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THEORY AND DESIGN OF A VARIABLE MACH NUMBER CORNER NOZZLE

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I Summary

Theoretical considerations are presented on the flow in an asymmetric variable Mach number nozzle in which change in Mach number is accomplished by translation of one contour relative to the other. Design criteria for a specific class of nozzles are set down and a procedure for designing a nozzle to cover any specified Mach number range is proposed. The procedure makes use of an iteration process with the method of characteristics. An approximate theory is included which indicates how the first step in the interation procedure may be chosen so as to bring about rapid convergence to a solution. The theory also predicts the overall length of nozzles of the class under consideration as a function of the design Mach number range and expansion angle.

The procedure is applied to the design of a nozzle which, in operation, is expected to cover the Mach number range 1.4 to 4.0. The length of the supersonic portion of this nozzle is approximately 8 test section heights. Considerations on the flow in the subsonic and transonic regions of this nozzle are included as well as the results of calculations of viscous effects for the case where the nozzle is operated with standard atmospheric stagnation conditions.

The nozzle will be evaluated by experimental investigation of test section flow. In order to be able to correct for viscous effects for the entire Mach number range and to have some control over the uniformity of test section flow the nozzle will be constructed with flexible surfaces. The amount and types of flexibility required are discussed.

II Introduction

Several types of asymmetric variable Mach number nozzles have been proposed. Allen (Ref. 1) investigated the flow in an asymmetric nozzle which he considered to be one-half of a two-dimensional plug-type nozzle proposed by A. Silberstein at the NACA Cleveland Laboratory. With this type of nozzle, the test section Mach number is varied by translation of one nozzle contour relative to the other.

Improvements in the method of obtaining the coordinates of the contour of such a nozzle were presented by Syvertson and Savin (Ref. 2), who suggested the use of an inclined and curved sonic line.

Evvard and Wyatt (Ref. 3) investigated the flow in a nozzle whose contours were based on the Prandtl-Meyer theory for flow around a corner.

Ferri, Burbank, and Byrne (Ref. 4) studied the flow in a nozzle similar to the type proposed by Allen with the exception that the sonic line was eliminated as a design parameter. This was accomplished by design of the subsonic inlet so that the sonic line was straight and perpendicular to the nozzle wall at the throat over the design Mach number range.

Nozzles designed by the above procedures have been evaluated by experimental investigation of the test section flow, and were found to perform satisfactorally over a limited Mach number range (Ref. 2,4).

At the time the present investigation was initiated, the upper limit of the Mach number range which had been obtained was approximately Mach 3. The investigation was undertaken to make a study of the general characteristics of variable Mach number asymmetric nozzles, and in particular to obtain a design which would cover the Mach number range 1.4 to 4.0. This report presents the progress made between April and October, 1951.

III Supersonic Portion of the Nozzle.

(a) Design Criteria.

A preliminary investigation (Ref. 6) of the two design methods given in Ref. 2 and 4, showed that the overall length of a nozzle designed by either procedure for a given Mach number range and expansion angle is approximately the same for Mach numbers up to 3. The essential difference between the two design procedures is associated with the shape of the sonic line.

The nozzles considered in this report are of the type proposed by Allen and use translation of the lower contour as means of changing the Mach number. They incorporate the following criteria some of which were suggested by A. Ferri.

1. The sonic line should be straight and perpendicular to the nozzle

wall at the throat.

2. The nozzle contour should have no inflection points i.e., the second derivative should not change sign in the supersonic flow region.

3. The variation of the first derivative of the contour should be continuous and smooth over the nozzle length.

4. No compression wavelets should be introduced in the theoretical design of the contour.

In order to cover a specified Mach number range (M_1' to M_2'), two design Mach numbers must be chosen. Let M_1 and M_2 represent the lower and upper design Mach numbers respectively. M_1 and M_2 satisfy the relations $M_1 > M_1'$: $M_2 < M_2'$. Experimental results (Ref. 4) indicate that the interval (M_1 to M_1') can be made larger than the interval (M_2' to M_2). The respective values for that nozzle are:

$$M_1 = 1.71; \; M_1' = 1.27$$

$$M_2 = 2.63; \; M_2' = 2.75$$

After a suitable choice of design Mach numbers is made, it is assumed that the lower design Mach number M_1 is obtained by a simple wave flow in the nozzle. Therefore, all waves originating at the lower contour are cancelled upon striking the upper contour.

This assumption fixes the expansion angle of the nozzle as that angle of turning required to expand a Mach 1 flow to M_1. Let this angle be called θ_1. Also, let the amount of turning required to expand a Mach 1 flow to the upper design Mach number M_2 in a simple wave flow be called θ_2. Then with the nozzle in position to give M_2, $(\theta_2 - \theta_1)$ degrees of expansion are required by reflected waves to obtain M_2. The assumption is made that the reflection of waves takes place along a straight wall on the upper surface. Both of the above assumptions were shown to be valid in Ref. 4.

On the basis of the above criteria a variable Mach number nozzle can be designed as discussed in the subsequent paragraphs. An approximate theory is developed which provides an initial estimate of the nozzle shape and predicts the overall length of a nozzle.

(b) Initial Approximation of Nozzle Shape.

The supersonic portion of the nozzle in the two design positions is shown schematically in Fig. 1.

Let the distances along the upper and lower contour of the nozzle be designated in the following manner:

L_1 = length of curved portion of lower contour where expansion waves originate.

L_u = length of curved portion of upper contour where expansion waves are cancelled.

R = axial distance covered by the last simple wave in the nozzle in the Mach M_2 position.

P = axial distance covered by the last simple wave in the nozzle in Mach M_1 position.

Z = length of straight portion of upper contour where waves are reflected. i.e., distance between points O and F (Fig. 1a)

h = test section height

$W = h \tan \theta_1$

β_1 = Mach angle corresponding to M_1

β_2 = Mach angle corresponding to M_2

s = distance along nozzle contour

Therefore

(1) $$P = \frac{h}{\tan \beta_1}, \quad R = \frac{h}{\tan \beta_2}$$

Neglecting second order differences we may write for the nozzle in the two design positions:

(2) $$Z + L_u = R + L_1 + W$$

(3) $$L_u = P + L_1 + W$$

Subtracting (3) from (2) yields:

(4) $$Z = R - P = h\left(\frac{1}{\tan \beta_2} - \frac{1}{\tan \beta_1}\right)$$

This determines the position of point F relative to the throat for M_2

Now consider the nozzle in position to give M_2, Fig. 2. The Mach number at the point F on the upper surface, M_F, is determined as the Mach number obtained by an expansion of $(\theta_2 - \theta_1)$ from Mach 1. The Mach number at point B on the lower surface is M_2. We consider the case where the last reflected wave leaves the upper surface at F, strikes the lower surface at B and is reflected from the lower wall as the last simple wave in the nozzle.

Consider the wave crossing the nozzle from F to B (Fig. 2). It can be shown that to a fair degree of approximation, the curved wave can be replaced by a straight wave making the angle,

(5) $$\beta_3 = \frac{\beta_F + \beta_2 + \theta_1}{2} \quad (\beta_F = \text{Mach Angle for } M_F)$$

with the direction $\overline{OF}$ and having the length ℓ. The length ℓ can be determined from the condition of continuity.

Let the length of the last wave in the nozzle be ℓ_2. It is determined by the relation

$$\sin \beta_2 = \frac{h}{\ell_2} = \frac{1}{M_2}$$

or

(6) $$\ell_2 = hM_2$$

The mass crossing the wave ℓ_2 per second is

(7) $$Q_2 = hM_2\rho_2 c_2$$

where

ρ_2 = density at M_2

c_2 = speed of sound at M_2

The mass crossing the wave l per second, to a sufficient approximation, is given by

(8) $$Q = \frac{1}{2} (\rho_{1/4}\, c_{1/4} + \rho_{3/4}\, c_{3/4})$$

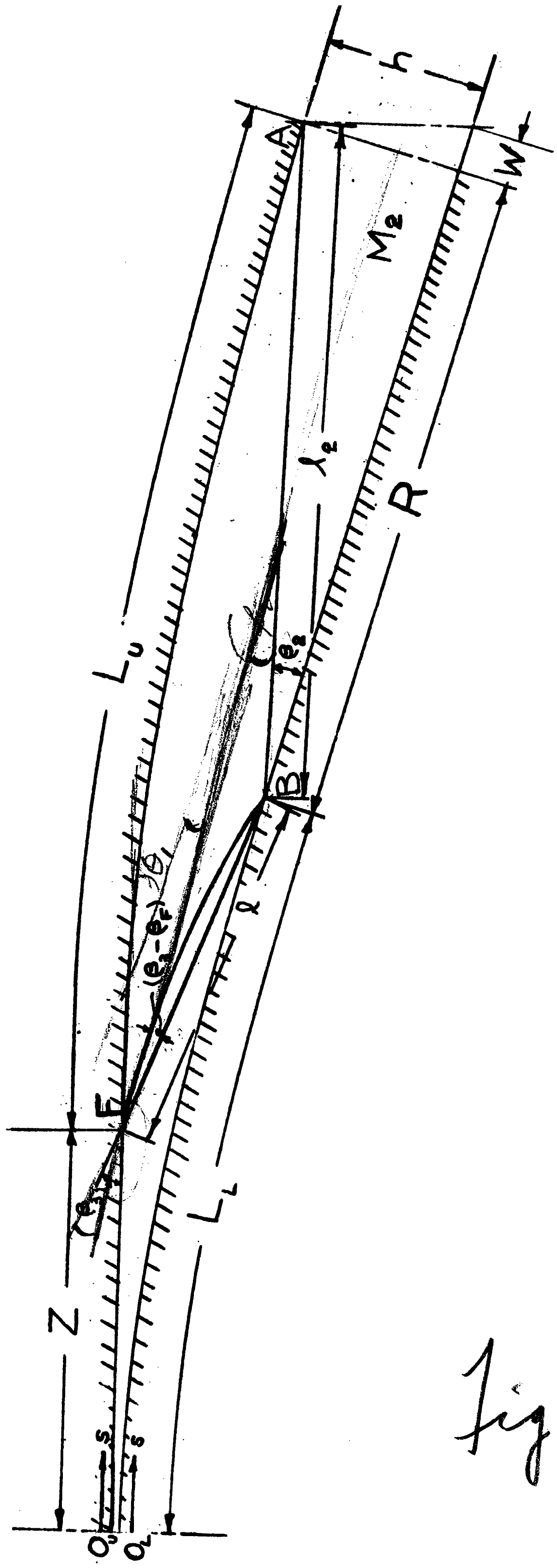


Fig 2

where

$\rho_{1/4}$ = density at $M = M_2 - 1/4(M_2 - M_F)$

$c_{1/4}$ = speed of sound at M = "

$\rho_{3/4}$ = density at $M = M_2 - 3/4(M_2 - M_F)$

$c_{3/4}$ = ~~density~~ speed of sound at M = "

Since by continuity $Q = Q_2$ we have

(9) $$\ell = \frac{2hM_2\rho_2c_2}{\rho_{1/4}c_{1/4} + \rho_{3/4}c_{3/4}}$$

Hence the position of point B relative to point F is known from eqs. (5) and (9). Equation (6) defines the location of point A relative to B. Point A is located at the nozzle exit.

The overall length, L, of the nozzle is then given by the equation:

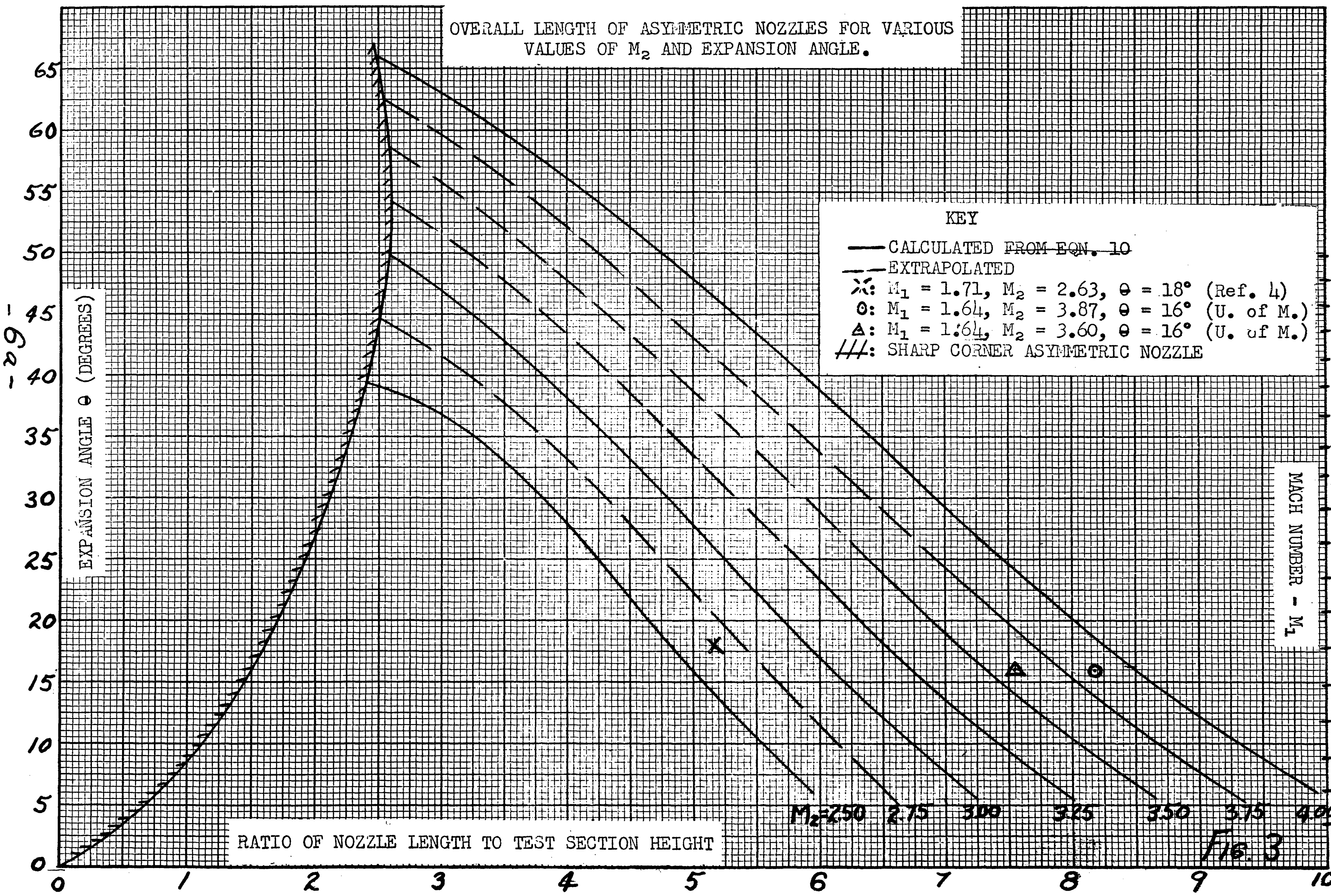
(10) $$\frac{L}{h} = \frac{Z}{h} + \frac{1}{h}\cos\beta_3 + M_2\cos(\theta_1 - \beta_2)$$

