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JHEP07(2009)049

(http://iopscience.iop.org/1126-6708/2009/07/049)

Published by IOP Publishing for SISSA

Received: *June 5, 2009*
Accepted: *June 22, 2009*
Published: *July 14, 2009*

Flux attractors and generating functions

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Abstract: We use the flux attractor equations to study IIB supergravity compactifications with ISD fluxes. We show that the attractor equations determine not just the values of moduli fields, but also the masses of those moduli and the gravitino. We then show that the flux attractor equations can be recast in terms of derivatives of a single generating function. We also find a simple expression for this generating function in terms of the gravitino mass, with both quantities considered as functions of the fluxes. For a simple prepotential, we explicitly solve the attractor equations. We conclude by discussing a thermodynamic interpretation of this generating function, and possible implications for the landscape.

Keywords: Flux compactifications, Superstring Vacua, Black Holes in String Theory

ArXiv ePrint: 0905.2130

doi:10.1088/1126-6708/2009/07/049

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Contents

1 Introduction

The compactification of string theory from 10 to 4 dimensions is a subject of both formal and phenomenological interest. Many methods of compactification result in moduli: massless 4D scalar fields which correspond to deformations of the compactification geometry.

Given the observed absence of massless scalars, these moduli are phenomenologically undesirable. As a result, much attention has been focused on the question of how moduli can be *stabilized*, i.e. how features can be added to a simple compactification so that most or all of the 4D scalar fields become massive. We can consider this question in three different levels of detail:

1. Is the proposed stabilization method consistent? That is, does the stabilized compactification still solve the 10D equations of motion?
2. Which moduli are stabilized, and what are their VEVs?
3. What are the masses of the moduli?

In this paper we study compactifications of IIB string theory on Calabi-Yau orientifolds, with RR and NS 3-form flux in the compact directions. The *flux attractor equations* [1] describing the stabilization of the moduli strongly resemble black hole attractor equations, and we will exploit this similarity to address the questions above.

We will focus our attention on one of the 10D equations of motion. If the (real) 3-form RR flux is F_3, the (real) 3-form NS flux is H_3, and the complex axio-dilaton is τ, we define the complex 3-form flux

$$G_3 \equiv F_3 - \tau H_3 \,. \tag{1.1}$$

For large classes of compactifications to 4D Minkowski space, the 10D equations of motion require [2, 3] that G_3 be imaginary self dual (ISD):

$$*_6 \, G_3 = i G_3 \,. \tag{1.2}$$

Because $*_6$ involves the metric, a non-zero G_3 stabilizes some or all of the complex structure moduli and τ. Specifically, the complex structure of the Calabi-Yau is fixed so that G_3 has only $(0, 3)$ and/or $(2, 1)$ components. If no such combination of complex structure and τ exists, the choice of F_3 and H_3 is not consistent with compactification to Minkowski space. In order to analyze (1.2) in detail we may expand G_3 and the holomorphic 3-form, Ω_3, on a judiciously chosen basis of 3-cycles. This procedure results in the flux attractor equations, as we review in section 2.

The resulting algebraic equations suffer an apparent inconsistency, in that there are many more equations than moduli. If $n = b_3/2 - 1$ is the number of $(2, 1)$ cycles on the Calabi-Yau, we will find $4n + 4$ different (real) equations and only $2n + 2$ (real) moduli. While this mismatch suggests that the system of equations is overconstrained, we will show that this is not the case. In section 3 we will show that the $4n + 4$ attractor equations determine both the VEVs of the moduli *and* the independent parameters of their mass matrix, as well as the gravitino mass. All of these outputs together constitute $4n + 4$ parameters, the same as the number of input fluxes.

Having established that the flux attractor equations determine both the moduli VEVs *and* certain mass parameters, in section 4 we develop an algorithm to find them. We take inspiration from OSV [4], who solved the black hole attractor equations by introducing a mixed ensemble. Accordingly, we first solve the "magnetic" half of the attractor equations,

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writing our $4n + 4$ parameters in terms of the $2n + 2$ magnetic fluxes and $2n + 2$ as-yet-undetermined electric potentials. We then show that the "electric" attractor equations can be rewritten in terms of a generating function, and that they can be formally solved by a simple Legendre transform.

The existence of the generating function $\mathcal{G}$ is the principal result of this paper. If one can determine it as a function of arbitrary fluxes, its derivatives will give back the moduli VEVs and the mass parameters. Thus $\mathcal{G}$ provides a compact summary of the flux attractor behavior, and this suggests that we study the properties of $\mathcal{G}$ directly. We initiate such a study in section 5, where we find a general formula for $\mathcal{G}$:

$$\mathcal{G} = \int F_3 \wedge H_3 - 2\mathrm{Vol}^2 m_{3/2}^2. \tag{1.3}$$

Here the gravitino mass is considered as a function of arbitrary fluxes.

We proceed in section 6 by considering an explicit example. We use the prepotential $F = Z^1 Z^2 Z^3 / Z^0$, a setting with sixteen distinct fluxes. For a reduced set of eight of these fluxes we are able to completely solve the flux attractor equations. We then argue that the general case can be solved as well, by appealing to duality transformations.

For the sake of simplicity, we will discuss many of our results in the context of large-volume, unwarped compactifications. These lead to relatively well-understood 4D theories, and we can easily translate our findings about the 10D geometry into statements about 4D physics. However, our 10D reasoning applies equally well to strongly-warped compactifications and some non-geometric compactifications [5]. Since we are analyzing the ISD condition, which is quite robust, we expect our qualitative understanding of the flux attractor behavior, such as the existence of a generating function, to be similarly robust. On the other hand the detailed mass spectrum depends on the Kähler potential, and is therefore less robust.

As we have mentioned above, the solution of the flux attractor equations is controlled by a single generating function, which depends on the fluxes alone. In the case of the black hole attractor, the analogous function turned out to the the equilibrium value of the black hole entropy. It is tempting to speculate that the flux attractor equations also describe a thermodynamic system. Ultimately, the underlying statistical system may be related to a classical measure on this patch of the string theory landscape. We conclude in section 7 by summarizing the issues that must be resolved in order to make this interpretation sound.

2 From the ISD condition to attractor equations

In this section we review some basic aspects of special geometry and flux compactifications. We then provide a simple derivation of the flux attractor equations.

2.1 Special geometry

Most of the objects we are interested in, including F_3, H_3, and Ω_3, are 3-forms on the compact space. It is useful to expand these 3-forms on a real basis $\{\alpha_I, \beta^I\}$, $I = 0, \ldots, n$,

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satisfying

$$\int \alpha_I \wedge \beta^J = \delta_I^J\,, \tag{2.1}$$

$$\int \alpha_I \wedge \alpha_J = \int \beta^I \wedge \beta^J = 0\,. \tag{2.2}$$

We specify the NS fluxes H_3 and RR fluxes F_3 with respect to this basis as

$$H_3 = m_h^I \alpha_I - e_I^h \beta^I\,, \tag{2.3}$$

$$F_3 = m_f^I \alpha_I - e_I^f \beta^I\,. \tag{2.4}$$

There is an $Sp\,(2n+2,\mathbb{R})$ symmetry[1] that corresponds to a change in the basis $\left\{\alpha_I, \beta^I\right\}$. The fluxes $\left\{m_h^I, e_I^h\right\}$ and $\left\{m_f^I, e_I^f\right\}$ transform in the fundamental of $Sp\,(2n+2,\mathbb{R})$, and objects with an index $I, J, K \ldots$ transform in the fundamental of $SO\,(n+1,\mathbb{R}) \subset Sp\,(2n+2,\mathbb{R})$.

We can also expand the holomorphic 3-form with respect to the real basis,

$$\Omega_3 = Z^I \alpha_I - F_I \beta^I\,. \tag{2.5}$$

The combination $\left\{Z^I, F_I\right\}$ is called a *symplectic section* [6], and also transforms in the fundamental of $Sp\,(2n+2,\mathbb{R})$. While the fluxes $e_I^{h,f}$ and $m_{h,f}^I$ were all independent parameters, the F_I and Z^I are holomorphic functions of the complex structure moduli. For our purposes, it is sufficient to treat the F_I as functions that are holomorphic and homogeneous of degree 1 in the Z^I. The functional form of the F_I is the only information about the Calabi-Yau geometry that we will use.

The holomorphic 3-form is only defined up to a holomorphic rescaling,

$$\Omega_3 \to f\left(Z^I\right)\Omega_3\,. \tag{2.6}$$

These are the *Kähler transformations.* If, under Kähler transformations, an operator is simply multiplied by h powers of $f\left(Z^I\right)$ and $\overline{h}$ powers of $\overline{f\left(Z^I\right)}$, we will say that it is Kähler covariant with weight $\left(h, \overline{h}\right)$. For example, Ω_3 has weight $(1,0)$.

Physical moduli must be invariant under Kähler transformations. For example, on a patch where $Z^0 \neq 0$ we may use the ratios

$$z^i \equiv \frac{Z^i}{Z^0}\,, \tag{2.7}$$

where $i = 1, \ldots, n$. The z^i are clearly Kähler invariant. Unfortunately, this breaks the $SO\,(n+1)$ symmetry enjoyed by the Z^I, so we will sometimes use an alternative approach to formulating Kähler invariant quantities. We will utilize a coefficient C which has weight $(-1,0)$, so that the products CZ^I are Kähler invariant.

[1]Dirac quantization conditions require the magnetic fluxes $m_{h,f}^I$ and electric fluxes $e_I^{h,f}$ to take integer values, breaking $Sp\,(2n+2,\mathbb{R})$ to a discrete subgroup.

Because Kähler transformations are local, ordinary derivatives of Kähler covariant functions do not give new Kähler covariant functions. We introduce the Kähler potential

$$K_z = -\log i \int \Omega_3 \wedge \overline{\Omega}_3 \,, \tag{2.8}$$

which generates the metric on moduli space,

$$g_{i\bar{j}} = \partial_i \overline{\partial}_{\bar{j}} K_z \,. \tag{2.9}$$

By construction, e^{K_z} has weight $(-1,-1)$. This motivates the definition of the Kähler covariant derivative of an operator of weight $(h, \overline{h})$,

$$D_i \mathcal{O}^{(h,\overline{h})} \equiv e^{-hK_z} \partial_i \left(e^{hK_z} \mathcal{O}^{(h,\overline{h})} \right) = \partial_i \mathcal{O}^{(h,\overline{h})} + h \mathcal{O}^{(h,\overline{h})} \partial_i K_z \,. \tag{2.10}$$

We note that since the Kähler potential is real, the Kähler covariant derivative of a holomorphic object is not itself holomorphic.

It is especially interesting to consider derivatives of the holomorphic 3-form. An ordinary derivative with respect to the complex structure moduli gives a sum of $(3,0)$ and $(2,1)$ forms,

$$\partial_i \Omega_3 = k_i \Omega_3 + \chi_i \,. \tag{2.11}$$

If we instead use a Kähler covariant derivative, the Kähler potential is constructed so that the $(3,0)$ piece cancels and we are left with only a $(2,1)$ form,

$$D_i \Omega_3 = \chi_i \,. \tag{2.12}$$

This establishes a convenient *complex* basis for 3-forms on the Calabi-Yau, $\{\Omega_3,\ D_i\Omega_3,\ \overline{D_i\Omega_3},\ \overline{\Omega}_3\}$ [7]. The intimate connection between the complex structure of a Calabi-Yau and its cohomology will be the primary tool that we use to analyze the ISD condition (1.2).

2.2 S-Duality

In addition to Kähler transformations, S-duality helps organize the flux attractor equations. Type IIB supergravity has an $SL(2,\mathbb{R})$ symmetry,[2] under which

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d} \,, \tag{2.13}$$

$$\begin{pmatrix} F_3 \\ H_3 \end{pmatrix} \rightarrow \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} F_3 \\ H_3 \end{pmatrix} , \tag{2.14}$$

with the constraint

$$ad - bc = 1 \,. \tag{2.15}$$

The transformation of the complex flux G_3 under S-duality can be deduced from the transformations of F_3, H_3, and τ :

$$G_3 \rightarrow \frac{G_3}{c\tau + d} \,. \tag{2.16}$$

[2]Quantum effects break this to $SL(2,\mathbb{Z})$, but the distinction between the two groups will not be relevant to our analysis.

We will frequently encounter $\mathrm{Im}\,(\tau)$, which transforms as

$$\mathrm{Im}\,(\tau) \to \frac{\mathrm{Im}\,(\tau)}{|c\tau + d|^2} \,. \tag{2.17}$$

2.3 4D physics of large volume compactifications

The flux attractor equations are simply a rephrasing of the ISD condition (1.2). We could discuss the ISD condition entirely from the 10D point of view, but we find it useful to make reference to the resulting 4D effective theory. As long as the volume of the Calabi-Yau is large relative to the string scale, and regions of strong warping are all string scale, the result is a 4D, $\mathcal{N} = 1$ theory with the GVW superpotential [8, 9],

$$W = \int_{\mathrm{CY}} G_3 \wedge \Omega_3 \,, \tag{2.18}$$

and Kähler potential,[3]

$$\begin{aligned} K &= K_z + K_\tau + K_t \qquad (2.19) \\ &= -\log\left[i\int_{\mathrm{CY}} \Omega_3 \wedge \overline{\Omega}_3\right] - \log\left[2\mathrm{Im}\,(\tau)\right] - 2\log\left[\mathrm{Vol}\right] . \qquad (2.20) \end{aligned}$$

These compactifications are reviewed in e.g. [10–13]. While both the superpotential and Kähler potential receive a variety of phenomenologically interesting corrections [14–16], we will not consider their effects here. Note that the 4D Kähler potential contains the Kähler potential (2.8) that we introduced earlier for the Calabi-Yau. This relationship between the 4D kinetic terms and the Calabi-Yau geometry is a special characteristic of the large-volume limit, and breaks down in the presence of significant warping (see e.g. [17–22]).

In addition to the complex structure moduli z^i and τ, the 4D theory also contains a number of Kähler moduli t^a. Rather than depending on the holomorphic volumes of three-cycles, these measure the actual volumes of two- and four-cycles. Since the Kähler moduli do not appear in the superpotential, their F-terms are just

$$F_a = D_a W = W \partial_a K_t \,. \tag{2.21}$$

When summed up they give $\sum_a |F_a|^2 = 3\,|W|^2$, so the standard expression for the scalar potential simplifies to

$$\begin{aligned} V &= e^K \left[\sum_{A=i,\tau,a} |D_A W|^2 - 3\,|W|^2\right] \qquad (2.22) \\ &= e^K \left(\sum_i |D_i W|^2 + |D_\tau W|^2\right) . \qquad (2.23) \end{aligned}$$

When $W \neq 0$ the F-terms for the Kähler moduli (2.21) are non-vanishing, so SUSY is broken. However, the potential (2.22) is positive definite and has a global minimum when

[3] The volume of the Calabi-Yau is determined by the Kähler moduli, which are not stabilized by 3-form fluxes. We have little to say about the factors of the volume that appear, but include them for completeness.

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$F_i = D_i W = 0$ and $F_\tau = D_\tau W = 0$. Because of this, we require that $F_i = F_\tau = 0$, regardless of whether SUSY is broken.

