Comorbidity of Reading and Mathematics Disabilities: Genetic and Environmental Etiologies

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Although children with learning disabilities frequently manifest comorbid reading and mathematics deficits, the cause of this comorbidity is unknown. To assess the extent to which comorbidity between reading and mathematics deficits is due to genetic and environmental influences, we conducted a twin study of reading and mathematics performance. Data from 148 identical and 111 fraternal twin pairs in which at least one member of the pair had a reading disability were subjected to a cross-concordance analysis and also fitted to a bivariate extension of the basic multiple regression model for the analysis of selected twin data. Results of these analyses suggest that genetic and shared-environmental influences both contribute to the observed covariance between reading and mathematics deficits.

Approximately 80% of students with learning disabilities (LD) experience reading problems (Lerner, 1989), and most of these students also evidence comorbid deficits in mathematics (Fletcher & Loveland, 1986; Kosc, 1974; McLeod & Armstrong, 1982). In contrast, relatively few children with learning problems have specific deficits in mathematics and quantitative reasoning.

Although the relationship between reading and mathematical deficits in children with LD is well established (Culyer, 1988; Lansdown, 1978; Rourke & Strang, 1983; Share, Moffitt, & Silva, 1988; Siegel & Linder, 1984), the etiology of this comorbidity is unknown. Comorbidity may be due to the co-occurrence of two independent disorders, two disorders having a common underlying etiology, or two disorders that are causally related. The present study is the first attempt to assess the genetic and environmental etiologies of comorbid reading and math deficits.

Reading Disability

Early in this century, Thomas (1905) noted the familial nature of reading disability (RD), and subsequent genetic analyses have employed increasingly sophisticated designs and methodologies (e.g., case studies, pedigree analysis, family and twin studies, and linkage analysis) (DeFries & Gillis, 1993). Nevertheless, definitive evidence for a genetic etiology of reading disability has only recently been reported (DeFries, Fulker, & LaBuda, 1987). For example, data collected from 133 identical (monozygotic; MZ) and 98 same-sex fraternal (dizygotic; DZ) twin pairs with reading deficits (DeFries & Gillis, 1993) were recently fitted to the basic regression model for the analysis of selected twin data (DeFries & Fulker, 1985). Results of this statistically powerful analysis indicated that approximately 50% of the participants' reading deficits was due to genetic factors. Moreover, when an augmented regression model was fit to the same data set, results suggested that about three fourths of the variance in reading performance within the affected sample was due to genetic influences, whereas only about 10% was caused by environmental influences shared by members of a twin pair.

Mathematics Disability

Terms such as *learning disabilities in mathematics, arithmetic disorders, math disabilities,* and *specific math disabilities* refer to a variety of deficits in mathematical ability (Keller & Sutton, 1991). In this article, we employ the term *mathematical disability.* Different subtypes of mathematical disability may occur. For example, neuropsychologists often differentiate between *acalculia* and *dyscalculia* (Keller & Sutton, 1991; Kosc, 1974). Acalculia refers to a condition in individuals who once mastered mathematical ability but subsequently lost it (e.g., as a result of brain injury), and dyscalculia (or developmental dyscalculia) refers primarily to a failure to develop mathematical competence.

Several hypotheses have been proposed to account for mathematical disabilities, including poor motivation (see Geary, 1993); deficits in verbal ability (Lansdown, 1978; McLeod & Crump, 1978; Muth, 1984); automaticity deficits in basic arithmetic oper-

JOURNAL OF LEARNING DISABILITIES
VOLUME 28, NUMBER 2, FEBRUARY 1995
PAGES 96-106

ations (Kirby & Becker, 1988); and genetic predisposition (Barakat, 1951; Burt, 1949; Husen, 1959; Kosc, 1974).

Although no behavioral genetic studies of mathematical disability have been reported to date, results of twin and family studies of mathematics performance suggest that individual differences in arithmetical ability within the normal range of variation are due, at least in part, to genetic influences (see Plomin, DeFries, & McClearn, 1990; Thompson, Detterman, & Plomin, 1991).

Comorbidity

Comorbidity refers to the co-occurrence of at least two different disorders in the same individual. The term has become increasingly common in the psychiatric literature but has only recently been applied to learning disabilities. For example, several recent studies have concerned the comorbidity between reading disability and attention-deficit/hyperactivity disorder (ADHD) (August & Garfinkel, 1990; Hinshaw, 1992; Semrud-Clikeman et al., 1992), and between comorbid learning disabilities and substance abuse disorders (Bentley & Conley, 1992; Fox & Forbing, 1991; Rhodes & Jasinski, 1990).

The concept of comorbidity has its origins in general medicine, whereby Feinstein (1970) defined comorbidity as any "distinct additional clinical entity that has existed or that may occur during the clinical course of a patient who has the index disease under study" (pp. 456–457). Thus, the original term referred to coexisting but distinct disorders—distinct in that the disorders may involve separate phenomenologies, pathologies, or etiologies (Shea, Widiger, & Klein, 1992). In reference to mental disorders and learning disabilities, however, distinctness among coexisting disorders often cannot be assumed because of substantial overlap in criteria and unknown etiology of the separate disorders. Thus, it is difficult to determine whether a disorder that exists in pure form or that coexists with another disorder represents the same syndrome. With regard to learning disabilities, knowledge pertaining to the etiology of comorbidity may influence the remediation strategy as well as the final outcome. For example, if reading disability causes mathematics deficits, then reading remediation may alleviate the math difficulties as well. On the other hand, if reading and mathematics disabilities coexist, but with no causal relation, then both disorders must be remediated (see Clarkin & Kendall, 1992).

The presence of comorbid disorders necessarily complicates studies of the individual conditions. In clinical research, investigators often limit their samples to patients who have just one disease. Although this approach is straightforward, it may result in the investigation of atypical samples. For example, in two studies of childhood depression, only 3 of 14 cases (Anderson, Williams, McGee, & Silva, 1987) and none of 12 cases (Kashani et al., 1987) of depression were "pure," that is, did not co-occur with anxiety disorders, conduct disorders, and/or attention-deficit disorders. In contrast, social and epidemiological researchers usually consider broader samples and then investigate the extent of comorbidity for various disabilities (Verbrugge, Lepkowski, & Imanaka, 1989).

Caron and Rutter (1992) suggested that researchers' failure to consider comorbidity in previous studies may have resulted in misleading findings for two main reasons: First, a study of one condition (e.g., condition X) may produce findings that are largely a consequence of an ignored disorder (e.g., condition Y). Second, when comorbidity is ignored, an implicit assumption is made that condition X is the same, regardless of the presence or absence of condition Y—a possible false assumption.

Observed comorbidity may be either "artifactual" or "true." For example, apparent comorbidity may be due to sample selection methods (e.g., the Berkson effect; see Berkson, 1946). Selection biases are particularly prevalent in clinically derived samples, which tend to contain the most severely affected children. Artifactual comorbidity could also occur because of overlapping diagnostic criteria, or because one disorder is an early manifestation of a second disorder or is actually a part of the second disorder.