Fig. 3 shows the variation of overall nozzle length as a function of expansion angle for various upper design Mach numbers, M_2. Since expansion angle determines the lower design Mach number M_1, the ordinate is shown as both expansion angle θ_1 and lower design Mach number M_1.

Since the locations of points F, B, and A are known relative to the throat, they serve as an initial approximation to the nozzle shape. The distribution of curvature along the upper surface between points F and A and along the lower surface between points O_1 and B remains to be determined.

(c) Curvature of Nozzle Contours.

The next step in the design is to determine the distribution of curvature along the lower surface from O_L to B and along the upper surface between points F and A, (see Fig. 2), such that the exit flow is uniform for the two design Mach numbers M_1 and M_2. This is accomplished by means of an iteration procedure using the method of characteristics (Ref. 6). A curvature distribution is assumed for the lower surface between points O_1 and B. Let this distribution be $K_1(s)$. Using $K_1(s)$ along the lower surface, a distribution of curvature is found along the upper surface between F and A which


OVERALL LENGTH OF ASYMMETRIC NOZZLES FOR VARIOUS VALUES OF M_2 AND EXPANSION ANGLE.
KEY
CALCULATED ~~FROM EQN. 10~~
EXTRAPOLATED
X: M_1 = 1.71, M_2 = 2.63, θ = 18° (Ref. 4)
O: M_1 = 1.64, M_2 = 3.87, θ = 16° (U. of M.)
Δ: M_1 = 1.64, M_2 = 3.60, θ = 16° (U. of M.)
///: SHARP CORNER ASYMMETRIC NOZZLE
EXPANSION ANGLE θ (DEGREES)
0
5
10
15
20
25
30
35
40
45
50
55
60
65
RATIO OF NOZZLE LENGTH TO TEST SECTION HEIGHT
0
1
2
3
4
5
6
7
8
9
10
MACH NUMBER - M_1
3.00
2.90
2.80
2.70
2.60
2.50
2.40
2.30
2.20
2.10
2.00
1.90
1.80
1.70
1.60
1.50
1.40
1.30
1.20
M_2=2.50
2.75
3.00
3.25
3.50
3.75
4.00
Fig. 3

gives uniform exit flow at M_2. Let this distribution be $K_u(s)$. With the nozzle in position to give M_1, $K_u(s)$ defines a new curvature distribution $K_o(s)$ along the lower surface which gives uniform exit flow at M_1. $K_o(s)$ and $K_1(s)$ are compared and form the basis for a new estimate of the curvature of the lower surface, say $K'_1(s)$. $K'_1(s)$ defines a new curvature distribution for the upper surface, $K'_u(s)$ to give uniform M_2 flow. $K'_u(s)$ defines a new lower surface curvature distribution $K'_o(s)$ to give uniform M_1 flow. $K'_o(s)$ is compared to $K'_1(s)$ and adjustments made so that they are the same, say $K''_1(s)$. This procedure is repeated n times until the curvature distributions $K^n_1(x)$ along the lower surface and $K^{n-1}_u(x)$ along the upper surface give uniform exit flow at both M_1 and M_2. Experience indicates that approximately 8 to 10 steps are necessary.

Each of the above steps is carried out by means of the method of characteristics. Ref. 4 indicates that the use of a 1° wave approximation in the iteration procedure is sufficiently accurate. Experience at the University of Michigan indicates that this is true, but that the last few steps in this process should be made with a 1/2" wave approximation.

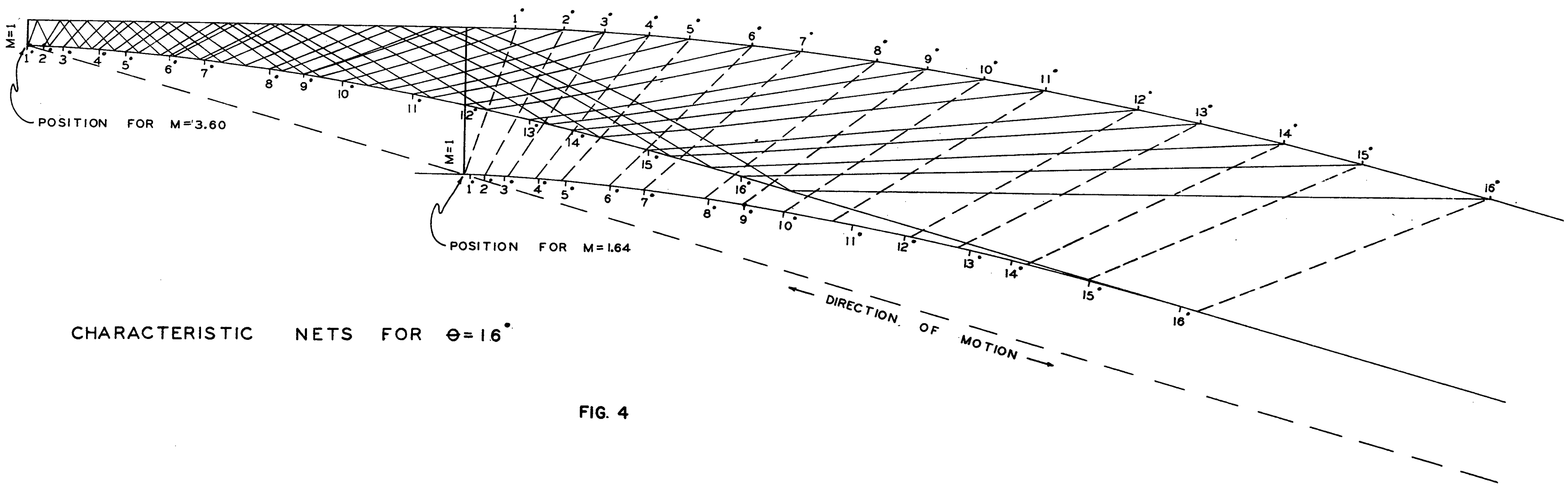
Figs. 4 and 5 show the characteristic diagrams of flow in two nozzles calculated in this manner. The first has design Mach numbers 1.64 and 3.60, and the second design Mach numbers 1.64 and 3.87. Both nozzles have an expansion angle of 16°. These nozzles are also shown in Fig. 3 as is the nozzle given in Ref. 4. It can be seen from this figure that the agreement between the approximate theory and the characteristic solutions is such that the approximate theory can be used to determine the overall length of a nozzle of the class considered.

Fig. 3 shows that for a given upper design Mach number M_2, overall nozzle length decreases as expansion angle increases. Consequently, since it is desirable to cover a given Mach number range with as short a nozzle as possible, a design was initiated with design Mach numbers, $M_1 = 1.845$, $M_2 = 3.87$ and $\theta_1 = 22°$, which in operation was expected to cover the Mach number range 1.4 to 4.0 with a saving in length of approximately 3/4h as compared with the $M_1 = 1.64$, $M_2 = 3.87$, $\theta_1 = 16°$ nozzle.

The results of the approximate theory determined the initial approximation used in the iteration procedure. However, the iteration procedure did not converge readily enough to allow completion of the design in the time available. The exact cause of the non-convergence to a solution is not known.

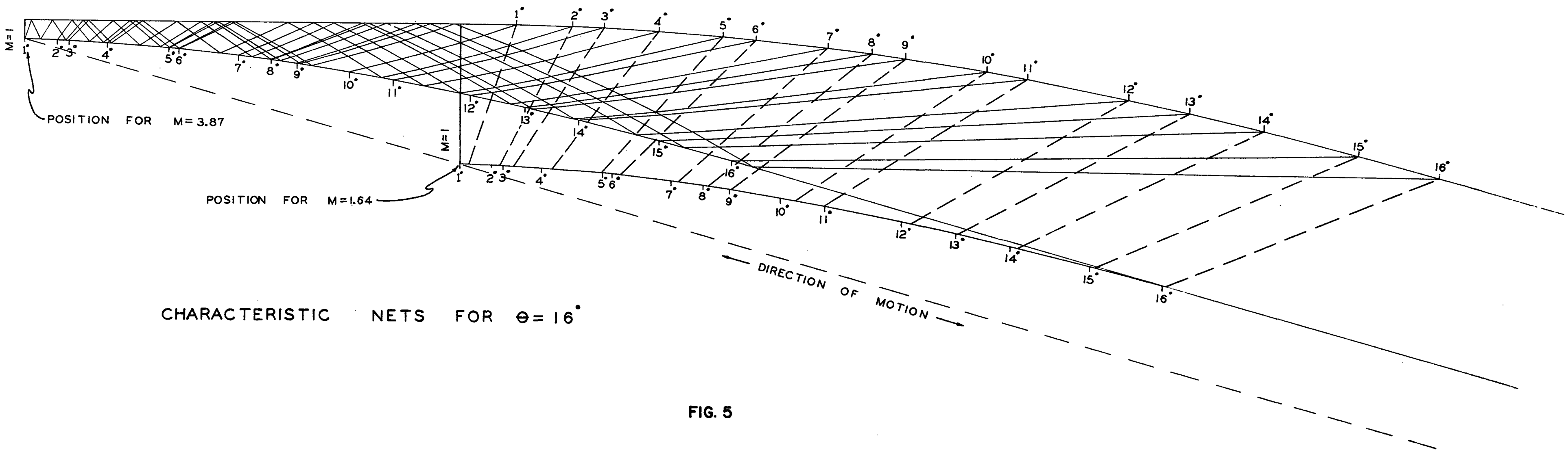
(d) Nozzle Coordinates.

Comparison of the two 16° nozzles show that the $M_2 = 3.60$ nozzle is shorter than the $M_2 = 3.87$ nozzle. However, it is believed that the $M_2 = 3.60$ nozzle would not give as uniform a M = 4 flow as the $M_2 = 3.87$ nozzle, because the interval $(M_2' - M_2) = .40$ is probably too large. The $M_2 = 3.60$ nozzle might be expected to give satisfactory flow at M = 3.80 and the $M_2 = 3.87$ is expected to give satisfactory flow at M = 4.00. Checks of the $M_1 = 1.64$, $M_2 = 3.87$, θ - 16° nozzle at Mach numbers M = 2.37, 3.23, 3.60 and 4.01 indicate that the test section flow is uniform within the approximations of the characteristic analysis at these intermediate Mach



CHARACTERISTIC NETS FOR $\theta = 1.6^\circ$

FIG. 4



CHARACTERISTIC NETS FOR $\Theta = 16^\circ$

FIG. 5

numbers.

Because of these considerations the coordinates have been computed only for the $M_2 = 3.87$, $\theta = 16°$ nozzle. The procedure followed in this computation is indicated below.

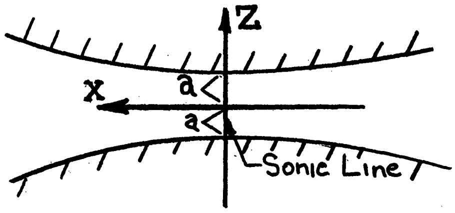
The results of the characteristic diagrams give the distribution of slope along the nozzle surface. This curve is faired and differentiated to obtain $\frac{d^2y}{dx^2}$ which is also faired, and then the curves are integrated to obtain $y = y_0(x)$ for the contour, which then has a smooth second derivative.

The coordinates of the nozzle contours, computed in this manner are given in Tables I and II for the $M_2 = 3.87$, $M_1 = 1.64$, $\theta = 16°$ nozzle.