The simple form of the Kähler potential gives the $F_i = F_\tau = 0$ conditions simple geometric interpretations. For the complex structure moduli we find

$$D_i W = \int_{\mathrm{CY}} G_3 \wedge D_i \Omega_3 = \int_{\mathrm{CY}} G_3 \wedge \chi_i \,, \tag{2.24}$$

so that setting $F_i = 0$ is equivalent to requiring that G_3 have no $(1,2)$ component. In addition one can verify that

$$D_\tau \int G_3 \wedge \Omega_3 = -\frac{1}{\tau - \overline{\tau}} \int \overline{G}_3 \wedge \Omega_3 \,, \tag{2.25}$$

so setting $F_\tau = 0$ is equivalent to requiring that G_3 have no $(3,0)$ component. Thus we have found that minimizing the potential (2.22) is *equivalent* to imposing the ISD condition (1.2). This is one of the reasons that the GVW superpotential is believed to accurately describe large-volume compactifications.

2.4 Flux attractor equations

The flux attractor equations were originally derived in [1] by considering F-theory compactified on $\mathrm{CY}_3 \times T^2$. For the sake of variety, we present a slightly different derivation which does not involve an explicit embedding in F-theory.

Our goal is to make the implications of the ISD condition (1.2) more explicit. Since an ISD 3-form can have only $(0,3)$ and $(2,1)$ pieces, we can expand it with respect to the complex basis introduced at the end of section (2.1) as:

$$G_3 = -i\mathrm{Im}\,(\tau)\, \left[\overline{C\Omega_3} + C^i D_i \Omega_3\right] \,. \tag{2.26}$$

The overall factor of $-i\mathrm{Im}\,(\tau)$ is included for convenience. Note that C and C^i both have weight $(-1,0)$ under Kähler transformations, and transform under S-duality as

$$C \to (c\tau + d)\, C \,, \tag{2.27}$$
$$C^i \to (c\overline{\tau} + d)\, C^i \,. \tag{2.28}$$

In order to make (2.26) completely explicit we must specify the symplectic section $\{Z^I, F_I\}$, as this determines how Ω_3 depends on the complex structure moduli. We can then compute the Kähler covariant derivatives $D_i\Omega_3$, so that (2.26) becomes an *algebraic* equation for the complex structure moduli and the axio-dilaton.

One undesirable aspect of (2.26) is that the l.h.s. contains both the real fluxes F_3 and H_3, which we think of us "inputs," and the axio-dilaton τ, which we think of as an "output." This is rectified by writing

$$\begin{pmatrix} G_3 \\ \overline{G}_3 \end{pmatrix} = \begin{pmatrix} 1 & -\tau \\ 1 & -\overline{\tau} \end{pmatrix} \begin{pmatrix} F_3 \\ H_3 \end{pmatrix} = -i\mathrm{Im}\,(\tau) \begin{pmatrix} \overline{C\Omega_3} + C^i D_i\Omega_3 \\ -C\Omega_3 - \overline{C}^i\, \overline{D_i\Omega_3} \end{pmatrix} , \tag{2.29}$$

which we can easily invert:

$$\begin{pmatrix} F_3 \\ H_3 \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} -\overline{\tau} & \tau \\ -1 & 1 \end{pmatrix}\begin{pmatrix} \overline{C\Omega_3} + C^i D_i\Omega_3 \\ -C\Omega_3 - \overline{C}^i\overline{D_i\Omega_3} \end{pmatrix} \tag{2.30}$$

$$= \begin{pmatrix} \mathrm{Re}\left[\tau\left(C\Omega_3 + \overline{C}^i\overline{D_i\Omega_3}\right)\right] \\ \mathrm{Re}\left[C\Omega_3 + \overline{C}^i\overline{D_i\Omega_3}\right] \end{pmatrix}. \tag{2.31}$$

Now the l.h.s. of the attractor equations consists entirely of quantities that define the vacuum (fluxes), while the r.h.s. depends on the moduli and the symplectic section (choice of $\{Z^I, F_I\}$).

The equations in (2.31) are equations for 3-forms, rather than for ordinary numbers. While this makes their geometric implications clear, if we want to actually solve the equations it will be helpful to integrate them against a real basis of 3-forms. We have already introduced the required notation in (2.3)–(2.5), so we simply quote the result,

$$m_f^I = \mathrm{Re}\left[\tau\left(CZ^I + \overline{C}^i\overline{D_iZ^I}\right)\right], \tag{2.32}$$

$$m_h^I = \mathrm{Re}\left[CZ^I + \overline{C}^i\overline{D_iZ^I}\right], \tag{2.33}$$

$$e_I^f = \mathrm{Re}\left[\tau\left(CF_I + \overline{C}^i\overline{D_iF_I}\right)\right], \tag{2.34}$$

$$e_I^h = \mathrm{Re}\left[CF_I + \overline{C}^i\overline{D_iF_I}\right]. \tag{2.35}$$

One benefit to writing the attractor equations in this form is that there is manifestly one real equation for each real flux, for a total of $4n + 4$ real equations. We will compare this to the number of moduli and other parameters quite carefully in the next section.

One may wonder to what extent it makes sense to call (2.32)–(2.35) "attractor equations." The word "attractor" implies some sort of flow along which all information about a set of initial conditions is lost, but we have not introduced any notion of attractor flow. We note that in the study of extremal black holes, there is a useful distinction between the entire attractor *flow*, which takes place between spatial infinity and the horizon, and the attractor *equations*, which describe how the moduli are stabilized at the horizon. Because (2.32)–(2.35) are closely analogous to the black hole attractor equations, we consider calling them "attractor equations" to be only a minor abuse of the term.

3 Attractor equations and mass matrices

In expanding out the flux attractor equations, we found $4n+4$ real equations[4] (2.32)–(2.35). This is many more than the $2n + 2$ real moduli VEVs we want to fix, the z^i and τ. The origin of this mismatch is that there are additional "outputs" of the attractor equations, namely the coefficients C and C^i. Including these outputs gives $4n + 4$ real variables, equal to the number of attractor equations. We will see that these coefficients determine the mass spectrum of the 4D theory.

[4] $n = b_3/2 - 1$, so that $n + 1$ is the number of $\mathcal{N} = 1$ vector multiplets in the 4D theory.

3.1 Black hole attractor equations and the entropy

While the C^i are a new feature of the flux attractor equations, the coefficient C also appears in the more familiar context of BPS black hole attractor equations. We begin by discussing the role it plays there. Suppose we have constructed a 4D BPS Reissner-Nordström black hole by wrapping D3 branes on the 3-cycles of a Calabi-Yau manifold. The charges of the black hole can be described by a 3-form, F_3. We can expand a general real 3-form either against a real basis, or against the complex basis introduced in section 2.1:

$$
\begin{aligned}
F_3 &= p^I \alpha_I - q_I \beta^I && (3.1)\\
&= \mathrm{Re}\left[C\Omega_3 + C^i D_i \Omega_3\right] \,. && (3.2)
\end{aligned}
$$

The expression for the spacetime central charge of the black hole is

$$
W_{\mathrm{BH}} = \int F_3 \wedge \Omega_3 \,, \tag{3.3}
$$

and the BPS conditions are $D_i W_{\mathrm{BH}} = 0$. Since F_3 does not depend on the moduli, the BPS conditions reduce to

$$
\int F_3 \wedge D_i \Omega_3 = 0 \,, \tag{3.4}
$$

i.e. they require that the $(1,2)$ piece of F vanishes. This simplifies the general expansion (3.2) to

$$
F_3 = 2\mathrm{Re}\left[C\Omega_3\right] \,. \tag{3.5}
$$

This is the standard black hole attractor equation, originally derived in [23–25] and reviewed in [26–28].

If we expand (3.5) on the real basis $\left\{\alpha_I, \beta^I\right\}$ we will find a counting problem. Although there are $2n+2$ real equations, there are only $2n$ real physical moduli, the z^i. In order to understand the mismatch, we first note that the righthand side of (3.5) contains $2n+4$ real parameters, $\left\{C, Z^I\right\}$. Since both C and Z^I transform under Kähler transformations we can eliminate one complex parameter, leaving $2n+2$ Kähler invariant parameters. For example, if we assume that $Z^0 \neq 0$, we can take the Kähler invariant parameters to be $\left\{CZ^0, z^i = Z^i/Z^0\right\}$. More generally, the number of Kähler invariant parameters is equal to the number of attractor equations. The non-trivial feature is that, in addition to determining the values of the moduli z^i, the black hole attractor equations fix the Kähler invariant quantity CZ^0.

It is natural to ask what the physical significance of the additional parameter is. One important place where it appears is in the black hole entropy,

$$
\begin{aligned}
\frac{S}{\pi} &= e^{K_z} \left|W_{\mathrm{BH}}\right|^2 && (3.6)\\
&= \frac{e^{-K_z}}{|Z^0|^2} \cdot \left|CZ^0\right|^2 \,, && (3.7)
\end{aligned}
$$

since (3.3) and (3.5) imply that $W_{\mathrm{BH}} = -i\overline{C}e^{-K_z}$. In the final expression we have written the black hole entropy as the product of two Kähler-invariant factors, with the first factor depending only on the moduli z^i. We see that a change in CZ^0 leads to a change in the entropy, with the moduli held fixed.

It is sometimes stated that solving the attractor equations is equivalent to minimizing an effective potential. Our analysis shows that, in fact, the attractor equations simultaneously determine both the values of the moduli *and* the value of the effective potential. Simply minimizing the effective potential with respect to the moduli would have given us $2n$ real equations, rather than $2n+2$, and we would have had to insert the solutions for the moduli back into the effective potential to find its value at the minimum.

3.2 Fermion masses

Let us now return to the flux attractor equations. (2.32)–(2.35) constitute $4n+4$ real equations, while the moduli z^i and τ constitute $2n+2$ real parameters. Our analysis of the black hole attractor equations revealed that CZ^0 contributes two more real independent parameters, but we are still left with $2n$ more equations than parameters. The new features in the flux attractor are the coefficients C^i, first introduced in (2.26). Including these in our set of Kähler-invariant parameters as $\left\{\tau, z^i, CZ^0, C^iZ^0\right\}$, we have accounted for everything that appears on the righthand side of (2.31), for a grand total of $4n+4$ parameters. Just as in the black hole case we found that different choices of charges could lead to the same moduli but different entropies, here different choices of the fluxes can lead to the same moduli, but different values of CZ^0 *and* C^iZ^0.

In large-volume compactifications, the role of the black hole entropy is played by the gravitino mass:

$$m_{3/2}^2 = e^K\,|W|^2\,, \tag{3.8}$$

Indeed, if we substitute in the expressions (2.18) for the superpotential and (2.20) for the Kähler potential, we find

$$m_{3/2}^2 = \frac{e^{-K_z}\mathrm{Im}\,(\tau)}{2\,|Z^0|^2\,\mathrm{Vol}^2}\cdot\left|CZ^0\right|^2. \tag{3.9}$$

Just as CZ^0 determined the entropy of the black hole attractor, it determines the gravitino mass for the flux attractor.

While we understand well enough what it means to solve for the VEVs of z^i and τ, and we know that C is related to the gravitino mass, we need to develop a physical interpretation of the C^i. We'll first observe that the C^i appear when we consider the second derivatives of the superpotential:

$$\begin{aligned} D_iD_jW &= \int G_3\wedge D_iD_j\Omega_3 && (3.10)\\ &= \int G_3\wedge\left(\mathcal{F}_{ijk}\overline{\chi}^k\right) && (3.11)\\ &= \mathrm{Im}(\tau)e^{-K_z}\mathcal{F}_{ijk}C^k\,, && (3.12)\end{aligned}$$

where [29]

$$\mathcal{F}_{ijk} = ie^{K_z}\int\Omega_3\wedge\partial_i\partial_j\partial_k\Omega_3 \tag{3.13}$$

depends on both the moduli and the symplectic section.[5] We also need the mixed derivatives,

$$
\begin{aligned}
D_\tau D_i W &= -\frac{\int \overline{G}_3 \wedge \chi_i}{\tau - \overline{\tau}} && (3.14)\\
&= -\frac{1}{2}\int \left(C\Omega_3 + \overline{C^j \chi_j}\right) \wedge \chi_i && (3.15)\\
&= \frac{i}{2}\overline{C}^{\overline{j}} g_{i\overline{j}} e^{-K_z}\,. && (3.16)
\end{aligned}
$$

Here we used (2.24) and (2.25). Also, in the last step we used the relationship between the metric on complex structure moduli space (2.9) and the $(2,1)$ forms (2.11),

$$
g_{i\overline{j}} = -\frac{\int \chi_i \wedge \overline{\chi}_{\overline{j}}}{\int \Omega_3 \wedge \overline{\Omega}_3}\,. \tag{3.17}
$$

The remaining second derivative vanishes,

$$
\begin{aligned}
D_\tau D_\tau W &= \frac{2}{(\tau-\overline{\tau})^2}\int \overline{G}_3 \wedge \Omega_3 && (3.18)\\
&= 0\,, && (3.19)
\end{aligned}
$$

since $\overline{G}_3$ has no $(0,3)$ piece.

The second derivatives of the superpotential generically determine the masses of the components of chiral multiplets. The standard expression [30] for the spinor mass matrix in 4D $\mathcal{N}=1$ supergravity is

$$
m_{\alpha\beta} = \left(D_\alpha D_\beta W - \frac{2}{3}\left(D_\alpha W\right)\left(D_\beta W\right) - \Gamma^c_{\alpha\beta} D_c W\right)\frac{m_{3/2}}{W}\,. \tag{3.20}
$$

Since the Kähler moduli are not stabilized, we will only consider $\alpha = i, \tau$. The moduli space factorizes, so the connection Γ^A_{BC} will have no mixed components, $\Gamma^a_{\alpha\beta} = 0$. Imposing the global minimum condition $D_i W = D_\tau W = 0$ reduces the mass matrix to

$$
m_{\alpha\beta} = e^{K/2}\sqrt{\frac{\overline{W}}{W}} D_\alpha D_\beta W\,. \tag{3.21}
$$

Note that the overall phase $\sqrt{\overline{W}/W}$ could be absorbed into the definition of the fermions, though we will not do so here. Substituting in the second derivatives computed above, the fermion mass matrix simplifies to

$$
\begin{pmatrix} m_{ij} & m_{i\tau} \\ m_{\tau i} & m_{\tau\tau} \end{pmatrix} = \frac{m_{3/2}}{\overline{C}}\begin{pmatrix} \mathcal{F}_{ijk}C^k & -\frac{1}{2i\mathrm{Im}(\tau)} g_{i\overline{j}}\overline{C}^{\overline{j}} \\ -\frac{1}{2i\mathrm{Im}(\tau)} g_{i\overline{j}}\overline{C}^{\overline{j}} & 0 \end{pmatrix}. \tag{3.22}
$$

Here we used (3.21) and (3.8), substituted in the second derivatives (3.12), (3.16), and (3.19), then simplified using (2.8), (2.18), (2.26), and (3.17). This demonstrates how,

[5] For cubic prepotentials and physical moduli $z^i = Z^i/Z^0$, $\int \Omega_3 \wedge \partial_i \partial_j \partial_k \Omega_3 = \left(Z^0\right)^2 C_{ijk}$.

in the large volume scenario, the C^i determine the structure of the fermion mass matrix. These masses remain finite even in the limit $m_{3/2} \sim |C| \to 0$, since the ratio $m_{3/2}/\overline{C}$ approaches a finite value.

