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UNIVERSITY OF MICHIGAN
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TECHNICAL REPORT NO. 3

A REDETERMINATION OF THE SOLAR CURVE OF GROWTH

BY

DONALD H. MENZEL

BARBARA BELL

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Since we had available a new theory of the solar spectrum[1] (D. H. Menzel, unpubl.) we considered it desirable to redetermine the fundamental curve of growth in accordance with this theory.

Theory

The basic formula for line profiles is

$$1 - r = \frac{R_c \beta X/p_c}{1 + \beta X/p_c} \tag{1}$$

where r is the ratio of intensity at any point in a line profile to the intensity in the continuum; R_c is a factor depending on the boundary temperature; p_c is the continuous absorption coefficient, referred arbitrarily to λ5063 as unity;

$$X = N \frac{\pi \varepsilon^2}{mc} f \frac{c}{\pi^{1/2} v \nu_o}, \tag{2}$$

the number of absorbing atoms; and

$$\beta = \left\{ e^{-\left[c(\nu - \nu_o)/v\nu_o\right]^2} + \left(\frac{2\pi\varepsilon^2}{hc}\right)^3 \frac{vq}{6\pi^{1/2}c} \frac{\nu_o{}^2}{(\nu - \nu_o)^2} \right\}; \tag{3}$$

q is a dimensionless parameter describing the departure of the damping from the classical value. The optical depth at any point is

$$\tau(\nu) = \beta X/p_c. \tag{4}$$

The theory indicates[2] (op. cit.) that in the construction of a curve of growth one should plot

$$\log \left(\frac{W}{\lambda}\right)\left(\frac{1}{R_c}\right) \text{ vs. } \log S' - \log p_c \qquad (5)$$

since X/p_c replaces the X_o of the earlier theory[3] (Ap. J. 87, 81, 1938). Thus where we had

$$\log X_o = L - \frac{5040}{T}\chi_{J'} + \log S' \qquad (6)$$

we now have

$$\log X/p_c = L - \frac{5040}{T}\chi_{J'} + \log S'/p_c \qquad (7)$$

where S' is the relative line strength, so that if empirical line strengths are used

$$\log S' = \log gf\lambda \ . \qquad (8)$$

The value of p_c, the continuous absorption, was taken from the table of theoretical values given for H^- by Chandrasekhar and Breen[4] (Ap. J. 104, 430, 1946) for wave lengths to the red of λ4800. To the violet of λ4800 we used the values determined empirically by Münch [5] (Ap. J. 102, 385, 1945).

$$p_c\lambda = \frac{\kappa_\lambda}{\kappa_{5063}} \qquad (9)$$

For the theoretical values we assumed T = 6300° but this is of little significance for, although the absolute value of the H^- absorption varies with temperature the ratios vary hardly at all. Figure 1 shows log p_c vs. λ.

Before proceeding to the construction of the actual curve of growth, we had also to determine R_c, the boundary temperature function. With this end in view, we took most of the lines in Allen's Tables of Equivalent Widths[6] (Mem. of Com. Solar Observ., vol. I No. 5, 1934 and vol. II No. 6, 1938), divided them into seven groups according to λ, 3924-4300, 4278-4500, 4500-4800, 4800-5200, 5200-5600, 5600-6100, 6100-6600, and plotted $(1 - r_c)$, where r_c is the central intensity, against log X_o as determined from Rubenstein's curve of growth(7) (Ap. J. 92, 114, 1940). Using the recent curve of growth by Goldberg and Pierce, we subsequently corrected our values of log X_o. The corrected values appear in the figures. The resulting family of curves (Fig. 2) enabled us to plot $(1 - r_c)$ vs. λ for various values of log X_o (Fig. 3).

Since r_c = intensity in the line center / intensity in the continuum, we find by use of the Planck law of black body radiation

$$r_c = \frac{\frac{2\pi h\nu^3}{c^2} \frac{1}{e^{h\nu/kT_t} - 1}}{\frac{2\pi h\nu^3}{c^2} \frac{1}{e^{h\nu/kT_p} - 1}} = \frac{e^{\frac{h\nu}{kT_p}} - 1}{e^{\frac{h\nu}{kT_t}} - 1} . \qquad (10)$$

T_p is the effective temperature of the photosphere, 6700° according to the recent work of Chalonge and Canavaggia(8) (Ann. d'Astrophysique 9, 143, 1946). T_t is the effective temperature of the atmospheric level producing the center of the line and ranging from 6500° to 4000°. More conveniently we can write (10) in the form

$$1 - r_c = \frac{e^{\frac{h\nu}{kT_t}} - e^{\frac{h\nu}{kT_p}}}{e^{h\nu/kT_t} - 1} . \qquad (11)$$

Comparison of the theoretical curves for $(1 - r_c)$ vs. λ with those determined empirically indicated 4000° as a reasonable choice for the boundary temperature. Thus we find

$$R_c = \frac{e^{\frac{h\nu}{4000k}} - e^{\frac{h\nu}{6700k}}}{e^{h\nu/4000k} - 1} . \qquad (12)$$

Figure 3 shows that the observed curves of $(1 - r_c)$ vs. λ are somewhat steeper than the theoretical, for reasons unknown. If we introduce X/p_c for X_o the correction is in the desired direction, but insufficient in amount.

There is, moreover, a conspicuous discrepancy between the set of curves from Allen's λ4277-6600 table and that for 3924-4300. We have compared the lines appearing in both tables and find that part of this discrepancy can be attributed to a difference in Allen's correction of the central intensity between the two tables. This explanation, however, is not entirely adequate. Note that if we were to connect the points around λ6000 to those around λ4100, without regard to the intervening regions, we should obtain slopes more closely approaching the theoretical values.

Since the equation (11) is perfectly general and not confined to the line center, we consider that the different points in a line, corresponding to different optical depths, are formed at regions of different effective temperature. Thus, the wings of a strong line are formed at deeper and hotter layers of the atmosphere than are the line centers, since the smaller absorption coefficient in the wings permits us to see further down into the atmosphere. Figure 4 illustrates this effect schematically. What we have essentially is a relation between effective temperature and optical depth at any point in the line profile, that is, between $\log \beta X/p_c$ and T_t.

Data

The equivalent widths we took from Allen's Tables[9] (op. cit.) with the modifications and additions described below. We used the values of W from line contours, whenever they were available, in preference to those derived from the central intensity.

In connection with our work on line profiles, to be described later in this paper, we determined the equivalent widths of a number of lines of Fe in the Utrecht Atlas in the region of λ5300 where most of the lines seem to be free of blends. For the strong lines of the multiplet $a^5F - z^5D^o$ we found equivalent widths that are systematically larger than the values given by Allen. For the most part, we determined our equivalent widths by the formula given by van de Hulst and Reesinck [10] (Ap. J. 106, 121, 1947).

$$A = chp, \qquad (13)$$

where A is the area or equivalent width, c is the central intensity, h is the half-width, and p is a parameter depending upon the ratio of half-width to tenth-width. In the few cases where we checked the results of formula (13) by counting squares in the Atlas, we found much closer agreement with the above formula than with Allen's values.

On the other hand, for lines of more moderate intensity, on the flat part of the curve of growth, we find the Utrecht Atlas gives equivalent widths in general agreement with Allen's values. The small differences that do occur are unsystematic in direction and generally such as to reduce the scatter in the curve of growth.

Because of the systematic effect found for the $a^5F - z^5D^o$ multiplet of Fe we sought to check the strong lines further to the violet. J. Houtgast[11] (The Variation in the Profiles of Strong Fraunhofer Lines along a Radius of the Solar Disc, Utrecht, 1942) has included in his study of line profiles six of the lines of the $a^3F - y^3F^o$ multiplet of Fe. His values of the equivalent widths are systematically larger than those of Allen. Our measures in the Atlas for strong violet lines support the equivalent widths of Houtgast. In Figure 5 we have plotted the Allen equivalent widths against those from our measures in the Atlas and those of Houtgast. From these we have deduced the correction curve shown in the figure. From this curve we have corrected Allen's equivalent widths of strong iron lines.

We have also added a number of strong lines in the ultraviolet. We determined their equivalent widths from the Atlas by means of formula (13), assuming p = 1.57 or a pure dispersion profile. To obtain the best possible estimate of the half-width, we either measured it directly or calculated it from van de Hulst's tables for Voigt profiles from measures at the points seeming most free from distortion by blends. For most lines, two or more points were measured as a check.

