Technical Report No. 135
4853-10-T

# IMPULSE RESPONSE SYNTHESIS FOR NETWORKS CHARACTERIZED BY FUNCTIONS OF A FIRST-ORDER, LINEAR OPERATOR

by

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for

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The University of Michigan
Department of Electrical Engineering
Ann Arbor

Contract No. DA-36-039 sc-89227
Signal Corps, Department of the Army
Department of the Army Project No. 3A99-06-001-01

October 1962

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## ABSTRACT

This report considers the use of integral transforms in the synthesis of time-varying networks for a given impulse response. The class of networks considered consists of all those containing a finite number of fixed resistances and time-varying reactances, with every reactive element varying in the same way. Such networks are characterized by finite linear combinations of a first-order, linear operator.

Using an integral transform developed by Wattenburg, network functions, $H(\lambda)$, which are rational functions of the transform variable, $\lambda$, can be obtained for networks in this class. Conversely, if a rational network function and the corresponding linear operator are given, a network realization can be obtained by well-known methods.

In synthesizing a network realizing a prescribed impulse response, $h(t, \tau)$, neither the rational network function nor the linear operator is known. This report presents a method of finding the operator corresponding to a given $h(t, \tau)$, and from this finding the network function $H(\lambda)$. Necessary conditions for the realizability of a given $h(t, \tau)$ by a network in the class considered here are presented.

## ACKNOWLEDGEMENT

The author wishes to express his appreciation to Prof. A. W. Naylor and Dr. G. R. Sell for many helpful discussions and comments.

## 1. INTRODUCTION

Recent years have seen a considerable growth of interest in time-varying networks, and, in particular, in the application of integral transform techniques to such networks. This latter interest is no doubt motivated to some extent by a desire to extend to time-varying networks the well-known and powerful methods of analysis and synthesis in the complex frequency domain.

Aseltine (Refs. 1, 2) has developed a transform applicable to certain second-order systems, while Ho and Davis (Ref. 3), Wattenburg (Ref. 4), and Kilmer and Johnson (Ref. 5) consider various types of first-order systems. Narendra (Ref. 6) and Zadeh (Ref. 7) have discussed transform techniques for higher-order systems; however, the usefulness of integral transform methods for high-order systems is limited by the fact that finding an appropriate transform may involve the solution of a differential equation of high order.

Ho and Davis (Ref. 3) have shown that the techniques of network synthesis in the complex frequency domain can be extended to time-varying networks without difficulty; however, they assume that the time variation of the network elements is known a priori. Kilmer and Johnson (Ref. 5), on the other hand, give an example of network design in which the topology is known and the time variation of the elements is determined.

An important problem to which the techniques mentioned above are not applicable is that of synthesizing a network for a prescribed impulse response, with neither the time variation of the elements nor the network topology known a priori. This report deals with an aspect of this synthesis problem. Specifically, we will consider the realization of a prescribed impulse response by a network characterized by a linear combination of a single first-order operator. It will be shown that such networks may be described mathematically in a particularly simple way. They are also of interest from a practical standpoint, since all the time-varying elements can be controlled by a single source.

## 2. INTEGRAL TRANSFORMS FOR A CLASS OF TIME-VARYING SYSTEMS

Let us consider the first-order linear operator $\mathcal{L}$, defined by

$$\mathcal{L}f(t) = \frac{d}{dt}\,[a(t)f(t)]\ , \tag{1}$$

where a(t) is piecewise continuous and has a piecewise-continuous derivative, and $0 < b \leq a(t) \leq B < \infty$ for $-\infty < t < \infty$. These conditions imply that

1) $\phi(t) = \int_0^t \frac{1}{a(x)}\,dx$ is strictly increasing for $-\infty < t < \infty$,

2) $\phi(0) = 0$,

3) $\phi(t) \to \pm\infty$ as $t \to \pm\infty$.

We will consider systems for which the excitation, e(t), and the response, r(t), are related by

$$\sum_{n=0}^{N} \alpha_n \mathcal{L}^n r(t) = \sum_{m=0}^{M} \beta_m \mathcal{L}^m e(t)\ , \tag{2}$$

where the $\alpha$'s and $\beta$'s are constants.