A few comments are in order. First, the fermion mass matrix has $2n+2$ real eigenvalues, two more than there are parameters C^i. This indicates that we cannot independently determine the masses of *all* of the moduli — for example, we could consider choosing the masses of the z^i, but then the mass of τ would be determined. It is also interesting that the form of m_{ij} suggests a generalized Higgs mechanism. If we think of the $\mathcal{F}_{ijk}$ as Yukawa couplings, than C^k appears to play the role of a Higgs vacuum expectation value. While the C^k do not correspond to the expectation values of any dynamical scalars, it is possible that they can be interpreted as the expectation values of auxiliary fields. Finally, if we can make $\mathrm{Im}\,(\tau) = 1/g_s$ large, then the smallest fermion mass will be roughly $m_{3/2}g_s^2$. It would be interesting to see if such a light mode is of phenomenological interest, perhaps at an intermediate scale.

3.3 Scalar masses

In supersymmetric vacua, the masses of scalar fields should match the masses of their fermionic partners. However, the no-scale vacua that we consider generically break supersymmetry. While the F-terms for the complex structure moduli and axio-dilaton vanish, $D_iW = D_\tau W = 0$, the F-terms for the Kähler moduli only vanish when $W = 0$, as shown in (2.21). In this case, the scalar mass-squared matrix takes the following form:

$$\mathcal{M}^2 = \begin{pmatrix} M_{\alpha\beta} & M_{\alpha\overline{\beta}} \\ \overline{M}_{\overline{\alpha}\beta} & \overline{M}_{\overline{\alpha\beta}} \end{pmatrix}, \tag{3.23}$$

$$M^2_{\alpha\beta} = e^K\overline{W}\left(D_\alpha D_\beta W + D_\beta D_\alpha W\right), \tag{3.24}$$

$$M^2_{\alpha\overline{\beta}} = e^K\left[g^{\gamma\overline{\delta}} D_\alpha D_\gamma W \overline{D_\beta D_\delta W} + |W|^2 g_{\alpha\overline{\beta}}\right]. \tag{3.25}$$

While (3.24) and (3.25) would be standard expressions for a theory with *only* the complex structure moduli and axio-dilaton, we verify in appendix A that they also hold when Kähler moduli are included, and supersymmetry is broken in that sector. Note that when $W = 0$, i.e. when supersymmetry is preserved, $M^2_{\alpha\beta}$ vanishes and $M^2_{\alpha\overline{\beta}} = g^{\gamma\overline{\delta}} m_{\alpha\gamma}\overline{m}_{\overline{\delta\beta}}$, as expected. When $W \neq 0$, the scalar masses are lifted above the fermion masses, and the splitting of the masses-squared is of order $m^2_{3/2} = e^K|W|^2 \sim \left|CZ^0\right|^2$.

4 A generating function for the flux attractor equations

In this section we develop an algorithm which, in principle, solves the flux attractor equations. To do so we adapt the OSV solution of the black hole attractor equations [4]. We begin with a change of variables designed to automatically solve the magnetic half of the attractor equations. Next, we rewrite the electric half of the attractor equations as derivatives of a generating function. Finally, a Legendre transform provides a formal solution of the attractor equations.

The generating function itself is quite interesting. In [4], the generating function governing the black hole attractor turned out to be the free energy of the black hole. Our interest in the generating function is not restricted to this section, rather we will discuss some of its general properties in section 5.

4.1 An alternative formulation of the attractor equations

The flux attractor equations (2.32)–(2.35) contain Kähler covariant derivatives, which we find much less convenient than ordinary derivatives. We therefore consider a modified version of (2.26) that does not have this problem:

$$G_3 = -i\mathrm{Im}\,(\tau)\left[\overline{C\Omega}_3 + L^I\partial_I\Omega_3\right], \tag{4.1}$$

where $\overline{C}$ and the L^I are coefficients. Note that we differentiate with respect to the Z^I, not the z^i.

The ISD condition (1.2) allows only $(2,1)$ and $(0,3)$ pieces in the complex flux G_3. While the *ansatz* (4.1) does not contain a $(1,2)$ piece, equation (2.11) shows that the $\partial_I\Omega_3$ term includes a $(3,0)$ piece. Since the ISD condition (1.2) forbids such a term, we must choose the L^I so that it is projected out. The appropriate condition on the L^I is

$$L^I\partial_I K_z = 0\,. \tag{4.2}$$

After imposing this condition, the resulting G_3 has only $(0,3)$ and $(2,1)$ pieces. We thus conclude that (4.1) and (4.2) together are equivalent to (2.26), with

$$C^i = \frac{\partial z^i}{\partial Z^I}L^I\,. \tag{4.3}$$

If we think of the C^i as given, then this fixes n of the $n+1$ components of L^I, and (4.2) fixes the final component.

As in section 2.4, we can expand (4.1) and find a set of real attractor equations. This is equivalent to replacing $C^iD_i \to L^I\partial_I$ in (2.32)–(2.35) and adding the constraint (4.2). The resulting attractor equations are:

$$\begin{aligned}
m_h^I &= \mathrm{Re}\left[CZ^I + L^I\right], && (4.4)\\
m_f^I &= \mathrm{Re}\left[\tau CZ^I + \overline{\tau}L^I\right], && (4.5)\\
e_I^h &= \mathrm{Re}\left[CF_I + L^JF_{IJ}\right], && (4.6)\\
e_I^f &= \mathrm{Re}\left[\tau CF_I + \overline{\tau}L^JF_{IJ}\right], && (4.7)\\
0 &= L^I\left(\overline{F}_I - \overline{Z}^JF_{IJ}\right), && (4.8)
\end{aligned}$$

where we have introduced $F_{IJ} \equiv \partial_I F_J$, and used (2.5) to make the constraint (4.2) more explicit. The magnetic attractor equations (4.4) and (4.5) are simpler than their counterparts (2.32) and (2.33), in that the $C^iD_iZ^I$ term reduces to L^I. Similarly, the electric attractor equations (4.6) and (4.7) are simpler than (2.34) and (2.35) since the Kähler covariant derivatives have been replaced with ordinary derivatives.

Another benefit of these reformulated attractor equations is that the L^I transform in the $n+1$ of $SO(n+1)$, just like the Z^I and the fluxes, and in contrast to the C^i. This suggests solving (4.4)–(4.7) for CZ^I and L^I, treating the L^I on an equal footing with the CZ^I, then solving (4.8) for τ. This procedure is more practical than solving (2.32)–(2.35) for the $n+1$ vector CZ^I, n vector C^i, and scalar τ, even though the results are equivalent. We will demonstrate this by completely solving an explicit example in section 6.

4.2 Magnetic attractor equations and the mixed ensemble

We now solve the flux attractor equations by adapting the OSV procedure for solving the black hole attractor equations [4]. We treat τ as a fixed variable while solving (4.4)–(4.7), then determine it at the very end by solving (4.8). The two sets of variables we have seen so far, $\{CZ^I, L^I, \tau\}$ and $\left\{m_h^I, m_f^I, e_I^h, e_I^f, \tau\right\}$, describe two different ensembles. Following OSV, we introduce a "mixed ensemble," $\left\{m_h^I, m_f^I, \phi_h^I, \phi_f^I, \tau\right\}$, where $\phi_{h,f}^I$ are potentials conjugate to the electric fluxes. When introducing these potentials, we require that:

1. The expressions for CZ^I and L^I in terms of $\left\{m_h^I, m_f^I, \phi_h^I, \phi_f^I, \tau\right\}$ automatically solve the "magnetic" attractor equations, (4.4) and (4.5).

2. The potentials $\left\{\phi_h^I, \phi_f^I\right\}$ transform like $\left\{m_h^I, m_f^I\right\}$ under S-duality.

3. The relationship between $\{CZ^I, L^I, \tau\}$ and $\left\{m_h^I, m_f^I, \phi_h^I, \phi_f^I, \tau\right\}$ is covariant under S-duality.

These conditions determine the relationship between $\{CZ^I, L^I, \tau\}$ and $\left\{m_h^I, m_f^I, \phi_h^I, \phi_f^I, \tau\right\}$ to be

$$CZ^I = \frac{1}{\tau-\overline{\tau}}\left(m_f^I - \overline{\tau} m_h^I\right) + \frac{1}{\tau-\overline{\tau}}\left(\phi_f^I - \overline{\tau}\phi_h^I\right), \tag{4.9}$$

$$L^I = -\frac{1}{\tau-\overline{\tau}}\left(m_f^I - \tau m_h^I\right) + \frac{1}{\tau-\overline{\tau}}\left(\phi_f^I - \tau\phi_h^I\right). \tag{4.10}$$

We will also want to know how derivatives with respect to Z^I and L^I are mapped into derivatives with respect to fluxes and the potentials. Here it is important to note that both sets of variables we are considering, $\{CZ^I, L^I, \tau\}$ and $\left\{m_h^I, m_f^I, \phi_h^I, \phi_f^I, \tau\right\}$, include τ as an *independent* variable. The derivatives are therefore related by

$$\frac{1}{C}\frac{\partial}{\partial Z^I} = \frac{1}{2}\left[\left(\frac{\partial}{\partial m_h^I} + \tau\frac{\partial}{\partial m_f^I}\right) + \left(\frac{\partial}{\partial \phi_h^I} + \tau\frac{\partial}{\partial \phi_f^I}\right)\right], \tag{4.11}$$

$$\frac{\partial}{\partial L^I} = \frac{1}{2}\left[\left(\frac{\partial}{\partial m_h^I} + \overline{\tau}\frac{\partial}{\partial m_f^I}\right) - \left(\frac{\partial}{\partial \phi_h^I} + \overline{\tau}\frac{\partial}{\partial \phi_f^I}\right)\right], \tag{4.12}$$

where all derivatives are taken with τ held fixed.

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4.3 Electric attractor equations and the generating function

In the previous section we solved the magnetic attractor equations, (4.4) and (4.5). We now introduce an auxiliary function,

$$\mathcal{V} = 2\mathrm{Im}\,(\tau)\, C F_I L^I\,, \tag{4.13}$$

that simplifies the electric attractor equations, (4.6) and (4.7). This new function plays a role analogous to that of the prepotential in the solution of the black hole attractor equations. It enjoys the following properties:

1. Derivatives of $\mathcal{V}$ with respect to L^I give CF_I, one of the terms that appears in the electric attractor equations:

$$\frac{1}{2\mathrm{Im}\,(\tau)}\frac{\partial \mathcal{V}}{\partial L^I} = CF_I\,. \tag{4.14}$$

2. Derivatives with respect to Z^I give $L^J F_{IJ}$, the other term that appears in the electric attractor equations:

$$\frac{1}{2C\mathrm{Im}\,(\tau)}\frac{\partial \mathcal{V}}{\partial Z^I} = L^J F_{IJ}\,. \tag{4.15}$$

3. The factor of C in (4.13)makes $\mathcal{V}$ invariant under Kähler transformations.

4. By (2.27), (2.28), and (2.17), the factor of $\mathrm{Im}\,(\tau)$ in (4.13) makes $\mathcal{V}$ invariant under S-duality.

5. $\mathcal{V}$ is holomorphic in L^I and Z^I.

The first two properties will allow us to replace the F_I and $L^J F_{IJ}$ terms in the electric attractor equations, (4.6) and (4.7), with derivatives of $\mathcal{V}$. This is analogous to the role played by the prepotential in the solution of the electric black hole equations. The invariance of $\mathcal{V}$ under Kähler transformations and S-duality (properties 3 and 4) will allow us to interpret it in terms of a physical quantity. Finally, we will make extensive use of holomorphy in the following manipulations.

As described above, we can rewrite the electric attractor equations (4.6) and (4.7) in terms of derivatives of $\mathcal{V}$,

$$e^h_I = \frac{1}{2\mathrm{Im}\,(\tau)}\mathrm{Re}\left[\left.\frac{\partial \mathcal{V}}{\partial L^I}\right|_{Z^J, L^{J\neq I}, \tau} + \frac{1}{C}\left.\frac{\partial \mathcal{V}}{\partial Z^I}\right|_{Z^{J\neq I}, L^J, \tau}\right], \tag{4.16}$$

$$e^f_I = \frac{1}{2\mathrm{Im}\,(\tau)}\mathrm{Re}\left[\tau\left.\frac{\partial \mathcal{V}}{\partial L^I}\right|_{Z^J, L^{J\neq I}, \tau} + \overline{\tau}\frac{1}{C}\left.\frac{\partial \mathcal{V}}{\partial Z^I}\right|_{Z^{J\neq I}, L^J, \tau}\right]. \tag{4.17}$$

We then use holomorphy of $\mathcal{V}$ to find

$$e^h_I = \frac{i}{2\mathrm{Im}\,(\tau)}\left(\frac{\partial}{\partial L^I} + \frac{1}{C}\cdot\frac{\partial}{\partial Z^I} - \frac{\partial}{\partial \overline{L}^I} - \frac{1}{\overline{C}}\frac{\partial}{\partial \overline{Z}^I}\right)\mathrm{Im}\,(\mathcal{V})\,, \tag{4.18}$$

$$e^f_I = \frac{i}{2\mathrm{Im}\,(\tau)}\left\{\tau\left(\frac{\partial}{\partial L^I} - \frac{1}{\overline{C}}\cdot\frac{\partial}{\partial \overline{Z}^I}\right) - \overline{\tau}\left(\frac{\partial}{\partial \overline{L}^I} - \frac{1}{C}\frac{\partial}{\partial Z^I}\right)\right\}\mathrm{Im}\,(\mathcal{V})\,. \tag{4.19}$$

Finally, we introduce derivatives with respect to the potentials using (4.11) and (4.12),

$$e_I^h = -\left[\frac{\partial}{\partial \phi_f^I}\mathrm{Im}\left(\mathcal{V}\right)\right]_{\phi_h^{J\neq I},\phi_f^J,m_h^J,m_f^J,\tau}, \tag{4.20}$$

$$e_I^f = \left[\frac{\partial}{\partial \phi_h^I}\mathrm{Im}\left(\mathcal{V}\right)\right]_{\phi_h^J,\phi_f^{J\neq I},m_h^J,m_f^J,\tau}, \tag{4.21}$$

Though we initially defined $\mathcal{V}$ in terms of L^I and Z^I, in this last step we simply substitute in (4.9) and (4.10) to make it a function of the magnetic fluxes and electric potentials.

It is remarkable that the electric attractor equations, which appear rather complex, reduce to derivatives of a single generating function! This is one of the principal results of this paper.

Since we have made a rather long chain of substitutions and redefinitions, we briefly summarize our procedure for solving the flux attractor equations:

1. Take as inputs the fluxes $\left\{m_f^I, m_h^I, e_I^f, e_I^h\right\}$ and the symplectic section $\left\{Z^I, F_I\right\}$.