IV Transonic and Subsonic Portion of the Nozzle.

(a) Transonic region.

Refs. 4, and 8 indicate that a sonic line which is straight and perpendicular to the contour can be obtained by proper shaping of the nozzle surface upstream of the throat. It is shown in Ref. 8 that a contour having the equation:



$$Z = a\left(1 + .1924\ x^6\right) \tag{11}$$

for the region upstream of the throat will give such a sonic line. (See accompanying sketch for definition of symbols) In practical terms, this equation indicates that the curvature of the wall should be essentially zero for a distance of 1 to 1 1/2 throat heights upstream of the sonic line in order to satisfy the condition used as a starting point in the characteristic analysis of the supersonic flow. This criterion was followed in design of the transonic region of the nozzle.

(b) Subsonic region.

The flow in this region of the nozzle was analyzed by means of one-dimensional flow theory. However, for the low Mach number positions of the lower contour, this method is not practical. Therefore a few exploratory experiments have been made in the 8 x 13 inch Supersonic Tunnel of the University to check out this portion of the nozzle as well as the transonic region. The results of these experiments are included in section VII of this report.

The entire nozzle contours obtained by the preceding perfect fluid analysis are shown in Fig. 9. These contours are designated as $y = y_0(x)$ in the figure which shows the nozzle in the Mach 4.0 and 1.35 positions.

V Boundary Layer Computations.

A series of computations have been made which give boundary layer displacement thickness along the supersonic contour for various test section Mach numbers (Ref. 10), for standard atmosphere stagnation conditions. The Tucker (NACA) method was followed. The results of these computations are presented in Figs. 6 and 7 which show displacement thickness, δ^*, for upper and lower supersonic contours respectively at Mach numbers 1.64, 2.37, 3.23, 3.60, 3.87, and 4.01. The abscissa is the distance along the contour measured downstream from the throat.

From Fig. 7 it can be seen that a cross plot of the values of δ^* at a given station for all Mach numbers between 1.6 and 4.01 can be prepared. Fig. 8 is an example of such a cross-plot for stations 15, 12.5, 10, 7.5, and 5 inches downstream of the sonic line along the lower surface, which shows that the data presented in Figs. 6 and 7 sufficiently difines δ^* at all points on the supersonic contour for all Mach numbers between 1.6 and 4.01.

These data, however, do not take into consideration secondary flows. It is expected that these flows will alter the values of δ^* given in Fig. 6 and 7. The extent of this alteration must be determined experimentally, at the present time.

VI Flexibility Criteria for Nozzle Contour.

The nozzle under investigation is to be used as an instrument to determine the degree of uniformity of the test section flow obtainable with asymmetric nozzles. Consequently, the surfaces will be flexible in order to correct for any flow non-uniformities resulting from the use of the contours found by the perfect fluid analysis described in sections III and IV. There are two types of flexibility required:

(1) large deviations from perfect fluid contour necessary to correct for viscous effects.

(2) small deviations from perfect fluid contour necessary to correct for any local non-uniformities which may occur in the test section flow at any Mach number, between 1.4 and 4.0.

(a) Corrections for viscous effects.

Let the deviations given under (1) be considered as deviations from the theoretical contour. Then we may write the equation of the mean contour in the following manner:

(12) $$y_m = y_o(x) + \Delta(x)$$

where y_o is given in Tables I and II, and $\Delta(x)$ is a deviation of the type

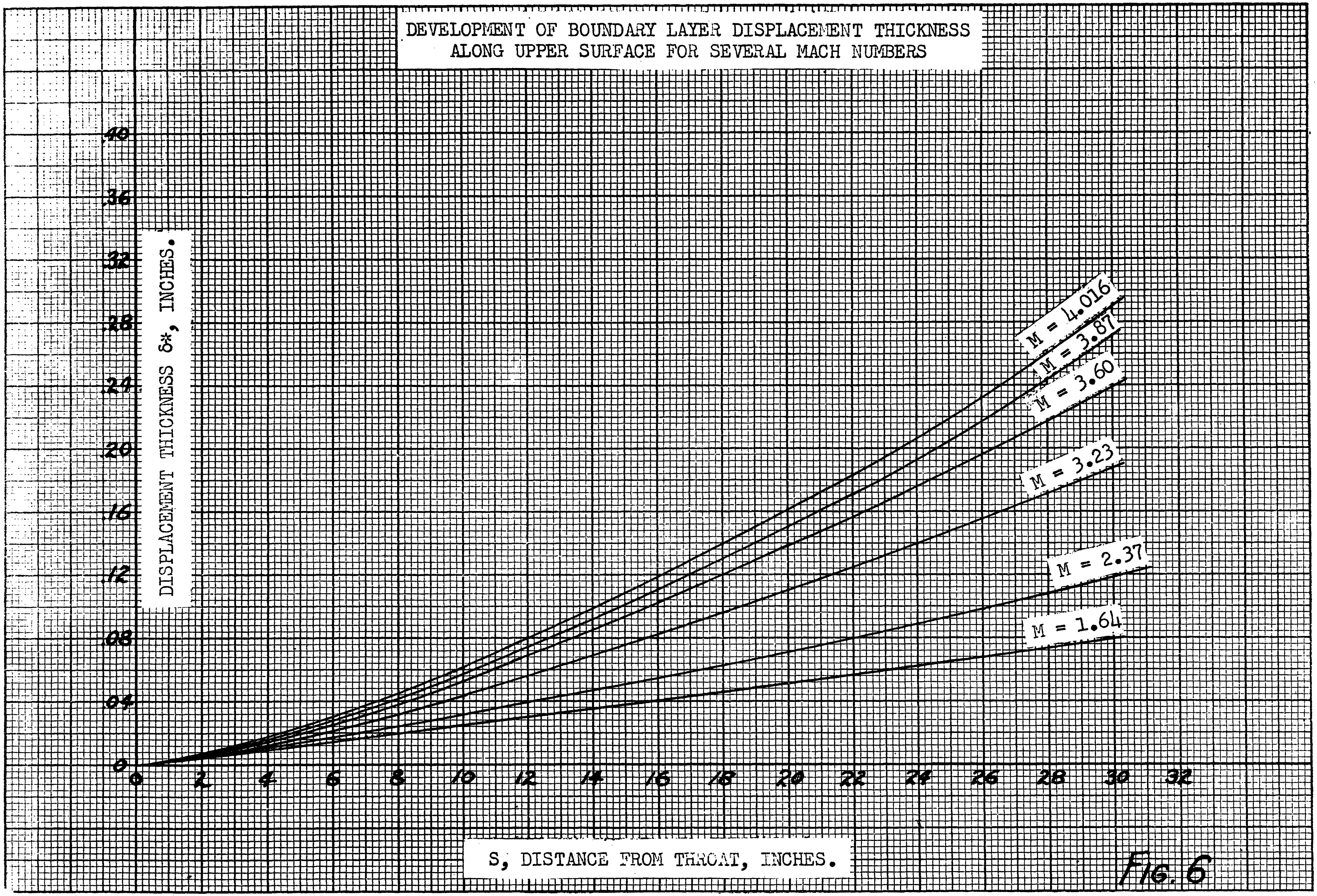

DEVELOPMENT OF BOUNDARY LAYER DISPLACEMENT THICKNESS
ALONG UPPER SURFACE FOR SEVERAL MACH NUMBERS
DISPLACEMENT THICKNESS δ*, INCHES.
.40
.36
.32
.28
.24
.20
.16
.12
.08
.04
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
M = 4.016
M = 3.87
M = 3.60
M = 3.23
M = 2.37
M = 1.64
S, DISTANCE FROM THROAT, INCHES.

FIG. 6

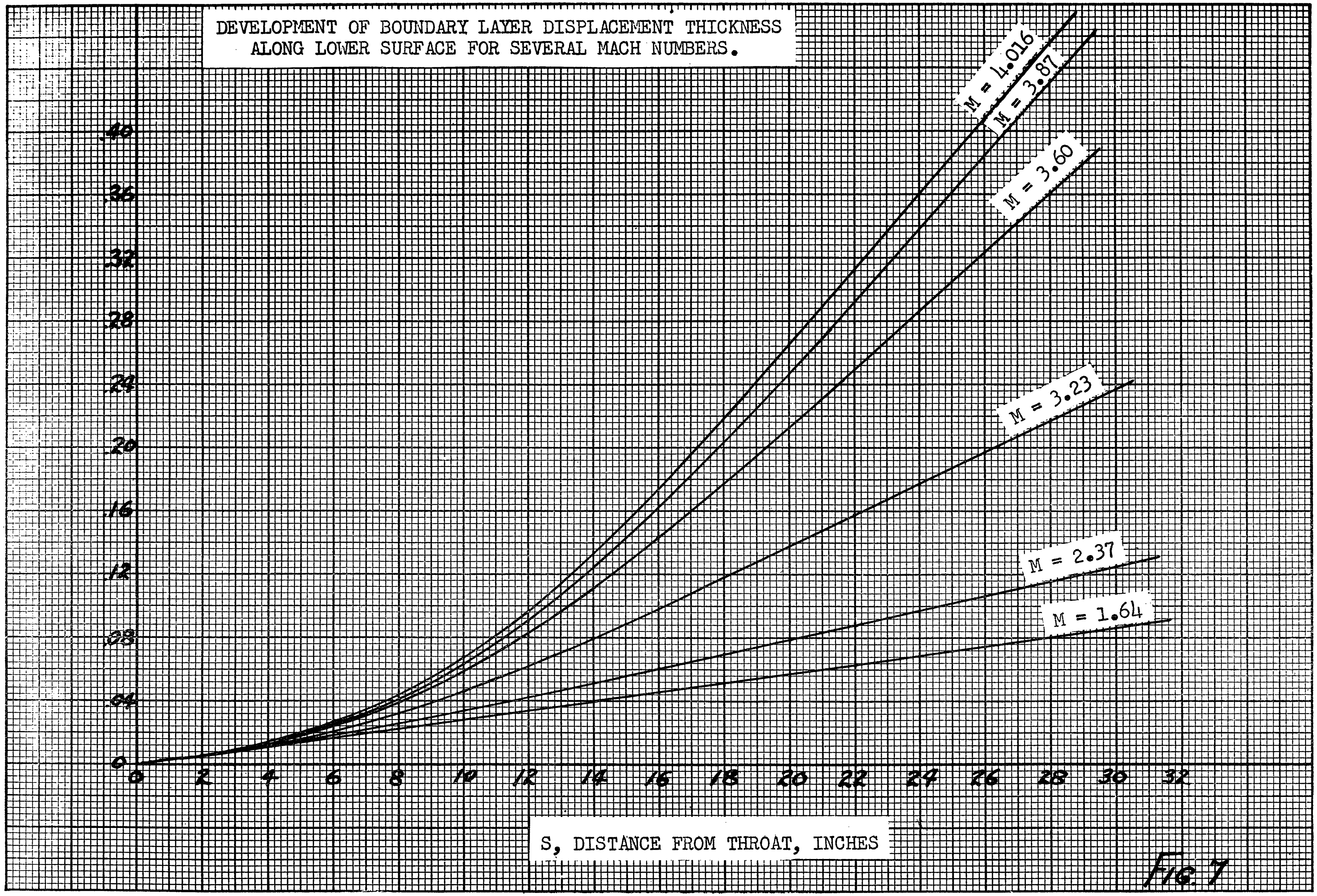
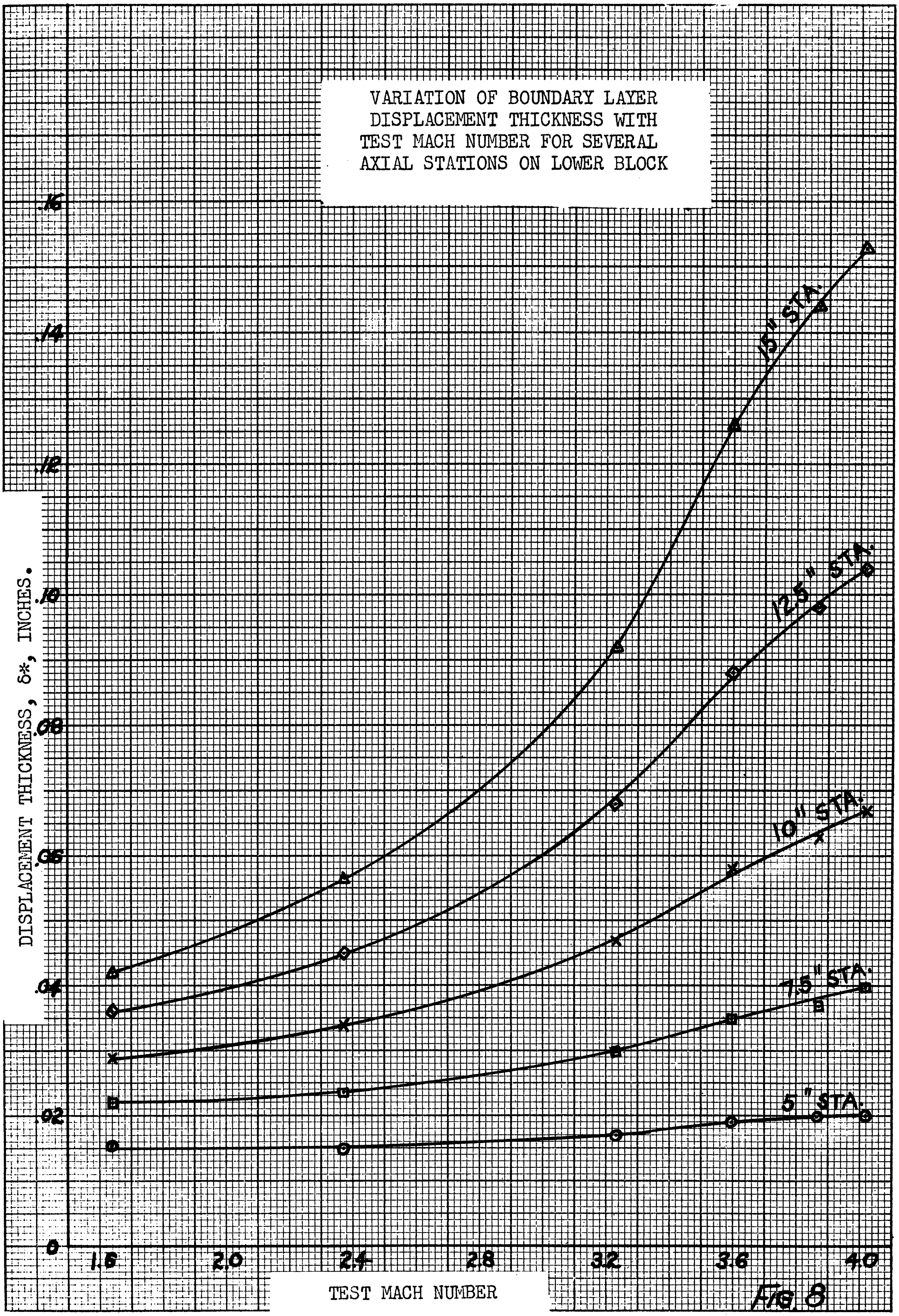

