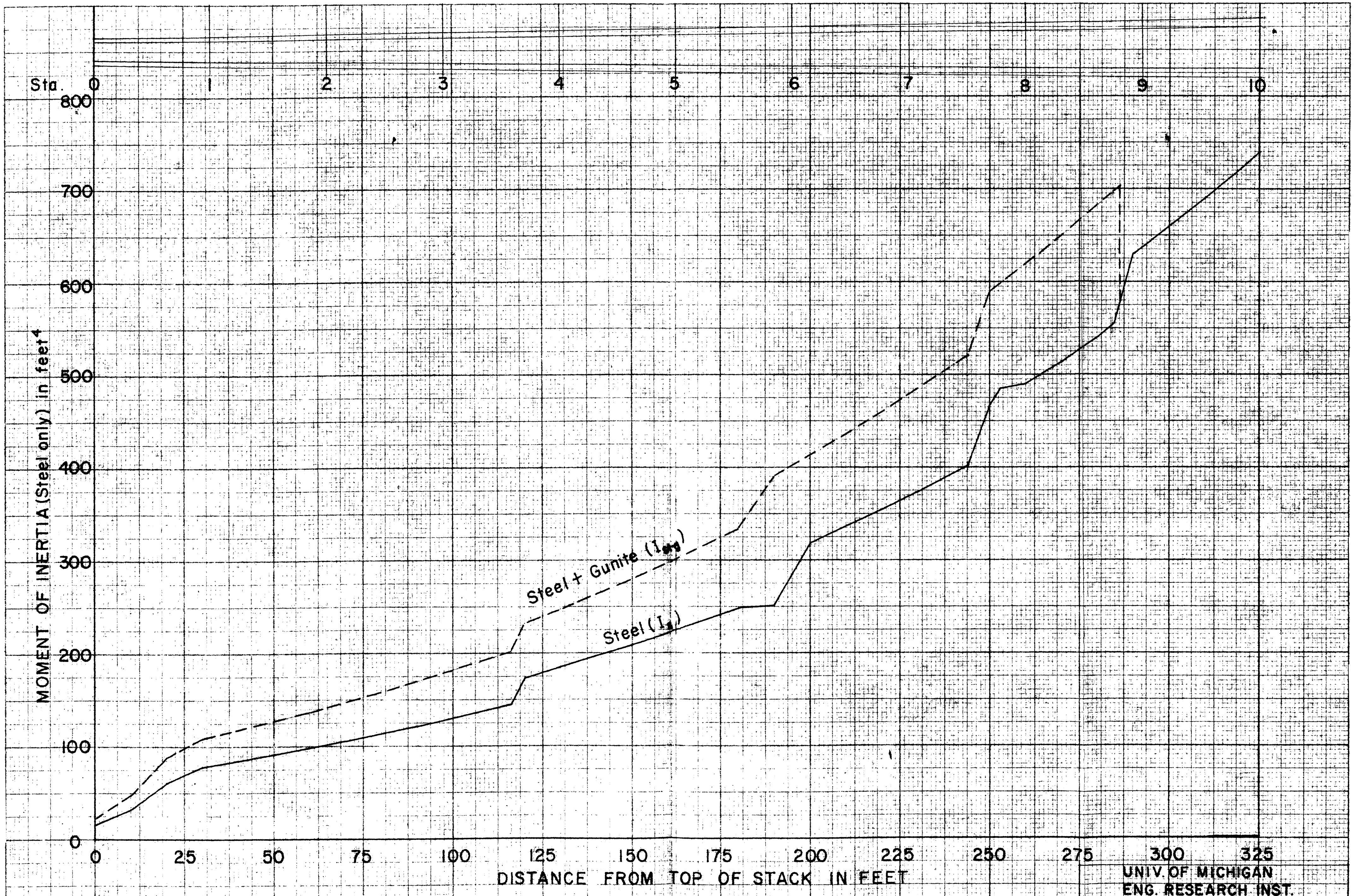
FIG. 8



MOMENT OF INERTIA CURVES

