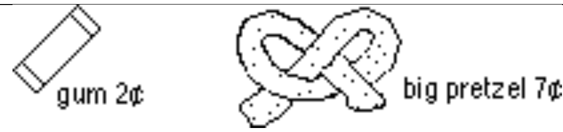
<u>Introduction to the *Sean*[1] *Numbers-Ofala* Video[2]</u>

The short classroom episode shown in this video occurred on January 19, 1990. This third grade class had been exploring and solving word problems with multiple solutions that were designed to elicit observations about even and odd numbers, and patterns in adding them. For example:



Mark has 30 cents in his pocket and he wants to spend it all and not have any change left in his pocket.

What can he buy for 30 cents?
What different choices does he have?
(from 1/10/90)

In this problem, the use of these specific numbers—2, 7, and 30—creates opportunities to notice patterns related to the addition of even and odd numbers. For instance, if Mark buys pretzels at 7¢ each, his purchase *must* include an even number of pretzels in order for the total to be 30¢, because 30 is an even number; an odd number of pretzels would cost an odd number of cents, and an odd number of cents plus any multiple of 2¢ cannot equal 30¢. The teacher had designed these problems to give the children practice with basic addition and multiplication as well as to create opportunities for working on multiples, even and odd numbers, and on mathematical reasoning and proof, such as how to show that one has found all possible solutions to the problem.

Problems such as this one led the class into interesting mathematical territory. The children began making conjectures about even and odd numbers. One girl, Betsy, proposed, "An odd number of odd numbers will *always* equal an odd number." Keith suggested, "If you add an odd number to an even number, the answer will always be an odd number." And Lindiwe noticed that, "No matter how many even numbers you add together, the answer will always be *even*." The pupils were engaged in exploring these conjectures. The teacher had them work in small groups, trying to prove that particular conjectures were true or finding exceptions that did not work. The children investigated these mathematical patterns, and began to develop an understanding of the role of definitions in mathematical reasoning. They also began to see what is involved in mathematical proof (e.g., how would you prove that an odd number plus an odd number *always* equals an even number).

The video shows an episode early in the class' work on even and odd numbers. The pupils are learning to use the working definitions they have been developing. The episode begins about seven minutes into the lesson with the teacher inviting comments about a meeting they had the day before with the fourth-grade class. This was to be followed with group work on the conjectures about adding even and odd numbers. Instead the discussion took an unexpected turn.

The episode opens with the teacher asking if anyone has any other comments about the meeting. Just before this, a boy named Nathan commented on an idea raised at this meeting and claimed that even numbers can be "made" from two other even numbers—e.g., eight can be "made from" 4 + 4; twelve

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[1] All names used are pseudonyms and are drawn appropriately, to the extent possible, from the individual children's actual linguistic and ethnic backgrounds. They also accurately reflect the pupils' gender.

[2] This video was collected as part of a project that was funded in part by the National Science Foundation (TPE-8954724). This third-grade mathematics class and a neighboring fifth grade mathematics class were documented for an entire school year, beginning in September 1989 through June 1990.

can be made from 6 + 6. The teacher next called on a boy named Sean, who figures prominently in the episode on the video. Sean says he has no comments about the meeting but he has noticed something "special" about the number 6." He claims that 6 could be even *and* it could be odd. This provokes disbelief among his classmates, who disagree. The video provides a small glimpse into the discussion of Sean's idea. It consists of three short segments, approximately 10 minutes in total.

Near the end of the video clip, Mei proposes that 10 has the same property that Sean has noticed about 6: Both these numbers comprise *an odd number of groups of two*. In the ensuing discussion (after the video ends) the children review and clarify their emerging definition of an odd number (i.e., that a whole number is called "odd" if there is one left over when it is grouped by twos). Many more children become involved in the discussion. They explore a bit further the pattern of numbers identified by Sean and Mei. The pupils discover that these occur every fourth number (i.e., . . . 6, 10, 14, 18, . . .). Although this may seem off-track, in fact, a set of numbers related to these were discussed in the ancient Euclid's *Elements*.[3] Moreover, in modular arithmetic, these are the numbers congruent to 2 mod 4, and they are also exactly those whole numbers not expressible as a difference of two squares. Sean and his classmates were observing a significant mathematical pattern, but did not yet know how to name it. Thus, Sean incorrectly tried to label these numbers "even *and* odd." Eventually, however, this special subset of the even numbers was referred to as the "Sean Numbers," and was defined as those even numbers made up of an odd number of groups of two. In the course of this episode the children practice mathematical reasoning, they develop mathematical arguments to justify their claims, and they learn to listen carefully to and evaluate the arguments of others.

[3] From Euclid's *Elements* VII: Definition 9—An **even-times odd number** is that which is measured by an even number according to an odd number [(Stamatis, 1970, p. 103]. The children were talking about numbers of the form *two times an odd number*, a subset of the "even-times odd numbers." See Stamatis, E. S. (Ed.). (1970). *Euclidis Elementa.* Leipzig: BSB B. G. Teubner.