DEVELOPMENT OF BOUNDARY LAYER DISPLACEMENT THICKNESS
ALONG LOWER SURFACE FOR SEVERAL MACH NUMBERS.
.40
.36
.32
.28
.24
.20
.16
.12
.08
.04
0
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
M = 4.016
M = 3.87
M = 3.60
M = 3.23
M = 2.37
M = 1.64
S, DISTANCE FROM THROAT, INCHES

FIG. 7



VARIATION OF BOUNDARY LAYER DISPLACEMENT THICKNESS WITH TEST MACH NUMBER FOR SEVERAL AXIAL STATIONS ON LOWER BLOCK
DISPLACEMENT THICKNESS, δ*, INCHES.
TEST MACH NUMBER
15" STA.
12.5" STA.
10" STA.
7.5" STA.
5" STA.
0
.02
.04
.06
.08
.10
.12
.14
.16
1.6
2.0
2.4
2.8
3.2
3.6
4.0

FIG. 8

(1). It follows from (12) that

$$(13) \qquad \frac{d^2y_m}{dx^2} = \frac{d^2y_o}{dx^2} + \frac{d^2\Delta}{dx^2} \text{ and } \frac{dy_m}{dx} = \frac{dy_o}{dx} + \frac{d\Delta}{dx}$$

The boundary layer displacement thickness computations have been used to calculate $\Delta(x)$. In order to correct for viscous effects in a manner which makes $\Delta(x)$ as small as possible, the following procedure has been used. The lower nozzle block is rotated about a point near the throat so that the point B is displaced normal to the original contour an amount δ^*_B, where δ^*_B = displacement thickness at point B. Then other points on the contour are moved by flexing the surface so that each point has moved a net amount δ^* from its original unrotated position where δ^* = boundary layer displacement thickness at the point under consideration.

The movement of the contour by flexing is denoted by $\Delta(x)$. The greatest amount of flexing is required at M = 4.0 and the curve of $\Delta(x)$ for this case is given in Table I, along with its first and second derivatives.

The flexing of type (1) at Mach number ≤ 1.6 is negligible, and that at any intermediate Mach number is less than $\Delta_{M=4}$. Consequently, the mean contour must assume the extreme shapes given by

$$(14) \qquad \begin{cases} y_m = y_o(x) \\ y_m = y_o(x) + \Delta(x)_{M=4} \end{cases}$$

and the shape at any intermediate Mach number lies somewhere between the extremes and is similar to one of the equations (14).

A similar procedure has been used for the upper surface except that the rotation is such that the contour is displaced normal to its original position an amount δ^* at the nozzle exit (point A). The values of Δ for the upper surface are given in Table II. Fig. 9 shows the contour in its rotated M = 4 position and in its rotated and flexed M = 4 position.

(b) Corrections for Local Non-Uniformities.

Deviations of the type (2) are restricted to the region just downstream of the throat on the lower surface and to that just upstream of the exit on the upper surface. Small deviations from the mean contour in these regions will give a control over the uniformity of test section flow at all Mach numbers.

Analysis of the design contours with the method of characteristics at off-design Mach numbers indicates that the deviations of type (2) are quite small. If we characterize these deviations as $\lambda(x)$, the shape of the contour is given by the equation

(15) $y = y_m(x) + \lambda(x)$

where y_m is determined by equation (14). It follows that

(16) $\frac{dy}{dx} = \frac{dy_m}{dx} + \frac{d\lambda}{dx}$; $\frac{d^2y}{dx^2} = \frac{d^2y_m}{dx^2} + \frac{d^2\lambda}{dx^2}$

The effect of deviations of the type λ on the test section flow can be found by the following analysis (Ref. 11). If we assume the non-uniformities in the test section flow to be characterized by a two dimensional perturbation potential

(17) $\varphi = \varphi\ (x_o, y_o)$

where x_o = rect. cart. coord. in direction of tunnel axis

y_o = " " " " " " " "

Then the non-uniformities in velocity are

(18) $u = \frac{\partial\varphi}{\partial x_o}$ in x_o direction

$v = \frac{\partial\varphi}{\partial y_o}$ in y_o direction.

and the non-uniformities in flow inclination are

(19) $\theta = \pm \frac{v}{U+u}$

where U = test stream velocity (See Fig. 10). Because of (18), we may consider the stagnation pressure p_o to be a constant.

Then the static pressure at a given point in the flow is related to Mach number at the point by

(20) $\frac{p}{p_o} = (1 + \frac{\gamma-1}{2} M^2)^{\frac{\gamma}{\gamma-1}}$

and static pressure deviations are given by

(21) $$\frac{\Delta p}{p} = \frac{-\gamma M^2}{1+\frac{\gamma-1}{2}M^2}\frac{dM}{M}$$

This pressure variation is related to the inclination deviations by the equation:

(22) $$\frac{\Delta p}{p} = \pm\frac{\gamma M^2}{\sqrt{M^2-1}}\Delta\theta$$

Now if a change in wall slope $\frac{d\lambda}{dx}$ occurs at some point Xu, Xu', Xu" or X_1, X_1', X_1'', etc., along the supersonic nozzle surface as shown in Fig. 10, the change in flow inclination at a point X_o along the tunnel centerline is given by

(23) $$\theta = \pm\frac{d\lambda}{dx}$$

and the change in pressure at X_o by

(24) $$\frac{\Delta p}{p} = \pm\frac{\gamma M^2 N}{\sqrt{M^2-1}}\frac{d\lambda}{dx}$$

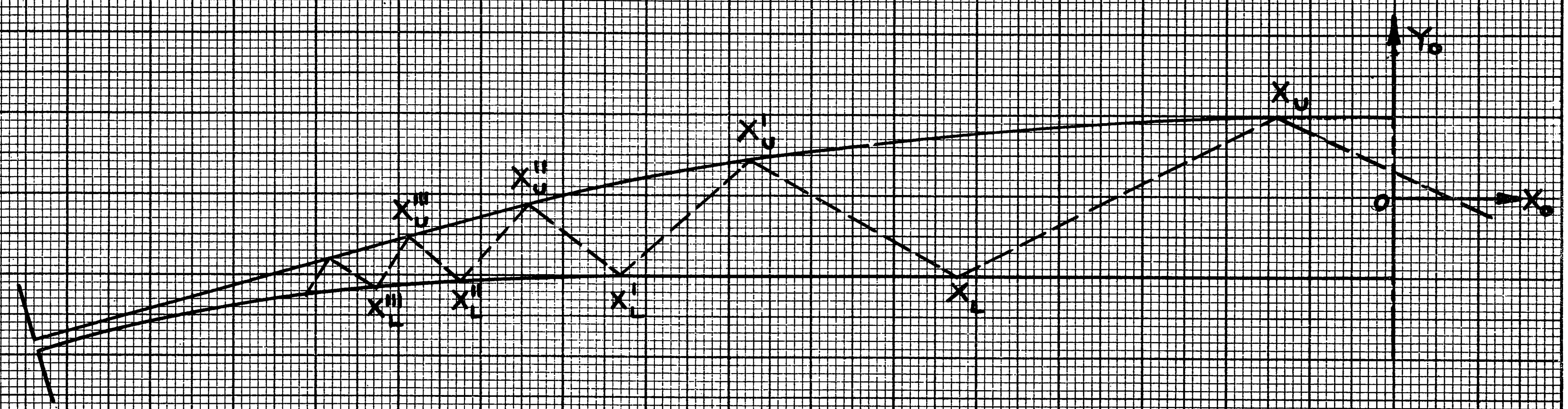
where N indicates the number of times that a wave from Xu, Xu', or X_1, X_1' etc., reflects before it reaches the point X_o.

To obtain an order of magnitude on the values of $\frac{d\lambda}{dx}$ which may be necessary in order to be able to correct for non-uniform flow in the test section, we consider a deviation in Mach number of .02. The change in pressure associated with this Mach number deviation is given by

(25) $$\frac{\Delta p}{p} = \frac{.02\gamma M^2}{1+\frac{\gamma-1}{2}M^2}\frac{1}{M}$$

which becomes by use of (24)

(26) $$\frac{.02\gamma M^2}{1+\frac{\gamma-1}{2}M^2}\frac{1}{M} = \frac{\gamma M^2 N}{\sqrt{M^2-1}}\frac{d\lambda}{dx}$$



COORDINATE SYSTEM FOR ANALYSIS OF
TEST SECTION FLOW.

FIG. 10

so that

$$(27) \qquad \frac{d\lambda}{dx} = \frac{.02\sqrt{M^2-1}}{MN(1+\frac{\gamma-1}{2}M^2)}$$

The values of $\frac{d\lambda}{dx}$ obtained by solution of (27) are shown in Fig. 11 as a function of Mach number for N = 1,5.

The extreme values of the function $\lambda(x)$ which may be necessary for the upper and lower surfaces are given in Table III, where the $\pm$ signs indicate that perturbations both above and below the mean contour are desirable.

VII Preliminary Experimental Results In The Subsonic And Transonic Regions Of The Nozzle.

As indicated in Section IV, a series of simple experiments have been made to check out the subsonic and transonic regions of the nozzle. In order to perform the tests economically, models of the nozzle contour up to the throat were constructed from wooden blocks. The blocks were fashioned in such a manner that they could be inserted in the existing 8" x 13" Supersonic Wind Tunnel, in the M = 3.87 and M = 1.40 positions relative to one another. Instrumentation for tests in these two positions was limited to schlieren photography, china-clay-film visualization studies, and a small amount of pressure measurement with a five-prong total head probe. The measurements obtained in this experimental work are reported fully in Ref. 9, but some of the more important results are reproduced here.

Investigations were conducted with two different contours for the subsonic regions. The results with the original contours showed that the sonic line was straight and perpendicular to the nozzle wall at both M = 3.87 and M = 1.40 positions. Evidence of this is shown in Fig. 12. Fig. 12a is a schlieren photograph of the nozzle in the M = 3.87 position, while Fig. 12b was taken with the nozzle in the M = 1.40 position. The Mach waves just downstream of the throat are slightly inclined and straight. Comparison of the experimental Mach wave distribution just downstream of the throat with that predicted theoretically in Ref. 8 (see Fig. 11 of that Ref.).for the case of a straight sonic line leads to the conclusion that a straight sonic line exists in Fig. 12 even though it is, of course, invisible. Another result with the original contour was that the viscous flow in the neighborhood of the M = 3.87 throat was adequately predicted by theory.