Several explanations may also exist for true comorbidity, that is, comorbidity that is not a statistical or diagnostic artifact (Caron & Rutter, 1992). First, the disorders may share the same risk factors. Many psychiatric disorders are multifactorial in origin and their causes are not specific to any one disorder. For example, variables such as overactivity, short attention span, and impulsivity have all been found to influence both conduct disorders and reading disability. Thus, these variables might account for comorbidity between conduct disorders and reading disability. Because genetic influences may be important in hyperactivity and impulsivity (see Stevenson, 1992), the observed comorbidity between conduct disorders and reading disability may be due, at least in part, to a shared risk factor that is genetic in origin. Alternatively, shared risk factors can also originate from environmental circumstances. For example, large family size and social disadvantage can also increase the risk for both conduct problems and reading problems (Richman, Stevenson, & Graham, 1982; Rutter & Giller, 1983).

A second possible explanation for true comorbidity is that the risk factors for two disorders may co-occur. For example, Caron and Rutter (1992) noted that parental depression is a risk factor for several childhood psychopathologies. Weissman et al. (1987) examined the occurrence of childhood depression in the offspring of depressed parents and suggested that the risk is genetically transmitted from parent to child. There also appears to be an increased risk of conduct disorders among offspring that seems to be a function of family discord, which is much more frequent when one or both

parents are depressed (see Rutter & Quinton, 1984). Thus, the comorbidity between childhood depression and conduct disorders may be due to both a genetic risk for depression and an environmental risk for conduct disorder as a result of family discord.

A third possible explanation of true comorbidity is that the comorbid symptoms represent a meaningful syndrome. For example, with regard to reading and mathematics deficits, individuals may be classified into one of three groups: (a) reading and math disabled (RDMD), (b) only reading disabled (RD), and (c) only math disabled (MD). Results obtained from several studies (e.g., Fletcher, 1985; Fletcher & Loveland, 1986; Rourke, 1985, 1993; Rourke & Finlayson, 1978; Rourke & Strang, 1983; Siegel & Linder, 1984) suggest that individuals with RDMD and RD have similar cognitive profiles, which differ from that of individuals with MD.

A fourth possible explanation for true comorbidity is that the presence of one disorder may increase the risk of a second disorder. For example, the presence of a reading disability may cause a greater risk for a mathematical disability. Individuals placed in remedial education classes may receive remediation in reading ability at the expense of time devoted to teaching mathematical concepts (Keller & Sutton, 1991; Kosc, 1974; McLeod & Armstrong, 1982).

Comorbid Reading and Mathematics Deficits

The overlap between reading and mathematics deficits has been well documented (Ackerman, Anhalt, & Dykman, 1986; Fletcher & Loveland, 1986; Kulak, 1993; McLeod & Armstrong, 1982). Ackerman et al. suggested that most children diagnosed early as reading disabled will eventually display a serious deficiency in arithmetic, despite having near-average arithmetic scores in the primary grades. In a sample of fifth-grade students with LD participating in the Florida Longitudinal Project, Fletcher and Loveland reported that only 1% showed specific deficits in reading, 18% evidenced specific deficits in mathematics, and 81% evidenced comorbid deficits in both reading and math. In a descriptive study of students served as learning disabled by Child Service Demonstration Centers, Norman and Zigmond (1980) found that 8% showed severe deficits solely in math, while 13% of their sample displayed deficits in both reading and mathematics. In a separate sample, Badian (1983) reported that 6% of elementary and junior high school students experienced math difficulties, while an additional 5% experienced difficulties with reading. Moreover, 56% of the children with reading problems also showed poor mathematics achievement, and 43% of the children with math deficits also showed poor reading achievement. In a series of studies of children with reading disabilities, Siegel and colleagues (Siegel & Linder, 1984; Siegel & Ryan, 1988, 1989) found few children that did not have accompanying severe arithmetic deficits, except at the youngest ages.

Etiology

Students who display comorbid reading and mathematics deficits probably do so for a variety of reasons. Although understanding of the mechanisms underlying the observed comorbidity may facilitate appropriate remediation strategies, little research has been conducted in this area. One possibility is that poor achievement in mathematics may be due to language-based deficits that also underlie poor reading achievement (Lansdown, 1978; McLeod & Crump, 1978; see also Satz, Taylor, Friel, & Fletcher, 1978; Share et al., 1988). Rourke and Strang (1983) suggested that children with RDMD have difficulties memorizing tables and procedural steps in problem solving because of general language and verbal impairments. Consistent with this view, Muth (1984) found that by experimentally manipulating the computational and reading demands of a series of arithmetic problems, 14% of the variance explained in performance scores was uniquely attributable to reading skills, 8% was attributable to computational skills, and 32% was attributable to joint variance. Alternative hypotheses to explain comorbid reading and math deficits include overall deficits in verbal short-term and/or long-term memory retrieval (Ackerman et al., 1986; Brandys & Rourke, 1991; Fletcher, 1985; Fletcher & Loveland, 1986; Lerner, 1989; Siegel & Linder, 1984; Webster, 1979).

The primary objective of the present study was to assess the genetic and environmental etiologies of comorbid reading and mathematics deficits. Data from twin pairs participating in the Colorado Learning Disabilities Research Center (LDC) in which at least one member of the pair had a reading disability were subjected to cross-concordance analyses and to a bivariate extension of the basic multiple-regression model for the analysis of selected twin data (DeFries & Fulker, 1985). The results of these analyses provide evidence concerning the extent to which deficits in reading are due to genetic and environmental influences that also cause mathematics deficits.

Method

Procedure

To minimize ascertainment bias, we systematically identified twin pairs from cooperating school districts within a 150-mile radius of Denver. Without regard to reading status, all twin pairs within a school district were identified, and permission was sought from their parents to review the twins' school records for evidence of reading problems. Records were screened for evidence of reading problems, which included low standardized achievement

test scores, referrals to special resource rooms or remedial reading programs, and comments from teachers or school psychologists. Pairs in which at least one member evidenced a school history of reading problems were then invited to the University of Colorado, where they were administered a battery of psychometric tests. A comparison sample of control twin pairs was also tested in which neither member of a pair had a history of reading problems in school. Control twins were matched to the probands by age, gender, and school district. By December 31, 1992, school information from nearly 2,000 twin pairs in 27 cooperating school districts had been obtained. From that sample, 463 twin pairs in which at least one member evidenced reading difficulties and 293 twin pairs in which both twins exhibited normal reading performance had been tested.

