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VERTICAL AND LATERAL STIFFNESS CHARACTERISTICS OF AIRCRAFT TIRES

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I. INTRODUCTION

Both analog and mathematical modeling of an aircraft operating in contact with the ground, either landing, taking off or taxiing, requires a reasonably definitive knowledge of tire stiffness characteristics. Such data, when it exists, is sparse and scattered in many locations. For many tire sizes stiffness properties have never been measured. Furthermore, both tire vertical and lateral stiffness are functions of the tire inflation pressure, so that an almost infinite variety of stiffness possibilities exist. In the case of large aircraft tires, much of this data is simply not available since mechanical equipment to generate it does not exist in sufficient quantity or size to produce the necessary experimental results. A definite need exists for some method of estimating aircraft tire stiffness properties, particularly in the preliminary design stage where it is not feasible to measure such properties experimentally.

This report presents methods for estimating both vertical and lateral stiffness characteristics of pneumatic tires, and compares such predictions with a large amount of existing experimental data which has been collected from various sources.

II. SUMMARY

Dimensional analysis is used as a basic tool for presenting collected experimental data on both the vertical and lateral stiffness of pneumatic tires. Such analysis is aided by a simplified force-deflection model for the tire in both vertical and lateral modes.

Using such techniques, vertical force-deflection data reduces quite well for an extremely large range of tire sizes, ply ratings and inflation pressures. Lateral force deflection data, on the other hand, only reduces well for aircraft tires whose inflation pressure is higher than about 65 psi and for which the thick shoulder typical of automobile tires is absent.

These restrictions on the types of tires which can be successfully represented in dimensionless form come about due to the basic nature of forces generated in pneumatic tires. These may be thought of as being due to two separate sources, the first being the membrane and bending stiffness of the tire carcass itself, while the second is the presence of the contained air. In case of vertical deflection, practically the entire force resultant comes about due to the presence of the contained air, so that very little compensation must be made for the effects of the carcass in bending. Unfortunately this is not quite the case in lateral deformation, since here at low pressures the tire carcass itself can contribute a substantial amount to the overall stiffness property. For that reason, the lateral data is most simply restricted to that lying above a certain level of inflation pressure, where the contained air represents the major part of the total stiffness. Fortunately, the modeling and computation

of the pressure forces due to inflation are much simpler than the modeling of the forces due to the elastic shell, since the former primarily depend upon the tire geometry and inflation level.

III. VERTICAL FORCE-DEFLECTION ANALYSIS

It has been shown in a previous report by one of the authors¹ that the vertical forces resulting from vertical deflection of a tire are primarily associated with the inflation pressure and tire size and geometry. It is possible to use such a simple theory as outlined in this reference to give a simple equation relating tire deflection to vertical force, and then to cast this equation into dimensionless form. It will be useful to give this derivation here, in order that the approximations made may be clearly shown.

The deformation process is first characterized by a geometric calculation of the contact area of an aircraft tire with a flat surface. From Figure 1, it is seen that the vertical deflection is given by

$$\Delta_v = \frac{d}{2} (1 - \cos \theta) \quad (1)$$

where

Δ_v = vertical deflection

d = outside tire diameter

θ = angle of geometric intersection of ground plane and tire

and the contact patch length is given by

$$L \sim d \sin \theta \quad (2)$$

where L = contact patch length as shown in Figure 1.

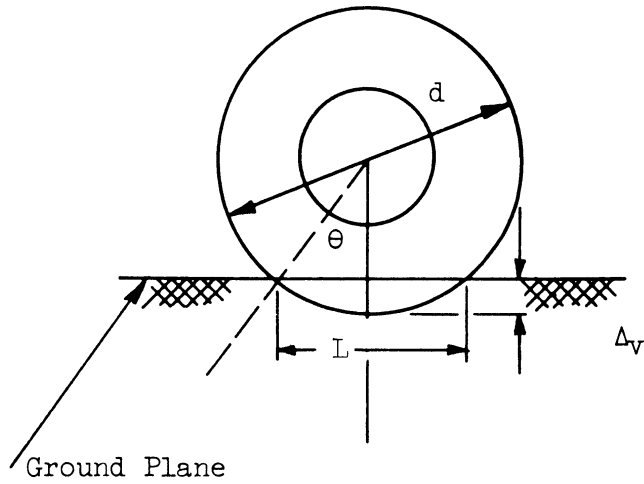


Figure 1. Side view of deformed tire.