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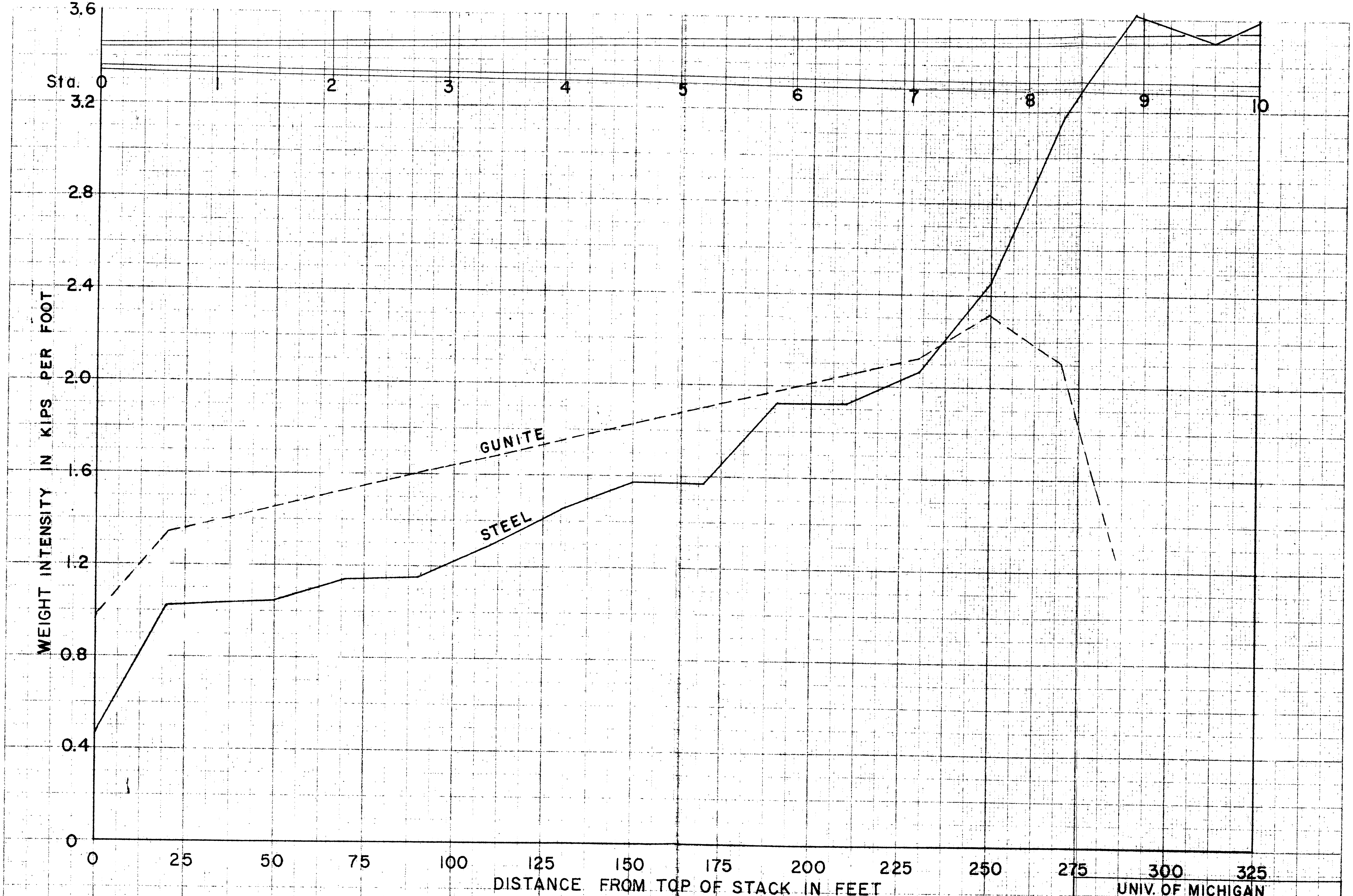
FIG. 9