All lines that deviated conspicuously from the curve of growth we discarded if King had indicated the line strength (gf) as uncertain or if we were able to find evidence that a blend was causing W to be uncertain. Rejected lines are indicated in Table I by an asterisk.

To summarize the sources of equivalent width data: for TiI and VI we used Allen's values, to which we added a few faint lines of VI not listed by Allen; Allen's values for most of the FeI lines on the flat portion of the curve of growth; our own measures from the Atlas for the $a^5F - z^5D^o$ multiplet and for a number of strong Fe lines to the violet of λ3924; Houtgast's values for the lines of $a^3F - y^3F^o$; and for the other strong lines of FeI we used Allen's values corrected by means of Figure 5. Table I gives the line strengths (log gfλ) and log W/λ with the source of the latter indicated.

The line strengths we used were those determined empirically by King at Mt. Wilson for FeI and TiI[12] (Ap. J. 87, 24, 1938) and VI[13] (Ap. J. 105, 376, 1947). We used the precise excitation potential for each line, reducing the lines to a mean for the multiplet. For this small shift, a temperature of 5040° was considered sufficiently exact. Use of the precise excitation potential significantly reduces the scatter for FeI. The effect is less marked for TiI and VI due to the lesser spread in EP of the various levels within a multiplet.

Discussion and Interpretation

The multiplets of a given element were combined by horizontal shifts in the usual way. The required shift in the case of a Boltzmann distribution of the population of the energy levels is given by

$$\Delta = \frac{5040}{T} \chi_{J'},$$

where $\chi_{J'}$ is the excitation potential of the lower level in the transition. For each energy level we plotted the excitation potential against the horizontal shift, Δ, required to bring the level into coincidence with the lowest energy level of the element. Figure 6 shows this plot. The slope of the best fitting line through the points gives the excitation temperature, T.

We were particularly interested to see if it was possible to combine each of the three elements at the same excitation temperature since a single temperature is a priori what one expects to find. And in fact, within the limits of the accuracy of the observations, we find no convincing evidence of different excitation temperatures. We have adopted T = 4600° as the temperature that best fits all three elements.

The empirical curves of growth for VI, TiI, FeI at 4600° excitation temperature appear in Figures 7-9. Figure 10 exhibits the combined curve for all three elements, fitted to a theoretical curve for T_v = 16000° and q = 10. By T_v we mean the effective velocity temperature. In our current work there is no basis for deciding whether T_v represents random kinetic (thermal) velocity or turbulence. However, in the light of the recent work of Goldberg and Pierce, we tend to accept the former view. A q = 10 corresponds to a damping 50 times the classical in the vicinity of λ4500. We computed the theoretical curve of growth by numerical integration over theoretical profiles.

VI appears in Figure 7, fitted to a theoretical curve of $T_v = 16000°$, $q = 10$. We find no convincing evidence for decreasing turbulence with increasing excitation potential. This result conflicts with that suggested by R. King and K. O. Wright on the basis of their recent work on the curve of growth for VI[14] (Ap. J. 106, 224, 1947). For a^4D ($\chi_{J'} = 1.06$) we have too few sufficiently strong lines to justify any conclusions about the turbulence. As between a^6D ($\chi_{J'} = 0.28$) and a^4F ($\chi_{J'} = 0.00$) our work indicates if anything a lower turbulence for a^4F, contrary to the conclusions of King and Wright. However, we do not feel that we have enough strong lines in a^4F to justify any conclusions of this nature.

Although a considerably higher excitation temperature would improve the fit of a^4F and a^6D with respect to one another, the higher temperature would have an unfavorable effect on the fit of a^4D with respect to a^6D. For the curve as a whole, we believe that $T = 4600°$ is as good as a higher temperature and has, in addition, the merit of being in agreement with TiI and FeI. Because of the small difference in EP between a^4F and a^6D we do not believe that their relative shift should be given undue weight in the determination of the excitation temperature.

Figure 8 shows TiI. The evidence of the strong lines here is for a somewhat smaller damping factor, perhaps 10-20 times the classical value. The scatter is conspicuously less than in the case of VI.

The case of FeI, considered alone in Figure 9, is exceedingly ambiguous since the empirical curve can be represented about equally well by a theoretical curve for $T_v = 5000°$, $q = 2$; or for $T_v = 16000°$, $q = 10$; and doubtless for various other combinations of T_v and q. For this reason, it is unfortunate that we have as yet no empirical line strengths for faint lines of FeI.

There is a tendency for the empirical curve to have a slope steeper than the theoretical value of 22°.5. Probably the most reasonable explanation for this departure from the theoretical slope is a damping factor increasing with line strength, a possibility that other workers have previously mentioned. The quantum mechanical expression for the damping factor is

$$\Gamma = \frac{1}{t_1} + \frac{1}{t_2}$$

where t_1 and t_2 are the mean lifetimes of the lower and upper levels involved in the transition. Since the lifetime is inversely proportional to the transition probability or line strength, as Goldberg and Pierce have called to our attention, the observed deviation from the theoretical slope is at least qualitatively, what we might expect. Also we must not overlook the possibility that insufficient allowance for blends may give erroneously large values of log W/λ for some of the strong lines to the violet of λ3900.

The complete curve of growth, shown in Figure 10, is quite insensitive to both kinetic temperature and damping factor. For instance, a theoretical curve having T_v = 10000° and q = 10 would fit the observations very nearly as well as T_v = 16000°. The only difference would be in the linear or lower part of the curve, where, because of the difference in the zero point of the absolute scale, the 10000° curve would fall slightly above the 16000° curve. The direction of this difference is contrary to what one might expect. However, we must shift the absolute scale of the 10000° curve to the left with respect to the 16000° curve for each to give the best possible fit with the empirical curve.

Also, a damping factor 30 times the classical value would fit the observations about as well as the factor of 50 that we have used. The damping factor that best represents the observations depends strongly on how Fe is combined with Ti and V. Since there is so little overlap of Fe with the other two elements the relative position of iron is necessarily rather uncertain. A damping as low as 10 times classical would however be quite unsatisfactory for the combined curve of growth of the three elements. With so small a damping factor, the stronger iron lines would fall markedly above the theoretical curve, as other workers have previously noted. We might explain this deviation by postulating a marked increase in damping with increased transition probability. To represent the curve as a whole with a single damping constant, however, we require a damping of at least 30 times the classical value.

Resolution of the various uncertainties discussed above would be greatly advanced if empirical line strengths for weaker iron lines were available. There would then be no "guess work" involved in combining Fe with Ti and V.

Profiles

In hope of throwing more light on the problem of the proper kinetic (or turbulent) temperature for iron, we have made some preliminary studies of line profiles. For this purpose, we selected relatively unblended lines of FeI on the transition and upper portion of the curve of growth.

The observed profiles we took from the Utrecht Atlas. We corrected for instrumental broadening using the method devised by van de Hulst and Reesinck[15] (op. cit.). For the lines considered, the Voigt profiles seem to give a satisfactory representation of the observed profiles. We used the instrumental profile determined by van de Hulst[16] (BAN No. 367, 80, 1946), which is somewhat narrower than the profile indicated in the Atlas itself.

Our corrected central intensities are often but not always slightly greater than Allen's values for the same lines. We have previously indicated that our areas are in satisfactory agreement with Allen's values for lines on the flat part of the curve of growth, and that for $a^5F - z^5D^o$ our areas were systematically larger than Allen's.

We used formula (1),

$$1 - r = R_c \frac{\beta X/p_c}{1 + \beta X/p_c},$$

to compute the theoretical profiles.

We exhibit in Figures 11a and 11b two lines for which we have obtained the most satisfactory profiles. Both lines belong to the multiplet $a^5F - z^5D^o$ of FeI. The points in the wings, $\Delta\lambda \geq .12$, can be represented by various combinations of q, T_v, and X/p_c, the effective number of atoms. We considered three cases: $T_v = 16000°$, q = 10; $T_v = 10000°$, q = 10; and $T_v = 5000°$, q = 2. In each case the value of X/p_c required to fit the observed profiles is in good agreement with the value taken from the relevant curve of growth. The figures show all three combinations of q, T_v, and X/p_c for $\Delta\lambda < .12$. The results, though suggestive of a steep turbulence gradient, are not at all conclusive.

Our work with other lines suggests that $T_v = 16000°$ is too high a value for the kinetic temperature of the layer at which the central part of these lines is produced. We find, however, some indications that a higher kinetic or turbulent velocity exists in the deeper layers of the atmosphere where the major part of the wings of strong lines and the entirety of weaker lines are produced. We suggest that the deeper levels of the reversing layer may be considerably more turbulent than the higher and less dense layers.