Assuming $\Delta_v \ll d$, one may write

$$\Delta_v = \frac{\theta^2}{2} \frac{d}{2}$$

and

$$\frac{L}{d} \sim \theta$$

which yields

$$\Delta_v \sim \frac{L^2}{4d} \quad (3)$$

Assuming a circular cross section, as shown in Figure 2, the vertical deflection and contact patch width are related by

$$\Delta_v = r_1(1 - \cos \beta) \quad (4)$$

where

r_1 = cross-sectional radius

β = angle of geometric intersection of ground plane and tire

and

$$\frac{b}{2r_1} \sim \sin \beta \quad (5)$$

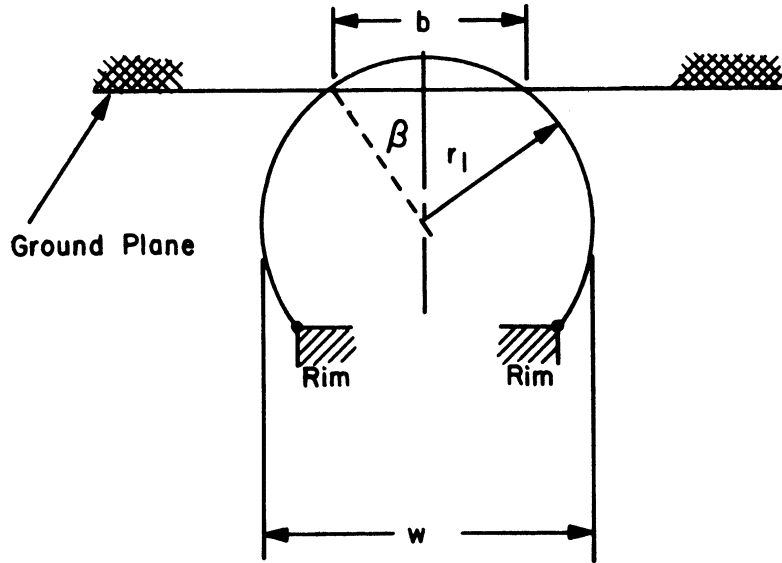


Figure 2. Cross section of aircraft tire.

Assuming $\Delta_v \ll r_1$, one may write as before

$$\Delta \sim r_1 \frac{\beta^2}{2} \quad (4')$$

$$\frac{b}{2r_1} \sim \beta \quad (5')$$

and

$$\Delta_v = \frac{b^2}{8r_1} \quad (6)$$

A further approximation assumes

$$r_1 \sim w/2 \quad (7)$$

where w = tire section width. Then Eq. (6) becomes

$$\Delta_v \sim \frac{b^2}{4w} \quad (8)$$

Now, assuming an elliptical contact patch with an average pressure value equaling the inflation pressure, the vertical load may be approximated by

$$P \sim \pi \frac{L}{2} \frac{b}{2} p_o$$

or

$$P \sim \pi p_o \Delta_v \sqrt{dw} \quad (9)$$

where

P = vertical load

p_o = inflation pressure

Δ_v = vertical deflection

d = outside tire diameter

w = tire section width

There are several possible dimensionless ways of expressing the vertical deflection. One possibility would be to use the ratio Δ_v/H , which is often done in rating tire deflection characteristics. Physically this represents the ratio of the vertical deflection to section height, and this ratio is undoubtedly important in fixing overall tire deflection because it determines to a great extent both the membrane and bending stresses and hence cord loads, set up in the deflected rotating tire. For this reason, the ratio Δ_v/H is valuable for load rating, and hence hysteresis characteristics in a tire. On the other hand, as far as the vertical load-carrying capabilities of a tire are concerned, it seems more reasonable to use the ratio Δ_v/d , which physically represents a ratio of vertical deflection to tire diameter. The reason for this is that the length of the contact patch is much more directly related to this ratio than to the ratio of Δ_v/H , so that physically the contact area can probably be better represented by Δ_v/d .