An integral transform which is useful in the analysis of systems described by (2) has been given by Wattenburg (Ref. 4). This transform is defined by[1]

$$T\left\{f(t)\right\} = F(\lambda) = \int_{-\infty}^{\infty} f(t)\epsilon^{-\lambda\phi(t)}dt\ , \tag{3}$$

where $\phi(t)$ is defined above. By an obvious extension of the existence condition for the two-sided Laplace transform, if

$$\int_{-\infty}^{\infty} \left|a(t)f(t)\right|\epsilon^{-\sigma\phi(t)}dt < \infty \tag{4}$$

[1]The transform given by Wattenburg is one-sided, but the extension is obvious.

for all real $\sigma$, $\sigma_1 \leq \sigma \leq \sigma_2$, then (3) converges absolutely and the transform exists for $\sigma_1 \leq \mathrm{Re}\lambda \leq \sigma_2$.

The transform of $\mathcal{L}f(t)$ is given by

$$T\left\{\mathcal{L}f(t)\right\} = \int_{-\infty}^{\infty} \frac{d}{dt}\left[a(t)f(t)\right] \epsilon^{-\lambda\phi(t)} dt \ . \tag{5}$$

Integrating by parts, we have

$$T\left\{\mathcal{L}f(t)\right\} = \left[a(t)f(t)\epsilon^{-\lambda\phi(t)}\right]_{-\infty}^{\infty} + \lambda \int_{-\infty}^{\infty} \phi'(t)a(t)f(t)\epsilon^{-\lambda\phi(t)}dt . \tag{6}$$

It is known that if a(t)f(t) is absolutely continuous and satisfies (4) and its derivative satisfies

$$\int_{-\infty}^{\infty} \left|(af)'\right| \epsilon^{-\sigma\phi(t)} dt < \infty \tag{4'}$$

for $\sigma_1 \leq \sigma \leq \sigma_2$, then $a(t)f(t)\epsilon^{-\lambda\phi(t)} \to 0$ as $t \to \pm\infty$, for $\sigma_1 \leq \mathrm{Re}\lambda \leq \sigma_2$. Thus, the first term on the right side of (6) vanishes at both limits. Then since $\phi'(t) = \frac{1}{a(t)}$, we have

$$T\left\{\mathcal{L}f(t)\right\} = \lambda \int_{-\infty}^{\infty} f(t)\epsilon^{-\lambda\phi(t)}dt = \lambda F(\lambda) . \tag{7}$$

By repeated application of this result, we obtain

$$T\left\{\mathcal{L}^n f(t)\right\} = \lambda^n F(\lambda) , \tag{8}$$

provided (af), (af)', ..., $(af)^{(n)}$ satisfy (4). Thus, operating with $\mathcal{L}$ and multiplying by $\lambda$ are equivalent operations in the t and $\lambda$ domains, respectively. Therefore, the transform of (2) is

$$\sum_{n=o}^{N} \alpha_n \lambda^n R(\lambda) = \sum_{m=o}^{M} \beta_m \lambda^m E(\lambda). \tag{9}$$

This is an algebraic relationship which can easily be solved for the "system function," which is defined by

$$H(\lambda) = \frac{R(\lambda)}{E(\lambda)} = \frac{\sum_{m=o}^{M} \beta_m \lambda^m}{\sum_{n=o}^{N} \alpha_n \lambda^n} . \tag{10}$$

The function $H(\lambda)$ characterizes the system in the complex-frequency domain in the same way that the system function H(s) characterizes an invariant system in the two-sided Laplace-transform domain.

Wattenburg (Ref. 4) gives an expression for the inverse transform. For the two-sided transform considered here, this expression becomes

$$\lim_{\epsilon \to 0} \frac{f(t+\epsilon) + f(t-\epsilon)}{2} = \frac{1}{2\pi j} \int_{Br} F(\lambda)\phi'(t)\epsilon^{\lambda \phi(t)} d\lambda , \qquad (11)$$

where Br is the Bromwich-Wagner contour in the strip $\sigma_1 < \mathrm{Re}\lambda < \sigma_2$. At points where f(t) is continuous, the left-hand side of (11) obviously reduces to f(t).