The twin pairs were administered a battery of psychometric tests measuring various cognitive abilities, including subtests (Reading Recognition, Reading Comprehension, Spelling, Mathematics) of the Peabody Individual Achievement Test (PIAT; Dunn & Markwardt, 1970), and the Wechsler Intelligence Scale for Children-Revised (WISC-R; Wechsler, 1974) or the Wechsler Adult Intelligence Scale-Revised (WAIS-R; Wechsler, 1981). A discriminant function reading score was computed for each individual, employing discriminant weights estimated from an analysis of PIAT Reading Recognition, Reading Comprehension, and Spelling data obtained from an independent sample of 140 nontwin children with diagnosed reading problems and 140 controls. A composite mathematics score was computed by summing standardized scores on the PIAT Mathematics and WISC-R Arithmetic subtests.

Pairs were included in the reading proband sample if at least one member of the pair (the proband) met the following five criteria: (a) a positive history of reading problems in school, (b) a negative discriminant reading score, (c) a Verbal or Performance IQ of at least 90, (d) no evidence of neurological or severe emotional problems, and (e) no uncorrected visual or auditory acuity deficits. Criteria for mathematics disability included a standardized math composite score of at least 1.5 standard deviations below that of the control twin sample, and criteria (c) through (e) for probands.

Zygosity of the twin pairs was determined using selected items from the Nichols and Bilbro (1966) zygosity questionnaire, which has a reported accuracy of 95%. In doubtful cases, zygosity was determined by the analysis of blood samples. By December 31, 1992, a total of 149 pairs of MZ twins (64 male, 85 female) and 111 pairs of same-sex DZ twins (64 male, 47 female) met the criteria for inclusion in the proband sample [i.e., at least one member of the pair met criteria (a) through (e)] and had complete reading and mathematics performance measures. The comparison sample of control twins included 134 MZ (57 male, 77 female) and 93 same-sex DZ (52 male, 41 female) pairs. The mean verbal and performance IQ scores were 96 and 102, respectively, for probands, and 112 and 112, respectively, for the matched control twins. All twins ranged in age from 8 to 20 years at the time of testing (mean age = 12.06 years) and all had been raised in middle class homes in which English was the primary language spoken. Although most twins in the current sample are White, recent efforts have been made by LDC staff members to increase the number of nonwhite twin pairs, in order to obtain a more representative sample.

Analyses

Cross-Concordance Rates. Several different methodologies can be used to assess the etiology of comorbid disorders such as reading and math disabilities. Just as a comparison of MZ and DZ concordance rates was employed to assess genetic etiology in early twin studies of reading disability (Bakwin, 1973; Hallgren, 1950; Stevenson, Graham, Fredman, & McLoughlin, 1984, 1987; Zerbin-Rüdin, 1967), a comparison of cross-concordance rates for MZ and DZ twin pairs can be used to assess the etiology between two comorbid disorders (Gilger, Pennington, & DeFries, 1992). A pair is defined as cross-concordant if Twin 1 manifests one disorder and Twin 2 manifests the second disorder. In the present study, a pair was defined as cross-concordant if Twin 1 displayed a reading disability and Twin 2 had a math disability. The logic of this design relies on the fact that MZ twins share all of their genes, whereas DZ twins, on average, share only 50% of their segregating genes. Thus, a higher cross-concordance among MZ twin pairs compared to DZ twin pairs would suggest a genetic etiology to the comorbidity between the two disorders.

Multiple Regression Analysis. An alternative method for assessing the etiology of comorbidity between two disorders is a bivariate extension of the basic multiple regression model for the analysis of selected twin data (DeFries & Fulker, 1985). In the univariate case, when probands are selected because of deviant scores on a continuous measure, the scores of both MZ and DZ co-twins should regress toward the mean of the unselected population. However, as shown in Figure 1, the regression of MZ co-twins should be less than that of DZ co-twins if the condition is heritable. When the MZ and DZ proband means are approximately equal, a simple *t* test between the MZ and DZ co-twin means provides a test of genetic etiology. However, DeFries and Fulker proposed that a multiple-regression analysis of such data, in which a co-twin's score is regressed on both the proband's score and the coefficient of relationship, facilitates a more flexible and statistically powerful test. Subsequently, DeFries and Fulker (1988) demonstrated that a simple transformation of the data prior to multiple-regression analysis yielded an

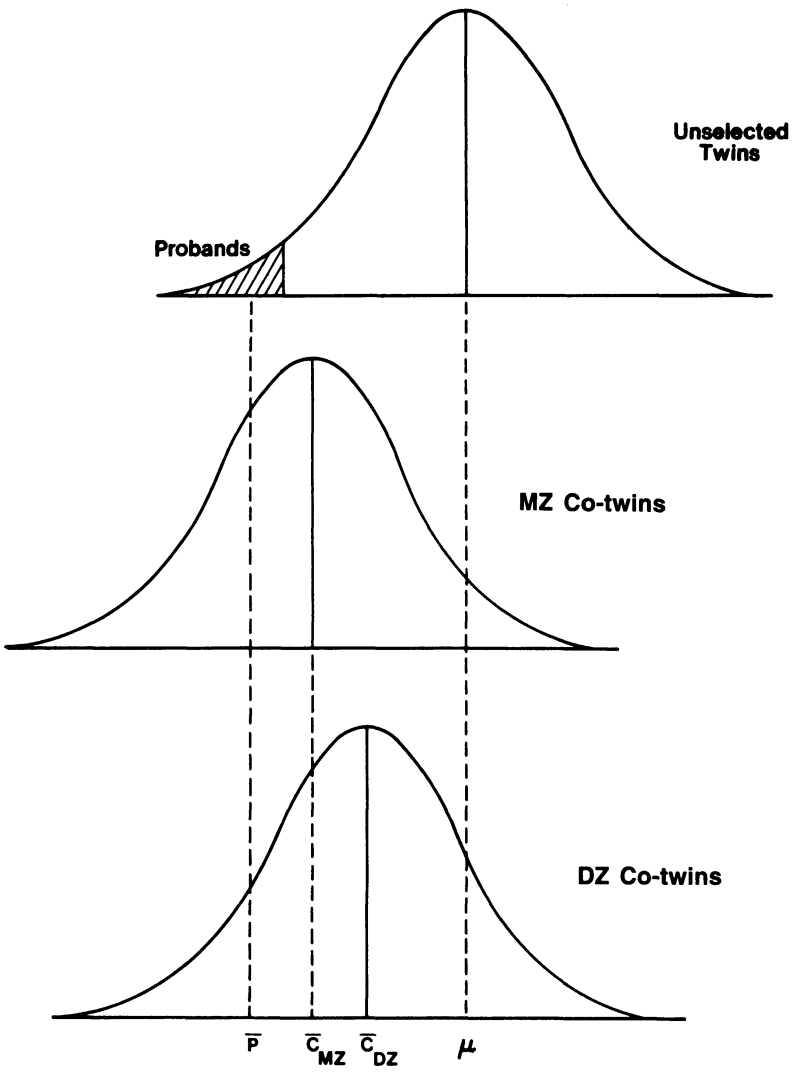

FIGURE 1. Hypothetical distribution for reading performance of an unselected sample of twins, and of the identical (MZ) and fraternal (DZ) co-twins of probands with reading disabilities. The differential regression of the MZ and DZ co-twins toward the mean of the unselected sample (μ) provides a test of genetic etiology. ($\bar{P}$, $\bar{C}_{MZ}$, and $\bar{C}_{DZ}$ represent the means for probands, MZ co-twins, and DZ co-twins, respectively.) Reprinted with permission from *Nature* ("Evidence for a Genetic Aetiology in Reading Disability of Twins," by J. C. DeFries, D. W. Fulker, and M. C. LaBuda, 1987, *Nature, 329,* p. 537). Copyright 1987 by Macmillan Magazines Limited.

estimate of h_g^2, an index of the extent to which the observed proband deficit is heritable.

