THE UNIVERSITY OF MICHIGAN

INDUSTRY PROGRAM OF THE COLLEGE OF ENGINEERING

RIBBON GENERATORS, WITH MHD GENERATOR AND OTHER IMPLICATIONS

A. D. Moore

November, 1960

IP-475

TABLE OF CONTENTS

Page

INTRODUCTION.......... 1

Definition and Origin.......... 2

RIBBON GENERATORS.......... 2

The Annulus Form.......... 2
Energy Conversion.......... 4
Annulus, with Non-Radial Ribbon.......... 6
Rectangular Sheet: Effect of Change of Ribbon Path.......... 7
Sheet as a Disk, with Ribbon Manipulations.......... 7
Net Zero Power for a Through-Ribbon or Continuous Ribbon (Electrodes Absent).......... 11
Infinite Sheet.......... 13
Imaging with Ribbon Generators.......... 14
RG Analysis of a Faraday Disk Generator.......... 15
Further Ramifications.......... 18

RIBBON STACKS.......... 19

Uniform Ribbon Stacks.......... 19

THE MHD GENERATOR.......... 23

The MHD.......... 23
RG Theory Applied to a Gas Element.......... 25
Losses, and Voltage Characteristic for the Gas Element.. 27
More General Analysis of the MHD.......... 29
Analysis Further Generalized.......... 31
No-Load Voltage Proportional to Average Velocity.......... 31
RG Analysis and the Electromagnetic Flowmeter.......... 33
MHD with Non-Rectangular Duct.......... 34
MHD Performance for a Uniform Block of Gas.......... 35
Efficiency Outlook for the MHD.......... 38

RG ANALYSIS AND PHYSICAL ANALOGS.......... 39

Adapting an RG Analysis for Use in Physical Analogs.......... 39
Other Applications.......... 44

CONCLUSIONS.......... 44

REFERENCES.......... 46

LIST OF FIGURES

Figure		Page
1	A basic ribbon generator. Uniform magnetic field crossing a uniform conducting sheet. Ribbon, of same material, fits a slot in the sheet and moves lengthwise..	3
2	Annulus form of sheet. (a) Radial ribbon sets up circular current. (b) Non-radial ribbon, same current. (c) Zero-current case, with equal ribbons, one moving out, the other in..	5
3	Rectangular sheet, ends shortcircuited. (a) Ribbon across one end. (b) Ribbon moved to center: no change in current field. (c) Ribbon slanted, same current. Potentials do change in area over which ribbon is translated....................................	8
4	Disk sheet. (a) Through-ribbon and continuous ribbon: set up potentials, but no current (electrodes absent). (b) Internal ribbon. (c....h) Various ribbon manipulations - see text. (i) Through-ribbon MN and continuous ribbon P do not alter original current due to ribbon AD...	9
5	Infinite sheet with ribbon. (a) AD ribbon makes double eccentric circle current pattern. (b) AD ribbon replaced by ribbons AB and CD, same current. (c) CD ribbon alone gives concentric current component. (d,e,f) Other manipulations...................................	12
6	A Farady Disk Generator. (a...d) RG arguments (see text) show that the load current field can be simply determined..	16
7	Disk cases used in the text to show that RG situations are analogs for ordinary ways of setting up fields......	17
8	Uniform ribbon stack. (a) Square conducting sheet, shortcircuited all around, uniform current. (b) As before, but with boundary having any curved shape: current still uniform....................................	20
9	Ribbon stack in rectangular sheet, with top and bottom electrodes. (a),(b) Single ribbons in the slices they occupy, and the potentials. (c) Foregoing combined; process continued leads to ribbon stack. (d) The stack sets up rectilinear currents (electrodes not connected).	22

LIST OF FIGURES CONT'D

Figure Page

10 The MHD generator. This figure to make analysis of potentials and currents due to a symmetrically located gas element AF, using RG theory.................. 24

11 The MHD. This figure used through RG theory, to derive more general analyses of effect of gas elements......... 32

12 Voltage characteristics and electrical efficiencies of a rectangular-duct MHD generator, as due to a block of gas of uniform velocity, of differing widths. The gas block is KLMN in the insert.................................. 36

13 Three ribbons, actually close together or superimposed, replaced by equal sources and sinks, prior to setting up a physical analog for solving for fields analyzed by RG theory.. 40

14 Using the MHD to illustrate routines for setting up the physical analog...................................... 42

INTRODUCTION

Ribbon generators are conceptual or theoretical devices. The concept of a ribbon generator definitely appears to be a new addition to electrical theory. When the rules or laws for ribbon generator behavior are developed, as they are herein, there are three resulting contributions.

First, we have a new set of electrical theorems, dealing with potentials and current fields set up in a conducting sheet, by emf developed in one or more thin ribbons.

Second, a new set of analogies appears. When ribbon generator phenomena are fitted into the existent family of potential fields (heat flow, magnetic, electrostatic, etc.) analogous situations become evident. Thus, there is the possibility that a field situation needing analysis may be analyzed in terms of ribbon generator theory. Not only is potential field theory enhanced: the further possibility exists that the new concepts may at times offer the simplest and easiest form of analysis.

Third, ribbon generator concepts can be applied to the analysis of the potentials and current fields in such items as the Faraday disk generator, the electromagnetic flowmeter, and the MHD (magnetohydrodynamic) generator. The MHD can be further analyzed for losses and voltage characteristic.

All of the ideas involved in ribbon generator theory are simple and easily grasped. The reader should know, however, that these ideas must be learned, from the earlier part of the paper, before he can proceed with the analogies and applications that appear later on.

Definition and Origin

A ribbon generator is a conceptual device. It consists of a uniform conducting sheet of uniform thickness having a narrow slot of constant width cut in it, with a ribbon of the same material exactly fitting the slot and making perfect electrical contact with the sheet. The ribbon is perfectly flexible, and it will slide along the slot without friction. When a uniform magnetic field is set up through the sheet and normal to it, and the ribbon is moved lengthwise along the slot, a uniform emf is induced across the ribbon, causing potential differences, and setting up currents if current paths exist.

The ribbon generator concept arose from the "flux band and pencil pair" concept, also originated by the writer.(1) By that concept, it became possible to deal in simple fashion with the damping (Foucault) currents of round disks, as in the watthour meter. The band-and-pair concept is sound, and it brought results; however, some who meet it for the first time have difficulty in accepting it. The writer has always wanted to find an explanation or further concept that would offer a more immediate appeal and insure a better acceptance. It was out of these struggles that the ribbon generator concept was born.

RIBBON GENERATORS

The Annulus Form

All of the elements needed for the simplest form of the RG, or ribbon generator, are shown in Figure 1, where the conducting sheet is an annulus, and the ribbon runs radially. With downward flux and outward movement of the ribbon, the crosswise emf of the ribbon is indicated by

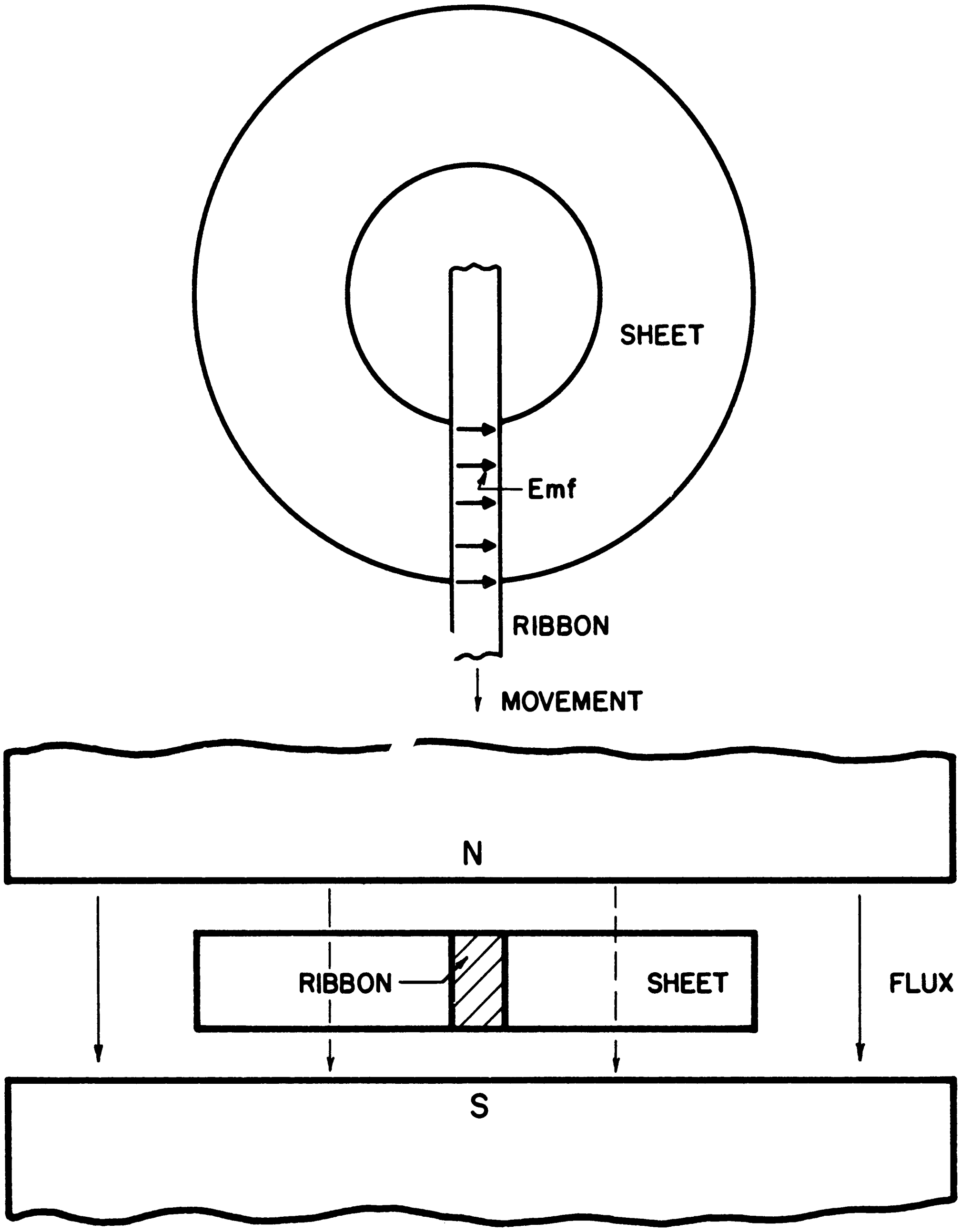


Figure 1. A basic ribbon generator. Uniform magnetic field crossing a uniform conducting sheet. Ribbon, of same material, fits a slot in the sheet and moves lengthwise.

the little arrows. Hereafter, the flux will always be taken downward, and is always present, even if not shown.

The ribbon is shown here with greatly exaggerated width. In the conceptual form, Figure 2a, the ribbon width is always taken to be so small that it is essentially infinitely thin. If we begin with a finite thickness and finite speed, a finite emf results. Then, as the ribbon is made thinner and thinner, we can make it go faster and faster. Conceptually then, a finite emf can be had with an exceedingly thin ribbon.

Because of symmetry, the current filaments in this case will be in concentric circles.

Energy Conversion

If this is a generator, there must be energy conversion. The electrical power in watts developed by a ribbon would be equal to the voltage developed, multiplied by the current it sets up and which crosses the ribbon. There is a back force on the ribbon due to the magnetic field acting on the current as it crosses the ribbon, and this force is the force required to pull the ribbon along and maintain its speed. Force times speed, converted to watts, would equal the electrical power developed.

Later, cases will appear where a current opposite to that which the ribbon would cause, will occur. The ribbon generator then becomes a ribbon motor: the force developed would help that ribbon to move the way it is going. The electrical input to the ribbon would be ribbon volts times current crossing the ribbon. The mechanical power produced (force times speed, converted to watts) would equal the electrical power input.