Based on this line of reasoning, Eq. (9) becomes

$$\frac{P}{p_o d \sqrt{dw}} = \pi \frac{\Delta_v}{d} . \quad (10)$$

These two quantities are plotted in Figure 3 with the slope of π superimposed as a solid line. The data points represent load-deflection data for a wide range of tires. The extremes of this range are an earthmover tire weighing 2650 lb. at one end and a 4.5-in. outside diameter model airplane tire weighing a few ounces at the other end. Both of these tires lie close to the line drawn representing a slope of π . The fact that most of the data falls close to this line suggests that the dimensionless parameters which have been chosen in Eq. (10) are well suited to represent vertical load deflection data.

From the large quantity of data which has been plotted here, it may be concluded that a good first approximation to the static vertical load deflection characteristics of a pneumatic tire is gotten by the use of Eq. (10). This means that preliminary estimates of such vertical spring rates can be obtained with relative ease for preliminary design purposes.

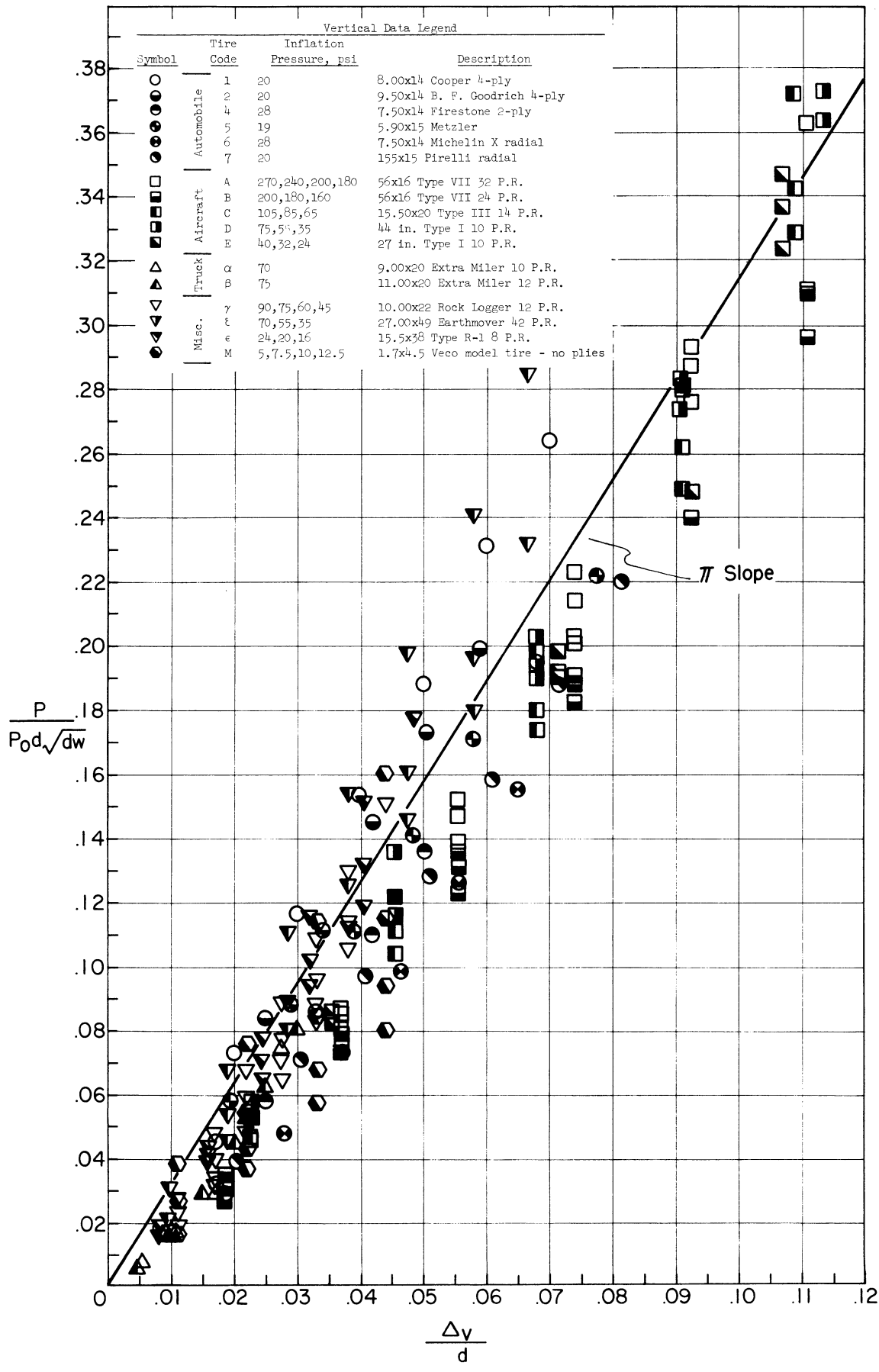


Figure 3. Static vertical load-deflection data in dimensionless form.

IV. LATERAL-FORCE DEFLECTION ANALYSIS

As previously discussed, the elastic stiffness of the carcass influences to some extent lateral-force deflection relationships. The interaction of the elastic and pressure effects has been discussed by Dodge, Clark, and Johnson,² who developed a rather complicated method of determining the contribution of each effect, and hence the total lateral spring constant. This analysis was primarily directed toward automotive tires, in the low pressure region. For aircraft tires with relatively high inflation pressures, the carcass or elastic forces are very small compared to the pressure forces, and hence can be ignored above a certain level of inflation pressure. With this assumption, the lateral force analysis becomes quite simple, since it is based on the concept of a string on an elastic foundation as used in Ref. 1. The general area of the tread is considered to be the string, while the sidewalls represent the elastic foundation. This rather drastic assumption, while quite crude, does succeed in simplifying the analysis substantially and fortunately yields fairly good results.

We first develop an equation relating the tire parameters to both the lateral elastic foundation modulus, or stiffness, required in the theory of a string on an elastic foundation, as well as relating the tire parameters to the tension in the string. This development was originally given in Ref. 1 but will be repeated here so that the nature of the necessary assumptions is clear.

Assuming the rather severe idealization of the tire cross section as shown in Figure 4, one obtains a laterally deformed cross section given in Figure 5.

Here the unit width of the tire is acted upon both by the inflation pressure and by an assumed side force Q .