The basic multiple-regression equation formulated by DeFries and Fulker (1985) is as follows:

$$C = B_1P + B_2R + A \qquad (1)$$

where C is the co-twin's score, P is the proband's score, R is the coefficient of relationship (R = 1.0 for MZ twins and 0.5 for DZ twins), and A represents the regression constant. The B_1 coefficient is the partial regression of the co-twin's score on the proband's score, a measure of average MZ and DZ twin resemblance (LaBuda, DeFries, & Fulker, 1986). The B_2 coefficient is the partial regression of the co-twin's score on the coefficient of relationship and equals twice the difference between the MZ and DZ co-twin means after covariance adjustment for any difference between MZ and DZ proband means. Thus, the B_2 coefficient provides a direct test for genetic etiology. When the data are transformed prior to multiple regression analysis (each score is expressed as a deviation from the mean of the unselected population and then divided by the difference between the proband and control means), B_2 directly estimates h_g^2 (the extent to which the observed proband deficit is heritable). Moreover, the difference between the transformed MZ co-twin mean and h_g^2 estimates c_g^2, a measure of the extent to which the proband deficit is due to environmental influences shared by members of twin pairs.

To assess the heritable nature of the comorbidity between reading and mathematics deficits, we fitted the following bivariate extension of the basic multiple regression model to proband discriminant reading scores and co-twin composite mathematics scores of the selected twin sample:

$$C_y = B_1P_x + B_2R + A \qquad (2)$$

where C_y is the co-twin's expected composite math score, P_x is the proband's discriminant reading score, R is the coefficient of relationship (the same as in the univariate case), and A is the regression constant. B_1 is the partial regression of the co-twin's math score on the proband's discriminant reading score—a measure of average MZ–DZ cross-variable twin resemblance (i.e., the extent to which co-twin scores on Y are related to proband scores on X across zygosity). B_2 is the partial regression of the co-twin's composite math score on the coefficient of relationship. Because the data were standardized and transformed prior to multiple regression analysis, the standard deviations of both the reading and math variables were equal to 1, and, consequently, the B_2 coefficient is a function of the group heritabilities for reading and math performance and the genetic correlation between them (see Appendix; see also Stevenson,

Pennington, Gilger, DeFries, & Gillis, 1993). Thus, B_2 provides an estimate of "bivariate heritability," an index of the extent to which the proband reading deficit is due to genetic factors that also influence math disability. Furthermore, the ratio of the B_2 estimate to the observed covariance between reading and math scores estimates the proportion of the observed covariance that can be attributed to genetic factors.

For the present analyses, data from twin pairs in which at least one member of the pair met the criteria for reading disability were used. Because truncate selection was employed when ascertaining the current twin sample (see Thompson & Thompson, 1986), pairs in which both members were diagnosed as reading disabled were double-entered for all regression analyses. This is analogous to the computation of proband-wise concordance rates, in which both affected members of concordant pairs are included as probands (DeFries & Gillis, 1991). Standard error estimates and tests of significance provided by conventional computer regression programs were adjusted accordingly.

Results

Concordance and Cross-Concordance

To test the hypothesis that math deficits displayed by individuals with reading disabilities are due at least in part to genetic factors that also influence reading performance, we conducted both concordance and regression analyses. In Table 1, MZ and DZ probands and co-twins are classified according to their reading and math status, with pairs concordant for RD being double-entered. As indicated by the numbers tabulated in columns 1 and 2, 68% (154/226) of the MZ co-twins and 40% (56/139) of the DZ co-twins of RD probands were also reading disabled. The difference between the MZ and DZ concordance rates is significant ($z = 4.46$, $p < .001$), strongly suggesting a genetic etiology for reading disability.

Of more relevance to the present study were the MZ and DZ cross-concordance rates between proband reading and co-twin math status. As shown in columns 3 and 4 of Table 1, 49% of the MZ co-twins and 32% of the DZ co-twins of RD probands were math disabled. These MZ and DZ cross-concordance rates are also significantly different ($z = 2.71$, $p < .01$), suggesting that genetic influences contribute to the observed comorbidity between reading and mathematics performance deficits.

Multiple Regression Analysis of Selected Twin Data

Table 2 summarizes the MZ and DZ proband and co-twin reading performance (READ) score means, standardized against the mean of 454 control twins. The MZ and DZ proband READ scores are highly similar and are over 2.5 standard deviations below the control twin mean. However, there is a differential regression of the MZ and DZ co-twin READ scores. Whereas MZ co-twins regress only 0.23 standard deviation units, DZ co-twins regress 0.90 standard deviation units toward the unselected population mean. As shown in Table 3, when proband and co-twin transformed READ scores were fitted to the univariate basic regression model (Equation 1), $B_2 = h_g^2 = 0.51 \pm 0.10$, suggesting that approximately 50% of the proband deficit in reading is due to genetic influences. The difference between the transformed co-twin mean (0.92) and h_g^2 yields an estimate of c_g^2—a measure of the extent to which the proband deficit in reading is due to environmental influences shared by twin pairs, that is, $c_g^2 = 0.41 \pm 0.40$. Thus, approximately 40% of the proband reading deficit can be attributed to environmental influences shared by members of twin pairs.

Table 4 presents the mean composite mathematics scores (MATH) for the MZ and DZ probands with RD and their co-twins. As was the case with

TABLE 1
Frequency of Reading Disability (RD) and Math Disability (MD) in Co-twins of RD Probands

	Co-twin			
RD Proband	RD	Non-RD	MD	Non-MD
Identical	154	72	110	116
Fraternal	56	83	45	94

TABLE 2
Mean Reading Performance (READ) Scores

		Standardized against controls[a]		Transformed READ scores[b]	
	N_{Pairs}	Proband	Co-twin	Proband	Co-twin
Identical	148	−2.77 ± 0.83	−2.54 ± 1.07	1.00 ± 0.30	0.92 ± 0.39
Fraternal	111	−2.65 ± 0.82	−1.75 ± 1.33	1.00 ± 0.31	0.66 ± 0.50

[a]Expressed in standard deviation units from the mean of a sample of 454 control twins. [b]Each score is expressed as a deviation from the READ mean of the unselected population and then divided by the difference between the proband and control mean READ scores.

the READ scores, there was differential regression of the MZ and DZ co-twin MATH scores. Whereas MZ co-twins, on average, did not regress toward the unselected population mean, the DZ co-twins regressed 0.33 standard deviation units.

