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IMPROVEMENTS IN THE CHARACTERISTICS OF A-C LEAD NETWORKS FOR SERVOMECHANISMS

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Electric lead networks are used quite extensively to improve the response of servomechanisms. Such lead networks may be required to operate on the servo error voltage, on the input or output signal voltages separately, on the feedback voltage from an output tachometer, on various, internal loop voltages, and on other signals. In many cases the input voltage to the lead network is a modulated, suppressed-carrier voltage, and it is necessary that the output voltage of the lead network also be a modulated, suppressed-carrier voltage of the same carrier frequency. Consequently, the basic d-c lead network of Figure 1A, which is normally used where the output signal may be d-c, is not applicable by itself to these problems, and an a-c lead network must be used.

The conventional a-c lead networks do not operate on their input signals so effectively as does the network of Figure 1A in a d-c servomechanism. It is the purpose of this article to suggest modifications in design and in the components used which will improve the characteristics of a-c lead networks.

Before considering improvements in the characteristics of a-c lead networks, it is desirable to summarize the general characteristics of these conventional a-c lead networks.

Conventional A-C Lead Networks

When the input and output voltages of a lead network must be modulated, suppressed-carrier voltages, two general classes of a-c lead networks are normally used. In the first class, illustrated by Figure 1, the a-c signal is demodulated, then the demodulated signal is passed through a d-c lead network, like the one of Figure 1A, and lastly, the carrier is modulated with the output voltage of the d-c lead network. This class of a-c lead networks will be called a Class I network.

Regardless of the type of lead network, it may be stated that the essential characteristic of an a-c lead network is that it produce a pronounced, positive phase shift of the modulating envelope over specific portions of the modulating frequency band. The analytical expressions (1A-2), (1A-3), and (1A-4) show that the Class I lead network of Figure 1 and Figure 1A does have a pronounced positive phase shift ϕ which reaches a maximum value of ϕ_{sm} at a signal frequency of w_{sm}. Graph 1 is a plot of the phase shift and magnitude characteristics of this lead network. In general, for all lead networks, either a-c or d-c, the ratio of the lead-network transfer-function at infinite signal frequency to the lead-network transfer-function at zero signal frequency (for example Equation (1A-5)) will be defined as the lead network attenuation.

The demodulator and modulator circuits of the first class of a-c lead networks produce noise. Filter circuits added to the demodulator and modulator tend to reduce the magnitude of this noise. Unfortunately the reduction in noise by this method is accompanied by a time delay or an effective negative phase shift of the modulating envelope. Such a negative phase shift may be partially compensated for by an increase in the attenuation of the d-c lead network. This increase in the attenuation will result in an increase of the overall noise-to-signal ratio. Hence, in the Class I a-c lead network, an engineering compromise must be reached between (1) the amount of noise suppressed by filters in the demodulator and modulator and (2) the increase in noise-to-signal ratio resulting from the increase in attenuation required to offset the effect of the filters. These several factors cause this Class I network to produce a smaller maximum positive phase shift than the d-c lead network can produce by itself.

An example of the second class of a-c lead networks is shown in Figure 2, and this class will be referred to as Class II networks. The parallel T network of Figure 3 is another simple a-c lead network of Class II.[1] Equations (2-2) through (2-5) and Equations (3-3) through (3-7) of Figures 2 and 3, respectively, demonstrate that these networks modify the envelope of the modulated signal in approximately the same fashion as does the circuit of Figure 1.

The application of these Class II networks is restricted by the following two factors:

1. A-C lead networks of the second class are quite sensitive to changes in the carrier frequency because a change in carrier frequency can reduce the effective lead developed by these networks and can, in some cases, produce lag instead of lead. This effect is more noticeable at 400 cps. than at 60 cps., because it is the absolute change in carrier frequency which affects the network, and, in general, 60 cps. sources are better frequency-regulated on an absolute basis than 400 cps. sources.
2. The maximum value of the lead time constant T of the second class of a-c lead networks is limited and this limits the minimum value of w_{sm}, the frequency at which the maximum positive phase shift of the modulating envelope occurs. (For the network of Figure 2, the maximum value of $T = 2 \frac{L}{R_2}$ and for the network of Figure 3, the maximum value of $T' = \frac{1}{w_c}$.) In some cases when the minimum value of w_{sm} is too large, it is possible to increase the phase shift at lower frequencies by increasing the attenuation. Unfortunately this increase in attenuation may increase the noise-to-signal ratio of the lead network sufficiently to offset any improvement derived from an increase in positive phase shift at lower frequencies.

[Figure 2A[2] shows a modified form of the basic Class II a-c lead network. As will be noted from Equations (2A-5), (2A-6), and (2A-7), when the maximum phase shift ϕ_{sm} is the same for both the circuits of Figures 2 and 2A, the minimum value of w_{sm} is smaller for Figure 2A than for Figure 2 by a factor of c. Inasmuch as the minimum value of r' is 1, the maximum value of c under the condition of Equation (2A-5) is r.]

Modifications of the Class I A-C Lead Network

As was mentioned before, the demodulator and modulator filters of Figure 1 produce time delays or effective negative phase shifts of the modulating envelope. Figure 4 shows a circuit where the demodulator and modulator filters are used to an advantage. The arrangement of the components of Figure 4 shall be referred to as a feedback type modification of Class I. To simplify the consideration of this circuit, the time delays or phase shifts created by the filters of the demodulator and modulator have been lumped in the single RC time lag network of Figure 4. Thus the minimum value of the time constant of the RC network of Figure 4 will be determined by the filters in the demodulator and modulator, but if required, additional filtering or delays may be added to increase this equivalent time constant.

The analytical expressions (4-10), (4-11), and (4-12) of Figure 4 show that the characteristics of this system are the same as for the basic Class I network. The attenuation r may be adjusted by adjusting the gains of the amplifier, demodulator or modulator. The lead time constant T may be made as large as desired but does have a minimum value determined by the equivalent time constant of the demodulator and modulator filters which produce an acceptable noise level.

Figure 4A shows another arrangement of the components of the basic Class I a-c lead network and will be referred to as a parallel type modification of Class I a-c lead network. The assumptions made in representing the circuit of Figure 4 are carried over into the circuit of Figure 4A. The analytical expressions (4A-9), (4A-10), and (4A-11) show that the response of this network is similar to that of Figure 4. The parallel type has the limitation that, as noted from Equation (4A-8), the minimum value of the lead time constant is equal to r times the minimum value of the equivalent time constant of the demodulator and modulator filters. Inasmuch as this value may be too large for some servomechanism applications, the parallel type may not always be as useful as the feedback type.

The effectiveness of the demodulator and modulator filters or the complexities of the demodulator and modulator plus filters may be such as to limit the degree of suppression of noise generated. Such a limitation will establish the noise-to-signal ratio and determine the applicability of these circuits.

By a change in the type of components as well as the arrangement used in the basic Class I network a system can be evolved having demodulators and modulators with excellent noise-to-signal ratios. Such a system will be referred to as an Electromechanical A-C Lead Network - Class I.

Electromechanical A-C Lead Networks - Class I

Figure 5 is an example of such a modified Class I network.[3] In its most general form, this network is equally applicable for d-c and a-c operation, but in this article, only its a-c form is discussed. In this system the motor functions as a demodulator and the generator as a modulator. Both of these functions are approximate with the limitations that w_s and the motor-generator speed be small compared to the carrier frequency and synchronous speeds, respectively. The analytical expressions of (5-11), (5-12), and (5-13) of Figure 5 show that the characteristics of this system are the same as for the basic Class I a-c lead network. By proper adjustment of K_1, K_2, and K_3, it is possible to establish any value of attenuation r. The lead time constant T can be made as large as desired by the addition of inertia to the common motor-generator shaft. Unfortunately the minimum value of T is $\frac{J_2 + J_3}{f_2}$ and this value may be too high for some servo applications.

The arrangement of the components of the circuit of Figure 5 is such that this type of electromechanical network will be referred to as a feedback type. Another arrangement of the components of Figure 5 is shown in Figure 6 and will be referred to as an electromechanical a-c lead network of the parallel type. This parallel type of electromechanical network has characteristics similar to those of the feedback type except that its lead time constant T is greater by a factor equal to the attenuation r. Consequently in many cases, the parallel type is not so useful as the feedback type of electromechanical a-c lead network. It is of interest to note that the amplifier K_1 can be omitted in the parallel type.

Figure 7 is a schematic of another electromechanical a-c lead network - Class I. It is of the feedback type but, as indicated by Equation (7-12), does not have the phase and amplitude characteristics of the basic Class I a-c network. This network is a resonant lead controller of Class I. Graphs 2, 3, 4, 5, and 6 show the phase and amplitude characteristics of this electromechanical system.

Figure 8 shows the parallel type counterpart of the electromechanical a-c lead network of Figure 7. Its characteristics are similar to those of the feedback type.

Of the several types of electromechanical a-c lead networks discussed, that of Figure 7 appears to have the most desirable characteristics and also to be the simplest to manufacture. Figure 9 is a cut-away view of a pilot design of this electromechanical system. The main components of the system are (1) an induction torque motor, (2) a combination induction pickoff and electric spring, (3) an eddy-current damper, and (4) inertia weights. The induction torque motor and the combination induction pickoff and electric spring are made of simple, punched laminations and bobbin-wound coils and are called Microsyn elements.[4]

The induction torque motor is excited by a fixed a-c reference voltage and by the signal voltage. It develops a torque proportional to the signal voltage which is virtually independent of the angular displacement of its rotor. The coils of the combination induction pickoff and electric spring are so arranged that there is a separate circuit for both the pickoff and electric spring. The pickoff circuit is excited with a-c voltage and the circuit of the electric spring is excited with d-c voltage. This combination then produces a signal voltage and a torque, both of which are proportional to the angular displacement of the rotor from neutral. By changing the d-c excitation of the electric spring circuit, the torque gradient of the electric spring may be adjusted.