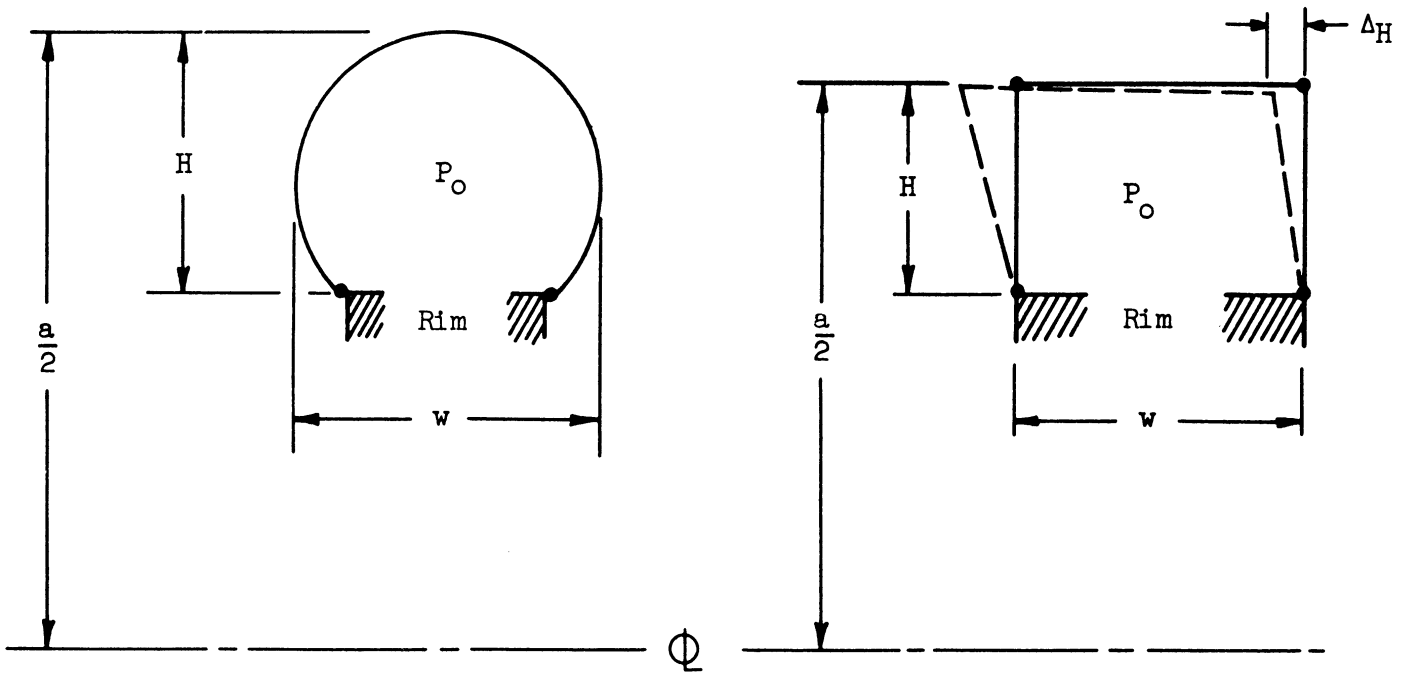


Figure 4. Idealized section of aircraft tire.

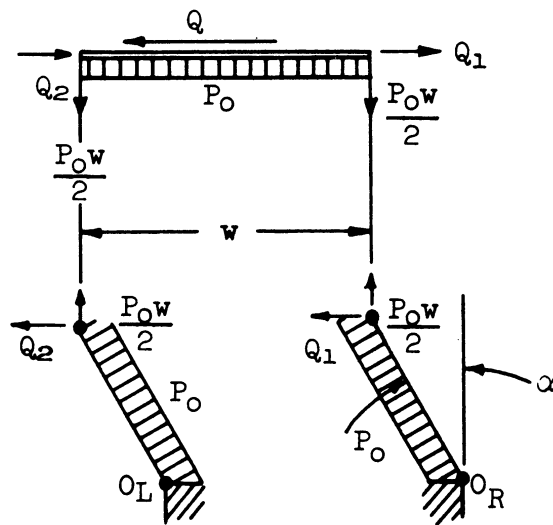


Figure 5. Free body diagram of deformed tire.

For the right-hand sidewall, taking moments about O_R gives

$$Q_1 H \cos \alpha = \frac{p_o w}{2} H \sin \alpha + p_o H \cdot H/2$$

For small α ,

$$Q_1 \sim p_o \frac{H}{2} + p_o \frac{w}{2} \frac{\Delta H}{H} \quad (11)$$

For the left-hand sidewall, taking moments about O_L gives

$$-Q_2 H \cos \alpha + \frac{p_o w}{2} H \sin \alpha = p_o H \cdot H/2 - Q_2 \sim p_o H/2 - p_o \frac{w}{2} \frac{\Delta H}{H} \quad (12)$$

The total side force Q is given by

$$Q = Q_1 + Q_2 = p_o \frac{w \Delta H}{H}$$

$$\frac{Q}{\Delta H} = \frac{p_o w}{H} = K_L \quad (13)$$

where K_L is the lateral foundation modulus for the string.

The other quantity needed for this analysis is the string tension. Pressure vessel theory can be used to get an approximate expression for this tension. Referring to Figure 6, membrane theory predicts

