Chapter IV

Articulation Between Elementary and Secondary Schools

The Importance of Articulation

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PROBABLY no experienced teacher or school administrator needs to be convinced that continuity and articulation in our school program are important. If so the fact that two substantial current publications[1] are largely devoted to these topics should emphasize that consideration of this problem is particularly timely, as well as important. Further, data gathered in connection with the first of these two studies shows that, of the various situations in which students reported help or hindrance in their progress through school, subject matter and moving to the next school level ranked third and fourth in importance as judged by the frequency with which they were mentioned. This seems to give particular importance to the problem of the articulation between elementary and secondary schools of their mathematics programs.

This observed importance is well explained by psychological and philosophical considerations which also give some clues as to how to work toward improved continuity and articulation. It seems well established that *meaningful learning,* learning with *understanding,* is easier, retained longer, and more likely to transfer to new situations. Meanings and understandings, however, are not absolutes or all-or-none insights which are achieved at some particular instant. Meanings and understandings grow, develop, and expand throughout one's life. Rarely if ever can one who pauses to think and say "I fully understand that. I comprehend all that there is to be understood about it." Meaning is a continuum, always capable of being extended or having gaps, perhaps unperceived at first, filled in. This expansion of one's understandings and insights develops as one perceives similarities, analogies, and relationships between new concepts and ones which have been met earlier. This growth takes place, then, as one meets old principles and processes in new

[1]Association for Supervision and Curriculum Development. *A Look at Continuity in the School Program.* 1958 yearbook. Washington, D.C.: the Association, a department of the NEA, 1958.
National Council of Teachers of Mathematics. *The Growth of Mathematical Ideas, Grades K-12.* Twenty-fourth yearbook. Washington, D.C. The Association, a department of the NEA, 1958.

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situations, sees new applications for old ideas, observes old ideas as special cases of new, or finds that a new and apparently distinct concept has connections with ones learned earlier. Even those sudden perceptions so delightful to both pupils and teacher in which "the light dawns" are probably never the bolts-from-the-blue which they seem, but happen only when a concept has been met often previously, though perhaps in different terms and contexts.

The implications of this for teachers and curriculum planners seem clear. In lesson planning teachers need continually to look both forward and backward. They need to look for those threads of continuity begun in earlier years to which they may attach the ideas which they are helping youngsters to develop. Further, they must look at concepts which are to be developed in later years and search out ways to prepare youngsters to understand them by directing their attention to elementary special cases and introducing, even though briefly and simply, ideas which will grow in importance later. For example, mathematicians find the distributive law to be a frequently recurring fundamental idea of more importance than can be explained in this short space. However, this law occurs and is used in some way at nearly all levels of instruction. This law stated in algebraic symbols is $a \cdot (b + c) = ab + ac$. Neither the words "distributive law" nor the symbolic representation need be used anywhere in the elementary school, but the principle and its frequent recurrence should be perceived by elementary-school students. They are likely to perceive it, however, only if they are led to do so by teachers who are aware of its importance and alert to its recurrences. Thus when a youngster sees that three toys priced at two dollars and ten cents each will cost six dollars and thirty cents, he is using this principle; $3 \text{ x } (2 + 10) = (3 \text{ x } 2) + (3 \text{ x } 10)$. Similarly the total weight of two packages containing one pound and six ounces each is two pounds and twelve ounces, or to cut three boards each 2′ 1″ he will need a board at least 6′ 3″ long. This same principle is used in multiplying 3×21. Since 21 really means $20 + 1$, 3×21 really means $3 \times (20 + 1) = (3 \times 20) + (3 \times 1) = 60 + 3 = 63$. Of course the student eventually learns and practices an algorism in the form

$$\begin{array}{r} 21 \\ \underline{3} \\ 63 \end{array}$$

But, if understanding is to precede the formulation of algorisms and drill, he must first understand and use that which years later he may learn to call the distributive law. This law is also the basis for many other processes from elementary school through advanced mathematics such as factoring, special products, and the explanation for the definition of the sign laws in junior high-school algebra. It is also to be found in relationships between areas which may be noted both in algebra and in the intuitive geometry of the earlier years.

For teachers to be able and willing to point out and use these central continuing themes, they need first be aware of their existence and scope, and secondly they must recognize that not all of these concepts can or should be taught at their first appearance for the mastery and understanding which ultimately we hope the student achieves. By this we do not mean that mastery and understanding are unimportant, that progress toward appropriate levels of achievement should not be measured, nor that we should be content with less achievement than that of which the individuals in our class are capable. We do mean that the possibility of complete mastery and understanding is not the sole criterion for the inclusion of a topic in a curriculum or the pointing out of a relationship in a daily lesson plan. Seeds which are not planted cannot be expected to grow, and teachers who are delighted by the progress of a class should not loose their joy in their own achievement, but should realize that just as they are reaping harvests sown by their predecessors, so they too must at times sow where they may not reap.