The eddy-current damper is made up of an aluminum damping disc and a multi-pole, Alnico permanent magnet with its poles parallel to the axis of rotation. This magnet is threaded axially into the housing; and therefore it is possible to adjust the spacing between the magnet and damping disc, thus changing the damping coefficient f.

The total moment of inertia of the system may be changed by adjusting the position of the inertia weights.

Thus as there are three continuously adjustable parameters--the spring constant, the damping, and the moment of inertia--it is possible to adjust the two variables w_n and c independently. The attenuation factor h^2 can also be adjusted independently by changing the pickoff sensitivity or the amplifier gain.

As planned, w_n and c are to be continuously adjustable from 1 cps. to 6 cps. and .5 to 1, respectively. Over these ranges of network parameters, the maximum power consumption of the torque motor should not exceed one-third of a watt.

(Addendum A)

Changes in carrier frequency which will not prevent proper operation of any electromechanical components in the complete servo loop (such as motors and indicators) will not affect the operation of the electromechanical a-c lead networks described. Graphs 2, 3, 4, 5, and 6 show that w_{sm} occurs near w_n, and, as noted above, w_n and hence w_{sm} may be adjusted to cover the region of frequencies 1 to 6 cps. Also the maximum value of phaseshift ϕ_{sm} may be made larger for the network of Figure 4 than for a simple lead network having the same attenuation or noise-to-signal ratio.

An example of the application of the electromechanical lead network of Figure 7 is shown in Figure 10. Here a two-phase induction servo is being stabilized by the use of this electromechanical a-c lead network. As will be noted, the amplifier driving the servo motor is also used to drive the torque motor of the lead network. This is possible because of the very low power consumption of the torque motor.

In closing, if the feedback paths of Figures 4, 5, and 7 are connected regeneratively, instead of degeneratively, and the parallel paths of Figures 4A, 6, and 8 are connected so as to add, instead of subtract, these systems will become lag or integral networks instead of lead networks.

In general, it may be stated that the modifications in the circuit arrangements and the change in the type of components of the basic Class I lead network produces an a-c lead network which has all of the advantages of the Class I lead network with its disadvantages kept to a minimum. The general approach used in modifying the basic Class I a-c lead network has been to place all of the elements with negative phase shifts in a parallel or feedback branch where they will effectively produce a lead instead of a lag. Thus by this procedure, the inherent and unavoidable delays that occur in many networks, such as in demodulators and modulators, can be put to an advantage instead of being a handicap.

The characteristics of the Class II a-c lead networks have been described in this article in order to contrast more effectively the two classes, but no attempt will be made in this article to describe methods of improving the characteristics of the Class II a-c lead networks.

Acknowledgements

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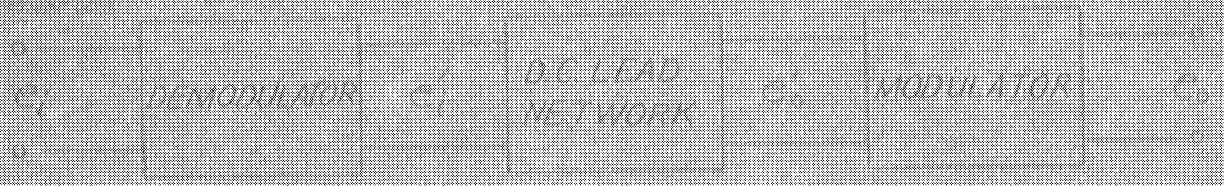
2. Differentiating Networks and Lead Networks by G. T. Coate, Department of Electrical Engineering, Massachusetts Institute of Technology

3. A New Servomechanisms Technique by D. McDonald Report UMM-24 Aeronautical Research Center, University of Michigan "Stabilizing Servomechanisms" Electronics November 1948

4. The Microsyn elements were developed by the Massachusetts Institute of Technology Instrumentation Laboratory and are more fully described in Electronic Instruments, McGraw-Hill Book Company, Inc., 1948, pp. 365-366. The combination of the induction pickoff and electric spring into one Microsyn element was developed by Mr. K. C. Mathews of the University of Michigan Aeronautical Research Center while he was a member of the Doelcam Corporation, Newton, Massachusetts.

Addendum A

If the electromechanical system of Figure 9 is used in the circuit of Figure 11, its natural frequency w_n can be increased and its design can be simplified. In Figure 11 the spring K_s of Figure 7 has been replaced by the amplifier K_4. As may be noted from Equations (11-2) and (11-3), when K_2 K_3 K_4 is substituted for K_s the ratio $\frac{e_o}{e_i}$, Equation (11-3), becomes the same as Equation (7-6). Hence the response of the circuit of Figure 11 is the same as the response of the circuit of Figure 7 and Equation (7-12) and Graphs 2, 3, 4, 5, and 6 apply for both circuits.

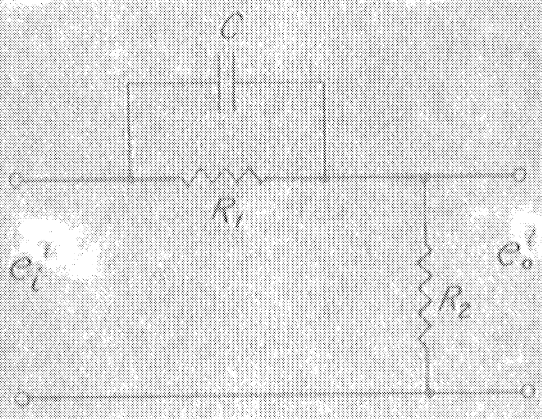


(1-1) if $e_i = e \sin w_m \sin w_c t$

(1-2) then $e_i' = e \sin w_m$

(1-3) and $e_o = e_o' \sin w_c t$

Fig. 1



$r = \dfrac{R_1 + R_2}{R_2}$

$T = R_1 C$

$\phi = \angle \dfrac{e_o'}{e_i'}$

(1A-1) if $e_i' = e \sin w_s t$

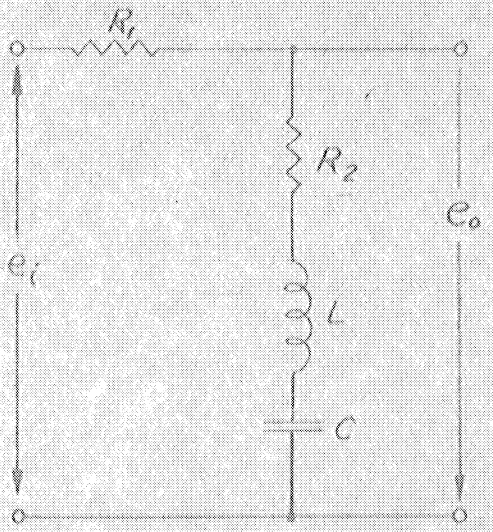
(1A-2) $$\frac{e_o'}{e_i'} = \frac{1}{r} \; \frac{1 + j\, w_s T}{1 + j\, w_s \frac{T}{r}}$$

(1A-3) where maximum phase shift $\phi_{sm} = \sin^{-1} \left[\dfrac{r - 1}{r + 1} \right]$

(1A-4) and occurs at $w_{sm} = \dfrac{\sqrt{r}}{T}$

(1A-5) $$\frac{\left.\frac{e_o'}{e_i'}\right]_{w_s = \infty}}{\left.\frac{e_o'}{e_i'}\right]_{w_s = 0}} = r = \text{attenuation}$$

Fig. 1A



$$r = \frac{R_1 + R_2}{R_2}$$

$$T = 2\,\frac{L}{R_2}$$

$$w_c = \sqrt{\frac{1}{LC}}$$

$$\phi = \angle \frac{e_o'}{e_i'}$$

(2-1) if $e_i = e \sin w_s t \sin w_c t$

(2-2) then $\dfrac{e_o'}{e_i'} = \dfrac{1}{r}\,\dfrac{1 + j\,w_s\,T}{1 + j\,w_s\,\frac{T}{r}}$ for $w_s << w_c$

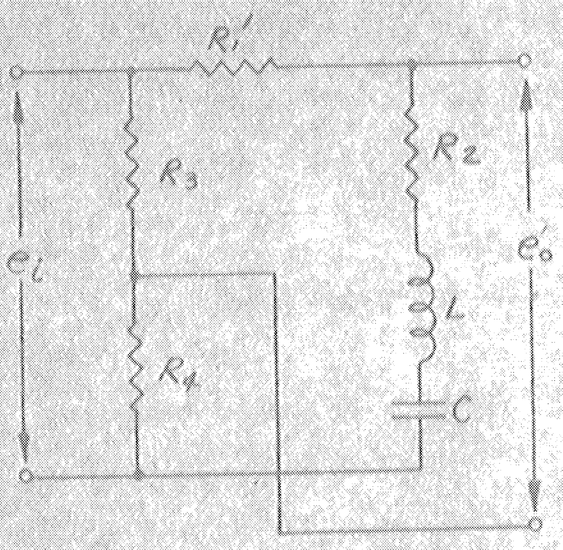
(2-3) the maximum phase lead (of the signal) $\phi_{sm} = \sin^{-1}\left[\dfrac{r - 1}{r + 1}\right]$

(2-4) and occurs at $w_{sm} = \dfrac{\sqrt{r}}{T}$

(2-5) $$\frac{\left.\frac{e_o'}{e_i'}\right]_{w_s = \infty}}{\left.\frac{e_o'}{e_i'}\right]_{w_s = 0}} = r$$

in general a primed variable denotes the envelope of the variable

FIG. 2



$r' = \dfrac{R_1' + R_2}{R_2}$ $\qquad T = 2\dfrac{L}{R_2}$ $\qquad w_c = \sqrt{\dfrac{1}{LC}}$

$b = \dfrac{R_3 + R_4}{R_4}$ $\qquad c = \dfrac{b - 1}{b - r'}$ $\qquad \phi' = \angle \dfrac{e_o'}{e_i}$

(2A-1) $\dfrac{e_o'}{e_i'} = \dfrac{b - r'}{br'} \; \dfrac{1 + j\, w_s\, c\, T}{1 + j\, w_s \frac{T}{r'}}$ $\qquad w_s << w_c$