WEIGHT INTENSITY CURVES

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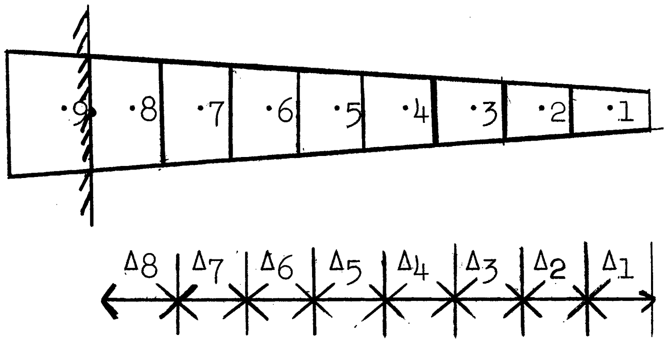
FIG. 10

APPENDIX B

Analog Computer Data



Equation of Motion

$$\frac{\partial^2}{\partial x^2}\left[M_{(x,t)}\right] + \rho_{(x)}\frac{\partial^2 u_{(x,t)}}{\partial t^2} + C_{(x)}\frac{\partial u_{(x,t)}}{\partial t} = f(x,t)$$

$f(x,t)$ = force per unit length

$\rho_{(x)}$ = mass per unit length

$C_{(x)} = \dfrac{\omega\,\rho_{(x)}\,\delta}{\pi}$

= damping force per unit length per unit transverse velocity,

where

ω = natural frequency (radians/sec)

δ = logarithmic decrement.

Writing the above equation in difference form we get:

$$\frac{M_2}{(\Delta_1)^2} + \rho_1\,\ddot{u}_1 + C_1\,\dot{u}_1 = f_1(t) \tag{1}$$

$$\frac{-2M_2 + M_3}{(\Delta_2)^2} + \rho_2\,\ddot{u}_2 + C_2\,\dot{u}_2 = f_2(t) \tag{2}$$

$$\frac{M_2 - 2M_3 + M_4}{(\Delta_3)^2} + \rho_3\,\ddot{u}_3 + C_3\,\dot{u}_3 = f_3(t) \tag{3}$$

$$\frac{M_3 - 2M_4 + M_5}{(\Delta_4)^2} + \rho_4\,\ddot{u}_4 + C_4\,\dot{u}_4 = f_4(t) \tag{4}$$

$$\frac{M_4 - 2M_5 + M_6}{(\Delta_5)^2} + \rho_5\,\ddot{u}_5 + C_5\,\dot{u}_5 = f_5(t) \tag{5}$$

$$\frac{M_5 - 2M_6 + M_7}{(\Delta_6)^2} + \rho_6 \ddot{u}_6 + C_6 \dot{u}_6 = f_6(t) \tag{6}$$

$$\frac{M_6 - 2M_7 + M_8}{(\Delta_7)^2} + \rho_7 \ddot{u}_7 + C_7 \dot{u}_7 = f_7(t) \tag{7}$$

$$\frac{M_7 - 2M_8 + M_9}{(\Delta_8)^2} + \rho_8 \ddot{u}_8 + C_8 \dot{u}_8 = f_8(t) \tag{8}$$

We know that $M_{(x,t)} = EI(x) \dfrac{\partial^2 u_{(x,t)}}{\partial t^2}$

The above can also be written in the difference form equations:

$$M_2 = \frac{E_2 I_2}{(\Delta_2)^2} (\ddot{u}_1 - 2u_2 + u_3) \tag{9}$$

$$M_3 = \frac{E_3 I_3}{(\Delta_3)^2} (u_2 - 2u_3 + u_4) \tag{10}$$

$$M_4 = \frac{E_4 I_4}{(\Delta_4)^2} (u_3 - 2u_4 + u_5) \tag{11}$$

$$M_5 = \frac{E_5 I_5}{(\Delta_5)^2} (u_4 - 2u_5 + u_6) \tag{12}$$

$$M_6 = \frac{E_6 I_6}{(\Delta_6)^2} (u_5 - 2u_6 + u_7) \tag{13}$$

$$M_7 = \frac{E_7 I_7}{(\Delta_7)^2} (u_6 - 2u_7 + u_8) \tag{14}$$

$$M_8 = \frac{E_8 I_8}{(\Delta_8)^2} (u_7 - 2u_8 + u_9) \tag{15}$$

Boundary Conditions:

$$M_1 = 0$$

The condition that the shear is zero at the end is satisfied by $M_0 = 0$, i.e., the change in bending moments is zero.

