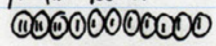
1/25/90, cont.

Mei: 18 is even because 10 is even and 8 is even so 18 is even.

Lucy: 114 is even because 100 is even, 10 is even, 4 is even.

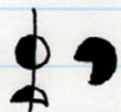
Maria and Devin both showed solutions (Maria: 8, Devin: 13

At first he did



and so, he said, it was even.

Nathan challenged and Haruon revised his answer.



I asked Betsy and Jeannie whether $1\frac{1}{2}$ is even or odd. A stumper.

"It's not even because you can't split in half without using halves."

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**Friday, 1/26/90**

Jim, Erin, Margery, Kara — Janine, Meredith

(Videotape sound quality terrible with remote mike. Gets fine at 1:41 when whole group discussion starts.)

Prove or revise:

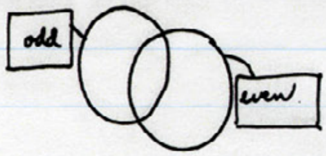
An odd number plus an odd number always equals an even number. } Came from Betsy's conjecture — (see p. 169 1/16/90)

Class began late — the PDS long lunch was followed by an assembly, so the kids weren't back in the room until about 1:10. We began with a couple of people giving examples to go with Betsy's conjecture, and Riba gave 9+9=18, Ofala 7+7=14. Riba "proved" hers — that meant showing that 9+9 was equal to 18. (The different things that the kids do and call "proving" is interesting: from showing that a given result is right, to explaining a method plus answer (solution), showing something is true.)

I was thinking that this should be the last day on odd and even numbers, but given the way it ended, I'm not sure. Two issues (at least!) are outstanding → is $1\frac{1}{2}$ even or odd? This seems an important question to explore because I'd like them to have a sense that not all numbers are —

1/26/90, cont.

either even or odd, then there are numbers that would go <u>outside</u> the strings here:




The kids worked in small groups for about half an hour. Some observations on particular children:

Cassandra and Lucy worked, trying lots of examples with double odd numbers. (With some kids, I just wanted to make sure they understood what the conjecture <u>said</u>.) Tembe asked if the two odd numbers had to be the <u>same</u> odd number - he was the only one who thought of that. Then he tried 7+9 and got 17 by adding wrong. I helped him discover his computational error and encouraged him to pursue his question.

Cassandra tried $\begin{array}{r} 35 \\ +35 \\ \hline 70 \end{array}$ as one of her examples. When I asked how she knew 70 was even, she said because 69 was odd. When I asked if she could explain this another way, she said because half of 70 was 35, but when I asked her how she knew that, she had trouble justifying that, even though $\begin{array}{r} 35 \\ +35 \\ \hline 70 \end{array}$ was right in front of her.

Sheena and Jeannie approached me at about 1:40 to say that this conjecture could not be proved to be <u>always</u> true "because numbers go on and on forever and you can't just keep writing." They said that finding all these examples just served to make the conjecture <u>maybe</u> be true, but it couldn't be proved to be <u>always</u> true.

1/26/90, cont.

Some kids worked on the board, some at the back portable chalkboard.

Maria worked alone but really was into the conjecture. She proved to me that

$$\begin{array}{r} 21 \\ +\ 21 \\ \hline 42 \end{array}$$

was an example because 21 was odd (she used the "one left over" definition) and 42 was even (she used the "groups of two" approach).

The group discussion lasted about 20 minutes. Cassandra showed

$$\begin{array}{r} 35 \\ +35 \\ \hline 70 \end{array}$$

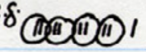
as another example. When asked how she knew that 70 was even, she relied on the even-odd alternating argument (even though the number line doesn't go past 56). Lindiwe showed her that she could use the meterstick as a number line.

Betsy suggested revising her conjecture to say that it had to be the <u>same</u> odd number. Tembe objected, using 7+9=16. But in showing it, ~~he~~ he became confused, thinking it refuted ~~the~~ the the conjecture. Lucy said that two odds were <u>supposed</u> to equal an even, given the conjecture. Betsy, though, was convinced by Tembe's example and decided to withdraw her revision. She offered that she'd tried this with numbers below zero.