(2A-2) where $\phi'_{sm} = \sin^{-1}\left[\dfrac{cr' - 1}{cr' + 1}\right]$

in general a primed variable denotes the envelope of the variable

(2A-3) $w'_{sm} = \sqrt{\dfrac{r'}{c}}\,\dfrac{1}{T}$

(2A-4) $\dfrac{\left.\dfrac{e_o'}{e_i'}\right]_{w_s = \infty}}{\left.\dfrac{e_o'}{e_i'}\right]_{w_s = o.}} = cr'$

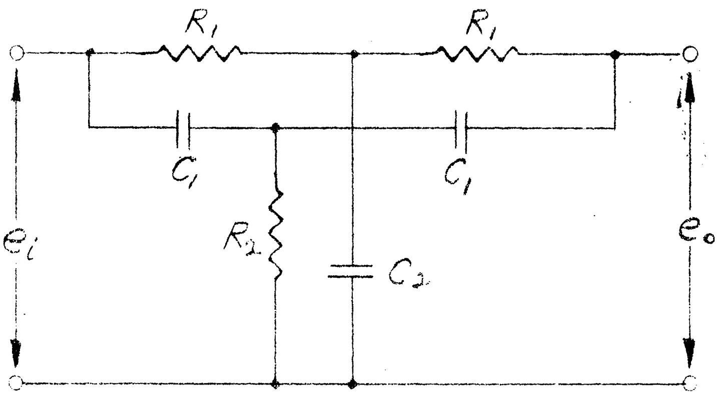
(2A-5) if $cr' = r$,

(2A-6) then $\phi'_{sm} = \phi_{sm}$

(2A-7) $w'_{sm} = \dfrac{\sqrt{r'}}{T}\;\dfrac{1}{c} = \dfrac{w_{sm}}{c}$

where r, ϕ_{sm}, and w_{sm} refer to Figure 2

FIG. 2A



(3-1) For balance: $R_1C_1 = \frac{1}{w_c}$

(3-2) then if $e_i = e \sin w_s t \sin w_c t$

(3-3) $$\frac{e_o'}{e_i'} = \frac{1}{r} \; \frac{1 + j\, w_s r T'}{1 + j\, w_s\, T'}$$ $w_s << w_c$

(3-4) $$\phi = \angle \frac{e_o'}{e_i'}$$

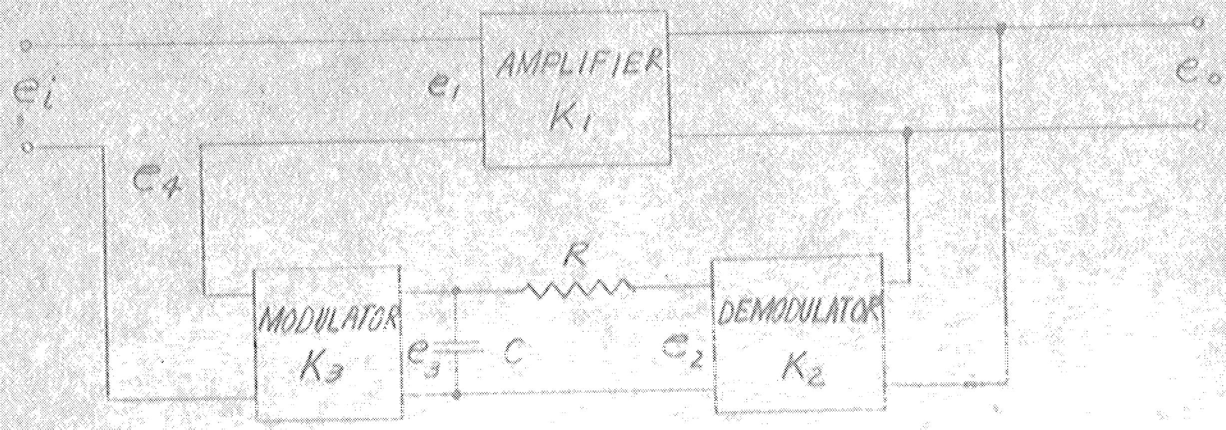
(3-5) $$\phi_{sm} = \sin^{-1}\left[\frac{r - 1}{r + 1}\right]$$

in general a primed variable denotes the envelope of the variable

(3-6) $$w_{sm} = \frac{1}{\sqrt{r}\,T'}$$

(3-7) $$\frac{\left.\frac{e_o'}{e_i'}\right]_{w_s = \infty}}{\left.\frac{e_o'}{e_i'}\right]_{w_s = 0}} = r$$

FIG. 3



(4-1) $e_i = e \sin w_s t \sin w_c t$

where in general a primed variable denotes the envelope of the variable

(4-2) $e_o = K_1 e_1$

(4-3) $e_1 = e_i - e_4$

(4-4) $e_4 = K_3 e_3'$

(4-5) $e_3' = \dfrac{e_2'}{1 + jw_s RC}$

(4-6) $e_2' = K_2 e_o$

(4-7) $\dfrac{e_o'}{e_i'} = \dfrac{K_1}{1 + K_1K_2K_3} \dfrac{1 + jw_s RC}{1 + jw_s \dfrac{RC}{1 + K_1K_2K_3}}$

(4-8) let $r = 1 + K_1 K_2 K_3$

(4-9) $T = RC$

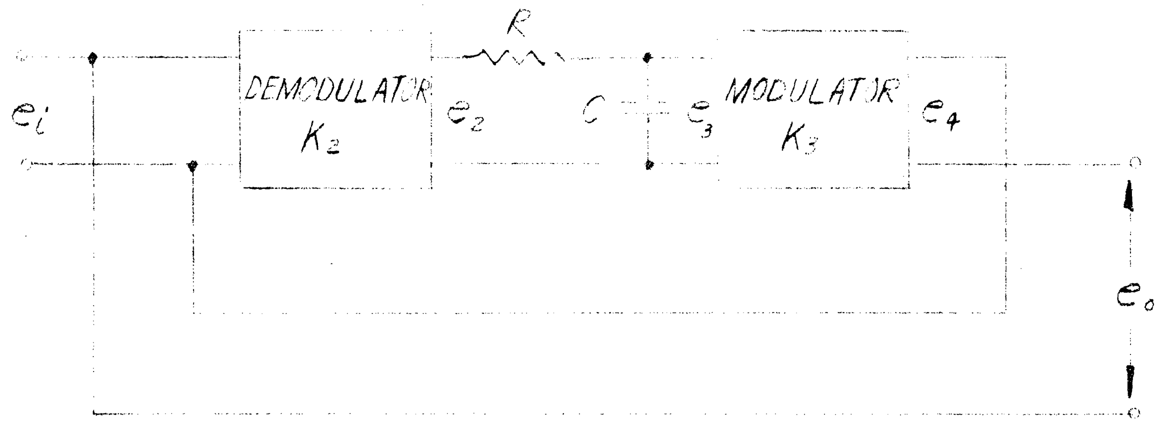
then

(4-10) $\dfrac{e_o'}{e_i'} = \dfrac{K_1}{r} \dfrac{1 + jw_s T}{1 + jw_s \dfrac{T}{r}}$

(4-11) $\phi_{sm} = \sin^{-1} \left[\dfrac{r - 1}{r + 1}\right]$

(4-12) $w_{sm} = \dfrac{\sqrt{r}}{T}$

FIG. 4



(4A-1) $e_i = e \sin w_s t \sin w_c t$

where in general a primed variable denotes the envelope of the variable

(4A-2) $e_o = e_i - e_4$

(4A-3) $e_4 = K_3 \, e_3'$

(4A-4) $e_3' = \dfrac{e_2'}{1 + j w_s T}$ $\qquad T' = RC$

(4A-5) $e_2' = K_2 \, e_i$

(4A-6) $\dfrac{e_o'}{e_i'} = (1 - K_2 K_3) \dfrac{1 + j w_s \dfrac{T'}{1 - K_2 K_3}}{1 + j \, w \, T'}$

let

(4A-7) $r = \dfrac{1}{1 - K_2 K_3}$

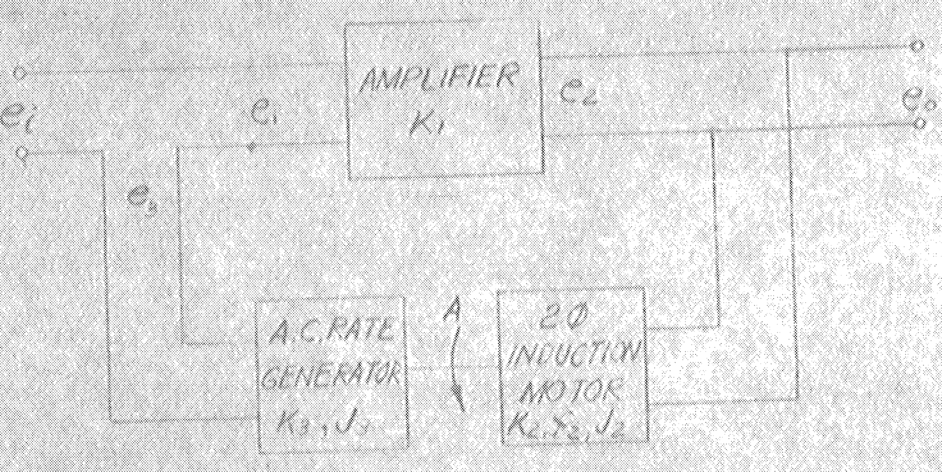
(4A-8) $T = r \, T' = r \, RC$

(4A-9) $\dfrac{e_o'}{e_i'} = \dfrac{1}{r} \; \dfrac{1 + j \, w_s \, T}{1 + j \, w \, \dfrac{T}{r}}$

(4A-10) $\phi_{sm} = \sin^{-1} \left[\dfrac{r - 1}{r + 1} \right]$

(4A-11) $w_{sm} = \dfrac{\sqrt{r}}{T}$

FIG. 4-A



(5-1) $e_o = e_2$

(5-2) $e_1 = e_i - e_3$

(5-3) $e_2 = K_1 e_1$

(5-4) $K_2 e_2' = \left[(J_2 + J_3)\, p^2 + f_2\, p \right] A$, where $p = \frac{d}{dt}$