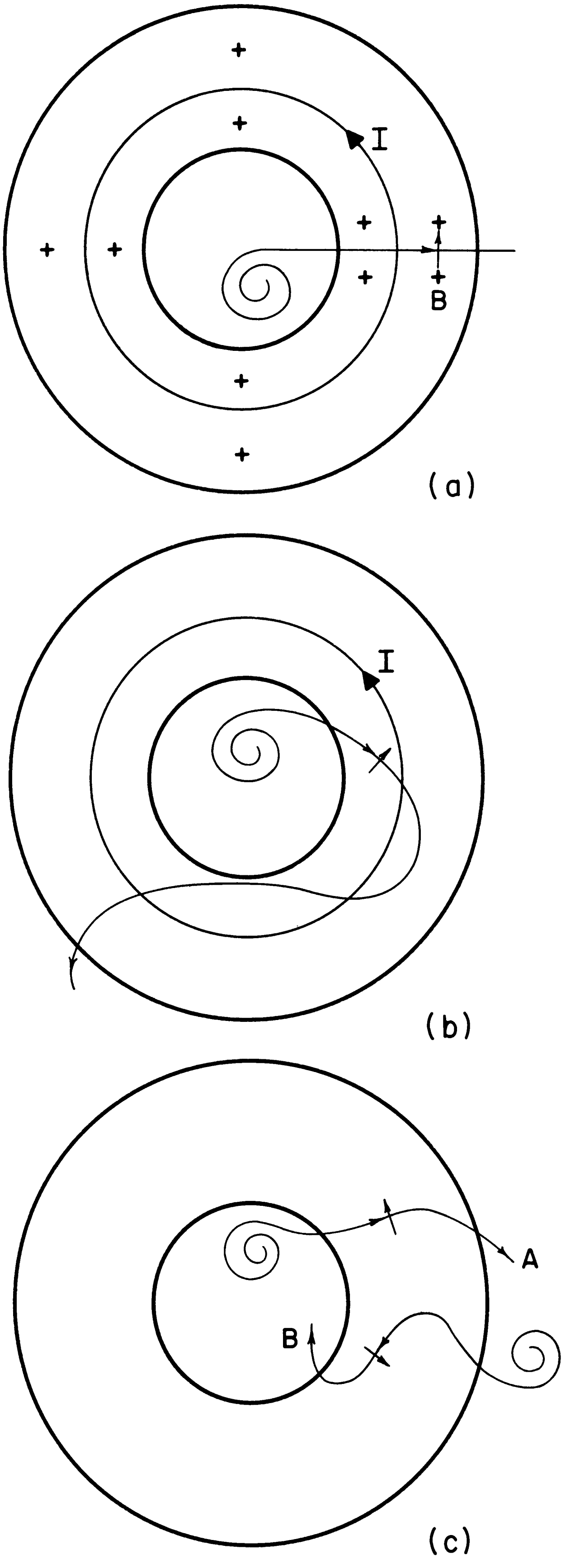


Figure 2. Annulus form of sheet. (a) Radial ribbon sets up circular current. (b) Non-radial ribbon, same current. (c) Zero-current case, with equal ribbons, one moving out, the other in.

Annulus, with Non-Radial Ribbon

In Figure 2b the ribbon is allowed to take any path out through the annulus. The drawing shows circular currents, as before. But is this the truth of it? This is a key question. The numerous workers to whom this question has been put, fell into three groups: those who held that the circular current remained; those who argued for at least some distortion, immediately next to and within the ribbon; and those who held out for major distortion in the whole general neighborhood of a highly distorted ribbon path. No one in any group was sure of his opinion.

It will now be proved that the circular current of Figure 2a remains, completely undistorted by the shift of the ribbon path in Figure 2b. In Figure 2c, furnish the annulus with two ribbons A and B, with A going out and B moving in. The ribbons generate equal voltages. Now, this is a zero-current case: no current path can be found anywhere with a net emf acting around it. Next, stop ribbon B, whereupon A sets up counterclockwise current. Now stop A and let B run: it produces clockwise current. The two current component fields must everywhere cancel if, with both ribbons running, it is a zero-current case. Remember that the ribbons may have any paths whatever. If A, running alone, had distorted currents in its own neighborhood, it would be impossible for B, in general, with its current, to make an exact cancellation of those currents; and the converse is true. Therefore, whether the conducting sheet has the annulus or any other form, the only possible current field components due to A and B would be opposite, and of identical conformation. In the annulus case, a ribbon taken by any path across the annulus will set up concentric current.

Rectangular Sheet: Effect of Change of Ribbon Path

The conducting sheet takes rectangular form in Figure 3, with the ends of the sheet shortcircuited by a yoke of zero resistance. In Figure 3a the 4-volt ribbon is taken through a slot cut next to the left end. The current would obviously have to be uniform, as shown, due to symmetry. Taking the shorting yoke at zero potential, the potentials along the sheet are shown at regular intervals. The jump from zero to +4 in passing through the ribbon makes no allowance for IR drop across the ribbon. For a ribbon approaching zero in thickness, no allowance is needed.

Next, Figure 3b, the ribbon is passed through the vertical center line. There is no change whatever in the current pattern. Moreover, there is no change in sheet potentials to the right of the new path; but all potentials to the left (except the yoke potential) have been changed, and the new values are shown. A like study, but of more general character, is seen with the bias path, Figure 3c.

Thus, moving the ribbon path sidewise does effect changes in potential within the area over which it is translated, but there is precisely no effect otherwise. Remaining potentials are unaffected, and currents are unchanged.

Sheet as a Disk, with Ribbon Manipulations

The great flexibility with which ribbon generators as conceptual tools can be manipulated will now begin to appear. In Figure 4 the sheet has the form of a disk.

A 4-volt "through" ribbon is shown above, Figure 4a, entering one side and making exit through the other. It sets up no current; but it does create potential plateaus, zero and +4, as shown. This figure

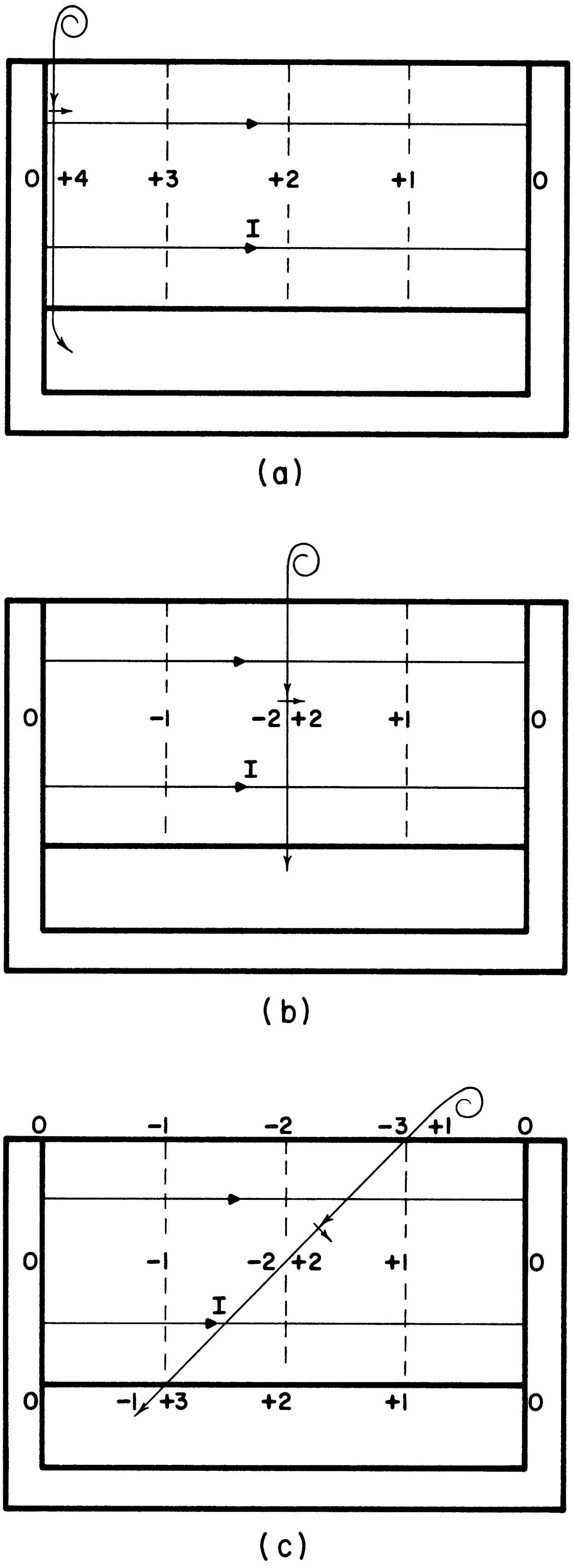


Figure 3. Rectangular sheet, ends shortcircuited. (a) Ribbon across one end. (b) Ribbon moved to center: no change in current field. (c) Ribbon slanted, same current. Potentials do change in area over which ribbon is translated.

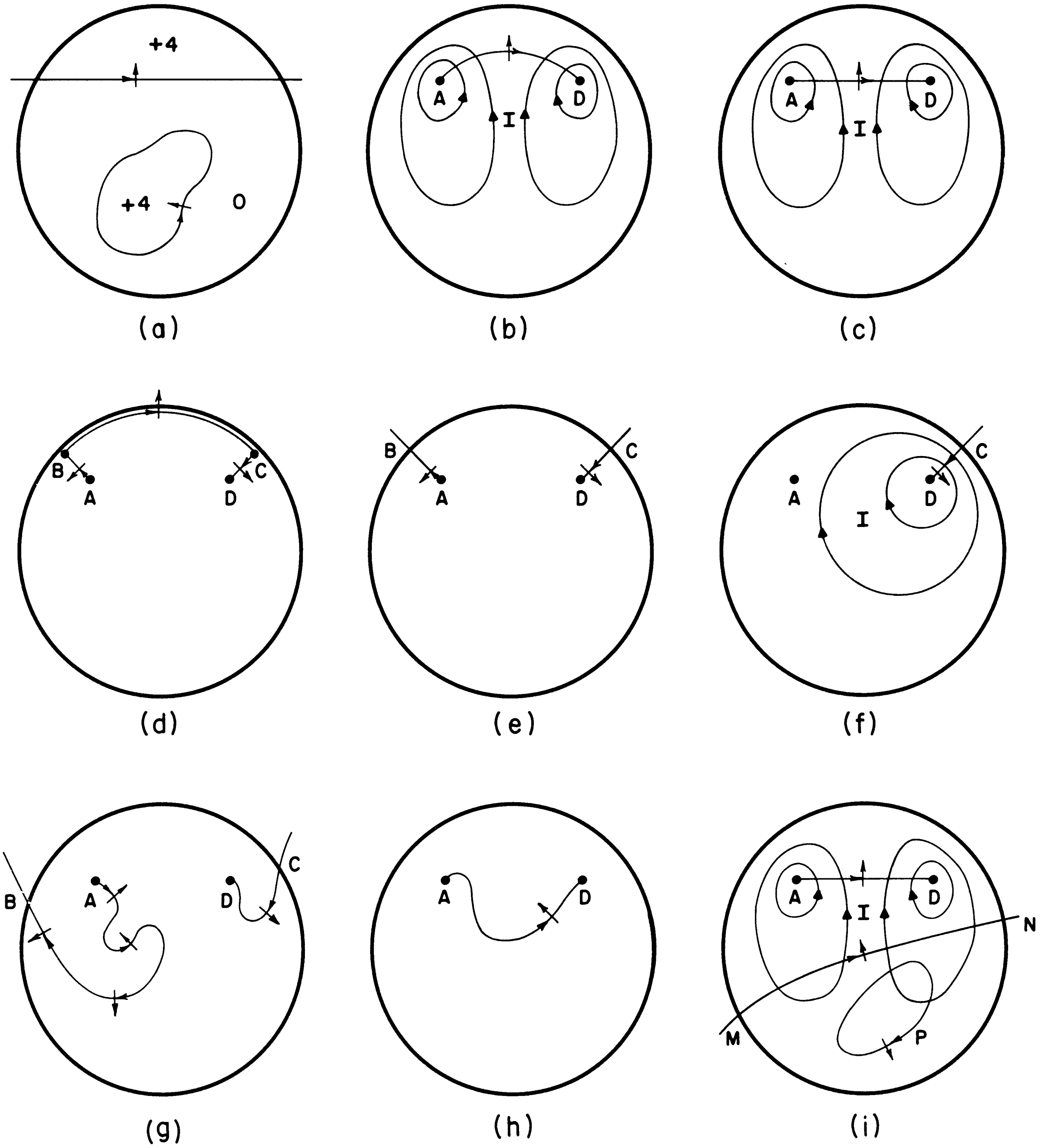


Figure 4. Disk sheet. (a) Through-ribbon and continuous ribbon: set up potentials, but no current (electrodes absent). (b) Internal ribbon. (c...h) Various ribbon manipulations - see text. (i) Through-ribbon MN and continuous ribbon P do not alter original current due to ribbon AD.

also shows a continuous ribbon: it causes no current, but makes potential plateaus. The plateaus are equipotential areas.