2. Insert the expressions for the F_I as functions of the Z^I into (4.13), giving $\mathcal{V}\left(L^I, CZ^I, \tau\right)$.

3. Substitute the expressions (4.9) and (4.10) into $\mathcal{V}\left(L^I, CZ^I, \tau\right)$ to get $\mathcal{V}\left(\phi_h^I, \phi_f^I, m_h^I, m_f^I, \tau\right)$.

4. Invert (4.20) and (4.21) to get expressions for ϕ_f^I and ϕ_h^I in terms of m_h^I, m_f^I, e_I^h, e_I^f, and τ.

5. Rewrite the constraint (4.8) in terms of m_h^I, m_f^I, e_I^h, e_I^f, and τ. Do this by substituting (4.9) and (4.10) into (4.8), then inserting the solutions for ϕ_f^I and ϕ_h^I in terms of m_h^I, m_f^I, e_I^h, e_I^f, and τ.

6. Solve the constraint (4.8) for τ as a function of the fluxes only. Substitute this back into the expressions for $\phi_{f,h}^I$ to get expressions for the potentials in terms of the fluxes only, and then insert τ and the potentials into the expressions (4.9) and (4.10) to get expressions for CZ^I and L^I in terms of the fluxes only.

The most difficult part of this procedure is step 4, which requires that we invert a system of $2n + 2$ equations. Even in simple cases, these result in polynomials of impractically high order.

The electric attractor equations (4.20) and (4.21) take the form of thermodynamic relations, indicating that the potentials $\phi_{f,h}^I$ are *conjugate* to the fluxes $e_I^{f,h}$. This suggests the Legendre transformation

$$\mathcal{G} = \mathrm{Im}\left(\mathcal{V}\right) + e_I^h\phi_f^I - e_I^f\phi_h^I\,, \tag{4.22}$$

so that the electric attractor equations become

$$\phi_h^I = -\left[\frac{\partial \mathcal{G}}{\partial e_I^f}\right]_{e_J^h, e^f_{J\neq I}, m_h^J, m_f^J, \tau}, \tag{4.23}$$

$$\phi_f^I = \left[\frac{\partial \mathcal{G}}{\partial e_I^h}\right]_{e^h_{J\neq I}, e_J^f, m_h^J, m_f^J, \tau}. \tag{4.24}$$

This means that we only need to know a single function, $\mathcal{G}$, which is in principle determined by steps 1–4 above.

In practice, this may not be the best way to proceed. The analogue of $\mathcal{G}$ for the black hole attractor equations is the entropy S, which can be computed by many different methods. For example, the requirement that S be invariant under duality transformations severely constrains, and sometimes completely determines, its functional form [31].

4.4 The constraint and the generating function

So far, we have demonstrated that the electric attractor equations (4.6) and (4.7) can be recast in terms of derivatives of a generating function. Indeed, we designed the generating function $\mathcal{G}$ specifically for this purpose. Next, we demonstrate a more surprising result: the constraint (4.8) can *also* be written in terms of derivatives of the same generating function.

We first compute $\tau-$derivatives of CZ^I and L^I in the $\left\{m_h^I, m_f^I, \phi_h^I, \phi_f^I, \tau\right\}$ ensemble, using (4.9) and (4.12):

$$\left.\frac{\partial Z^I}{\partial \tau}\right|_{m_h^I, m_f^I, \phi_h^I, \phi_f^I} = -\frac{Z^I}{\tau - \overline{\tau}}, \quad \left.\frac{\partial \overline{Z}^I}{\partial \tau}\right|_{m_h^I, m_f^I, \phi_h^I, \phi_f^I} = \frac{Z^I}{\tau - \overline{\tau}}, \tag{4.25}$$

$$\left.\frac{\partial L^I}{\partial \tau}\right|_{m_h^I, m_f^I, \phi_h^I, \phi_f^I} = \frac{\overline{L}^I}{\tau - \overline{\tau}}, \quad \left.\frac{\partial \overline{L}^I}{\partial \tau}\right|_{m_h^I, m_f^I, \phi_h^I, \phi_f^I} = -\frac{\overline{L}^I}{\tau - \overline{\tau}}. \tag{4.26}$$

Using these preliminary results, we find:

$$\frac{\partial}{\partial \tau}\left[\mathrm{Im}\left(\mathcal{V}\right)\right]_{m,\phi} = \frac{\partial}{\partial \tau}\left[2\mathrm{Im}\left(\tau\right)\mathrm{Im}\left(L^I F_I\right)\right]_{m,\phi} \tag{4.27}$$

$$\begin{aligned} &= -i\mathrm{Im}\left(L^I F_I\right) - i\mathrm{Im}\left(\tau\right)\left[\frac{\overline{L}^I}{\tau - \overline{\tau}}F_I + \frac{\overline{L}^I}{\tau - \overline{\tau}}\overline{F}_I\right] \\ &\quad -i\mathrm{Im}\left(\tau\right)\left[-L^I F_{IJ}\frac{Z^J}{\tau - \overline{\tau}} - \overline{L}^I\overline{F}_{IJ}\frac{Z^J}{\tau - \overline{\tau}}\right] \end{aligned} \tag{4.28}$$

$$= \frac{1}{2}\left[-L^I F_I - \overline{L}^I F_I + L^I F_{IJ} Z^J + \overline{L}^J \overline{F}_{IJ} Z^J\right] \tag{4.29}$$

$$= -\frac{1}{2}\overline{L}^I\left[F_I - \overline{F}_{IJ} Z^J\right], \tag{4.30}$$

using the homogeneity property $F_{IJ}Z^J = F_I$. The last line is proportional to the complex conjugate of the constraint (4.8). Setting $\partial \mathrm{Im}\left(\mathcal{V}\right)/\partial \tau = 0$ is thus equivalent to imposing (4.8). Notice that the overall factor of $\mathrm{Im}\left(\tau\right)$ included in $\mathcal{V}$, originally introduced to make $\mathcal{V}$ invariant under S-duality, is exactly what is required to recover the constraint (4.8) from $\partial \mathrm{Im}\left(\mathcal{V}\right)/\partial\tau$.

The Legendre transform that takes us from the $\left\{\phi_h^I, \phi_f^I, m_h^I, m_f^I, \tau\right\}$ ensemble to the $\left\{e_I^h, e_I^f, m_h^I, m_f^I, \tau\right\}$ ensemble does not change the equilibrium condition associated with τ. In the latter ensemble, the constraint (4.8) is equivalent to

$$\left.\frac{\partial\mathcal{G}}{\partial\tau}\right|_{e_I^h, e_I^f, m_h^I, m_f^I} = 0. \tag{4.31}$$

This completes our demonstration that the flux attractor equations can be interpreted as equilibrium conditions for a thermodynamic system. From the thermodynamic point of view, (4.30) indicates that τ is *conjugate* to the constraint (4.8).

While studying $\mathcal{G}$ in the $\left\{e_I^h, e_I^f, m_h^I, m_f^I, \tau\right\}$ ensemble may be conceptually clearer, there is a useful consequence of (4.31). Suppose we take derivatives of $\mathcal{G}$ *without* holding τ fixed. The result is:

$$\left.\frac{\partial\mathcal{G}}{\partial e_I^h}\right|_{e_{J\neq I}^h, e_J^f, m_h^J, m_f^J} = \left.\frac{\partial\mathcal{G}}{\partial e_I^h}\right|_{e_{J\neq I}^h, e_J^f, m_h^J, m_f^J, \tau} + \left.\frac{\partial\mathcal{G}}{\partial\tau}\right|_{e_I^h, e_I^f, m_h^I, m_f^I}\left.\frac{\partial\tau}{\partial e_I^h}\right|_{e_{J\neq I}^h, e_J^f, m_h^J, m_f^J} \tag{4.32}$$

$$= \left.\frac{\partial\mathcal{G}}{\partial e_I^h}\right|_{e_{J\neq I}^h, e_J^f, m_h^J, m_f^J, \tau}. \tag{4.33}$$

In other words, if we substitute the attractor value for τ into $\mathcal{G}$ we can simplify (4.23) and (4.24) to:

$$\phi_h^I = -\left[\frac{\partial\mathcal{G}}{\partial e_I^f}\right]_{e_J^h, e_{J\neq I}^f, m_h^J, m_f^J}, \tag{4.34}$$

$$\phi_f^I = \left[\frac{\partial\mathcal{G}}{\partial e_I^h}\right]_{e_{J\neq I}^h, e_J^f, m_h^J, m_f^J}. \tag{4.35}$$

If one can determine $\mathcal{G}$ as a function of arbitrary fluxes, then (4.34) and (4.35) determine the potentials $\phi_{h,f}^I$, (4.9) and (4.10) then determine the moduli Z^I and mass parameters L^I, and finally (4.8) determines the axio-dilaton τ. In this way the single function $\mathcal{G}$ determines the vacuum expectation values and masses of the moduli.

5 General properties of the generating function

The generating function $\mathcal{G}$ introduced in (4.22) is the function that controls the flux attractor, giving attractor values for scalars and other physical quantities upon differentiation. In this section we initiate a general study of the generating function by demonstrating a simple relationship between $\mathcal{G}$ and the gravitino mass:

$$\mathcal{G} = \int F_3 \wedge H_3 - 2\mathrm{Vol}^2 m_{3/2}^2\,. \tag{5.1}$$

Note that the gravitino mass is to be considered a function of arbitrary fluxes. We first introduce a condensed, complex notation for the fluxes and potentials. We then exploit the homogeneity properties of $\mathcal{G}$ to prove the relationship (5.1).

5.1 Complex fluxes and potentials

One of the results of section 4 is that we can solve the electric and magnetic attractor equations (4.4)–(4.7) treating τ as a constant, then determine τ by solving (4.8). This justifies the introduction of the following complex fluxes and potentials:

$$m^I \equiv m_f^I - \tau m_h^I \,, \tag{5.2}$$

$$e_I \equiv e_I^f - \tau e_I^h \,, \tag{5.3}$$

$$\varphi^I \equiv \phi_f^I - \tau \phi_h^I \,. \tag{5.4}$$

We can then use (5.2) and (5.4) to rewrite (4.9) and (4.10) as

$$CZ^I = \frac{1}{2i\mathrm{Im}\,(\tau)} \left[\overline{m}^I + \overline{\varphi}^I\right] , \tag{5.5}$$

$$L^I = \frac{1}{2i\mathrm{Im}\,(\tau)} \left[-m^I + \varphi^I\right] . \tag{5.6}$$

We also define derivatives with respect to the complex electric fluxes as

$$\frac{\partial}{\partial e_I} \equiv \frac{i}{2\mathrm{Im}\,(\tau)} \left(\frac{\partial}{\partial e_I^h} + \overline{\tau} \frac{\partial}{\partial e_I^f} \right) , \tag{5.7}$$

where the normalization is chosen so that $\partial e_I / \partial e_J = \delta_I^J$. Definitions for $\partial/\partial m^I$ and $\partial/\partial\varphi^I$ are completely analogous. We can then rewrite the electric attractor equations (4.34) and (4.35) as

$$\varphi^I = 2i\mathrm{Im}\,(\tau) \frac{\partial \mathcal{G}}{\partial \overline{e}^I} \,, \tag{5.8}$$

and the expressions for CZ^I and L^I as

$$CZ^I = \frac{1}{2i\mathrm{Im}\,(\tau)} \left[\overline{m}^I - 2i\mathrm{Im}\,(\tau) \frac{\partial}{\partial e_I} \mathcal{G} \right] , \tag{5.9}$$

$$L^I = \frac{1}{2i\mathrm{Im}\,(\tau)} \left[-m^I + 2i\mathrm{Im}\,(\tau) \frac{\partial}{\partial \overline{e}_I} \mathcal{G} \right] . \tag{5.10}$$

While (5.9) and (5.10) present a fairly compact version of the results of section (4), they treat the electric and magnetic fluxes quite differently. The generating function $\mathcal{G}$ is not homogeneous in either the electric or the magnetic fluxes *alone*, so a symplectic invariant version of (5.9) and (5.10) will be helpful. We formulate this by first introducing a new operator:

$$\partial \equiv \alpha_I \frac{\partial}{\partial e_I} + \beta^I \frac{\partial}{\partial m^I} \,, \tag{5.11}$$

which maps scalar functions of the fluxes to 3-forms. We then examine (4.4)–(4.7), and see that symplectic invariance requires

$$C\Omega_3 = \frac{1}{2i\mathrm{Im}\,(\tau)} \left[\overline{G}_3 - 2i\mathrm{Im}\,(\tau)\, \partial \mathcal{G} \right] , \tag{5.12}$$

$$L^I \partial_I \Omega_3 = \frac{1}{2i\mathrm{Im}\,(\tau)} \left[-G_3 + 2i\mathrm{Im}\,(\tau)\, \overline{\partial} \mathcal{G} \right] . \tag{5.13}$$

These are equivalent to the electric attractor equations, so they must be supplemented by the constraint (4.8). This amounts to some flexibility in our treatment of $\mathcal{G}$. We can either use $\mathcal{G}\left(e_I, m^I, \tau\right)$ and take all derivatives with τ held fixed, as in (4.23) and (4.24), or substitute in the attractor value of τ to find $\mathcal{G}\left(e_I, m^I\right)$ and differentiate as in (4.34) and (4.35).

5.2 General expression for the generating function

We now show that the relationship between the generating function $\mathcal{G}$ and the gravitino mass (5.1) holds for general compactifications. Our argument turns on a homogeneity property of the attractor equations that is evident from examining (4.4)–(4.8). These attractor equations are invariant under a uniform rescaling of the fluxes,

$$m^I_{h,f} \to e^{\lambda} m^I_{h,f}\,, \tag{5.14}$$

$$e_I^{h,f} \to e^{\lambda} e_I^{h,f}\,, \tag{5.15}$$

provided that we simultaneously rescale

$$CZ^I \to e^{\lambda} CZ^I\,, \tag{5.16}$$

$$L^I \to e^{\lambda} L^I\,. \tag{5.17}$$

If we then turn our attention to the expressions for the CZ^I and L^I in terms of fluxes and potentials,(4.9) and (4.10), we see that the potentials must transform as

$$\phi^I_{h,f} \to e^{\lambda} \phi^I_{h,f}\,. \tag{5.18}$$

Equations (4.34) and (4.35) then indicate that if the potentials are to be homogeneous of degree one in the fluxes, then $\mathcal{G}$ must be homogeneous of degree two in the fluxes. If we use the complex fluxes introduced in (5.2) and (5.3), we find that $\mathcal{G}$ is homogeneous of degree one in the complex fluxes and degree one in their conjugates. This homogeneity implies that

$$\int G_3 \wedge \partial\mathcal{G} = \left[m^I \frac{\partial}{\partial m^I} + e_I \frac{\partial}{\partial e_I}\right] \mathcal{G} = \mathcal{G}, \tag{5.19}$$

where we used the orthogonality relations (2.1) and (2.2) and expansions (2.3) and (2.4) to compute the integral. We will now use this result to compute the superpotential and Kähler potential at the attractor point, and finally the gravitino mass.