(5-5) $e_3 = K_3\, p\, A$

(5-6) $J = J_2 + J_3$, $f = f_2$ $w_s << w_c$

(5-7) $\frac{e_o'}{e_i'} = \frac{K_1 f}{f + K_1 K_2 K_3} \times \frac{1 + p \frac{J}{f}}{1 + p \frac{J}{f + K_1 K_2 K_3}}$ in general a primed variable denotes the envelope of the variable

(5-8)
let $\frac{f + K_1 K_2 K_3}{f} = r$ $T = \frac{J}{f}$

(5-9) $\frac{e_o'}{e_i'} = \frac{K_1}{r} \frac{1 + p T}{1 + p \frac{T}{r}}$

(5-10)
if $e_i' = e \sin w_s t . \sin w_c t$

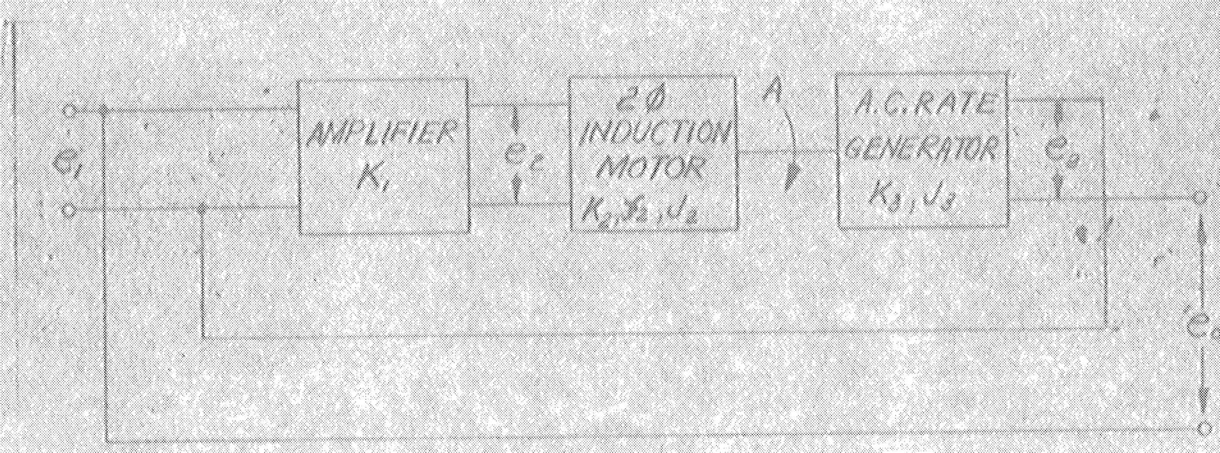
(5-11) $\frac{e_o'}{e_i'} = \frac{K_1}{r} \frac{1 + j w_s T}{1 + j w_s \frac{T}{r}}$

(5-12) $\phi_{sm} = \sin^{-1} \left[\frac{r - 1}{r + 1} \right]$

(5-13) $w_{sm} = \frac{\sqrt{r}}{T}$

(5-14) $\frac{\left.\frac{e_o'}{e_i'}\right]_{w_s = \infty}}{\left.\frac{e_o'}{e_i'}\right]_{w_s = 0}} = r$

Fig. 5



(6-1) $e_2 = K_1 e_1$

(6-2) $K_2 e_2' = \left[(J_2 + J_3)\, p^2 + f_2\, p \right] A$ where $p = \frac{d}{dt}$

(6-3) $e_3 = K_3\, p\, A$

(6-4) $e_o = e_i - e_3$ $w_s \ll w_c$

(6-5) $J = J_2 + J_3$, $f = f_2$

(6-6) $\frac{e_o'}{e_i'} = \frac{f - K_3 K_2 K_1}{f} \times \frac{1 + p\left(\frac{J}{f - K_1 K_2 K_3}\right)}{1 + p\frac{J}{f}}$ in general a primed variable denotes the envelope of the variable

(6-7) let $\frac{f}{f - K_1 K_2 K_3} = r$ $T' = \frac{J}{f}$

(6-8) $\frac{e_o'}{e_i'} = \frac{1}{r} \, \frac{1 + p\, r\, T'}{1 + p\, T'}$

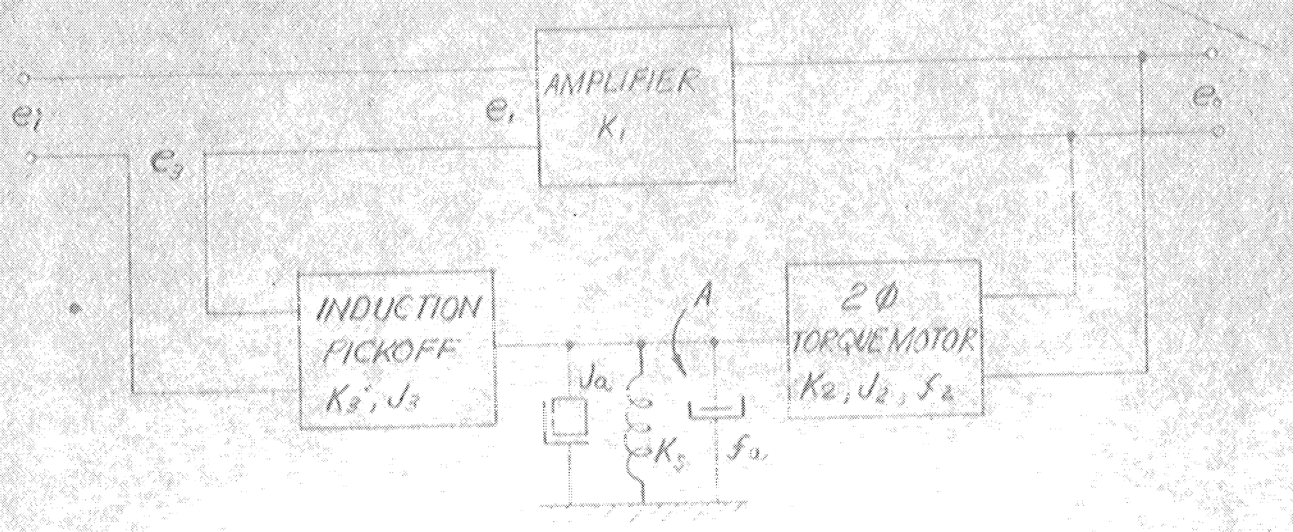
(6-9) or $\frac{e_o'}{e_i'} = \frac{1}{r} \, \frac{1 + j\, w_s r\, T'}{1 + j\, w_s T'} = \frac{1}{r} \, \frac{1 + j\, w_s T}{1 + j\, w_s \frac{T}{r}}$

(6-10) where $rT' = T$

(6-11) $\phi_{sm} = \sin^{-1}\left[\frac{r-1}{r+1}\right]$

(6-12) $w_{sm} = \frac{\sqrt{r}}{T}$

Fig. 6



(7-1) $$e_o = K_1 e_1$$

(7-2) $$e_1 = e_i - e_3$$

(7-3) $$e_3 = K_3 A$$

(7-4) $$K_2 e_o' = \left[(J_2 + J_3 + J_a) p^2 + (f_2 + f_a) p + K_s \right] A \text{ , where } p = \frac{d}{dt}$$

(7-5) $$J = J_2 + J_3 + J_a \quad , \quad f = f_2 + f_a$$

(7-6) $$\frac{e_o'}{e_i'} = K_1 \frac{J p^2 + f p + K_s}{J p^2 + f p + (K_s + K_1 K_2 K_3)} \qquad w_s << w_c$$

(7-7) let $$e_o = e \sin w_s t \sin w_c t$$

in general a primed variable denotes the envelope of the variable

(7-8) $$w_n^2 = \frac{K_s}{J}$$

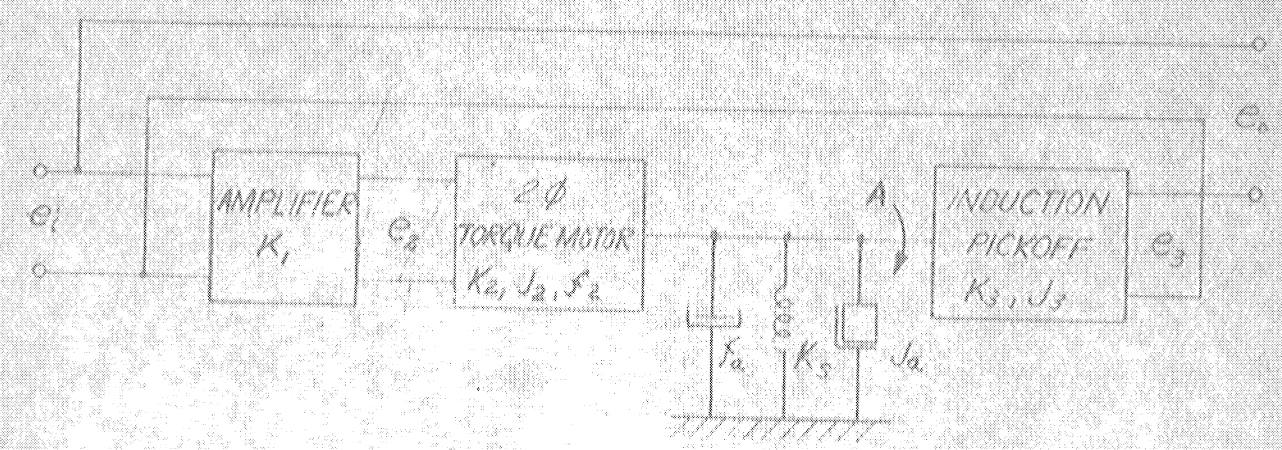
(7-9) $$c = \frac{f}{2\sqrt{K_s J}}$$

(7-10) $$h^2 = \frac{K_s + K_1 K_2 K_3}{K_s}$$

(7-11) $$u = \frac{w_s}{w_n}$$

(7-12) then $$\frac{e_o'}{e_i'} = \frac{K_1}{h^2} \frac{1 - u^2 + j\, 2\, c\, u}{1 - (\frac{u}{h})^2 + j\, 2(\frac{c}{h})(\frac{u}{h})} = K_L\, G_L(ju)$$