That (11) is indeed the inverse transform may be proved by arguments quite similar to those used in the Laplace-transform case. A rigorous proof is rather lengthy and therefore is omitted here.

It should be noted that (11) is simply the inverse Laplace transform of $F(\lambda)$, with t replaced by $\phi(t)$ and the result multiplied by $\phi'(t) = \frac{1}{a(t)}$ .

## 3. NETWORK RELATIONS IN THE $\lambda$-DOMAIN

Let us consider the class $C_a$ of all networks containing a finite number of fixed resistances and time-varying reactances, with every reactive element varying as a(t). That is

$$\ell_j(t) = L_j a(t) \quad , \quad \text{all } j \, ,$$

$$c_k(t) = C_k a(t) \quad , \quad \text{all } k \, , \tag{12}$$

where the $L_j$'s and $C_k$'s are positive constants. Every element in the network is, therefore, described by one of the three relations,

$$v_i(t) = R_i i_i(t) \, , \tag{13a}$$

$$v_j(t) = \frac{d}{dt}(\ell_j i_j) = L_j \frac{d}{dt}(a i_j) = L_j \mathcal{L} i_j(t) \, , \tag{13b}$$

$$i_k(t) = \frac{d}{dt}(c_k v_k) = C_k \frac{d}{dt}(a v_k) = C_k \mathcal{L} v_k(t) \, . \tag{13c}$$

This is a restricted class of time-varying networks, but, as pointed out earlier, it is one of interest both from a mathematical and a practical point of view.

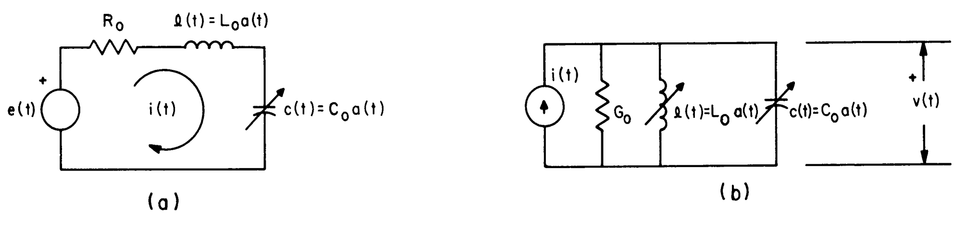


Fig. 1. Examples of networks in the class $C_a$.

Since Kirchhoff's laws are obviously valid for time-varying networks, they may be used to combine relations (13) to give an excitation-response relationship of the form (2), or, in the $\lambda$-domain, of the form (9). For example, for Fig. 1(a), we may write, using the inverse operator, $\mathcal{L}^{-1}$,

$$e(t) = v_L(t) + v_R(t) + v_c(t) = L_o \mathcal{L} i + R_o \mathcal{L}^o i + \frac{1}{C_o} \mathcal{L}^{-1} i \,, \tag{14}$$

or, operating on the equation term-by-term with $\mathcal{L}$,

$$\mathcal{L} e(t) = (L_o \mathcal{L}^2 + R_o \mathcal{L} + \frac{1}{C_o} \mathcal{L}^o) i(t) \,, \tag{15}$$

where $\mathcal{L}^o$ is the identity operator. For Fig. 1(b), we have, by a similar procedure

$$\mathcal{L} i(t) = (C_o \mathcal{L}^2 + G_o \mathcal{L} + \frac{1}{L_o} \mathcal{L}^o) v(t) \,. \tag{16}$$

Both (15) and (16) are obviously of the form (2).

If an excitation $\delta(t-\tau)$ is applied to an initially quiescent network in $C_a$, the response is, by definition, the impulse response $h(t,\tau)$. Thus

$$\sum_{n=o}^{N} \alpha_n \mathcal{L}^n h(t,\tau) = \sum_{m=o}^{M} \beta_m \mathcal{L}^m \delta(t-\tau) \,. \tag{17}$$

In the transform domain, since

$$T[\delta(t-\tau)] = \int_{-\infty}^{\infty} \delta(t-\tau) \epsilon^{-\lambda \phi(t)} dt = \epsilon^{-\lambda \phi(\tau)} \,, \tag{18}$$

we have

$$\sum_{n=o}^{N} \alpha_n \lambda^n T[h(t,\tau)] = \sum_{m=o}^{M} \beta_m \lambda^m \epsilon^{-\lambda \phi(\tau)} \,. \tag{19}$$