Due to the rotation at the base, $\ddot{u}_9 = -\ddot{u}_8$ (using $\Delta_8 = \Delta_9$) and thus Equation 15 will be written in the form

$$M_8 = \frac{E_8 I_8}{(\Delta_8)^2}(u_7 - 3u_8) \quad (16)$$

$$M_9 = k\theta \qquad \theta = \frac{2u_8}{\Delta_8}$$

$$M_9 = \frac{2ku_8}{\Delta_8} \quad (17)$$

Determination of the Constants

(Use kip-ft-sec) Units

$$\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = \Delta_7 = \Delta_8 = \underline{40.625 \text{ ft}}$$

$$\Delta_1^2 = \Delta_2^2 = \Delta_3^2 = \Delta_4^2 = \Delta_5^2 = \Delta_6^2 = \Delta_7^2 = \Delta_8^2 = \underline{1650}$$

To find mass intensities

Element	Steel Intensity k/ft	2-1/2" Gunite Intensity k/ft	Steel + Gunite Intensity k/ft	Steel + Gunite Mass Intensity w/g
1	0.99	2.11	3.10	0.0963
2	1.10	2.50	3.60	0.1118
3	1.24	2.74	3.98	0.1237
4	1.51	3.00	4.51	0.1400
5	1.77	3.25	5.02	0.1558
6	2.05	3.52	5.57	0.1730
7	2.93	3.42	6.35	0.1973
8	3.55	0.55*	4.10	0.1273

* Gunite on Platform 2-1/2"

To find the values of $C_{(x)}$

$$C_1 = \frac{(3.83)}{\pi} \times 0.0963\ \delta = 0.1175\ \delta$$

$$C_2 = \frac{(3.83)}{\pi} \times 0.1118\ \delta = 0.1364\ \delta$$

$$C_3 = \frac{(3.83)}{\pi} \times 0.1237\ \delta = 0.1510\ \delta$$

$$C_4 = \frac{(3.83)}{\pi} \times 0.1400\ \delta = 0.1708\ \delta$$

$$C_5 = \frac{(3.83)}{\pi} \times 0.1558\ \delta = 0.1902\ \delta$$

$$C_6 = \frac{(3.83)}{\pi} \times 0.1730\ \delta = 0.2110\ \delta$$

$$C_7 = \frac{(3.83)}{\pi} \times 0.1973\ \delta = 0.2410\ \delta$$

$$C_8 = \frac{(3.83)}{\pi} \times 0.1273\ \delta = 0.1554\ \delta$$

	$\delta = 0.15$	$\delta = 0.20$	$\delta = 0.25$	$\delta = 0.30$	$\delta = 0.35$	$\delta = 0.40$
C_1	0.01763	0.02350	0.02938	0.03525	0.04113	0.04700
C_2	0.02046	0.02728	0.03410	0.04092	0.04774	0.05456
C_3	0.02265	0.03020	0.03775	0.04530	0.05285	0.06040
C_4	0.02562	0.03416	0.04270	0.05124	0.05978	0.06832
C_5	0.02853	0.03804	0.04755	0.05706	0.06657	0.07608
C_6	0.03165	0.04220	0.05275	0.06330	0.07385	0.08440
C_7	0.03615	0.04820	0.06025	0.07230	0.08435	0.09640
C_8	0.02331	0.03108	0.03885	0.04662	0.05439	0.06216

<u>To find the values of $I_{(x)}$</u>

$$E_1I_1 = 29{,}500 \times 144 \times 105 = 446 \times 10^6$$
$$E_2I_2 = 29{,}500 \times 144 \times 163 = 692 \times 10^6$$
$$E_3I_3 = 29{,}500 \times 144 \times 218 = 926 \times 10^6$$
$$E_4I_4 = 29{,}500 \times 144 \times 311 = 1321 \times 10^6$$
$$E_5I_5 = 29{,}500 \times 144 \times 422 = 1793 \times 10^6$$
$$E_6I_6 = 29{,}500 \times 144 \times 543 = 2307 \times 10^6$$
$$E_7I_7 = 29{,}500 \times 144 \times 717 = 3046 \times 10^6$$
$$E_8I_8 = 29{,}500 \times 144 \times 675 = 2867 \times 10^6$$