In Figure 4b the internal ribbon AD is introduced from above or below the sheet by way of a small hole at A, moves along a circular arc slot to D, and is removed by way of a small hole at D. It produces the double whirl of current as shown. This case represents the genesis of the ribbon generator concept. Temporarily abandoning ribbon ideas, think of the disk as rotating about its axis, and of the motion being damped by a local uniform downward flux in the AD region. Consider the AD element of that flux, which is a uniform band, the band being an arc, and of uniform radial dimension. What component of damping current would the emf of this band contribute? In the earlier paper,(1) the writer replaced the flux band and disk rotation, with a pair of opposite flux pencils at A and D passing through a fixed disk; and showed that the pencils, if increased together, would set up the same currents as would the flux band. And now, with ribbon generator concepts at hand, we can replace the band with a ribbon generator AD, and set up these same currents. The ribbon concept can replace the flux-band and pencil-pair concept.

In Figure 4c the fact is illustrated that the AD ribbon may take a straight path rather than curved, if we wish, without in the least altering the current pattern.

Next, Figure 4d, we choose to take the ribbon by path ABCD, in which BC is infinitely close to the boundary. Now, BC is a boundary ribbon, and it can set up no current: all it does is to make a potential difference between the boundary on one side of it, and the disk area on the other side. Therefore, we now consider the ABCD ribbon as three ribbons, ignore the BC boundary ribbon, and pay attention to AB and CD. AB is an internal ribbon, setting up a double whirl as its current component; but if B is infinitely close to the boundary, the current going around B is confined to an infinitely small region at B. With B at the boundary, that current disappears, and the AB current pattern will be the same as if ribbon AB is extended out through the boundary, Figure 4e. Likewise for CD.

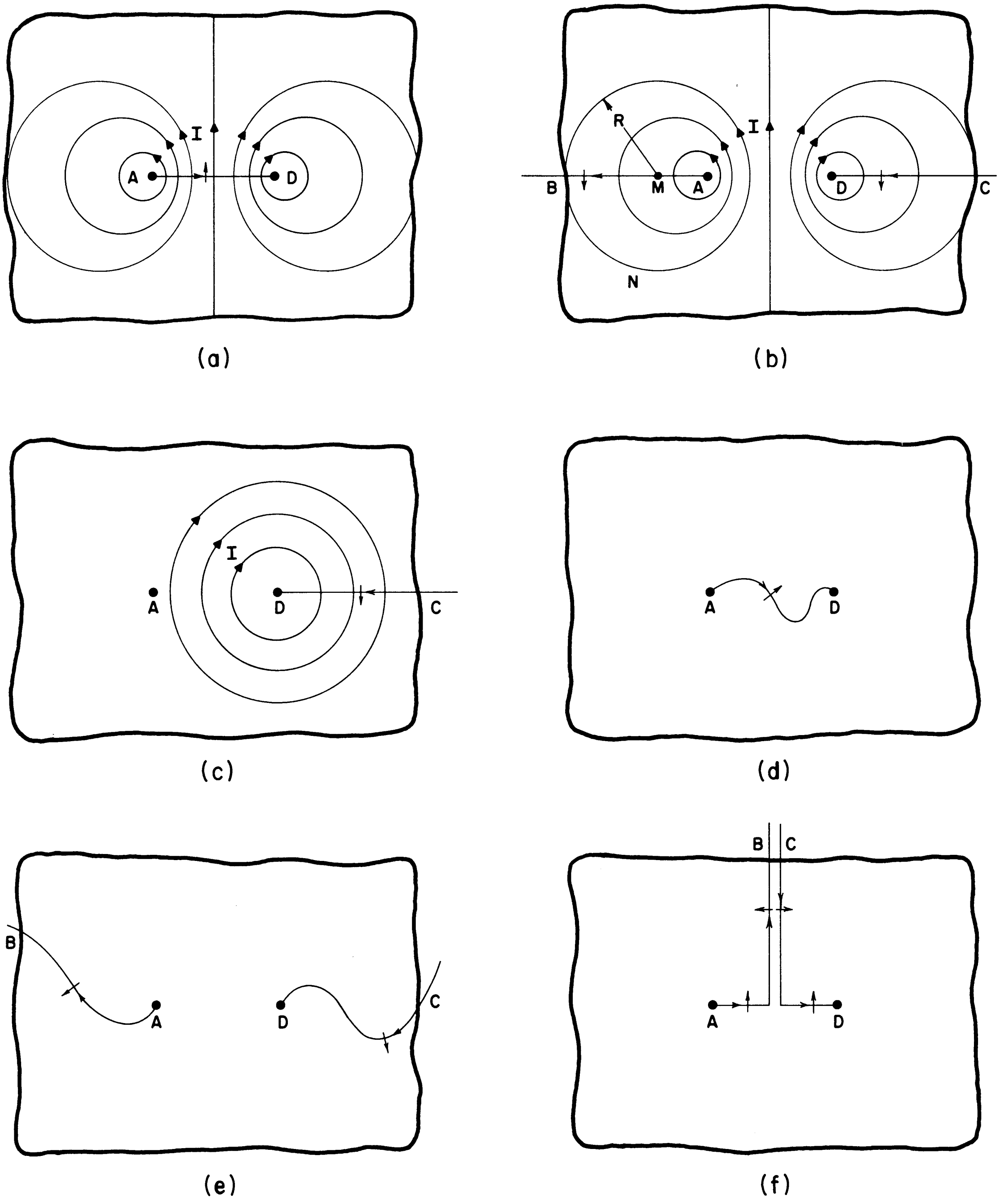
Now, taking either ribbon alone, for example CD, Figure 4f, it sets up a component current in the familiar form of eccentric filaments. Likewise for AB taken alone. The resultant of these two components is the original current, Figure 4b.

As to flexibility of treatment, Figure 4g, the AB and CD ribbons may be given any paths whatever, as long as they pierce the boundary, and respectively begin and end at A and D. Figure 4h indicates that the original AD ribbon could have taken any internal path.

In Figure 4i, the conditions of Figure 4c are repeated. Also, a through-ribbon MN has been added. MN alone produces no current. Therefore, it makes no change whatever in the AD current already flowing. It does set up its own component of potential plateau, and so does change the net potentials.

Net Zero Power for a Through-Ribbon or Continuous Ribbon (Electrodes Absent)

In Figure 4i, the AD current field already existed when the MN through-ribbon was introduced. It is important to recognize that no power is required to operate the MN ribbon. All of the current that crosses it somewhere and which, say, gives it forward or motor forces, crosses back again somewhere else and develops backward or generator forces; and these



(a) (b) (c) (d) (e) (f)

Figure 5. Infinite sheet with ribbon. (a) AD ribbon makes double eccentric circle current pattern. (b) AD ribbon replaced by ribbons AB and CD, same current. (c) CD ribbon alone gives concentric current component. (d,e,f) Other Manipulations.

forces cancel out. No net energy conversion occurs in operating a through-ribbon. The same is true of a continuous ribbon such as P, Figure 4i.

These findings hold, in the absence of electrodes.

Infinite Sheet

In the infinite sheet of Figure 5a, an internal ribbon runs straight from A to D. It sets up the double whirl of current as shown. Next, if we are to pass a through-ribbon across the sheet, it would set up potential plateaus, but no current; and it would not disturb the existing current. Let such a through-ribbon be installed which is straight, parallel to AD, and infinitely close to it. Let it run to the left. If it generates the same voltage as AD, that part of it from D to A would cancel the AD ribbon, and leave, Figure 5b, the new replacement ribbons AB and CD. Potentials are changed, but currents are unimpaired.

Now consider the component current of each of these new ribbons. In Figure 5c, the CD ribbon alone would produce the concentrically circular currents shown. The resultant of the two concentric current conformations would yield the original current pattern of eccentrically circular current.

Figure 5d indicates that the original ribbon could take any path from A to D. In Figure 5e, the replacement ribbons AB and CD can have any such paths without disturbing the current. These rules of path flexibility tie together nicely in Figure 5f, in which the replacement ribbons have parallel vertical parts. If these parts are taken infinitely close together they cancel, and there remains only the original internal ribbon AD, as in Figure 5a.

It can now be noted that the current conformation of Figure 5a precisely agrees with the lines of a magnetic field set up by equal and opposite currents in long parallel conductors, the conductors being normal to the paper and passing through it at A and D. Likewise, the RG currents coincide with the equipotential surfaces of the electric field set up by two such wires if they have a potential difference. Or again, if the wires are made into a heat source and a heat sink and are surrounded by a heat conducting medium, the ribbon current filaments correspond to the heat case isotherms. Likewise for other potential field analogies. Thus, ribbon generators offer a new analog for other potential fields.

Imaging with Ribbon Generators

Imaging methods apply to RG concepts, just as they do to other means of field analysis. For example, Figure 5b, suppose that the circle N is the boundary of a round disk, and that at A, a small pencil of flux goes through the disk. If the flux is changed, eddy currents of eccentrically circular form will flow in the disk. The disk center is at M. Let R be the disk radius. Let us now go to an infinite sheet, draw on it the circle N, mark the center M, and the point A. Install the ribbon generator, AB. The next step is to locate the image ribbon. The point D is located by making the product MA times MD equal to the square of R. Install the CD ribbon. The AB and CD ribbons acting together will set up the currents shown in the sheet; and within the N-circle that was drawn, the currents will be identical in form with the eddy currents due to the flux pencil through the real disk. Moreover, if the sheet and the real disk are of identical materials and dimensions, and if the ribbons develop the same emf as made by the changing flux, the current densities are identical.

RG Analysis of a Faraday Disk Generator

A Faraday disk generator, Figure 6a, has a round disk on a shaft, and is rotating clockwise. The brush above is an electrode making contact with part of the boundary. If a uniform downward magnetic field is put through the disk everywhere, emf's will be produced throughout the disk, and these can be considered as radial. The question is - What is the conformation of the current flowing in the disk, if the generator is connected to a load? RG concepts easily and rigorously lead to the answer.

Consider any circular element of metal such as that at the A-circle, of constant radial dimension. Rotating at constant velocity through a uniform field, it contributes outwardly its small component of emf. Now step the disk, and consider this element as a continuous ribbon generator. The continuous ribbon of Figure 4a sets up no current; but this ribbon will, since it goes around an electrode, the shaft. Next, consider this continuous ribbon as two ribbons end-to-end: the AB ribbon, and the BA ribbon. The AB ribbon can take any path from A to B such as the straight path, Figure 6b, and yet set up its own unchanging component of current. The BA ribbon can also take any path, such as that shown, provided it is carried around the electrode. In Figure 6c we make a fresh start with the original continuous ribbon, now choosing A and B infinitely close together; also the part of BA around the shaft is taken infinitely close to it. The vertical parallel parts of BA cancel each other. This leaves only the new circular ribbon wrapped around the shaft, and developing precisely the same emf as the original ribbon. All other circular elements respond to the same treatment. Whereupon, the total emf is now produced by one circular ribbon wrapped on the shaft.