Fig. 7



(8-1) $e_o = e_i - e_3$

(8-2) $e_3 = K_3 A$

(8-3) $K_2 e_2' = \left[(J_2 + J_3 + J_a)\, p^2 + (f_2 + f_a)\, p + K_s\right] A$, where $p = \frac{d}{dt}$

(8-4) $e_2 = K_1 e_i$

(8-5) $J = (J_2 + J_3 + J_a)$, $f = f_2 + f_a$

(8-6) $$\frac{e_o'}{e_i'} = \frac{J\,p^2 + f\,p + (K_s - K_1 K_2 K_3)}{J\,p^2 + f\,p + K_s}$$ $w_s << w_c$

(8-7) let $e_o = e \sin w_s t \sin w_c t$

in general a primed variable denotes the envelope of the variable

(8-8) $w_n^{\,2} = \frac{K_s}{J}$

(8-9) $c = \frac{f}{2\sqrt{K_s J}}$

(8-10) $q^2 = \frac{K_s}{K_s - K_1 K_2 K_3}$

(8-11) $u = \frac{w_s}{w_n}$

then (8-12) $$\frac{e_o'}{e_i'} = \frac{1}{q^2}\;\frac{1 - (q\,u)^2 + j\,2\,(q\,c)\,(q\,u)}{1 - u^2 + j\,2\,c\,u}$$

Fig. 8

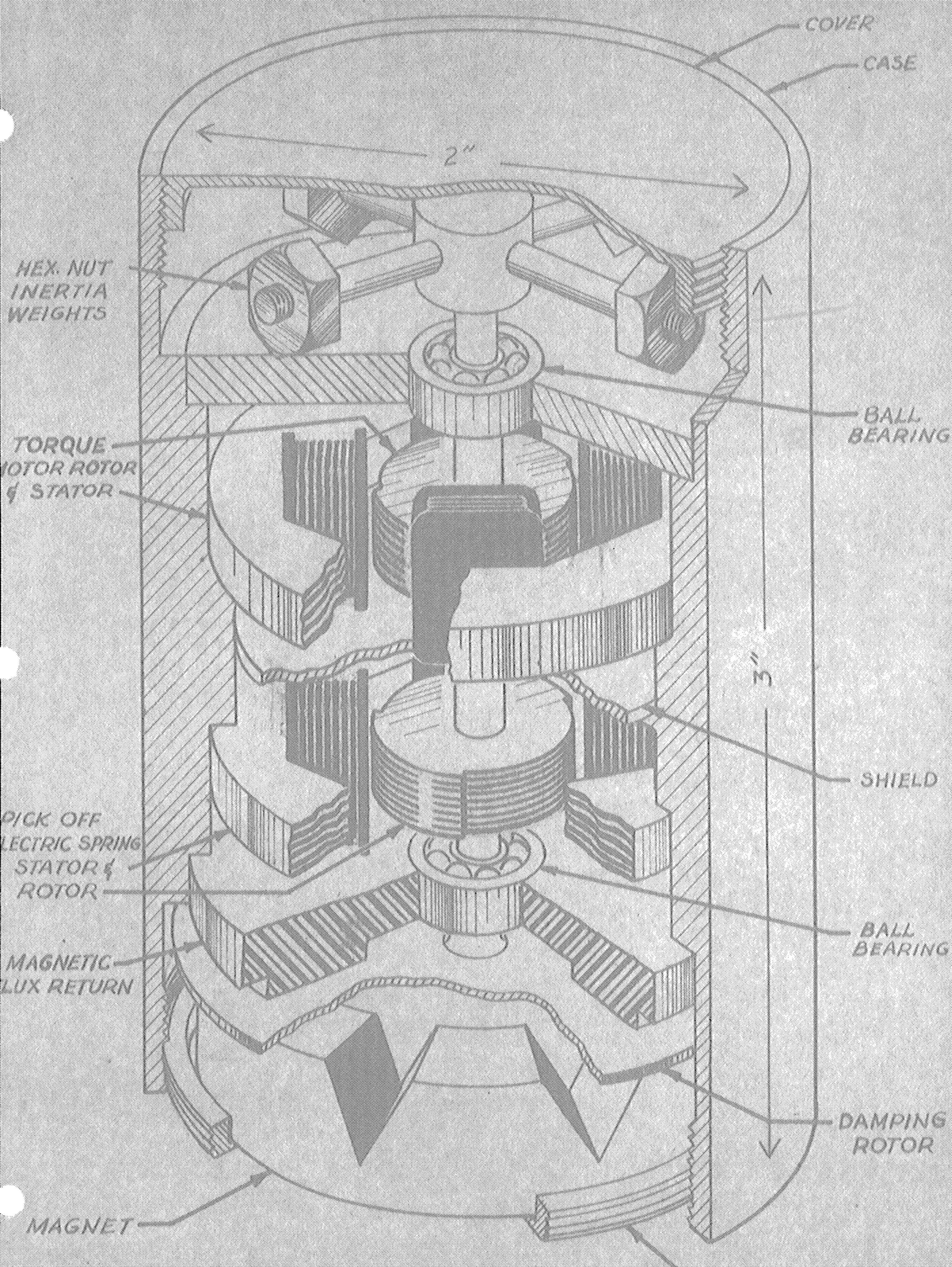

COVER
CASE
2"
HEX. NUT
INERTIA
WEIGHTS
BALL
BEARING
TORQUE
MOTOR ROTOR
& STATOR
3"
SHIELD
PICK OFF
LECTRIC SPRING
STATOR &
ROTOR
BALL
BEARING
MAGNETIC
LUX RETURN
DAMPING
ROTOR
MAGNET
MAGNET HOLDER

FIG. 9

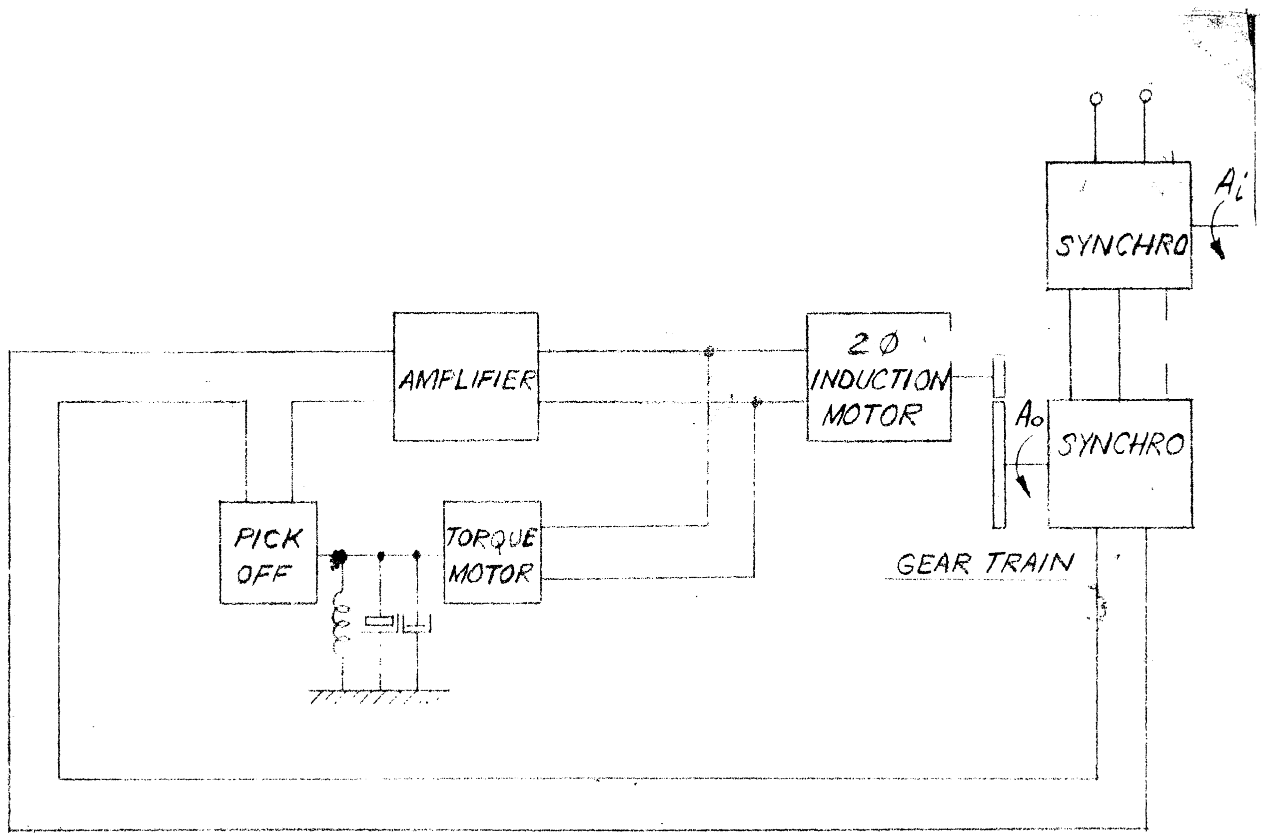
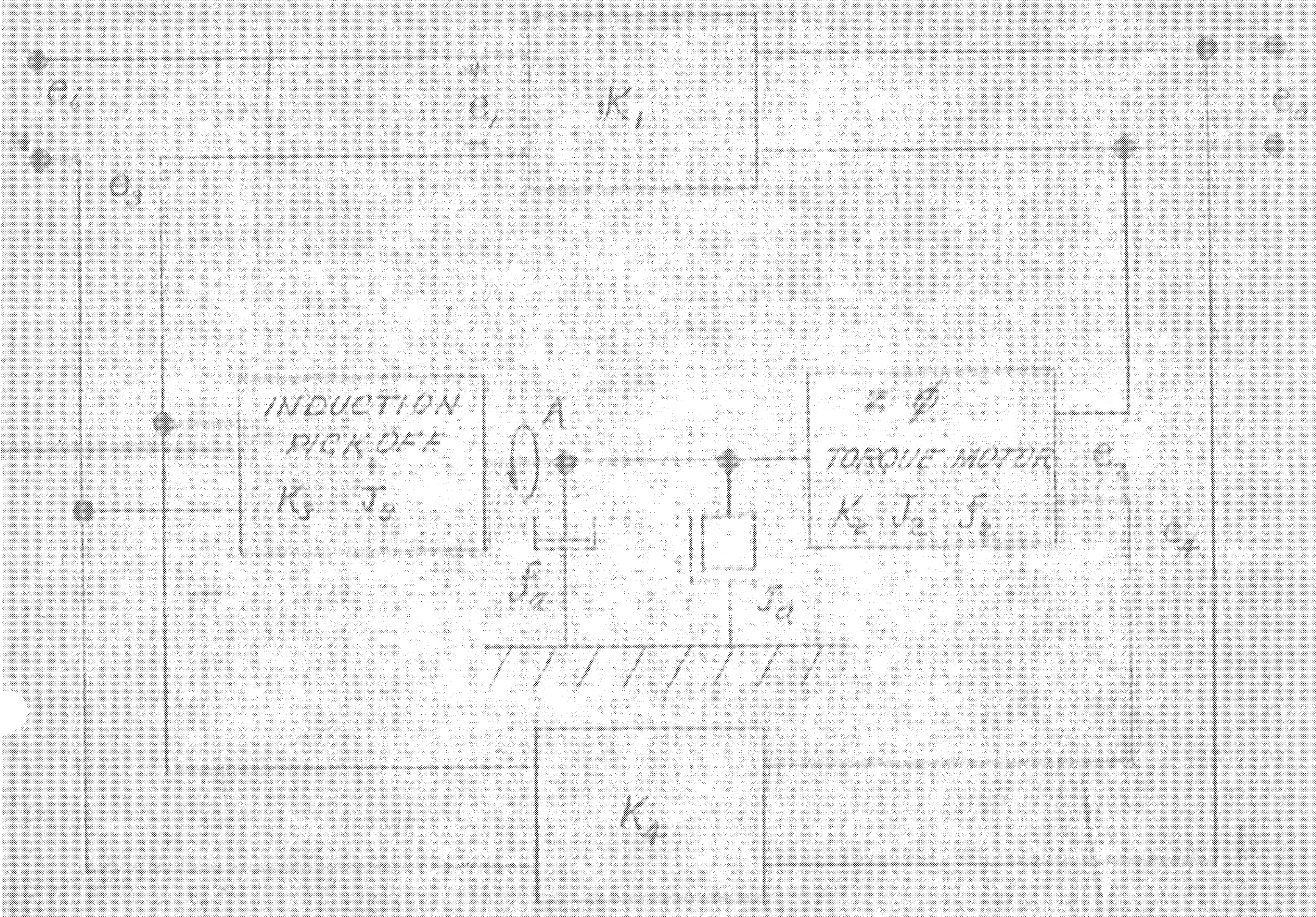