Summary and Conclusion

Although many aspects of our investigation are necessarily inconclusive, we feel that considerable progress has been made. By an empirical procedure we have shown that the boundary temperature of the solar reversing layer is close to 4000° K.

We have found that the equivalent widths of strong FeI lines in the Utrecht Atlas are significantly greater than the values given by Allen. This conclusion is supported by the recent work of Houtgast. We have added a number of strong ultraviolet iron lines to the curve of growth. This addition gives more overlap in the portions of the curve covered by the different energy levels and hence should give a more reliable value for the excitation temperature of iron.

We have found that all three elements can be satisfactorily represented by a single excitation temperature, T = 4600°. And finally, we have found strong evidence for a damping factor of 30 to 50 times the classical value. This large damping is contrary to the widely accepted value of 10 times the classical value.

Still somewhat uncertain is the precise value of both the damping factor and the kinetic or turbulent temperature, T_v. These uncertainties remain as subjects requiring further research.

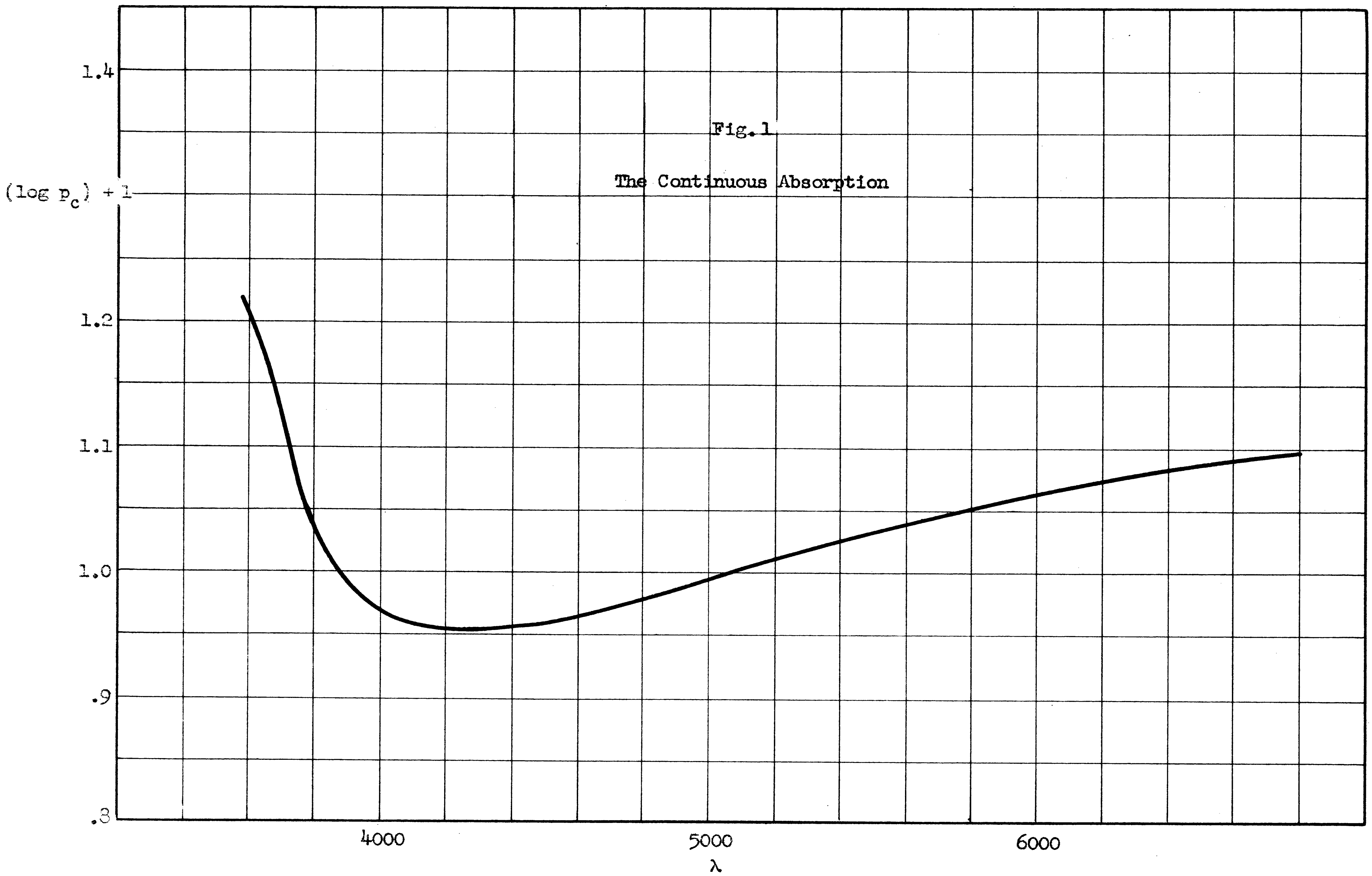


Fig. 1

The Continuous Absorption

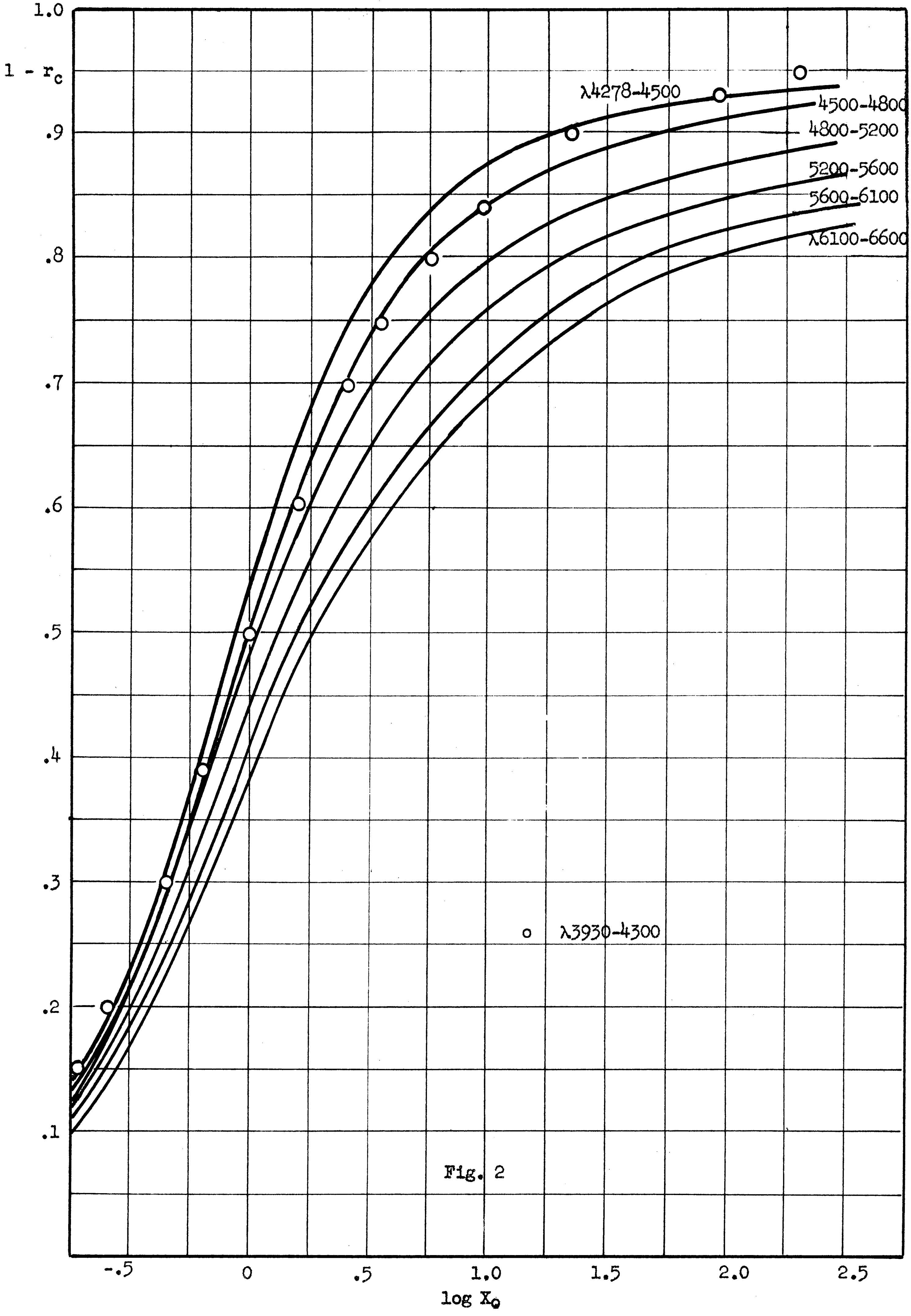
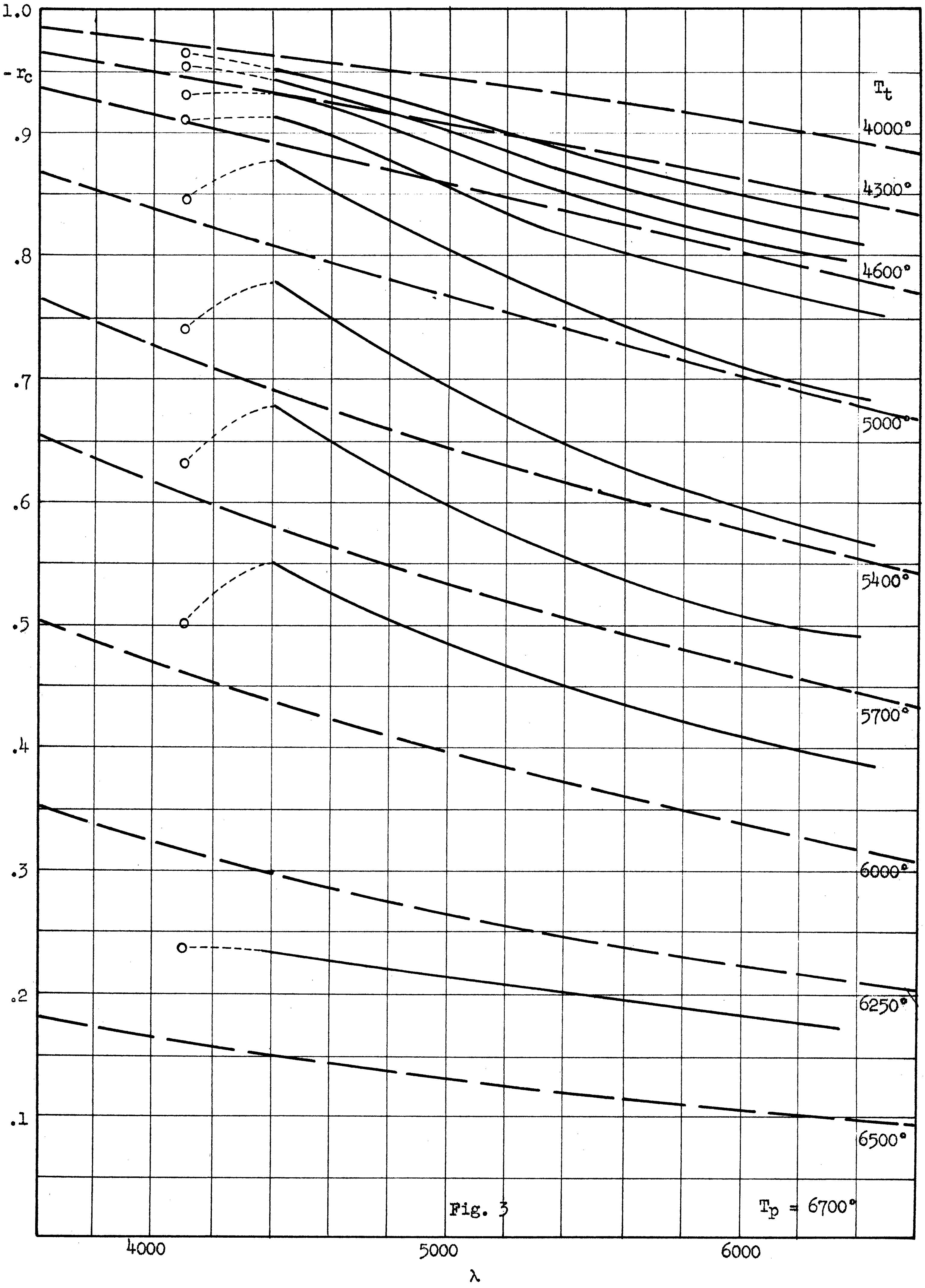
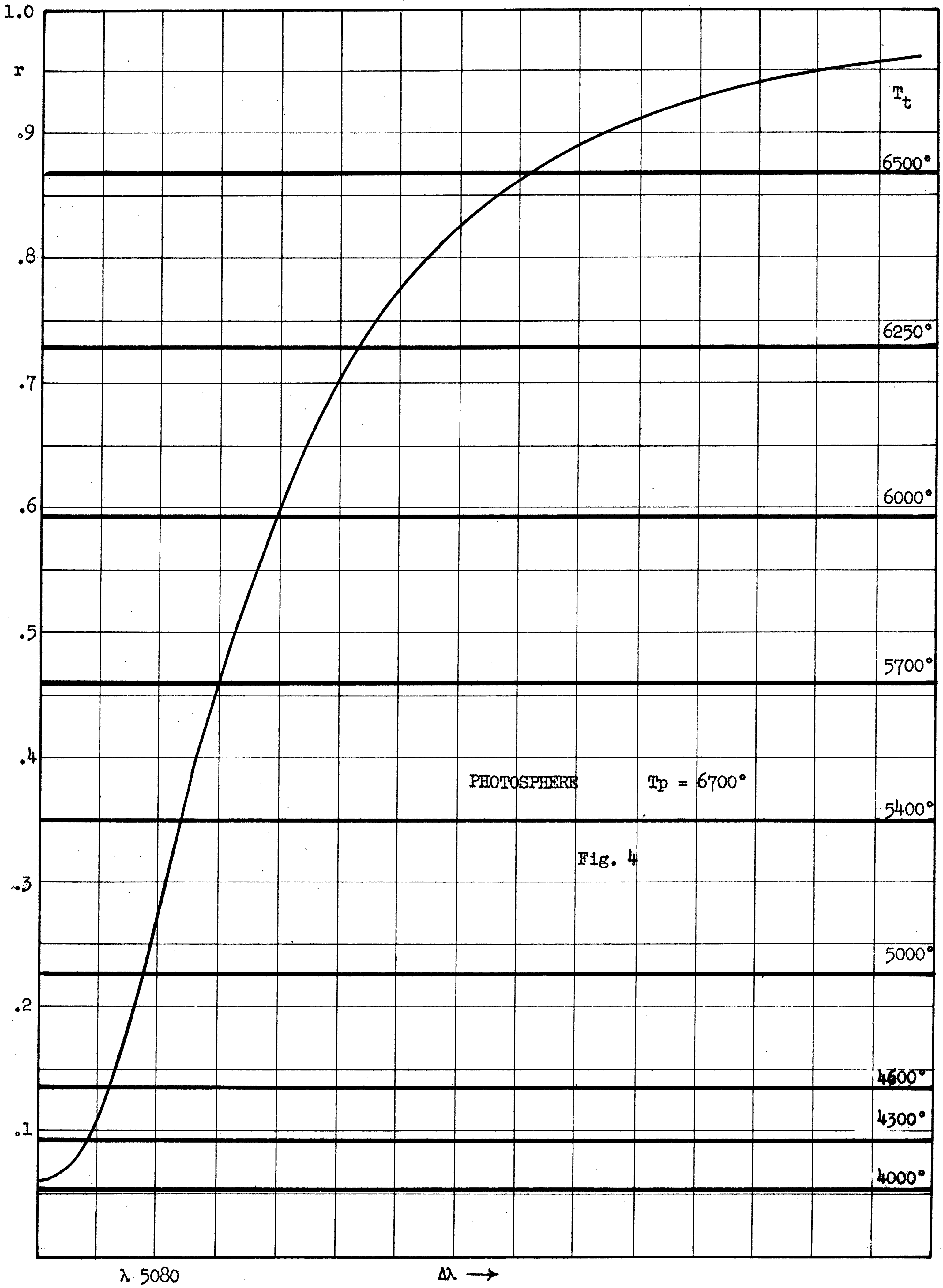

1.0
1 - r_c
.9
.8
.7
.6
.5
.4
.3
.2
.1
λ4278-4500
4500-4800
4800-5200
5200-5600
5600-6100
λ6100-6600
λ3930-4300
-.5
0
.5
1.0
1.5
2.0
2.5
log X_0

Fig. 2



1.0
- r_c
.9
.8
.7
.6
.5
.4
.3
.2
.1
T_t
4000°
4300°
4600°
5000°
5400°
5700°
6000°
6250°
6500°
T_p = 6700°
4000
5000
6000
λ