$$\frac{N_\phi}{r} + \frac{N_\theta}{r_o} = \frac{p_o}{t} \quad (14)$$

where

N_ϕ = force/unit area in

the sectional circumferential

direction

r = sectional tire radius

N_θ = force/unit area in the

tire's overall circumferential

direction

r_o = overall tire radius

p_o = inflation pressure

t = membrane thickness

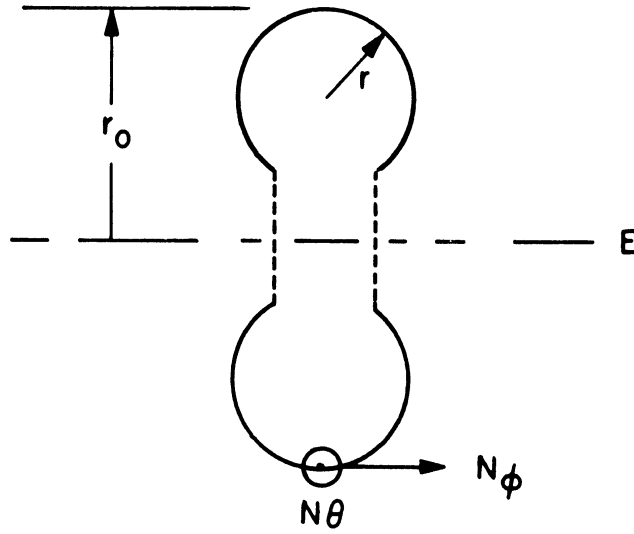


Figure 6. The tire membrane.

Referring to Figure 7, N_ϕ acts over an area $2\pi r_0 t$ and p_0 acts on an area $\pi [r_0^2 - (r_0 - r)^2]$. Equating these two forces gives

$$N_\phi t 2\pi r_0 = \pi p_0 [r_0^2 - (r_0 - r)^2]$$

or

$$N_\phi = \frac{p_0 r}{2r_0 t} (2r_0 - r) \quad (15)$$

Substituting this expression for N_ϕ into the membrane Eq. (14) N_θ becomes

$$\frac{p_0}{2r_0 t} (2r_0 - r) + \frac{N_\theta}{r} = \frac{p_0}{t}$$

or

$$N_\theta = \frac{p_0 r_0}{t} - \frac{p_0}{2t} (2r_0 - r) = \frac{p_0 r}{2t} \quad (16)$$

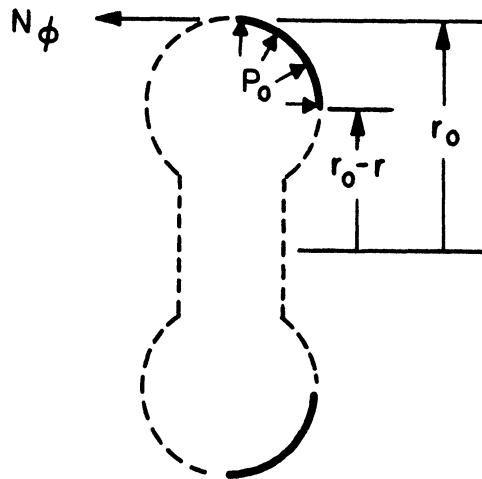


Figure 7. Tire section showing N_ϕ .

Referring to Figure 8 it is seen that the area over which N_θ acts can be approximated by

$$\text{Area} = Ct$$

where C = actual tire sectional circumference.

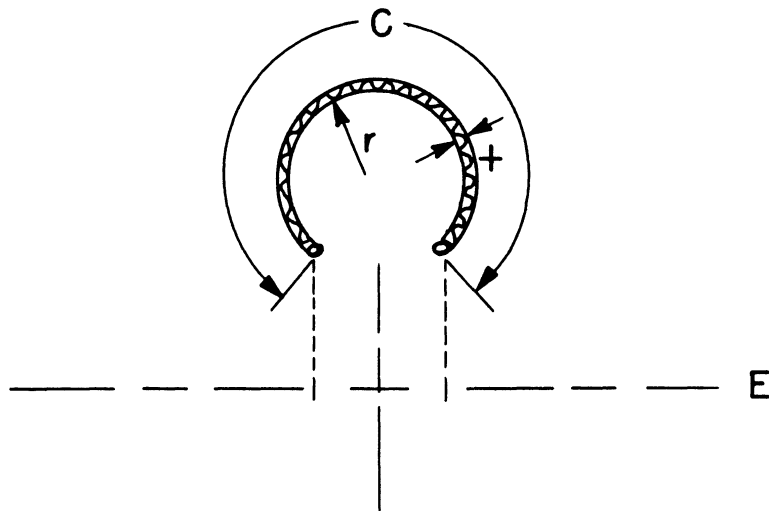


Figure 8. Tire sectional area.