However, a separation occured approximately two inches upstream of the lower block leading edge in the M = 1.40 position. Fig. 13a shows the extent of the separation; while Fig. 13b shows that the separated flow re-attached to the lower surface approximately 3 1/2" upstream of the M = 1.40 throat. The streaks visible near the lower block leading edge in Fig. 13a are the results of application of a small amount of China-clay-film on the

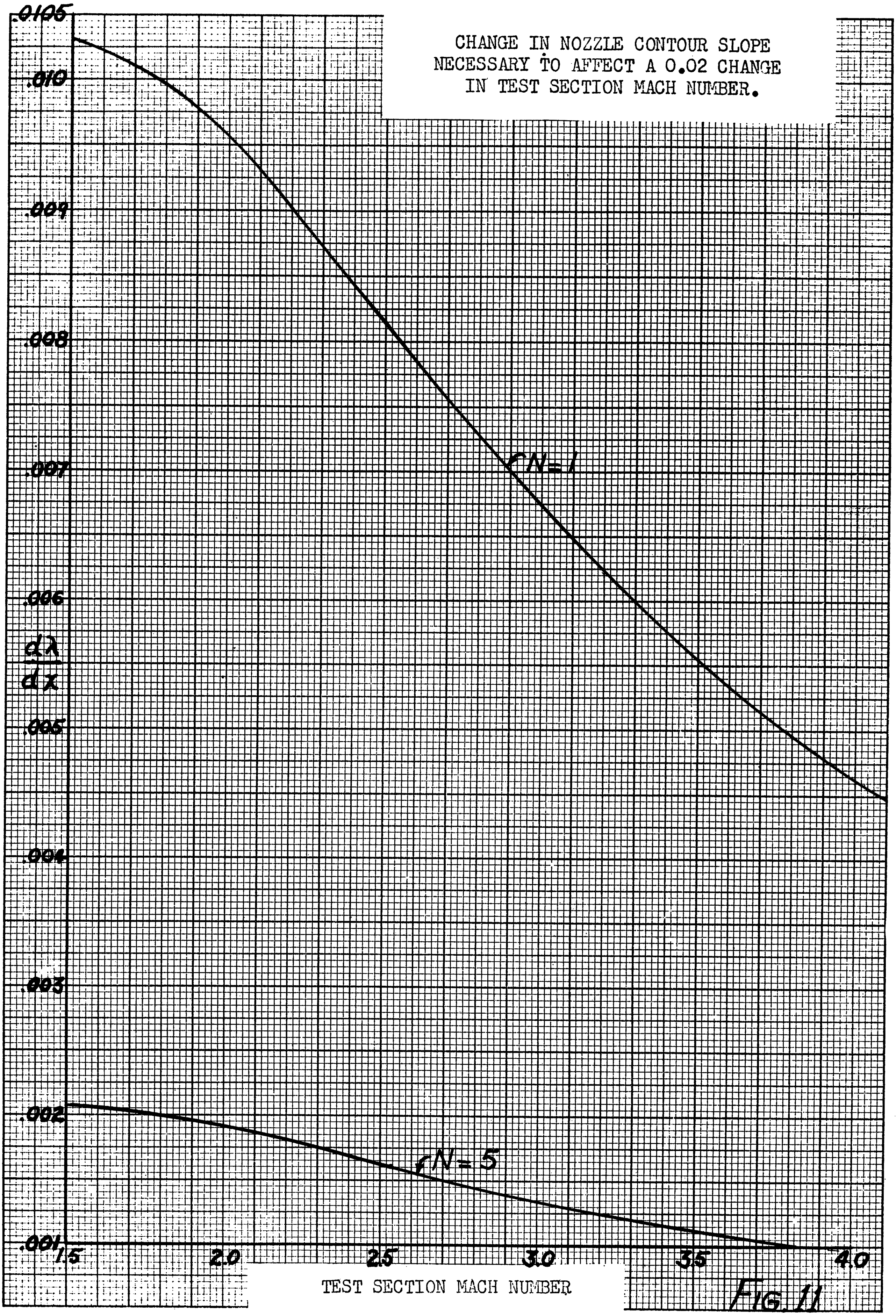

CHANGE IN NOZZLE CONTOUR SLOPE NECESSARY TO AFFECT A 0.02 CHANGE IN TEST SECTION MACH NUMBER.
.0105
.010
.009
.008
.007
.006
dλ/dx
.005
.004
.003
.002
.001
1.5
2.0
2.5
3.0
3.5
4.0
TEST SECTION MACH NUMBER
N=1
N=5

FIG. 11

sidewall. They represent essentially the stream line directions in the boundary layer near the sidewall. The separation seemed to be due to two-dimensional effects associated with the asymmetry of the channel in the M = 1.40 position.

As a result of the separation, a modification of the subsonic contours was considered necessary. The modification essentially decreased the curvature of the subsonic inlet; and consequently led to an inlet approximately one test section height longer than the original inlet. Since the modification did not extend into the transonic region of the nozzle, it did not affect the sonic line. The coordinates of the modified contour are given in Tables I and II. Fig. 14 is a photograph of the modified nozzle blocks; while Fig. 15 shows the method of attachment of the blocks to the top and bottom sidewalls of the 8" x 13" channel. The plug was attached to the block by 6 wood-screws, and was inserted in a hole in the tunnel wall. A bolt was inserted through the channel (which rested against the outside tunnel wall) and fitted the plug tightly in place.

An indication of the fact that the modification eliminated the separation is shown in Fig. 16. Fig. 16 is a photograph of the Mach 1.4 throat region after the modification. It was made with a highly sensitive setting of the schlieren apparatus in order to bring out the flow details. The boundary layer on the lower surface appears to behave in a satisfactory manner, although the secondary or cross-flows in the sidewall boundary layer appear to be rather large near the leading edge of the lower block.

These preliminary experimental results thus indicate that with the modified inlet contours:

(a) a straight sonic line is obtained as predicted by theory, and

(b) viscous effects up to the throat should not materially influence the flow in the supersonic portion of the nozzle.

PERSONNEL

The following persons have been connected with the work included in this report:

J. S. Murphy....................................Full Time

H. Buning.......................................Part Time

D. V. Black.....................................Part Time

C. W. Donnar....................................Part Time

R. P. Schulze...................................Part Time

L. Talbot.......................................Part Time

R. T. Wagner....................................Part Time

In addition H. P. Liepman, Director, Supersonic Wind Tunnel and Professors A. M. Kuethe and M. V. Morkovin of the Aeronautical Engineering Department have made valuable suggestions. Discussions with personnel at the Langley Field and Ames Aeronautical Laboratories, NACA, were quite helpful during the initial phases of the work.

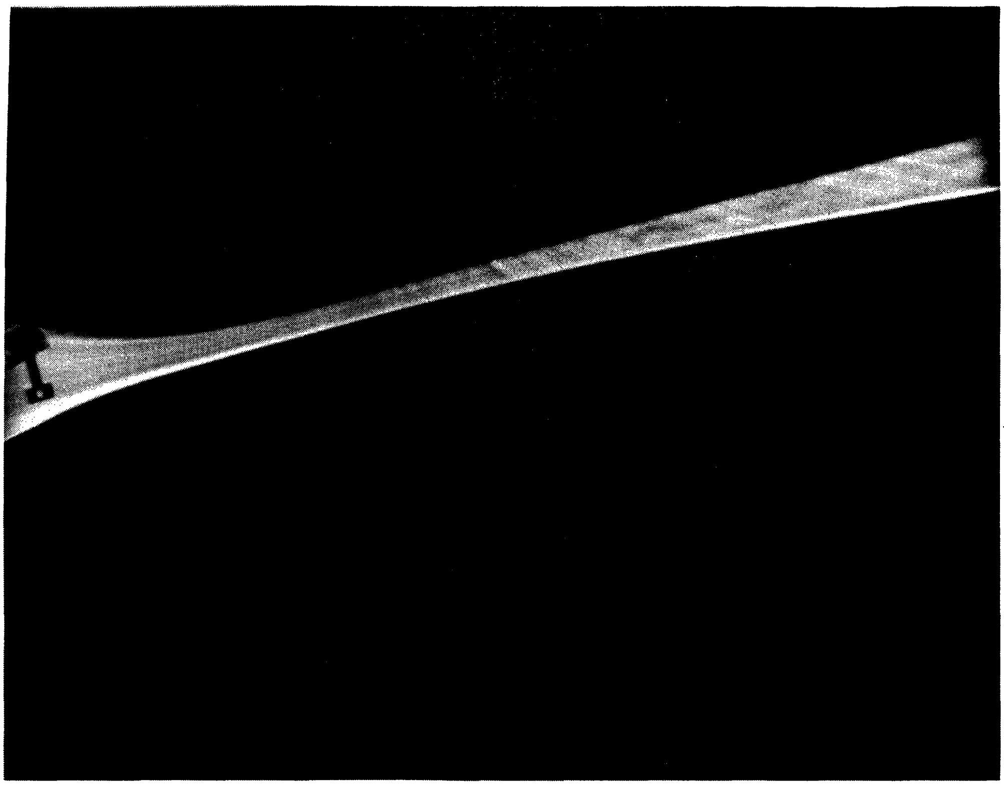


FIG. 12a TRANSONIC REGION, M=3.87 POSITION (ORIGINAL CONTOURS)

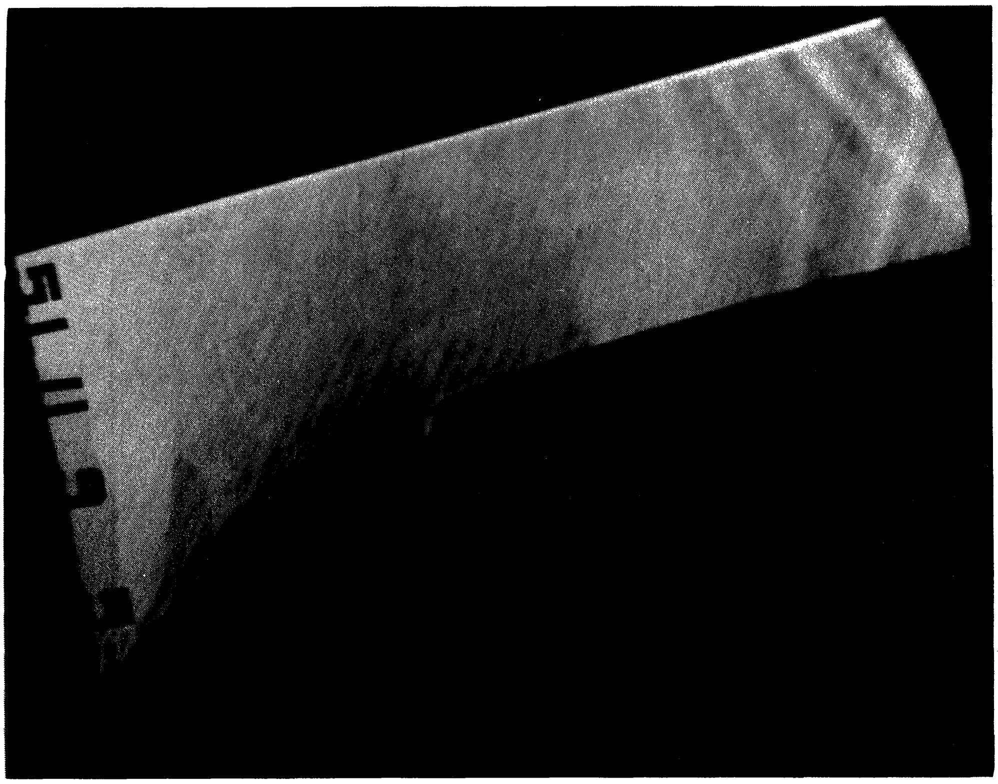


FIG. 12b TRANSONIC REGION, M=1.40 POSITION (ORIGINAL CONTOURS)

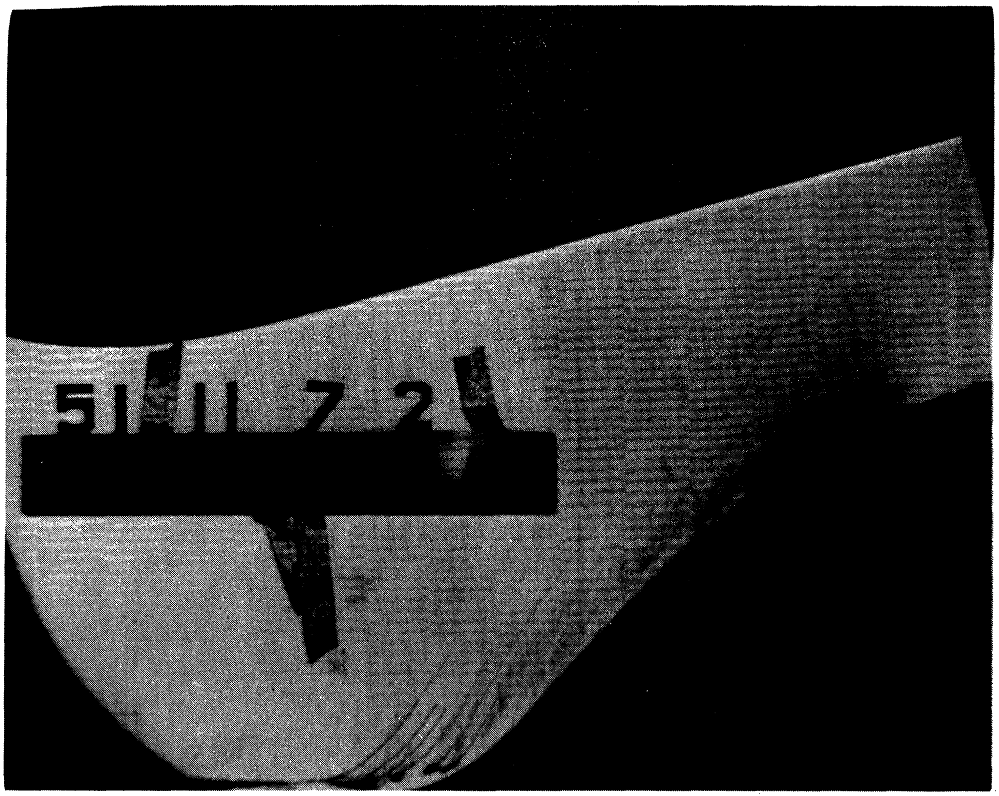


FIG. 13a SUBSONIC REGION, M = 1.40 POSITION (ORIGINAL CONTOURS)