We begin with the superpotential (2.18), then substitute in (5.12):

$$\begin{aligned} CW &= \int G_3 \wedge C\Omega_3 && (5.20)\\ &= \frac{1}{2i\mathrm{Im}\,(\tau)} \left[\int G_3 \wedge \overline{G}_3 - 2i\mathrm{Im}\,(\tau) \int G_3 \wedge \partial\mathcal{G}\right] && (5.21)\\ &= \int F_3 \wedge H_3 - \mathcal{G}. && (5.22) \end{aligned}$$

In order to determine the Kähler potential we need to compute

$$|C|^2 \int \Omega_3 \wedge \overline{\Omega}_3 = \frac{1}{4\mathrm{Im}\,(\tau)^2} \int \left(\overline{G}_3 - 2i\mathrm{Im}\,(\tau)\,\partial\mathcal{G}\right) \wedge \left(G_3 + 2i\mathrm{Im}\,(\tau)\,\overline{\partial}\mathcal{G}\right) \tag{5.23}$$

$$= \frac{1}{4\mathrm{Im}\,(\tau)^2} \Bigg[-\int G_3 \wedge \overline{G}_3 + 2i\mathrm{Im}\,(\tau) \left(\int G_3 \wedge \partial\mathcal{G} + \int \overline{G}_3 \wedge \overline{\partial}\mathcal{G} \right) + 4\mathrm{Im}\,(\tau)^2 \int \partial\mathcal{G} \wedge \overline{\partial}\mathcal{G} \Bigg] \tag{5.24}$$

$$= -\frac{i}{\mathrm{Im}\,(\tau)} \left[\int F_3 \wedge H_3 - \mathcal{G} \right]. \tag{5.25}$$

In the last step we used (5.19) and

$$4\mathrm{Im}\,(\tau)^2 \int \partial\mathcal{G} \wedge \overline{\partial}\mathcal{G} = -\int G_3 \wedge \overline{G}_3\,, \tag{5.26}$$

which we prove as follows. $L^I \partial_I \Omega_3$ contains only $(3,0)$ and $(2,1)$ pieces, so if we integrate it against Ω_3 the result must vanish:

$$0 = \int C\Omega_3 \wedge L^I \partial_I \Omega_3 \tag{5.27}$$

$$= -\frac{1}{4\mathrm{Im}\,(\tau)^2} \int \left(\overline{G}_3 - 2i\mathrm{Im}\,(\tau)\,\partial\mathcal{G}\right) \wedge \left(-G_3 + 2i\mathrm{Im}\,(\tau)\,\overline{\partial}\mathcal{G}\right) \tag{5.28}$$

$$= -\frac{1}{4\mathrm{Im}\,(\tau)^2} \Bigg[\int G_3 \wedge \overline{G}_3 + 2i\mathrm{Im}\,(\tau) \left(\int \overline{G}_3 \wedge \overline{\partial}\mathcal{G} - \int G_3 \wedge \partial\mathcal{G} \right) + 4\mathrm{Im}\,(\tau)^2 \int \partial\mathcal{G} \wedge \overline{\partial}\mathcal{G} \Bigg] \tag{5.29}$$

$$= -\frac{1}{4\mathrm{Im}\,(\tau)^2} \left[\int G_3 \wedge \overline{G}_3 + 4\mathrm{Im}\,(\tau)^2 \int \partial\mathcal{G} \wedge \overline{\partial}\mathcal{G} \right], \tag{5.30}$$

which implies (5.26).

We now write out the gravitino mass (3.8) with the full Kähler potential (2.20):

$$\mathrm{Vol}^2 m_{3/2}^2 = \frac{|CW|^2}{2i\mathrm{Im}\,(\tau)\,|C|^2 \int \Omega_3 \wedge \overline{\Omega}_3} \tag{5.31}$$

$$= \frac{1}{2} \left[\int F_3 \wedge H_3 - \mathcal{G} \right]. \tag{5.32}$$

Reorganizing this we find the generating function,

$$\mathcal{G} = \int F_3 \wedge H_3 - 2\mathrm{Vol}^2 m_{3/2}^2\,, \tag{5.33}$$

as we wanted to show. We also point out a curious relationship:

$$\mathrm{Vol}^2 m_{3/2}^2 = \frac{1}{2} CW\,, \tag{5.34}$$

where both quantities are evaluated at the attractor point. One could have imagined that other duality-invariant quantities, e.g. eigenvalues of the mass matrix, would appear in one or more of these expressions, but they do not. We also point out that the combination $\mathrm{Vol}^2 m_{3/2}^2$ is independent of the Kähler moduli, which cannot be stabilized by turning on 3-form fluxes.

As a side product of our derivation, we find another interesting identity. While one combination of (5.12) and (5.13) gives (2.26), another combination appears more novel:

$$\overline{\partial}\mathcal{G} = \frac{1}{2}\left[L^I \partial_I \Omega_3 - \overline{C\Omega}_3\right]. \tag{5.35}$$

The operator introduced in (5.11) is nilpotent,

$$\int \partial \wedge \partial = \frac{\partial}{\partial e_I}\frac{\partial}{\partial m^I} - \frac{\partial}{\partial m^I}\frac{\partial}{\partial e_I} = 0\,, \tag{5.36}$$

so we find that

$$\int \overline{\partial} \wedge \left[L^I \partial_I \Omega_3 - \overline{C\Omega}_3\right] = 0\,, \tag{5.37}$$

in other words $L^I \partial_I \Omega_3 - \overline{C\Omega}_3$ is $\overline{\partial}-$closed. Indeed, according to (5.35) it is $\overline{\partial}-$exact. This observation may motivate the introduction of the generating function $\mathcal{G}$ even in cases where the F_I are not globally well-defined.

6 An explicit solution of the attractor equations

In this section we find an explicit solution to the attractor equations for a particular prepotential:

$$F = \frac{Z^1 Z^2 Z^3}{Z^0}\,. \tag{6.1}$$

This prepotential appears frequently in the supergravity literature as the STU model [32–35], while in the flux compactification literature it appears as the untwisted sector of a $T^6/\mathbb{Z}^2 \times \mathbb{Z}^2 \approx T^2 \times T^2 \times T^2$ orbifold [36, 37]. Because it is a truncation of $\mathcal{N} = 8$ supergravity it has a number of useful symmetries. On the other hand, it shares many features with more generic prepotentials, and so is of broader interest than the pure $\mathcal{N} = 8$ model.

We first write down the attractor equations explicitly for an arbitrary set of fluxes. For a *subset* of all possible fluxes, we are able to solve the attractor equations, finding explicit expressions for the complex structure moduli and τ. We then compute the generating function $\mathcal{G}$ and the gravitino mass, and verify that the proposed relationship between them (5.1) holds in this case. We conclude with a discussion of the U-duality group for this model, and describe how to generalize the solution for our subset of fluxes to a solution for general fluxes.

6.1 Symplectic section and electric attractor equations

In order to make the attractor equations (4.4)–(4.8) completely explicit, we need to specify the symplectic section $\{Z^I, F_I\}$. In the present case the F_I are just derivatives of the prepotential (6.1):

$$F_I = \frac{\partial F}{\partial Z^I}\,, \tag{6.2}$$

with $I = 0,1,2,3$. We substitute (6.2) into (4.13) to find the generating function in the mixed ensemble:

$$\begin{aligned}\mathcal{V}\left(m^I,\varphi^I,\tau\right) &= 2\mathrm{Im}\left(\tau\right) C\left[-L^0\frac{Z^1Z^2Z^3}{\left(Z^0\right)^2}+L^1\frac{Z^2Z^3}{Z^0}+L^2\frac{Z^3Z^1}{Z^0}+L^3\frac{Z^1Z^2}{Z^0}\right]. &(6.3)\\ &= \frac{1}{2\mathrm{Im}\left(\tau\right)\left(\overline{m}^0+\overline{\varphi}^0\right)}\left\{\frac{-m^0+\varphi^0}{\overline{m}^0+\overline{\varphi}^0}\left[\overline{m}^1+\overline{\varphi}^1\right]\left[\overline{m}^2+\overline{\varphi}^2\right]\left[\overline{m}^3+\overline{\varphi}^3\right]\right.\\ &\quad -\left[-m^1+\varphi^1\right]\left[\overline{m}^2+\overline{\varphi}^2\right]\left[\overline{m}^3+\overline{\varphi}^3\right]-\left[\overline{m}^1+\overline{\varphi}^1\right]\left[-m^2+\varphi^2\right]\left[\overline{m}^3+\overline{\varphi}^3\right]\\ &\quad \left.-\left[\overline{m}^1+\overline{\varphi}^1\right]\left[\overline{m}^2+\overline{\varphi}^2\right]\left[-m^3+\varphi^3\right]\right\}. &(6.4)\end{aligned}$$

Since $\mathcal{V}$ is a function of magnetic charges and electric potentials, we substituted in (5.5) and (5.6) for the Z^I and L^I. The electric attractor equations (4.20) and (4.21) require that we differentiate[6] $\mathrm{Im}\left(\mathcal{V}\right)$:

$$\begin{aligned}\overline{e}_0 &= -2i\mathrm{Im}\left(\tau\right)\frac{\partial}{\partial\varphi^0}\frac{\mathcal{V}-\overline{\mathcal{V}}}{2i} &(6.5)\\ &= -\frac{1}{2\left(\overline{m}^0+\overline{\varphi}^0\right)^2}\left[\overline{m}^1+\overline{\varphi}^1\right]\left[\overline{m}^2+\overline{\varphi}^2\right]\left[\overline{m}^3+\overline{\varphi}^3\right]\\ &\quad -\frac{1}{2\left(m^0+\varphi^0\right)^2}\left\{2\frac{-\overline{m}^0+\overline{\varphi}^0}{m^0+\varphi^0}\left[m^1+\varphi^1\right]\left[m^2+\varphi^2\right]\left[m^3+\varphi^3\right]\right.\\ &\quad -\left[-\overline{m}^1+\overline{\varphi}^1\right]\left[m^2+\varphi^2\right]\left[m^3+\varphi^3\right]-\left[m^1+\varphi^1\right]\left[-\overline{m}^2+\overline{\varphi}^2\right]\left[m^3+\varphi^3\right]\\ &\quad \left.-\left[m^1+\varphi^1\right]\left[m^2+\varphi^2\right]\left[-\overline{m}^3+\overline{\varphi}^3\right]\right\}, &(6.6)\\ \overline{e}_1 &= -2i\mathrm{Im}\left(\tau\right)\frac{\partial}{\partial\varphi^1}\frac{\mathcal{V}-\overline{\mathcal{V}}}{2i} &(6.7)\\ &= \frac{1}{2\left(\overline{m}^0+\overline{\varphi}^0\right)}\left[\overline{m}^2+\overline{\varphi}^2\right]\left[\overline{m}^3+\overline{\varphi}^3\right]\\ &\quad +\frac{1}{2\left(m^0+\varphi^0\right)}\left\{\frac{-\overline{m}^0+\overline{\varphi}^0}{m^0+\varphi^0}\left[m^2+\varphi^2\right]\left[m^3+\varphi^3\right]\right.\\ &\qquad \left.-\left[-\overline{m}^2+\overline{\varphi}^2\right]\left[m^3+\varphi^3\right]-\left[m^2+\varphi^2\right]\left[-\overline{m}^3+\overline{\varphi}^3\right]\right\}, &(6.8)\end{aligned}$$

where the φ^I-derivatives are defined analogous to e_I-derivatives (5.7). The equations for $\overline{e}_2$ and $\overline{e}_3$ are cyclic permutations of (6.8), so we have a system of four complex equations.

We also need to make the constraint (4.8) explicit. For the prepotential (6.1), it

[6] We could also have substituted our F_I directly into the electric attractor equations (4.6) and (4.7), then made the change of variables (4.9) and (4.10). This gives an identical result, indicating that our $\mathrm{Im}\left(\mathcal{V}\right)$ correctly generates the electric attractor equations.

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reduces to

$$0 = L^I \overline{F}_I - L^I F_{IJ} \overline{Z}^J \tag{6.9}$$

$$= -L^0 \frac{\overline{Z}^1 \overline{Z}^2 \overline{Z}^3}{\left(\overline{Z}^0\right)^2} + \left[L^1 \frac{\overline{Z}^2 \overline{Z}^3}{\overline{Z}^0} + \text{cyc.}\right] - 2L^0 \frac{Z^1 Z^2 Z^3}{(Z^0)^3} \overline{Z}^0 + \left[L^0 \frac{Z^1 Z^2}{(Z^0)^2} \overline{Z}^3 + \text{cyc.}\right]$$
$$+ \left[L^1 \frac{Z^2 Z^3}{(Z^0)^2} \overline{Z}^0 + \text{cyc.}\right] - \left[L^1 \frac{Z^2}{Z^0} \overline{Z}^3 + L^1 \frac{Z^3}{Z^0} \overline{Z}^2 + \text{cyc.}\right]. \tag{6.10}$$

After we substitute in (4.9) and (4.10) this expands out to

$$\begin{aligned}
0 =& -\left(-m^0 + \varphi^0\right) \frac{\left(m^1 + \varphi^1\right)\left(m^2 + \varphi^2\right)\left(m^3 + \varphi^3\right)}{\left(m^0 + \varphi^0\right)^2} \\
&+ \left[\left(-m^1 + \varphi^1\right) \frac{\left(m^2 + \varphi^2\right)\left(m^3 + \varphi^3\right)}{m^0 + \varphi^0} + \text{cyc.}\right] \\
&-2\left(-m^0 + \varphi^0\right) \frac{\left(\overline{m}^1 + \overline{\varphi}^1\right)\left(\overline{m}^2 + \overline{\varphi}^2\right)\left(\overline{m}^3 + \overline{\varphi}^3\right)}{\left(\overline{m}^0 + \overline{\varphi}^0\right)^3} \left(m^0 + \varphi^0\right) \\
&+ \left[\left(-m^0 + \varphi^0\right) \frac{\left(\overline{m}^1 + \overline{\varphi}^1\right)\left(\overline{m}^2 + \overline{\varphi}^2\right)}{\left(\overline{m}^0 + \overline{\varphi}^0\right)^2} \left(m^3 + \varphi^3\right) + \text{cyc.}\right] \\
&+ \left[\left(-m^1 + \varphi^1\right) \frac{\left(\overline{m}^2 + \overline{\varphi}^2\right)\left(\overline{m}^3 + \overline{\varphi}^3\right)}{\left(\overline{m}^0 + \overline{\varphi}^0\right)^2} \left(m^0 + \varphi^0\right) + \text{cyc.}\right] \\
&- \left[\left(-m^1 + \varphi^1\right) \frac{\overline{m}^2 + \overline{\varphi}^2}{\overline{m}^0 + \overline{\varphi}^0} \left(m^3 + \varphi^3\right) + \left(-m^1 + \varphi^1\right) \frac{\overline{m}^3 + \overline{\varphi}^3}{\overline{m}^0 + \overline{\varphi}^0} \left(m^2 + \varphi^2\right) + \text{cyc.}\right].
\end{aligned} \tag{6.11}$$

This appears to be another high-order polynomial equation in many variables.