Then the tension T is given by

$$T = \frac{p_o r}{2t} (Ct) = \frac{p_o r C}{2} \quad (17)$$

Now assume that

$$r = w/2$$

where

w = section width

r = section radius

so that the effective tread length is a fraction f of the complete tire circumference

$$C = 2\pi r(f) \quad (18)$$

We now have expressions for the two quantities needed for equation governing the string on the elastic foundation. This equation of equilibrium is

$$T \frac{d^2u}{dx^2} - K_L u = 0 \quad (19)$$

where

T = string tension

u = displacement, lateral

x = coordinate, circumferential

K_L = foundation modulus

Letting $\lambda^2 = \frac{K_L}{T}$, the solution is

$$u = Ae^{-\lambda x} + Be^{\lambda x} \quad (20)$$

Considering the tire diameter large compared with the contact patch length,

one obtains B=0 and

$$u = Ae^{-\lambda x}$$

The other boundary condition is

$$(u)_{x=0} = \Delta H$$

so that

$$A = \Delta H$$

and

$$u = \Delta H e^{-\lambda x} \quad (21)$$

$$\frac{du}{dx} = -\Delta H \lambda e^{-\lambda x} \quad (22)$$

The force component perpendicular to the string under a point load is given by

$$S = -2T \left(\frac{du}{dx} \right)_{x=0} \quad (23)$$

as shown in Figure 9.

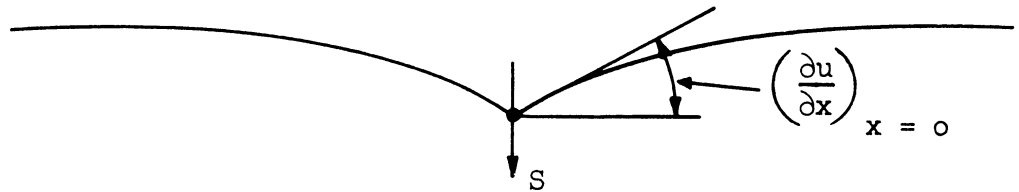


Figure 9. String under point load.

Using Eq. (22)

$$S = 2T \Delta H \lambda = 2 \Delta H \sqrt{TK_L} \quad (29)$$

Substituting the expressions for T and K_L obtained previously, the expression for the lateral point load in terms of the tire parameters is

$$S = \Delta_H \cdot p_o w \sqrt{\frac{w}{H}} \sqrt{f\pi} \quad (25)$$

where

S = lateral point load

Δ_H = lateral deflection

p_o = inflation pressure

w = section width

H = section height

f = dimensionless fraction representing ratio of actual
trend width to the tire circumference.

Equation (25) may be viewed as a relation between dimensionless quantities.

It could be written as

$$\frac{S}{p_o w^2} = \sqrt{f\pi} \cdot \frac{\Delta H}{H} \left(\frac{H}{w}\right)^{1/2} \quad (26)$$

For almost all tires considered in this survey, the section height to width ratio does not vary much, lying between 0.8 to 1.0. For this reason $\left(\frac{H}{w}\right)^{1/2}$ is nearly a constant, and one may finally arrive at the most convenient set of dimensionless parameters in which to plot data. These are

$$\frac{S}{p_o w^2} \text{ vs } \frac{\Delta H}{H} \quad (27)$$

The left-hand term is the ratio of lateral or side force to the tension force in a square tube of cross-sectional dimensions w by w inflated to pressure p_o . The right-hand side represents the tangent of the angle of inclination of the tire due to lateral deflection.

The results of reducing lateral stiffness data to the parameters of Eq. (27) are shown in Figure 10. Since pressure effects are presumed to be predominant, only data is taken from aircraft tires inflated to 65 psi or over. Figure 10 shows that for such tires these parameters succeed in grouping data for a wide range of tire geometries and sizes. Because the 4.5-in. model air-