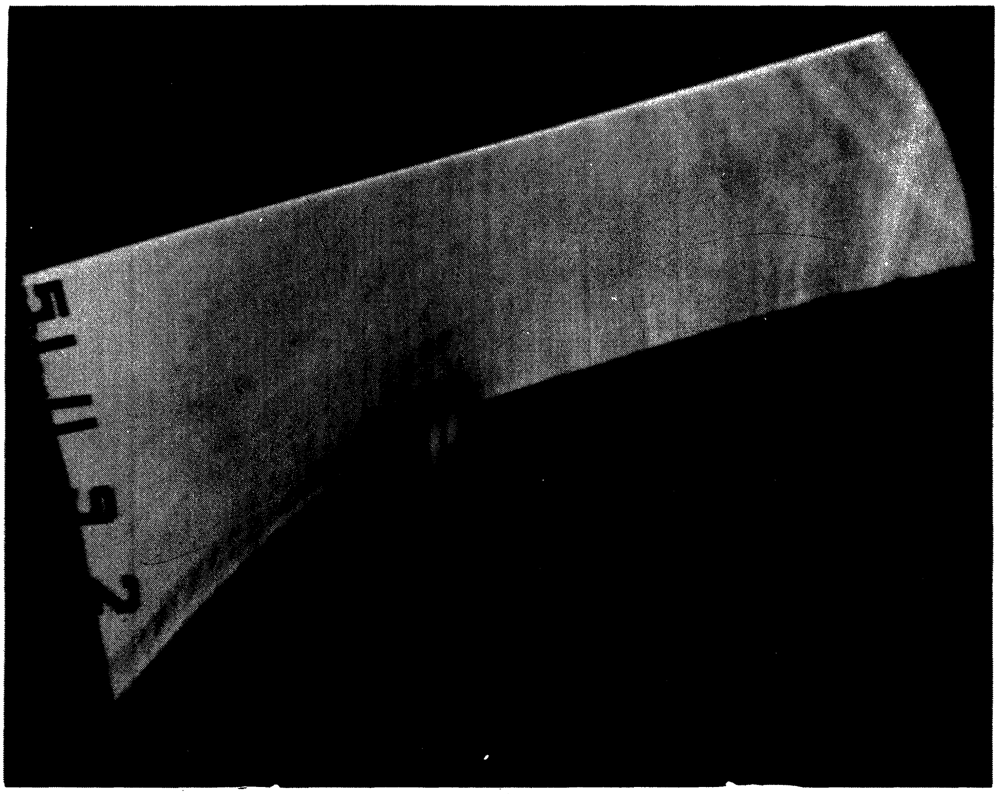


FIG. 13b TRANSONIC REGION, M = 1.40 POSITION (ORIGINAL CONTOURS)

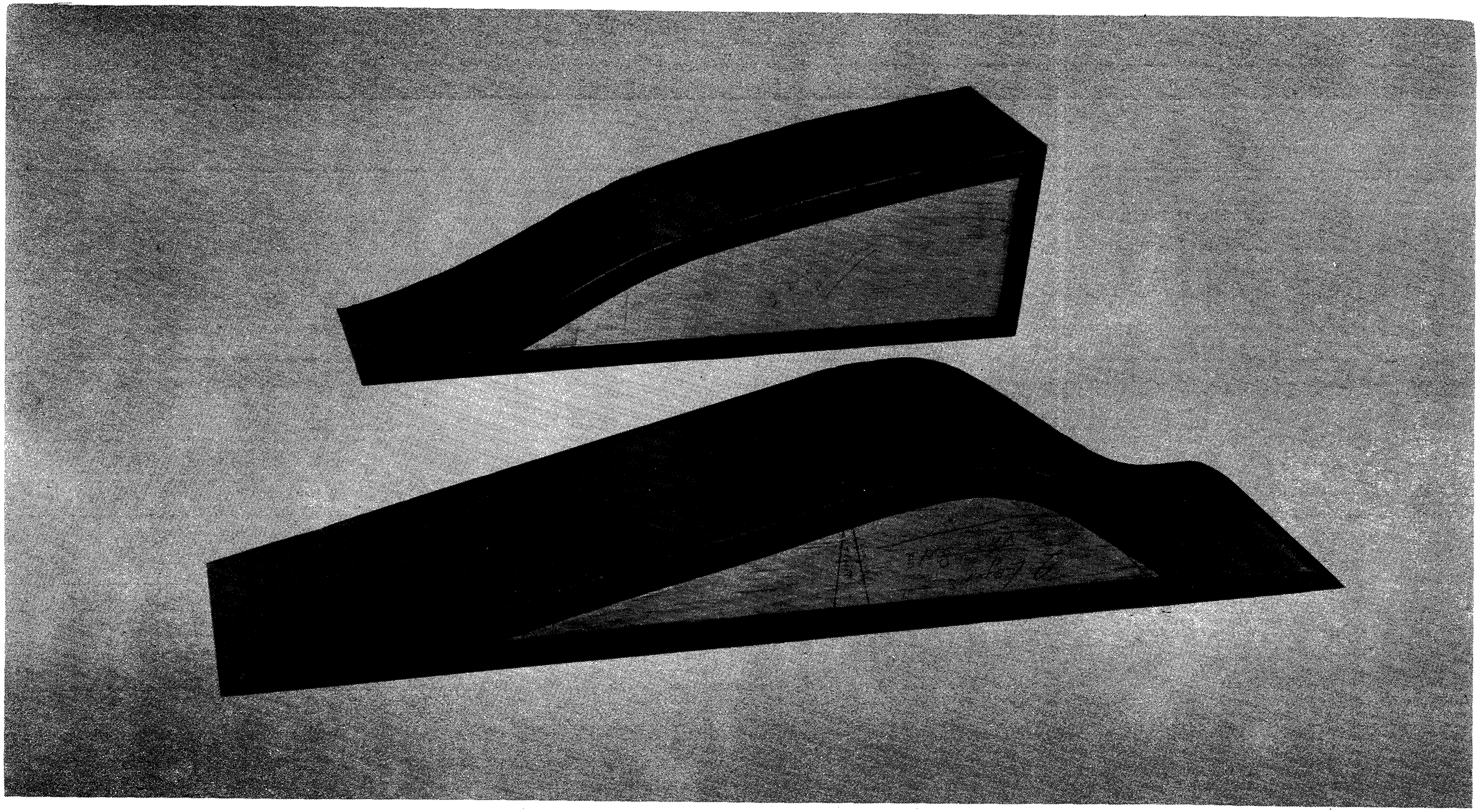


FIG. 14

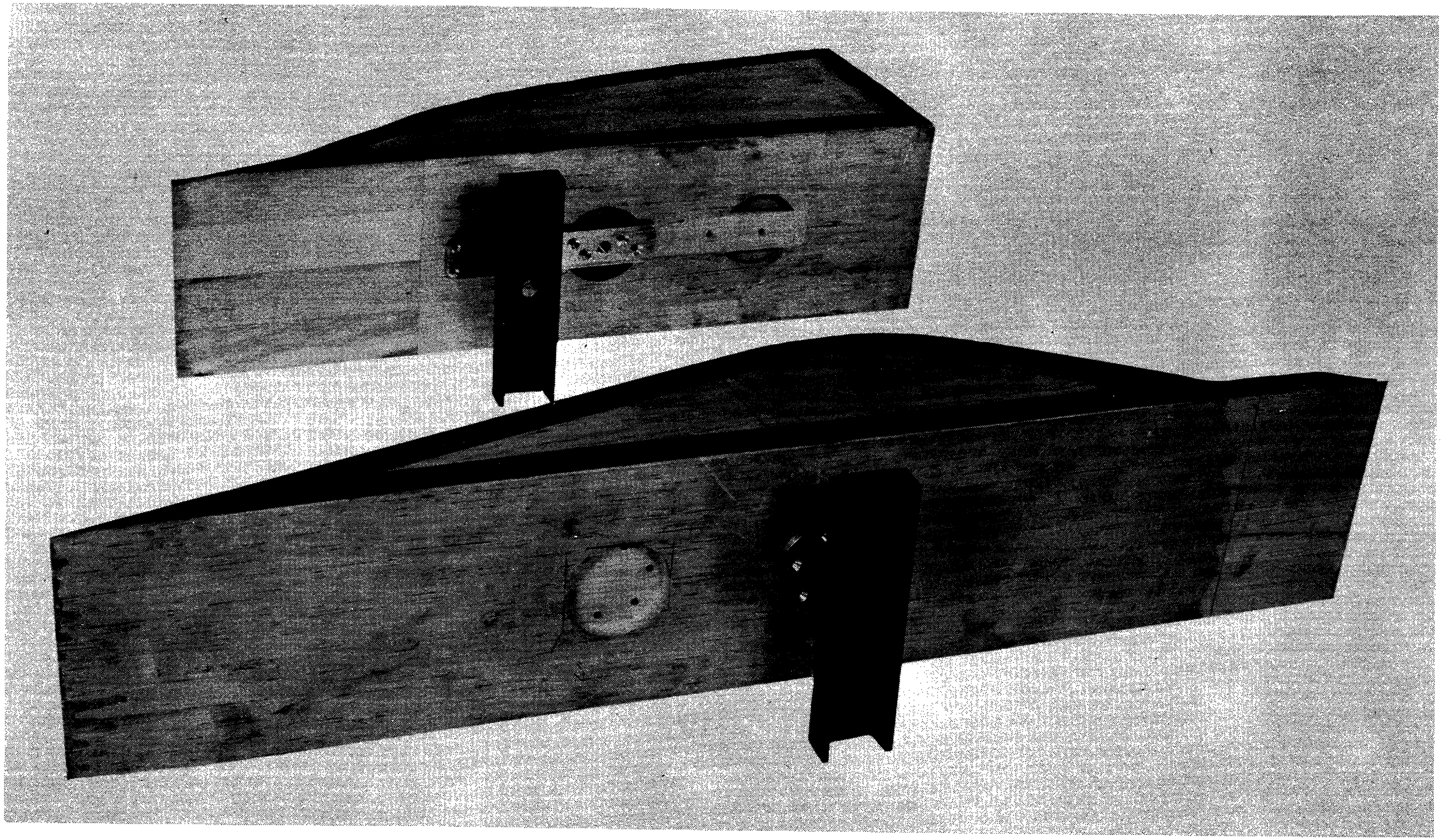


FIG. 15

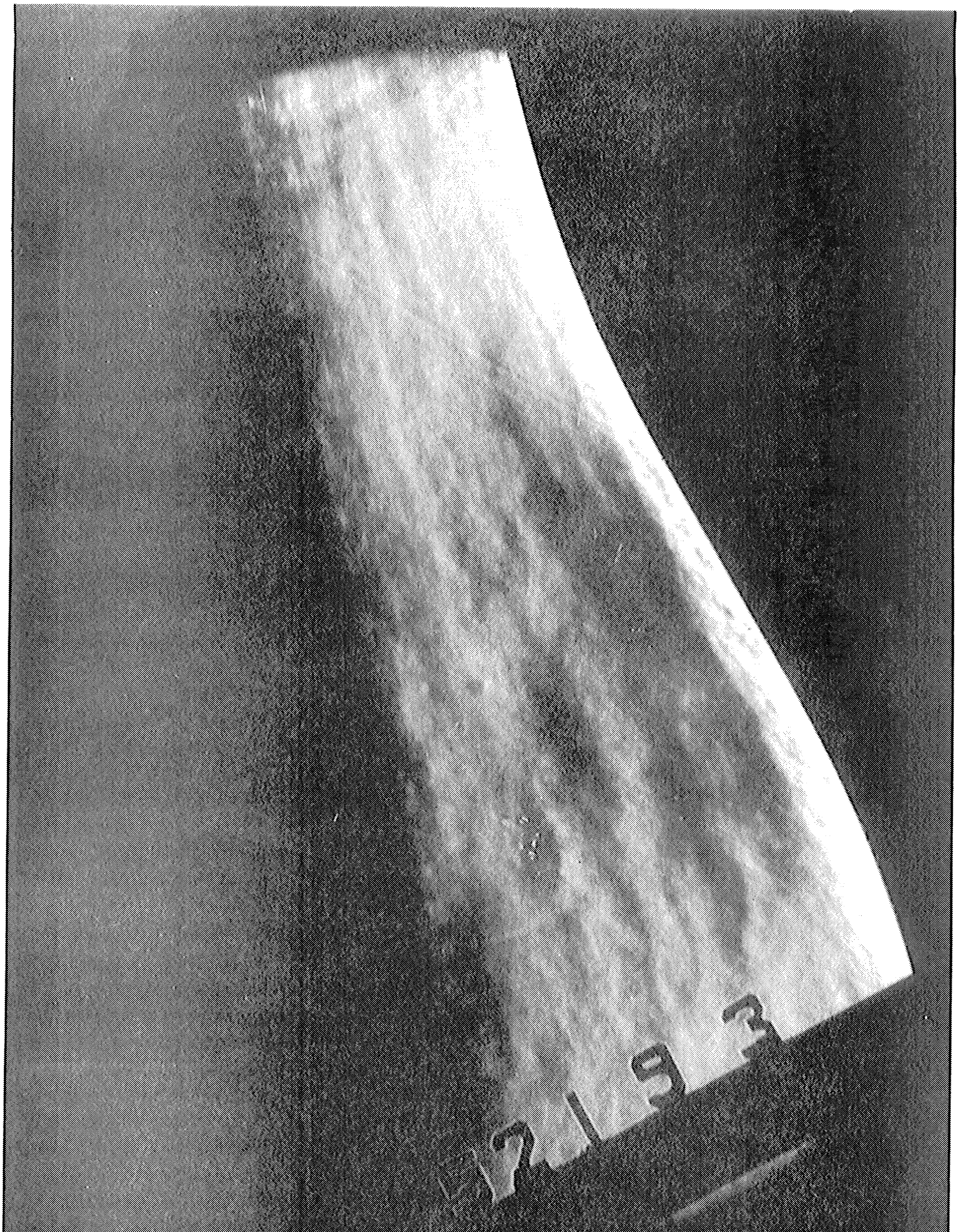


FIG. 16

REFERENCES

1. Allen, H. Julian, "The Asymmetric Adjustable Supersonic Nozzle for Wind-Tunnel Application". NACA RM No. A8E17, July 23, 1948.