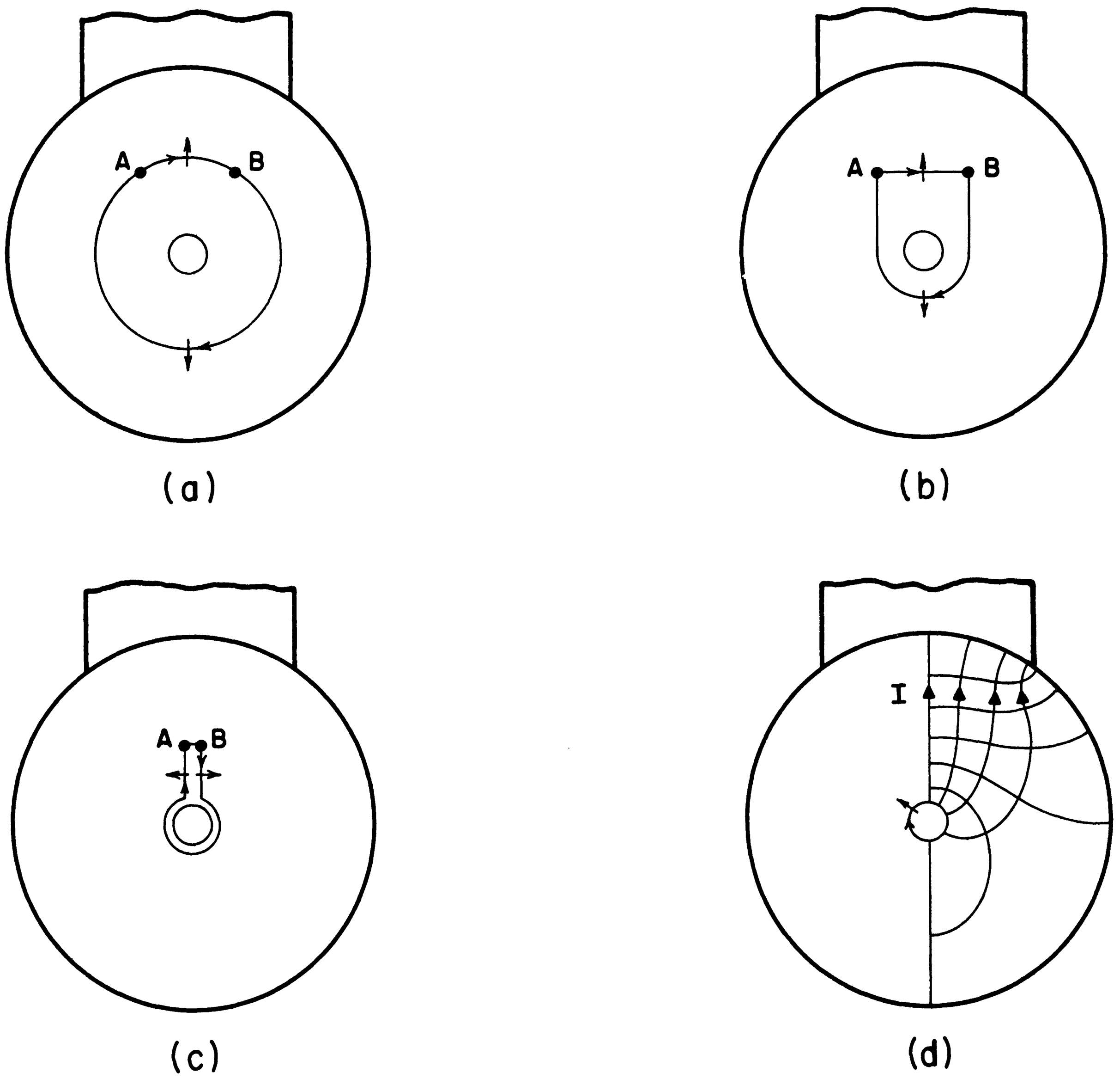

(a) (b) (c) (d)

Figure 6. A Faraday Disk generator. (a...d) RG arguments (see text) show that the load current field can be simply determined.

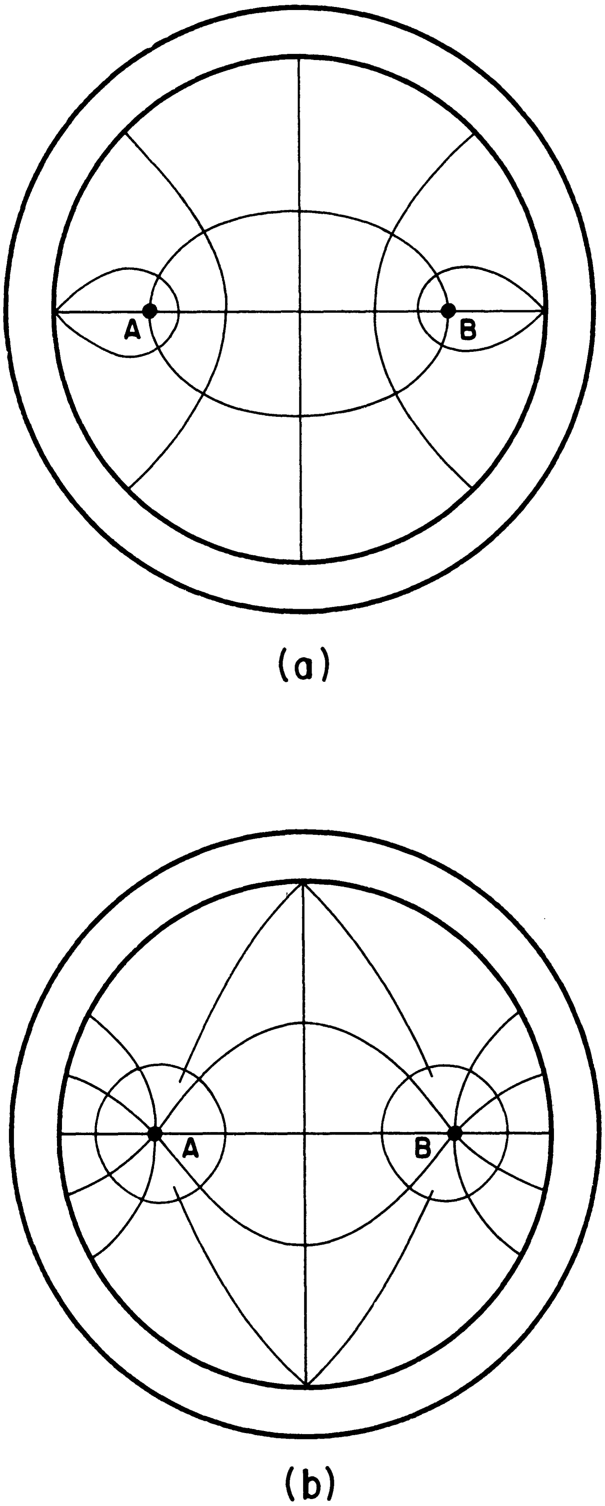


Figure 7. Disk cases used in the text to show that RG situations are analogs for ordinary ways of setting up fields.

The disk current is now easily described. It is shown in Figure 6d, being merely the Laplacian field between the shaft surface as an equipotential, and the brush electrode as the other equipotential.

Further Ramifications

In Figure 7a, consider the case as a round disk surrounded by an insulating band, with electrodes A and B touching the disk. Apply an external voltage and set up the current field shown between A and B. This current can be duplicated in RG terms. Remove the external voltage source, short circuit the electrodes, and run a continuous ribbon around either electrode by any path; or instead, run a ribbon from any point on the boundary, by any path between the electrodes, and back to the boundary again. The current will be duplicated.

Now invert the case. Replace the insulating band with an electrode that short circuits the whole boundary. Install a ribbon generator from A to B by any path: its current filaments would conform to what had been the equipotentials of the previous current.

Passing to Figure 7b, if the disk has a short-circuiting ring to begin with, and an external voltage is applied to electrodes A and B, the current shown between A and B is produced. To make this same current, short circuit the electrodes, and run a ribbon generator around either electrode; or, run it from anywhere on the boundary, between electrodes, to any other point on the boundary.

To invert the case, let the shorting electrode around the disk be replaced by an insulating ring. Install a ribbon from A to B by any path: the currents now going around A and B conform to the previous equipotentials.

RIBBON STACKS

Uniform Ribbon Stacks

A square sheet in Figure 8a has its entire boundary shortcircuited by an electrode. A very large number of equally spaced ribbons is installed, running to the right. The emf is upward, and the current flow is uniform. Only a few of the many ribbons are shown. It will be noted that there would be local non-rectilinear current conformations in the immediate vicinity of the ribbon terminals. However, with a very large number of ribbons, the local effects can be admitted, but their existence ignored.

Such a uniform stack, infinite in number, can, in effect, be had. First, let the bounding electrode rest on the sheet and make perfect contact with it without being attached to it. Second, let the sheet have ample extent beyond the electrode. Third, retain the uniform downward magnetic field common to all ribbon generators. Fourth, do not install any ribbons. Fifth, start moving the sheet to the right, sliding it along under the electrode. The vertical rectilinear currents would flow.

Returning to the form first described, Figure 8a, draw any closed curve on the sheet, such as the dash line shown. Now extend the electrode, Figure 8b, into this new boundary, and terminate the ribbons at the new boundary. The remaining sheet area would carry rectilinear current, it being a completely undisturbed portion of the original current.

The description of the magnetic analog for this case may be of interest. Let Figure 8b be viewed in new terms. We have here now a long iron pipe of infinite permeability: the previous electrode becomes the section of the of the pipe wall. The inner medium is air. Run a system of small parallel wires along the wall, a wire being placed at each former terminal of a ribbon. Let all wires at the right carry equal current away from us,

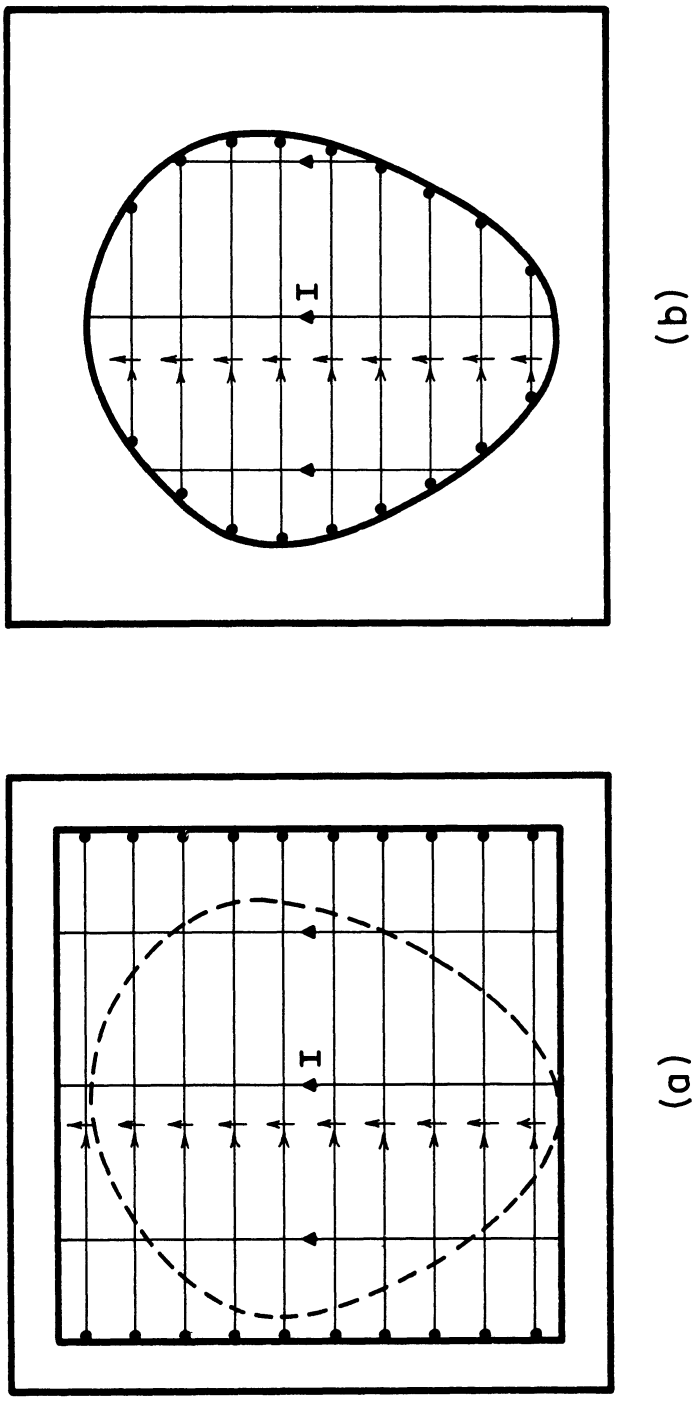


Figure 8. Uniform ribbon stack. (a) Square conducting sheet, shortcircuited all around, uniform current. (b) As before, but with boundary having any curved shape: current still uniform.

while equal currents return in the wires at the left. The currents would produce a uniform vertical magnetic field in the air. This is true, quite irrespective of the shape given to the pipe's inner boundary.