SYNCHRO
A_i
AMPLIFIER
2 Ø
INDUCTION
MOTOR
A_o
SYNCHRO
PICK
OFF
TORQUE
MOTOR
GEAR TRAIN

Fig. 10

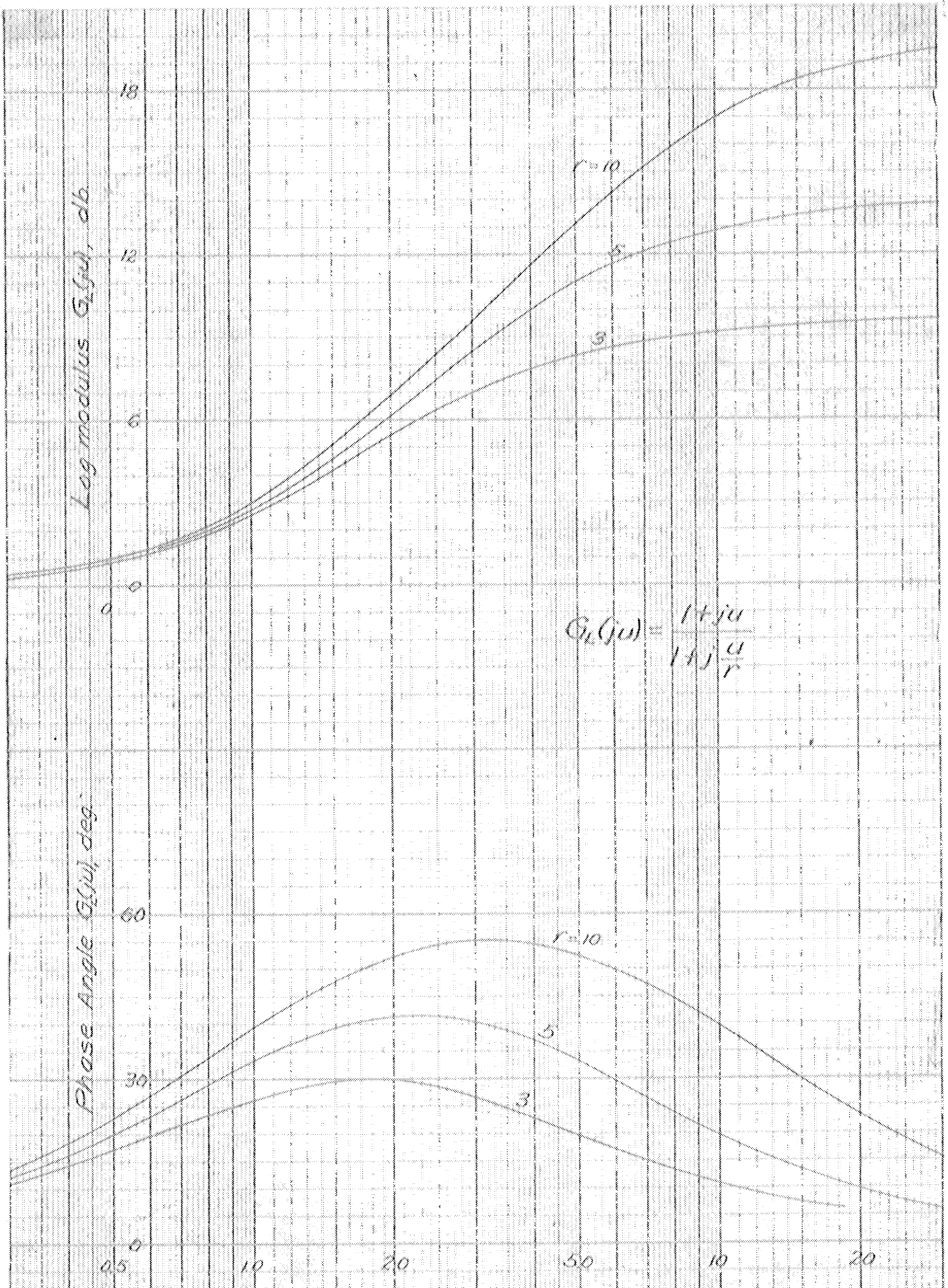


(11-1) $$\frac{e_o'}{e_i'} \cong K_1 \frac{J p^2 + f p + K_2 K_3 K_4}{J p^2 + f p + (K_2 K_3 K_4 + K_1 K_2 K_3)}$$