The results of fitting proband READ scores and co-twin MATH scores to the bivariate regression model (Equation 2) are shown in Table 5. The B_1 estimate of 0.11 ± 0.07 (p = .06) is a measure of average MZ–DZ cross-variable twin resemblance, that is, the extent to which proband READ scores are related to co-twin MATH scores in MZ and DZ twin pairs, on average. The B_2 estimate of 0.26 ± 0.09 (p = .015) provides a direct estimate of bivariate heritability, a measure of the extent to which the proband deficit in reading is due to genetic factors that also influence composite MATH scores. Furthermore, the ratio of B_2 to the observed covariance (0.47) between transformed READ and MATH scores (estimated from the regression of MATH on READ in the proband sample) indicates the proportion of the phenotypic relationship between reading and math that can be attributed to genetic influences. The magnitude of this ratio (i.e., 0.26/0.47 = 0.55) suggests that about half of the observed covariance between reading and math deficits is due to genetic influences. In addition, because the expected transformed MZ co-twin mean for MATH within the RD sample equals bivariate h_g^2 + bivariate c_g^2 (see Appendix), subtracting bivariate B_2 from the MZ co-twin transformed MATH mean provides a direct estimate of bivariate c_g^2, a measure of the extent to which the proband reading deficit is due to environmental factors that influence composite MATH scores and are shared by members of twin pairs. In the present study, bivariate c_g^2 = 0.51 − 0.26 = 0.25, suggesting that proband reading deficits are also substantially due to shared environmental factors that influence MATH performance. Thus, both genetic and shared environmental factors appear to contribute to comorbid reading and math performance deficits.

TABLE 3
Fit of Basic Model to Proband and Co-twin Transformed READ Scores

Coefficient	Estimate ± *SE*	*t*	*p*[a]
B_1	0.69 ± 0.08	8.84	< .001
$B_2 = h_g^2$	0.51 ± 0.10	5.26	< .001

[a]One-tailed.

TABLE 4
Mean Mathematics Performance (MATH) Scores

		Standardized against controls[a]		Transformed MATH scores[b]	
	N_{Pairs}	Proband	Co-twin	Proband	Co-twin
Identical	148	−1.40 ± 0.86	−1.40 ± 0.89	0.51 ± 0.31	0.51 ± 0.32
Fraternal	111	−1.33 ± 0.80	−1.00 ± 1.02	0.50 ± 0.30	0.38 ± 0.38

[a]Expressed in standard deviation units from the mean of a sample of 454 control twins. [b]Each standardized score is divided by the difference between the proband and control mean READ scores.

Discussion

A major objective of the Colorado Reading Project (DeFries, Olson, Pennington, & Smith, 1991) was to assess the genetic and environmental etiologies of reading disability. Subsequent funding by the National Institute of Child Health and Human Development has provided the LDC coinvestigators the opportunity to assess the genetic and environmental etiologies of both reading and mathematics disabilities, as well as their covariation with measures of reading and language processes, ADHD, and executive functions. Although previous studies have indicated that children with learning disabilities often experience comorbid reading and mathematics deficits, the nature of this observed comorbidity is not well understood (Fletcher & Loveland, 1986; Kosc, 1974; McLeod & Armstrong, 1982). The present study employed behavioral genetic analyses of reading and math performance data collected from twin pairs in which at least one member of the pair evidenced a reading problem, in order to assess the extent to which the observed comorbidity between reading and mathematics deficits is due to common genetic and environmental influences.

Based on previous findings, it was hypothesized that math deficits displayed by individuals with reading disabilities are due at least in part to genetic factors that also influence these individuals' reading performance. One approach we employed to test this hypothesis was to compare MZ and DZ cross-concordance rates between proband reading status and co-twin math status. In the present study, 49% of the MZ co-twins and 32% of the DZ co-twins were found to be math disabled. The difference between the cross-concordance rate of MZ twins (who shared all of their genes with an RD proband) and that of DZ twins (who shared, on average, only 50% of their segregating genes with a proband) was significant. Thus, comorbid reading and math deficits are appar-

ently due at least in part to genetic influences.

DeFries and Fulker (1985, 1988) suggested that when probands are selected because of extremely high or low performance on a continuous measure, multiple regression analysis provides a more flexible and statistically powerful method of testing for genetic etiology. Applying this methodology to a composite measure of reading collected from MZ and DZ probands and co-twins, an h_g^2 estimate of 0.51 ± 0.10 was obtained, suggesting that approximately 50% of the group reading deficit displayed by probands is due to genetic influences. In the present study, a bivariate extension of this basic multiple regression model was used to assess the etiology of comorbid reading and math deficits. Subjects for this analysis included 148 MZ and 111 same-sex DZ twin pairs in which at least one member of the pair had a reading disability. When the bivariate regression model was fitted to proband reading scores and co-twin math scores, the resulting estimate of bivariate h_g^2 suggested that approximately 26% of the proband reading deficit is due to genetic factors that also influence math performance. A corresponding estimate of bivariate c_g^2 (0.25) was also obtained. These results suggest that genetic and shared environmental influences contribute almost equally to the observed covariance between reading and math scores in twin pairs exhibiting reading disabilities.

One limitation of the present study is the small number of measures of mathematical ability that were administered. Because both the WISC-R Arithmetic and PIAT Mathematics subtests primarily involve word problems, it perhaps is not surprising that a sample of twins with reading problems would also manifest problems on these tests. A small subsample of the LDC twin pairs (58 MZ and 39 DZ) were also administered the Wide Range Achievement Test–Revised (WRAT-R; Jastak & Wilkinson, 1984), a paper-and-pencil measure of mathematical ability. When these data were subjected to regression analysis, the resulting estimate of bivariate h_g^2 (0.35) was surprisingly similar to that for the WISC-R and PIAT composite measure. The estimate of bivariate c_g^2 (0.41 − 0.35 = 0.06) was lower, however, suggesting that when math ability is assessed using a paper-and-pencil measure, comorbid reading and math deficits may be due primarily to genetic factors. Because the sample size for this analysis was small, this hypothesis will be tested again when a larger number of twin pairs in the LDC project have been administered the WRAT-R.

TABLE 5
Fit of Bivariate Model to Proband READ Scores and Co-twin MATH Scores

Coefficient	Estimate ± *SE*	*t*	*p*[a]
B_1	0.11 ± 0.07	1.59	.06
B_2 = bivariate heritability	0.26 ± 0.09	2.94	< .002

[a]One-tailed.