Therefore,

$$T[h(t,\tau)] = \epsilon^{-\lambda \phi(\tau)} \frac{\Sigma \, \beta_m \lambda^m}{\Sigma \, \alpha_n \lambda^n} = \epsilon^{-\lambda \phi(\tau)} H(\lambda) \,, \tag{20}$$

where the system function, $H(\lambda)$, is defined by (10). From (20) it follows that the time-domain and $\lambda$-domain descriptions of the network are related by

$$h(t,\tau) = \frac{1}{2\pi j} \int_{Br} \epsilon^{-\lambda \phi(\tau)} H(\lambda) \phi'(t) \epsilon^{\lambda \phi(t)} d\lambda \ , \tag{21}$$

$$H(\lambda) = \epsilon^{\lambda \phi(\tau)} \int_{\tau}^{\infty} h(t,\tau) \epsilon^{-\lambda \phi(t)} dt \ . \tag{22}$$

The lower limit of integration in (22) is $\tau$ rather than $-\infty$, since a physically realizable system cannot respond before it is excited; i. e., $h(t,\tau) = 0$ for $t < \tau$.

If $h(t,\tau)$ is the impulse response of a network in $C_a$ with $a(t) = \frac{1}{\phi'(t)}$, then the right side of (22) will not depend on $\tau$. However, if $h(t,\tau)$ and $\phi(t)$ are not so related, the right side of (22) will in general be a function of $\tau$ as well as of $\lambda$.

## 4. THE SYNTHESIS PROBLEM

Given an impulse response $h(t, \tau)$, zero for $t < \tau$, the problem of realizing it with a network in $C_a$ involves several steps:

1) Determine whether $h(t, \tau)$ is realizable as the impulse response of a network in $C_a$.

2) Find $a(t)$, or equivalently, $\phi(t)$, corresponding to this network.

3) Determine $H(\lambda)$ for the network.

4) Synthesize $H(\lambda)$ in the $\lambda$ domain by the well-known techniques of synthesis in the complex frequency domain.

5) Convert the network realized in (4) to the time domain by means of the pairs

$$Z(\lambda) = \lambda L_o \longleftrightarrow \ell(t) = L_o a(t) , \tag{23a}$$

$$Z(\lambda) = R_o \longleftrightarrow R = R_o , \tag{23b}$$

$$Z(\lambda) = \frac{1}{\lambda C_o} \longleftrightarrow c(t) = C_o a(t) . \tag{23c}$$

The preceding steps described an ideal realization procedure. In practice, we will carry out this procedure in the following way:

a) <u>Assume</u> that the given $h(t, \tau)$ is realizable. (Realizable here means realizable by a network in $C_a$.)

b) From $h(t, \tau)$, compute $\phi(t)$. This also gives $a(t) = \frac{1}{\phi(t)}$.

c) Substitute the given $h(t, \tau)$, and $\phi(t)$ as computed in b), in (22). If the result is a rational function of $\lambda$, then the original assumption [that $h(t, \tau)$ is realizable by a network in $C_a$] was correct, and this rational function is $H(\lambda)$. If, on the other hand, (22) gives a function of $\lambda$ and $\tau$, or a nonrational function of $\lambda$, the original assumption was incorrect, and $h(t, \tau)$ is not realizable.

d) Having obtained $\phi(t)$, $a(t)$, and a rational $H(\lambda)$, the remaining steps are straightforward. It should be noted that, if $H(\lambda)$ is a transfer ratio, it can in general be realized only to within a constant multiplier.