We need to invert (6.6), (6.8), and (6.11) and find both the electric potentials φ^I and τ as functions of the electric and magnetic fluxes. Doing this by brute force would be quite challenging, as each equation is at least cubic in the potentials. Although we have written the attractor equations in terms of complex potentials and fluxes they are clearly not holomorphic in the potentials, so even counting the number of distinct solutions (sometimes called "area codes" [26, 38–41]) for general fluxes appears difficult. In the following we will find a solution to these equations using the ideas developed in section (4).

6.2 Reduction to eight fluxes

Much of the difficulty in solving (6.6), (6.8), and (6.11) arises from their dependence on both m^I, φ^I, *and* $\overline{m}^I$, $\overline{\varphi}^I$. Things simplify quite a bit if we set $m_h^0 = m_f^i = e_0^f = e_i^h = 0$, and make the *ansatz* that $\mathrm{Re}\,(\tau) = \phi_h^0 = \phi_f^I = 0$, so that the complex fluxes and potentials become:

$$m^0 = m_f^0\,, \tag{6.12}$$
$$m^i = -i\mathrm{Im}\,(\tau)\, m_h^i\,, \tag{6.13}$$
$$e_0 = -i\mathrm{Im}\,(\tau)\, e_0^h\,, \tag{6.14}$$
$$e_i = e_i^f\,, \tag{6.15}$$
$$\varphi^0 = \phi_f^0\,, \tag{6.16}$$
$$\varphi^i = -i\mathrm{Im}\,(\tau)\, \phi_h^i\,. \tag{6.17}$$

This makes it easy to take the complex conjugate of a flux or potential: $\overline{m}^0 = m^0$, $\overline{e}_i = e_i$, $\overline{\varphi}^0 = \varphi^0$, $\overline{m}^i = -m^i$, $\overline{e}_0 = -e_0$, and $\overline{\varphi}^i = -\varphi^i$.

If we apply these restrictions to (6.6), (6.8), and (6.11) we find:

$$e_0 = -\frac{\left(m^1+\varphi^1\right)\left(m^2+\varphi^2\right)\left(m^3+\varphi^3\right)}{2\left(m^0+\varphi^0\right)^2}\left\{1 - 2\frac{-m^0+\varphi^0}{m^0+\varphi^0} - \frac{-m^1+\varphi^1}{m^1+\varphi^1} - \frac{-m^2+\varphi^2}{m^2+\varphi^2} - \frac{-m^3+\varphi^3}{m^3+\varphi^3}\right\}, \tag{6.18}$$

$$e_1 = \frac{\left(m^2+\varphi^2\right)\left(m^3+\varphi^3\right)}{2\left(m^0+\varphi^0\right)}\left\{1 + \frac{-m^0+\varphi^0}{m^0+\varphi^0} + \frac{-m^2+\varphi^2}{m^2+\varphi^2} + \frac{-m^3+\varphi^3}{m^3+\varphi^3}\right\}, \tag{6.19}$$

$$0 = \frac{\left(m^1+\varphi^1\right)\left(m^2+\varphi^2\right)\left(m^3+\varphi^3\right)}{2\left(m^0+\varphi^0\right)^2}\left[\frac{-m^0+\varphi^0}{m^0+\varphi^0} + \frac{-m^1+\varphi^1}{m^1+\varphi^1} + \frac{-m^2+\varphi^2}{m^2+\varphi^2} + \frac{-m^3+\varphi^3}{m^3+\varphi^3}\right]. \tag{6.20}$$

Note that the same prefactor appears in (6.18) and (6.20). So long as $e_0 \neq 0$, we conclude that the factor in square brackets in (6.20) must vanish. We can apply this to (6.18) and (6.19) to arrive at a simpler set of equations:

$$e_0 = -\frac{\left(m^1+\varphi^1\right)\left(m^2+\varphi^2\right)\left(m^3+\varphi^3\right)}{\left(m^0+\varphi^0\right)^3}m^0\,, \tag{6.21}$$

$$e_1 = \frac{\left(m^2+\varphi^2\right)\left(m^3+\varphi^3\right)}{\left(m^0+\varphi^0\right)\left(m^1+\varphi^1\right)}m^1\,, \tag{6.22}$$

$$0 = \frac{-m^0+\varphi^0}{m^0+\varphi^0} + \frac{-m^1+\varphi^1}{m^1+\varphi^1} + \frac{-m^2+\varphi^2}{m^2+\varphi^2} + \frac{-m^3+\varphi^3}{m^3+\varphi^3}\,. \tag{6.23}$$

As usual, expressions for e_2 and e_3 arise from cyclic permutations of (6.22). In the next section we will explicitly invert these equations.

6.3 Moduli, potentials, and mass parameters (reduced fluxes)

We begin by solving for the physical complex structure moduli,

$$z^i \equiv \frac{Z^i}{Z^0} = \frac{\overline{m}^i+\overline{\varphi}^i}{\overline{m}^0+\overline{\varphi}^0} = -\frac{m^i+\varphi^i}{m^0+\varphi^0}\,. \tag{6.24}$$

The ratio of (6.21) and (6.22) can be solved for the z^i :

$$\frac{e_i}{e_0} = -\left(\frac{m^0+\varphi^0}{m^i+\varphi^i}\right)^2 \frac{m^i}{m^0} = -\frac{1}{(z^i)^2}\frac{m^i}{m^0}\,. \tag{6.25}$$

In order to avoid awkward branch cuts when we take the square root, we will carefully analyze the signs on the charges. If we insert the real charges and potentials into the previous expression,

$$\frac{e_i^f}{e_0^h} = -\frac{m_h^i}{m_f^0}\left(\frac{m_f^0+\phi_f^0}{m_h^i+\phi_h^i}\right)^2 , \tag{6.26}$$

we find that $e_i^f m_f^0/e_0^h m_h^i < 0$, and thus that $e_i m^0/e_0 m^i > 0$. We must also consider the Kähler potential (2.8) with the prepotential (6.1). Evaluating it, we find

$$K_z = -\log\left|Z^0\right|^2 - \log\left[-8\mathrm{Im}\left(z^1\right)\mathrm{Im}\left(z^2\right)\mathrm{Im}\left(z^3\right)\right]. \tag{6.27}$$

The condition that the volume of each of the underlying T^2's is positive requires $\mathrm{Im}\left(z^i\right) < 0$, which in turn implies that K_z is real. This determines the expression for the modulus:

$$z^i = -i\sqrt{\frac{e_0 m^i}{m^0 e_i}} = -i\mathrm{Im}\left(\tau\right)\sqrt{-\frac{e_0^h m_h^i}{m_f^0 e_i^f}}\,. \tag{6.28}$$

In order to make this completely explicit we must solve for $\mathrm{Im}\left(\tau\right)$, so we will do that next.

We can use (6.28) to simplify (6.21):

$$e_0 = z^1 z^2 z^3 m^0 = i\sqrt{\frac{\left(e_0\right)^3 m^1 m^2 m^3}{\left(m^0\right)^3 e_1 e_2 e_3}}\,m^0\,. \tag{6.29}$$

All dependence on the potentials has been eliminated, so this is a single equation that determines $\mathrm{Im}\left(\tau\right)$. Substituting in real quantities, we find

$$1 = -\mathrm{sgn}\left(m_f^0 e_0^h\right)\sqrt{-\mathrm{Im}\left(\tau\right)^4\frac{e_0^h m_h^1 m_h^2 m_h^3}{m_f^0 e_1^f e_2^f e_3^f}}\,. \tag{6.30}$$

Note that the $\mathrm{sgn}\left(m_f^0 e_0^h\right)$ appeared when we pulled the factor of m_f^0/e_0^h under the square root. We now find that

$$\mathrm{Im}\left(\tau\right) = \left(-\frac{m_f^0 e_1^f e_2^f e_3^f}{e_0^h m_h^1 m_h^2 m_h^3}\right)^{1/4} , \tag{6.31}$$

where the physical condition $\mathrm{Im}\left(\tau\right) = e^{-\phi}$ dictates that we use the real, positive branch, and implies that K_τ (2.20) is real.[7]

[7] It is somewhat awkward that our Kähler potential requires $\mathrm{Im}\left(\tau\right) > 0$ but $\mathrm{Im}\left(z^i\right) < 0$, especially if we want to consider this model as a compactification of F-theory. On the other hand, our conventions are self-consistent, and chosen to agree with the bulk of the literature on flux compactifications.

Equation (6.30) also implies that $\operatorname{sgn}\left(m_f^0 e_0^h\right) = -1$. We can combine this with our earlier result that $\operatorname{sgn}\left(m_f^0 e_0^h m_h^i e_i^f\right) = -1$ to find a complete set of sign restrictions:

$$-\operatorname{sgn}\left(m_f^0 e_0^h\right) = \operatorname{sgn}\left(m_h^1 e_1^f\right) = \operatorname{sgn}\left(m_h^2 e_2^f\right) = \operatorname{sgn}\left(m_h^3 e_3^f\right) = +1. \tag{6.32}$$

Only 1/16 of the possible fluxes satisfy the physical conditions we have imposed. It is interesting to consider what might happen if we relaxed these sign restrictions. Suppose we chose signs that violated some of the conditions in (6.32), but satisfied the *product* of those conditions. The Kähler potential (2.20) would still be real, so we would still have solutions to the ISD condition, at least formally. The caveat is that the complex structures of some of the T^2's would no longer be in the upper half-plane and/or the sign of the string coupling would be negative. At a minimum, then, we would have to give up the conventional geometrical interpretation of the moduli. Going even further, we can consider signs such that the *product* of the conditions in (6.32) are violated. Then the Kähler potential (2.20) would not be real and it is not clear that the proposed solution would, in fact, be a solution. Indeed, for such flux assignments there may not be any solutions to the ISD conditions at all. In the following we will analyze only the clearly physical solutions that satisfy (6.32).

We can compare our restrictions with a more familiar one [12]. If we assume that the attractor equations can be satisfied, i.e. (2.26), then

$$\int F_3 \wedge H_3 = \frac{1}{2i\operatorname{Im}(\tau)} \int G_3 \wedge \overline{G}_3 \tag{6.33}$$

$$= \frac{e^{-K_z}}{2\operatorname{Im}(\tau)} \left[|C|^2 + \left|C^i\right|^2 \right], \tag{6.34}$$

and thus $\int F_3 \wedge H_3$ is positive. The sign restrictions (6.32) are consistent with this, but stronger. If we evaluate $\int F_3 \wedge H_3$ for our reduced fluxes,

$$\int F_3 \wedge H_3 = -e_0^h m_f^0 + e_i^f m_h^i\,, \tag{6.35}$$

we see that the sign restrictions require that *each term* be positive.

Having determined $\operatorname{Im}(\tau)$ and the sign restrictions on the various fluxes, (6.28) gives explicit expressions for the complex structure moduli:

$$z^1 = -i\left[\left(-\frac{e_0^h}{m_f^0}\right)\left(\frac{m_h^1}{e_1^f}\right)\left(\frac{e_2^f}{m_h^2}\right)\left(\frac{e_3^f}{m_h^3}\right)\right]^{1/4}, \tag{6.36}$$

and cyclic permutations. These explicit expressions for the physical moduli, along with the dilaton (6.31) and the restrictions on the fluxes (6.32), are some of the principal results of this example.

Up to this point we have solved for the moduli and derived a set of restrictions on the fluxes, but we haven't yet solved for the potentials. The only equation that we haven't

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solved is the constraint (6.23), so let's turn our attention there. We can rewrite that equation as

$$m^0+\varphi^0=\frac{m^0}{2}\left\{1+\frac{m^1}{m^0}\frac{m^0+\varphi^0}{m^1+\varphi^1}+\frac{m^2}{m^0}\frac{m^0+\varphi^0}{m^2+\varphi^2}+\frac{m^3}{m^0}\frac{m^0+\varphi^0}{m^3+\varphi^3}\right\}. \tag{6.37}$$

Combining (6.24) and (6.28), we find

$$m^0+\varphi^0=\frac{m^0}{2}\left\{1-i\frac{m^1}{m^0}\sqrt{\frac{m^0e_1}{e_0m^1}}-i\frac{m^2}{m^0}\sqrt{\frac{m^0e_2}{e_0m^2}}-i\frac{m^3}{m^0}\sqrt{\frac{m^0e_3}{e_0m^3}}\right\}. \tag{6.38}$$

We now rewrite this in terms of real quantities:

$$\phi^0_f=\frac{m^0_f}{2}\left\{-1-\text{sgn}\left(m^0_fm^1_h\right)\sqrt{-\frac{m^1_he^f_1}{e^h_0m^0_f}}-\text{sgn}\left(m^0_fm^2_h\right)\sqrt{-\frac{m^2_he^f_2}{e^h_0m^0_f}}-\text{sgn}\left(m^0_fm^3_h\right)\sqrt{-\frac{m^3_he^f_3}{e^h_0m^0_f}}\right\}. \tag{6.39}$$

If we again use the relation between $m^0+\varphi^0$ and $m^1+\varphi^1$, (6.24), we find the following expression for ϕ^1_h :

$$\phi^1_h=\frac{m^1_h}{2}\left\{-1-\text{sgn}\left(m^1_hm^0_f\right)\sqrt{-\frac{m^0_fe^h_0}{m^1_he^f_1}}+\text{sgn}\left(m^1_hm^2_h\right)\sqrt{\frac{m^2_he^f_2}{m^1_he^f_1}}+\text{sgn}\left(m^1_hm^3_h\right)\sqrt{\frac{m^3_he^f_3}{m^1_he^f_1}}\right\}. \tag{6.40}$$

This completes our inversion of (6.21), (6.22), and (6.23).