Then Jeannie brought up this point that she and Sheena found — that this conjecture could not be proved. Ofala disagreed because she tried 18 examples (and "even a Sean number")

1/26/90, cont.

and they were always even. Stan said something similar – he'd kept trying different odd numbers and same odd numbers and they kept turning out even. Riba, too, concurred. Mei challenged Jeannie: "Why did you say that all those were true?" (pointing to the conjectures above the chalkboard that we've all been using) Jeannie responded that she hadn't thought of it and Sheena, coming to Jeannie defense, turned to Mei and said, "She just thought of it today."

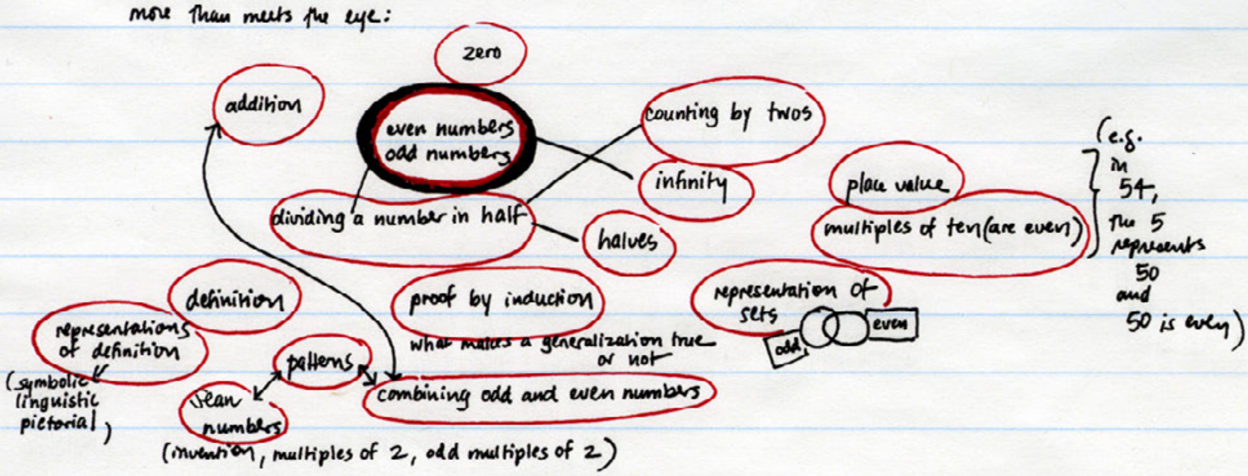
An interesting issue is whether or not eight- and nine-year-olds can accept that, since an odd number will always be of the form $2n+1$ (represented by them as, e.g. ), then an odd number plus an odd number will always be even? Or will they see any such drawing as an example of a specific case? (because the two "loose" leftovers will combine to make a new group of 2) Also interesting is the representational issue: How good/problematic is the correspondence between the drawing of the form of an odd or even number and the algebraic expression ($2n$ or $2n+1$)? Since one has to have some number of lines in the drawing, does that make it more difficult to see it as a representation of the general form?

(Sluggish as this discussion was (probably) due to my cold and laryngitis, [Note after having recently reviewed tape of first day of school!!] there is certainly a marked difference between this and the teacher-student patterns of discourse seen on Sept. 11! I really notice kids talking more to one another, wanting to respond to one another, etc.!)

1/26/90, cont.

So why is this area – ostensibly even and odd numbers — worth spending time on?

* One issue is to unravel the mathematics of this investigation, for it includes more than meets the eye:




(This kind of analysis would be useful for prospective teachers to undertake: What mathematical opportunities exist within the territory we have been exploring? ).

(I wish I knew how to create a map that would pictorially represent the map (or a map!) of these (and other) ideas available in the kids' experiences. This map does not represent the relationships among or any hierarchy or relative emphasis among and between them.

* Another issue is how enthusiastic the kids are about this content. Why is that? Last year I was struck by the same thing. Is this generally true or is just that I like the content? (Once again, I wonder about qualitative differences in the) discourse from when we were doing addition and subtraction with regrouping. I remember musing about this last year, too.)

* A closely related issue is the opportunities kids have to invent or discover mathematics – e.g. the "Sean numbers". This domain allows for such pattern-finding and conjecturing. (Jeanie's puzzle about "we can not prove it") (Sheena and Jeanne went back into that issue, concerned less for whether it was right or wrong but more for whether it could be proved!)