Fig. 3


1.0
r
.9
.8
.7
.6
.5
.4
.3
.2
.1
T_t
6500°
6250°
6000°
5700°
5400°
5000°
4600°
4300°
4000°
PHOTOSPHERE Tp = 6700°
λ 5080
Δλ →
Fig. 4

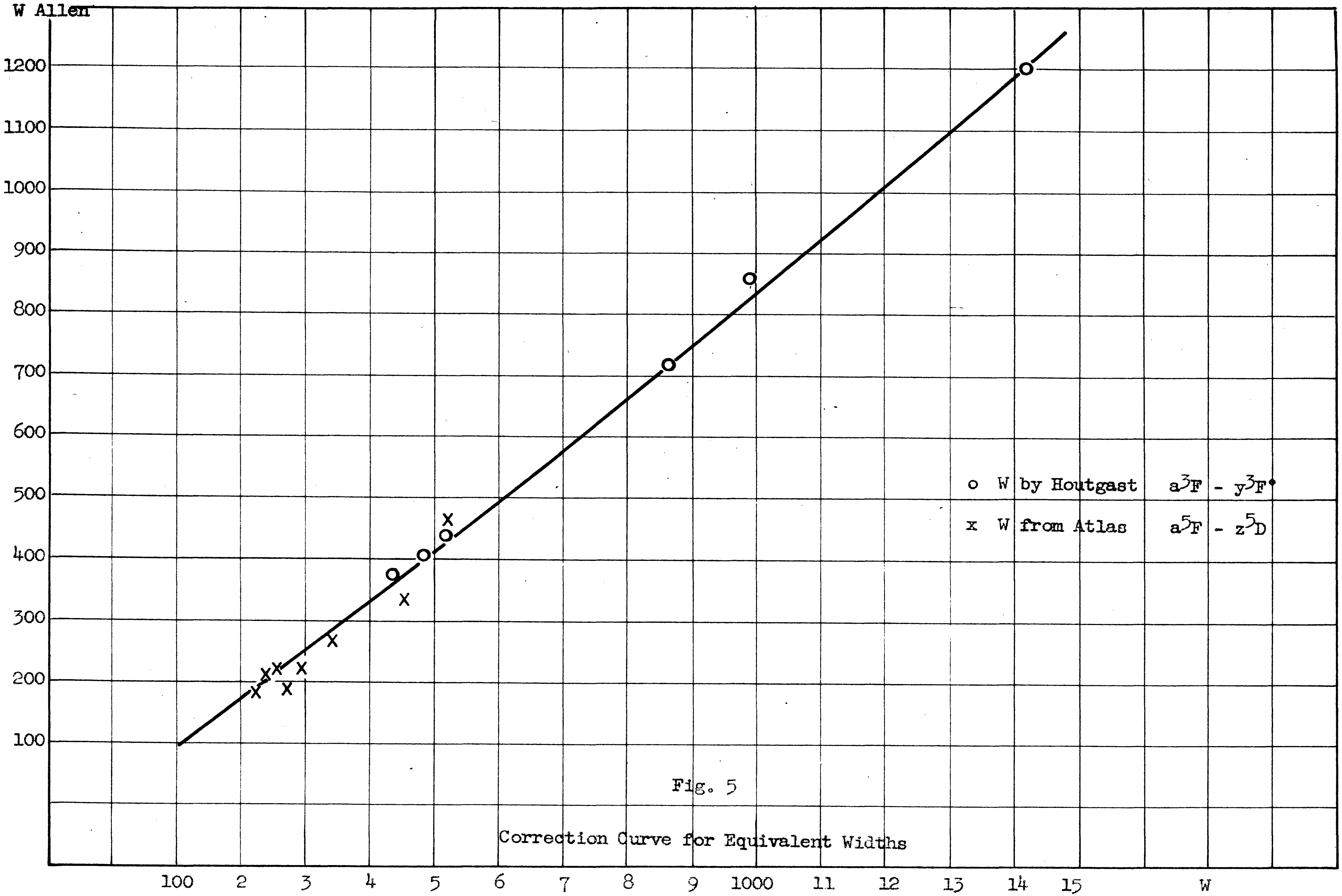


Fig. 5

Correction Curve for Equivalent Widths

