

ENGINEERING RESEARCH INSTITUTE
UNIVERSITY OF MICHIGAN
ANN ARBOR

STATUS REPORT NO. 7

STRESS-STRAIN RELATIONS IN PLASTICITY
AND RELATED TOPICS

P. M. NAGHDI
Professor of Engineering Mechanics

Project 2027

DEPARTMENT OF THE ARMY
DETROIT ORDNANCE DISTRICT
CONTRACT NO. DA-20-018-12099
PROJECT NO. TB20001
DA PROJECT 599-01-004, OOR PROJECT NO. 234

November 1, 1954

STRESS-STRAIN RELATIONS IN PLASTICITY
AND RELATED TOPICS

During the last period (August 1 to November 1, 1954) work was continued as follows:

(a) With reference to our work on experimental plasticity, the detailed design of the extensometer [see our OOR Technical Report No. 2, December, 1953] has been modified in order that (1) the interdependence of the axial and circumferential displacement measurements may be minimized, (2) the range of axial displacement may be increased, and (3) the rotation may be measured in both directions. The modifications that have been carried out are as follows:

The lower disk has been replaced and both the upper face of the lower disk and the lower face of the upper disk have been machined to plane surfaces to accuracies of better than $1/10,000$ inch. While this contributes nothing to the functioning of the extensometer it greatly facilitates the construction of the instrument in that reference planes are clearly prescribed.

Three spacer bars which are doweled to the lower plate (and perpendicular to the lower plate) and fastened with removable screws prescribe the gage length and insure the parallelism of the reference planes. The zero point on the twist measuring potentiometer is also prescribed by these spacer bars and has been located to permit measurement of twist in both directions.

The aligning screws have been replaced with screws of finer thread and provided with stops so that in mounting the extensometer on a 0.900 inch O.D. specimen all that is required is to tighten the aligning screws to the stops and the extensometer is aligned symmetrically about the axis of the specimen with the reference planes perpendicular to the axis of the specimen. The accuracy of this procedure is the accuracy which dial gages measuring to $1/10,000$ inch will permit but considerable improvement over the alignment blocks previously employed is afforded.

Whereas previously only one of each group of three clamping studs was spring loaded, all six studs have been provided with springs. This provides for a retention of axial alignment throughout diameter changes of the specimen during the course of the experiment.

The four phosphor bronze springs have been replaced with three steel leaf springs. This permits displacements up to $3/8$ inch or provides for axial strain measurements of approximately 0.14 in./in. based on a 2-5/8-inch gage length. It is apparent from geometrical considerations that an even number of symmetrically situated springs (provided with strain gages) would compensate any errors in perpendicularity of the reference planes provided that all spring-strain-gage combinations had identical characteristics. Under the present set up, since errors in perpendicularity of the reference planes are minimized, three spring-strain-gage combinations should suffice and no interaction between axial and radial displacement measurements will be introduced even though there is a deviation between the characteristics of the individual spring-strain-gage combinations.

(b) Considerable efforts were made for the interpretation of our previous experimental results [Technical Report Nos. 2 and 5] concerning stress-strain relations in plasticity and comparison of test data with predictions of existing theories of plasticity. It may be mentioned here that prediction of the slip theory of plasticity [Reference 9 of Technical Report No. 2] compared quite poorly with experimental data of two typical specimens.

During the last two weeks, however, in the light of some analytical considerations, some progress has been made along the lines mentioned and it appears at this time that a favorable comparison between theory and experiment may be possible.

(c) Since Technical Report No. 6 (entitled "On Elastic Ellipsoidal Shells of Revolution") was issued, it was felt that the analysis of ellipsoidal shells of variable thickness (where a drastic variation in thickness is present) is of considerable, practical, and theoretical value. To this end the results given previously in Technical Report Nos. 3 and 6 were extended to shells of variable thickness. For purposes of comparison the solution is being obtained for the case treated in Technical Report No. 6, but with variable thickness.