2. Syvertson, C. A., and Savin, R. C. "The Design of Variable Mach Number Asymmetric Supersonic Nozzles by Two Proceedures Employing Inclined and Curved Sonic Lines", NACA RM No. A51A19, April 11, 1951.

3. Envard, John C., and Wyatt, D. D. "Investigation of a Variable Mach Number Supersonic Tunnel with Non-Intersecting Characteristics", NACA RM No. E8J13, November 15, 1948.

4. Burbank, P. B., and Byrne, R. W. "The Aerodynamic Design and Calibration of an Asymmetric Variable Mach Number Nozzle with a Sliding Block for the Mach Number Range 1.27 to 2.75". NACA RM No. L50L15, March 15, 1951.

5. Murphy, James S. "Description of Method Used to Determine Contour of Variable Mach Number Nozzle", Univ. of Mich. WTM-217, October 10, 1951.

6. Schulze, R. P., and Talbot, L. "Design of Variable Mach Number Supersonic Nozzles Employing the Method of Syvertson and Savin", Univ. of Mich. WTM-211, August, 1951.

7. Puckett, A. E. "Supersonic Nozzle Design", Transactions of the ASME, December, 1946.

8. Görtler, H. "On the Transition from Subsonic to Supersonic Velocities in Channels". Zeit. Ang. Math. und Mech., Dez. 1939. Translated by H. Buning at Univ. of Mich. WTM-214, September, 1951.

9. Murphy, J. S. "Experiments on the Subsonic and Transonic Regions of the 4 x 4 Inch Variable Mach Number Nozzle".

10. Buning, H. "Boundary Layer Thickness Computations". Univ. of Mich. WTM-216, September, 1951.

11. Puckett, Allen E. "Design and Operation of a 12" Supersonic Wind Tunnel", Preprint No. 160, Inst. of the Aero. Sciences, July 16, 1948.

TABLE I

COORDINATES OF LOWER CONTOUR

X"	Y_o"	Δ"	dY_o/dx	$d\Delta/dx$	d^2Y_o/dx^2	$d^2\Delta/dx^2$
-4.00	2.178					
-3.75	2.248					
-3.50	2.292					
-3.25	2.328					
-3.00	2.353					
-2.75	2.371					
-2.50	2.386					
-2.25	2.393					
-2.00	2.398					
-1.75	2.398					
-1.50	2.388					
-1.25	2.378					
-1.00	2.361					
- .75	2.338					
- .50	2.312					
- .25	2.280					
.00	2.246		-.150			
.25	2.207		-.167			
.50	2.163		-.183			
.75	2.115		-.199			
1.00	2.064		-.214			
1.25	2.009		-.228			
1.50	1.950		-.241			
1.75	1.888		-.253			
2.00	1.824		-.264			
2.25	1.756		-.274			
2.50	1.687		-.283			
2.75	1.615		-.290			
3.00	1.542		-.297			
3.25	1.467		-.302			
3.50	1.391		-.306			
3.75	1.314		-.308			
4.00	1.237		-.309			
4.25	1.159		-.308			
4.50	1.083		-.306			
4.75	1.006		-.303			
5.00	.931		-.298			
5.25	.858		-.292			
5.50	.785		-.285			
5.75	.715		-.276			
6.00	.647		-.2665			
6.25	.582		-.2555			
6.50	.520		-.244			
6.75	.460		-.231			
7.00	.404		-.217			
8.25	.352		-.203			
7.50	.303		-.188			
7.75	.258		-.1725			
8.00	.216		-.157			
8.25	.179		-.1415			

COORDINATES OF LOWER CONTOUR (CONT.)

X"	Y_o"	Δ"	dY_o/dx	$d\Delta/dx$	d^2Y_o/dx^2	$d^2\Delta/dx^2$
8.25	.179		-.1415			
8.50	.146		-.1260			
8.75	.116		-.1105			
9.00	.090		-.0955			
9.25	.068		-.0815			
9.50	.050		-.0675			
9.75	.035	.0260	-.054	.00000		-.00344
10.00	.023	.0258	-.0415	-.00116		-.00292
10.25	.014	.0255	-.030	-.00212		-.00257
10.50	.0075	.0249	-.020	-.00300		-.00229
10.75	.0035	.0242	-.012	-.00374		-.00204
11.00	.0010	.0233	-.006	-.00441		-.00185
11.25	.0000	.0243	-.002	-.00501		-.00168
11.50		.0216	.000	-.00552		-.00150
11.75		.0198		-.00599		-.00136
12.00		.0182		-.00637		-.00121
12.25		.0167		-.00670		-.00108
12.50		.0150		-.00698		-.00095
12.75		.0132		-.00721		-.00080
13.00		.0114		-.00739		-.00068
13.25		.0095		-.00753		-.00056
13.50		.0076		-.00763		-.00042
13.75		.0057		-.00771		-.00031
14.00		.0038		-.00778		-.00021
14.25		.0019		-.00780		-.00011
14.50	.0000	.0000	.0000	-.00782	.0000	-.00002
14.75	.0004	-.0021	.0012	-.00781	.0097	.00006
15.00	.0015	-.0040	.0047	-.00779	.0173	.00013
15.25	.0039	-.0059	.0095	-.00774	.0225	.00020
15.50	.0079	-.0078	.0157	-.00769	.0266	.00028
15.75	.0136	-.0098	.0228	-.00760	.0293	.00034
16.00	.0212	-.0117	.0302	-.00750	.0291	.00040
16.25	.0304	-.0135	.0373	-.00739	.0279	.00045
16.50	.0415	-.0154	.0442	-.00725	.0269	.00051
16.75	.0545	-.0172	.0511	-.00701	.0260	.00055
17.00	.0608	-.189	.0575	-.00695	.0252	.00060
17.25	.084	-.0207	.0637	-.00679	.0244	.00064
17.50	.102	-.0224	.0697	-.00660	.0263	.00069
17.75	.121	-.0240	.0754	-.00640	.0229	.00072
18.00	.141	-.0256	.0810	-.00620	.0223	.00077
18.25	.163	-.0270	.0865	-.00599	.0218	.00080
18.50	.186	-.0285	.0920	-.00577	.0214	.00083
18,75	.210	-.0299	.0972	-.00554	.0210	.00087
19.00	.236	-.0313	.1023	-.00530	.0208	.00090
19.25	.262	-.0326	.1075	-.00507	.0206	.00094
19.50	.291	-.0338	.1128	-.00481	.0204	.00098
19.75	.320	-.0350	.1180	-.00456	.0202	.00101
20.00	.351	-.0361	.1230	-.00429	.0201	.00103
20.25	.383	-.0372	.1280	-.00402	.0200	.00108
20.50	.416	-.0382	.1330	-.00375	.0198	.00110
20.75	.450	-.0391	.1380	-.00348	.0197	.00111

COORDINATES OF LOWER CONTOUR (CONT.)

X"	Y_o"	Δ"	dY_o/dx	$d\Delta/dx$	d^2Y_o/dx^2	$d^2\Delta/dx^2$
20.75	.450	-.0391	.1380	-.00348	.0197	.00111
21.00	.486	-.0399	.1428	-.00320	.0195	.00113
21.25	.524	-.0407	.1475	-.00291	.0194	.00115
21.50	.561	-.0414	.1524	-.00261	.0192	.00118
21.75	.600	-.0419	.1570	-.00231	.0190	.00119
22.00	.641	-.0425	.1618	-.00201	.0188	.00120
22.25	.683	-.0429	.1664	-.00171	.0185	.00121
22.50	.726	-.0433	.1710	-.00140	.0183	.00122
22.75	.769	-.0436	.1755	-.00110	.0180	.00123
23.00	.814	-.0438	.1800	-.00079	.0176	.00124
23.25	.860	-.0440	.1843	-.00047	.0172	.00126
23.50	.908	-.0441	.1887	-.00015	.0168	.00128
23.75	.956	-.0441	.1929	.00016	.0164	.00129
24.00	1.005	-.0440	.1968	.00047	.0159	.00130
24.25	1.055	-.0438	.2008	.00079	.0154	.00130
24.50	1.106	-.0436	.2044	.00110	.0149	.00130
24.75	1.158	-.0434	.2080	.00141	.0145	.00131
25.00	1.211	-.0429	.2116	.00174	.0141	.00132
25.25	1.265	-.0425	.2149	.00205	.0136	.00132
25.50	1.320	-.0418	.2180	.00238	.0133	.00133
25.75	1.375	-.0413	.2213	.00270	.0130	.00134
26.00	1.431	-.0405	.2246	.00302	.0127	.00135
26.25	1.488	-.0397	.2277	.00335	.0124	.00135
26.50	1.545	-.0388	.2308	.00367	.0123	.00136
26.75	1.604	-.0378	.2338	.00400	.0122	.00137
27.00	1.664	-.0368	.2368	.00431	.0121	.00137
27.25	1.724	-.0357	.2397	.00466	.0120	.00137
27.50	1.784	-.0345	.2428	.00498	.0120	.00137
27.75	1.846	-.0332	.2456	.00531	.0120	.00137
28.00	1.908	-.0318	.2485	.00565	.0120	.00138
28.25	1.970	-.0305	.2515	.00598	.0120	.00138
28.50	2.034	-.0289	.2545	.00630	.0120	.00138
28.75	2.098	-.0273	.2575	.00664	.0120	.00138
29.00	2.164	-.0256	.2605	.00697	.0120	.00139
29.25	2.229	-.0238	.2633	.00730	.0120	.00139
29.50	2.296	- 0219	.2664	.00763	.0120	.00139
29.75	2.364	-.0200	.2695	.00798	.0120	.00139
30.00	2.432	-.0179	.2723	.00831	.0120	.00139
30.25	2.500	-.0158	.2752	.00866	.0117	.00139
30.50	2.570	-.0136	.2781	.00899	.0112	.00138
30.75	2.570	-.0136	.2781	.00899	.0112	.00138
30.75	2.640	-.0113	.2810	.00932	.0100	.00130
31.00	2.711	-.0088	.2832	.00969	.0083	.00123
31.25	2.782	-.0064	.2850	.01000	.0060	.00115
31.50	2.854	-.0038	.2861	.01026	.0030	.00102
31.75	2.925	-.0013	.28675	.01045	.0005	.00089
32.00	STRAIGHT LINE AT $dY_o/dx = .28675$ ↓	.0013	↓	.01060	.0000	.00074
32.25		.0040		.01069	↓	.00060
32.50		.0067		.01072		.00042
32.75		.0094		↓		.00028
33.00		.0120				.00014
33.25		.0146				.00003
33.50		.0174				.00000
⋮		LINEAR DEFLECTION, Δ, AT $d\Delta/dx = .01072$ ↓				↓
51.50						

TABLE II
COORDINATES OF UPPER CONTOUR

X"	Y_o"	Δ"	dY_o/dx	$d\Delta/dx$	d^2Y_o/dx^2	$d\Delta/dx^2$
-3.83	8.982					
-3.75	8.960					
-3.50	8.890					
-3.25	8.818					
-3.00	8.746					
-2.75	8.673					
-2.50	8.600					
-2.25	8.528					
-2.00	8.450					
-1.75	8.359					
-1.50	8.250					
-1.25	8.122					
-1.00	7.963					
-.75	7.775					
-.50	7.561					
-.25	7.313					
.00	7.047					
00.25	6.765					
00.50	6.471					
00.75	6.162					
1.00	5.846					
1.25	5.522					
1.50	5.200					
1.75	4.870					
2.00	4.545					
2.25	4.220					
2.50	3.913					
2.75	3.600					
3.00	3.300					
3.25	3.010					
3.50	2.723					
3.75	2.450					
4.00	2.182					
4.25	1.936					
4.50	1.694		.915			
4.75	1.478		.818			
5.00	1.286		.730			
5.25	1.114		.650			
5.50	.960		.580			
5.75	.823		.520			
6.00	.701		.460			
6.25	.593		.408			
6.50	.497		.360			
6.75	.413	-.0243	.318	.00000		-.00600
7.00	.340	-.0241	.273	-.00145		-.00500
7.25	.276	-.0236	.235	-.00255		-.00415
7.50	.222	-.0228	.200	-.00337		-.00348
7.75	.175	-.0219	.170	-.00404		-.00292
8.00	.136	-.0208	.145	-.00458		-.00246
8.25	.103	-.0197	.120	-.00506		-.00209
8.50	.076	-.0183	.098	-.00545		-.00175
8.75	.055	-.0169	.078	-.00578		-00147
9.00	.0375	-.0154	.060	-.00608		-.00123