To identify these central or continuing themes in mathematics instruction and to show clearly and concretely how they can be a conscious part of the instructional program at all grade levels is not an easy task. The second of the two works cited at the beginning of this article attempts to do this. It discusses *number and operation, relation and function, proof, measurement and approximation, probability and statistics, symbolism, mathematical modes of thought, psychological teaching-learning principles*. It is clear that not all of these central concepts are or should be equally prominent at all grade levels. Number and operation are probably more all-pervading vertically than are probability and statistics. This latter topic is a relative newcomer to the roll of central ideas in elementary- and secondary-school mathematics. Wo do not yet really have well developed definitions of the amount of it to be taught, or when or how. However, its rapidly expanding role in both pure and applied mathematics, even daily living, has convinced many that it belongs in this category.

Many of the concepts listed above have for years been taught at some and perhaps all grade levels. But even where this has been done, teachers may deliberately point out the importance and recurrence of basic ideas more frequently than they do. For example there is room for much more attention to be paid to the notion of proof and logical structure in arithmetic and algebra. We can check subtraction (division) by addition (multiplication) not by a happy accident, but by virtue of the fact that subtraction is *defined* as the inverse of addition. Thus six minus two really means, what added to two gives six? If this were stressed all along from the earliest grades, we would be teaching basic ideas which extend to new situations as well as a mechanical device for checking.

To get our school systems to improve in this respect (much good work is being done) we must (1) be sure that mathematics teachers understand the basic ideas themselves, (2) encourage them consciously and deliberately to seek out and use continuing threads in daily teaching, (3) provide for joint vertical staff meetings where talk of the nature and role of these continuing ideas is discussed in an interested and purposeful atmosphere. In other words, looking for the nature of the central ideas and the steps in their continuing development may provide a perspective view of the entire mathematics program which may effect both teaching and the content and organization of the curriculum.

This viewpoint may explain the apparent contradiction in the first of the two works cited in our introductory paragraph. On page 88 it says, "Curriculum sequences based on the logic of subject matter may be inadequate as a basis for the continuity of learning which our schools should provide," while on page 133 it says, "An answer seems to lie in the identification of certain basic concepts, . . .," and on page 93, "New material to be learned will be meaningful insofar as the learner is able to see relationships between the elements of the new learning and the things he has previously understood or experienced. He must also have a mastery of abstractions, . . . Most teachers know that the learning of any school subject, particularly one with a great deal of logical relatedness, like science or mathematics, is more meaningful and permanent if the student understands it *as he goes*." These two statements may be reconciled by our earlier observations that: (1) understanding, and in mathematics this means understanding of logical relationships as well as social applications, is important, but that, (2) an order and technique of presentation which is determined solely by the final and most abstract formulation of a subject may not be the best pedagogically.

The essence of mathematics is abstractness; a single mathematical concept may be interpreted and applied in a variety of different ways. Even the idea of cardinal numbers, 1, 2, 3, 4, . . ., from sets of objects (candy, marbles, dolls, *etc.*), is abstract. Further the foundations of mathematics (and I am deliberately using mathematics not arithmetic here because, even at an elementary level, the objectives should be much broader in both content and concept than the mere manipulation of numbers which some people associate with arithmetic) lie in its logical structure and the process of proof. However, understanding of these will be achieved neither by imposing on elementary mathematics the logical structure of the graduate school nor by ignoring abstractness and structure entirely or even in part. An understanding of both the logical structure and the way in which ideas grow and develop will enable teachers to plan a continuing emphasis on the central themes of mathematics.

These themes include mathematical modes of thought; *i.e.*, problem solving and reflective thinking. This process is aided and tested by a knowledge of the structure of mathematics and its logic, but it also requires intuition, insight, intelligent trial and error which cannot be taught in a unit or by a chapter in a text. These are taught by continual use of "discovery" teaching techniques, continual stimulus to the pupil to ask "why?" and continual giving of assistance and direction to him in his search for "why?" Students and teachers find it difficult to turn on and off at will, as one goes from grade to grade or class to class, an inquiring attitude and a thoughtful developmental approach to new situations. Continuity, not uniformity, in the pursuit of the goal of developing pupils' ability to think is, therefore, most essential. This says that continuity, not uniformity, in teaching methods or at least in the spirit of inquiry and discovery is important. Both elementary-school teachers who give only rules or examples followed by drill and secondary-school teachers who do all the telling exist. Both need to turn to the better practices of the many fine teachers who exist at both levels. Further, all can profit from exploring together vertically the objectives of our instruction and the continuing central themes which should exist and be exploited throughout grades K-9 with respect to both content and teaching approach.

Mathematics As a Man-Made Invention in the Elementary Classroom

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WE ARE living in an increasingly complex scientific age. It is an age in which the quantitative aspects of technology touch each of us. Effective living requires an understanding of both the technology and the language man has developed to communicate its characteristics—mathematics. In this brief article, the author discusses some of the ways in which elementary teachers are contributing to the development of pupils' understanding of mathematics as a man-made invention.

Quantitative relationships exist in our environment. The "threeness" of a clover leaf, the shape of a crystal, the division of a cell, all exist independent of man. Throughout the elementary curriculum, primary importance is placed on increasing the pupil's awareness of the quantitative nature of his many experiences. Man's invention begins with his attempt to communicate to others the quantitative characteristics of his environment and experience.

Three important routes through which teachers are approaching an understanding of the man-madeness of mathematics are: (1) an understanding of symbolism in mathematics, (2) an understanding of the systematic organization of quantitative symbols, (3) the story of man's creation and use of symbols and organization of symbols. Elementary classroom teachers contribute to each of these objectives in a variety of ways.

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