In Figure 9d the rectangular sheet has top and bottom electrodes, and a uniform ribbon stack installed at the left in only part of the sheet. With ribbons moving to the left, their emfs would be down. The current is shown as uniform and down through the stack; and uniform and up at the right of the stack. Can this simple conformation be validated?

It does call for justification, and for a very good reason. Sketch any closed curve in the central region, with the line passing through some of the ribbons, but otherwise passing through the blank area to the right. It is a tentative current path, and it does have a net emf acting around it. Thus, it is entirely in order to assert that the current conformation may be curvilinear, not rectilinear. A proof to the contrary is called for.

At least two proofs of rectilinearity can be adduced, one of which makes use of the other sketches of this figure. Take a "one-ribbon stack", Figure 9b. Let it be a 6-volt ribbon. Assign zero potential to the lower electrode. To make the values easy to follow, let the ribbon run for one-third the width, and the blank part be two-thirds of the width. Let the total current path resistance be 6 ohms. There would be 2 ohms offered to the return current flowing up at the right, and 4 ohms to the current down through the ribbon. Starting at the right with zero electrode potential and going with the current, potentials at important locations are seen tobe zero, -1, -2 at the upper electrode; coming down at the left, -4 just above the ribbon, +2 just below it, and back to zero at the lower electrode. An

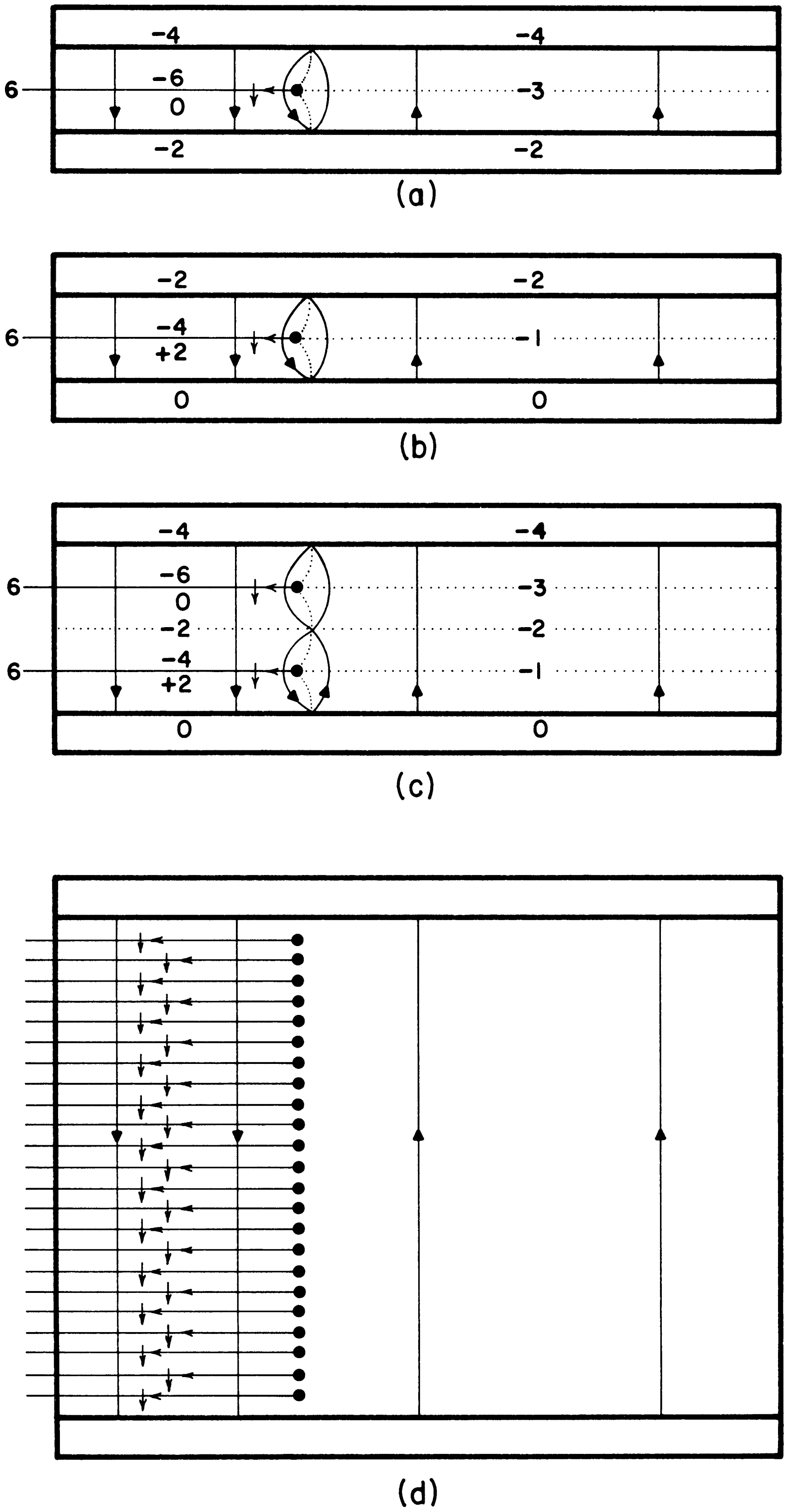


Figure 9. Ribbon stack in rectangular sheet, with top and bottom electrodes. (a),(b) Single ribbons in the slices they occupy, and the potentials. (c) Foregoing combined; process continued leads to ribbon stack. (d) The stack sets up rectilinear currents (electrodes not connected).

identical stack is seen in Figure 9a, except that we give its lower electrode the potential of -2. Then its potentials would be as shown.

It is now evident that if the upper electrode of Figure 9b and the lower one of Figure 9a are removed, and these two stacks are fitted together, the common new boundary will have, for each, the same potential as before, namely, -2. Nothing would change anywhere, as to potential or current. Thus a stack having any number of ribbons can be built up, and its main current pattern would be rectilinear. As the sketches show, there would be local curvilinear currents in the immediate vicinity of the ribbon terminals, but their importance in the whole case diminishes toward zero as the number of ribbons approaches infinity.

The other proof involves starting with the rectilinear current assumption. Going down through the stack, an equation for the potential at any level can be written. Then, going down through the righthand part, the potential equation can also be written. These equations are identities. Therefore, if the two parts, with their equal currents, were set up independently, they could be joining the potentials at all levels would match, and no change in currents would occur.

THE MHD GENERATOR

The MHD

The MHD generator is a possible power source of the future, and is presently the subject of active interest and research. In Figure 10a (not to scale) MNOP is the section of a long duct, a hot ionized gas being forced along the duct. The generator's magnetic field (not shown) would be set up between pole pieces, across the duct from right to left. The ceiling NO and the floor MP are the electrodes.

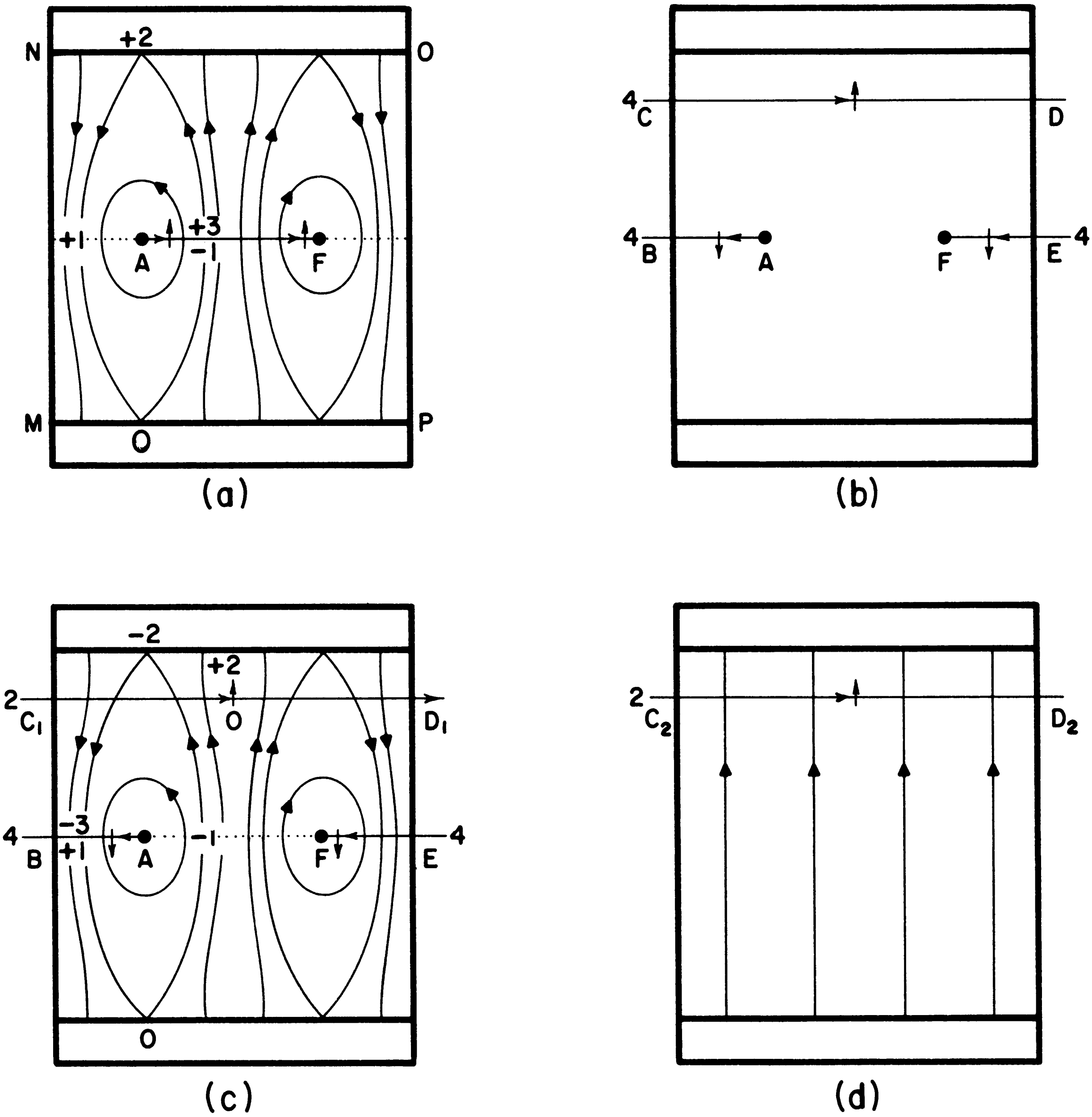


Figure 10. The MHD generator. This figure to make analysis of potentials and currents due to a symmetrically located gas element AF, using RG theory.

If the entire gas flow moved ahead as a block, with velocity everywhere the same, a uniform vertical emf per inch would everywhere be induced. Then there would be no circulating or eddy currents, any load current would be rectilinear in going up through the duct, and the analysis would be very simple indeed.

However, the laws of fluid mechanics will continue to apply. With the magnetic field turned off, fluid mechanics only will dictate the flow. In turbulent flow, maximum gas velocity is at the center; the velocity falls off slowly going from the center until the walls are approached; then falls off faster and faster, to become zero at the walls. The analysis of electromagnetic phenomena will be quite complicated, and the prospect of making an analysis depends on being able to analyze the phenomena due to a simple gas element. RG concepts offer a clear-cut way to make the analysis. (With field on, forces developed by currents in the gas will modify the foregoing flow description. This will be mentioned later.)

RG Theory Applied to a Gas Element

The gas element chosen for analysis, Figure 10a, is a thin layer of gas AF, uniform in thickness, and moving at uniform velocity. Such an element develops a uniform emf everywhere within itself. The MHD case will now be converted to a RG case. The element of gas becomes a ribbon generator; the ductful of gas becomes the RG conducting sheet; the flux of the MHD is wiped out, and replaced by the RG flux field into the paper through the sheet. The electrodes remain.

At no load, the ribbon AF would set up the eddy currents shown in this figure, upward through the ribbon, downward outside of the ribbon, and with some of the current using the electrodes as return paths. This case is to become a simple numerical example. Therefore, A and F were taken at the centers of the two halves, thereby reaping the advantage of symmetrical current in each half: it becomes easy to study the potentials.