<u>To find the force $f_i(t)$</u>

$$V = \frac{fD_{av.}}{S}$$

f = frequency of vortex shedding (cycles/sec)
S = Strouhal number = 0.2 approx.
$D_{av.}$ = Average diameter of the stack.
V = Velocity of wind in fps.

$$f = \frac{VS}{D_{av.}} = \frac{V}{5D_{av.}} = \frac{V}{5 \times 21.277} = \frac{V}{106.385}$$

$$f = \frac{P}{2\pi}$$

P is in radians per second

$$P = 2\pi f \qquad P = \frac{2\pi V}{106.385}$$

$$F_i(t) = C_L \, \rho \, \frac{V^2}{2} \, D_i \sin P t$$

$$F_i(t) = 1.000 \times \frac{0.002378}{1000} \times 1/2 \, V^2 \, D_i \sin \frac{2\pi V}{106.385} t$$

$$F_i(t) = 0.000001189 \, V^2 \, D_i \sin 0.059061 \, Vt$$

$F_i(t)$ in k/ft
D_i in ft
V in ft/sec

v mph	V fps	V^2	$0.000001189\ V^2$	0.059061 V
30	44.00	1,936.00	0.002302	2.599
35	51.33	2,634.77	0.003133	3.032
40	58.67	3,442.17	0.004093	3.465
45	66.00	4,356.00	0.005179	3.898
50	73.33	5,377.29	0.006394	4.331
55	80.67	6,507.65	0.007738	4.764
60	88.00	7,744.00	0.009208	5.197
65	95.33	9,087.81	0.010805	5.630
70	102.67	10,541.13	0.012533	6.064
75	110.00	12,100.00	0.014387	6.497
80	117.33	13,766.33	0.016368	6.930
85	124.67	15,542.61	0.018480	7.363
90	132.00	17,424.00	0.020707	7.796
95	139.33	19,412.85	0.023082	8.229
100	146.67	21,512.09	0.025578	8.662

$$+\ddot{u}_1 = -1.220\,\delta\,\dot{u}_1 - 0.00629\,(\qquad + M_2) + 10.384\,f_1(t)$$
$$-\ddot{u}_2 = +1.220\,\delta\,\dot{u}_2 + 0.00542\,(\quad - 2M_2 + M_3) - 8.945\,f_2(t)$$
$$+\ddot{u}_3 = -1.220\,\delta\,\dot{u}_3 - 0.00490\,(M_2 - 2M_3 + M_4) + 8.084\,f_3(t)$$
$$-\ddot{u}_4 = +1.220\,\delta\,\dot{u}_4 + 0.00433\,(M_3 - 2M_4 + M_5) - 7.143\,f_4(t)$$
$$+\ddot{u}_5 = -1.220\,\delta\,\dot{u}_5 - 0.00389\,(M_4 - 2M_5 + M_6) + 6.418\,f_5(t)$$
$$-\ddot{u}_6 = +1.220\,\delta\,\dot{u}_6 + 0.00350\,(M_5 - 2M_6 + M_7) - 5.780\,f_6(t)$$
$$+\ddot{u}_7 = -1.220\,\delta\,\dot{u}_7 - 0.00307\,(M_6 - 2M_7 + M_8) + 5.068\,f_7(t)$$
$$-\ddot{u}_8 = +1.220\,\delta\,\dot{u}_8 + 0.00476\,(M_7 - 2M_8 + M_9) - 7.855\,f_8(t)$$

$$M_2 = 419 \times 10^3\,(u_1 - 2u_2 + u_3)$$
$$M_3 = 561 \times 10^3\,(u_2 - 2u_3 + u_4)$$
$$M_4 = 801 \times 10^3\,(u_3 - 2u_4 + u_5)$$

$$M_5 = 1087 \times 10^3 \, (u_4 - 2u_5 + u_6)$$
$$M_6 = 1398 \times 10^3 \, (u_5 - 2u_6 + u_7)$$
$$M_7 = 1846 \times 10^3 \, (u_6 - 2u_7 + u_8)$$
$$M_8 = 1738 \times 10^3 \, (u_7 - 3u_8)$$
$$M_9 = 1717 \times 10^3 \, u_8$$

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