We emphasized earlier in this paper that the attractor equations include the mass parameters C^i on equal terms with the moduli z^i. With (6.39) and (6.40) in hand, it is straightforward to compute the C^i. We first insert our $z^i=Z^i/Z^0$ into (4.3) to make the relationship between the C^i and L^I explicit:

$$C^iZ^0=-z^iL^0+L^i\,. \tag{6.41}$$

Note that the combination C^iZ^0 is Kähler-invariant, while C^i alone is not. If we substitute (4.10) and (6.28) into (6.41), we find

$$\begin{aligned}
C^1Z^0&=\sqrt{-\frac{e^h_0m^1_h}{m^0_fe^f_1}}\frac{1}{2}\left(-m^0_f+\phi^0_f\right)-\frac{1}{2}\left(-m^1_h+\phi^1_h\right) &(6.42)\\
&=\frac{1}{2}\left[\text{sgn}\left(m^0_fm^1_h\right)\frac{m^1_h}{m^0_f}\sqrt{-\frac{m^0_fe^h_0}{m^1_he^f_1}}\left(-m^0_f+\phi^0_f\right)-\left(-m^1_h+\phi^1_h\right)\right] &(6.43)\\
&=\frac{m^1_h}{4}\text{sgn}\left(m^0_fm^1_h\right)\sqrt{-\frac{m^0_fe^h_0}{m^1_he^f_1}}\left[-3-\sum_{i=1}^{3}\text{sgn}\left(m^0_fm^i_h\right)\sqrt{-\frac{m^i_he^f_i}{e^h_0m^0_f}}\right]\\
&\quad-\frac{m^1_h}{4}\left[-3-\text{sgn}\left(m^1_hm^0_f\right)\sqrt{-\frac{m^0_fe^h_0}{m^1_he^f_1}}+\text{sgn}\left(m^1_hm^2_h\right)\sqrt{\frac{m^2_he^f_2}{m^1_he^f_1}}+\text{sgn}\left(m^1_hm^3_h\right)\sqrt{\frac{m^3_he^f_3}{m^1_he^f_1}}\right]
\end{aligned}$$

$$
\begin{aligned}
&= \frac{m_h^1}{4}\left[-3\,\mathrm{sgn}\left(m_f^0 m_h^1\right)\sqrt{-\frac{m_f^0 e_0^h}{m_h^1 e_1^f}} - 1 - \mathrm{sgn}\left(m_h^1 m_h^2\right)\sqrt{\frac{m_h^2 e_2^f}{m_h^1 e_1^f}} - \mathrm{sgn}\left(m_h^1 m_h^3\right)\sqrt{\frac{m_h^3 e_3^f}{m_h^1 e_1^f}}\right] \\
&\quad -\frac{m_h^1}{4}\left[-3 - \mathrm{sgn}\left(m_h^1 m_f^0\right)\sqrt{-\frac{m_f^0 e_0^h}{m_h^1 e_1^f}} + \mathrm{sgn}\left(m_h^1 m_h^2\right)\sqrt{\frac{m_h^2 e_2^f}{m_h^1 e_1^f}} + \mathrm{sgn}\left(m_h^1 m_h^3\right)\sqrt{\frac{m_h^3 e_3^f}{m_h^1 e_1^f}}\right] \\
&= \frac{m_h^1}{2}\left[1 - \mathrm{sgn}\left(m_f^0 m_h^1\right)\sqrt{-\frac{m_f^0 e_0^h}{m_h^1 e_1^f}} - \mathrm{sgn}\left(m_h^1 m_h^2\right)\sqrt{\frac{m_h^2 e_2^f}{m_h^1 e_1^f}} - \mathrm{sgn}\left(m_h^1 m_h^3\right)\sqrt{\frac{m_h^3 e_3^f}{m_h^1 e_1^f}}\right]. \qquad (6.44)
\end{aligned}
$$

If one wishes to compute the fermion and scalar mass matrices explicitly, these expressions can be substituted into (3.22), (3.24), and (3.25).

6.4 Generating functions (reduced fluxes)

One of the principal results of this paper is that the attractor behavior of these flux compactifications is governed by a single function $\mathcal{G}$. In this section we compute this function for our reduced fluxes. We will then verify the simple relationship between $\mathcal{G}$ and the gravitino mass.

We begin with $\mathrm{Im}\,(\mathcal{V})$. If we substitute our F_I into (4.13), we find

$$
\mathrm{Im}\,(\mathcal{V}) = 2\mathrm{Im}\,(\tau)\,\mathrm{Im}\left\{C\frac{Z^1Z^2Z^3}{Z^0}\left[-\frac{L^0}{Z^0}+\frac{L^1}{Z^1}+\frac{L^2}{Z^2}+\frac{L^3}{Z^3}\right]\right\} \qquad (6.45)
$$

$$
\begin{aligned}
&= 2\mathrm{Im}\,(\tau)\,\mathrm{Im}\left\{-C^2\frac{Z^1Z^2Z^3}{Z^0}\left[\frac{-m^0+\varphi^0}{m^0+\varphi^0}+\frac{-m^1+\varphi^1}{m^1+\varphi^1}\right.\right. \\
&\qquad\qquad \left.\left. +\frac{-m^2+\varphi^2}{m^2+\varphi^2}+\frac{-m^3+\varphi^3}{m^3+\varphi^3}\right]\right\}. \qquad (6.46)
\end{aligned}
$$

The term in square brackets is just the constraint (6.23) so $\mathrm{Im}\,(\mathcal{V}) = 0$. If we substitute this into (4.22), we find for our reduced fluxes

$$
\mathcal{G} = e_0^h \phi_f^0 - e_i^f \phi_h^i\,. \qquad (6.47)
$$

We compute each term separately:

$$
\begin{aligned}
e_0^h\phi_f^0 &= \frac{1}{2}\left\{-e_0^h m_f^0 + \mathrm{sgn}\left(m_f^0 m_h^1\right)\sqrt{-e_0^h m_f^0}\sqrt{m_h^1 e_1^f}\right. \\
&\qquad \left. +\mathrm{sgn}\left(m_f^0 m_h^2\right)\sqrt{-e_0^h m_f^0}\sqrt{m_h^2 e_2^f} + \mathrm{sgn}\left(m_f^0 m_h^3\right)\sqrt{-e_0^h m_f^0}\sqrt{m_h^3 e_3^f}\right\}, \qquad (6.48) \\
e_1^f\phi_h^1 &= \frac{1}{2}\left\{-e_1^f m_h^1 - \mathrm{sgn}\left(m_f^0 m_h^1\right)\sqrt{-e_0^h m_f^0}\sqrt{m_h^1 e_1^f}\right. \\
&\qquad \left. +\mathrm{sgn}\left(m_h^1 m_h^2\right)\sqrt{m_h^1 e_1^f}\sqrt{m_h^2 e_2^f} + \mathrm{sgn}\left(m_h^1 m_h^3\right)\sqrt{m_h^1 e_1^f}\sqrt{m_h^3 e_3^f}\right\}. \qquad (6.49)
\end{aligned}
$$

Putting this together yields

$$\begin{aligned}\mathcal{G} = \frac{1}{2}\left[-e_0^h m_f^0 + e_i^f m_h^i\right] &+ \mathrm{sgn}\left(m_f^0 m_h^1\right)\sqrt{-e_0^h m_f^0}\sqrt{m_h^1 e_1^f} + \mathrm{sgn}\left(m_f^0 m_h^2\right)\sqrt{-e_0^h m_f^0}\sqrt{m_h^2 e_2^f} \\ &+\mathrm{sgn}\left(m_f^0 m_h^3\right)\sqrt{-e_0^h m_f^0}\sqrt{m_h^3 e_3^f} - \mathrm{sgn}\left(m_h^1 m_h^2\right)\sqrt{m_h^1 e_1^f}\sqrt{m_h^2 e_2^f} \\ &-\mathrm{sgn}\left(m_h^1 m_h^3\right)\sqrt{m_h^1 e_1^f}\sqrt{m_h^3 e_3^f} - \mathrm{sgn}\left(m_h^2 m_h^3\right)\sqrt{m_h^2 e_2^f}\sqrt{m_h^3 e_3^f}\,. \end{aligned} \tag{6.50}$$

The term in square brackets is just $\int F_3 \wedge H_3$ (6.35), while the remainder is less familiar. It is precisely what is required so that $\partial\mathcal{G}/\partial e_0^h = \phi_f^0$ and $\partial\mathcal{G}/\partial e_i^f = -\phi_h^i$, as can be readily verified. It is also closely related to the gravitino mass, as we will now see.

In order to compute the gravitino mass we substitute (6.27), (6.31), (6.36), and (6.39) into (3.9) and simplify

$$\mathrm{Vol}^2 m_{3/2}^2 = -\frac{8\mathrm{Im}\,(\tau)\,\mathrm{Im}\left(z^1\right)\mathrm{Im}\left(z^2\right)\mathrm{Im}\left(z^3\right)}{2}\left(\frac{1}{2\mathrm{Im}\,(\tau)}\right)^2\left(m_f^0+\phi_f^0\right)^2 \tag{6.51}$$

$$= -\frac{e_0^h m_f^0}{4}\left\{1 - \mathrm{sgn}\left(m_f^0 m_h^1\right)\sqrt{-\frac{m_h^1 e_1^f}{e_0^h m_f^0}} - \mathrm{sgn}\left(m_f^0 m_h^2\right)\sqrt{-\frac{m_h^2 e_2^f}{e_0^h m_f^0}} - \mathrm{sgn}\left(m_f^0 m_h^3\right)\sqrt{-\frac{m_h^3 e_3^f}{e_0^h m_f^0}}\right\}^2 \tag{6.52}$$

$$\begin{aligned} = \frac{1}{2}\Bigg\{\frac{1}{2}\left[-e_0^h m_f^0 + e_i^f m_h^i\right] &- \mathrm{sgn}\left(m_f^0 m_h^1\right)\sqrt{-e_0^h m_f^0}\sqrt{m_h^1 e_1^f} \\ &-\mathrm{sgn}\left(m_f^0 m_h^2\right)\sqrt{-e_0^h m_f^0}\sqrt{m_h^2 e_2^f} - \mathrm{sgn}\left(m_f^0 m_h^3\right)\sqrt{-e_0^h m_f^0}\sqrt{m_h^3 e_3^f} \\ &+\mathrm{sgn}\left(m_h^1 m_h^2\right)\sqrt{m_h^1 e_1^f}\sqrt{m_h^2 e_2^f} + \mathrm{sgn}\left(m_h^1 m_h^3\right)\sqrt{m_h^1 e_1^f}\sqrt{m_h^3 e_3^f} \\ &+\mathrm{sgn}\left(m_h^2 m_h^3\right)\sqrt{m_h^2 e_2^f}\sqrt{m_h^3 e_3^f}\Bigg\}\,, \end{aligned} \tag{6.53}$$

If we compare this with our expression for $\mathcal{G}$ (6.50), we see that they are related by

$$\mathcal{G} = \int F_3 \wedge H_3 - 2\mathrm{Vol}^2 m_{3/2}^2\,, \tag{6.54}$$

in accord with the general relationship (5.1).

6.5 $U-$invariants for $F = Z^1Z^2Z^3/Z^0$

The model we are considering enjoys a large set of duality symmetries. We have not made explicit use of these dualities so far, but in this section we will show how they may be used to generalize our solution with only eight fluxes to a solution for the full set of sixteen fluxes. We take inspiration here from the STU black hole, where consideration of duality-invariant combinations of the black hole charges led to a simple expression for the generating function of the potentials [31, 33].

One part of the duality group is easily identified if we think of our prepotential as arising from compactification on $T^2 \times T^2 \times T^2$. We can interpret each z^i as the modular parameter of the ith torus, and consider modular transformations on each torus. Since the tori and their associated modular transformations factorize, their contribution to the U-duality group is just $SL(2)^3$. This is the symmetry group of the STU black hole [33], whose charges transform[8] in the $(2,2,2)$ of $SL(2)^3$.

IIB theories also enjoy an $SL(2)$ S-duality, independent of the $SL(2)^3$ that we have already discussed. This does not factor into discussions of the STU black hole in the IIB picture,[9] as the D3-branes that one uses to construct the black hole (see section (3.1)) are invariant under S-duality. The fluxes H_3 and F_3, however, transform under S-duality, so we must consider the larger duality group $SL(2)^4$, under which our fluxes transform as $(2,2,2,2)$.

The discussion of STU black holes in terms of $SL(2)^3$ invariants is relatively straightforward because there is a single $SL(2)^3$-invariant that one can construct from the charges [31]. This essentially determines the black hole entropy, which in turn is the generating function for the electric and magnetic potentials. On the other hand, one can construct *four* invariants[10] from the $(2,2,2,2)$ of $SL(2)^4$ [43]. The quadratic $I_2 = \int F_3 \wedge H_3$ appears in most studies of IIB flux compactifications, while the other three are less familiar. Considered as polynomials in the fluxes, there are also two quartics, $I_4^{(1)}$ and $I_4^{(2)}$, and a sextic, I_6.

In section 6.2 we chose a reduced set of fluxes that allowed us to explicitly solve the attractor equations. One of our motivations in choosing these particular fluxes was to choose a combination that left all four $SL(2)^4$ invariants non-zero and independent. While the general expressions for these invariants are quite complicated (see [43] for details), they simplify considerably for our reduced fluxes:

$$I_2 = \int F_3 \wedge H_3 = \left(-m_f^0 e_0^h\right) + \left(e_1^f m_h^1\right) + \left(e_2^f m_h^2\right) + \left(e_3^f m_h^3\right), \tag{6.55}$$

$$I_4^{(1)} = -\left(-m_f^0 e_0^h\right)\left(e_1^f m_h^1\right) + \left(-m_f^0 e_0^h\right)\left(e_2^f m_h^2\right) + \left(e_1^f m_h^1\right)\left(e_3^f m_h^3\right) - \left(e_2^f m_h^2\right)\left(e_3^f m_h^3\right), \tag{6.56}$$

$$I_4^{(2)} = -\left(-m_f^0 e_0^h\right)\left(e_2^f m_h^2\right) + \left(-m_f^0 e_0^h\right)\left(e_3^f m_h^3\right) + \left(e_1^f m_h^1\right)\left(e_2^f m_h^2\right) - \left(e_1^f m_h^1\right)\left(e_3^f m_h^3\right), \tag{6.57}$$

$$\begin{aligned} I_6 = {} & \left(-m_f^0 e_0^h\right)^2\left(e_1^f m_h^1\right) + \left(-m_f^0 e_0^h\right)\left(e_1^f m_h^1\right)^2 + \left(e_2^f m_h^2\right)^2\left(e_3^f m_h^3\right) + \left(e_2^f m_h^2\right)\left(e_3^f m_h^3\right)^2 \\ & -4\left(e_1^f m_h^1\right)^2\left(e_2^f m_h^2\right) - 4\left(e_1^f m_h^1\right)\left(e_2^f m_h^2\right)^2 - 4\left(-m_f^0 e_0^h\right)^2\left(e_3^f m_h^3\right) \\ & -4\left(-m_f^0 e_0^h\right)\left(e_3^f m_h^3\right)^2 + 3\left(-m_f^0 e_0^h\right)\left(e_1^f m_h^1\right)\left(e_2^f m_h^2\right) + 3\left(-m_f^0 e_0^h\right)\left(e_1^f m_h^1\right)\left(e_3^f m_h^3\right) \\ & +3\left(-m_f^0 e_0^h\right)\left(e_2^f m_h^2\right)\left(e_3^f m_h^3\right) + 3\left(e_1^f m_h^1\right)\left(e_2^f m_h^2\right)\left(e_3^f m_h^3\right). \end{aligned} \tag{6.58}$$

[8] For details of the action of $SL(2)^3$ on the charges, see e.g. [42].

[9] One can also discuss this entirely in the language of $\mathcal{N}=2$ supergravity. In the STU black hole all of the hypermultiplets, including the universal hypermultiplet, decouple from the attractor flow. On the other hand the axio-dilaton, which descends from the universal hypermultiplet, does *not* decouple from the flux attractor.

[10] More precisely, one can construct exactly four invariants from the $(2,2,2,2)$ of $SL(2,\mathbb{C})^4$. These are also invariants of $SL(2,\mathbb{R})^4$ but additional invariants might arise when we restrict to the subgroup. Possible examples include sgn $\left(m_f^0 m_h^i\right)$. We also expect some number of discrete invariants to appear upon further restriction to $SL(2,\mathbb{Z})^4$.

