TO ENHANCE LEARNING
The Snuffing Version of EQUATIONS

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In a series of studies it has alredy been determined that playing the EQUATIONS game as part of the regular classroom curriculum results in increased achievement in mathematics (Allen and Ross, 1975, 1977; Edwards et al., 1972) and more favorable attitudes toward school in general (Allen and Main, 1976). It has also been determined that varying the method of scoring to provide incentive for paying attention to selected ideas results in even further achievement gains (Allen et al., forthcoming). The hypothesis that underlies the "snuffing" version of EQUATIONS is that it will produce still further gains in achievement compared to the standard version game. The snuffing version provides incentive for each player in the course of play to reveal to the other players mathematical ideas that he or she is thinking about in seeking to win. In the standard game such exchanges occur only one time in a single match (when a challenge has been made or a force-out declared), whereas in the snuffing version the idea-swapping can occur on every move throughout the match. The

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pair of rules that are added to the standard version of EQUATIONS to produce the snuffing version are these:

(1) Before each move is made, any player may write a possible Solution that is concealed until after the move. The mover must give the other players at least one minute to write their possible Solutions on their SNUFFING SHEETS. After a player has written a possible Solution, the player should place the SNUFFING SHEET face down in the center of the table. Once the SNUFFING SHEETS of all of the other players are in the center of the table, the mover can move before the one-minute deadline and the other players can call a stall on the mover. After the move, all of the possible Solutions written are shown to all players. If a player's possible Solution is extinguished by the move, that player receives an extra point in the scoring.

(2) When there are more than two remaining resources, a regular move by a player consists of transferring two of them to either the same (or different) section(s) of the playing mat (Forbidden, Permitted, or Required).

SAMPLE GAME

A sample game of snuffing EQUATIONS is presented so that readers may judge for themselves the reasonableness of the hypothesis. The ultimate test, of course, will be at some stage in the conducting of a carefully controlled experiment.

The match started with the following 21 resources:

0	3	3	3	4	+	+	−	x	÷	÷
5	6	6	9	9	√	√	√	*	*	

from which Player 1 (P1) used the three circled resources to set a goal of $3\sqrt{9}$ (interpreted in the game as the third root of nine). After the players made their snuffing predictions, P1 chose to move, forbidding the 9 and requiring a ÷, and the playing mat looked as follows:

Players	Predictions	Bonuses
P1	6√[9x(5+4)]	1
P2	3√(5+4)	1
P3	3√(5+4)	1

SOLUTION = GOAL: 3 √9								
RESOURCES								
0		3	3	4	+	+	–	x ÷
5	6	6				√	√	* *
FORBIDDEN			PERMITTED			REQUIRED		
9						$		

All three of the possible Solutions predicted to be "snuffed out" were, in fact, extinguished by the requiring of the ÷, because none of them contains a ÷.

After the players' predictions, P3's move of forbidding the x and permitting one of the 3s left the mat as follows:

Players	Predictions	Bonuses
P1	3*[(6-4)÷3]	0
P2	[3√(5+4)]x(6÷6)	1
P3	3√[6x6)÷4]	1

SOLUTION = GOAL: 3 √9								
RESOURCES								
0			3	4	+	+	–	÷
5	6	6				√	√	* *
FORBIDDEN			PERMITTED			REQUIRED		
9	x		3			÷		

The possible Solutions of P2 and P3 were snuffed by the forbidding of the x because the only x was used in their possible Solutions. However, P1's prediction is still a possible Solution after the move; so no bonus point for P1. (Note that in the game, "*" denotes exponentiation. So, 3*2 = 3^2 = 9.)

After the players' predictions and P1's forbidding the 5 and permitting the 4, the mat looked as follows:

Players	Predictions	Bonuses
P1	[3√(5+4)]+(0÷6)	11
P2	[3√(5+4)]*(6÷6)	1
P3	[6√(4*3)]+(0÷5)	0

_______ = 3 √9

SOLUTION GOAL

RESOURCES

0 3 + + – ÷

6 6 √ √ * *

FORBIDDEN			PERMITTED		REQUIRED
9	x	5	3	4	÷

The possible Solutions of P1 and P2 were snuffed by the forbidding of the 5; so they each get a bonus point. P3's expression was also snuffed by the move, but it was not a possible Solution: it did not equal the goal. Hence, no bonus point for P3 on this move. (Apparently, what P3 was thinking of as a Solution was [6√(3*4)] + (0÷5).)

Before P2's second turn, the players each still made a prediction of a possible Solution that would be snuffed by the move. P2 then forbade the ÷ and one of the +s, and the mat was as follows:

Players	Predictions	Bonuses
P1	3√[6÷(4÷6)]	1
P2	[3*(4÷6)]÷(6*0)	1
P3	[6+(0÷6)]√(3*4)	0

_______ = 3 √9

SOLUTION GOAL

RESOURCES

0 3 + –

6 6 √ √ * *

FORBIDDEN			PERMITTED		REQUIRED
9	x	5	3	4	÷
÷	+				

The forbidding of the only remaining ÷ snuffed those possible Solutions that used two ÷s. So P1 and P2 each got a bonus point on this move. But P3's prediction was not snuffed; it was still a possible Solution after the move.