The entire process depends on finding a way of computing $\phi(t)$ from a realizable $h(t,\tau)$. The key to this is (21), which holds if $h(t,\tau)$ is realizable. Letting $\tau$ approach t from the left, (21) becomes

$$\lim_{\epsilon \to 0} h(t, t-\epsilon) = \frac{1}{2\pi j} \int_{Br} \phi'(t) H(\lambda) d\lambda = K\phi'(t) . \qquad (24)$$

If $K \neq 0$, it may be normalized to unity without affecting the final result.[2] Therefore,

$$\phi(t) = \int_0^t h(x,x) dx . \qquad (25)$$

If $h(t,t) = 0$, this implies that $K = 0$ in (24), and therefore (25) will not give $\phi(t)$. This case can be handled by differentiating (21) with respect to $\tau$, giving

$$\frac{\partial h(t,\tau)}{\partial \tau} = - \frac{\phi'(t)\phi'(\tau)}{2\pi j} \int_{Br} \lambda H(\lambda) \epsilon^{\lambda[\phi(t) - \phi(\tau)]} d\lambda \qquad (26)$$

Again letting $\tau$ approach t from the left, we obtain

$$\lim_{\epsilon \to 0} \left. \frac{\partial h(t,\tau)}{\partial \tau} \right|_{\tau = t-\epsilon} = - \frac{[\phi'(t)]^2}{2\pi j} \int_{Br} \lambda H(\lambda) d\lambda = K'[\phi'(t)]^2 . \qquad (27)$$

Since $\phi'(t) = \frac{1}{a(t)}$ is positive for all t, (27) uniquely defines $\phi(t)$, provided $K' \neq 0$. As before, K' may be normalized to unity.

If both K and K' are zero, an expression for $\phi(t)$ can be found by further differentiation of (21); however, such cases appear to be unlikely.

The preceding steps have been carried out under the assumption that $h(t,\tau)$ is realizable. This assumption is now checked by substituting $\phi(t)$, from (25) or (27), in (22).

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[2] Multiplying $\phi(t)$ by a constant K has the effect of multiplying $L_0$ and $C_0$ by K and also of dividing $a(t)$ by K. The values of $\ell(t)$ and $c(t)$ are thus unchanged.

If the result is a function of $\lambda$ only, the assumption is verified, $H(\lambda)$ has been determined, and the realization can proceed in a straightforward manner.

Although this procedure is simple, it is obviously desirable to be able to eliminate unrealizable impulse responses at the outset, thus avoiding unnecessary and useless computation. To this end, certain necessary (but not sufficient) conditions on $h(t,\tau)$ can be stated:

1) Both $h(t,t-)$ and $\left.\frac{\partial h(t,\tau)}{\partial\tau}\right|_{\tau=t-}$ must be bounded and piecewise continuous on $-\infty < t < \infty$, with no sign changes, and if either function is zero anywhere, it must be identically zero.

2) If $\int_{\tau}^{\infty} h(t,\tau)dt$ exists, it must be a constant.

The first condition follows at once from (24) and (37), since $h(t,t-)$ and $\left.\frac{\partial h(t,\tau)}{\partial\tau}\right|_{\tau=t-}$ are proportional, respectively, to $\frac{1}{a(t)}$ and $\frac{1}{a^2(t)}$. To demonstrate the second condition, we let $\lambda = 0$ in (22), obtaining

$$H(0) = \int_{\tau}^{\infty} h(t,\tau)\, dt\,. \tag{28}$$

If H(0) exists, it is, of course, a constant.

These two conditions will also be satisfied by a sum of impulse responses from networks in $C_a$ with different a(t)'s and are thus not sufficient. They are, nevertheless, useful as a preliminary check of a given impulse response.

## 5. EXAMPLES

A. We will consider first an example for which the procedure of Section 4 is not applicable. Consider the problem of realizing the impulse response

$$h(t,\tau) = \epsilon^{-(t^2-\tau^2)} u(t-\tau) . \tag{29}$$

That this is not realizable is seen by computing

$$\int_\tau^\infty \epsilon^{-(t^2-\tau^2)} dt = \frac{\sqrt{\pi}}{2} \epsilon^{\tau^2} \operatorname{erfc}(\tau) \neq \text{const.} \tag{30}$$

In order to see why this impulse response is not realizable, let us attempt to carry out the realization procedure. From (25),

$$\phi(t) = \int_0^t \epsilon^{-(x^2-x^2)} dx = t . \tag{31}$$

Substituting this in (22), we obtain

$$\epsilon^{\lambda\tau} \int_\tau^\infty \epsilon^{-(t^2-\tau^2)} \epsilon^{-\lambda t} dt = \frac{\sqrt{\pi}}{2} \epsilon^{(\tau+\frac{\lambda}{2})^2} \operatorname{erfc}\,(\tau + \frac{\lambda}{2}) . \tag{32}$$

Since this is a function of $\lambda$ and $\tau$, we have not obtained $H(\lambda)$, and the procedure has failed [as predicted by (30)] .

