AFOSR TR-59-33
ASTIA AD-212 470

THE UNIVERSITY OF MICHIGAN RESEARCH INSTITUTE
ANN ARBOR, MICHIGAN

Final Report

THERMO-ELASTICITY AND VISCO-ELASTICITY IN STRUCTURES

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UNITED STATES AIR FORCE
AIR RESEARCH AND DEVELOPMENT COMMAND
AIR FORCE OFFICE OF SCIENTIFIC RESEARCH
PROJECT NO. 1750-17500-717
CONTRACT NO. AF 18(603)-47
WASHINGTON, D. C.

February 1959

The results presented in this paper were obtained in the course of research sponsored by the Air Force Office of Scientific Research under Contract No. AF 18(603)-47, when the author was at The University of Michigan.

During the term of the contract (initiated on March 1, 1956), five aspects of the program of research were completed, published, and issued as Technical Notes as follows:

1. Technical Note No. 1 (AFOSR TN-57-618): "On Elastic Plates of Variable Thickness," by F. Essenburg and P. M. Naghdi, *Proc. 3rd U.S. Nat'l Congr. Appl. Mech.*, 313-319 (1958).

This paper contains a derivation of *suitable* stress-strain relations for elastic isotropic plates of variable thickness which include the effects of transverse shear deformation and normal stress, as well as the variation in thickness. The significance of the results is examined in the light of some simple examples, where in particular for torsion of plates whose thickness is such as to give rise to equilateral triangular and elliptic cross sections, exact agreement is obtained for the stress distribution with the corresponding results of Saint-Venant's theory of torsion.

2. Technical Note No. 2 (AFOSR TN-58-174): "On Axially Symmetrical Plates of Variable Thickness," by F. Essenburg, J. Appl. Mech., 25, 625-626 (1958).

In this paper, axially symmetric plates of variable thickness are considered in the light of observations made in (1). With the thickness treated as a function of the middle plane coordinate, it is shown that the prediction of the classical theory of plates of variable thickness (where the effect of transverse shear deformation, as well as normal stress, is neglected) might be substantially in error.

3. Technical Note No. 3 (AFOSR TN-58-904): "On Thermo-Elastic Stress-Strain Relations for Thin Isotropic Shells," by P. M. Naghdi, J. Aero/Space Sciences, 26, p. 125 (1959).

The usual formulation of problems of thermo-elastic shells of isotropic materials (e.g., as in Love's first approximation) is defective in the sense that it does not conform to the requirement that the initially stress-free isotropic shells (in the absence of suitable edge constraints), when subjected to a uniform temperature field, should remain stress-free and undergo only a uniform dilatation. To remedy this situation, thermo-elastic stress-strain relations were derived which are free from the defect mentioned above. The results which may be regarded as a generalization of earlier work ["On the Theory of Thin Elastic Shells, by P. M. Naghdi, Quart. Appl. Math., 14, 369-380 (1957)], assume an especially simple form for the theory of Love's first approximation.

4. Technical Note No. 4 (AFOSR TN-58-993): "Response of Shallow Visco-elastic Spherical Shells to Time-Dependent Axisymmetric Loads," by P. M. Naghdi and W. C. Orthwein.

This paper is concerned with the response of shallow viscoelastic spherical shells to arbitrary time-dependent axisymmetric loads; the medium is assumed homogeneous and isotropic. Although emphasis is placed on unlimited shallow spherical shells, shallow spherical shell segments are also considered and discussed. The solutions, employing the differential equations governing the transverse motion of thin shallow elastic shells, are obtained (within the scope of the linear theory of viscoelasticity) with the joint use of the Laplace and the Hankel transforms which, by interchanging the order of the inversions, avoids an otherwise intricate task of contour integration in the complex Laplace transform-plane. Explicit results in integral form are deduced for viscoelastic shells under instantaneous pulse loading (including those uniformly distributed about and concentrated at the apex), and are particularized to the cases of Maxwell and Kelvin solids. The solutions for a shallow elastic shell and for the case of a flat plate are also given as by-products of the general solution and comparison is made with known results.

5. Technical Note No. 5 (AFOSR TN-59-109): "On Axisymmetric Vibrations of Thin Shallow Viscoelastic Spherical Shells," by P. M. Naghdi and W. C. Orthwein.

This investigation is concerned with transverse vibrations of shallow viscoelastic spherical shells subjected to axisymmetric loads which are harmonic in time. While the solutions due to these types of loading may be obtained in integral form as a special case of the general results in (4), for such solutions (which include the steady state solutions) it is desirable from a practical point of view to obtain an alternative closed representation. Thus, the steady state solution, within the scope of the linear theory of viscoelasticity, is deduced in terms of Kelvin functions and is valid for finite shell segments as well as unlimited shallow spherical shells. In particular, the solution is applied to an unlimited viscoelastic shallow spherical shell subjected to an oscillating load uniformly distributed over a small circular region about the apex. Numerical results for axial displacements and stresses are obtained for two special viscoelastic materials (Maxwell and Kelvin) as well as for the elastic shell, and comparison is made with known results.