After three more predictions, P3 forbade one of the *s and one of the √ s, leaving the situation looking like this:

Players	Predictions	Bonuses
P1	(6*0)√[3*(4÷6)]	1
P2	(3*4)*[(6*0)÷6]	0
P3	[(6÷4)√3]+(0*6)	0

	______ = 3 √9	
	SOLUTION GOAL	
	RESOURCES	
0 6 6	3 +	– √ *
FORBIDDEN	PERMITTED	REQUIRED
9 x 5	3 4	÷
÷ + *		
√		

Moving in haste, P3 failed to realize that forbidding only one of the *s did not snuff the predicted possible Solution, because it used only one *. P2's expression was snuffed, but it was not a possible Solution because it used three *s. P1 got the only bonus point.

P2 failed to get a prediction written on the SNUFFING SHEET before the one-minute deadline expired. P1 then quickly made a move to cut off any further opportunity for P2 to make a prediction. After P1 forbade the 0 and the remaining +, the playing mat looked like this:

Players	Predictions	Bonuses
P1	3*(4÷6+0	1
P2		0
P3	(6÷4)√3+0	1

	______ = 3 √9	
	SOLUTION GOAL	
	RESOURCES	
6 6 3		– √ *
FORBIDDEN	PERMITTED	REQUIRED
9 x 5	3 4	÷
÷ + *		
√ 0 +		

The possible Solutions of P1 and P3 were snuffed by the forbidding of the 0 and the +.

P2 predicted a slightly simplified version of P3's prior prediction, and then snuffed it out by forbidding the remaining √ and requiring the remaining 3. The playing mat then looked like this:

Players	Predictions	Bonuses
P1	3*(4÷6)	0
P2	(6÷4)√3	1
P3	(6÷4)√3	1

Both P2 and P3 got bonus points for predicting possible Solutions that were snuffed by the forbidding of the √. P1's prediction remained a possible Solution after the move.

SOLUTION	=	GOAL
		3 √9

RESOURCES

–

6 6 *

FORBIDDEN	PERMITTED	REQUIRED
9 x 5	3 4	÷ 3
÷ + *		
√ 0 +		
√		

Each of the three players predicted the same possible Solution before what turned out to be the final move of the game. When P3 forbade a 6 and required the –, P2 thought that the move (P)revented all possible solutions and quickly challenged it as a P-flub. The situation was this:

Players	Predictions	Bonuses
P1	3*(4÷6)	1
P2	3*(4÷6)	1
P3	3*(4÷6)	1

The requiring of the – snuffed all three predictions; so, all three players received bonus points.

SOLUTION	=	GOAL
		3 √9

RESOURCES

6 *

FORBIDDEN	PERMITTED	REQUIRED
9 x 5	3 4	÷ 3 –
÷ + *		
√ 0 +		
√ 6		

P1 joined the mover, and each of them sustained the burden of proof with:

3*[6-4)÷3].

CHANGES IN SCORING

So that the bonus points achieved for snuffing predictions on moves throughout the game will not "swamp" the end-game scoring in importance, the end-game scoring has been changed to achieve an appropriate balance.

OLD END-GAME SCORING:	-1	0	1	2
NEW END-GAME SCORING:	0	6	8	10

So the scoring for this match was:

Player	End-Game Score	Bonus Score	Final Score
P1	10	6	16
P2	6	6	12
P3	10	5	15

CONJECTURES

From this example it is clear that the introduction of the snuffing rule increases the feedback possibilities enormously among the players about Solutions involving different mathematical ideas. Instead of showing each other a Solution only once—at the end of the play of a game when one of the players has the burden of proof, as is the case in Basic EQUATIONS—in the snuffing version there is incentive for players to show a Solution to each other on every turn. It also has the effect of getting rid of the Solutions that involve only relatively easy ideas early in the play of a game and gently nudges the players in later play to explore more subtle Solutions involving more complex ideas. Still another effect of the snuffing version is to decrease the importance of penalty points that are sometimes imposed in tournament play for actions by players that are "illegal procedures." All three of these effects seem likely to be in the desired direction of further improving the learning behavior of participants, the first two by directly channeling attention on more relevant information and the third by indirect effects on attitudes. But whether the switch to snuffing will have the dramatic impact of doubling or tripling learning increments, the way that some other changes in the EQUATIONS game have (see Allen et al., forthcoming), remains to be determined. That experiment is currently being designed and scheduled. Until the results of the experiment are available, interested readers may wish to examine evidence of a type that is less conclusive in nature, but to experienced eyes likely to be highly persuasive. The reference is to a series of sample games (like the one above) of the snuffing version of EQUATIONS played by inner-city Detroit junior high school students. For those familiar with the quality of thinking about mathematical problems exhibited customarily by students at this level, the handling of mathematical ideas exhibited in this series of games is likely to be surprising.[1]

NOTE

1. Copies of these game summaries are available from the senior author, Layman E. Allen, MHRI, University of Michigan, Ann Arbor, Michigan 48109.

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