B. Find a one-port network whose voltage response to a current input $\delta(t-\tau)$ is

$$h(t,\tau) = (1 + \rho\cos t)\epsilon^{-(t-\tau)} \epsilon^{-\rho(\sin t - \sin\tau)} u(t-\tau), \; (\rho < 1) . \tag{33}$$

It can be verified easily that this function satisfies the necessary conditions given above. Since $h(t,t) \neq 0$, we have from (25)

$$\phi(t) = \int_0^t (1 + \rho\cos x)dx = t + \rho\sin t , \tag{34}$$

$$a(t) = \frac{1}{\phi'(t)} = \frac{1}{1+\rho\cos t} . \tag{35}$$

Substituting $\phi(t)$ in (22), we obtain

$$H(\lambda) = \epsilon^{\lambda(\tau+\rho\sin\tau)} \int_\tau^\infty (1 + \rho\cos t)\epsilon^{-(t-\tau)}\epsilon^{-\rho(\sin t-\sin\tau)}\epsilon^{-\lambda(t+\rho\sin t)}dt =$$

$$\epsilon^{(1+\lambda)(\tau+\rho\sin\tau)} \int_\tau^\infty \epsilon^{-(1+\lambda)(t+\rho\sin t)}(1+\rho\cos t)dt = \frac{1}{\lambda+1} . \tag{36}$$

Since we have obtained a rational function of $\lambda$, independent of $\tau$, we have shown that $h(t,\tau)$ is realizable.


Fig. 2. Network realizations in the $\lambda$ and t domains for Example B.

$H(\lambda)$ as obtained from (36) is, in this problem, a driving-point impedance which can be realized by inspection. This realization is shown in Fig. 2, along with the corresponding time-domain realization. The differential equation for this network is

$$i(t) = \frac{d}{dt}\left\{c(t)v(t)\right\} + v(t) . \tag{37}$$

It is easily verified that $h(t,\tau)$ as given by (33) is a solution of this equation for $i(t) = \delta(t-\tau)$.

C. Find a two-port network whose voltage response to a voltage input $\delta(t-\tau)$ is

$$h(t,\tau) = (1 + 3t^2)\sin(t+t^3 - \tau-\tau^3)u(t-\tau) . \tag{38}$$

[redacted]

## 6. CONCLUSION

By the use of an integral transform, the techniques of network synthesis in the complex frequency domain have been extended to a class of time-varying networks, characterized by a linear combination of a single first-order operator. A simple method for finding an appropriate transform for a given impulse response has been shown.

## REFERENCES


1. J. A. Aseltine, Transforms for Linear Time-Varying Systems, Technical Report No. 52-1, Department of Engineering, UCLA, February 1952.

2. J. A. Aseltine, "A Transform Method for Linear Time-Varying Systems," JAP 25, 6, June 1954, pp. 761-764.

3. E. C. Ho and H. Davis, Generalized Operational Calculus for Time-Varying Networks, Report No. 54-71, Department of Engineering, UCLA, July 1954.

4. W. H. Wattenburg, Transform Methods and Time-Varying Systems, Electronic Research Lab., University of California, Berkeley, Report No. 321, September 1960.

5. G. W. Johnson and F. G. Kilmer, "Integral Transforms for Algebraic Analysis and Design of a Class of Linear Variable and Adaptive Control Systems," IRE Trans. on Automatic Control, Vol. AC-7, March 1962, pp. 97-106.

6. K. S. Narendra, "Integral Transforms for a Class of Time-Varying Linear Systems," IRE Trans. on Automatic Control, Vol. AC-6, September 1961, pp. 311-318.

7. L. A. Zadeh, "Time-Varying Networks, I," Proc. IRE, 49-10, October 1961, pp. 1488-1502.

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