A second limitation of the present study concerns the small number of twin pairs with specific math disability in the LDC. Because the primary objective of the Colorado Reading Project was to study reading disability, twin pairs were selected with regard to their reading status. More recently, LDC staff members have identified twins on the basis of either reading or math deficits. However, the overwhelming majority of twins tested to date are pairs in which at least one member has evidenced difficulty in reading or in both reading and math. Thus, the present sample is not adequate for regression analysis of specific math deficits. Because data from a sample of twins with math disabilities will facilitate unique analyses of subtype validity (e.g., children with specific math deficits vs. those with comorbid reading problems; Fletcher, 1985; Fletcher & Loveland, 1986; Rourke, 1985, 1993; Rourke & Finlayson, 1978; Rourke & Strang, 1983; Siegel & Linder, 1984), we will undertake such analyses with a larger sample of children with math disability in the near future.

These findings address an important issue concerning the observed comorbidity between reading and mathematics deficits within populations with learning problems, namely, whether comorbid reading and math deficits are independent disorders or whether there is some shared etiology. Results obtained from a comparison of MZ and DZ cross-concordance rates and a bivariate extension of the basic multiple regression analysis (DeFries & Fulker, 1985, 1988) both suggest that the math deficits of probands with reading problems are due, in part, to genetic factors that also influence reading performance. In addition, there is evidence to suggest that comorbidity between reading and math disabilities is also due to environmental influences shared by members of twin pairs. These tentative conclusions will be tested more rigorously during the next several years when the sample of twins in the Colorado Learning Disabilities Research Center with math disabilities becomes substantially larger.

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AUTHORS' NOTES

1. *This work was supported in part by program project and center grants from NICHD (HD-11681 and HD-27802) to John C. DeFries. This report was prepared while Jacquelyn Gillis Light was supported by an NICHD training grant, No. HD-07289.*
2. *The invaluable contributions of staff members of the many Colorado school districts and of the families who participated in this study are gratefully acknowledged. We also thank Judith Voress and two anonymous reviewers for helpful comments on an earlier version of this article.*

REFERENCES

Ackerman, P. T., Anhalt, J. M., & Dykman, R. A. (1986). Arithmetic automatization failure in children with attention and reading disorders: Associations and sequela. *Journal of Learning Disabilities, 19,* 222-232.

Anderson, J. C., Williams, S., McGee, R., & Silva, P. A. (1987). DSM-III disorders in pre-adolescent children: Prevalence in a large sample from the general population. *Archives of General Psychiatry, 44,* 69-78.

August, G. J., & Garfinkel, B. D. (1990). Comorbidity of ADHD and reading disability among clinic-referred children. *Journal of Abnormal Child Psychology, 18,* 29-45.

Badian, N. A. (1983). Dyscalculia and nonverbal disorders of learning. In H. R. Myklebust (Ed.), *Progress in learning disabilities* (Vol. 5, pp. 235-264). New York: Stratton.

Bakwin, H. (1973). Reading disability in twins. *Developmental Medicine and Child Neurology, 15,* 184-187.

Barakat, M. K. (1951). A factorial study of mathematical abilities. *The British Journal of Psychology, Statistical Section, 4,* 137-156.

Bentley, J. L., & Conley, M. W. (1992). Making connections between substance abuse and literacy difficulties. *Journal of Reading, 35,* 386-389.

Berkson, J. (1946). Limitations of the application of fourfold table analysis to hospital data. *Biometrics, 2,* 47-53.

Brandys, C. F., & Rourke, B. F. (1991). Differential memory abilities in reading-and-arithmetic-disabled children. In B. F. Rourke (Ed.), *Neuropsychological validation of learning disability subtypes* (pp. 73-96). New York: Guilford.

Burt, C. (1949). The structure of the mind: A review of the results of factor analysis. *British Journal of Educational Psychology, 19,* 100-111, 176-199.

Caron, C., & Rutter, M. (1992). Comorbidity in child psychopathology: Concepts, issues, and research strategies. In M. E. Hertzig & E. A. Farber (Eds.), *Annual progress in child psychiatry and child development-1992* (pp. 467-486). New York: Brunner/Mazel.

Clarkin, J. F., & Kendall, P. C. (1992). Comorbidity and treatment planning: Summary and future directions. *Journal of Consulting and Clinical Psychology, 60,* 904-908.

Culyer, R. C. (1988). Reading and mathematics go hand in hand. *Reading Improvement, 25,* 189-195.

DeFries, J. C., & Fulker, D. W. (1985). Multiple regression analysis of twin data. *Behavior Genetics, 15,* 467-473.

DeFries, J. C., & Fulker, D. W. (1988). Multiple regression analysis of twin data: Etiology of deviant scores versus individual differences. *Acta Geneticae Medicae et Gemellolgiae, 37,* 205-216.

DeFries, J. C., Fulker, D. W., & LaBuda, M. C. (1987). Evidence for a genetic aetiology in reading disability of twins. *Nature, 329,* 537-539.

DeFries, J. C., & Gillis, J. J. (1991). Etiology of reading deficits in learning disabilities: Quantitative genetic analysis. In J. E. Obrzut & G. W. Hynd (Eds.), *Neuropsychological foundations of learning disabilities: A handbook of issues, methods, and practice* (pp. 29-47). Orlando, FL: Academic Press.

DeFries, J. C., & Gillis, J. J. (1993). Genetics of reading disability. In R. Plomin & G. E. McClearn (Eds.), *Nature, nurture, and psychology* (pp. 121-145). Washington, DC: American Psychological Association.

DeFries, J. C., Olson, R. K., Pennington, B. F., & Smith, S. D. (1991). Colorado Reading Project: Past, present, and future. *Learning Disabilities, 2,* 37-46.

Dunn, L. M., & Markwardt, F. C. (1970). *Examiner's manual: Peabody Individual Achievement Test.* Circle Pines, MN: American Guidance Service.

Feinstein, A. R. (1970). The pre-therapeutic classification of comorbidity in chronic disease. *Journal of Chronic Disease, 23,* 455-468.

Fletcher, J. M. (1985). External validation of learning disability typologies. In B. P. Rourke (Ed.), *Neuropsychology of learning disabilities: Essentials of subtype analysis* (pp. 187-211). New York: Guilford.

Fletcher, J. M., & Loveland, K. A. (1986). Neuropsychology of arithmetic disabilities in children. *Focus on Learning Problems in Mathematics, 8,* 23-40.

Fox, C. L., & Forbing, S. E. (1991). Overlapping symptoms of substance abuse and learning handicaps: Implications for educators. *Journal of Learning Disabilities, 24,* 24-31.

Geary, D. C. (1993). Mathematical disabilities: Cognitive, neuropsychological, and genetic components. *Psychological Bulletin, 114,* 345-362.