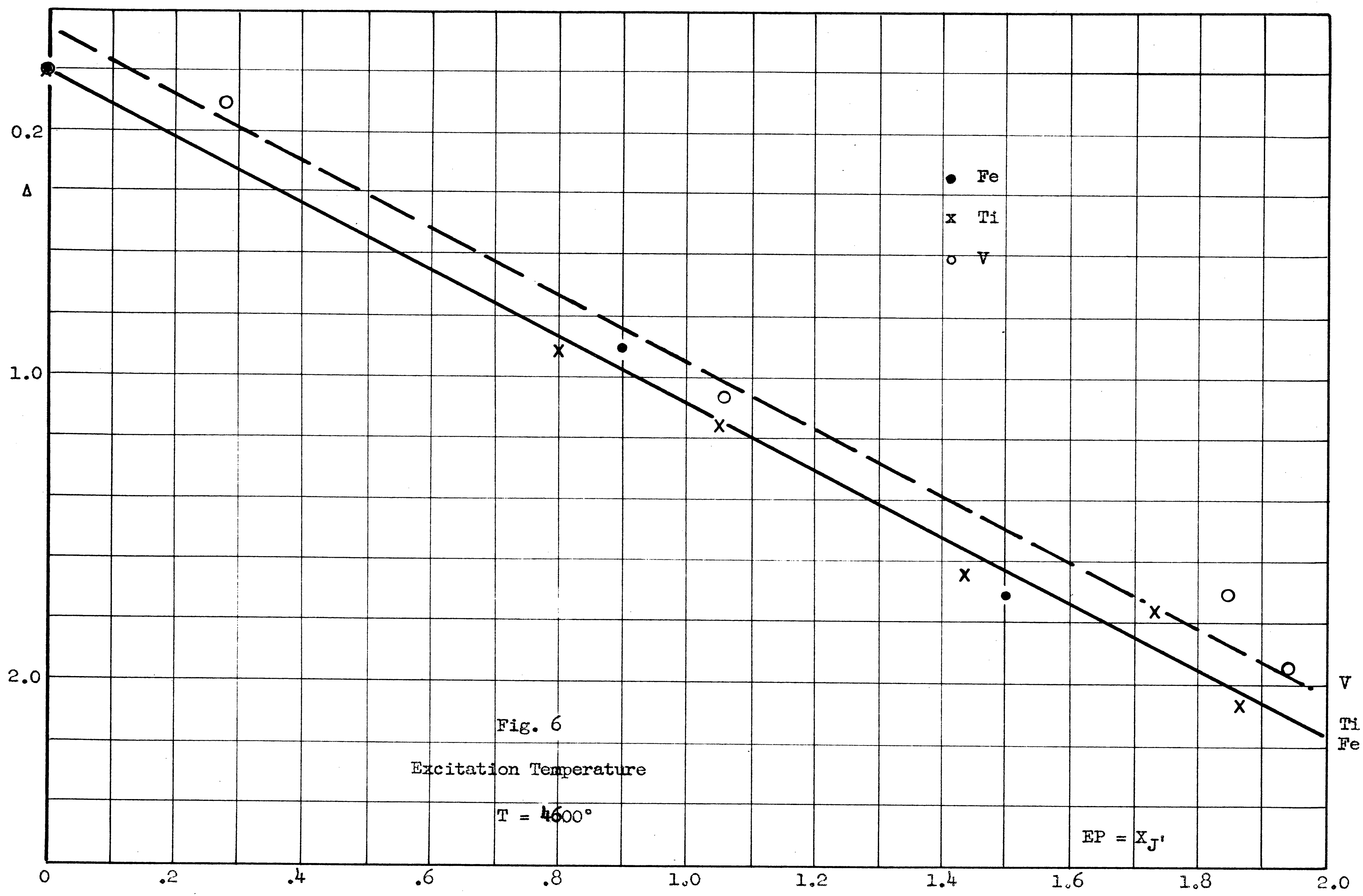


Fig. 6

Excitation Temperature

T = 4600°

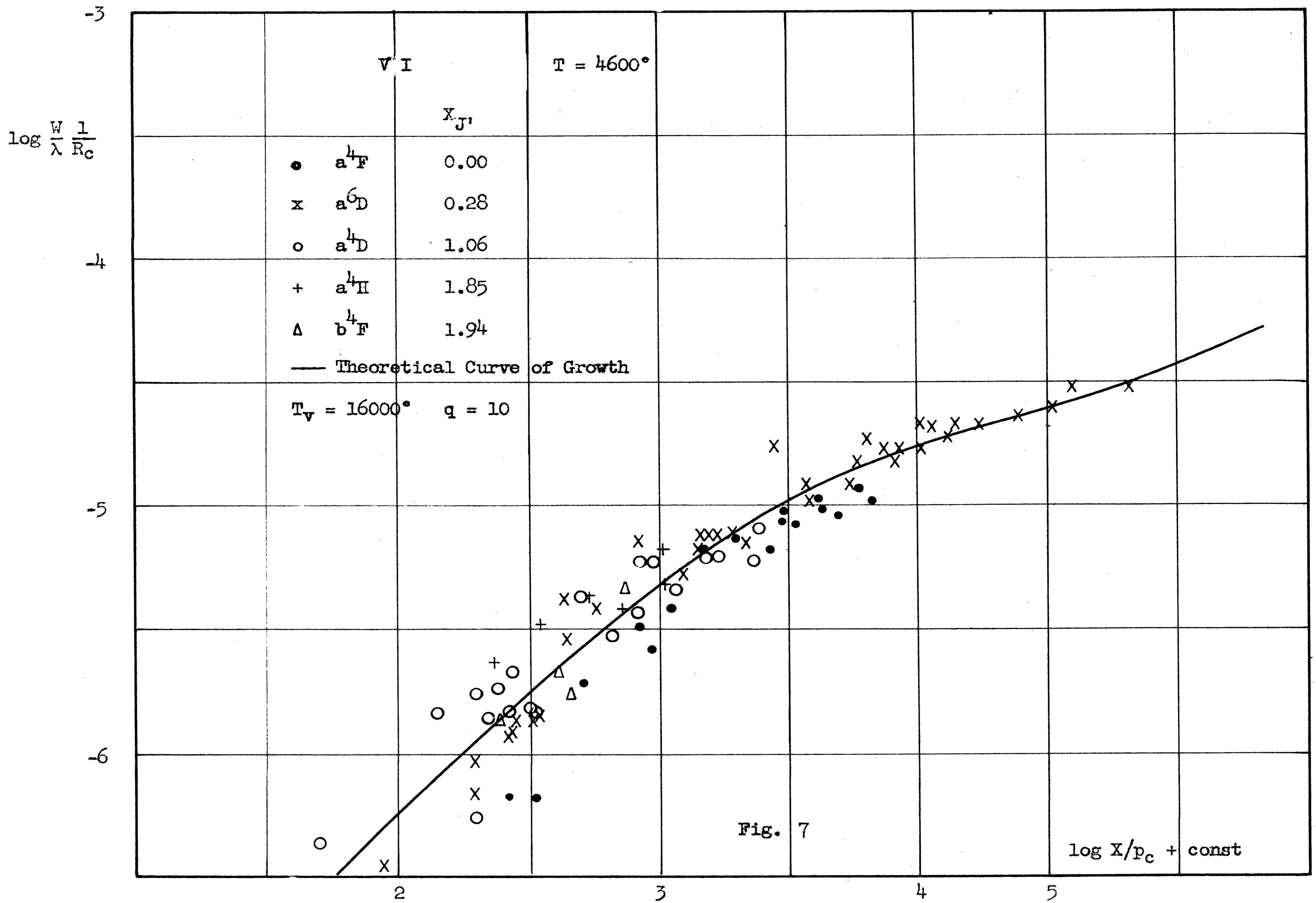
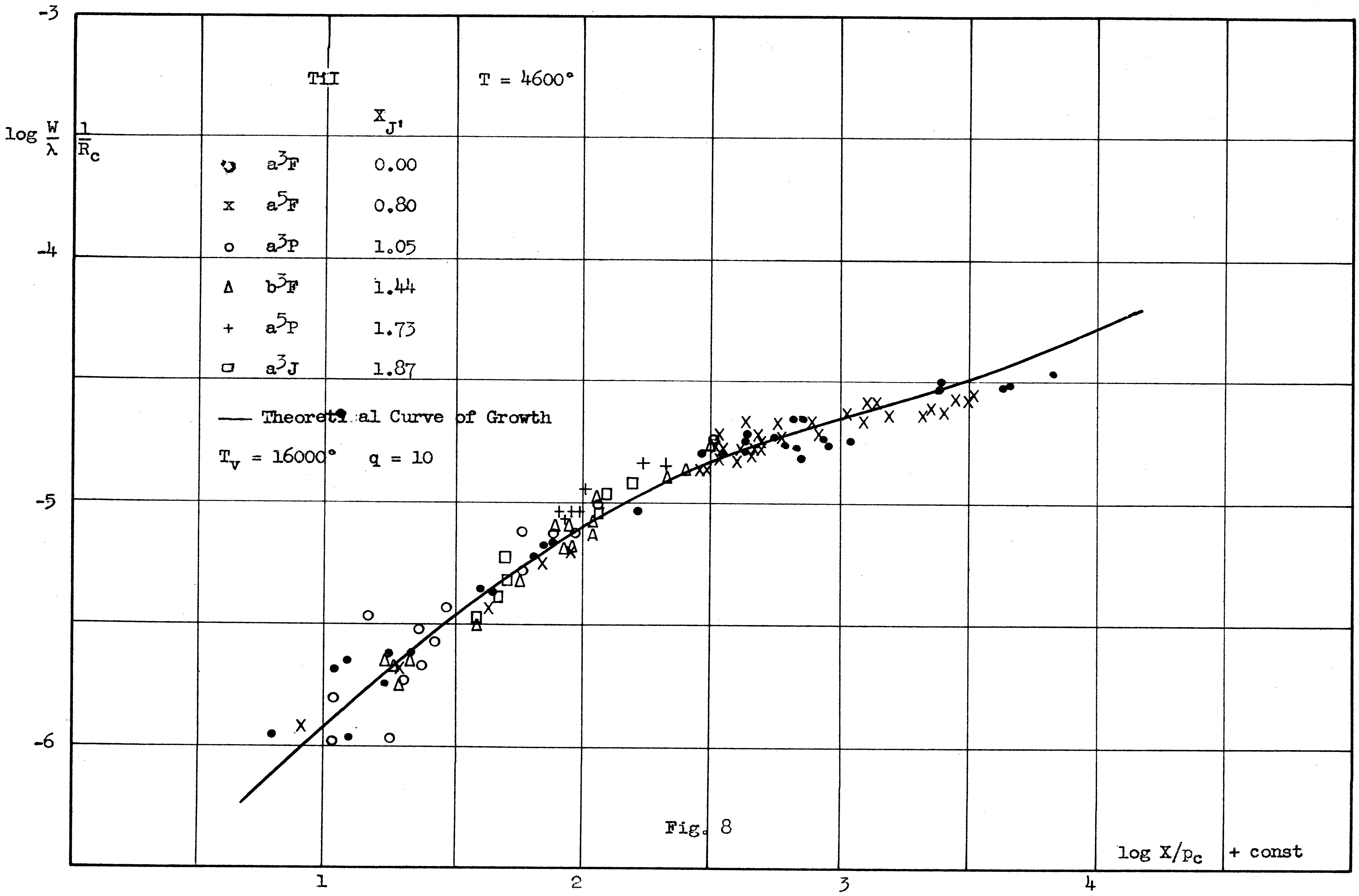
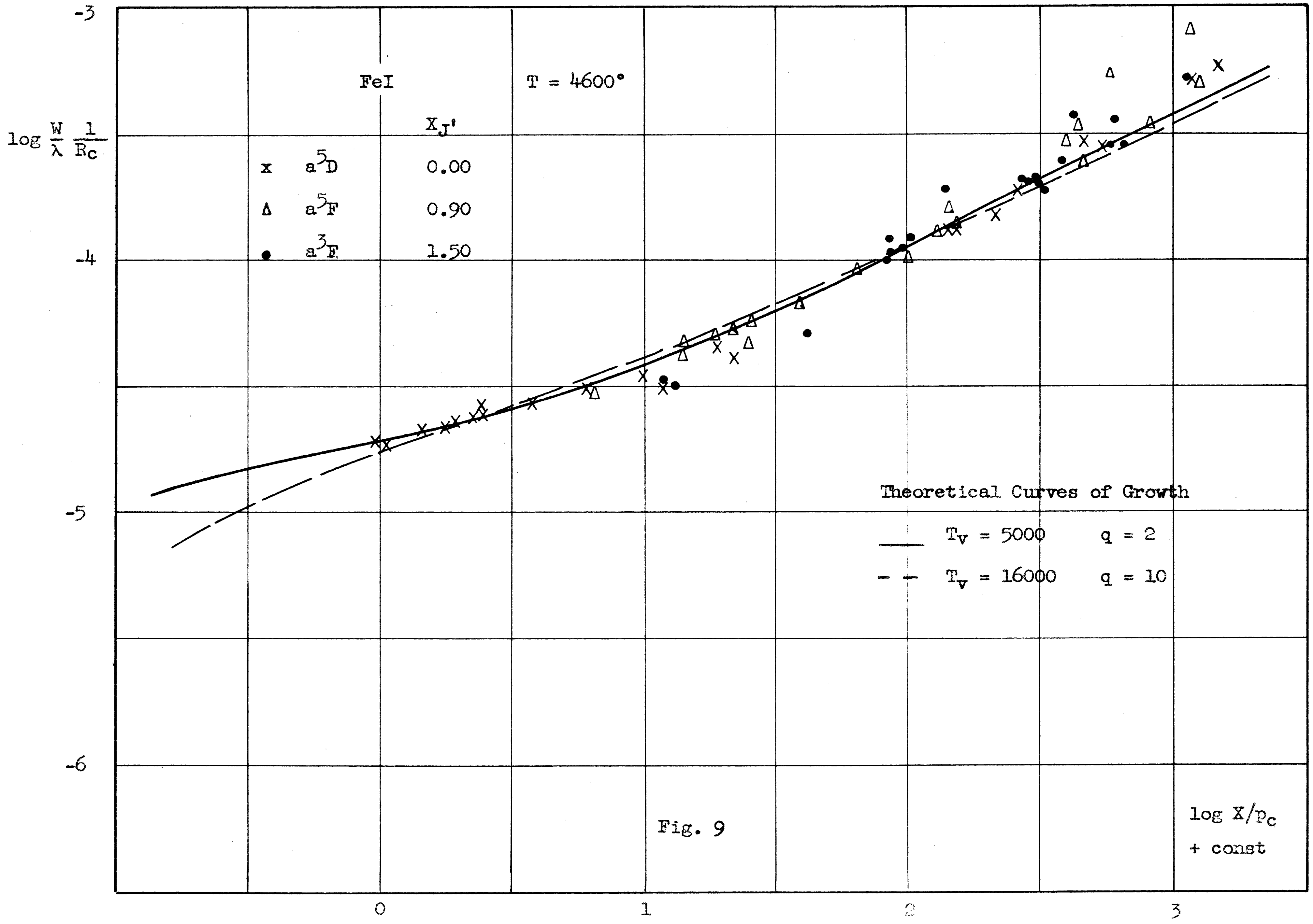