COORDINATES OF UPPER CONTOUR

X"	Y_o"	Δ"	dY_o/dx	$dΔ/dx$	d^2Y_o/dx^2	$d^2Δ/dx^2$
9.00	.0375	-.0154	.060	-.00608		-.00123
9.25	.0245	-.0139	-.045	-.00632		-.00100
9.50	.0150	-.0123	-.030	-.00653		-.00080
9.75	.0082	-.0107	-.020	-.00670		-.00063
10.00	.0040	-.0089	-.012	-.00685		-.00046
10.25	.0015	-.0072	-.008	-.00696		-.00031
10.50	.0005	-.0054	-.003	-.00705		-.00018
10.75	.0000	-.0037	-.000	-.00712		-.00005
11.00		-.0018		-.00716		.00006
11.25		.0000		-.00719		.00016
11.50		.0017		-.00720		.00025
11.75		.0036		-.00717		.00035
12.00		.0054		-.00714		.00042
12.25		.0072		-.00708		.00050
12.50		.0089		-.00702		.00055
12.75		.0107		-.00694		.00061
13.00		.0124		-.00685		.00066
13.25		.0140		-.00673		.00071
13.50		.0157		-.00661		.00075
13.75		.0173		-.00648		.00079
14.00		.0189		-.00633		.00082
14.25		.0206		-.00617		.00085
14.50		.0221		-.00600		.00087
14.75		.0236		-.00583		.00090
15.00		.0250		-.00564		.00092
15.25		.0264		-.00543		.00093
15.50		.0277		-.00520		.00095
15.75		.0290		-.00495		.000955
16.00		.0302		-.00470		.00096
16.25		.0313		-.00445		.00096
16.50		.0324		-.00420		.00096
16.75		.0334		-.00395		.00096
17.00		.0343		-.00372		.000955
17.25		.0353		-.00348		.00095
17.50		.0362		-.00324		.00094
17.75		.0368		-.00300		.00092
18.00	.0000	.0376	.000	-.00279		.00090
18.25		.0383		-.00258		.00087
18.50		.0388		-.00236		.00085
18.75		.0394		-.00218		.00082
19.00		.0399		-.00200		.00078
19.25		.0404		-.00182		.00075
19.50		.0408		-.00165		.00070
19.75		.0412		-.00150		.00065
20.00		.0417		-.00135		.00061
20.25		.0420		-.00123		.00056
20.50		.0423		-.00110		.00051
20.75		.0426		-.00100		.00046
21.00		.0428		-.00090		.00041
21.25		.0430		-.00083		.00036
21.50	.0000	.0432	.0000	-.00075	.0000	.00031
21.75	.0000	.0434	.0000	-.00070	-.0026	.00026
22.00	-.0001	.0436	-.0012	-.00066	-.0064	.00023

COORDINATES OF UPPER CONTOUR

X''	Y_o''	Δ''	dY_o/dx	$d\Delta/dx$	d^2Y_o/dx^2	$d^2\Delta/dx^2$
22.00	-.0001	.0436	-.0012	-.00066	-.0064	.00023
22.25	-.0006	.0437	-.0031	-.00064	-.0093	.00018
22.50	-.0017	.0439	-.0058	-.00063	-.0116	.00015
22.75	-.0036	.0441	-.0090	-.00062	-.0135	.00012
23.00	-.0063	.0442	-.0126	-.00062	-.0152	.00008
23.25	-.0099	.0444	-.0165	-.00062	-.0165	.00006
23.50	-.0145	.0446	-.0208	-.00062	-.0173	.00005
23.75	-.0203	.0448	-.0252	-.00062	-.0176	.00003
24.00	-.0271	.0449	-.0296	-.00062	-.0177	.00002
24.25	-.0351	.0451	-.0340	-.00061	-.0176	.00001
24.50	-.0441	.0452	-.0384	-.00060	-.0175	.00001
24.75	-.0543	.0454	-.0428	-.00060	-.0173	.00001
25.00	-.0655	.0456	-.0470	-.00059	-.0170	.000025
25.25	-.0778	.0457	-.0513	-.00058	-.0168	.00004
25.50	-.0911	.0458	-.0554	-.00057	-.0166	.00005
25.75	-.105	.0459	-.0596	-.00055	-.0165	.00006
26.00	-.121	.0460	-.0637	-.00054	-.0163	.00007
26.25	-.137	.0461	-.0678	-.00051	-.0163	.00010
26.50	-.055	.0462	-.0718	-.00049	-.0161	.00011
26.75	-.173	.0464	-.0758	-.00047	-.0160	.00013
27.00	-.192	.0465	-.0797	-.00044	-.0160	.00015
27.25	-.213	.0466	-.0837	-.00040	-.0160	.00017
27.50	-.235	.0467	-.0877	-.00035	-.0161	.00019
27.75	-.257	.0467	-.0917	-.00031	-.0162	.00021
28.00	-.280	.0468	-.0956	-.00030	-.0163	.00023
28.25	-.304	-.0468	-.0996	-.00020	-.0164	.00025
28.50	-.330	-.0469	-.1037	-.00015	-.0164	.00027
28.75	-.356	-.0469	-.1076	-.00008	-.0165	.00029
29.00	-.384	-.0469	-.1117	-.00001	-.0165	.00031
29.25	-.412	-.0470	-.1158	.00005	-.0165	.00033
29.50	-.442	-.0470	-.1198	.00014	-.0165	.00035
29.75	-.472	-.0470	-.1239	.00021	-.0164	.00036
30.00	-.503	-.0469	-.1280	.00030	-.0163	.00037
30.25	-.536	-.0469	-.1321	.00039	-.0162	.00038
30.50	-.569	-.0468	-.1361	.00047	-.060	.00040
30.75	-.604	-.0467	-.1402	.00056	-.0158	.00041
31.00	-.640	-.0466	-.1440	.00065	-.0155	.00042
31.25	-.677	-.0464	-.1480	.00075	-.0152	.00043
31.50	-.714	-.0462	-.1517	.00085	-.0149	.00044
31.75	-.752	-.0460	-.1553	.00095	-.0146	.00045
32.00	-.792	-.0457	-.1589	.00105	-.0143	.00046
32.25	-.832	-.0455	-.1623	.00115	-.0140	.00047
32.50	-.872	-.0452	-.1659	.00125	-.0137	.00048
32.75	-.914	-.0449	-.1692	.00135	-.0135	.00050
33.00	-.957	-.0446	-.1726	.00146	-.0132	.00051
33.25	-1.001	-.0442	-.1758	.00157	-.0129	.00052
33.50	-1.045	-.0438	-.1790	.00168	-.0126	.00053
33.75	-1.090	-.0434	-.1821	.00180	-.0124	.00054
34.00	-1.136	-.0430	-.1852	.00190	-.0121	.000545
34.25	-1.183	-.0425	-.1881	.00202	-.0118	.00055
34.50	-1.230	-.0420	-.1911	.00214	-.0116	.00055
34.75	-1.279	-.0415	-.1939	.00225	-.0114	.00055
35.00	-1.327	-.0410	-.1967	.00237	-.0111	.00056

COORDINATES OF UPPER CONTOUR

X"	Y_o"	Δ"	dY_o/dx	$d\Delta/dx$	d^2Y_o/dx^2	$d^2\Delta/dx^2$
35.00	-1.327	-.0410	-.1967	.00237	-.0111	.00056
35.25	-1.377	-.0404	-.1993	.00250	-.0109	.00056
35.50	-1.427	-.0398	-.2021	.00261	-.0107	.00056
35.75	-1.478	-.0392	-.2050	.00272	-.0106	.00056
36.00	-1.529	-.0385	-.2076	.00285	-.0105	.00056
36.25	-1.581	-.0378	-.2102	.00298	-.0104	.00056
36.50	-1.634	-.0371	-.2128	.00310	-.0104	.00056
36.75	-1.688	-.0363	-.2156	.00232	-.0104	.00055
37.00	-1.742	-.0356	-.2182	.00336	-.0103	.00055
37.25	-1.797	-.0347	-.2208	.00352	-.0103	.00055
37.50	-1.853	-.0339	-.2234	.00362	-.0103	.00055
37.75	-1.909	-.0330	-.2261	.00375	-.0102	.00055
38.00	-1.966	-.0321	-.2286	.00388	-.0102	.00055
38.25	-2.023	-.0311	-.2312	.00400	-.0102	.00055
38.50	-2.082	-.0301	-.2337	.00415	-.0101	.000545
38.75	-2.140	-.0291	-.2362	.00427	-.0101	.00054
39.00	-2.199	-.0280	-.2387	.00441	-.0100	.00053
39.25	-2.259	-.0268	-.2413	.00455	-.0100	.000525
39.50	-2.320	-.0258	-.2438	.00470	-.0100	.00052
39.75	-2.381	-.0246	-.2463	.00483	-.0100	.00051
40.00	-2.443	-.0235	-.2489	.00497	-.0100	.000505
40.25	-2.506	-.0222	-.2513	.00510	-.0100	.00050
40.50	-2.569	-.0210	-.2540	.00525	-.0100	.00049
40.75	-2.633	-.0197	-.2565	.00537	-.0100	.00047
41.00	-2.697	-.0183	-.2589	.00550	-.0100	.00045
41.25	-2.762	-.0168	-.2614	.00563	-.0100	.00042
41.50	-2.820	-.0155	-.2640	.00574	-.0100	.00040
41.75	-2.894	-.0142	-.2667	.00582	-.0100	.00035
42.00	-2.961	-.0127	-.2692	.00590	-.0100	.00031
42.25	-3.029	-.0113	-.2718	.00595	-.0100	.00026
42.50	-3.097	-.0098	-.2742	.00598	-.0100	.00021
42.75	-3.166	-.0084	-.2768	.00600	-.0100	.00017
43.00	-3.235	-.0069	-.2790	.00600	-.0096	.00013
43.25	-3.306	-.0054	-.2816	.00600	-.0088	.00009
43.50	-3.376	-.0040	-.2837	.00600	-.0075	.00005
43.75	-3.447	-.0024	-.2853	.00600	-.0054	.00002
44.00	-3.519	-.0009	-.2864	.00600	-.0029	.00000
44.125	-3.554	-.0000	-.28675	.00600	-.0000	.00000

TABLE III

LOCAL PERTURBATIONS IN CONTOUR

LOWER SURFACE X''	λ''	$\frac{d\lambda}{dx}$	UPPER SURFACE X''	λ''	$\frac{d\lambda}{dx}$
15.00	———	———	36.5	∓.00632	∓.000
15.25	———	———	36.75	∓.00626	∓.00100
15.50	±.00030	±.00000	37.00	∓.00603	∓.00165
15.75	±.00110	±.00355	37.25	∓.00563	∓.00215
16.00	±.00206	±.00395	37.50	∓.00510	∓.00255
16.25	±.00285	±.00325	37.75	∓.00447	∓.00283
16.50	±.00331	±.00185	38.00	∓.00377	∓.00300
16.75	±.00381	∓.00020	38.25	∓.00303	∓.00292
17.00	±.00358	∓.00150	38.50	∓.00232	∓.00263
17.25	±.00276	∓.00330	38.75	∓.00166	∓.00226
17.50	±.00163	∓.00990	39.00	∓.00111	∓.00177
17.75	±.00073	∓.00405	39.25	∓.00065	∓.00110
18.00	±.00065	∓.00010	39.50	∓.00038	∓.00010
18.25	±.00124	±.00275	39.75	∓.00037	±.00115
18.50	±.00182	±.00242	40.00	∓.00067	±.00200
18.75	±.00206	±.00110	40.25	∓.00114	±.00243
19.00	±.00186	∓.00095	40.50	∓.00172	±.00235
19.25	±.00128	∓.00250	40.75	∓.00229	±.00195
19.50	∓.00053	∓.00310	41.00	∓.00277	±.00140
19.75	∓.00010	∓.00255	41.25	∓.00315	±.00077
20.00	∓.00038	+.0012	41.50	∓.00340	±.00010
20.25	∓.00024	±.00065	41.75	∓.00342	∓.00055
20.50	±.00024	±.00200	42.00	∓.00327	∓.00105
20.75	±.00090	±.00285	42.25	∓.00301	∓.00145
21.00	±.00151	±.00250	42.50	∓.06266	∓.00178
21.25	±.00193	±.00170	42.75	∓.00223	∓.00194
21.50	±.00215	±.00090	43.00	∓.00175	∓.00194
21.75	±.00221	±.00000	43.25	∓.00128	∓.00178
22.00	———	———	43.50	∓.00083	∓.00150
———	———	———	43.75	∓.00044	∓.00108
			44.00	∓.00012	∓.00045
			44.125	∓.00000	.000000