1/19/90, Friday

I opened class with the intention of "interviewing" the class about the meeting — but they were not to be diverted into talking about mere process! They kept wanting to talk more about the *issues* raised at the meeting, not what they thought *about* the meeting! It was kind of funny: Here I was, asking what they thought about having the meeting, about what they noticed, etc. and Sean would say something like, "I just want to say something about zero being even. You can't *prove* that because what are you going to make zero out of?" And someone would respond and then I would persist in trying to get them to reflect on the meeting and then someone else would go back to the substantive issues. Near the end of the class I did return one more time to such a question, asking them why they thought I had asked them how many people thought zero was even, how many thought it was neither even or odd, etc. Betsy said she thought it was so I could see how they were thinking. I asked them if they thought that the idea was whichever point of view had more people, it was right? No one seemed to think so. Jennie said, rather articulately that she "didn't care if a different view had more people," that she'd agree with a different position — only if someone *convinced* her. It was pretty neat.

One of Mei's comments: "After the discussion yesterday, I was mixed up." She said different people's arguments were all convincing. I asked her what she was going to do about that. She said she was going to listen to the discussion and think about it some more.

The substantive focus today: The definition of an *odd* number. Sean argued that "6 could be even" because

oo|oo|oo it takes *3* groups of two and 3 is an odd number.

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I said that we needed to discuss what our definition of an odd number was — that although we'd discussed even numbers and come up with a "working definition" of an even number, we'd not discussed <u>odd</u> numbers. So Ofala introduced the idea that an odd number was one where if you circle all the twos there would be one left

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Ofala had a <u>stunning</u> day today: She was articulate, confident, and insistent. She defended and illustrated her definition against Sean's insistence that some numbers <u>could</u> be "even or odd" — e.g. 6, 10. Sean, too, was willing to keep explaining his idea, even in the face of my question: "Why would it be useful to have a definition that some numbers would end up both even and odd?" He said it wasn't necessarily <u>useful</u>, but he was just thinking it <u>could</u> be. We exploited his idea: Was there a pattern to the ones he was considering "even or odd"? It turned out there was: Riba identified it as 2, 6, 10, 14.... Sean said it was every four numbers. When Riba said that 2 was such an example, Sean disagreed until she argued that it was an <u>odd</u> number of groups of 2 and he agreed.

At first he said only 6, then 6 and 10. Then I asked about 14, drawing 14 lines on the board. He said that was one. That's when I asked if there was a pattern.

Sean said he had changed his mind re: 1 being even because his mom had told him that 1 was odd, not even. I asked how (or why) she had been successful in convincing him and he said * that he could <u>trust</u> his mom. He couldn't necessarily trust everyone in our class because, for example, if you told someone a secret, they would sometimes tell other people. But he could trust

(see further discussion, p. 185)

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some people because he'd known them so long — like Mei, because he'd known her since first grade. Mei nodded assent.

Ofala argued that her definition (although she called it, proudly, a "conjecture") didn't say anything about the number of groups, (which is what Sean was focusing on) just whether, when you grouped by twos, there was one left over. Sean maintained a stance that a number was both even and odd if it, when grouped by twos, had an odd number of groups.

(I'm wondering if I should introduce the idea that Sean has identified (discovered?) a new category of numbers — those that have this property he has noted. Maybe they could be named something. Or maybe this is silly — will just confuse kids since it's nonstandard knowledge — i.e., not part of the wider mathematical community's shared knowledge. I have to think about this.)

(It has the potential to enhance what kids are thinking about "definition" and its role, nature, purpose in mathematical activity and discourse, which, after all, has been the substantive point of spending so much time on this this week.) What should a definition do? Why is it needed? For example, a definition of even numbers that says it's every other (whole) number starting with 0 (or 2) is pretty useless for dealing with Lucy's example of 1,421. Even the grouping by 2 definition (corollary to Ofala's odd number definition) is not too helpful for 1,421. But a definition centered on divisibility is (They all relate to this, of course, to a greater or lesser

get Phil's paper on definition!

1/19/90, cont.

degree) because then you can show that 1,421 is odd because 1,000 is even, 400 is even, 20 is even, but 1 is odd. That leads to the proof of Lucy's conjecture that "you only have to look at the last number." (Many adults know this rule but do not know why it works.)

A second aspect of definition is that it facilitates discourse. That was where we started, because people were meaning different things by "even number" and that was going to make debates over the four conjectures (see p. 169) difficult — impossible.

Another, I think, is logical partitioning? Well, in a case like this, maybe, but certainly not always, or there wouldn't be intersecting sets in number theory (e.g. prime numbers, odd numbers).

* That's where Sean's discovery fits, maybe.

So I would hope that, in addition to learning (meaningfully) what even and odd numbers are, and what are some logical patterns about how they behave (e.g. Keith's conjecture), the kids are also developing some appreciation for the importance of definition in a mathematical community and in mathematical discourse. Although perhaps not explicit, it might show up in future discourse where definition emerges as either problematic or central to the discussion.

* On trusting parents to tell you the truth: I asked, carefully, had anyone had the experience of arguing or challenging something and managing to convince their parents. I meant in math, thinking that things might have come up based on what we've been doing in class, but what I got were non-math examples. Ofala said her father had told her she couldn't play soccer

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because "in our country girls don't play soccer" but she "proved" to him that they do and in America they do too" and now he agreed to let her play soccer. (!!)

An interesting mismatch between the two definitions we're using for even and odd numbers. Our "working definition" of even numbers is numbers that are divisible (evenly) by two while our odd number definition is numbers that when you form groups of 2, you get one left over. Our even number definition is based on a partitive meaning for division by 2 (i.e., form two groups) while our odd number definition is based on a measurement meaning (i.e. form groups of 2). I wonder if this is just chance? It seems rather easier to divide even numbers in half because kids are so aware of doubles, while with odd numbers, maybe it seems easier to try making groups of 2? Also, on another note, we haven't tried to integrate the other "pieces" of their mathematical knowledge about even and odd numbers → e.g., counting by twos (starting with 0 or 2) or that even and odd numbers alternate ("even, odd, even, odd"). Could I get the kids to see that these are not stipulative definitions, but merely descriptive (obviously, not in these terms! — but the point that these don't say what's special, what makes a given number even or odd)?