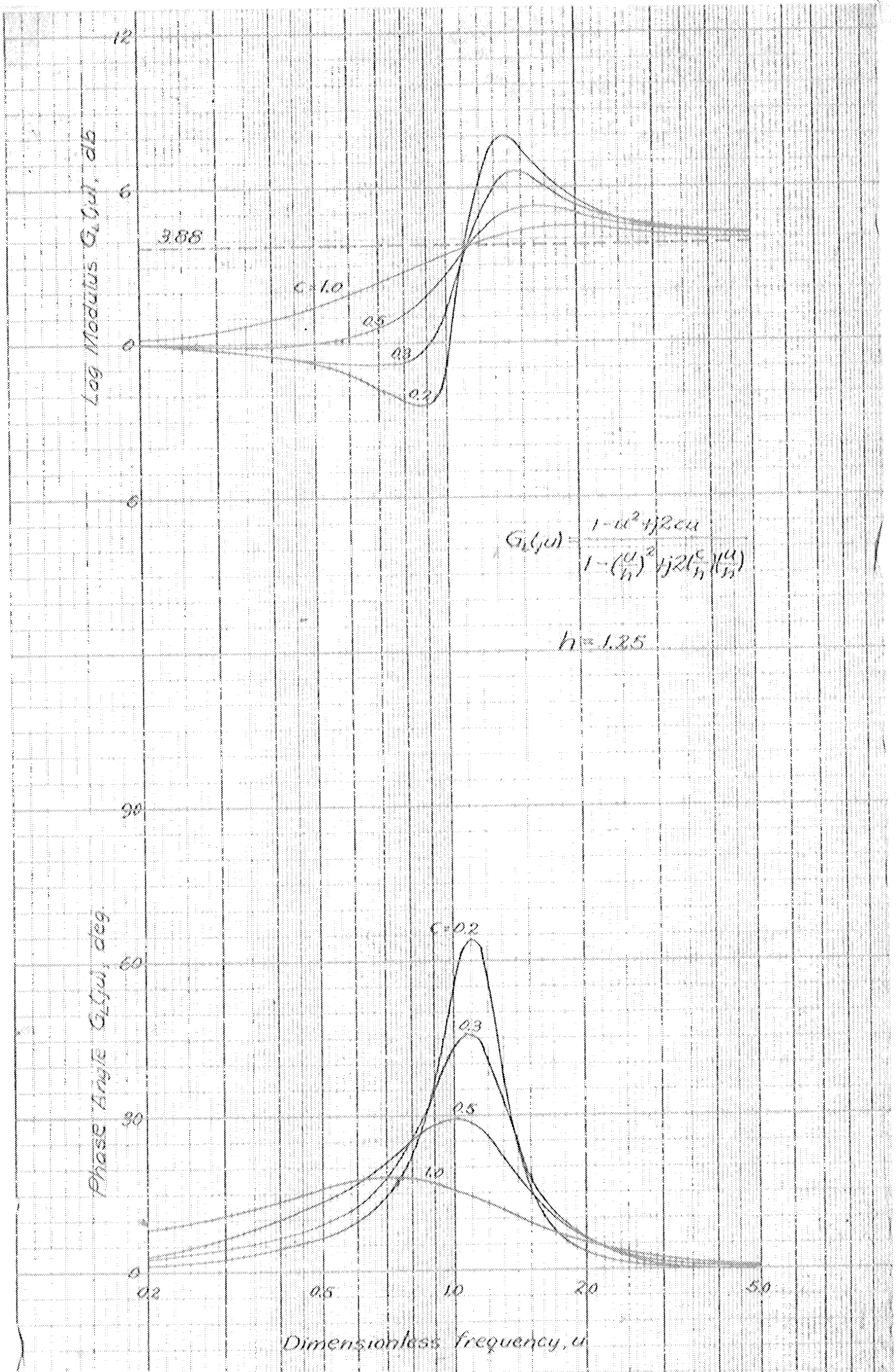
(11-2) Let $K_2 K_3 K_4 = K_s$

(11-3) Then $$\frac{e_o'}{e_i'} \cong K_1 \frac{J p^2 + f p + K_s}{J p^2 + f p + (K_1 K_2 K_3 + K_s)}$$

Fig. 11

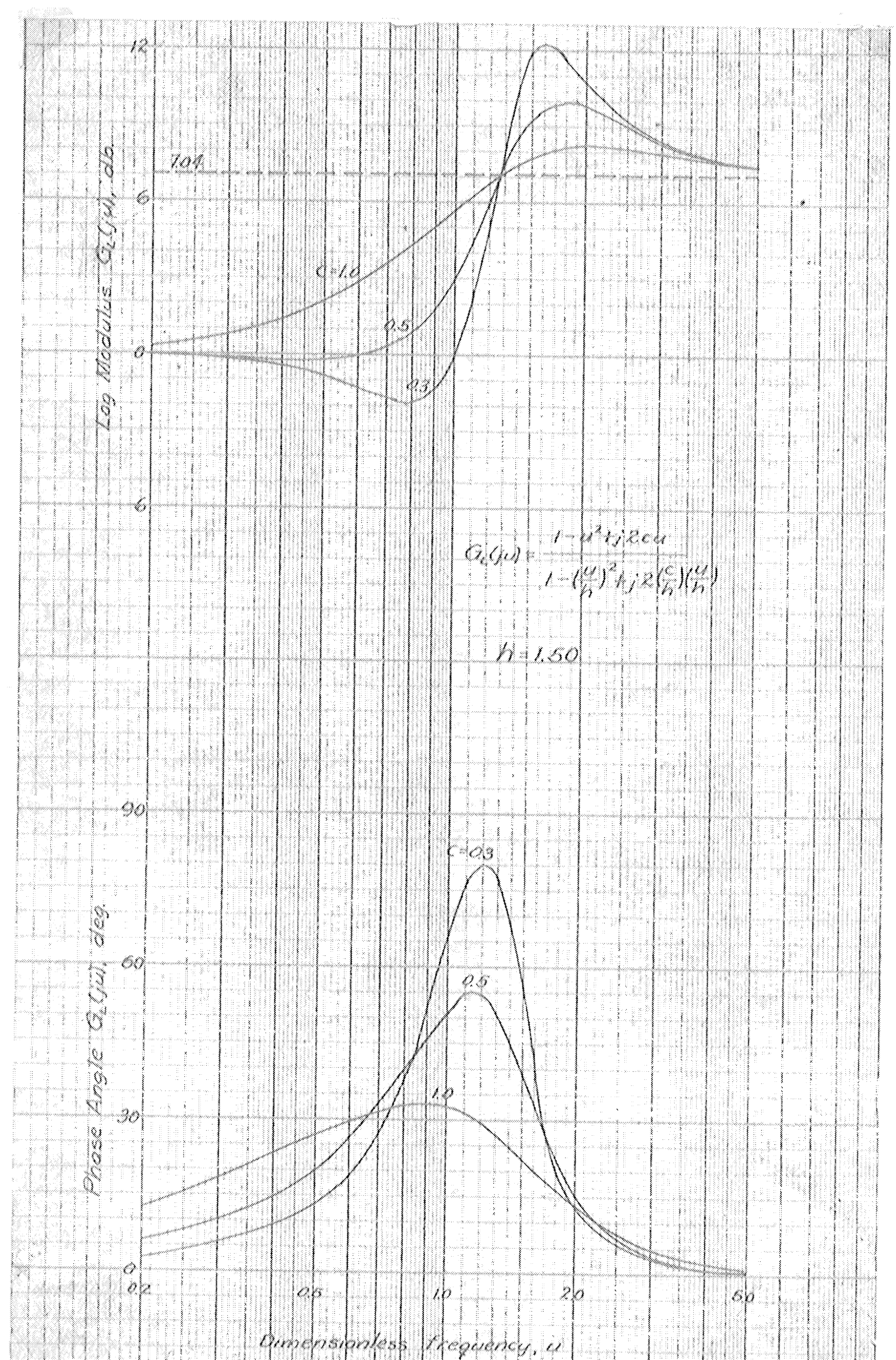

Log modulus $G_L(ju)$, db
18
12
6
0
r=10
5
3
$G_L(ju) = \frac{1+ju}{1+j\frac{u}{r}}$
Phase Angle $G_L(ju)$, deg.
60
30
0
r=10
5
3
0.5
1.0
2.0
5.0
10
20
Dimensionless frequency u

Graph 1



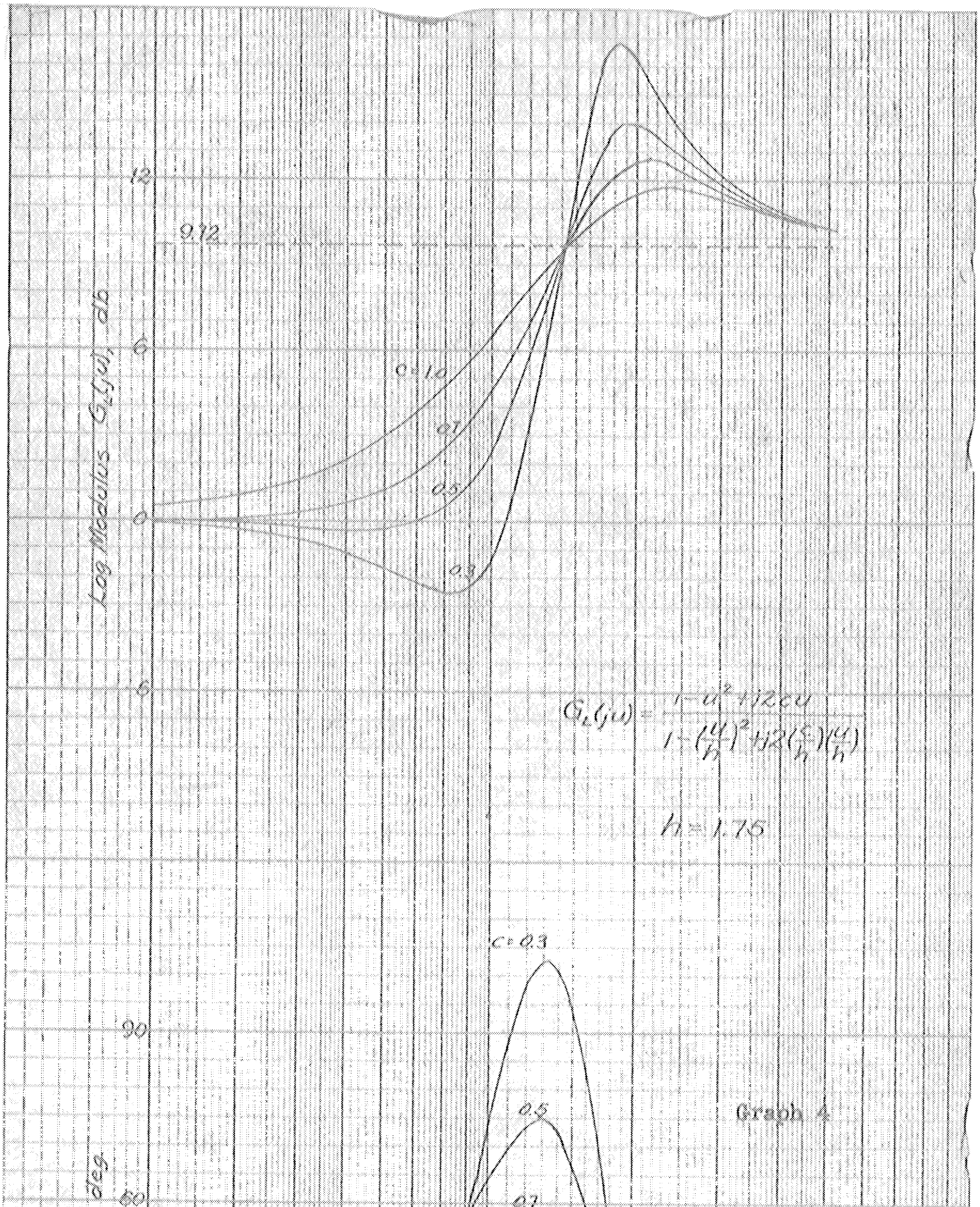
12
6
3.88
0
6
c=1.0
0.5
0.3
0.2
Log Modulus G1(ju), db
G1(ju) = (1 - u² + j2cu) / (1 - (u/h)² + j2(c/h)(u/h))
h = 1.25
90
60
30
0
c=0.2
0.3
0.5
1.0
Phase Angle G1(ju), deg
0.2
0.6
1.0
2.0
5.0
Dimensionless frequency, u

Graph 2.