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Note that given the sign restrictions in (6.32), each term in parentheses is positive-definite. Also, note that exactly four distinct *products* of pairs of fluxes appear in the expressions for the invariants. Duality orbits of our reduced fluxes therefore sweep out a codimension 0 volume in the full space of fluxes. It is more difficult to say whether pairs of fluxes satisfying the sign constraints (6.32) span the physically allowed values of the invariants (6.55)–(6.58).

The explicit form (6.50) of the generating function $\mathcal{G}$ raises an interesting question. Three independent signs appear, $\operatorname{sgn}\left(m_f^0 m_h^1\right)$, $\operatorname{sgn}\left(m_f^0 m_h^2\right)$, and $\operatorname{sgn}\left(m_f^0 m_h^3\right)$. One can readily verify that duality transformations that leave the subspace of reduced fluxes invariant also leave these signs, and only these signs, invariant. Although we are not certain that these signs lift to invariants of the full $SL\left(2\right)^4$, it is possible that they label different octants of the full space of fluxes, with distinct expressions for e.g. the gravitino mass in each octant.

We can use these facts to generalize our solution of the $F = Z^1Z^2Z^3/Z^0$ model with eight fluxes to a solution with all sixteen fluxes. We propose the following procedure:

1. Consider (6.55)–(6.58) to be a set of implicit functions for each pair of fluxes in terms of $I_2 = \int F_3 \wedge H_3$, $I_4^{(1,2)}$, and I_6.

2. Substitute these functions into (6.50) to get $\mathcal{G}$ as a function of the invariants.

3. Substitute the full expressions for $I_2 = \int F_3 \wedge H_3$, $I_4^{(1,2)}$, and I_6 into $\mathcal{G}$ to get an expression for $\mathcal{G}$ as a function of general fluxes.

4. Derivatives of $\mathcal{G}$ with respect to the fluxes will then give the potentials, and in turn the values of the complex structure moduli and mass parameters.

5. Solve (4.8) to determine the value of τ.

This procedure will certainly work if the eight additional fluxes are small. As they become large, global properties of the space of fluxes may present an obstruction, for example one of $\operatorname{sgn}\left(m_f^0 m_h^1\right)$, $\operatorname{sgn}\left(m_f^0 m_h^2\right)$, or $\operatorname{sgn}\left(m_f^0 m_h^3\right)$ might effectively flip. It is also possible that there are other branches of solutions that we have not identified.

Though considerations of duality-invariance have not yet led us to a complete solution of the flux attractor equations with $F = Z^1Z^2Z^3/Z^0$, we hope that future work will make our understanding of flux compactifications on this geometry as detailed as the modern understanding of the STU black hole.

7 Thermodynamics, stability, and the landscape

One of the goals of this paper was to determine how much of the analysis of flux compactifications could be done directly on the space of input fluxes. We demonstrated that local properties of the compactification are completely determined by a single generating function $\mathcal{G}$ defined on the space of fluxes. Although we have been conservative in describing $\mathcal{G}$ as a "generating function," we hope that future analysis will reveal that it is a proper thermodynamic function, and that we can think of the fluxes themselves as the parameters

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of an underlying thermodynamic system. At the same time, we might worry that our success in constructing $\mathcal{G}$ hinged only on the Kähler structure of the moduli space, and that no thermodynamic interpretation exists. We now outline some of the principal challenges surrounding a thermodynamic interpretation of flux attractors.

Is $\mathcal{G}$ a Thermodynamic Function? Equations (4.34) and (4.35) look like equilibrium relations between the fluxes and their thermodynamic conjugates. In addition to equilibrium relations, thermodynamic functions also obey a set of stability conditions. For a sensible thermodynamic interpretation, we would require that stable and unstable thermodynamic equilibria correspond to stable and unstable minima of the traditional spacetime potential (2.22). Here we find an apparent mismatch between the two Hessians. While the field-theoretic mass matrix has $2n+2$ eigenvalues, the matrix of second derivatives of $\mathcal{G}$ has $4n+4$ eigenvalues. For guidance we might study the analogous issue in the black hole attractor. There, the Hessian of the effective potential has $2n$ eigenvalues, while the second derivatives of the entropy lead to $2n+2$ eigenvalues.

What Kind of Thermodynamic Function is $\mathcal{G}$? In thermodynamic problems, the energy and the entropy are treated rather differently. In particular, energies are *minimized* at stable equilibria, while entropies are *maximized.* In other ensembles the energy is mapped to a free energy and the entropy to a generalized Massieu function, but free energies are still minimized and Massieu functions are still maximized. The interpretation of $\mathcal{G}$ hinges on whether it is minimized, in which case it might be interpreted as the tension of a dual domain wall [44], or maximized, in which case it could be interpreted as an entropy. Determining this requires that we fix the overall sign of $\mathcal{G}$. Doing this might be as simple as requiring that $\mathcal{G}$ be positive for stable configurations, but it could be more subtle.

What Does This Imply for the Landscape? If we can establish that $\mathcal{G}$ is an entropy, it becomes quite natural to propose $e^{\mathcal{G}}$ as a classical measure on the string theory landscape. Presumably such a measure would be related to the number of microscopic realizations of a given set of fluxes. We can go on to ask if there are any geometries for which this measure becomes strongly peaked, or whether consistency conditions (such as the tadpole constraint) require that $\mathcal{G}$ be $\mathcal{O}(1)$.

Clearly many potential obstacles lie between the generating function introduced in this paper and a *predictive* measure on the landscape. However the prospect of such a measure is quite exciting, and so worthy of some attention.

Acknowledgments

We thank Sera Cremonini for collaboration in the initial stages of this project. It is a pleasure to acknowledge helpful discussions with Konstantin Bobkov, Alejandra Castro, Gianguido Dall'Agata, Josh Davis, Sergio Ferrara, Jason Kumar, Jan Louis, Gary Shiu, Eva Silverstein, Alessandro Tomasiello, and Brett Underwood. FL thanks CERN for hospitality during part of this project. This work was supported by the DoE under grant DE-FG02-95ER40899.

A Scalar mass matrix in no-scale compactifications

In this appendix we present an explicit computation of the scalar mass matrix for no-scale compactifications.

We divide the scalar potential into two terms as follows:

$$\begin{aligned} V_{\rm tot} &= V + V_0 & \text{(A.1)}\\ &= e^K g^{\alpha\overline{\beta}} D_\alpha W \overline{D_\beta W} + e^K \left(g^{a\overline{b}} D_a W \overline{D_b W} - 3\left|W\right|^2 \right). & \text{(A.2)} \end{aligned}$$

The indices $\alpha, \beta, \gamma \ldots$ run over the complex structure moduli $i, j, k \ldots$ and axio-dilaton τ, and $a, b, \ldots$ run over the Kähler moduli. Because the superpotential is independent of the Kähler moduli, their F-terms are (2.21)

$$D_a W = W \partial_a K\,. \tag{A.3}$$

The inverse metric is such that

$$g^{a\overline{b}} \partial_a K \overline{\partial}_{\overline{b}} K = 3\,, \tag{A.4}$$

so that $V_0 = 0$. The remaining term V is positive semi-definite, so the absolute minima of the scalar potential all have vanishing cosmological constant. This is why these solutions are called “no-scale.”

Since $V_0 = 0$, we do not expect this term to make a contribution to the mass matrix. We now show explicitly that this is the case, beginning with the contribution to $M^2_{\alpha\beta}$ from V_0 :

$$\begin{aligned} \partial_\beta \partial_\alpha V_0 &= \partial_\beta \left\{ e^K \left[g^{a\overline{b}} \left(D_\alpha D_a W \overline{D_b W} + D_a W D_\alpha \overline{D_b W} \right) + D_a W \overline{D_b W} \partial_\alpha g^{a\overline{b}} \right] - 3\overline{W} D_\alpha W \right\} \\ &= e^K \left[g^{a\overline{b}} \left(D_\beta D_\alpha D_a W \overline{D_b W} + D_\alpha D_a W D_\beta \overline{D_b W} \right) + \left(\partial_\beta g^{a\overline{b}} \right) D_\alpha D_a W \overline{D_b W} \right. \\ &\quad + g^{a\overline{b}} \left(D_\beta D_a W D_\beta \overline{D_b W} + D_a W D_\beta D_\alpha \overline{D_b W} \right) + \left(\partial_\beta g^{a\overline{b}} \right) D_a W D_\alpha \overline{D_b W} \\ &\quad \left. + \partial_\alpha g^{a\overline{b}} \left(D_\beta D_a W \overline{D_b W} + D_a W D_\beta \overline{D_b W} \right) + D_a W \overline{D_b W} \partial_\beta \partial_\alpha g^{a\overline{b}} - 3\overline{W} D_\beta D_\alpha W \right]. \end{aligned}$$

Since the Kähler potential factorizes into $K = K_z\left(z^i, \overline{z}^{\overline{i}}\right) + K_\tau\left(\tau, \overline{\tau}\right) + K_t\left(t^a, \overline{t}^{\overline{a}}\right)$ we find that $\partial_\alpha g^{a\overline{b}} = 0$, and simplify further:

$$\begin{aligned} \partial_\beta \partial_\alpha V_0 &= e^K \left[g^{a\overline{b}} \left(D_\beta D_\alpha D_a W \overline{D_b W} + D_\alpha D_a W D_\beta \overline{D_b W} \right) \right. \\ &\quad \left. + g^{a\overline{b}} \left(D_\beta D_a W D_\beta \overline{D_b W} + D_a W D_\beta D_\alpha \overline{D_b W} \right) - 3\overline{W} D_\beta D_\alpha W \right]. \end{aligned} \tag{A.5}$$

Since $\partial_\alpha \partial_a K = 0$, we have

$$\begin{aligned} D_\alpha D_a W &= D_\alpha \left(W \partial_a K \right) & \text{(A.6)}\\ &= \left(D_\alpha W \right) \partial_a K & \text{(A.7)} \end{aligned}$$

and

$$D_\alpha \overline{D_b W} = D_\alpha \left(\overline{W} \partial_b K \right) \tag{A.8}$$
$$= 0 \,. \tag{A.9}$$

This, combined with (A.4), gives

$$\partial_\beta \partial_\alpha V_0 = e^K \left[\left(g^{a\bar{b}} \partial_a K \overline{\partial}_{\bar{b}} K \right) \overline{W} D_\beta D_\alpha W - 3 \overline{W} D_\beta D_\alpha W \right] \tag{A.10}$$
$$= 0 \,, \tag{A.11}$$

so V_0 indeed makes no contribution to $M^2_{\alpha\beta}$.

The contributions to $M^2_{\alpha\overline{\beta}}$ from V_0 simplify in a similar way:

$$\begin{aligned}\overline{\partial}_{\overline{\beta}} \partial_\alpha V_0 &= \overline{\partial}_{\overline{\beta}} \left\{ e^K \left[g^{a\bar{b}} \left(D_\alpha D_a W \overline{D_b W} + D_a W D_\alpha \overline{D_b W} \right) + D_a W \overline{D_b W} \partial_\alpha g^{a\bar{b}} \right] - 3 \overline{W} D_\alpha W \right\} \\ &= e^K \left[g^{a\bar{b}} \left(\overline{D}_{\overline{\beta}} D_\alpha D_a W \overline{D_b W} + D_\alpha D_a W \overline{D_\beta D_b W} \right) \right. \\ &\qquad \left. -3 \left(\overline{W D}_{\overline{\beta}} D_\alpha W + D_\alpha W \overline{D_\beta W} \right) \right] \qquad \text{(A.12)} \\ &= 0 \,. \qquad \text{(A.13)}\end{aligned}$$

So our expectations were correct, and V_0 makes no contribution to the scalar mass matrix.

We emphasize that in computing the contributions from V_0 to the mass matrix we have not set $D_\alpha W = 0$, we have only used the factorization of the Kähler potential. Our conclusion that V_0 makes no contribution to the scalar mass matrix thus holds for metastable local minima, where $D_\alpha W \neq 0$, as well as absolute minima, where $D_\alpha W = 0$.

Next we compute the contributions to the mass matrix from V. Since we are interested in absolute minima of the potential, we will set $D_\alpha W = 0$. We begin with contributions to $M^2_{\alpha\beta}$:

$$\begin{aligned}\partial_\beta \partial_\alpha V &= \partial_\beta \left\{ e^K \left[g^{\gamma\bar{\delta}} \left(D_\alpha D_\gamma W \overline{D_\delta W} + D_\gamma W D_\alpha \overline{D_\delta W} \right) + D_\gamma W \overline{D_\delta W} \partial_\alpha g^{\gamma\bar{\delta}} \right] \right\} \\ &= e^K g^{\gamma\bar{\delta}} \left(D_\alpha D_\gamma W D_\beta \overline{D_\delta W} + D_\beta D_\gamma W D_\alpha \overline{D_\delta W} \right) \qquad \text{(A.14)}\end{aligned}$$

We can eliminate the mixed derivatives using

$$D_\alpha \overline{D_\beta W} = D_\alpha \left(\overline{\partial_\beta W} + \overline{W} \overline{\partial}_{\overline{\beta}} K \right) \tag{A.15}$$
$$= \overline{W} \partial_\alpha \overline{\partial}_{\overline{\beta}} K \tag{A.16}$$
$$= \overline{W} g_{\alpha\overline{\beta}} \,, \tag{A.17}$$

so that (A.14) simplifies to

$$M^2_{\alpha\beta} = \partial_\beta \partial_\alpha V = e^K g^{\gamma\bar{\delta}} \left[D_\alpha D_\gamma W D_\beta \overline{D_\delta W} + D_\beta D_\gamma W D_\alpha \overline{D_\delta W} \right] \tag{A.18}$$
$$= e^K \overline{W} \left(D_\alpha D_\beta W + D_\beta D_\alpha W \right) . \tag{A.19}$$

We'll follow the same procedure for $M^2_{\alpha\overline{\beta}}$,

$$\begin{aligned} M^2_{\alpha\overline{\beta}} = \overline{\partial}_{\overline{\beta}}\partial_\alpha V &= \overline{\partial}_{\overline{\beta}}\left\{e^K\left[g^{\gamma\overline{\delta}}\left(D_\alpha D_\gamma W\overline{D_\delta W}+D_\gamma W D_\alpha\overline{D_\delta W}\right)+D_\gamma W\overline{D_\delta W}\partial_\alpha g^{\gamma\overline{\delta}}\right]\right\} \\ &= e^K g^{\gamma\overline{\delta}}\left[D_\alpha D_\gamma W\overline{D_\beta D_\delta W}+\overline{D}_{\overline{\beta}}D_\gamma W D_\alpha\overline{D_\delta W}\right] \\ &= e^K\left[g^{\gamma\overline{\delta}}D_\alpha D_\gamma W\overline{D_\beta D_\delta W}+|W|^2 g_{\alpha\overline{\beta}}\right]. \end{aligned} \tag{A.20}$$

Our results for the scalar mass matrices, (A.19) and (A.20), agree with the standard results for $\mathcal{N}=1$ supergravity, e.g. eq. 23.27 in [30]. We have verified that the Kähler moduli do not make any additional contributions.

We also see that when $W \neq 0$, i.e. when SUSY is broken, the scalar masses-squared are lifted above the fermion masses-squared by $\mathcal{O}\left(m^2_{3/2}\right)$.

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