Gilger, J. W., Pennington, B. F., & DeFries, J. C. (1992). A twin study of the etiology of comorbidity: Attention-deficit hyperactivity disorder and dyslexia. *Journal of the American Academy of Child and Adolescent Psychiatry, 31,* 343-348.

Hallgren, B. (1950). Specific dyslexia: A clinical and genetic study. *Acta Psychiatrica et Neurologica Scandinavica, 65* (Suppl.), 1-287.

Hinshaw, S. P. (1992). Academic underachievement, attention deficits, and aggression: Comorbidity and implications for intervention. *Journal of Consulting and Clinical Psychology, 60,* 893-903.

Husen, T. (1959). *Psychological twin research: A methodological study.* Uppsala, Sweden: Almquist & Wiksells.

Jastak, S., & Wilkinson, G. S. (1984). *Wide Range Achievement Test-Revised: Administration manual.* Wilmington, DE: Jastak Associates.

Kashani, J. H., Beck, N. C., Hoeper, E. W., Fallahi, C., Corcoran, C. M., McAllister, J. A., Rosenberg, T. K., & Reid, J. C. (1987). Psychiatric disorders in a community sample of adolescents. *American Journal of Psychiatry, 144,* 584-589.

Keller, C. E., & Sutton, J. P. (1991). Specific mathematics disorders. In J. E. Obrzut & G. W. Hynd (Eds.), *Neuropsychological foundations of learning disabilities: A handbook of issues, methods, and practice* (pp. 549-571). Orlando, FL: Academic Press.

Kirby, J. R., & Becker, L. D. (1988). Cognitive components of learning problems

in arithmetic. *Remedial and Special Education, 9*(5), 7-15, 27.

Kosc, L. (1974). Developmental dyscalculia. *Journal of Learning Disabilities, 7,* 164-177.

Kulak, A. G. (1993). Parallels between math and reading disability: Common issues and approaches. *Journal of Learning Disabilities, 26,* 666-673.

LaBuda, M. C., DeFries, J. C., & Fulker, D. W. (1986). Multiple regression analysis of twin data obtained from selected samples. *Genetic Epidemiology, 3,* 425-433.

Lansdown, R. (1978). Retardation in mathematics: A consideration of multi-factorial determination. *Journal of Child Psychology and Psychiatry, 19,* 181-185.

Lerner, J. W. (1989). Educational interventions in learning disabilities. *Journal of the American Academy of Child and Adolescent Psychiatry, 28,* 326-331.

McLeod, T., & Armstrong, S. (1982). Learning disabilities in mathematics-skill deficits and remedial approaches at the intermediate and secondary level. *Learning Disability Quarterly, 5,* 305-311.

McLeod, T. M., & Crump, W. D. (1978). The relationship of visuospatial skills and verbal abilities to learning disabilities in mathematics. *Journal of Learning Disabilities, 11,* 53-57.

Muth, K. D. (1984). Solving arithmetic word problems: Role of reading and computational skills. *Journal of Educational Psychology, 76,* 205-210.

Nichols, R. C., & Bilbro, W. C. (1966). The diagnosis of twin zygosity. *Acta Genetica et Statistica Medica, 16,* 265-275.

Norman, C. A., & Zigmond, N. (1980). Characteristics of children labeled and served as learning disabled in school systems affiliated with Child Service Demonstration Centers. *Journal of Learning Disabilities, 13,* 542-547.

Plomin, R., DeFries, J. C., & McClearn, G. E. (1990). *Behavior genetics: A primer.* New York: Freeman.

Rhodes, S. S., & Jasinski, D. R. (1990). Learning disabilities in alcohol-dependent adults: A preliminary study. *Journal of Learning Disabilities, 23,* 551-556.

Richman, N., Stevenson, J., & Graham, P. J. (1982). *Pre-school to school: A behavioral study.* London: Academic Press.

Rourke, B. P. (1985). Overview of learning disability subtypes. In B. P. Rourke (Ed.), *Neuropsychology of learning disabilities: Essentials of subtype analysis* (pp. 3-14). New York: Guilford.

Rourke, B. P. (1993). Arithmetic disabilities, specific and otherwise: A neuropsychological perspective. *Journal of Learning Disabilities, 26,* 214-226.

Rourke, B. P., & Finlayson, M. A. J. (1978). Neuropsychological significance of variations on patterns of academic performance: Verbal and visual-spatial abilities. *Journal of Abnormal Child Psychology, 6,* 121-133.

Rourke, B. P., & Strang, J. D. (1983). Subtypes of reading and arithmetic disabilities: A neuropsychological analysis. In M. Rutter (Ed.), *Developmental neuropsychiatry* (pp. 473-488). New York: Guilford.

Rutter, M., & Giller, H. (1983). *Juvenile delinquency: Trends and perspectives.* New York: Guilford.

Rutter, M., & Quinton, D. (1984). Parental psychiatric disorder: Effects on children. *Psychological Medicine, 14,* 853-880.

Satz, P., Taylor, H. G., Friel, J., & Fletcher, J. M. (1978). Some developmental and predictive precursors of reading disabilities: A six-year followup. In A. L. Benton & D. Pearl (Eds.), *Dyslexia* (pp. 313-347). New York: Oxford University Press.

Semrud-Clikeman, M., Biederman, J., Sprich-Buckminster, S., Lehman, B., Faraone, S. V., & Norman, D. (1992). Comorbidity between ADDH and learning disability: A review and report in a clinically referred sample. *Journal of the American Academy of Child and Adolescent Psychiatry, 31,* 439-448.

Share, D. L., Moffitt, T. E., & Silva, P. A. (1988). Factors associated with arithmetic-and-reading disability and specific arithmetic disability. *Journal of Learning Disabilities, 21,* 313-320.

Shea, M. T., Widiger, T. A., & Klein, M. H. (1992). Comorbidity of personality disorders and depression: Implications for treatment. *Journal of Consulting and Clinical Psychology, 60,* 857-868.

Siegel, L. S., & Linder, B. (1984). Short-term memory processes in children with reading and arithmetic learning disabilities. *Developmental Psychology, 20,* 200-207.

Siegel, L. S., & Ryan, E. B. (1988). Development of grammatical-sensitivity, phonological, and short-term memory skills in normally achieving and learning disabled children. *Developmental Psychology, 24,* 28-37.

Siegel, L. S., & Ryan, E. B. (1989). The development of working memory in normally achieving and subtypes of learning disabled children. *Child Development, 60,* 973-980.

Stevenson, J. (1992). Evidence for a genetic etiology in hyperactivity in children. *Behavior Genetics, 22,* 337-344.

Stevenson, J., Graham, P., Fredman, G., & McLoughlin, V. (1984). The genetics of reading disability. In C. J. Turner & H. B. Miles (Eds.), *The biology of human intelligence.* Nafferton Books.

Stevenson, J., Graham, P., Fredman, G., & McLoughlin, V. (1987). A twin study of genetic influences on reading and spelling ability and disability. *Journal of Child Psychology and Psychiatry, 28,* 229-247.