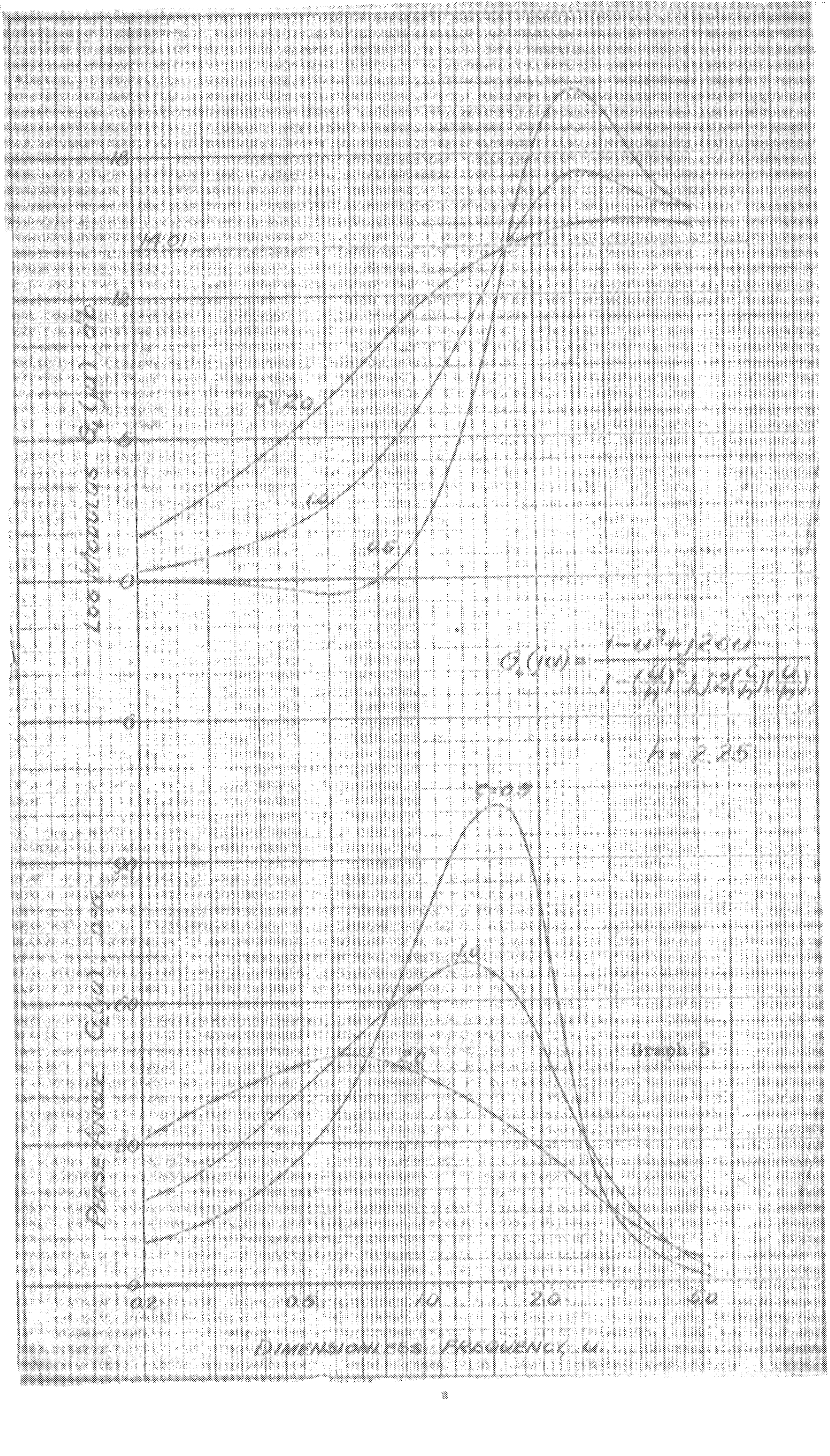
12
7.04
6
0
6
Log Modulus G_L(ju), db
c=1.0
0.5
0.3
G_L(ju) = (1 − u² + j2cu) / (1 − (u/h)² + j2(c/h)(u/h))
h=1.50
90
60
30
0
c=0.3
0.5
1.0
Phase Angle G_L(ju), deg.
0.2
0.5
1.0
2.0
6.0
Dimensionless frequency, u

Graph 3.

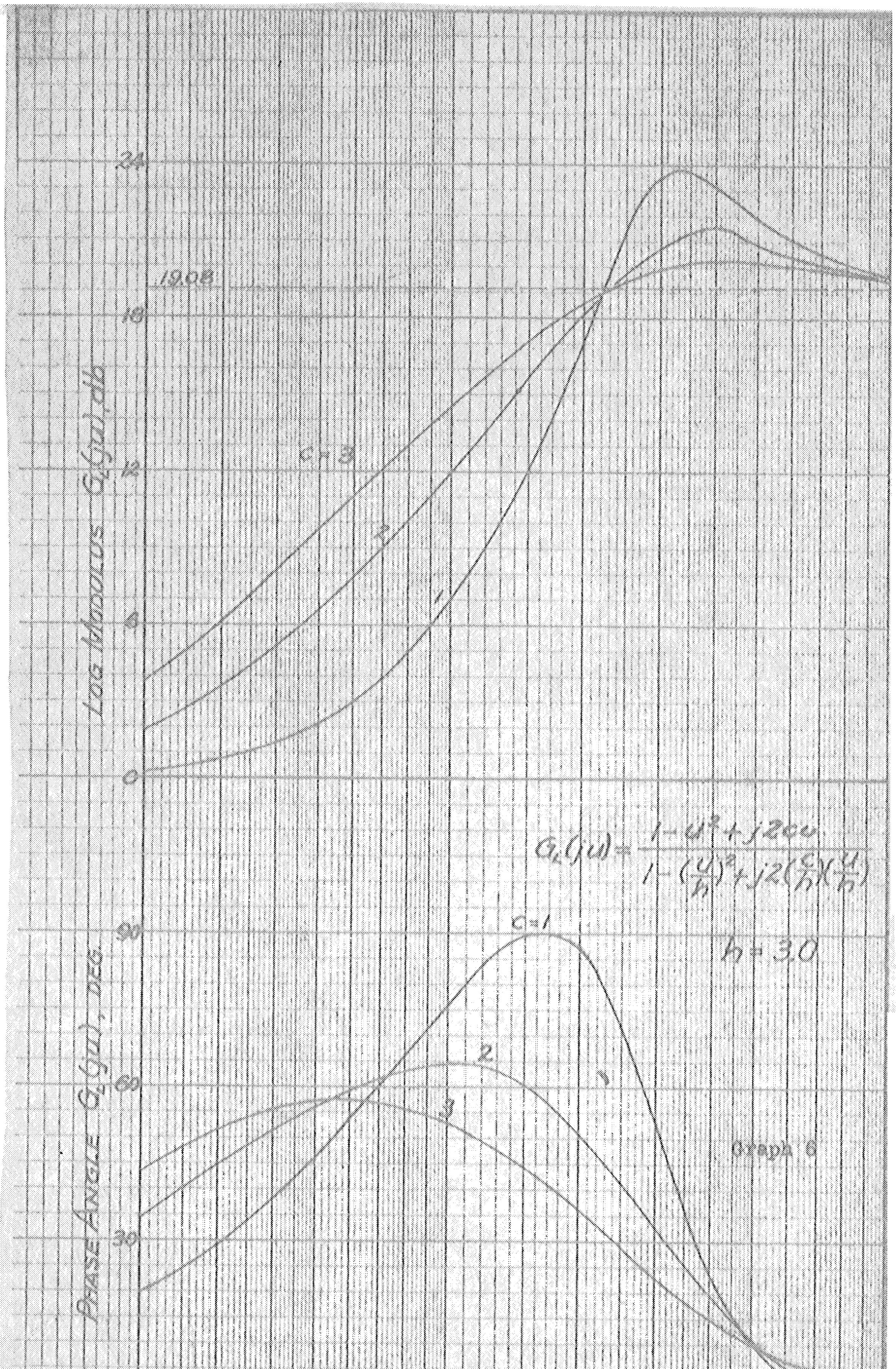


Log Modulus $G_L(j\omega)$, db
12
9.12
6
0
6
c=1.0
0.7
0.5
0.3
$G_L(j\omega) = \dfrac{1-u^2+j2cu}{1-\left(\frac{u}{h}\right)^2+j2\left(\frac{c}{h}\right)\left(\frac{u}{h}\right)}$
h = 1.75
c=0.3
90
0.5
60
0.7
1.0
30
0
Phase Angle $G_L(j\omega)$, deg
0.1
0.5
1.0
2.0
5.0
Dimensionless frequency, u

Graph 4



LOG MODULUS $G_4(j\omega)$, db
18
14.01
12
6
0
6
c=2.0
1.0
0.5
$G_4(j\omega) = \dfrac{1-\omega^2+j2c\omega}{1-(\frac{\omega}{h})^2+j2(\frac{c}{h})(\frac{\omega}{h})}$
h = 2.25
PHASE ANGLE $G_4(j\omega)$, DEG
90
60
30
0
c=0.5
1.0
2.0
Graph 5
0.2
0.5
1.0
2.0
5.0
DIMENSIONLESS FREQUENCY, ω



LOG MODULUS $G_2(ju)$, db
PHASE ANGLE $G_2(ju)$, DEG
DIMENSIONLESS FREQUENCY, U
19.08
c=3
2
1
c=1
2
1
3
$G_2(ju) = \frac{1-u^2+j2cu}{1-(\frac{u}{h})^2+j2(\frac{c}{h})(\frac{u}{h})}$
h = 3.0
Graph 6
0.2
0.5
1.0
2.0
5.0