V I
T = 4600°
$X_{J'}$
● a^4F 0.00
x a^6D 0.28
o a^4D 1.06
+ a^4H 1.85
Δ b^4F 1.94
—— Theoretical Curve of Growth
T_v = 16000° q = 10
$\log \frac{W}{\lambda} \frac{1}{R_c}$
-3
-4
-5
-6
2
3
4
5
$\log X/p_c$ + const

Fig. 7



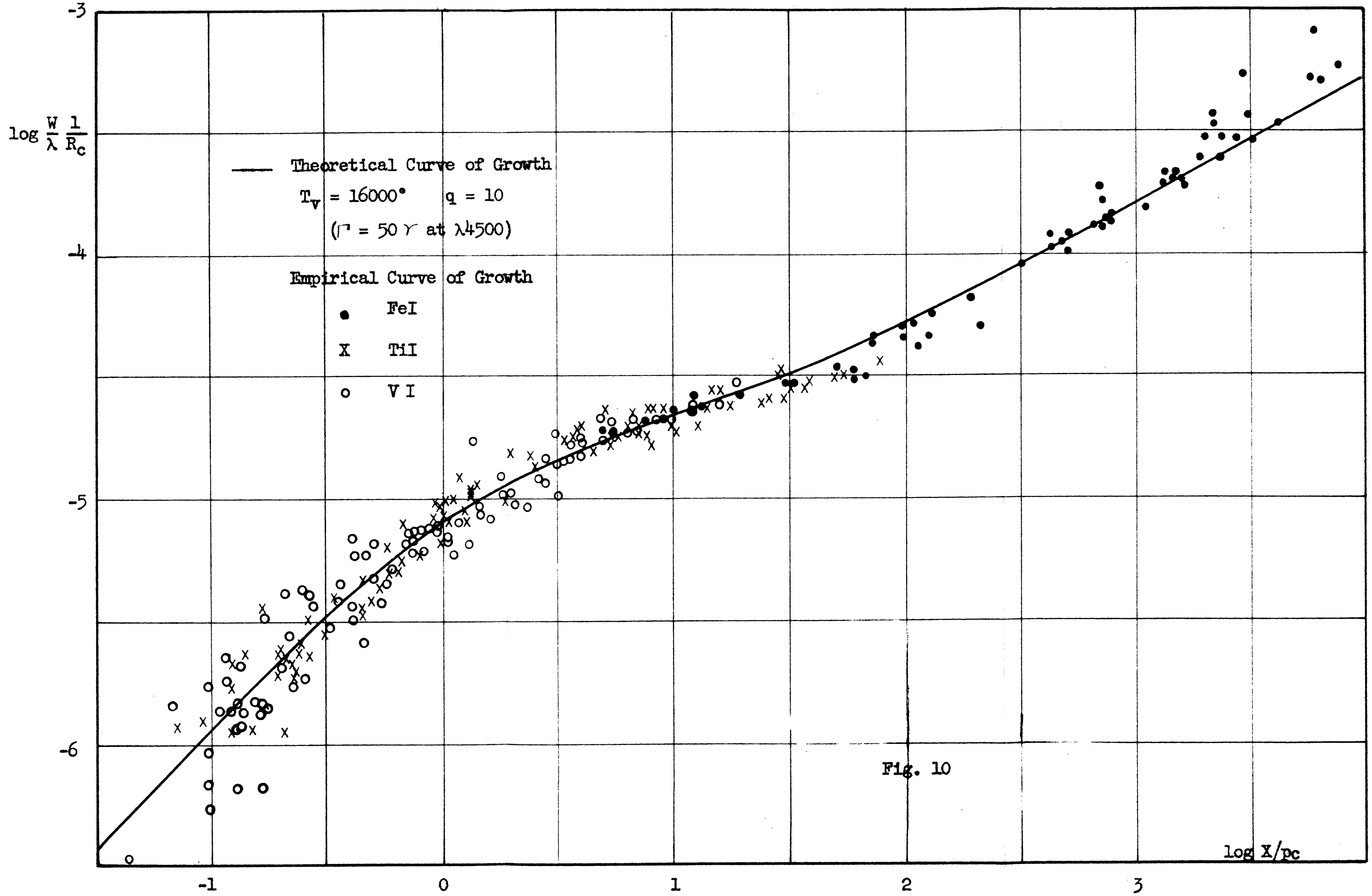
-3
-4
-5
-6
log W/λ 1/Rc
TiI
T = 4600°
XJ'
a3F 0.00
x a5F 0.80
o a3P 1.05
Δ b3F 1.44
+ a5P 1.73
□ a3J 1.87
Theoretical Curve of Growth
Tv = 16000° q = 10
1
2
3
4
log X/pc + const