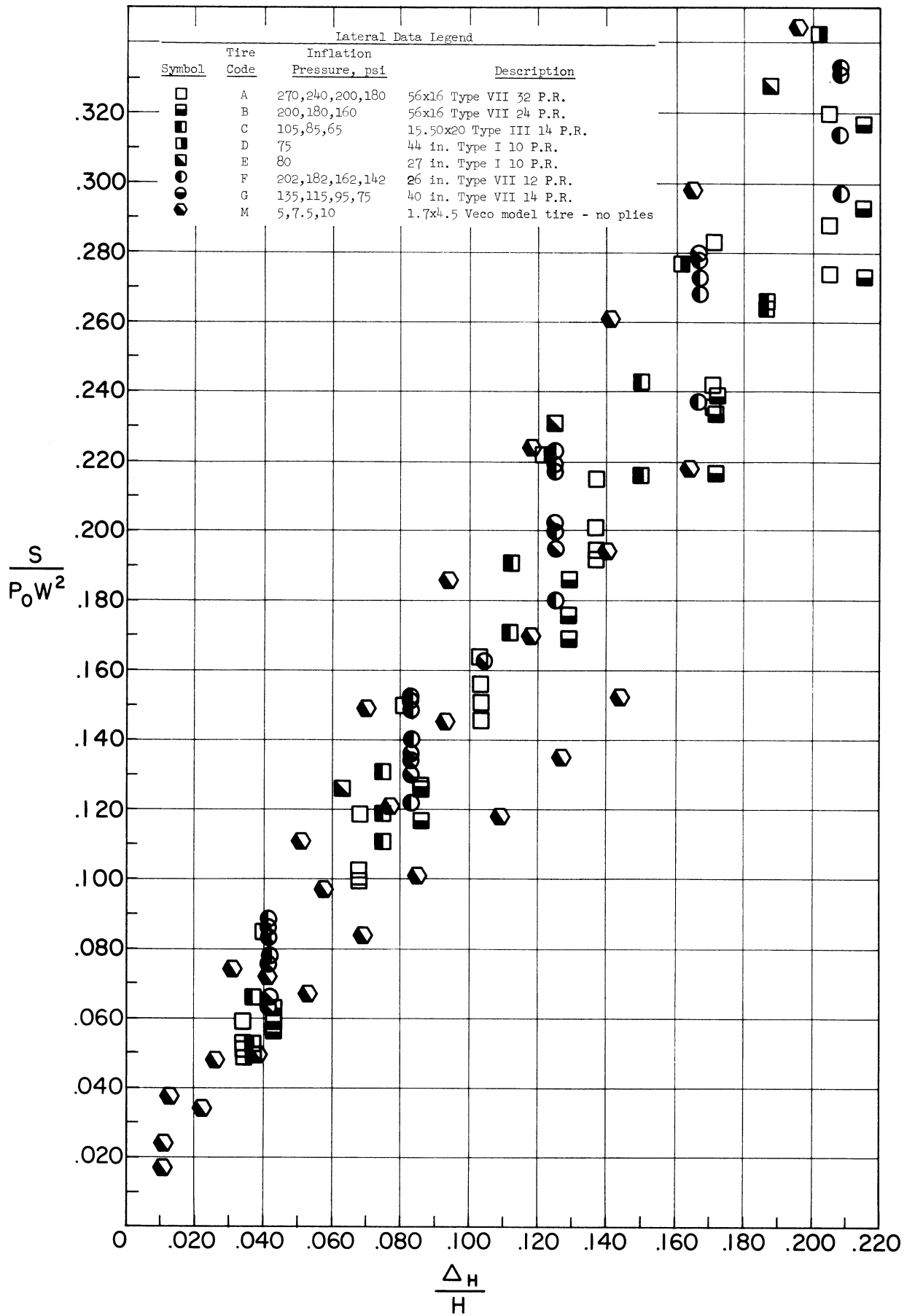


Figure 10. Static lateral force-deflection data in dimensionless form.

craft tire has no ply structure, this tire's data falls into the same group as the data for the full size aircraft tires.

V. REFERENCES

1. Clark, S. K., "Simple Approximations for Force-Deflection Characteristics of Aircraft Tires," The University of Michigan, ORA Technical Report 05608-8-T, December 1965.
2. Dodge, R. M., S. K. Clark and K. L. Johnson, "An Analytical Model for Lateral Stiffness of Pneumatic Tires," The University of Michigan, ORA Technical Report 02457-30-T, February 1967.

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