Let AF be a 4-volt ribbon, and assign zero potential to the lower electrode. By symmetry, we can see that the total IR drop in the left current circuit divides into one-volt drops in each of the 4 quarters of the current field. Thus, the potentials displayed in the figure are easily found.

Now, the AF ribbon may take any path from A to F (if electrodes are respected) and not change the current pattern. Suppose we take the ribbon from A to the left horizontally to just inside the boundary; up along the boundary a short distance; back horizontally across the sheet to just inside the right boundary; down inside the boundary to the middle; then horizontally left again to F. But the two boundary ribbon parts can set up no current, and can be discarded. As to the remaining ribbons, extend them through the boundaries, Figure 10b, and any currents will be the same as if produced by the original AF ribbon. Now, at no load, electrodes unconnected, CD is a through-ribbon, setting up no current. Thus, all of the current shown in Figure 10a and repeated in Figure 10c is due entirely to AB and EF.

The potentials due to AB and EF are next found, Figure 10c. Since the ribbon path was changed, some new potential values are to be expected. The new values are shown: taking the same route as before, we read zero, -1, -2 for the top electrode, -3 and +1 for the two sides of AB, and back to zero again.

The electrodes again have a potential difference, and if AB and EF only were operated, they would send current through a load if the load were connected to the electrodes. In order to take advantage of a simplification that will appear later, this must be prevented. It can be prevented. In Figure 10c, install the 2-volt C_1D_1 ribbon that is shown. This is half of the full CD ribbon; it can be thought of as being half as thick as the CD ribbon. It will be called the *no-load ribbon*. At no load, as a through-ribbon, it sets up no current, but does make potential plateaus of its own: zero below it, +2 above it. Whereupon, this no-load combination, as we will call it, now brings the upper electrode to zero potential. *Therefore, the no load combination* (AB, C_1D_1, EF) *can put no current through a load*, even if the load is connected. All it can do is to circulate the component field of current shown here.

It remains, Figure 10d, to complete the story by installing the other half of CD, the C_2D_2 ribbon, also a 2-volt ribbon. First, it is a through-ribbon, and can make no current at no load. Second, it can only cause load current, and hence will be called the *load ribbon*. Third, *the load current component is all uniform*, as shown.

Losses, and Voltage Characteristic for the Gas Element

The real, or resultant current pattern, would be the resultant of the circulating current and the rectilinear load current. With every increase of load current, a new conformation would appear. Any attempt to analyze a whole series of different current conformations in order to find the terminal voltages and losses, would involve much work. That work can be avoided. By the foregoing treatment, the circulating currents have become a fixed entity as to shape and amount; and the load current has the simple rectilinear conformation.

The no-load combination, Figure 10c, places both electrodes at zero potential, thus making no contribution to terminal voltage. It is the C_2D_2 load ribbon only, a 2-volt ribbon, that gives the 2 volts of no-load voltage. Loaded, the terminal voltage would be 2 volts, minus the IR drop up through the RG sheet due to load current. It is a straight line voltage characteristic.

To get at losses, the question is now raised - With the no load combination of Figure 10c in action, will those ribbons, now accounting for power lost at no load, exhibit any new net power occasioned by letting load current flow through? The answer to negative. The 2-volt C_1D_1 ribbon will act with all of the load current to have generator effect, and extra power would be required to move it. The two 4-volt ribbons, AB and EF, act with half of the load current to have motor effect, with the motor power helping to move them. Comparing the lengths of the ribbons involved, and also the thicknesses, it is seen that the mechanical powers described are equal, and no new net power appears. (These powers can also be compared in electrical terms, as will be done later.)

Therefore, when we let the 2-volt load ribbon C_2D_2 operate to set up load current, the only new power that appears is due to it: 2 volts, times load current. Part of that power is loss, being I^2R.

Thus RG theory effects a striking simplification. Instead of making detailed studies of new current conformations, it comes out that the total loss at any load is merely the loss at no load, plus I^2R due to load current.

More General Analysis of the MHD

In the numerical example, the original AF ribbon was taken half as long as the duct is wide. Let half the MHD duct, Figure 11a, or the corresponding RG sheet be taken as unity. And now, the gas element or its corresponding AF ribbon may have any horizontal dimension. Let the AB and EF lengths each be x. In this figure, the AF ribbon has already been replaced by AB, CD and EF, except that of the CD ribbon, only the no-load ribbon C_1D_1 is shown.

Also in the numerical example, the zero net power effect of load current on the no-load combination was argued via mechanical power. It can just as well be argued in terms of electrical power, as is done below. The problem next is to find the important relationships in general terms for any value of x.

Let E = emf of AF, or of AB, CD, EF

E_1 = emf of C_1D_1, the no-load ribbon

E_2 = emf of C_2D_2, the load ribbon

V_o = no load terminal volts

V_t = terminal volts at any load

I = load current

R = vertical resistance of duct or sheet.

Now, Ix = the part of load current crossing AB and EF,

EIx = motor power, watts, developed by AB and EF.

E_1I = generator power, watts, developed by C_1D_1, since all of the load current crosses it.

In the no-load combination these two powers are equal and cancel out to zero. To find E_1 of the no load combination, place them equal:

$$E_1 I = EIx \qquad (1)$$
$$E_1 = Ex$$

Since the CD ribbon of E volts is split into the no-load ribbon of E_1 volts and the load ribbon of E_2 volts, and since the no-load combination contributes zero potential difference to the electrodes, the terminal voltage V_o at no load is due only to C_2D_2, or

$$V_o = E_2 = E - E_1 = E(1-x) \qquad (2)$$

and the terminal voltage at any load, V_t, is

$$V_t = V_o - IR \qquad (3)$$

In the numerical example, it is possible to write the several potentials around the circulating current path, and show in detail that two outcomes of adopting the no-load combination were first, that this combination contributes no potential difference to the electrodes; and second, that its total interaction with the load current yields zero net power. It was possible to do so, even without knowing the curvilinear current conformation in detail, because of symmetry. That symmetry is now lost. So now, in order to find the upper electrode potential directly from the current conformation, it would first be necessary to solve for that conformation. But the point is, we need not do that: the RG treatment derived above automatically yields a no-load combination having zero potential between electrodes. This is true because, if the no-load combination did set up such a potential difference, then the load current would be bound to interact with it to produce or consume power. Therefore, the RG arguments are able to derive the above relationships without having to deal in any way with the actual no-load circulating current conformation.

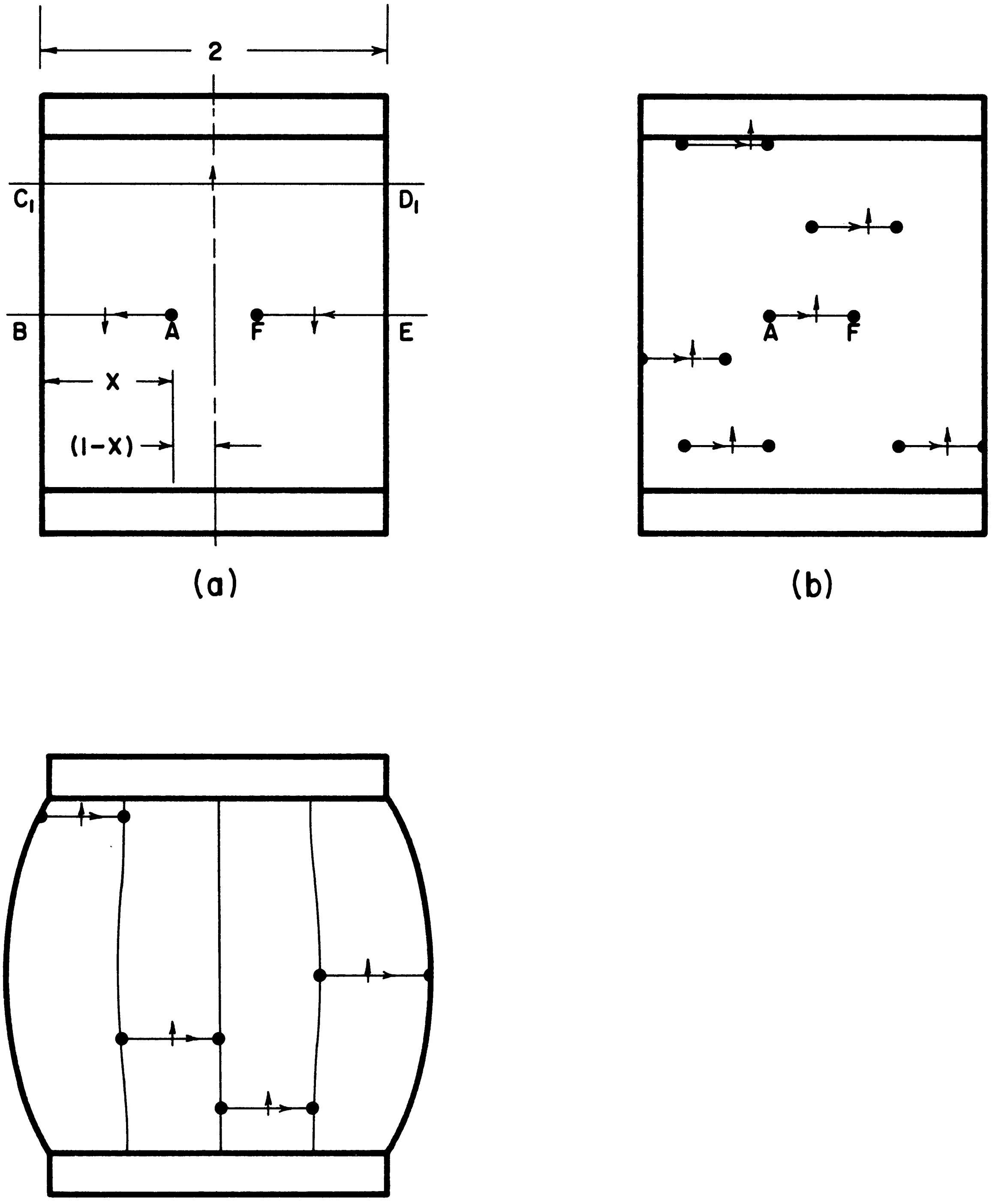
Analysis Further Generalized

In the foregoing, the AF ribbon could have any span, but it was kept to the central and symmetrical location of Figure 11a. What would happen if it is now displaced horizontally, or vertically, or both? In Figure 11b, a number of AF ribbons, all alike, are shown at radically different locations. Anyone having much familiarity with fields would hardly expect all of these ribbons to deliver the same no-load voltage at the terminals, and to have the same voltage characteristic. It is surprising, then, to find that this is to be the outcome. It is easily seen that the derivations of the preceding section as applied to Figure 11a, apply also to any of the new ribbon locations. If the length of AF remains constant, then the total length of AB and EF as crossed by the load current remains constant, the no-load combination is unchanged, and each of these AF ribbons contributes the same potential difference to the electrodes. This is all the more interesting, in that as AF is relocated, the circulating current component may change radically; and along with that, the no-load loss can change very considerably.

A completely independent RG proof of the above finding has been worked out by the writer. Space does not permit its inclusion here.

No-Load Voltage Proportional to Average Velocity

In the MHD generator, Figure 11b, let the gas velocity at first be the same everywhere. The gas flow can be divided into an infinite number of differential elements, all alike. Any one, in the RG analysis, is now an AF ribbon generator, where AF is infinitely short. Each contributes the same infinitesimal potential difference to the electrodes, irrespective of position.



(a)

(b)

(c)

Figure 11. The MHD. This figure used through RG theory, to derive more general analyses of effect of gas elements.

Going back to the MHD, we now move the little flow elements, any number of them to any new positions selected: any new velocity distribution can thus be achieved, without changing the average gas velocity. Making the corresponding changes in the RG set-up, it comes out that the no-load voltage between electrodes is unchanged.

That is, the no-load voltage of the MHD depends only upon the average gas velocity. This, be it noted, depends on having a rectangular duct. With such a duct, it turns out that the MHD at no load can be considered as an electromagnetic flowmeter.

RG Analysis and the Electromagnetic Flowmeter

The electromagnetic flowmeter harks back to Faraday as to basic ideas, but actual development took place only in recent years. Alexander Kolin pioneered its use for blood flow in physiological research. The typical form is a round duct of insulating material with a conducting fluid flowing along it. There is a uniform crosswise magnetic field. The tube is pierced by small electrodes at top and bottom. Potential difference picked up at the electrodes is proportional to average fluid velocity, provided the velocity distribution is symmetrical about the tube axis.

