Introduction to the *BetsyProof-Start*[1] Video[2]

The short classroom episode shown in this video occurred on January 26, 1990. This third grade class had been exploring and solving word problems with multiple solutions that were designed to elicit observations about even and odd numbers, and patterns in adding them. For example:

gum 2¢

big pretzel 7¢

Mark has 30 cents in his pocket and he wants to spend it all and not have any change left in his pocket.

What can he buy for 30 cents?
What different choices does he have?
(from 1/10/90)

In this problem, the use of these specific numbers—2, 7, and 30—creates opportunities to notice patterns related to the addition of even and odd numbers. For instance, if Mark buys pretzels at 7¢ each, his purchase *must* include an even number of pretzels in order for the total to be 30¢, because 30 is an even number; an odd number of pretzels would cost an odd number of cents, and an odd number of cents plus any multiple of 2¢ cannot equal 30¢. The teacher had designed these problems to give the children practice with basic addition and multiplication as well as to create opportunities for working on multiples, even and odd numbers, and on mathematical reasoning and proof, such as how to show that one has found all possible solutions to the problem.

Problems such as this one led the class into interesting mathematical territory. The children began making conjectures about even and odd numbers. Among the conjectures proposed were:

1. An odd number plus an odd number equals an even number.
2. An even number plus an even number equals an even number.
3. An odd number plus an even number equals an odd number.

The teacher had them work in small groups, trying to prove that particular conjectures were true for all numbers or finding exceptions that did not work. The children investigated these mathematical patterns, and began to develop an understanding of the role of definitions in mathematical reasoning. They also began to see what is involved in mathematical proof (e.g., how would you prove that an odd number plus an odd number *always* equals an even number).

The video shows an episode that occurred near the end of a class when the pupils had been working on the different conjectures. One girl, Jeannie, explained that she and her partner, Sheena, had been working together on the Betsy's conjecture (an odd number plus an odd number equals an even number), but they couldn't find an example that didn't work. So, she explains, they then tried "to prove that you *can't* prove that Betsy's conjecture always works."

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[1] All names used are pseudonyms and are drawn appropriately, to the extent possible, from the individual children's actual linguistic and ethnic backgrounds. They also accurately reflect the pupils' gender.

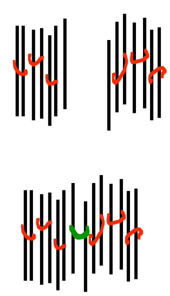
[2] This video was collected as part of a project that was funded in part by the National Science Foundation (TPE-8954724). This third-grade mathematics class and a neighboring fifth grade mathematics class were documented for an entire school year, beginning in September 1989 through June 1990.

The episode opens with the teacher asking whether anyone has any comments about the conjectures, or whether anyone has come up with an example that did not work (a counterexample). Jeannie makes her announcement, which is greeted with some disagreement by other students. The basis for Jeannie and Sheena's claim that the conjecture cannot be proved is that "numbers go on and on forever and that means odd numbers and even numbers go on forever and so you couldn't prove that all of them work." This statement represents an important mathematical insight—namely, the infinite quantification underlying the conjecture. The claim is about *all* odd and even numbers, two infinite sets. Ofala disagrees, explaining that she thinks the conjecture "*can* work" because she had tried many examples and they all worked. Ofala displays here the tendency to check mathematical claims by testing examples, which does not address the girls' argument that you could never check *all* odd and even numbers. Mei challenges the logical inconsistency of Jeannie and Sheena's argument, pointing out that the class had agreed to other claims in the past without checking *all* cases.

The episode reveals the mathematical depth of the issues that can arise within the ordinary content of the elementary curriculum, and the sophistication of which young pupils are capable. These four girls are grappling with an important mathematical challenge: how to prove something in general without being able to test all cases. Mathematical proof is powerful enough to deal with this challenge, and they are at the threshold of learning this.

The segment comes near the end of the class period, and the teacher asks the children to think about whether it will be possible to prove whether or not Betsy's conjecture is true.

In a subsequent class, Betsy offers a proof of her conjecture. Using this picture:



She argues "If you add two odd numbers together you can add the ones left over and it would always equal an even number." This line of reasoning is sometimes called a "generic proof" because, although it uses a particular case, it is an attempt at a general argument. Here she relies on implicit definitions of odd and even numbers. For example, a whole number is odd if it can be circled by twos with one left over. Because this definition itself is infinitely quantified (it is a property of <u>all</u> odd numbers) it can support a conclusion that is of similarly infinite purview.

The transition from explaining by providing examples to proving in general is a major watershed for learners. This episode provides a glimpse of young pupils' encounter with this significant aspect of mathematical reasoning, one that is important to the development of their ability to think mathematically.