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ANN ARBOR

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P. M. Naghdi

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1. INTRODUCTION

This is the Final Report on Contract No. DA-20-018-ORD-12099 between the Office of Ordnance Research and The University of Michigan from February 1, 1952, to January 31, 1957. The results of research carried out during this period (5 years) have been issued as eight Technical Reports and have also been published in technical journals, as follows:

	Technical Report No.	Title of Report (or Paper)	Author(s)	Publication Medium
(A)	1 and 2	An Experimental Study of Biaxial Stress-Strain Relations in Plasticity	Naghdi and Rowley	J. Mech. Phys. Solids, Vol. 3, pp. 63-80 (1954)
(B)	3	On the Deformation of Elastic Shells of Revolution	Naghdi and DeSilva	Quart. Appl. Math., Vol. 12, pp. 369-374 (1954)
(C)	4	An Experiment on Circular Plates in the Plastic Range	Cooper and Shifrin	Proc. 2nd U. S. Nat'l. Congr. Appl. Mech., pp. 527-534 (1955)
(D)	5	Experiments Concerning the Yield Surface and the Assumption of Linearity in the Plastic Stress-Strain Relations	Naghdi, Rowley, and Beadle	J. Appl. Mech., Vol. 22, pp. 416-420 (1955)
(E)	6	Deformation of Elastic Ellipsoidal Shells of Revolution	Naghdi and DeSilva	Proc. 2nd U. S. Nat'l. Congr. Appl. Mech., pp. 333-343 (1955)
(F)	7	Asymptotic Solutions of a Class of Shells of Revolution with Variable Thickness	DeSilva and Naghdi	To appear in Quart. Appl. Math.
(G)	8	An Experimental Study of Initial and Subsequent Yield Surfaces in Plasticity	Naghdi, Essenburg, and Koff	To appear in J. Mech. Phys. Solids.

As may be seen from the above table, two different programs of research were carried out simultaneously: (1) an experimental investigation in plasticity [identified by (A), (C), (D), and (G)]; and (2) on some aspects of the theory of elastic shells of revolution [identified by (B), (E), and (F)].

2. EXPERIMENTAL INVESTIGATION IN PLASTICITY

We first recall that while considerable progress has continuously been made in the development of the isotropic theory of plasticity for strain-hardening materials, relatively little is known about theories of plasticity which account for initial anisotropy and for such phenomena as Bauschinger effect. Furthermore, even to date, not only are very few specific elastic-plastic solutions available, but also the available experimental data and their correlation with the mathematical theory of plasticity are sparse.

In Ref. (A), experimental results for ten tubular 24-S-T-4 aluminum-alloy specimens which possessed severe initial anisotropy are reported. The tests were performed by subjecting a specimen initially to tension and later to torsion with increasing or decreasing tension. This procedure produced loading paths which should give strain paths, all having initial shear moduli (at the initiation of twist) equal to the elastic shear modulus, if the material were isotropic. By performing such tests on ten specimens, the initial shear modulus was found to be much less than that predicted by isotropic theory of plasticity. This behavior could not be explained by invoking an unsymmetrical smooth yield locus, although one having a corner at the chosen yielding in tension would suffice. Subsequently, this question of the existence of corners on the yield loci of material tested was answered affirmatively in Ref. (D).

Although it is well known that under a uniaxial state of stress, most materials exhibit the Bauschinger effect, i.e., possess a lower yield stress upon the reversal of the load, under more general circumstances (such as a biaxial state of stress), no experimental evidence has been available with regard to the shape of subsequent yield surfaces (or loading functions) beyond the initial yield surface. In this connection, experimental results for 27 tubular specimens made of 24-S-T-4 aluminum alloy [this material is not the same as that employed in Refs. (A) and (D)] which were initially (reasonably) isotropic were reported in Ref. (G). These specimens were subjected to combined torsion-tension-reversed torsion with variable loading paths in a study of initial and two subsequent yield surfaces covering the first and the fourth quadrants of the axial stress-shear stress plane. Among the conclusions reached in Ref. (G), the following may be mentioned: (1) the initial yield surface is essentially symmetric about the axial stress-axis; (2) the first and second subsequent yield surfaces, in the neighborhood of the shear stress-axis, display a pronounced Bauschinger effect which gradually vanishes as the curves approach the axial stress-axis; (3) the initial yield surface, except for a slight deviation in the vicinity of the axial stress-axis, is almost identical with the Mises yield condition; (4) all yield surfaces are convex with respect to origin; and (5) the existence of regions of high curvature on the subsequent yield surfaces in the vicinity of shear stress-axis is reminiscent of the slip theory of plasticity, which may also be viewed as a special incremental theory.

In addition to the above, an experiment on the load-carrying capacities on circular plates in the plastic range [Ref. (C)] was carried out, and comparison was made with the theoretically predicted limit loads given by Prager and Hopkins.

3. INVESTIGATION ON ELASTIC SHELLS OF REVOLUTION

For purposes of clarity, we first recall that the differential equations of the general linear (classical) theory of thin elastic shells in the form given by Love were not even in special cases amenable to analytical treatment. In 1912, H. Reissner, by a judicious choice of the dependent variables, expressed the deformation of a spherical shell under axisymmetric loading in terms of two ordinary second-order differential equations and indicated their solution by means of series involving exponential functions. In the following year, Meissner noted that these equations of H. Reissner were of the hypergeometric type and could be generalized to include all shells of revolution generated by curves of constant radii of curvature. In the following year, Meissner showed that a similar reduction of the general fourth-order differential equation to two second-order equations was possible for any shell of revolution, provided the thickness of the shell varied in a prescribed manner (Meissner's condition). H. Reissner, in addition to observing that the nonhomogeneous solution of the differential equations (due to the presence of surface load) may be approximated by a corresponding membrane solution, had also suggested (as noted above) the possibility of an asymptotic solution of the equations in powers of the ratio of shell thickness to a representative dimension of the shell. The integration of the H. Reissner-Meissner differential equations for the general shell of revolution, by classical methods of asymptotic integration, have been given by Hildebrand (1949). Such solutions, although very useful in many cases, are not valid whenever the region of interest includes a point (such as the apex of spherical shell) at which a singularity occurs in the differential equations.

The derivation of the axisymmetric deformation of shells of revolution with small displacements and the resulting differential equations were reconsidered in 1949 by E. Reissner. His formulations, although differing only slightly from those of H. Reissner-Meissner, offer certain advantages not revealed in earlier ones. Subsequently, the differential equations of the linear theory in the form given by E. Reissner were combined in Ref. (B) into a single complex differential equation which is valid if the "Meissner condition" holds.

In this form, the differential equations of shells of revolution are readily amenable to treatment by a method of asymptotic integration due to Langer, and yield solutions valid at singularities of the equations.

Utilizing the results given in Ref. (B), the deformation of thin elastic ellipsoidal shells of revolution of uniform thickness under axisymmetric loading was considered at length in Ref. (E). The solution obtained is valid at the apex of the shell and is of considerable practical interest in connection with design of pressure vessels and similar structural components where the region of interest includes the apex of the shell. Subsequently, the scheme suggested in Ref. (B) was also employed in Ref. (F) to obtain general asymptotic solutions for a class of shells of revolution of both uniform and variable thickness. Reference (F) also includes a study of asymptotic solutions of shallow shells of revolution together with a number of examples of practical interest.

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