Extension of the meter to industrial use is still more recent. As far as this writer knows, the round duct has been retained. If the rectangular duct has not been given attention, it might well be, for the above analysis of the rectangular-duct MHD shows that it would measure the average velocity, even when the flow is not symmetrical about the duct axis. This would be true for purely axial flow. If there are crosswise components, the horizontal components, in line with the field, would produce no emf. Vertical flow components would produce emf; these, if symmetrical, would introduce no error; if unsymmetrical, proper design might make any error negligible.

The rectangular-duct design, with flat electrodes at top and bottom, offers attractive advantages. Replacing the round duct with a tall, narrow rectangular duct of the same section, the same average velocity would produce considerably more potential difference at the same field density; and the shorter gap would permit a higher density to be used.

Incidentally, the vertical crosswise velocity components in a MHD would add to the no-load loss.

MHD with Non-Rectangular Duct

It is impossible at this early stage, to foretell the optimum design features of the MHD. It may turn out to have an odd shape of duct section: perhaps pinched in; or, as in Figure 11c, bulged out. If so, the argument above, using Figure 11b can be broadened. In Figure 11b the like gas elements contributed equally to the no-load voltage, irrespective of location. This would not be true in Figure 11c. But if we map the load current which is now curvilinear, and, say, divide it into four equal parts as shown, then we can compare the several gas elements that are also shown. They are alike except for horizontal dimensions. What they have in common is that each spans the same amount of load current. If so, they contribute equally to the no load voltage. And if, as shown, each spans 1/4 of the no-load current, and each generates E volts, each contributes E/4 volts to the no-load voltage.

MHD Performance for a Uniform Block of Gas

Before expanding the RG treatment of the MHD to go beyond the single gas element, it is in order to discuss velocity distribution more explicitly. The first thing to recognize is, that within the RG analysis offered here, the load current is all uniform, Figure 10d; it would develop a uniformly distributed back pressure on the gas throughout the duct; and therefore, would have no effect whatever in changing the flow from that which is dictated by other considerations.

It is the circulating component, Figure 10c, that would effect a redistribution of velocities from that dictated by fluid mechanics alone. The upward currents between A and F, Figure 10c, would give back pressures, slowing down the gas; and the downward currents yield forward pressures, speeding up the otherwise lagging velocities toward the sides. Thus, electromagnetics will no doubt radically affect the flow pattern that otherwise would occur. It will tend to yield a more uniform set of velocities. Near the electrodes, this smoothing-out effect will be less pronounced. Since circulating current distribution and gas velocity distribution are interdependent, the business of trying to predict the ultimate distribution will no doubt be rather involved.

To investigate the MHD as a whole, consider its performance in terms of a uniform block or mass or slug of gas. In the insert, Figure 12 (not to scale) the duct section is GHIJ. The block of gas has the section KLMN. In the following study the block of gas has the same velocity throughout this section; and gas velocity is zero between it and the side walls.

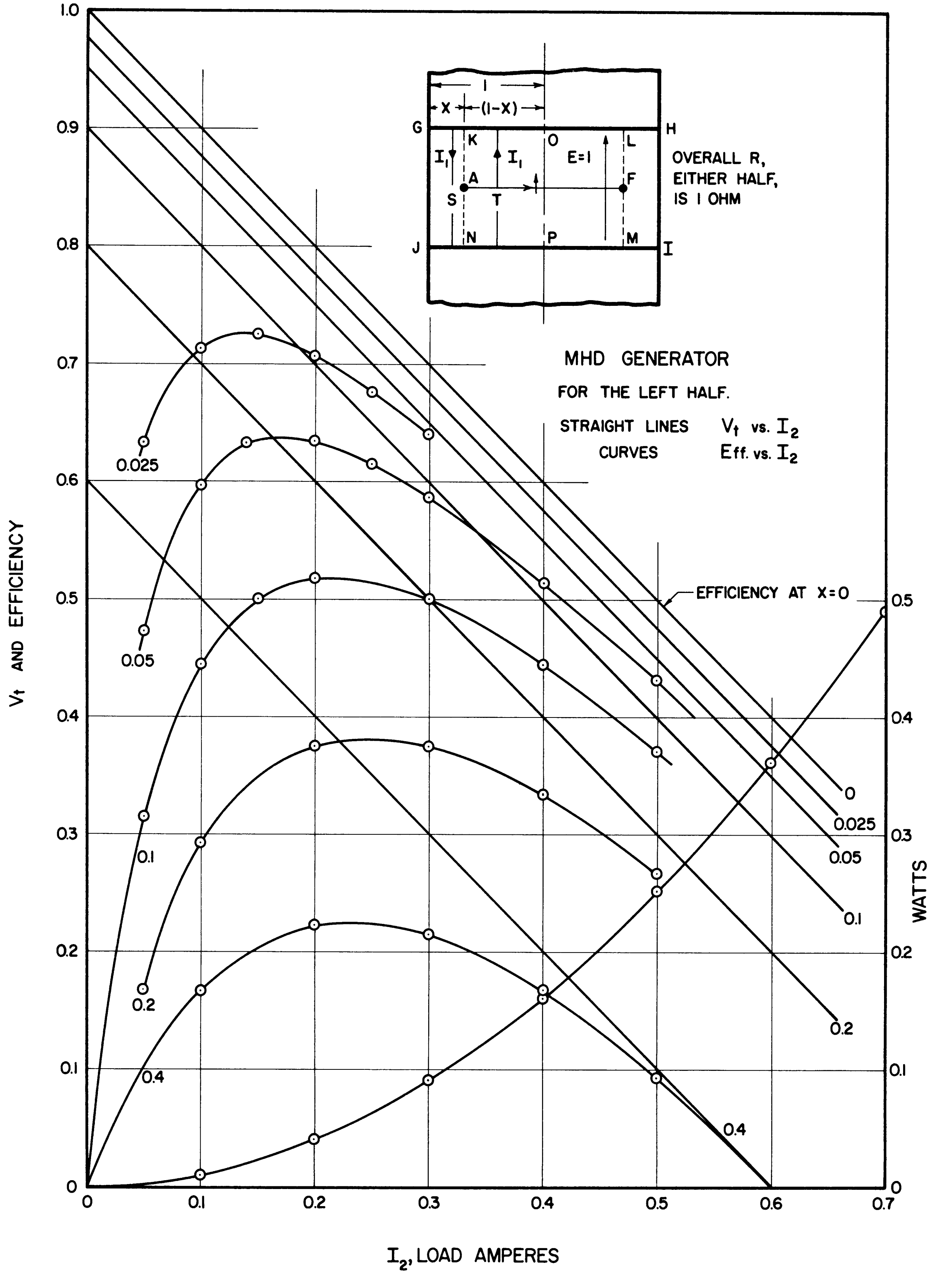


Figure 12. Voltage characteristics and electrical efficiencies of a rectangular-duct MHD generator, as due to a block of gas of uniform velocity, of differing widths. The gas block is KLMN in the insert.

The study will be carried out in the left half only. The half-duct width is taken as unity. The width KO in half the block is (1-x), and the side remainder of no flow, GK, is x. The vertical resistance R is now the resistance up through half of the duct or sheet. The half-duct is divided by KN into two columns, S and T.

Converting the MHD into a RG situation, any single element of gas AF, Figure 12 insert, becomes the ribbon AF moving to the right. The entire block of gas corresponds to a complete and uniform ribbon stack reaching from one electrode to the other, the bottom ribbon being along NM, and the top ribbon along KL. As already developed herein, such a ribbon stack, at no load, would set up rectilinear current going up through the stack, and returning rectilinearly down at both sides outside of the stack. This is the circulating current.

Each AF ribbon in the stack is now replaced by the ribbons AB, C_1D_1, C_2D_2, and EF, as in the treatment of Figure 11a. If the entire original AF stack develops a total of E volts, then the entire stacks AB and EF also develop E volts. The left half of the case looks like Figure 9d.

Let R = vertical resistance, left half

I_1 = circulating current, left half

I_2 = load current, left half

P_1 = no-load loss, watts, left half

P_2 = load loss, watts, left half

W = output, watts (total MHD output would be 2W)

Columns S and T have resistances, respectively, of R/x and R/(1-x) ohms. The resistance offered to I_1 is the sum of the two.

$$I_1 = \frac{E}{R/x + R/(1-x)} = \frac{E}{R} x(1-x) \qquad (4)$$

$$P_1 = EI_1 = \frac{E^2}{R} x(1-x) \qquad (5)$$

$$P_2 = I_2^2 R \qquad (6)$$

$$E_1 = Ex \qquad (1)$$

$$V_o = E(1-x) \qquad (2)$$

$$V_t = V_o - I_2 R \qquad (7)$$

$$W = V_t I_2 \qquad (8)$$

$$\% \text{ Efficiency} = 100 \frac{W}{P_1 + P_2 + W} \qquad (9)$$

If, in the above equations, unit value is assigned to E and R, the important performance characteristics can be worked out for selected values of x, on a per-unit basis. The curves resulting therefrom are shown in Figure 12.

The topmost straight line shows the limiting case, when x = zero. The block of gas then completely fills the duct, there is no circulating current and no loss at no load; and this line gives both the terminal voltage and the efficiency. For all other values of x, the straight lines give terminal voltage, and the curves give the electrical efficiency.

Efficiency Outlook for the MHD

The actual velocity pattern in the MHD may turn out to be quite complex. However, it can be seen from the Figure 12 curves that if the equivalent of a uniform block as described can be achieved, with dead spaces at each side in which x = 0.05, a maximum electrical efficiency of 64% can be had. If x = 0.1, the maximum falls to 52%.

The gas block described leaves no dead space at top and bottom. But there will inevitably be reduced velocities near the electrodes, and these will further reduce the efficiency by adding to no-load losses.

RG ANALYSIS AND PHYSICAL ANALOGS

Adapting an RG Analysis for Use in Physical Analogs

After a given situation has been analyzed in terms of RG theory, there may still remain the problem of somehow getting a complete solution. The MHD, for example, has been converted above into a RG case; but since ribbon generators are conceptual, there is little or no hope of building the RG equivalent itself and making it give the answers - such as, what are the circulating currents like, and what are the no-load losses, for a given velocity distribution of complicated character? But if the RG analysis can, in turn, be converted into a feasible physical analog, then solutions can be achieved.

In Figure 13a are three ribbons of different length, but alike in developing equal emfs. They are intended to be very close together, or even superimposed; but they are vertically spread apart here so they can be seen. Taking the top ribbon for discussion, it sets up a current field component, the form of which would depend on the boundary and boundary conditions: see Figures 4c, 5a, 7a and 7b for examples. Also, as indicated by these figures, and as brought out in the discussion of Figures 7a and 7b, a ribbon can be replaced by a source at one terminus and a sink at the other, these being equal; and thus, we may either set up the same current conformation; or, as the case may be, set up the conjugate system. Thus ribbons may be replaced by paired sources and sinks.