Fig. 8



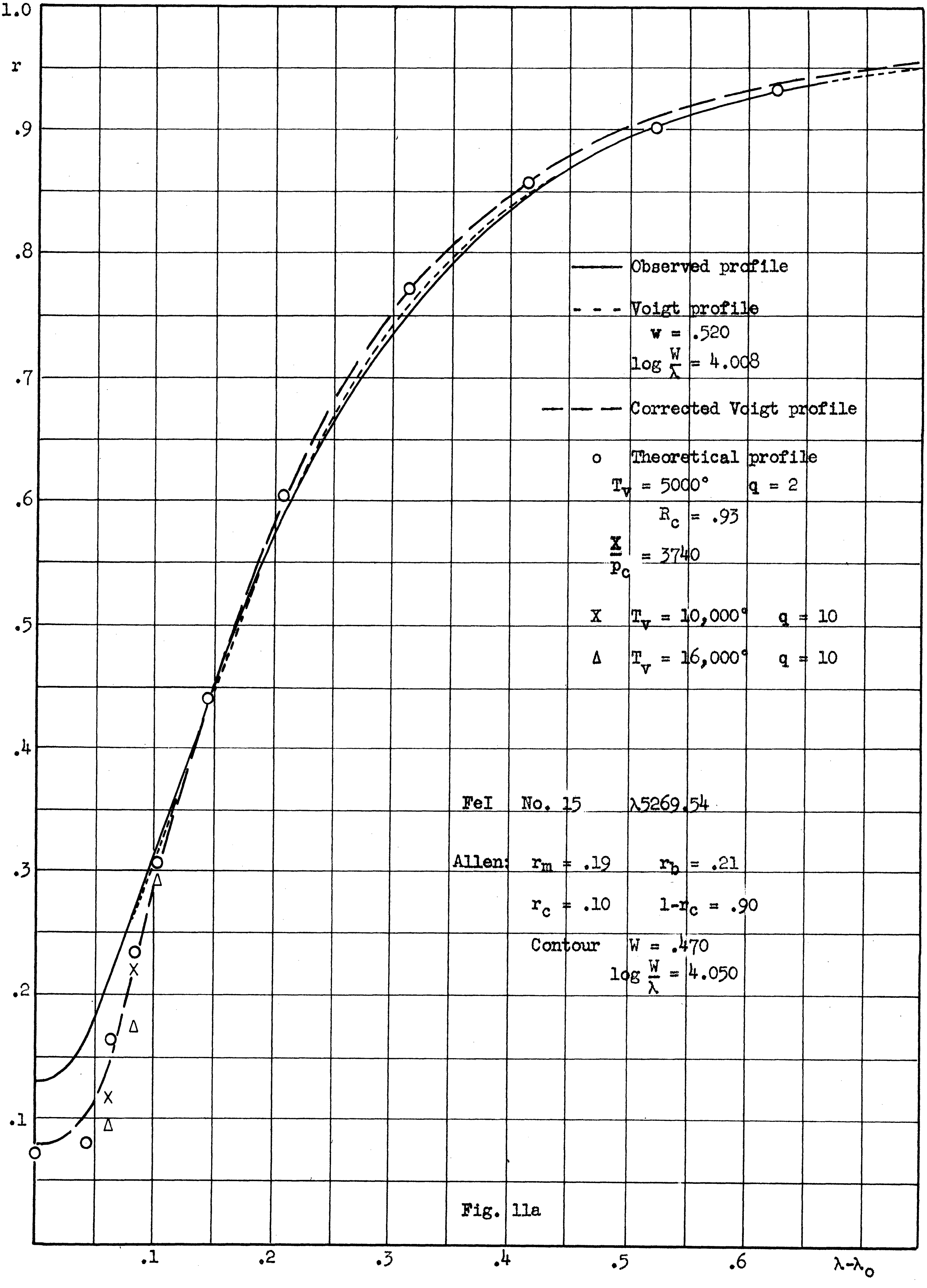
-3
-4
-5
-6
log W/λ 1/Rc
FeI
T = 4600°
X_J'
x a5D 0.00
Δ a5F 0.90
● a3E 1.50
Theoretical Curves of Growth
Tv = 5000 q = 2
Tv = 16000 q = 10
0
1
2
3
log X/pc
+ const

Fig. 9



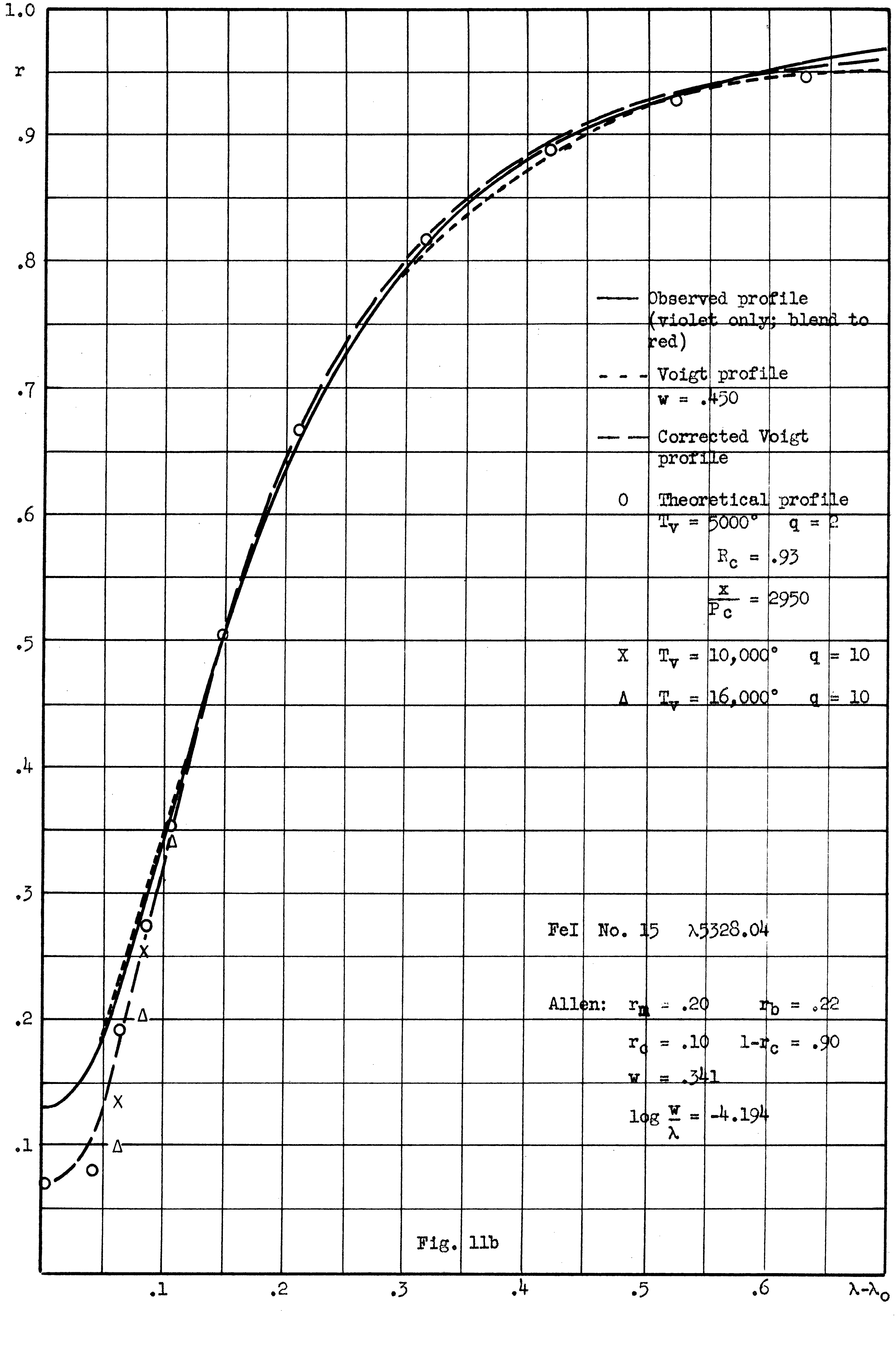
-3
-4
-5
-6
log $\frac{W}{\lambda}\frac{1}{R_c}$
Theoretical Curve of Growth
T_v = 16000° q = 10
(Γ = 50 γ at λ4500)
Empirical Curve of Growth
FeI
TiI
V I
-1
0
1
2
3
log X/pc

Fig. 10



1.0
r
.9
.8
.7
.6
.5
.4
.3
.2
.1
.1 .2 .3 .4 .5 .6 λ-λo
Observed profile
Voigt profile
w = .520
log W/λ = 4.008
Corrected Voigt profile
o Theoretical profile
Tv = 5000° q = 2
Rc = .93
X/pc = 3740
X Tv = 10,000° q = 10
Δ Tv = 16,000° q = 10
FeI No. 15 λ5269.54
Allen: rm = .19 rb = .21
rc = .10 1-rc = .90
Contour W = .470
log W/λ = 4.050

Fig. 11a



1.0
r
.9
.8
.7
.6
.5
.4
.3
.2
.1
.1
.2
.3
.4
.5
.6
λ-λo
Observed profile (violet only; blend to red)
Voigt profile w = .450
Corrected Voigt profile
O Theoretical profile Tv = 5000° q = 2
Rc = .93
x/Pc = 2950
X Tv = 10,000° q = 10
Δ Tv = 16,000° q = 10
FeI No. 15 λ5328.04
Allen: rm = .20 rb = .22
rc = .10 1-rc = .90
w = .341
log w/λ = -4.194

Fig. 11b