Stevenson, J., Pennington, B. F., Gilger, J. W., DeFries, J. C., & Gillis, J. J. (1993). ADHD and spelling disability: Testing for shared genetic aetiology. *Journal of Child Psychology and Psychiatry, 34,* 1137-1152.

Thomas, C. J. (1905). Congenital "word-blindness" and its treatment. *Opthalmoscope, 3,* 380-385.

Thompson, J. S., & Thompson, M. W. (1986). *Genetics in medicine.* Philadelphia: Saunders.

Thompson, L. A., Detterman, D. K., & Plomin, R. (1991). Associations between cognitive abilities and scholastic achievement: Genetic overlap but environmental differences. *Psychological Science, 2,* 158-165.

Verbrugge, L. M., Lepkowski, J. M., & Imanaka, Y. (1989). Comorbidity and its impact on disability. *The Milbank Quarterly, 67,* 450-484.

Webster, R. E. (1979). Visual and aural short-term memory capacity deficits in mathematics disabled students. *Journal of Educational Research, 72,* 272-283.

Wechsler, D. (1974). *Examiner's Manual: Wechsler Intelligence Scale for Children-Revised.* San Antonio, TX: Psychological Corp.

Wechsler, D. (1981). *Examiner's Manual: Wechsler Adult Intelligence Scale-Revised.* San Antonio, TX: Psychological Corp.

Weissman, M. M., Gammon, G. D., John, K., Merikangas, K. R., Warner, V., Prusoff, B. A., & Sholomskas, D. (1987). Children of depressed parents: Increased psychopathology and early onset of major depression. *Archives of General Psychiatry, 44,* 847-853.

Zerbin-Rüdin, E. (1967). Kongenitale wortblindheit oder spezifische dyslixic [Congenital word-blindness]. *Bulletin of the Orton Society, 17,* 47-56.

APPENDIX

Expectations for the Bivariate Extension of the Basic Multiple Regression Model

The bivariate extension of the basic multiple regression model for the analysis of selected twin data (DeFries & Fulker, 1985) is as follows:

$$C_y = B_1P_x + B_2R + A \quad (1)$$

where C_y is the co-twin's expected score on variable y, P_x is the proband's observed score on variable x, R is the coefficient of relationship (1.0 and 0.5 for MZ and DZ twin pairs, respectively), and A is the regression constant.

Following the derivation of the partial regression coefficients for the basic model by LaBuda, DeFries, and Fulker (1986), the normal equations can be formulated in terms of expected variances and covariances as follows:

$$S_X \cdot \mathbf{B} = S_{XY} \quad (2)$$

where S_X is the expected variance–covariance matrix among the independent variables, S_{XY} is a column vector of expected covariances of the independent variables with the dependent variable, and **B** is a column vector of unstandardized regression coefficients. The expected regression coefficients can then be derived by solution of Equation 2:

$$\mathbf{B} = S_X^{-1} \cdot S_{XY}. \quad (3)$$

As shown by LaBuda et al. (1986), the covariances and variances can be expressed as functions of means, phenotypic variance (V_P), additive genetic variance (V_A), and variance that is due to environmental influences shared by members of twin pairs (V_C).

Assuming that the means of MZ and DZ probands are equal, S_X becomes:

	P_x	R
P_x	V_{Px}	0
R		1/16

For the bivariate extension of the basic model, S_{XY} is as follows:

	C_y
P_x	$[(n_1 + n_2/2)/N]\ \sigma_{AxAy} + \sigma_{CxCy}$
R	$(\bar{C}_{MZy} - \bar{C}_{DZy})/8$

where σ_{AxAy} is the additive genetic covariance between x and y, σ_{CxCy} is the shared environmental covariance between x and y, $\bar{C}_{MZy}$ and $\bar{C}_{DZy}$ are the mean y scores for MZ and DZ co-twins, n_1 and n_2 are the number of MZ and DZ pairs, respectively, and $N = n_1 + n_2$.

Upon substituting these matrices into Equation 3, the following expected partial regression coefficients are derived:

$$B_1 = [(n_1 + n_2/2)/N]\ \sigma_{AxAy}/\sigma_{PxPy} + \sigma_{CxCy}/\sigma_{PxPy}$$

$$B_2 = 2\ (\bar{C}_{MZy} - \bar{C}_{DZy})$$

Thus, B_1 is a weighted average of the MZ–DZ cross-variable twin resemblance (i.e., to what extent co-twin scores on y are related to proband x scores across zygosity). Of greater interest is the expectation of B_2, which can be shown to be a function of bivariate heritability as follows:

$$\begin{aligned}
B_2 &= 2\ (\bar{C}_{MZy} - \bar{C}_{DZy}) \\
&= 2\ [\mathbf{b}C_{MZy}P_{MZx}\ (\bar{P}_x - \mu) - \mathbf{b}C_{DZy}P_{DZx}\ (\bar{P}_x - \mu)] \\
&= 2\ [\sigma C_{MZy}P_{MZx}/\sigma^2P_{MZx}\ (\bar{P}_x - \mu) - \\
&\quad \sigma C_{DZy}P_{DZx}/\sigma^2P_{DZx}\ (\bar{P}_x - \mu)] \\
&= 2\ [((\sigma_{AxAy} + \sigma_{CxCy})/V_{Px})(\bar{P}_x - \mu) - \\
&\quad ((.5\sigma_{AxAy} + \sigma_{CxCy})/V_{Px})(\bar{P}_x - \mu)] \\
&= 2\ (.5\sigma_{AxAy}/V_{Px})(\bar{P}_x - \mu) \\
&= (\sigma_{AxAy}/V_{Px})(\bar{P}_x - \mu) \\
&= (\sigma_{Ax}/\sigma_{Px})\ (\sigma_{Ay}/\sigma_{Px})\ (\sigma_{AxAy}/\sigma_{Ax}\sigma_{Ay})\ (\bar{P}_x - \mu).
\end{aligned}$$

After multiplying by $(\sigma_{Py}/\sigma_{Py})$ and rearranging the terms,

$$B_2 = h_x\ h_y\ r_{Gxy}\ (\bar{P}_x - \mu)\ (\sigma_{Py}/\sigma_{Px})$$

Thus, when data are standardized and transformed by dividing each score by $(\bar{P}_x - \mu)$ prior to multiple regression analysis,

$$B_2 = h_x\ h_y\ r_{Gxy}$$

That is, B_2 is an index of the extent to which the deficit of probands for character y, on average, is due to genetic factors that also cause deficits in x. Furthermore, because r_{Pxy} in unselected samples equals $h_x\ h_y\ r_{Gxy} + e_xe_yr_{Exy}$, the ratio of B_2 to r_{Pxy} provides a measure of the proportion of the observed covariance that can be attributed to genetic factors.