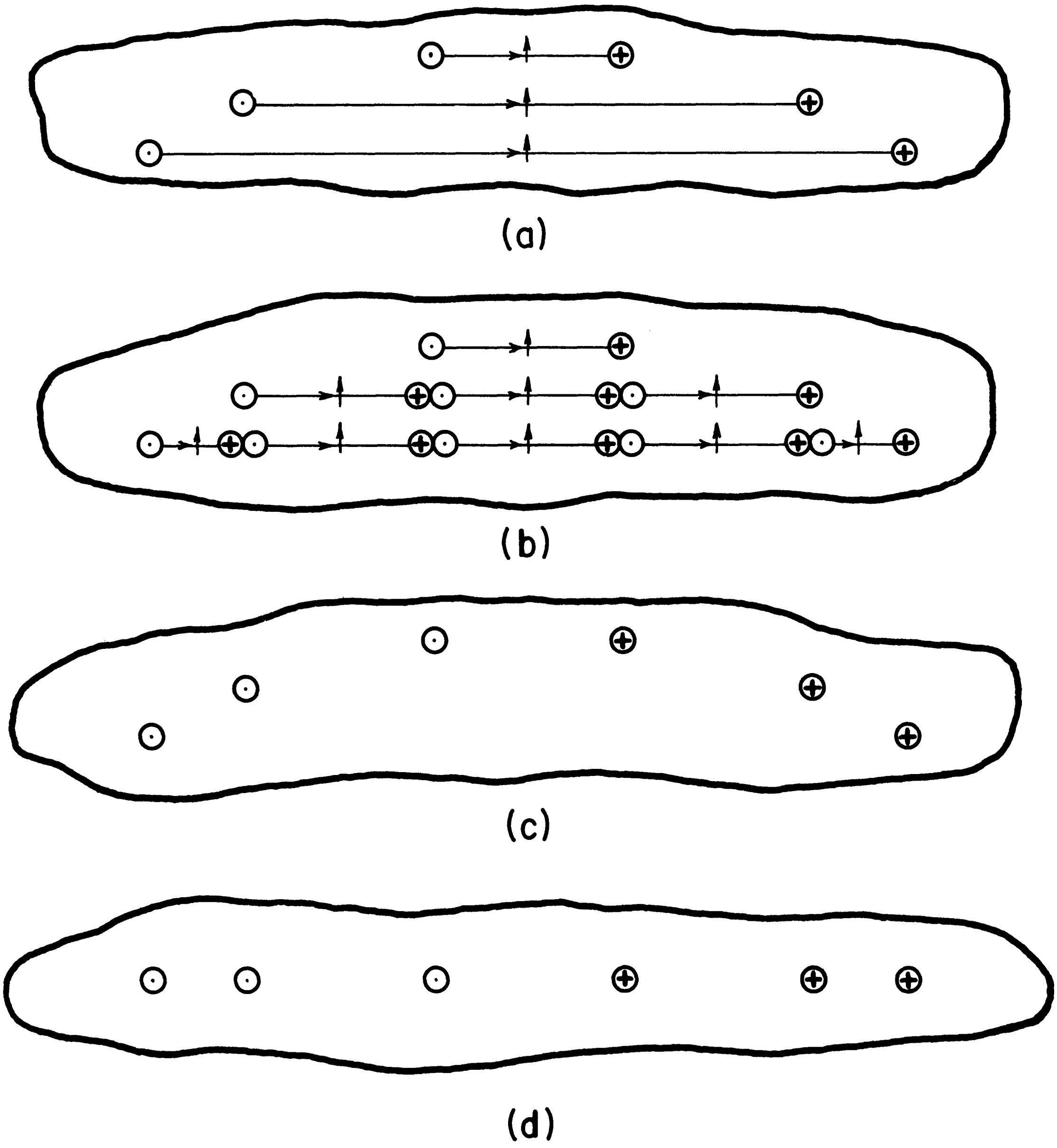


Figure 13. Three ribbons, actually close together or superimposed, replaced by equal sources and sinks, prior to setting up a physical analog for solving for fields analyzed by RG theory.

If this replacement is done immediately, we go directly from the ribbon array of Figure 13a to the source and sink array of Figure 13c. However, cases may arise in which an intermediate step may aid the understanding. In this step, Figure 13b, the middle ribbon has been divided as shown into 3 segments, and the lower one into 5 segments. This makes no change of current or potential. New additional sources and sinks have appeared. But if those shown close together are really superimposed, then, in any group, sources and sinks cancel out to leave only a single one at each position. The result is Figure 13c. Next, with ribbons close together or superimposed, the sources and sinks would be in line, Figure 13d. If these sources and sinks amount to equal currents forced to appear by way of small electrodes on the conducting sheet, it is now equipped with electrode arrays rather than ribbon arrays. Exploration for equipotentials would yield a pattern which duplicates, say, the circulating current of an MHD.

The analog can be dry, using conducting paper; or wet, using the electrolytic tank. Or again, it can be a fluid mapper, in which the electrodes are replaced by tiny wells of equal-flow rate: in which case, the flow pattern gives equipotentials, and the conjugate system to be drawn in would show the circulating MHD currents.(2)

To illustrate the foregoing and develop the routine, the MHD generator, Figure 14a, is taken as the example. The closed curves in this sketch, be and cd, roughly represent boundaries of isovelocity areas is the rectangular duct GHIJ, for flow unchanged by electromagnetic forces. They are chosen so as to divide the flow section into areas 1, 2 and 3, each of which is taken as an isovelocity section having uniform velocities proportional to the values, respectively, 1, 2 and 3. Next, the dotted horizontal lines, equally spaced, divide the crosswise MHD flux into equal increments.

The central increment is chosen for illustration, and repeated in Figure 14b. The emf's due to the several gas velocities can be duplicated when the case is converted to RG terms, by 3 ribbons, such as are shown in Figure 13a. When these ribbons are replaced by paired sources and sinks, the resulting array is the ABCDEF array, Figure 14b.

Now, in this inversion of ribbon to source-and-sink, the whole case must be inverted. Former current filaments become equipotentials, and vice versa. And there must be a boundary inversion: a boundary part having an electrode attached must have it removed; and a boundary part without an electrode, must have one attached. If there is a hole in the conducting sheet, it must be replaced by a resistanceless electrode, or vice versa; and so on. So now, Figure 14a, the electrodes are removed from top and bottom, and new electrodes appear all along the two sides. Whereupon, the two outer electrodes a and f, source and sink, being attached to the side electrodes, have no influence on the sheet's current, and are discarded. The source and sink array reduces to bcde.

The routine is now clear. Prepare the physical analog (say, conducting paper), Figure 14a, by noting where the center lines of the MHD flux increments cross the curves, and make a small dot at each intersection. All lefthand dots are now turned into equal-current sources; all righthand dots, into equal-current sinks. In an actual case, more iso-velocity areas than are shown here could be used, for higher accuracy. When the analog is explored for equipotentials, the pattern will appear on which to draw the conjugate system - the circulating current field.

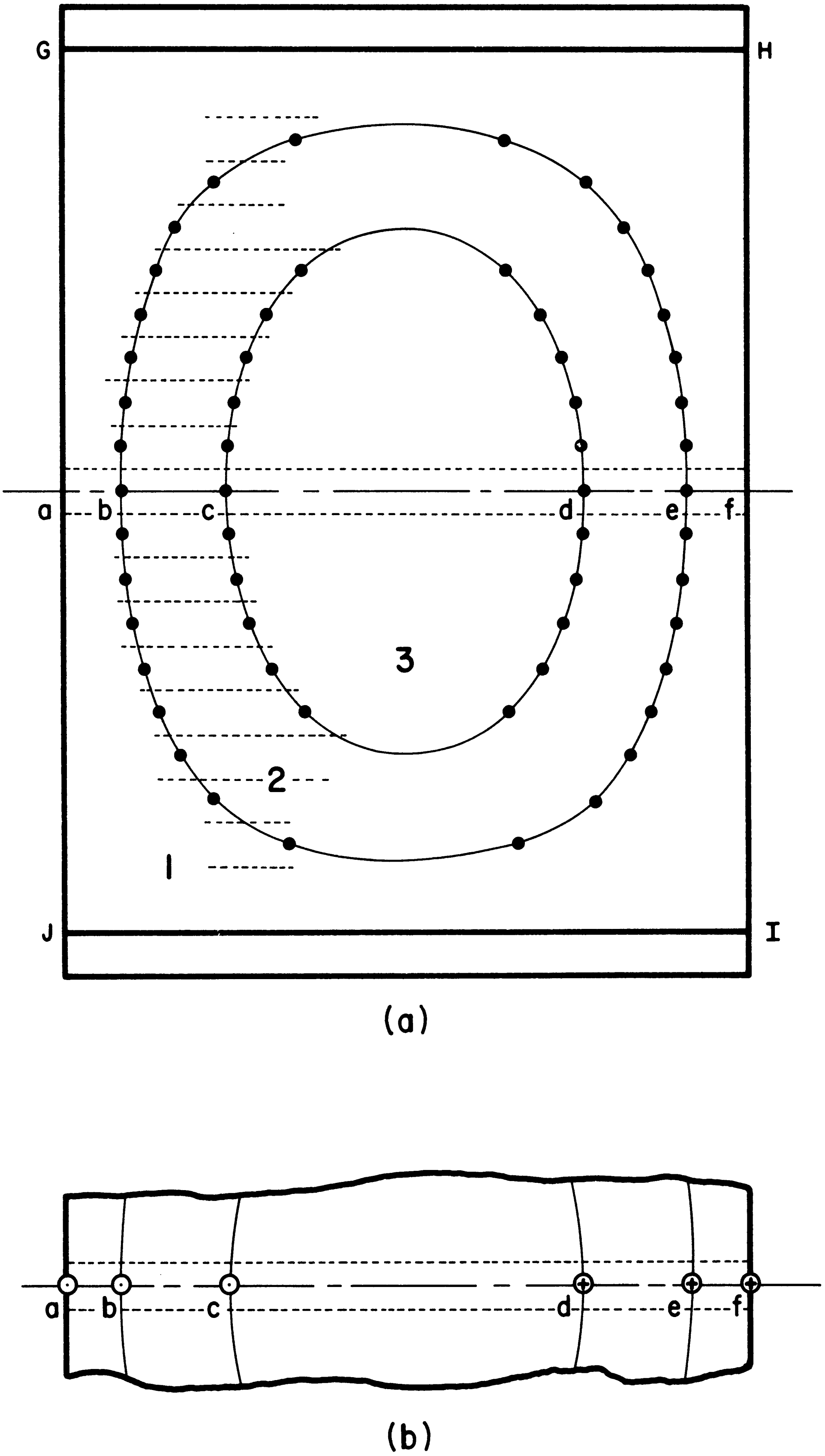


Figure 14. Using the MHD to illustrate routines for setting up the physical analog.

It can then be studied in detail sufficient to find the circulating current loss. Using a fluid mapper as the analog, the top and bottom would have barriers, and the sides would be open to tray water.

Other Applications

The electromagnetic pump and the electromagnetic flowmeter also are devices in which emf's distributed throughout a conducting mass set up circulating currents affecting the design and the performance. The unipolar generator is coming to life again, with at least two companies now making low voltage, extremely high current generators for research purposes; and it has a circulating current problem. The engineering design of such devices, including the MHD, calls at the least for a qualitative understanding of these phenomena; and at the best, asks for numerical evaluations. It may be that analysis by RG concepts may facilitate both.

A case of fluid circulation occurs in the aluminum reduction furnace, in which the bed of molten aluminum under the cryolite blanket is circulated by electromagnetic forces caused by the very heavy currents. Some circulation may be desirable; too much can lead to trouble. In this case, it is distributed forces making metal flow, rather than distributed emfs making current flow; but the writer has been able to stretch RG theory to cover the situation, and by means of a special fluid mapper, determine the general pattern of circulation with what seems to be fair accuracy.

CONCLUSIONS

1. Ribbon generators are electrical concepts that appear to be new. They can be manipulated with great flexibility for setting up potentials and current fields in conducting sheets.

2. RG concepts offer a means of grasping the meaning of potentials in potential fields, and the implications of the Principal of Superposition, perhaps better than by most other processes.

3. The several potential fields, such as the heat conduction, the damping eddy current, the electrostatic, and the magnetic field, can analogously be studied and analyzed by RG concepts.

4. Experienced teachers know that the student finds the magnetic field the most difficult to grasp. There is reason to hope that RG interpretations may ease the path to understanding.

5. In devices such as the MHD generator, the unipolar generator, the electromagnetic pump, the electromagnetic flowmeter, and disks damped by eddy currents, emfs set up throughout some or all of a conducting mass give rise to circulating currents which, in turn, affect the design and the performance of the device. The problems presented by such currents have been largely avoided in the past. RG concepts open up the analysis of these relatively obscure phenomena by methods that are simple, cleancut, and rigorous.

6. Ribbon generators, being conceptual, may never become realities, to use as physical analogs to reach solutions. But analyses reached through their use are readily converted into the physical analog forms of the conducting paper type, the electrolytic tank, and the fluid mapper.

REFERENCES

1. Moore, A. D. "Eddy Currents in Disks: Driving and Damping Forces and Torques," AIEE Transactions, 66 (1947) 1-11.

2. Moore, A. D. "Fluid Mappers as Visual Analogs for Potential Fields," Annals of The New York Academy of Sciences, 60, Article 6, (1955) 948-62.